Two-Sided Platforms: Pricing and Social Efficiency *

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Abstract

This paper models two-sided market platforms, which connect developers of many different products and services to users who demand a variety of these products. We determine the optimal platform structure as a function of user preferences for diversity and show that the use of royalties stems from two conflicting objectives: providing developers with adequate innovation incentives and reducing a hold-up problem by the platform. These results suggest simple explanations for the contrasting platform pricing structures observed across industries such as videogames and software for computers and other electronic devices. From a normative perspective, we show that the increasingly popular public policy presumption that open platforms and platform competition are inherently more efficient than monopoly proprietary platforms -in terms of induced product diversity, user adoption and total social welfare- is not justified in our framework. Our model reveals a fundamental welfare tradeoff between the extent to which proprietary platforms internalize indirect network effects through profit-maximizing pricing and the two-sided deadweight loss created.

Keywords: Two-Sided Markets, Platforms, Indirect Network Effects, Product Variety, Social Efficiency.

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1 Introduction

An increasing number of industries in today’s economy are organized around platforms, which enable consumers to purchase, access and use a great variety of products. These platforms and the markets in which they operate are said to be "multi-sided" because the vast majority of products is generally supplied by third-party (or independent) producers\(^1\), so that in order to thrive platforms have to attract, through adequate pricing, both consumers and product suppliers.

A classic example is shopping malls: the mall developer has to attract retailers (with which he signs lease contracts) and shoppers. However, it is in industries at the core of the "new economy" that this form of market organization has become most important: to a certain extent one may think of them as "digital shopping malls". For example, in the computer industry, operating system vendors such as Microsoft, Apple, Sun, IBM, Novell, etc. control the software platform which allows computer users to access the large variety of applications supplied by independent developers, who also have to gain access to it. An ever-increasing number of consumer electronics products such as personal digital assistants, smart mobile phones, television sets and car navigation systems are also built around operating system platforms such as Palm OS, Symbian and Linux, which likewise allow consumers to acquire and use thousands of applications from many third-party developers. Internet sites such as Priceline.com allow users to select from a variety of products and services offered by companies having obtained the right to be listed on the site. In the videogame market, users have to purchase consoles such as Sony’s Playstation, Microsoft’s XBox and Nintendo’s Gamecube in order to have access to hundreds of games supplied by independent publishers. Digital media platforms, from wireless networks such as Vodafone Live and NTT DoCoMo’s i-mode, to software media players such as Real’s Real Player, to on-demand and interactive cable television platforms such as TiVo and Sky Plus, enable users to access a variety of content (games, news, music, movies, etc.) from thousands of independent providers.\(^2\)

\(^1\)By contrast, products supplied by the platforms themselves are "first-party".
\(^2\)There are more than 70,000 applications developed for Windows; the Palm OS is supported by over 22,000 applications and its large community of developers is
This paper is the first to model two-sided platforms connecting buyers and sellers in markets in which product variety is important and to show that the latter is a key factor in determining the optimal platform pricing structures. In particular, in an empirical survey of software platforms central to computer-based industries, Evans Hagiu and Schmalensee (2004) document that platforms in this family of markets, economically very similar, have chosen strikingly different pricing structures in order to get the two sides -consumers and independent producers- "on board". At one end of the spectrum, all platforms in the markets for computers, handheld devices and mobile phones have chosen to subsidize or earn little if any profits on the developer side of the market and make virtually all of their profits on users, while on the other hand, in the videogame market, all console manufacturers make the bulk of their profits through royalties charged to third-party game publishers\(^3\) and sell their consoles at or below cost to users.

From a positive perspective, our model predicts that a higher intensity of users' preference for product diversity shifts the optimal pricing structure towards making a higher share of profits on third-party producers relative to users. Since both common intuition and empirical studies suggest that users care more about product variety in entertainment-oriented markets such as videogames than in professional productivity-oriented markets such as computer software, this prediction constitutes a plausible explanation for the observed differences in pricing structures.

We also use our model in order to propose a way of rationalizing a second important strategic pricing decision facing the platforms we study: the choice between fixed fees and variable fees (royalties) on the developer side of the market. We show that the optimal royalty rate is determined by two conflicting objectives: providing third-party producers with appropriate inno-

\(^3\)They also make money through sales of first-party games. However, the proportion of first-party games has decreased significantly over time: it is less than 20% for Playstation and XBox today.
vation incentives when the quality of their products is not contractible and reducing a hold-up problem by the platform, which arises when developers make their platform adoption decisions before users and the platform cannot credibly commit ex-ante to user prices. The first objective tends to require a low royalty rate whereas the second objective requires a high royalty. There are good reasons to believe that application/game quality is non-contractible and a significant number of developers have to be attracted before platforms are launched to users both in the videogame and in other computer-based industries. However, one major difference between videogame consoles and other software platforms is that the former are vertically integrated into hardware, whereas the latter generally license their platform to many competing hardware manufacturers, which can be viewed as creating a credible way to commit ex-ante to a certain level of user adoption for the platform. Based on our model, one would then expect videogame consoles to charge positive royalties and the other software platforms not to use variable fees, which is once again consistent with what we observe.

From a normative perspective, our model reveals a fundamental welfare tradeoff between two-sided profit-maximizing (proprietary) platforms and two-sided open platforms, which allow "free entry" on both sides of the market. On the one hand, a profit-maximizing platform creates two-sided deadweight loss through monopoly pricing, unlike an open platform which essentially prices at marginal cost on both sides. On the other hand however, precisely because it sets prices in order to maximize profits, a proprietary platform internalizes at least partially the positive indirect network externalities between users and third-party product suppliers and the direct negative externalities between producers, whereas an open platform does not. Therefore it is by no means obvious which platform will create higher product variety, user adoption and total social welfare. The same tradeoff arises when comparing a situation with competing platforms and one with a single monopoly platform. In this context, there is a sense in which platform competition is undesirable because it prevents platforms from sufficiently internalizing indirect network effects and therefore from inducing the appropriate levels of product variety.

This insight has important public policy implications. Indeed, the in-
creasing popularity of the open-source software movement with open platforms such as the Linux operating system or the Apache web-server, has given rise to a heated debate among economists and policy-makers regarding the efficiency merits of open versus proprietary platforms. In fact, an increasing number of governments around the world are considering or already enacting policies promoting open source software systems at the expense of proprietary systems. Oftentimes these policies stem from the conviction that open software platforms are inherently more efficient than their proprietary counterparts and therefore are more appropriate for stimulating industrial development. Our model makes it clear that the welfare analysis in two-sided markets is very different from the one in classic, one-sided contexts and it implies that in our framework an a priori preference of open platforms over proprietary platforms (or the other way around) is not economically justified.

**Related literature**

Our paper belongs to the very recent and quickly growing economics literature on two-sided markets, pioneered by Armstrong (2002), Caillaud and Jullien (2003) and Rochet and Tirole (2003) and (2004). A market is said to be two-sided if firms serve two distinct types of customers, who depend on each other in some important way, and whose joint participation makes platforms more valuable to each. In other words, there are indirect network externalities between the two different customer groups. One of the main insights which has emerged from this literature is the importance of platforms’ choice of pricing structures in "getting the two sides on board".

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4Hahn et al. (2002) contains a representative sample of the opposing views on this issue.

5For instance, Brazil has passed legislation mandating open source solutions be given preference in municipal governments and France has passed a parliamentary bill forbidding government-related institutions from using anything but open-source software. See Hahn et al. (2002) for a comprehensive overview of such policies.

6This is the definition offered by Evans (2003). Rochet and Tirole (2004) use a slightly different one: for them, a necessary and sufficient condition for a market to be two-sided is that the volume of transactions be sensitive to the distribution of total costs between the two sides.

Both definitions imply that a platform can improve upon the market outcome through a pricing structure that rebalances costs between the two sides by internalizing (to a certain extent) the indirect externalities.
Similar to Armstrong (2002) and Rochet and Tirole (2003) and (2004), our model emphasizes the role of elasticities of demand for the platform on both sides of the market.

The innovation of our model is to introduce competition among members of one side of the market (developers) and to determine the optimal platform pricing structure as a function of the intensity of users’ preferences for variety. This enables us to propose an explanation for the somewhat puzzling empirical finding of radically different pricing structures across a set of otherwise very similar industries\textsuperscript{7}. By contrast, most of the two-sided markets literature up to now has either focused on individual industries such as credit cards (Rochet and Tirole (2002) and (2003))\textsuperscript{8}, Schmalensee (2002), Wright (2003)), intermediaries (Caillaud and Jullien (2003), Baye and Morgan (2001)), Yellow Page directories (Rysman (2003)) and broadcasting (Anderson and Coate (2003)), or has provided general and essentially symmetric models\textsuperscript{9}, inadequate for undertaking the type of cross-industry comparison we make here. One exception is Nocke, Peitz and Stahl (2004), who build a model of two-sided platforms very similar to ours. The key difference however is that they focus on issues of platform ownership and abstract from the two-sided pricing problems we explore by assuming platforms can only charge the producer side of the market (one-sided pricing).

Second, our welfare comparison between open and proprietary platforms relates our paper to the literature on product variety, free entry and social efficiency, in particular Mankiw and Whinston (1986). Their paper is concerned with the inefficiencies associated with free-entry in product markets and shows that the sign of the inefficiency (i.e. whether there is excessive or insufficient entry) depends on the interplay between two opposite effects: the business-stealing effect and the product-diversity effect. Our

\textsuperscript{7}Hagiu (2004b) also studies platforms of the type we are interested in here. However, that paper abstracts from product diversity by assuming independent demand functions for applications and focuses instead on the issue of commitment and the use of variable fees (or royalties) in order to curb developer market power.

\textsuperscript{8}The model contained in this paper is inspired by and primarily destined to credit cards but the authors show that some of the general insights they offer also apply to other industries.

\textsuperscript{9}That is, demand functions on the two sides are symmetric and there is no competition within either side (Armstrong (2002), Rochet and Tirole (2004)).
paper can be viewed as an extension of their analysis in two important dimensions. First, Mankiw and Whinston’s model is "one-sided" in the sense that the number of consumers participating in the market is fixed and only the number of producers is variable. This allows them to focus exclusively on direct externalities on the producer side and abstract from the positive indirect network externalities between consumer entry and producer entry, which are central to our paper. Thus, our two-sided open platforms are similar but more general than the free-entry regimes studied by Mankiw and Whinston (1986), Kiyono and Suzumura (1987), Spence (1976), Dixit and Stiglitz (1977) and Salop (1979) because user participation in the market is endogenous in our model. Second and most important, our two-sided proprietary platforms controlling market access through prices charged to both users and independent product suppliers constitute a novel form of market organization, which has not been analyzed by the literature on product variety.

Finally, our paper is linked to the literature on indirect network effects, especially Church and Gandal (1992) and Church Gandal and Krause (2002). Both papers study two-sided technology (or platform) adoption, however in both models, the platform is assumed to be entirely passive, i.e. there is no strategic pricing on either side of the market. This is equivalent to an open platform in our model.

The remainder of the paper is organized as follows: the next section presents the model and sets up the optimization problem for a monopoly two-sided proprietary platform. Section 3 first derives the optimal platform pricing structure for a monopoly platform. It then extends the basic model in two directions: developer investment incentives and platform competition. Section 4 analyzes social efficiency, by comparing product variety, user adoption and social welfare under a monopoly proprietary platform, an open platform, competing platforms and and a benevolent social planner. Section 5 concludes.
2 Modelling framework

We are interested in modelling a two-sided platform whose value to users is increasing in the number of developers it supports and whose value to developers is increasing in the number of users who adopt it. The platform controls the extent of adoption on both sides of the market through prices.

Net surplus for a user indexed by $\theta$ from buying a platform which charges her $P^U$ and is supported by $n$ applications is:

$$u(n) - P^U - \theta$$

where $u(n)$ is the surplus obtained from the $n$ applications, net of the prices charged application developers and the parameter $\theta$ is the user’s intrinsic "distance" in preference space to the system comprised by the platform and the applications$^{11}$. It is distributed over a support $[\theta_L, \theta_H]$ (we allow $\theta_H$ to be infinite). The number of users "closer" than $\theta$ (i.e. characterized by $\theta' \leq \theta$) is $F(\theta)$, where $F$ is a differentiable and strictly increasing function with continuously differentiable derivative $f$, mapping $[\theta_L, \theta_H]$ into $[0, +\infty]$ and such that $F(\theta_L) = 0$. We denote by $\varepsilon_F$ the elasticity of $F$, which is to be interpreted as the "elasticity" of user demand for the platform$^{12}$:

$$\varepsilon_F(\theta) = \frac{\theta f(\theta)}{F(\theta)} > 0$$

Similarly, net profits for a developer indexed by $\phi$ from supporting a platform which charges $P^D$ and is adopted by all users $\theta \leq \theta^m$ are$^{13}$:

$$\pi(n) F(\theta^m) - P^D - \phi$$

$^{10}$Developers are third-party product suppliers: developers of software applications or games, content providers, etc. For simplicity and ease of interpretation throughout the paper we will use the blanket term "developers" instead of third-party producers and "applications" in order to refer to their products.

$^{11}$For example, it can be interpreted as the difference between the fixed (sunk) cost of learning how to use the system and the standalone value of the platform (in case it comes bundled with some applications).

$^{12}$Note that elasticity here is defined with respect to net utility rather than to price as is usually the case. Armstrong (2003) uses a similar notion of elasticity, whereas Rochet and Tirole (2003) and (2004) use price elasticities.

$^{13}$Indeed, given the structure of user preferences assumed above, if user $\theta$ adopts the platform given $n$ and $P^U$ then all users $\theta' \leq \theta$ will also adopt.
where \( \pi(n) \) is the profit per platform user net of variable costs and the parameter \( \phi \) is the fixed cost of writing an application, distributed on \([0, \phi_H]\) (we allow \( \phi_H \) to be infinite). The number of developers with fixed costs less than or equal to \( \phi \) is \( H(\phi) \), where \( H \) is a differentiable and strictly increasing function with continuously differentiable derivative \( h \), mapping \([0, \phi_H]\) into \([0, +\infty] \) and such that \( H(0) = 0 \). The elasticity of developer demand for the platform is:

\[
\varepsilon_H(\phi) = \frac{\phi h(\phi)}{H(\phi)} > 0
\]

As suggested by this formulation we will ignore integer constraints and treat \( n \) as a continuous variable throughout the paper. The reason is that in the markets we have in mind there are hundreds or even thousands of applications as explained in the introduction. Continuity also renders the analysis very convenient by allowing us to reason in terms of demand elasticities.

There are three important assumptions embedded in the expressions of user surplus and developer profits above. First, all users are assumed to have the same marginal valuation for applications, i.e. there is no vertical differentiation among them. Second, all applications are assumed to be identical and fully interchangeable from the point of view of every user and developers are also differentiated only horizontally by their fixed development cost. These assumptions greatly simplify the analysis, however our main insights hold for more general formulations\(^{14}\). Third, platforms charge only fixed "access" fees and no variable fees. In the basic version of the model, nothing would change if we allowed the platform to also charge variable fees: the two pricing instruments are perfect substitutes. In section 3.1 however, we show that royalties play an important role when developer innovation incentives are necessary.

Let:

\[
V(n) = u(n) + n\pi(n)
\]

denote the gross surplus created by \( n \) applications for each platform user.

We make the following assumption:

\(^{14}\)In section 4 below we provide an example with vertical developer differentiation. In Hagiu (2004c) we introduce vertical differentiation on the user side of the market. The formal analysis is slightly more complex but the main conclusions are unchanged, which is why we have chosen to focus on the simplest formulation.
**Assumption 1** $u(n)$ is strictly increasing, $\pi(n)$ is strictly decreasing and $V(n)$ is strictly increasing and concave.

This assumption is quite reasonable: it simply says that net user surplus $u(n)$ is increasing in the number of applications used, that each developer’s profits per user are decreasing in $n$ (crowding effect) and that the gross user surplus created by $n$ applications is increasing at a decreasing rate (the 100th application is less valuable than the 10th).

Let us denote by $\varepsilon_V$ the elasticity of $V$:

$$\varepsilon_V(n) = \frac{nV''(n)}{V'(n)} \in [0, 1]$$

The elasticity $\varepsilon_V$ plays a central role in our model: it measures the intensity of users’ preference for variety. The higher $\varepsilon_V$, the less concave $V(.)$ and therefore the higher the marginal contribution of an additional application to gross surplus per platform user.

Also, it will prove useful to define:

$$\lambda(n) = \frac{\pi(n)}{V'(n)}$$

the ratio between developer profits and the marginal contribution of an additional developer to gross surplus per platform user. Intuitively, when $\lambda(n) > 1$, each developer is gaining more than his marginal contribution, therefore one would expect a bias towards excessive entry on the developer side of the market under an open platform (or free entry regime), and vice versa, when $\lambda(n) < 1$, an open platform regime contains a bias towards insufficient developer entry. A two-sided proprietary platform may either correct or exacerbate this bias to a certain extent through its prices.

Let us clearly specify the timing of the pricing game we consider throughout the paper. There are 3 stages:

- **Stage 1)** The platform sets prices $P^U$ and $P^D$ for consumers and developers simultaneously

- **Stage 2)** Users and developers make their adoption decision simultaneously
Stage 3) Developers set prices for consumers and those consumers who have acquired the platform in the second stage decide which applications to buy.

The slightly odd-sounding assumption that users decide whether or not to buy the platform before developers set their prices is made in order to simplify the analysis of the two-sided pricing game. Given that developers are atomistic in our model, it is entirely harmless: developers ignore the effect of their pricing decision on total consumer demand for the platform anyway.

In order to illustrate how the reduced forms $u(n)$, $\pi(n)$ and $V(n)$ are obtained, we provide two specific examples, both of which satisfy assumption 1 and which we will use throughout the paper.

**Example 1** Suppose users’ gross benefits have the Spence-Dixit-Stiglitz form $G(\sum_i v(q_i))$, where $q_i$ is the "quantity" of application $i$ consumed, $v(0) = 0$, $v'(.) > 0$ and $v''(.) < 0$ and $G''(.) > 0$, $G'''(.) < 0$.

User maximization implies that the quantity $q_k$ demanded by each platform user from developer $k$ charging $p_k$ satisfies:

$$p_k = v'(q_k) G'(\sum_i v(q_i))$$

Each developer takes the market price $G'(\sum_i v(q_i))$ as given when setting his price. Consequently, the stage 3 pricing equilibrium among developers is symmetric and defined by:

$$v'(q_n) G'(nv(q_n)) = p_n = \arg \max_p \left\{ \left( p - c \right) v^{\prime -1} \left( \frac{p}{G'(nv(q_n))} \right) \right\}$$

Then: $\pi(n) = (p_n - c) q_n$, $u(n) = G(nv(q_n)) - np_n q_n$ and $V(n) = G(nv(q_n)) - ncq_n$. Letting $v(q) = q^\alpha$ and $G(z) = z^{\frac{\alpha}{\sigma}}$, with $0 < \alpha < \sigma < 1$, we obtain:

$$\pi(n) = (1 - \sigma) \alpha \left( \frac{\alpha \sigma}{c} \right)^{\frac{1}{\alpha}} n^{-\frac{\sigma - \alpha}{\sigma(\alpha - 1)}}$$

15 This is because all users "agree" on the incremental benefits offered by applications.
\[ u(n) = (1 - \alpha) \left( \frac{\sigma \alpha}{c} \right)^{\frac{\alpha(1 - \sigma)}{n(1 - \alpha)}} \]
\[ V(n) = (1 - \sigma \alpha) \left( \frac{\sigma \alpha}{c} \right)^{\frac{\alpha}{n(1 - \alpha)}} \]
\[ \varepsilon_v = \frac{\alpha (1 - \sigma)}{\sigma (1 - \alpha)} \in [0, 1] \]
\[ \lambda = \frac{\sigma (1 - \alpha)}{1 - \sigma \alpha} \in [0, 1] \]

**Example 2** Suppose users have unitary demand for applications (i.e. buy either 0 or one unit of each application) and gross benefits from using \( n \) applications are \( V(n) \) with \( V'(\cdot) > 0, V''(\cdot) < 0 \). In this case the stage 3 price equilibrium is: \( p_n = V'(n) \) leading to\(^{16}\): \( \pi(n) = V'(n) \), \( u(n) = V(n) - nV'(n) > 0 \) and \( \lambda = 1 \). Letting \( V(n) = An^\beta \), with \( 0 < \beta < 1 \), we obtain\(^ {17}\):

\[ \pi(n) = \beta An^{\beta - 1} \]
\[ u(n) = (1 - \beta) An^\beta > 0 \]
\[ \varepsilon_v = \beta \]

### 2.1 Set-up of the platform’s pricing problem

Let us now set up the optimization program for a two-sided profit-maximizing platform. Given the platform’s prices \( P_U \) and \( P_D \), it is indeed an (interior) equilibrium for \( n \) developers and \( F(\theta_m) \) users to adopt the platform in stage 2 only if the following two conditions hold:

\[ \pi(n) F(\theta^n) - P_D - H^{-1}(n) = 0 \quad (1) \]
\[ u(n) - P_U = \theta^n \quad (2) \]

The first condition says that in equilibrium all profit opportunities are exhausted for developers (assuming the supply of developers is large enough).

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\(^{16}\)Here we assume developers have 0 marginal costs: many of the real-life platforms we have in mind support digital applications whose marginal costs are virtually 0. In example 1 marginal costs are necessarily positive in order to keep prices and profits finite.\(^{17}\)This example is used by Church Gandal and Krause (2002).
and the second condition says that the marginal user \( \theta^m \) must be indifferent between adopting and not adopting the platform.

Equation (1) determines developer demand \( n \) as a function \( N(\theta^m, P^D) \) of user demand and the price charged to developers, whereas equation (2) determines the marginal user \( \theta^m \) (and therefore user demand \( F(\theta^m) \)) as a function \( \Theta(n, P^U) \) of developer demand and the price charged to users. Note that these two-way demand interdependencies or indirect network externalities are positive: \( N(\cdot, P^D) \) and \( \Theta(\cdot, P^U) \) are both increasing.

Plugging (2) into (1), we obtain \( n \) as an implicit function of the platform’s prices \( P^D \) and \( P^U \):

\[
\pi(n) F(u(n) - P^U) = H^{-1}(n) + P^D
\]

This expression makes clear that on the developer side of the market there are both positive indirect network effects contained in the term \( F(u(n) - P^U) \) and negative direct network effects contained in the term \( \pi(n) \).

Throughout the paper we normalize for simplicity and without any loss of substance the platform’s marginal costs on both sides to 0. The expression of platform profits is then:

\[
\Pi^P = P^U F(\theta^m) + nP^D
\]

Using (1) and (2) we obtain:

\[
\Pi^P = (V(n) - \theta^m) F(\theta^m) - nH^{-1}(n)
\]

which depends only on \((\theta^m, n)\). Therefore, rather than maximizing platform profits over \((P^U, P^D)\) we will do so directly over \((\theta^m, n)\)\(^{18}\).

The first-order conditions determining the optimal \((\theta^m_p, n_p)\) are:

\[
\frac{V(n) - \theta^m}{\theta^m} = \frac{1}{\varepsilon_F(\theta^m)}
\]

\[
V'(n) F(\theta^m) = nH^{-1}(n) + H^{-1}(n)
\]

\(^{18}\)A similar "trick" is used by Armstrong (2003) in a linear model. Below we discuss necessary conditions for this transformation to be legitimate in our model.
Given the profit-maximizing \((n_p, \theta^m_p)\), the corresponding profit maximizing prices \((P^U_{2sp}, P^D_{2sp})\) are then uniquely determined by (1) and (2).

There are however three issues we need to address before proceeding in order to be completely rigorous. First, we need to ensure that the first-order conditions (5) and (6) define indeed a global maximum for \(\Pi^P\).

**Assumption 2** The supports of \(F\) and \(H\) are wide enough so that (5) and (6) have a unique interior solution \((\theta^m_p, n_p)\) and the hessian matrix of \(\Pi^P(\theta^m, n)\) evaluated at \((n_p, \theta^m_p)\) is semi-definite negative.

Second, we also need to make sure that given \((P^U_p, P^D_p)\), \((n_p, \theta^m_p)\) arises as a stable market configuration. Graphically, stability means that at the point \((n_p, \theta^m_p)\), the curve \(n = N(\theta^m, P^D_p)\) crosses the curve \(\theta^m = \Theta(n, P^U_p)\) from below in a \((n, \theta^m)\) plane. Formally, this is justified by postulating a dynamic adjustment process for fixed \((P^U_p, P^D_p)\) of the following type. Starting from any \((n_0, \theta^m_0)\), the market configuration \((n_t, \theta^m_t)\) evolves according to\(^19\):

\[
\begin{pmatrix}
\dot{n}_t \\
\dot{\theta}^m_t
\end{pmatrix} = \begin{pmatrix}
B \left( \pi(n_t) F(\theta^m_t) - P^D_p - H^{-1}(n_t) \right) \\
A \left( u(n_t) - P^U_p - \theta^m_t \right)
\end{pmatrix}
\]

where \(A\) and \(B\) are two positive constants.

In words, the number of users who adopt the platform increases as long as the marginal user derives positive net utility and decreases when the marginal user derives negative net utility\(^20\). Similarly, developers enter as long as there are positive profit opportunities, i.e. as long as the marginal developer (the least efficient one) makes positive profits, and exit otherwise\(^21\). Figure 1 contains the phase diagram of this process: graph a) corresponds to the case in which the process converges to \((n_p, \theta^m_p)\), i.e. to the point where \(\left(\dot{n}_t, \theta^m_t\right) = (0, 0)\); graph b) corresponds to the case in which any perturbation from the point \(\left(\dot{n}_t, \theta^m_t\right) = (0, 0)\) diverges.

\(^{19}\)This formulation is a two-dimensional version of the one-dimensional process postulated by Kiyono and Suzumura (1987) in analyzing the social efficiency of free-entry.

\(^{20}\)The "closest" users, i.e. those with the lowest \(\theta\), are assumed to enter first.

\(^{21}\)Like users, the most efficient developers enter first whereas the least efficient developers exit first.
\[ \frac{dn(t)}{dt} = 0 \]

\[ \frac{d\theta(t)}{dt} = 0 \]

Figure 1:

(a) 

(b) 

Figure 1:
The necessary and sufficient condition for stability is therefore:

\[ \frac{\partial \Theta}{\partial n} (n_p, \pi(n_p) F(\theta^m_p) - H^{-1}(n_p)) < \frac{1}{\frac{\partial N}{\partial \theta^m}(\theta^m_p, \theta^m_i(n_p) F(\theta^m_p) - H^{-1}(n_p))} \]

Using the implicit function theorem, this condition is equivalent to:

\[ u'(n_p) < \frac{H^{-1}(n_p) - \pi'(n_p) F(\theta^m_p)}{\pi(n_p) f(\theta^m_p)} \]  

(7)

**Assumption 3** The profit-maximizing market configuration \((n_p, \theta^m_p)\) is stable (i.e. (7) holds).

Third and last, given \((P^U, P^D) = (P^U_p, P^D_p)\), (1) and (2) may have multiple stable solutions \((\theta^m, n)\) as illustrated by figure 2.

This is a well-known feature in markets with indirect network effects\(^{22}\). Thus, we also need the following assumption.

**Assumption 4** If there are multiple stable market configurations, solutions to (1) and (2) given \((P^U_p, P^D_p) = (u(n_p) - \theta^m_p, \pi(n_p) F(\theta^m_p) - H^{-1}(n_p))\), then the platform is able to coordinate users and developers on its most preferred solution, i.e. \((\theta^m_p, n_p)\).

\(^{22}\)See for example Church and Gandal (1992).
This assumption is less restrictive than it might appear at first glance. Even when there are multiple stable equilibria, it is reasonable to expect users and developers will coordinate on the stable equilibrium with the highest levels of entry on both sides of the market\(^{23}\), otherwise, in the absence of any entry restrictions, there are strictly positive rents available to coalitions of users and developers which are left out of the market. Therefore the only potentially problematic case is when the platform’s preferred stable equilibrium is not the one with the highest levels of entry. But then the platform can simply adopt a policy of entry restriction on either side, inducing both sides to coordinate on its most preferred equilibrium.

The following lemma, proven in the appendix, provides an example of explicit functional forms satisfying the assumptions made above.

**Lemma 0** Assume \( F, H \) and \( V \) are defined on \([0, +\infty]\) and have constant elasticities, i.e. \( F(\theta) = B\theta^F, H^{-1}(\phi) = C\phi^H, V(n) = An^V, \) \( \pi(n) = \lambda V'(n) \), where \( \varepsilon_V = \frac{\alpha(1-\sigma)}{\sigma(1-\alpha)} \), \( \lambda = \frac{\alpha(1-\alpha)}{1-\sigma\alpha} \) in example 1 and \( \varepsilon_V = \beta, \lambda = 1 \) in example 2. Then assumptions 2 and 3 are respectively equivalent to:

\[
\varepsilon_V (1 + \varepsilon_F) \leq 1 + \frac{1}{\varepsilon_H} \quad (8)
\]

\[
\left[ \frac{1 - \lambda \varepsilon_V}{1 - \varepsilon_V} \varepsilon_V (1 + \varepsilon_F) - \frac{1}{1 + \varepsilon_H} \right] \leq (9)
\]

### 3 Platform pricing structures

In an empirical survey of computer-based industries, Evans Hagiu and Schmalensee (2004) document that despite numerous economic similarities, software platforms operating in these markets have chosen radically different pricing structures. On the one hand, vendors of operating systems for computers and many other consumer electronics products (handheld digital assistants, smartphones, television sets) have chosen to subsidize or earn little

\(^{23}\)Note that because \( N(P^D) \) and \( \Theta(P^U) \) are increasing, if \((\theta_m, n)\) and \((\theta'_m, n')\) are two equilibria given the same prices \((P^U, P^D)\) then \( \theta_m < \theta'_m \) if and only if \( n < n' \), so that it makes sense to talk about the highest level of entry on both sides.
if any profits on the developer side of the market, be it applications or hardware complements. Despite investing large amounts of money every year in "developer support", Microsoft, Apple, Symbian, Palm, Novell, Sun, all make virtually all of their profits by selling their platforms to users. At the other end of the spectrum, in the videogame market, all console manufacturers without exception since the introduction of the first Nintendo Entertainment System in the United States in 1988 make the bulk of their profits through per-game royalties charged to publishers-developers\(^\text{24}\) and sell their consoles at or below cost to users\(^\text{25}\). Finally, pricing structures of digital media platforms seem to lie somewhere in-between these two extremes: for example, i-mode makes profits both on users and on content providers through variable fees based on the intensity of usage of the network and Real’s revenues come from both subscription fees charged to users and access fees charged to "non-premium" content providers\(^\text{26}\).

It should be stressed that there is absolutely nothing that prevents any of these platforms from charging developers, either fixed or variable fees, except of course business rationality. In fact, this pricing "puzzle" is all the more striking as it can be found within the same firm, Microsoft, which has two entirely opposite business models for Windows and XBox.

More generally, this discussion can be extended to include other two-sided platforms: shopping malls charge nothing for access to consumers and recoup their initial layout by collecting rent from retailers\(^\text{27}\); Priceline.com allows Internet users to access a variety of services and product offerings for free, while charging sponsors of these services and products for the right to be listed\(^\text{28}\); Ticketmaster pays venues or promoters a small fee per ticket sold and recoups by charging users $3 to $6 in addition to the ticket’s face

\(^{24}\)For example, Sony’s Playstation 2, Nintendo’s GameCube and Microsoft’s XBox charge $8-$10 royalties per game to independent game publishers.

\(^{25}\)See for example Clements and Ohashi (2004).

\(^{26}\)A few premium content providers are paid by Real. The company paid, for instance, the National Basketball Association $20 million and a share of subscription revenues for the rights to stream NBA games for three seasons (Sloan (2003)).

\(^{27}\)Pashigian and Gould (1998).

\(^{28}\)Ideally, one should distinguish between pure advertisers and genuine product/service offerings (trips, hotels, flights, etc.), however, from the broad perspective we take here, these two types of "products" can be considered approximately similar.
value\textsuperscript{29}.

How can one make sense of these contrasting pricing structures? In this section we show that our model yields an explanation based on the intensity of users’ preferences for variety. Of course, there are many other factors, specific to each industry, which have a significant influence on platforms’ pricing structures, however the intuitive explanation we propose has the merit of being applicable to a broad range of industries and, as we argue below, is quite plausible empirically, at least in a first-order approximation.

Throughout the paper we will calculate the pricing structure as the ratio between the portion of total profits $\Pi^P$ which is made on developers, $\Pi^{PD}$, and the portion which is made on users, $\Pi^{PU}$:

$$\Pi^P = \frac{\Pi^{PU} F(\theta^m)}{\Pi^{PD}}$$

Then, making use of (1) and (2), we can write:

$$\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{n \pi(n) F(\theta^m) - n H^{-1}(n)}{(u(n) - \theta^m) F(\theta^m)}$$

Using (6) and simplifying by $F(\theta^m)$:

$$\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{n V'(n) \left( \frac{\pi(n)}{V'(n)} - 1 + \frac{n H^{-1}(n)}{V'(n)} \right)}{V(n) \left( 1 - \frac{\pi(n)}{V'(n)} \frac{n V'(n)}{V(n)} - \frac{\theta^m}{V(n)} \right)}$$

But the first order conditions (5) and (6) imply\textsuperscript{30}:

$$\theta^m = \frac{\varepsilon F(\theta^m) V(n)}{1 + \varepsilon F(\theta^m)}$$

$$\frac{V'(n) F(\theta^m)}{n H^{-1}(n)} = 1 + \varepsilon F(\theta^m)$$

Combining the last three expressions and using $\varepsilon V(n) = n V'(n) / V(n)\textsuperscript{29}$ and $\lambda(n) = \frac{\pi(n)}{V'(n)}$, we obtain the following proposition.

\textsuperscript{29}Bilodeau (1995).
\textsuperscript{30}We use the fact that $\frac{n H^{-1}(n)}{H^{-1}(n)} = \frac{1}{\varepsilon n H^{-1}(n)}$. 
Proposition 1. Suppose assumptions 1 through 4 hold. Then the optimal platform pricing structure is given by:\(^{31}\):

\[
\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{\varepsilon_V (1 + \varepsilon_F) (1 - (1 - \lambda) (1 + \varepsilon_H))}{(1 + \varepsilon_H) (1 - \lambda \varepsilon_V (1 + \varepsilon_F))}
\]

(10)

If \(\lambda \leq \frac{\varepsilon_H}{1 + \varepsilon_H}\) then the platform subsidizes developers \((P^{D} < 0)\) and recoups on users.

If \(\lambda \geq \frac{1}{\varepsilon_V (1 + \varepsilon_F)}\) then the platform subsidizes users \((P^{U} < 0)\) and recoups on developers.

If \(\frac{\varepsilon_H}{1 + \varepsilon_H} < \lambda < \frac{1}{\varepsilon_V (1 + \varepsilon_F)}\)\(^{32}\) then the platform makes positive profits on both sides of the market and its optimal pricing structure is such that the share of profits made on developers relative to the share of profits made on the user side of the market is decreasing in the elasticity of developer demand \(\varepsilon_H\) and increasing in the elasticity of user demand for the platform \(\varepsilon_F\), in the elasticity of user demand for applications \(\varepsilon_V\) and in \(\lambda\), the ratio of developer profits per user over the marginal contribution of an additional developer to surplus per user.

\[\blacksquare\]

The result that \(\frac{\Pi^{PD}}{\Pi^{PU}}\) is increasing in \(\varepsilon_F\) and decreasing in \(\varepsilon_H\) is not surprising and has been obtained - albeit under different forms\(^{33}\) - in other theoretical models of two-sided markets, Armstrong (2002) and Rochet and Tirole (2004) in particular. It simply says that the share or profits made on one side of the market relative to the other side is higher the easier it is to attract the former and the more difficult it is to attract the latter.

Our model however yields two new results. First, the platform makes relatively more profits on the developer side of the market when developers extract a larger share \(\lambda\) of their marginal contribution to social surplus (per platform user). In particular, if this share is large enough \((\lambda \geq \frac{1}{\varepsilon_V (1 + \varepsilon_F)})\) then the platform may even find optimal to subsidize the participation of users and make all of its profits on developers. Conversely, if this share is

\[^{31}\text{We omit function arguments in order to avoid clutter.}\]

\[^{32}\text{Recall from lemma 0 that when elasticities are constant, assumption 2 is equivalent to } \varepsilon_V (1 + \varepsilon_F) < \frac{1 + \varepsilon_H}{\varepsilon_H}.\]

\[^{33}\text{Note in particular that our notion of "demand elasticity" is slightly different from the one which is commonly used.}\]
too low \((\lambda \leq \frac{\varepsilon_H}{1 + \varepsilon_H})\) - this happens for example when competition among developers is too strong - then the platform will subsidize developers and recoup on users.

Second and most important, developers pay relatively more when the "intensity" of users’ preferences for variety \(\varepsilon_V\) is higher. To see this more clearly, let us use the formulation of user preferences from example 2, with \(V(n) = A n^\beta, \pi(n) = V'(n)\), so that \(\lambda = 1\). Then the optimal pricing structure is:

\[
\frac{\Pi_{PD}}{\Pi_{PU}} = \frac{\beta (1 + \varepsilon_F)}{(1 + \varepsilon_H)(1 - \beta (1 + \varepsilon_F))}
\]

This expression contains an interesting explanation for the different pricing structures we observe, which appears quite plausible if we focus on software platforms. Indeed, while we are aware of no empirical evidence that the elasticities of developer and user demand for platforms are markedly different across computer-based industries, there are good reasons to believe that user demand for product variety is significantly higher for videogames than for productivity-oriented or professional software (for computers and PDAs for example). The most important such reason is durability: by definition, games get "played out"\(^{34}\), whereas professional software is theoretically infinitely durable (technological obsolescence notwithstanding of course). Consequently, users of videogame consoles will demand a constant stream of games throughout a console’s lifecycle, whereas computer users will generally stick to a few applications that they always use. As pointed out by Campbell-Kelly (2003):

"[...] Thus, while the personal computer market could bear no more than a few word processors or spreadsheet programs, the teenage videogame market could support an indefinite number of programs in any genre. In this respect, videogames were, again, more like recorded music or books than like corporate software..."\(^{35}\)

Given this difference, our model predicts that the pricing structures should be such that videogame platforms make a larger relative share of

\(^{34}\)Coughlan (2001).
profits on developers than the other software platforms, which is precisely what we observe.

More generally, our model implies that the platform pricing structure will "favor" users (i.e. developers will account for a larger share of profits) in industries in which the intensity of user preferences for diversity is inherently high. The table in figure 3 contains some of the industries/platforms mentioned above with their corresponding pricing structures, organized by increasing order of user demand for variety. It appears that in a first-order approximation, the prediction of our model is rather consistent with what we observe in reality.

Figure 3:

<table>
<thead>
<tr>
<th>Platform</th>
<th>Pricing structure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Developers</td>
</tr>
<tr>
<td>Ticketmaster</td>
<td>=0</td>
</tr>
<tr>
<td>Computer Operating Systems</td>
<td>=0</td>
</tr>
<tr>
<td>PDA operating systems</td>
<td>=0</td>
</tr>
<tr>
<td>Smart-phone operating systems</td>
<td>=0</td>
</tr>
<tr>
<td>Priceline.com</td>
<td>&gt;0</td>
</tr>
<tr>
<td>Shopping malls</td>
<td>&gt;0</td>
</tr>
<tr>
<td>Digital media (i-mode)</td>
<td>&gt;0</td>
</tr>
<tr>
<td>Video Games</td>
<td>&gt;0</td>
</tr>
</tbody>
</table>

36 Given the lack of precise data, we have contented ourselves with providing the sign of profits made on each side of the market.
37 Although this ranking is quite intuitive, it should be noted that it is based solely on casual empiricism and not on rigorous econometric analysis, which is the next logical step of this research.
In the following two subsections we extend our model in two important directions: developer innovation incentives and platform competition.

3.1 The role of royalties: commitment and innovation incentives

There are two important ingredients we have omitted from the basic model presented above: developer innovation incentives (i.e. the incentives to invest in application quality) and the use of variable fees or royalties by the platform on the developer side of the market.\(^{38}\)

In reality, particularly in the context of the software platforms which constitute the motivation for this paper, investments by third-party developers in order to enhance the quality of their products are quite important. The platform’s success in the marketplace depends heavily on the quality of the applications they support, which is why they try to provide adequate incentives through their pricing structure. In fact, there is a second interesting dimension to the variety of pricing structures documented by Evans Hagiu and Schmalensee (2004), aside from the relative shares of profits made on each side of the market: some platforms such as videogame consoles rely almost exclusively on royalties, whereas others, such as virtually all operating system vendors (Microsoft Windows, Palm, Symbian, etc.), only charge (moderate) fixed fees and no variable fees.

In this subsection we extend our model by endogenizing the choice of product quality by developers and allowing platforms to charge both fixed and variable fees (or royalties). We show that the optimal royalty rate for a two-sided platform is determined by two conflicting purposes: mitigating a hold-up problem by the platform when developers arrive before users and providing adequate innovation incentives to developers.

We model the quality \( q(i) \) of application \( i \) as the proportion of users which are interested in it. Suppose then that the platform is supported by \( n \) applications and \( F(\theta^m) \) users. Then each platform user is interested in \( \int_0^n q(i) \, di \) applications and values them at \( V(\int_0^n q(i) \, di) \). Conversely, each

\(^{38}\)In the absence of innovation incentives concerns, royalties and fixed access fees are perfect pricing substitutes when developers are price-takers, therefore there was no loss of generality in focusing on fixed fees in the previous section.
application developer $i$ knows that his application is desired by $q(i) F(\theta^m)$ users. We assume developers compete as in example 2, so that in equilibrium each developer will charge exactly $39 V' \left( \int_0^n q(i) di \right)$ and sell to $q(i) F(\theta^m)$ users.

We assume developers’ fixed cost of producing an application of quality $q$ is $c(q) + \phi$, where, as before, the number of developers with quality-independent fixed cost lower than $\phi$ is $H(\phi)$. $H$ is increasing and has a continuously differentiable derivative $h$. However, we assume for simplicity $c(q)$ is the same for all developers, i.e. all developers have the same marginal cost of quality provision. We assume $c(.)$ is strictly increasing and convex and $c(0) = 0^{40}$.

The platform can charge developers both fixed fees $P^D$ and royalties $\rho$, which for clarity of exposition we assume are proportional to the price charged by developers to users$^{41}$. Thus, a developer with fixed cost $\phi$ choosing quality $q$ when $n$ developers with respective qualities $q(i)$, $i \in [0, n]$, enter and $F(\theta^m)$ users adopt the platform, makes profits:

$$ (1 - \rho) V' \left( \int_0^n q(i) di \right) qF(\theta^m) - P^D - \phi - c(q) $$

### 3.1.1 Simultaneous entry and first-best choices

Let us start by assuming the same timing of the pricing game as above, i.e. that developers and users make their platform adoption decisions simultaneously and then those developers who enter choose the qualities of their respective products simultaneously and non-cooperatively. Then, given $P^D$, $\rho$, $n$, $\theta^m$ and the other developers’ quality choices, the optimal quality choice $q$ for a developer who enters is defined by:

$$ (1 - \rho) V' \left( \int_0^n q(i) di \right) F(\theta^m) = c'(q) $$

---

39 Given that all other developers charge this price, an individual developer cannot charge a price higher than the marginal increase in utility his product offers to users which are interested in it. A lower price would not make sense, since it already sells to all potentially interested users.

40 In other words, we assume quality is deterministic: choosing the level of investment is equivalent to choosing the quality level directly.

41 In our model proportional and nominal royalties are perfect substitutes as pricing instruments for the platform because developers are price-takers.
In the symmetric equilibrium, all developers who enter choose the same quality \( q \), defined implicitly by:

\[
(1 - \rho) V'(nq) F(\theta^m) = c'(q)
\]  

(11)

Thus, in the adoption stage, users and developers enter until the following two equations hold, where \( q \) is a function of \((n, \theta^m, \rho)\) defined by (11):

\[
(1 - \rho) V'(nq) qF(\theta^m) - P^D - c(q) - H^{-1}(n) = 0
\]  

(12)

\[
V(nq) - nqV'(nq) - P^U - \theta^m = 0
\]  

(13)

Consequently, given \((P^U, \rho, P^D)\), \((n, \theta^m, q)\) are simultaneously given by (11), (12) and (13). Platform profits are then:

\[
\Pi^P = P^U F(\theta^m) + \rho nqV'(nq) F(\theta^m) + P^D n
\]

\[
= (V(nq) - \theta^m) F(\theta^m) - nc(q) - nH^{-1}(n)
\]  

(14)

which the platform maximizes over \((P^U, \rho, P^D)\) subject to (11), (12) and (13) or, equivalently, over \((n, \theta^m, \rho)\), subject to (11)\textsuperscript{42}. The two first order conditions in \( \theta^m \) and \( n \) are:

\[
(V(nq) - \theta^m) f(\theta^m) - F(\theta^m) + (nV'(nq) - nc'(q)) \frac{\partial q}{\partial \theta^m} = 0
\]  

(15)

\[
qV'(nq) F(\theta^m) - c(q) - nH^{-1}(n) - H^{-1}(n) + (nV'(nq) - nc'(q)) \frac{\partial q}{\partial n} = 0
\]  

(16)

Note first that in the absence of quality investment concerns, i.e. if \( q \) were exogenously given (say, equal to 1), then constraint (11) would disappear and, since expression (14) does not depend on \( \rho \), (12) implies that \( \rho \) and \( P^D \) are perfect pricing substitutes for the platform. This case is identical to the one studied in the basic model. Here however, although \( \rho \) does not appear in expression (14), it determines \( q \) as a function of \( \theta^m \) and \( n \) through (11), whereas \( P^D \) (together with \( \rho \)) determines \( n \).

\textsuperscript{42} Naturally, in order to be completely rigorous, one would have to make sure that this change of variables is legitimate and discuss stability and concavity. This analysis would clearly be very similar to the one conducted in the previous section, therefore we omit it here.
Let us first determine the platform’s first best levels of user adoption, product variety and quality, i.e. the triplet \((\theta_{fb}^m, n_{fb}, q_{fb})\) which maximizes its profits if it were able to choose product quality directly, or equivalently, if the latter were contractible. The first–order conditions determining \((\theta_{fb}^m, n_{fb}, q_{fb})\) are:

\[
\begin{align*}
\theta^m &= \frac{\varepsilon_F V'(nq)}{1 + \varepsilon_F} \\
qV'(nq) F'(\theta^m) &= c(q) + nH^{-1}(n) + H^{-1}(n) \\
V'(nq) F'(\theta^m) &= c'(q)
\end{align*}
\]

(17)  
(18)  
(19)

It is then easily seen that in this simple model the platform can attain the first-best level of profits even when product quality is not contractible, by setting:

\[
\rho = 0
\]

Indeed, this implies that (11) is identical to the first-order condition (19), so that given \((\theta^m, n)\), the choice of product quality by developers is optimal from the platform’s point of view. Then, plugging (11) with \(\rho = 0\) into the first-order conditions (15) and (16) defining the second-best, they are also identical to their first-best counterparts (17) and (18) respectively.

**Fact** When users and developers adopt the platform simultaneously, the optimal royalty rate is \(\rho = 0\) and the platform achieves the first-best level of profits, i.e. that which it would obtain if it were able to choose the quality of developers’ products directly or if the latter were contractible.

Of course, the fact that the platform achieves the first-best level of profits is due to our simplifying assumption that developers have the same marginal cost of quality provision and that the marginal cost is known by the platform. This is not robust to more general formulations. However, the main point we wish to emphasize here is that when investment in product quality by developers is important, it is optimal for the platform to leave at least a share of variable revenues to developers in order to provide them with ex-ante optimal innovation incentives. This seems to be the reasoning behind Microsoft Windows’, Palm OS’ and Symbian’s platform pricing strategies for their respective developer communities: it is probably no coincidence that
all of these platforms have resisted the temptation to share in the success of their developers by charging them variable fees.

But this seems to also apply very well to the case of videogame platforms, which, to the contrary, rely heavily on royalties. How can one explain these apparently contradictory facts?

The answer we propose is based on a hold-up problem which arises when developers arrive before users. Indeed, in all of the markets mentioned above platform vendors have to secure the participation of a significant number of developers long before the platform and the applications become available to users, for obvious technological reasons: the development of videogames or computer applications can take from 6 months to more than a year, whereas the development of a new platform (operating system or videogame console) takes several years.

Once developer participation has been secured, the issue is that the platform does not internalize developer profits, therefore it sets its user price $P_U$ too high relative to the price that would maximize joint profits and restricts user adoption too much. Consequently, ex-ante the platform has to lower the fixed fee it charges to developers. This indirect hold-up problem arises when the platform is neither able to commit ex-ante to the price it will charge to users, nor can it negotiate with all of its developers ex-post in order to set the user price which maximizes joint profits\textsuperscript{43}. In this context, royalties can mitigate the hold-up problem, because they are an effective way of committing the platform to charging a lower user price, by allowing it to internalize a part of developer profits and thereby giving it incentives to induce more user adoption.

### 3.1.2 Developers enter before users

In order to formalize this insight, we now assume the following timing:

- **Stage 1)** The platform sets its fixed ($P_D$) and variable ($\rho$) charges for developers; developers decide whether or not to enter

- **Stage 2)** Developers who have entered decide their level of investment in product quality

\textsuperscript{43}It is quite realistic to assume that high transaction costs preclude this negotiation.
• Stage 3) The platform sets its price $P^U$ for users and the latter decide whether or not to adopt the platform.

• Stage 4) Developers set prices for users and those users who have acquired the platform in the previous stage decide which applications to buy.

We will solve the pricing game by proceeding backwards starting with the fourth stage. In this stage a developer having joined the platform and chosen product quality $q$ charges $V^0 \left( \int_0^n q(i) \, di \right)$ and makes revenues $(1 - \rho) \left( V^0 \left( \int_0^n q(i) \, di \right) - \rho \, V^0 \left( \int_0^n q(i) \, di \right) \right)\, F(\theta^m)$. In stage 3 the platform takes as given the number $n$ of developers having entered, their product quality choices $q(i)$ for $i \in [0, n]$ and the royalty rate $\rho$ it had set in stage 1. It maximizes its profits from stage 3 onwards:

$$\max_{P^D} \{ (P^U + \rho Q V'(Q)) F(\nu) - Q V'(Q) - P^U ) \}$$

where $Q = \int_0^n q(i) \, di$. This maximization problem is equivalent to:

$$\max_{\theta^m} \{ (V(Q) - (1 - \rho) Q V'(Q) - \theta^m) \, F(\theta^m) \}$$

yielding:

$$\theta^m = \frac{\varepsilon F(V(Q) - (1 - \rho) Q V'(Q))}{1 + \varepsilon_F} = \theta^m(Q, \rho) \quad (20)$$

In stage 2, each individual developer takes $Q$ as given and chooses $q$ to maximize his future profits. Therefore, in equilibrium all developers choose the same quality $q$, defined implicitly by:

$$(1 - \rho) V'(nq) \, F(\theta^m(nq, \rho)) = c'(q) \quad (21)$$

Together, (20) (with $Q = nq$) and (21) determine the equilibrium product quality and user adoption as functions of developer demand $n$ and the royalty rate $\rho$: $q(n, \rho)$ and $\theta^m(n, \rho)$.

Finally, in stage 1, given $P^D$ and $\rho$, developer demand $n$ is implicitly determined by:

$$(1 - \rho) V'(nq(n, \rho)) q(n, \rho) \, F(\theta^m(n, \rho)) - c(q(n, \rho)) - h^{-1}(n) - P^D = 0 \quad (22)$$
Platform profits from the perspective of stage 1 are the sum of the fixed fees collected from developers who enter at this stage and the profits made from stage 3 onwards through royalties and sales to users:

$$\Pi^P = nP^D + (V(nq) - (1 - \rho) nqV'(nq) - \theta^m) F(\theta^m)$$

where \( q = q(n, \rho), \theta^m = \theta^m(n, \rho) \) and \( n \) is given by (22).

Plugging in (22), we obtain that the platform maximizes:

$$\left(V(nq(n, \rho)) - \theta^m(n, \rho)\right) F(\theta^m(n, \rho)) - nc(q(n, \rho)) - nH^{-1}(n)$$

over \((P^D, \rho)\) subject to (22), or equivalently, over \((n, \rho)\) directly.

This time it is clear that the platform is unable to achieve the first-best level of profits. The reason is that now \( \rho \) plays two conflicting roles: on the one hand it helps mitigate the platform’s hold-up problem in stage 3 (this calls for a high \( \rho \)), while on the other hand it has to preserve developer innovation incentives (this calls for a low \( \rho \)).

First, assume that product quality is not an issue, i.e. it is contractible ex-ante for example. Then (21) disappears and it is easily seen from (20) that the platform can still achieve the first best by charging \( \rho = 1 \). In other words, by committing to take over all developer revenues in stage 4, it effectively commits itself to the first-best user price \( P^U \)-or equivalently the first-best level of user adoption \( \theta^m_{fb} \)-from the perspective of stage 1, which in turn allows it to induce the first-best level of developer adoption \( n_{fb} \).

Second, assume the platform were able to credibly commit in stage 1 to the user price it will charge in stage 3 to users or that it could get together will all developers in stage 3 and negotiate the user price in order to maximize joint profits. In this case the first-best level of profits is once again attainable by setting \( \rho = 0 \), just like in the previous situation, when users and developers arrived simultaneously. This is because (20) disappears and the platform can guarantee in stage 1 the first-best level of user adoption \( \theta^m = \frac{\varepsilon F V(nq[n, \rho])}{1 + \varepsilon F} \) given \( n \) and \( \rho \).

In the case when quality is not contractible and the platform is neither able to credibly commit to its user price ex-ante, nor can it negotiate with all developers in stage 3, the platform will have to settle for a second-best level of profits. The corresponding optimal royalty rate will then be strictly between 0 and 1, reflecting the tradeoff we have just explained.
The following proposition summarizes this discussion and provides a specific example:

**Proposition 2** Assume developers have to make their platform adoption decision before users. Then the platform can achieve the first-best level of profits (i.e. the level of profits it obtains when developers and users adopt simultaneously) if and only if the quality of developers’ products is contractible, in which case the optimal royalty rate is \( \rho = 1 \), or the platform is able to either credibly commit ex-ante to its user price or negotiate this price ex-post with all its developers, in which case the optimal royalty rate is \( \rho = 0 \).

Otherwise, the first-best level of profits is not attainable and the optimal royalty rate \( \rho \) is strictly between 0 and 1. In the case when user demand for the platform is linear, i.e. \( F(\theta) = B\theta \), the cost of quality provision is quadratic, i.e. \( c(q) = \frac{\alpha q^2}{2} \), and \( V(Q) = AQ^\beta, \beta \in [0, 1] \), the optimal royalty rate is \( \rho = \frac{\beta}{1+\beta} \).

**Proof** See appendix.
they lack a credible way of committing to low user prices when contracting with third-party game publishers, so that, following the logic of our model, one would expect them to resort to royalties in order to attenuate the hold-up problem, which is precisely what they do.

3.2 Platform competition

In this subsection we study the effect of introducing competition between platforms on the optimal pricing structure. Platform competition is modeled in the following way. The user horizontal differentiation parameter $\theta$ is assumed to be uniformly distributed on a Hotelling segment $[0, 1]$; unit transportation costs are $t$ and there is one platform situated at each of the two extremities. Thus, the utility of a user located at $\theta \in [0, 1]$ from adopting platform 1 is $u_0 + u(n_1) - t\theta - P^U_1$, whereas that from adopting platform 2 is $u_0 + u(n_2) - t(1 - \theta) - P^U_2$, where $u_0$ is the standalone value of each platform for users. In all that follows we assume $u_0$ is large enough so that the user market is always entirely covered. For $i = 1, 2$, $n_i$ denotes the number of developers supporting platform $i$. Then, denoting by $D^U_i$ the total user demand for platform $i$, we have $D^U_1 + D^U_2 = 1$ and:

$$D^U_1 = \frac{1}{2} + \frac{u_1 - u_2}{2t}$$

(24)

where:

$$u_i = u_0 + u(n_i) - P^U_i$$

is the utility gross of transportation costs offered by platform $i$ to its users.

Meanwhile we assume there is no differentiation between platforms from the developers’ perspective, i.e. a developer with fixed development cost $\phi$ makes profits $\pi(n_1)D^U_1 - P^D_1 - \phi$ by joining platform $i$ exclusively and $\pi(n_1)D^U_1 + \pi(n_2)D^U_2 - P^D_1 - P^D_2 - 2\phi$ from multihoming. Thus, for each developer, the decision to adopt platform 1 is independent of his decision to adopt platform 2, given $D^U_1$ and $D^U_2$, so that developer demand $n_i$ for

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44 We also implicitly assume the development cost is platform-independent and there are no economies of "platform scale", i.e. the cost of development for an additional platform does not depend on having or not developed for another platform.

45 This is because developers are atomistic, so that each individual developer does not take into account the effect of his adoption decision on $D^U_1$ and $D^U_2$ through the indirect network effect mechanism.
platform $i \in \{1, 2\}$ is implicitly defined by:

$$
\pi (n_i) D_i^U - P_i^D - H^{-1} (n_i) = 0
$$

(25)

The assumption that platforms are differentiated from the point of view of users but are perfect substitutes for developers simplifies the analysis and is also quite realistic. Indeed, at equal platform quality, developers care only about the respective installed bases of users and, compared to the latter, they are relatively less likely to have intrinsic preferences for one platform over the other (i.e. being die-hard MacIntosh or Nintendo fans for example).

Although in principle both users and developers are allowed to multihome, we focus on the symmetric equilibrium, in which each platform attracts half the users exclusively, whereas all developers who enter multihome. If the user differentiation parameter $t$ is large enough, this is the only symmetric equilibrium.

Platform 1’s profits are then:

$$
\Pi_1^P = P_1^U D_1^U + P_1^D n_1 = \left( P_1^U + n_1 \pi (n_1) \right) \left( \frac{1}{2} + \frac{u_1 - u_2}{2t} \right) - n_1 H^{-1} (n_1)
$$

$$
= (V (n_1) - u_1) \left( \frac{1}{2} + \frac{u_1 - u_2}{2t} \right) - n_1 H^{-1} (n_1)
$$

In order to find the symmetric pricing equilibrium without explicitly deriving the two-dimensional best-response functions, we use a "trick" developed by Choi (2004). In the symmetric equilibrium, $u_1 = u_2 = u$ and $D_1^U = D_2^U = \frac{1}{2}$. Consider then varying $P_1^D$ while maintaining $u (n_1) - P_1^U$ fixed equal to $u$:

$$
\Pi_1^P = (V (n_1) - u) \frac{1}{2} - n_1 H^{-1} (n_1)
$$

Meanwhile, (25) defines a 1-to-1 relationship between $n_1$ and $P_1^{D46}$:

$$
\pi (n_1) \frac{1}{2} - P_1^D = H^{-1} (n_1)
$$

so that we can optimize directly over $n_1$. We obtain that the equilibrium number $n_c$ of developers who enter (and multihome) in the symmetric equi-

\footnote{This is because $\pi$ is strictly decreasing and $H^{-1}$ strictly increasing.}
Equilibrium is defined by\( (26)\):
\[
V'(n_c) \frac{1}{2} = n_c H^{-1}(n_c) + H^{-1}(n_c)
\]
and the corresponding price charged by platforms to developers is:
\[
P_D^c = \pi(n_c) \frac{1}{2} - H^{-1}(n_c)
\]

In order to determine the equilibrium price for users, consider varying \(P_U^1\) keeping all other prices constant. Recalling that \(D_U^i = 1 - D_1^U\), (25) applied for \(i = 1, 2\) defines \(n_1\) and \(n_2\) as strictly increasing, respectively decreasing functions of \(D_1^U\). Indeed, using the implicit function theorem:
\[
\frac{dn_1}{dD_1^U} = \frac{\pi(n_1)}{H^{-1}(n_1) - \pi'(n_1) D_1^U}
\]
\[
\frac{dn_2}{dD_1^U} = \frac{-\pi(n_2)}{H^{-1}(n_2) - \pi'(n_2) D_2^U}
\]

Then, plugging these functions into (24), one obtains a 1-to-1 relation between \(D_1^U\) and \(P_U^1\) holding all other prices constant. It then turns out to be more convenient to optimize \(\Pi_1^P\) over \(D_1^U\) rather than over \(P_U^1\):
\[
\max_{D_1^U} \left\{ (V(n_1) - u_1) D_1^U - n H^{-1}(n_1) \right\}
\]
yielding the first-order condition:
\[
V(n_1) - u_1 + D_1^U \left( V'(n_1) \frac{dn_1}{dD_1^U} - \frac{du_1}{dD_1^U} \right) = (n_1 H^{-1}(n_1) + H^{-1}(n_1)) \frac{dn_1}{dD_1^U}
\]

From (24), we have:
\[
\frac{du_1}{dD_1^U} = 2t - u'(n_2) \frac{\pi(n_2)}{H^{-1}(n_2) - \pi'(n_2) D_2^U}
\]

Plugging this expression into the first order condition above and using the fact that in equilibrium \(D_1^U = \frac{1}{2}\) and \(n_1 = n_2 = n_c\), where \(n_c\) satisfies (26), we obtain the symmetric equilibrium user price:
\[
P_U^c = t - n_c \pi(n_c) - \frac{u'(n_c) \pi(n_c)}{2H^{-1}(n_c) - \pi'(n_c)}
\]

\(^{47}\)If the right-hand side of (26) is increasing then \(n_c\) is unique. This is the case when \(H\) has constant elasticity.
Finally, to see that this is indeed an equilibrium, note that since users singlehome and split equally among the two platforms, a developer with fixed cost $\phi$ makes the same profits on both platforms, therefore if he enters he necessarily multihomes. Conversely, given that developers multihome, the net gain to a user located at $\theta \leq \frac{1}{2}$ from multihoming rather than adopting platform 1 situated at 0 exclusively is $-P_U^t - t (1 - \theta)$ and is maximum for $\theta = \frac{1}{2}$. Therefore, if $t$ is large enough, all users do indeed singlehome.

**Proposition 3** Assume there are two competing platforms differentiated a la Hotelling for users and identical for developers. Then, if the standalone platform value for users and the user differentiation parameter $t$ are large enough, in the symmetric equilibrium users singlehome and split equally among the two platforms and there are $n_c$ developers who enter and multihome (i.e. support both platforms), where $n_c$ is given by (26). The symmetric platform prices $P_D^t$ charged to developers and $P_U^t$ charged to users are given by (27) and (28) respectively.

![Image]

Note that the equilibrium price on the user side of the market with competing platforms is equal to the standard Hotelling price $t$ discounted by two positive terms reflecting the internalization of indirect network effects. The first term, $n_c \pi(n_c)$, is the positive benefit (profits) created for the other side of the market (developers) by the gain of an additional user. The second term is the positive benefit created by the same additional user for members of the same side, i.e. other users: it is the product between the total marginal user benefits from an additional developer $\frac{1}{2} u'(n_c)$ and the number of additional developers attracted through the addition of the marginal user, $\pi(n_c) \frac{V^0(n_c)}{n_c - V^0(n_c)}$.

In order to obtain a tractable expression for the equilibrium platform pricing structures, we assume constant elasticity of developer demand and focus on example 2.

**Corollary** Assume in addition that developers for the same platform compete as in example 2, with $V(n) = An^\beta$, $\pi(n) = V'(n)$, and $H$ has \footnote{48Recall indeed that this factor is equal to $\frac{dn_1}{n_1^2}$.}
constant elasticity, i.e. \( H^{-1}(n) = Cn^{-1/H} \). Then, in the symmetric equilibrium we have \( n_c = \left[ \frac{\beta A_c H}{2C(1+\varepsilon_H)} \right]^{1+\frac{1}{H}-\beta} \) (increasing in \( \beta \)), \( P^D_c = \frac{\beta A_n^{\beta-1}}{2(1+\varepsilon_H)} \) and \( P^U_c = t - \beta A_n^\beta \left( 1 + \frac{1}{1+\frac{1}{(1-\beta)(1+\varepsilon_H)}} \right) \), yielding the following pricing structure:

\[
\frac{\Pi^{PD}}{\Pi^{PU}} = \frac{\beta A_n^\beta}{(1+\varepsilon_H)} \left( 1 + \frac{1}{1+\frac{1}{(1-\beta)(1+\varepsilon_H)}} \right)
\]

(29)

It is increasing in \( \beta \) for \( t \) and \( \varepsilon_H \) large enough or \( \beta \) small enough.

As is made clear by (29), the intensity of user preferences for variety has now two effects on the equilibrium pricing structure. The first effect is positive and comes from \( n_cV'(n_c) \): these are the revenues extracted by developers from users. The higher they are (i.e. the higher \( \beta \)), the more the platform will charge developers relative to users: this effect is essentially the same as with a monopoly platform. The second effect of \( \beta \) however is new and goes in the opposite direction, through the factor \( \frac{1}{1+\frac{1}{(1-\beta)(1+\varepsilon_H)}} \). Recall that, say for platform 1, this factor is due to the indirect competitive effect of a decrease in \( P^U_1 \) contained in the term \( \frac{u'(n_2)\pi(n_2)}{2(H^{-1}(n_2)-\pi'(n_2)D^U_2)} \). That is, in addition to the usual direct competitive effect \( \frac{1}{2} \), a price cut by platform 1 on the user side, by stealing users from platform 2, induces less developers to support platform 2, which in turn induces even more users to shift from 2 to 1 and so on and so forth. Through this effect, \( \beta \) tends to increase the share of profits made on users relative to developers.

The first effect dominates when platforms are sufficiently differentiated and the elasticity of developer demand is large enough. More importantly, it should be noted that the second effect would be reduced further if we allowed for user market expansion, by introducing hinterlands at each end of the Hotelling segment for example. Thus, in most realistic scenarios, the intensity of user preferences for variety has the same effect on the equilibrium platform pricing structure with competing platforms as in the monopoly platform case.
4 Proprietary platforms, open platforms, product diversity and social efficiency

In this section, we turn our attention to social efficiency considerations. Up to here we have focused exclusively on two-sided proprietary platforms. However, given the increasing popularity of open platforms such as Linux, Apache and other open source software systems, it is interesting from an economic theory perspective and important from an economic policy perspective to compare proprietary, profit-maximizing platforms to open platforms, in terms of induced product variety, user adoption and total social welfare. We will also compare a situation with competing proprietary platforms and a situation with a single monopoly platform.

4.1 Monopoly proprietary platform vs. open platform

In our framework, an open platform is characterized simply by free-entry of both users and developers, i.e. it charges prices equal to its marginal costs (0) on both sides of the market. Although this may be a very simplified conceptualization of, say, the open source software form of market organization, we believe it is sufficient for revealing a fundamental welfare tradeoff between the two types of platform. An open platform avoids the two-sided deadweight loss due to monopoly pricing but at the same time leaves uninternalized the positive indirect network effects between users and developers, whereas a proprietary platform has an incentive to internalize them precisely because it sets its prices in order to maximize profits.

Note that in a one-sided market the comparison is trivial: a firm pricing at marginal cost always entails higher output and higher social welfare than a profit-maximizing monopolist who cannot price-discriminate. By contrast, in a two-sided context, things are more complex: as we show below, a proprietary platform may in fact induce more developer entry (i.e. product variety), user adoption and higher total social welfare than an open platform and it may even result in socially excessive product variety (and user

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49In particular, "free entry" of users and developers is certainly not a perfect representation of the licensing agreements characteristic of open source software (BSD or GPL).
adoption).

A benevolent social planner maximizes total welfare, which in our framework is the difference between total surplus from indirect network effects and the costs of entry on the two sides of the market:

\[ W(\theta^m, n) = V(n) F(\theta^m) - \int_0^{\theta^m} \theta f(\theta) d\theta - \int_0^{H^{-1}(n)} \phi h(\phi) d\phi \] (30)

By contrast, we have shown in section 3 that a two-sided proprietary platform maximizes:

\[ \Pi^P(\theta^m, n) = (V(n) - \theta^m) F(\theta^m) - n H^{-1}(n) \]

In the case of an open platform (or free entry regime) \( n \) and \( \theta^m \) are determined as follows:

\[ \pi^D_m(n) = \pi(n) F(\theta^m) - H^{-1}(n) = 0 \] \[ \theta^m = u(n) = V(n) - n \pi(n) \] (31) (32)

where \( \pi^D_m(n) \) are net profits of the marginal developer when \( n \) developers have entered.

Consider first the developer side of the market. The derivative of total social welfare with respect to \( n \) is:

\[ \frac{\partial W}{\partial n} = V'(n) F(\theta^m) - H^{-1}(n) = \pi^D_m(n) + (V'(n) - \pi(n)) F(\theta^m) \] (33)

Thus, if one looks only at the developer side of the market, what drives a wedge between the levels of product diversity under an open platform relative to the socially optimal level is the term \( (V'(n) - \pi(n)) F(\theta^m) \). If developer profits per platform user \( \pi(n) \) exceed the marginal contribution of an additional developer to social welfare per platform user \( V'(n) \) (i.e. \( \lambda > 1 \)), then \( \frac{\partial W}{\partial n} < \pi^D_m(n) \) and therefore an open platform tends to induce excessive entry of developers all other things equal. And vice versa. This is precisely the insight of Mankiw and Whinston (1986). To see this more clearly, consider example 1:

\[ V'(n) - \pi(n) = n (G' v' - c) \frac{\partial q_n}{\partial n} + G' \times (v - v' q_n) \]

business-stealing product diversity
The first term represents the business stealing effect and is negative as long as $\frac{\partial q}{\partial n} < 0$ and the price $G'v'$ is above marginal cost, whereas the second term is the product diversity effect and is positive since $v$ is concave. The inefficiency of an open platform on the developer side depends on which of these two effects dominates. In example 2 we have $\pi (n) = V' (n)$, so that the open platform introduces no bias with respect to developer entry all other things equal.

But of course, all other things are not equal in our model, since developer entry depends on user entry and vice versa. As we show below, the open platform induces too little user entry, which in turn leads to too little developer entry, an indirect effect which does not exist in Mankiw and Whinston (1986).

Consider now the derivative of a two-sided platform’s profits with respect to $n$:

$$\frac{\partial \Pi}{\partial n} = V' (n) F (\theta^m) - H^{-1} (n) - nH^{-1} (n)$$

$$= \pi^D_m (n) + (V' (n) - \pi (n)) F (\theta^m) - nH^{-1} (n) \quad (34)$$

Comparing (34) with (33), the proprietary platform introduces no inefficiency through the business stealing and the product diversity effects. This is due to the fact that in our model both users and developers are differentiated only horizontally, so that the platform can fully internalize developer revenues $n\pi (n)$ and user gross surplus $V (n) - n\pi (n)^{50}$. What does induce a bias however is the proprietary platform’s inability to perfectly price discriminate among developers: it consequently discounts the total social value created by an additional developer by $nH^{-1} (n)$, the revenues lost on existing developers by reducing the price $P^D$ in order to accommodate the additional developer. Since this bias is negative, the proprietary platform tends to induce too little entry on the developer side, keeping everything else constant.

Turning now to the user side of the market, the respective first order conditions with respect to $\theta^m$ are:

$$\theta^m = V (n) \quad (35)$$

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50 Below we provide an example with vertical developer differentiation.
for the social planner and:

\[ u(n) - P^U = \theta^m = \frac{\varepsilon_F V(n)}{1 + \varepsilon_F} \]

for the proprietary platform.

Comparing (35) to (32), the open platform tends to induce too little user adoption all other things equal, because each developer who enters does not take into account the effect of his price on total user demand for the platform. Comparing (36) to (35), the proprietary platform also tends to induce too little user entry: it perceives the benefits of an additional user as the difference between the extra revenues \( P^U + n\pi(n) = V(n) - \theta^m \), which are exactly equal to the total social value created by the additional user\(^{51}\), and \( \varepsilon_{F(\theta^m)} \), the revenues lost on existing users by reducing the price \( P^U \) in order to accommodate the additional user.

Comparing (32) and (36), it is not possible to say in general which of the open platform or the proprietary platform restricts user adoption more. It depends on the sign of \( P^U \): all other things equal, the proprietary platform induces less restriction of user entry if and only if it subsidizes users, i.e. sets \( P^U < 0 \). This stresses the importance of the choice of pricing structure for overall efficiency: by being able to balance the interests of the two sides, a proprietary platform may come closer to the socially optimal level of adoption than a platform simply pricing at marginal cost on both sides.

Thus, given that a proprietary platform induces a bias towards insufficient entry on both sides of the market, the combination of the two leads unambiguously to insufficient product diversity and user adoption relative to the socially optimal level. This of course is not a robust conclusion: it is due to our assumption of horizontal differentiation on both sides. Below we show that introducing vertical developer differentiation is sufficient for overturning this result.

However, even in this simple horizontal differentiation framework, it is not possible to say in general which of the open or proprietary platforms induces a higher level of product variety and is more efficient in terms of total social welfare. To see this more clearly, we can combine the first-order conditions above in order to obtain:

\(^{51}\)Once again, this is because users are differentiated only horizontally.
• the level of product variety $n_p$ induced by a proprietary platform solves:

$$V'(n) F \left( \frac{\varepsilon_F V(n)}{1+\varepsilon_F} \right) = n H^{-1}(n) + H^{-1}(n)$$  \hspace{1cm} (37)

• the level of product variety $n_{fe}$ induced by an open platform solves:

$$\pi(n) F(u(n)) = H^{-1}(n)$$  \hspace{1cm} (38)

• the level of product variety $n_{so}$ chosen by the social planner solves:

$$V'(n) F(V(n)) = H^{-1}(n)$$  \hspace{1cm} (39)

Under sufficient regularity conditions, $n_p$, $n_{fe}$, $n_{so}$ are well-defined, i.e. (37), (38) and (39) each have a unique positive solution. Then, since, $\frac{\varepsilon_F}{1+\varepsilon_F} < 1$ and $H^{-1}(n) > 0$, we have $n_p < n_{so}$. However, comparing (37) and (38), it is not possible to say in general whether $n_p \gtrless n_{fe}$. Figure (4) illustrates (37), (38) and (39). In graph a) the positive indirect network effects are outweighed by the negative direct network effects on the developer side so that the left-hand sides of (37), (38) and (39) are decreasing in $n$, whereas graph b) depicts the case in which the positive indirect effects are stronger$^{52}$.

The following proposition derives these results rigorously:

**Proposition 4**  Assume $V$, $F$ and $H$ are defined on $[0, +\infty]$ with constant elasticities, $F(\theta) = B\theta^{\varepsilon_F}$, $H^{-1}(n) = Cn^{\frac{1}{\varepsilon_F}}$, $V(n) = An^{\varepsilon_V}$, $\pi(n) = \lambda V'(n)$, $u(n) = (1 - \lambda \varepsilon_V) V(n)$ and competition among developers is either as in example 1, with $\varepsilon_V = \frac{\alpha(1-\sigma)}{\sigma(1-\alpha)}$ and $\lambda = \frac{\sigma(1-\alpha)}{1-\sigma \alpha}$, or as in example 2, with $\varepsilon_V = \beta$ and $\lambda = 1$. Also, assume the concavity and stability conditions (8) and (9) hold. Then:

i) $n_p$, $n_{fe}$, $n_{so}$ are well-defined, i.e. (37), (38) and (39) each have a unique positive solution; the profit-maximizing $\left(n_p, \theta^m_p = \frac{\varepsilon_F V(n_p)}{1+\varepsilon_F} \right)$ and the open platform (or free-entry) $\left(n_{fe}, \theta^m_{fe} = u\left(n_{fe}\right) \right)$ market configurations are stable, whereas the configuration $\left(n_{so}, \theta^m_{so} = V\left(n_{so}\right) \right)$ is a global maximizer for $W$.

$^{52}$Using the terminology of Nocke, Peitz and Stahl (2004), in graph a) network effects are "weak", while in graph b) they are "strong".
Figure 4:
ii) Both the proprietary and the open platforms induce socially insufficient product variety and user adoption: $n_p, n_{fe} < n_{so}$ and $\theta^m_p, \theta^m_{fe} < \theta^m_{so}$

iii) Suppose in addition that $\varepsilon_F = 1$, all developers have the same fixed cost $\phi$ (i.e. $\varepsilon_H = +\infty$, $C = \phi$) and compete as in example 1. Then:

- $n_p > n_{fe}$ if and only if $(1 - \sigma \alpha)^2 > 2\sigma (1 - \alpha)^2$.

- Total social welfare can be higher with either type of platform: $\frac{W(n_p, \theta^m_p)}{W(n_{fe}, \theta^m_{fe})} \to +\infty$ when $\alpha \to 0$, $\sigma \to 0$ and $\frac{\alpha}{\sigma} \to k < 1$; $\frac{W(n_p, \theta^m_p)}{W(n_{fe}, \theta^m_{fe})} \to \frac{3}{4}$ when $\sigma \to 1$.

Proof  See appendix.

The most substantial part of proposition 4 is part iii): it exhibits specific cases in which a proprietary platform dominates an open platform both in terms of product variety and total social welfare.

4.2 Developer vertical differentiation and socially exclusive product variety

Despite its tractability, one shortcoming of the two-sided horizontal differentiation model we have used up to now is that it cannot generate cases in which proprietary platforms induce socially excessive levels of product variety\(^{53}\), as was made clear in the discussion above. This is because when the two sides of the market are differentiated only horizontally, a two-sided platform fully internalizes the indirect network effects between users and developers. The only distortions which arise are the deadweight losses due to monopoly pricing on both sides of the market and they lead to insufficient entry of both users and developers.

Intuitively however, it should be clear that this feature cannot be robust to more general formulations of user and developer demand. Even though a two-sided platform extracts only a part of total user and developer surplus,\(^{53}\)By contrast, it is easy to construct cases in which the level of product variety generated by an open platform is socially excessive: it suffices to assume inelastic user demand and use our example 1 with the functional forms provided in section 4 of Mankiw and Whinston (1986): $G(z) = \frac{1}{z}$, $v(q) = aq - \left(\frac{b}{2}\right)q^2$. 

53
there is no reason why the marginal contribution of an additional developer to platform profits should always be lower than the marginal contribution of that developer to total social surplus, so that the platform necessarily restricts entry too much relative to the social optimum. In particular, if developers are sufficiently vertically differentiated by the benefits they offer users (as opposed to being simply heterogeneous in their fixed costs) and if the platform is unable to perfectly price discriminate, then it might overestimate the value of the positive indirect network effects with respect to the value of negative direct network effects and therefore induce socially excessive entry.

To formalize this insight, let us consider the model with heterogeneous application qualities briefly introduced in section 3.2. For simplicity we assume here that quality (i.e. the probability that a given user is interested in a particular application) is cast in stone, rather than being endogenously determined by developers’ investments. Specifically, we assume that all developers have the same fixed cost $\phi > 0$ and that they are exogenously differentiated by the quality $q$ of their applications, where $q$ is distributed over a finite support $[q_L, q_H]$, such that the number of developers with quality lower than $q$ is $H(q)$, where $H$ is an increasing function with continuous derivative $h$ and satisfying $H(q_L) = 0$ and $H(q_H) < +\infty$.

Although developers are vertically differentiated, we assume for simplicity that the platform is restricted to charging only fixed access fees on both sides\(^\text{54}\). Given platform prices $(P^U, P^D)$ and user demand $F(\theta^m)$, only high-quality developers enter, i.e. those with $q \geq q_m$, where $q_m$ is the quality of the marginal developer defined by:

$$V'(\int_{q_m}^{q_H} q h(q) \, dq) q_m F(\theta^m) - P^D - \phi = 0$$

The marginal user $\theta^m$ is then given by:

$$V(Q(q_m)) - Q(q_m) V'(Q(q_m)) - \theta^m - P^U = 0$$

\(^\text{54}\)Our insights remain valid when the platform is also allowed to charge variable fees. The only necessary condition is that the platform should be unable to fully extract all developer revenues through its prices. This may be justified for example by the need to provide sufficient innovation incentives to developers, as we have shown in section 3.2.
where:

\[ Q(q^m) = \int_{q_m}^{q_H} q h(q) dq \]

is the number of applications bought by each platform user. Platform profits are:

\[ \Pi^P = P^U F(\theta^m) + P^D (H(q_H) - H(q^m)) \]

\[ = [V(Q(q^m)) - E(q^m) - \theta^m] F(\theta^m) - (H(q_H) - H(q^m)) \phi \]

where:

\[ E(q^m) = V'(Q(q^m)) (Q(q^m) - (H(q_H) - H(q^m)) q^m) > 0 \]

is the difference between total developer revenues and the portion thereof which is extracted by the platform. In other words, it is the part of developer gross surplus uninternalized by the platform. Note that when all developers have the same quality, \( E(q^m) = 0 \), which brings us back to the horizontal differentiation case, in which the two-sided platform fully internalizes both developer revenues and user surplus.

Assuming the appropriate second order and stability conditions hold, the profit-maximizing marginal product quality \( q^m_p \) and user \( \theta^m_p \) are the solutions to the first-order conditions:

\[ (V(Q) - E(q^m) - \theta^m) f(\theta^m) = F(\theta^m) \]  
\[ \left( \frac{dQ}{dq^m} V'(Q) - E'(q^m) \right) F(\theta^m) + h(q^m) \phi = 0 \]

Social welfare on the other hand has the following expression:

\[ W = V(Q) F(\theta^m) - \int_0^{\theta^m} \theta f(\theta) d\theta - (H(q_H) - H(q^m)) \phi \]

so that the socially optimal marginal product quality \( q^m_{so} \) and marginal user \( \theta^m_{so} \) are the solutions to:

\[ V(Q) - \theta^m = 0 \]
\[ \frac{dQ}{dq^m} V'(Q) F(\theta^m) + h(q^m) f = 0 \]
Comparing (42) and (43) to (40) and (41), it is no longer obvious whether the two-sided proprietary platform will induce too little \( q_m^m > q_{sa}^m \) or too much \( q_p^m < q_{so}^m \) variety (and user adoption). Indeed, while the monopoly pricing distortion on the user side still tends to render user adoption sub-optimal\(^{55} \), on the developer side it all depends on the sign of \( E'(q^m) \). Specifically, if \( E'(q^m) > 0 \), then the left hand side of (41) is lower than the left-hand side of (43) and consequently, since both expressions are decreasing in \( q^m \) (required by our assumption that the maximization problems are well-defined), it might turn out that \( q_p^m < q_{so}^m \). This case there is an excessive variety bias on the developer side, which may exceed the insufficient user adoption bias. The following proposition provides and example in which this happens:

**Proposition 5** Assume there is a mass \( B \) of identical users (i.e. user demand for the platform is inelastic), \( H(q) = C(q - q_L) \), \( V(Q) = V - \frac{A}{q^m} \) and:

\[
\beta q_L > 2q_H
\]

Then \( \Pi(q^m) \) and \( W(q^m) \) are concave and the proprietary platform induces socially excessive product diversity, i.e. \( q_p^m < q_{so}^m \) or \( H(q_H) - H(q_H^m) > H(q_{sa}^m) - H(q_{so}^m) \).

**Proof** In the appendix.

\[ \blacksquare \]

### 4.3 Monopoly platform vs. competing platforms

Let us now conduct the same welfare comparisons between a situation with two competing platforms differentiated a la Hotelling on the user side of the market and a situation with a single platform situated at one extremity of the Hotelling segment.

We have shown in section 3.3 that two competing platforms split the market for users equally and each induces a level of product diversity equal to \( n_c \) (recall that the \( n_c \) developers multihome), where \( n_c \) is given by:

\[
V'(n_c) \frac{1}{2} = n_cH^{-1}(n_c) + H^{-1}(n_c)
\]

\[ (44) \]

\(^{55}\)To see this, note that given the same \( q^m \), (40) yields a lower \( \theta^m \) than (42).
Let us assume for simplicity of exposition that the standalone user valuation for the platform \( u_0 \) is high enough so that a monopoly platform situated at one extremity of the Hotelling segment covers the entire market for users. It does so by charging \( P^U = u_0 + u(n) - t \). Developer demand \( n \) is given by:

\[
\pi(n) - P^D - H^{-1}(n) = 0
\]

Therefore platform profits are:

\[
\Pi^P = P^U + P^D n = V(n) + u_0 - t - nH^{-1}(n)
\]

so that the level of product diversity \( n_p \) chosen by the monopoly platform is defined by:

\[
V'(n_p) = n_pH^{-1'}(n_p) + H^{-1}(n_p)
\] (45)

Comparing (45) and (44), it is clear that the monopoly platform induces more product diversity than the competing platforms:

\( n_p > n_c \)

However, since we are dealing with two-sided platforms, comparing the respective levels of product diversity arising under a monopoly platform and competing platforms is not sufficient for our welfare analysis.

The welfare tradeoff is the following. With a monopoly platform situated at one extremity (say 0) of the Hotelling segment, there is less platform diversity so that users situated further than \( x = \frac{1}{2} \) incur additional transportation costs (\( \frac{t}{2} \) overall). On the other hand however, the monopoly platform offers more product diversity to its users than any of the two competing platforms because it is able to internalize a larger share of user benefits. Additionaly, there is no duplication of fixed costs for the developers who enter, since they only support one platform rather than two.

Formally, the social welfare gain from having one platform rather than two is:

\[
V(n_p) - V(n_c) - \frac{t}{4} + 2 \int^{H^{-1}(n_c)}_0 \phi h(\phi) d\phi - \int^{H^{-1}(n_p)}_0 \phi h(\phi) d\phi
\]

The following proposition shows that this expression can be either positive or negative depending on parameter values, thus confirming that it
may be socially optimal to have a single monopoly platform rather than two competing platforms.

**Proposition 6**  Assume $u_0$ is high enough so that both the competing platforms and a single monopoly platform cover the user market entirely in equilibrium, that all developers have the same fixed development cost $\phi$ (i.e. developer demand is inelastic, $\epsilon_H = +\infty$) and that developers for the same platform compete as in example 2 (i.e. $V(n) = A n^\beta$, $\pi(n) = \beta A n^{\beta-1}$, $u(n) = (1-\beta) A n^\beta$). Then there exists a non-empty interval $[t_L, t_H]$ so that total social welfare is higher with a single monopoly platform than with two competing platforms if and only if $t \in [t_L, t_H]$.

**Proof**  (45) and (44) become:

$$n_c = \left( \frac{\beta A}{2\phi} \right)^{\frac{1}{1-\beta}}$$

$$n_p = \left( \frac{\beta A}{\phi} \right)^{\frac{1}{1-\beta}}$$

The expression of the social welfare gain from having a single platform rather than two becomes:

$$A (1-\beta) n_p^\beta - A (1-\beta) n_c^\beta - \frac{t}{4} = A (1-\beta) \left( \frac{\beta A}{2\phi} \right)^{\frac{\beta}{1-\beta}} \left( 2^{\frac{\beta}{1-\beta}} - 1 \right) - \frac{t}{4}$$

and it is positive if and only if $t \leq 4A (1-\beta) \left( \frac{\beta A}{2\phi} \right)^{\frac{\beta}{1-\beta}} \left( 2^{\frac{\beta}{1-\beta}} - 1 \right) = t_H$.

On the other hand we need to make sure that in the symmetric equilibrium with two competing platforms they make non-negative profits and all users do indeed singlehome, which will yield a lower bound on $t$. We have:

$$P_c^D = \pi(n_c) \frac{1}{2} - \phi = \frac{1}{2} \beta A n_c^{\beta-1} - \phi = 0$$

$$P_c^U = t - n_c \pi(n_c) + \frac{u'(n_c) \pi(n_c)}{\pi'(n_c)} = t - 2\beta A n_c^\beta$$

Hence, the two platforms make non-negative profits and all users single-home if and only if $P_c^U \geq 0$, i.e.if and only if:

$$t \geq 2\beta A n_c^\beta = 2\beta A \left( \frac{\beta A}{2\phi} \right)^{\frac{\beta}{1-\beta}} = t_L$$
Finally, we need to verify that $t_L \leq t_H$, which is equivalent to:

$$\frac{\beta}{1 - \beta} \leq 2 \left(2^{\frac{\beta}{1 - \beta}} - 1 \right)$$

and this inequality holds for all $\beta \geq 0$.

This result shows that there is a sense in which platform competition may be undesirable because it prevents the competing platforms from sufficiently internalizing positive indirect network effects, so that they do not have enough incentives to provide product variety. Therefore, a monopoly platform has the potential of being more efficient, despite creating more deadweight loss. Although here we have focused on the simplest case, in which both the competing platforms and a monopoly platform cover the user market entirely, it should be clear that our insight is valid in more general settings, with partial coverage of the user market.

5 Conclusion

This paper has presented a model of two-sided platforms which -we hope- contributes to throwing some light on the economic mechanisms at work in an increasing number of industries central to the new economy, such as the Internet, software for computers and other electronic devices, videogames, digital media and others.

From a positive perspective we have identified the intensity of user preferences for variety as a key factor driving platform pricing structures. We have shown that when users care more about diversity the optimal pricing structure shifts towards making more profits on developers. We have also rationalized the use of royalties as stemming from the conflict of two objectives: providing appropriate innovation incentives to developers and reducing a hold-up problem by the platform, which arises when developers make their platform adoption decisions before users and the platform cannot credibly commit to user prices \textit{ex-ante}. Combining these two predictions, we were able to propose simple and plausible explanations for the contrasting choices of platform pricing structures observed, in particular videogame consoles relative to software platforms in other computer-based industries.
From a normative perspective, we have shown that the levels of product variety and user adoption can be higher under a monopoly two-sided proprietary (profit-maximizing) platform than under an open platform, competing platforms and even than the socially optimal levels. In particular, a monopoly proprietary platform can be more efficient than both an open platform and competing platforms because it is better able to internalize indirect network effects.

Clearly, this paper constitutes only an initial formal exploration of the economic issues raised by the category of two-sided market platforms on which we have focused and which we believe should be the topic of promising future research. On the theoretical side, our model can be extended to tackle the important issues of exclusive contracts and the efficiency of alternative forms of platform governance. On the empirical side, our model can provide the starting point for a more rigorous cross-industry analysis of pricing structures and other important business decisions such as the extent of vertical (dis)integration defining "platform scope".

References


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56 One interesting example is for-profit joint ventures between third-party developers, a form of platform governance similar in many respects to the patent pools studied by Lerner and Tirole (2004). Nocke, Peitz and Stahl (2004) contains an initial exploration of this case.


6 Appendix

Proof of Lemma 0  With the functional forms assumed, (5) and (6) are equivalent to:

$$\theta^m_p = \frac{\varepsilon_F}{1 + \varepsilon_F} A n^\varepsilon_V$$

$$\varepsilon_V A^{1 + \varepsilon_F} B \left( \frac{\varepsilon_F}{1 + \varepsilon_F} \right)^{\varepsilon_F} n^\varepsilon_V (1 + \varepsilon_F) = C \left( 1 + \frac{1}{\varepsilon_H} \right) n^\frac{1}{\varepsilon_H}$$

and they admit a unique solution interior to $[0, +\infty]^2$ if $\varepsilon_V (1 + \varepsilon_F) - 1 < \frac{1}{\varepsilon_H}$. This solution is a global maximum for $\Pi^P (\theta^m, n)$ if and only if the second order condition holds, i.e. if and only if the Hessian matrix of $\Pi^P (\theta^m, n)$ evaluated at $(\theta^m, n_p)$ is semi-definite negative.

We have:

$$\frac{\partial^2 \Pi^P}{(\partial \theta^m)^2} (\theta^m, n_p) = \left( V' (n_p) - \theta^m_p \right) f' (\theta^m_p) - 2 f (\theta^m_p)$$

$$= f (\theta^m_p) \left( V' (n_p) \frac{\theta^m_p}{\theta^m_p} f' (\theta^m_p) - 2 \right)$$

$$= -f (\theta^m_p) \left( 1 + \frac{1}{\varepsilon_F} \right) < 0$$

$$\frac{\partial^2 \Pi^P}{\partial n^2} (\theta^m, n_p) = V'' (n_p) F' (\theta^m_p) - (nH^{-1} (n))'' (n_p) < 0$$

because $V$ is concave and $(nH^{-1} (n))'' (n_p) = C \left( 1 + \frac{1}{\varepsilon_H} \right) \frac{1}{\varepsilon_H} n^{\frac{1}{\varepsilon_H} - 1} > 0$.

$$\frac{\partial^2 \Pi^P}{\partial \theta^m \partial n} (\theta^m, n_p) = V' (n_p) f (\theta^m_p) > 0$$
It therefore remains to check that \(
abla^2 \Pi (\theta^m_p, n_p) < \nabla^2 \Pi (\theta^m_p, n_p) \times \nabla^2 \Pi (\theta^m_p, n_p),\)
which, using the expressions above and omitting arguments of some functions in order to avoid clutter, is equivalent to:

\[
V'^2 f < \left(1 + \frac{1}{\varepsilon_F} \right) \left(-V''F + C \left(1 + \frac{1}{\varepsilon_H} \right) \frac{1}{n_p} n_p^{\frac{1}{n_p} - 1} \right)
\]

But (6) implies:

\[
C \left(1 + \frac{1}{\varepsilon_H} \right) \frac{1}{n_p} n_p^{\frac{1}{n_p} - 1} = \frac{1}{\varepsilon_H} V'(n_p) F (\theta^m_p)
\]

so that the inequality above is equivalent to:

\[
V'^2 < \left(1 + \frac{1}{\varepsilon_F} \right) \left(-\frac{V'' \theta^m_p}{\varepsilon_F} + \frac{1}{\varepsilon_H} \frac{V' \theta^m_p}{n_p} \frac{1}{\varepsilon_F} \right)
\]

or, using (5):

\[
\varepsilon_F < \frac{-V''}{V'^2} + \frac{1}{\varepsilon_H} \frac{V}{n_p} \frac{V'}{V'^2}
\]

Noting that \(\frac{V''}{V'^2} = \frac{\varepsilon_V - 1}{\varepsilon_V},\) this is finally equivalent to:

\[
\varepsilon_V (1 + \varepsilon_F) < 1 + \frac{1}{\varepsilon_H}
\]

ii) Using the first order conditions (5) and (6), (7) is equivalent to\(^{57}\):

\[
(1 + \varepsilon_F) \frac{n_p u' \pi}{VV'} + \frac{\pi' n_p}{V'} \frac{1}{1 + \varepsilon_H}
\]

Since \(\pi (n) = \lambda V' (n)\) and \(u (n) = V (n) - \lambda n V' (n) = (1 - \lambda \varepsilon_V) V (n),\)
the inequality above is equivalent to:

\[
(1 + \varepsilon_F) \frac{n_p (1 - \lambda \varepsilon_V) \lambda V'}{V} + \frac{\lambda V' n_p}{V'} \frac{1}{1 + \varepsilon_H}
\]

or:

\[
(1 + \varepsilon_F) \lambda \varepsilon_V (1 - \lambda \varepsilon_V) - \lambda (1 - \varepsilon_V) < \frac{1}{1 + \varepsilon_H}
\]

which, after factoring \(\lambda (1 - \varepsilon_V),\) is exactly the condition (9) given in the text.

\(^{57}\) We omit functional arguments and use the fact that \(\frac{n_H - V'(n)}{H - (n)} = \frac{1}{\varepsilon_H (n)} \).
Proof of Proposition 2  The only part of the proposition which was not proven in the text is the specific example. The equations defining \( \theta^m (n, \rho) \) and \( q (n, \rho) \) (20) and (21) become:

\[
\theta^m = \frac{A (1 - \beta (1 - \rho)) (nq)^\beta}{2} \\
A\beta (1 - \rho) (nq)^\beta - 1 B \theta^m = cq
\]

Letting \( \lambda = \beta (1 - \rho) \) and solving these two equations simultaneously, we get:

\[
q (n, \lambda) = \left( \frac{AB}{2c} \right)^{\frac{1}{2-2\beta}} n^{\frac{2\beta-1}{2-2\beta}} \left[ \lambda (1 - \lambda) \right]^{\frac{1}{2-2\beta}}
\]

(23) then becomes:

\[
A (nq (n, \lambda))^\beta \left( 1 - \frac{1 - \lambda}{2} \right) \frac{cq (n, \lambda)}{A\lambda (nq)^{\beta-1}} - n \frac{cq (n, \lambda)^2}{2} - nH^{-1} (n)
\]

\[
= \frac{n cq (n, \lambda)^2}{2\lambda} - nH^{-1} (n)
\]

\[
= \frac{c}{2} \left( \frac{AB}{2c} \right)^{\frac{2}{2-2\beta}} n^{\frac{2\beta}{2-2\beta}} \lambda^{\frac{2\beta}{2-2\beta}} (1 - \lambda)^{\frac{2}{2-2\beta}} - nH^{-1} (n)
\]

which has to be maximized over \( \lambda \) and \( n \). Maximizing over \( \lambda \) is equivalent to maximizing \( \lambda^\beta (1 - \lambda) \), which yields the optimal solution \( \lambda = \frac{\beta}{1+\beta} \), or \( \rho = \frac{1}{1+\beta} \).

Proof of Proposition 4  

i) (37), (38) and (39) are respectively equivalent to:

\[
BA^{1+\varepsilon_F} \varepsilon_V \left( \frac{\varepsilon_F}{1 + \varepsilon_F} \right)^{\varepsilon_F} n^{\varepsilon_V (1 + \varepsilon_F) - 1} = C \left( 1 + \frac{1}{\varepsilon_H} \right) n^{\frac{1}{\varepsilon_H}} \quad (46)
\]

\[
BA^{1+\varepsilon_F} \lambda \varepsilon_V (1 - \lambda \varepsilon_V)^{\varepsilon_F} n^{\varepsilon_V (1 + \varepsilon_F) - 1} = C n^{\frac{1}{\varepsilon_H}} \quad (47)
\]

\[
BA^{1+\varepsilon_F} \varepsilon_V n^{\varepsilon_V (1 + \varepsilon_F) - 1} = C n^{\frac{1}{\varepsilon_H}} \quad (48)
\]

Since \( \varepsilon_V (1 + \varepsilon_F) - 1 < \frac{1}{\varepsilon_H} \), each of these three equations admits a unique positive solution. Note that we are in the case depicted graph a) of figure 4 if \( \varepsilon_V (1 + \varepsilon_F) < 1 \) and in the case depicted in graph b) if \( \varepsilon_V (1 + \varepsilon_F) > 1 \).
If (8) and (9) hold, we know from Lemma 0 that the profit-maximizing market configuration \( \left( n_p, \theta^m_p = \frac{\pi V(n_p)}{1+\pi} \right) \) is stable given the corresponding prices \( (P^U_p, P^D_p) \). The market configuration under an open platform \( \left( n_{fe}, \theta^m_{fe} = u(n_{fe}) \right) \) is the intersection of the following two curves defined in the \((n, \theta^m)\) plane:

\[
\begin{align*}
  &u(n) - \theta^m = 0 \\
  &\pi(n) F(\theta^m) - H^{-1}(n) = 0
\end{align*}
\]

Stability means that, at the intersection point, the slope of the first curve defining \( \theta^m \) as a function of \( n \) is lower than the slope of the second curve defining \( n \) as a function of \( \theta^m \), which is equivalent to:

\[
  u'(n_{fe}) \pi(n_{fe}) f(\theta^m_{fe}) + \pi'(n_{fe}) F(\theta^m_{fe}) \leq H^{-1}(n_{fe})
\]

Using the definition of \( (n_{fe}, \theta^m_{fe}) \) and the fact that \( H \) has constant elasticity, this condition can be rewritten as:

\[
  \frac{u'(n_{fe}) n_{fe} f(\theta^m_{fe})}{u(n_{fe})} \frac{\theta^m_{fe}}{F(u(n_{fe}))} + \frac{\pi'(n_{fe}) n_{fe}}{\pi(n_{fe})} \leq \frac{1}{\varepsilon_H}
\]

or, since \( u(n) = (1 - \lambda \varepsilon_V) V(n) \), \( \pi(n) = \lambda V'(n) \):

\[
\varepsilon_V \varepsilon_F + \varepsilon_V - 1 \leq \frac{1}{\varepsilon_H}
\]

which is equivalent to (8).

Finally, given that \( F \) and \( H \) have infinite supports, social welfare is maximized by \( (n_{so}, \theta^m_{so} = V(n_{so})) \) if and only if the Hessian matrix of \( W(n, \theta^m) \) evaluated at this point is semi-definite negative. We have:

\[
\frac{\partial^2 W}{\partial \theta^m \partial n} (n_{so}, \theta^m_{so}) = (V(n_{so}) - \theta^m_{so}) f(\theta^m_{so}) - f(\theta^m_{so}) < 0
\]

\[
\frac{\partial^2 W}{\partial n^2} (n_{so}, \theta^m_{so}) = V''(n_{so}) F(\theta^m_{so}) - H^{-1}(n_{so}) < 0
\]

\[
\frac{\partial^2 W}{\partial \theta^m \partial n} (n_{so}, \theta^m_{so}) = V'(n_{so}) f(\theta^m_{so})
\]

and \( \left( \frac{\partial W}{\partial n} (n_{so}, \theta^m_{so}) \right)^2 < \frac{\partial^2 W}{\partial \theta^m \partial n} (n_{so}, \theta^m_{so}) \frac{\partial^2 W}{\partial \theta^m^2} (n_{so}, \theta^m_{so}) \) is equivalent to:

\[
V'(n_{so})^2 f(\theta^m_{so}) < -V''(n_{so}) F(\theta^m_{so}) + H^{-1}(n_{so})
\]
or, using $V'(n_{so}) F(\theta_{so}^m) = H^{-1}(n_{so})$:

$$\frac{-V'(n_{so})^2}{V''(n_{so}) V(n_{so})} \frac{\theta_{so}^m}{F(\theta_{so}^m)} < 1 - \frac{V'(n_{so})}{V''(n_{so}) n_{so} \varepsilon_H}$$

or:

$$\frac{\varepsilon_V \varepsilon_F}{1 - \varepsilon_V} < 1 + \frac{1}{(1 - \varepsilon_V) \varepsilon_H}$$

which is in turn equivalent to (8).

ii) Since $\lambda, \varepsilon_V < 1$, (46), (47) and (48) clearly imply that $n_{fe}, n_p < n_{so}$.

Then:

$$\theta_{fe}^m = (1 - \lambda \varepsilon_V) V(n_{fe}) < V(n_{fe}) < V(n_{so}) = \theta_{so}^m$$

$$\theta_{p}^m = \frac{\varepsilon_F}{1 + \varepsilon_F} V(n_{p}) < V(n_{p}) < V(n_{so}) = \theta_{so}^m$$

iii) Let $\beta = \frac{\sigma - \alpha}{\sigma (1 - \alpha)} = 1 - \varepsilon_V$ and recall that $\lambda = \frac{\sigma (1 - \alpha)}{1 - \sigma \alpha} \in [0, 1]$. Then, since $\varepsilon_H = +\infty$, (8) is implied by (9), which is equivalent to:

$$2 (1 - \beta) < \frac{\beta}{1 - \lambda (1 - \beta)}$$

Together with $\lambda < 1$, this implies that $1 > \beta > \frac{1}{2}$. (46), (47) and (48) become:

$$\frac{(1 - \beta)}{2} n_{2sp}^{1-2\beta} = \frac{C}{BA^2}$$

$$\frac{\alpha (1 - \sigma)(1 - \alpha)}{(1 - \sigma \alpha)^2} n_{fe}^{1-2\beta} = \frac{C}{BA^2}$$

$$(1 - \beta) n_{so}^{1-2\beta} = \frac{C}{BA^2}$$

Then, since $1 - 2\beta < 0$ and $\frac{(1-\beta)(1-\sigma\alpha)^2}{\alpha (1 - \sigma)(1 - \alpha)} = (\frac{1-\sigma\alpha}{1-\alpha})^2 \frac{1}{\sigma} > 1$, we have $n_p, n_{fe} < n_{so}$. Moreover $n_p > n_{fe}$ if and only if $\frac{(1-\beta)(1-\sigma\alpha)^2}{\alpha (1 - \sigma)(1 - \alpha)} > \alpha (1 - \sigma)(1 - \alpha)$, which is equivalent to $(1 - \sigma \alpha)^2 > 2\sigma (1 - \alpha)^2$. It remains to be verified that this inequality may hold or not, while still satisfying (49). If $\alpha \to 0$ then $\beta \to 1$ and $\lambda \to \sigma$, so that (49) is satisfied, and in the limit $n_p > n_{fe}$ if and only if $\sigma < \frac{1}{2}$, so that both cases are possible.

Total social welfare has the following expression:

$$W(n, \theta^m) = BV(n) \theta^m - \frac{B(\theta^m)^2}{2} - nC$$

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Using (50), (51) and \( \theta_{fe}^m = (1 - \lambda (1 - \beta)) V (n_{fe}) \), \( \theta_p^m = \frac{V(n_{2p})}{2} \) and \( V(n) = An^{1-\beta} \) we obtain:

\[
W (n_p, \theta_p^m) = BA^2 \left( \frac{1}{2} - \frac{1}{8} \right) n_p^{2-2\beta} - \frac{BA^2 (1 - \beta)}{2} n_p^{2-2\beta} = \frac{BA^2}{2} \left( \beta - \frac{1}{4} \right) \left( \frac{BA^2 (1 - \beta)}{2C} \right)^{2-2\beta}
\]

and:

\[
W (n_{fe}, \theta_{fe}^m) = BA^2 \left[ \left( 1 - \lambda (1 - \beta) - \frac{(1 - \lambda (1 - \beta))^2}{2} \right) - \frac{\alpha (1 - \sigma) (1 - \lambda)}{(1 - \sigma \alpha)^2} \right] n_{fe}^{2-2\beta} = \frac{BA^2 (1 - \alpha)^2}{2 (1 - \sigma \alpha)^2} \left( \frac{BA^2 \alpha (1 - \sigma) (1 - \alpha)}{C (1 - \sigma \alpha)^2} \right)^{2-2\beta}
\]

Finally:

\[
\frac{W (n_p, \theta_p^m)}{W (n_{fe}, \theta_{fe}^m)} = \left( \beta - \frac{1}{4} \right) \left( \frac{\sigma}{\lambda} \right)^{2-2\beta} \frac{1}{(2\sigma)^{2-2\beta}}
\]

Let \( \sigma = x \), \( \alpha = kx \) with \( 0 < k < \frac{1}{3} \) and \( x \to 0 \). Then \( \lambda \to 0 \) and \( \beta \to 1 - k \) so that (49) is satisfied in the limit and at the same time \( \frac{\sigma}{\lambda} \to 1 \) and therefore \( \frac{W_{2p}}{W_{fe}} \to +\infty \).

Now let \( \sigma \to 1 \) keeping \( \alpha \) fixed: \( \lambda, \beta \to 1 \) so that (49) is satisfied and \( \frac{W_{2p}}{W_{fe}} \to \frac{3}{4} \).

**Proof of Proposition 5**  The expression of platform profits is:

\[
\Pi^P (q^m) = B (V (Q (q^m)) - E (q^m)) - C (q_H - q^m) \phi
\]

and that of social welfare:

\[
W (q^m) = BV (q^m) - C (q_H - q^m) \phi
\]

where \( Q = \frac{C}{\beta} (q_H^2 - (q^m)^2) \) and \( E (q^m) = \frac{CV'(Q)}{2} (q_H - q^m)^2 \).

\( q^m_p \) solves:

\[
-BV' (Q (q^m)) C q^m - BE' (q^m) + C \phi = 0
\]
whereas \( q_{so}^{m} \) is the solution to:

\[-BV'(Q(q^m))Cq^m + C\phi = 0\]

We assume that the parameters \( A, B \) and \( C \) are such that both solutions are interior, so that in order to prove \( q_p^m < q_{so}^m \) it is sufficient to prove that \( E'(q^m) > 0 \) for all \( q^m \in [q_L, q_H] \) and the derivatives of the two expressions above are both negative. We have:

\[
E'(q^m) = -\frac{C^2V''(Q(q^m))q^m}{2}(q_H - q^m)^2 - CV'(Q(q^m))(q_H - q^m)
\]

\[
= -\frac{C^2}{2}V''(Q)(q_H - q^m)^2 \left[ q^m + \frac{V'(Q)}{QV''(Q)}(q_H + q^m) \right]
\]

\[
= -C^2V''(Q)(q_H - q^m)^2 \left( q^m - \frac{q_H + q^m}{\beta + 1} \right)
\]

so that \( E'(q^m) > 0 \) is equivalent to:

\[ \beta q^m > q_H \]

which is true since \( \beta q_L > 2q_H \).

The second derivative of total social welfare with respect to \( q^m \) is:

\[
BV''(Q(q^m))(Cq^m)^2 - BV'(Q(q^m))C
\]

and is clearly negative. Therefore, we are done if we show that \( E''(q^m) > 0 \). We have:

\[
E''(q^m) = -\frac{C^2V''(Q)}{\beta + 1} \left[ \beta(q_H - q^m)^2 - 2(q_H - q^m)(\beta q^m - q_H) \right]
\]

\[
+ \frac{C^3V'''(Q)}{\beta + 1}q^m(q_H - q^m)^2(\beta q^m - q_H)
\]

Using \( \frac{-V'''(Q)}{V''(Q)} = \beta + 2 \), the condition \( E''(q^m) > 0 \) is equivalent to:

\[ \beta(q_H - q^m) - 2(\beta q^m - q_H) + \frac{2(\beta + 2)}{q_H + q^m}q^m(\beta q^m - q_H) > 0 \]

or:

\[ ((\beta + 2)q_H - 3\beta q^m)(q_H + q^m) + 2(\beta + 2)q^m(\beta q^m - q_H) > 0 \]

or:

\[ (\beta + 2)q_H^2 + (2\beta + 1)q^m(\beta q^m - 2q_H) > 0 \]

which is true since \( \beta q_L > 2q_H \).