SELLING ONLINE VERSUS LIVE

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ABSTRACT. Any seller choosing between auctioning online and live faces a tradeoff: lower transaction costs online against more rents left with the bidders. We model this tradeoff, and apply the theory to auctions of art. The crucial variable in determining whether the seller does better online is not the expected price but the extent of valuation uncertainty.

Online auction sales topped \$50 billion in 2001. Cut flowers, seafood, classic cars, jewelry, coins, antiques, and art are now auctioned online as well as in traditional live sales. In what follows we model the seller's choice of sale venue, online versus live, and apply the theory to auctions of art, where prices and valuation uncertainties are the biggest of all.

Are there limits to what can be sold online? Industry wisdom says the internet is not suitable for selling highly valuable items, because potential buyers will resist bidding large sums for goods they have not seen. Counterexamples exist, however. The town of Bridgeville, California was auctioned on eBay for \$1.8 million. According to a news report, "Most of the 249 bidders made their offers based solely on digital photographs and descriptions of the town on a Web site."[1] A Ferrari sold on eBay Motors for \$330,000; a 1909 baseball card sold on eBay for \$1.3 million; and an original of the Declaration of Independence sold on Sothebys.com for \$8.1 million.

There are two main differences between online and live auctions. First, less information is available for bidders in online auctions. In a live auction, bidders inspect the item at the preview exhibition and (in the case of art auctions) can ask questions of the auction-house experts present. In an online auction, bidders get only what information is posted on a web page. The fuzziness of pictures on a computer screen means an online bidder's information is limited.

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[1] San Jose Mercury News, December 28 2002, p. 1A.

Second, transaction costs are higher in live auctions. Online, the seller's costs are those of running a web site. In live auctions, the auctioneer pays the costs of mounting the pre-sale exhibition and in running the theatrical performance that makes up a live auction. The seller must wait until a suitable auction is scheduled, bringing costs of delay. Bidders have negligible transaction costs online, whereas to bid live they incur travel and other participation costs.

In the auctions of tuna at Tokyo's Tsukiji market, where a single fish goes for up to $15,000, buyers spend half an hour examining the tuna to assess their suitability for sashimi and sushi. This adds up to hundreds of person-hours of transaction costs in these live auctions. In online seafood auctions in Europe, by contrast, the auctioneer uses a letter-grade system to stipulate the freshness of the fish, economizing on the bidders' time, possibly at the cost of inferior information. When Dutch flower auctions went online, prices fell—arguably because the internet bidders found it hard to assess the flowers' quality.[2]

Any seller choosing between auctioning online and live, then, faces a tradeoff. On the one hand, the costs to the seller and the bidders of participating are lower online. On the other hand, online bidders, worrying more about the winner's curse, bid lower. We will model the tradeoff of lower transaction costs online against more rents left with the bidders.

The internet has made markets more efficient by making it easier for buyers to learn what is available where for how much. The lowering of these search frictions has made pricing more competitive, as various empirical studies have found (Baye, Morgan, and Scholten, 2001, Brown and Goolsbee, 2002, Brynjolfsson and Smith, 2000, Clay, Krishnan, and Wolff, 2001, Ellison and Ellison, 2001). There is one set of frictions that the internet has not reduced, however: those of verifying quality. Such frictions mean, we will argue, that online prices tend to be *less* competitive.

The winning bidder does better in an online auction than in a live one, in our model, despite being more uncertain about the artwork's value (because competitors also bid lower). The gains can be mutual: in certain cases the seller, too, does better online.

What are the limits of online auctions? The common belief among online sellers seems to be that it is the item's expected price that determines whether selling online works better for the seller; high-value

[2]On the Tsukiji fish auctions, see McMillan (2002, p. 65); on the online European fish auctions, www.pefa.com/pefaportal/en/index.htm; on the Dutch flower auctions, Koppius, Van Heck, and Wolters (2002).

items should be sold live. Our theory suggests, to the contrary, that the crucial variable is not the expected price but the extent of valuation uncertainty. Valuable items can be successfully auctioned online if their variance of value is low.

1. Art Auctions

In 2000, Sotheby's revamped its time-honored auction mechanism, when it started selling online via Sothebys.com. Any online seller of heterogeneous goods faces questions of quality. The success of eBay, for example, is in part due to its seller-reputation mechanism, reassuring bidders by tracking previous buyers' experiences (Baron, 2001, Houser and Wooders, 2000, Lucking-Reiley, Bryan, Prasad, and Reeves, 2000, McDonald and Slawson, 2000, Resnick and Zeckhauser, 2001). More than anything else that is sold online, art is prone to vast uncertainties about authenticity and merit. Art therefore tests the scope of online sales.

Sotheby's live auctions have four steps: consignment, cataloguing, exhibition, and sale. After a prospective seller contacts Sotheby's, its experts check the item's authenticity and appraise its value. The owner then consigns the item to Sotheby's, which puts it in an upcoming auction. Sotheby's auctions have a theme, such as "20th Century Works of Art" or "Egyptian, Classical, and Western Asiatic Antiques," so the sale waits until there is an auction that it fits. The auction catalogue, available about one month ahead of the auction, contains a description of the item, its history, reference notes, and an upper- and lower-bound estimated selling price. A few days before the auction, Sotheby's mounts an exhibition of the items to be sold. The exhibition is open the public and Sotheby's specialists are on hand to answer questions.[3] Finally there is the auction itself, run with open ascending bidding. Any major sale is a social event for the glitterati. Space in the auction room is scarce so it is rationed (Watson, 1992, p. 11), desirable seats being assigned to clients who have spent heavily in the past.

Sotheby's online auctions have a checkered history. Sothebys.com began operations in January 2000 as a joint venture between Sotheby's

[3] It is a case of buyer beware. A catalogue states, "Neither we nor the Consignor make any warranties or representations of the correctness of the catalogue or other description of physical condition size, quantity, rarity, importance, medium, provenance, exhibitions, literature or historical relevance of the property and no statement anywhere, whether oral or written, shall be deemed such a warranty or representation. Prospective bidders should inspect the property before bidding to determine its condition, size or whether or not it has been repaired or restored" (Sotheby's, 1988).

and Amazon.com. This alliance was dissolved after nine months. (The end came five days after Sotheby's pleaded guilty to colluding with Christie's to fix fees in their live auctions.) Sotheby's operated the online auctions its own from October 2000 to June 2002. During this time it sold goods worth over $100 million in a period when the art market in general was depressed, but lost money because of the web site's high set-up costs.[4] Then Sotheby's formed a joint venture with eBay. The joint venture replaced eBay Premiere, eBay's own site for high-end auctions, while Sotheby's online business moved to eBay's web site, allowing Sotheby's to cut staff and expenditures in its online division.

The online auctions differ from the live auctions in several respects. First, Sotheby's is the consignor in only a fraction of Sothebys.com auctions. It has formed partnerships with about 5,000 "internet associates," independent dealers who offer goods for sale at Sothebys.com. The associates authenticate their own goods and make their own appraisals. Second, rather than large auction events with specific themes at discrete times, miscellaneous items are continuously on offer via the web site. (At any time the site lists around 13,000 items for sale.) Third, there is no printed catalogue and no exhibition; the descriptions are solely online. Fourth, the auction is run by eBay's rules; in particular, each auction has a fixed end-time (and so there is the possibility of last-second bidding, analyzed in eBay auctions by Bajari and Hortacsu, 2002, and Ockenfels and Roth, 2002). Unlike eBay's regular auctions but like Sotheby's live auctions, however, the online art auctions come with expert appraisals.

Sotheby's main cost of running online auctions is in producing the web-site descriptions. This costs no more, presumably, than printing the catalogue for a live auction. The other live-auction costs are avoided. For bidders, participation is easier online. The live bidders' transaction costs go beyond travel and other pecuniary costs. Sotheby's London and New York salerooms have an elitist ambiance, said to be daunting to newcomers. "This world has been too intimidating and rarified," said Sotheby's president Diana D. Brooks when Sothebys.com was starting up. "We want our site to be a friendly place to the potentially millions of people who've never bought art before."[5]

An online bidder sees pictures of the item for sale, a paragraph of biography of the artist, a statement of the work's condition, and a

[4]Sotheby's incurred costs of $60 million in the first two years of its online operation, mostly in set-up costs (Wall Street Journal, January 31, 2002, p. B4).

[5]Economist, January 27, 2000.

lower and upper estimate. Figures 1 and 2 exemplifiy how much—or how little—information the bidders get. (To see more such examples, go to www.sothebys.com and click on a few of the items for sale.)

The owner of a piece of art has a choice, then between selling it in a live auction and online. Is the internet right for selling art? Sotheby's

move to online auctions was greeted skeptically. Christie's, Sotheby's rival in the global art-market duopoly, has deliberately stayed away from online auctions. An unconvinced art dealer remarked, "What we sell is something that needs to be looked at and discussed." The chief executive of the US traditional art auctioneer Butterfields, Geoff Iddison, said, "Buyers expect to touch the works of art in person."[6] Next we develop a model that both sharpens and modifies the observation that buyers prefer to see the art before bidding.

2. The Model and Equilibrium

The decision makers: The seller is defined by a unit endowment of an asset with zero valuation and risk neutral preferences. A bidder i is defined by a scalar signal x_i and risk-neutral preferences with an expost payoff function $s - p$ where s is the common value, and p is the payment. (Risk aversion would reinforce the argument.) Define $x = (x_1, ..., x_n)$. The seller and the bidders share a common prior on s and x.

We model an online auction as a standard ascending auction–that is, frictionless–and a live auction as having endogeneous entry of bidders and in which the seller and the buyers incur transaction costs.

The decision process: First, nature decides a realization of s. Neither the seller nor the bidders learns the realization of s. Second, the seller decides whether to sell the asset in online market or in offline market. Third, if the seller decides to sell the asset in online market. Then:

- The seller reveals information on the asset on the webpage. The seller reveals information truthfully because of its guarantee[7] and eBay's feedback rating system.
- Bidder i obtains the signal x_i^{on} from the distribution F^{on}.
- Bidders compete in an ascending auction. The seller put the reserve price $r^{on} \in \mathbb{R}$.

[6]The first quote is from a London oriental art dealer, Giuseppe Eskenazi: Economist, January 27, 2000. Iddison quote: San Jose Mercury News, Aug. 2, 2002.

[7]"Each seller guarantees that the authorship, period, culture or origin of the lot is as set out in the Guaranteed sections of the View item page in the description of the lot." and this guarantee is valid for three years for the bidder who purchases the asset. http://pages.sothebys.ebay.com/help/rulesandsafety/guarantee.html

Fourth, if the seller decides to sell the asset in a live auction:

- The seller pays a cost $c_s \in (0, C)$ to hold a live auction.
- The seller provides information on the asset in the auction catalogue. The information revelation is truthful[8].
- Each potential bidder decides sequentially[9] whether to enter the auction. At the time of the choice, each potential bidder reads the catalogue. Each bidder learns the number of bidders who have already entered the auction[10]
- If potential bidder i decides to enter, bidder i pays cost $c_b \in (0, C)$ for some $C \in \mathbb{R}_{++}$. In an actual auction process, the bidders have to arrange the visit to the preview in New York before inspecting the asset.
- Each bidder attends the preview and estimates the signal x_i^{off} from the distribution F^{off}. Bidders learn the signal after the entry decision.
- Bidders compete in an ascending auction with a reserve price r^{off}.

The distributional assumptions: Each of X_i^{off} and X_i^{on} belongs to a mean-dispersion family with a mean zero base random variable Z with a distribution function F^Z and continuously differentiable density f^Z and with mean and dispersion parameters (μ, σ^{off}) and (μ, σ^{on}). (For example, if X is normal, the base random variable is *N(0, 1).*) The assumption implies

$$F^{on}(x) = F^Z(\frac{x-\mu}{\sigma^{on}}), F^{off}(x) = F^Z(\frac{x-\mu}{\sigma^{off}})$$

Intuitively, a member of mean–dispersion family is obtained by a shift in mean μ and dispersion σ of the base distribution. Many distributions, such as Gaussian, Poisson, uniform, lognormal, and extremum value distribution, satisfies this condition.

[8]An offline auction catalogue states, " We guarantee the authenticity of Authorship of each lots contained in this catalogue... 'Authorship', locations the identity of the creator, the period, culture, source of origin of property, as the case may be, as set forth in the Bold Type Heading of such catalogue entry."(Sotheby's (1988))

[9]Levin and Smith (1994) modelled simultaneous entry. There are no essential differences between two formulations since both analysis are driven by bidder's zero profit conditions. Other models include French and McCormick (1984), Hausch and Li (1993), McAfee, Quan, and Vincent (2002), and Ye (2001).

[10]This information which might be available from the conversations with Sotheby's specialists when the potential bidders try to arrange a visit to the preview.

The signal $(X_1^j, ..., X_N^j)$, $j =$*on, off* is independent. Further, we assume the sensitivity of expected value to signal does not vary excessively with the level of the signal: that is, S and X^j, $j =$*on, off* are such that there exists $m > 0, M < \infty$ such that for all x,

$$m \leq \frac{\partial}{\partial x_i} E[s|X = x] \leq M \tag{2.1}$$

(Recall affiliation already implies that $E[s|X = x]$ is nondecreasing in each arguments.) For example, if $m = M$, a \$1 increase in the signal will increase the bidder's expected value by the same amount regardless of the level of the signal. We assume both of X_i^{off}, and X_i^{on}satisfy increasing the hazard rate condition[11].

Note that, if $Y_{i,n}$ is the ith highest realization out of n iid samples, then,

$$E(Y_{i,n}^{off}) = \mu^{off} + \sigma^{off} E(Y_{i,n}^z),\ E(Y_{i,n}^{on}) = \mu^{on} + \sigma^{on} E(Y_{i,n}^z).$$

For example, see David (1981, p. 129).

The crucial parameter in our model, k, represents the extent to which bidders are better informed about the asset's value in a live auction than an online one. The live auction, we assume, provides a more precise signal: $\sigma^{off} = k\sigma^{on}$ for some $0 < k < 1$. Here k measures the lower dispersion in live auctions compared with online; $k = 0$ implies complete elimination of dispersion and $k = 1$ implies zero reduction of dispersion.

The first result identifies the equilibrium of the model.

Lemma 1: *In online markets without reserve prices, each bidder's ex ante payoff is,*

$$\pi_B^{on} = \int H^{on}(x_i, N)(1 - F^{on}(x_i))dx_i$$

where

$$\begin{aligned} H^{on}(x_i) &\equiv \int_{X_{-i}} \frac{\partial v_i(x, x_{-i})}{\partial x}|_{x=x_i} f(x_{-i}|x_j \leq x_i, j \neq i)dx_{-i}, \\ v_i(x) &= E[s|X = x]. \end{aligned}$$

The seller's ex ante pected payoff is

$$\pi_S^{on} = Es - N \int H^{on}(x_i, N)(1 - F^{on}(x_i))dx_i.$$

[11]The hazard rate of the random variable x is $h(x) = f(x)/(1 - F(x))$.

In a live auction, the seller sets the reserve price equal to zero, and each bidder's ex ante payoff is zero. The seller's expected payoff is

$$\begin{aligned}\pi_S^{off} &= Es - n^{off} c_b - c_s \\ &= Es - n^{off} \int H^{off}(x_i, n^{off})(1 - F^{off}(x_i))dx_i - c_s.\end{aligned}$$

where the number of bidders n^{off} is determined endogenously by

$$\int H^{off}(x_i, n^{off})(1 - F^{off}(x_i))dx = c_b.$$

Recall the concept of marginal revenue introduced by Bulow and Roberts (1989) and Bulow and Klemperer (1996). In our setting, it is

$$\mathrm{MR}_i(x_i) = \frac{-1}{f(x_i)} \frac{d}{dx_i}[v_i(x_{i,}x_{-i})(1 - F(x_i))].$$

We here suppress dependence on n unless needed. By our assumption, a bidder's marginal revenue is monotone increasing in own signal. Thus the seller sells the asset to the bidder with the highest signal. Since we assume independent signals, the revenue equivalence theorem (Bulow and Klemperer (1996)) holds, so our results for online auctions apply not only for ascending auctions, but also other standard auction formats such as first price auctions and second price auctions.

In a live auction, the number of bidders is determined so that the ex ante profit is equal to zero. Otherwise, potential bidders will keep entering in the auction, and since bidder's expected profit is strictly decreasing in the number of bidders, the number of bidders is determined (ignoring integer constraints) by the zero profit condition.

The tradeoff between online and live auctions is as follows. In a live auction, the number of bidders, n^{off}, can be less than the number of bidders in an online auction, N. This will imply less competition. On the other hand, in the live auction, the information available to the bidders, X^{off}, is more precise than in the online auction, X^{on}.

Next, we provide three comparative-statics results on payoffs. These results serve as a preliminary for the analysis in the next section and are of some theoretical interest by themselves. We first examine online auctions (i.e., standard auctions) in terms of dispersion parameters.

Proposition 1: *In an online auction, for any σ_1 and σ_2 such that $\sigma_1 \leq (m/M)\sigma_2$, the bidder's expected profit is lower and the seller's expected profit is higher in an auction with σ_1.*

An intuition is as follows: as σ falls, the bidders have a more precise estimate of the asset, so they bid more aggressively. As a result, the seller's expected profit rises and the bidder's expected profit decreases. A more quantitative intuition is that, the bidder's expected profit is s function of the difference between the expected value of the highest signal and the second highest signal. This difference is, by the formula $(EY_{1,n} - EY_{2,n}) = \sigma(EZ_{1,n} - EZ_{2,n})$, monotone decreasing in σ.

This approach will provide a quantitative estimate of the change in the expected profit as a function of dispersion. In contrast, previous results such as Milgrom and Weber (1982b), Persico (1999), and Athey and Levin (2001) are order-based.

Next, we examine online auctions in terms of mean parameters.

Proposition 2: *In an online auction, the seller's expected profit and the bidder's expected profit are independent of* μ.

An intuition is that bidder's expected profit is determined by the relative competition with other bidders. As a result, the mean shift common to all bidders will not affect in the bidder's payoff. Thus, the seller benefits totally from the increase in the mean parameter.

Next, we examine live auctions (that is, auctions with endogenous entry).

Proposition 3: *In a live auction, for any* σ_1 *and* σ_2 *such that* $\sigma_1 \leq (m/M)\sigma_2$*, the number of bidders will be monotone increasing and the seller's profit will be monotone decreasing in* σ*. The number of bidders and the seller's expected profit are independent of* μ.

The argument for the dispersion parameter is straightforward: when the signal is less accurate, bidders bid more aggressively, so a bidder's ex ante profit increases. Anticipating this, more bidders enter the auction. The added bidders do not, however, increase the seller's revenue: the zero-profit condition means the seller's payoff is equal to the value of the asset minus the bidder's entry costs. The argument for the mean parameter is similar. Since the bidder's ex ante expected payoff is independent of the mean parameter, so is the number of bidders.

3. Live versus Online

Our main result shows that the choice between selling live and online markets depends upon three paramaters: μ, the item's expected value, σ, the estimate variance, and k, the extent to which this variance is lower live than online.

Proposition 4: *There exists* $k^* < 1$ *such that for all* $k \leq k^*$*, there exist* σ^*, μ^* *such that for all* $\sigma \geq \sigma^*$ and $\mu \geq \mu^*$*, the seller's expected profit from selling live is higher than that from selling online.*

For an asset with a given mean and variance, therefore, an online auction generates a higher return for the seller if k is close enough to one, meaning the online bidders' information is not too far inferior to the live bidders' information. The intuition is that, if the asset has high valuation risk, then the benefit from information revelation is higher in a live auction. Further, in this case, the difference in the number of bidders between the live and the online auction is smaller. Thus the seller prefers the live auction. Another way of saying this is that the transaction cost is lower relative to the value of the asset. In a live auction, the seller has to pay not only her own transaction costs but also, indirectly, the bidders' participation costs (French and McCormick, 1984; McAfee and McMillan, 1987). If the expected price from a live auction does not cover these transaction costs, then the seller will not sell live.

It is the nature of the item for sale that determines k: there is more to be learned from seeing, say, an oil painting live rather than on a computer screen reproduction than, say, a print. Thus the useful interpretation of Proposition 4 is that, for categories of assets with a given k parameter, online sales are justified for assets whose mean and variance of value are not too high. High-value items can be sold online if k is close enough to one.

To see the role of the parameter k, consider how art differs from financial assets. In the sale of a piece of art, the preview reduces the valuation risk ($k < 1$). By contrast, staring at a 10,000 yen note will not tell you anything about the risk of holding yen. As a result, there is no merit in trading financial assets live, given the transaction costs saving from online trading.

The force of the parameter k suggests that the limits of online auctions differ for different categories of assets. The reproduction on a computer screen of an oil painting, for example, loses much of the quality of the picture: the texture of the oil paint and the subtleties of the colors. For oil paintings therefore, k is likely to be close to zero, so online bidders would bid cautiously to avoid the winner's curse, and live auctions would tend to yield better returns for the seller. With assets like coins and stamps, arguably, something closer to full information could be conveyed to online bidders, so k is close to one and the limits of online auctions are reached later; an expensive asset with a small valuation risk can be successfully sold online if its k is not far below one. An example is the auction of an original of the Declaration of Independence for a record \$8.14 million at Sothebys.com in 2000: a printed document arguably has a k close to one.

We now provide two simple numerical examples to explain our argument. Suppose the seller has an asset and wonders whether to sell it online or liv. There are three bidders interested in the asset and each bidder values the asset as an mean common value model $\sum x_i/3$.

Example 1: The seller estimates that the bidders will have signals distributed uniformly on [2,000, 18,000], based on the information the seller provides in the Internet. Alternatively, the seller could hold a live auction. In the live auction, the bidders need to pay \$100 to attend the preview. The seller expects that information will be 75% more accurate in the sense that bidders will have signals distributed uniformly on [8,000, 12,000]. Should the seller hold a live auction ?

Solution: First, we compute the seller and bidder profit from live auctions. Consider a symmetric equilibrium where each bidder drops out at the price equal to be value of the asset given the signal of the bidders who have already dropped out, and assuming all other remaining bidders having the same signal with that bidder. In expectation, bidder 1 has the signal \$14,000, bidder 2 has \$10,000, and bidder 3 has \$6,000. Bidder 3 will drop out at \$6000 (this simplification is not without loss of generality due to the linear structure of the model.) Bidder 3 will drop out at \$6,000. Bidder 2 will drop out at (6,000+10,000*2)/3 = \$8,667. The seller's expected price is \$8,666 and bidder's ex ante expected profit is (10,000-8,667)/3 = \$433.

Second, we compute the profits from a live auction. Suppose all three bidders choose to enter the auction. In expectation, bidder 1 has the signal of \$11,000, bidder 2 \$10,000, and bidder 3 \$9,000. Bidder 3 should drop out at \$9,000. Then bidder 2 drops out at (9,000+10,000*2)/3 = \$9,666 dollars. The seller's revenue is \$9,666. The bidder's ex ante expected payoff is 334/3 = \$111. Given that, each of three bidders will enter the auction, since the ex ante expected profit of \$111 is higher than the entry cost of \$100.

Third, we compute the seller's decision. In an online auction, the expected price is \$8,667. In a live auction, the expected price is 9,667-100 = \$9,556. Thus the seller will hold a live auction.

Example 2: The seller estimates that the bidder will have signals distributed uniformly on [9,200, 10,800], based on the information on the Internet. The seller can reduce the dispersion 75% to [9,800, 10,200] in a live auction. Should the seller hold a live auction?

Solution: First, we compute the equilibrium payoffs. On average, bidder 1 will have \$10400, bidder 2 \$10000 and bidder 3 \$9600. Bidder 2 will drop out at (9,600+20,000)/3 = \$9,867. Thus the seller's expected price is \$9,867 and the bidder's ex ante profit is 133/3 = \$43.

Second, we compute payoffs from a live auction [9,800, 10,200]. On average, bidder 1 has \$10,100, bidder 2 \$10,000 and bidder 3 \$9,900. Bidder 3 will drop out at (9,900+10,000*2)/3 = \$9,966. The seller's expected price is \$9,966 and the ex ante profit of the bidder is 34/3 =\$11. Given this profit, the third bidder will not enter. Then will two bidders enter? The price is \$9,933, and the expected profit of each bidder is 67/2 = \$34. Thus the second bidder will not enter. Only one bidder will enter, and may bid a price of ε (depending on the formulation of bargaining between the seller and the bidder).

Third, we derive the seller's choice. The seller's price from the online auction is \$9.867 and ϵ from the live auction. Thus the seller sells online.

These examples show that dispersion of the estimates has a major effect on the seller's profit in common-value auctions. If the dispersion is large, the seller may hold a live auction, even given the participation costs. If the dispersion is small, it may not pay to hold a live auction.

Finally we look at differences in sale rates (that is, the percentage of items that pass the reserve price and are sold):

Proposition 5: *The sale rate is higher in live auctions than in online auctions.*

The intuition is that, in an online auction, since it is a standard ascending auction, the seller sets nontrivial reserve price to increase the sale price, thus creating a positive probability of no sale. In a live auction, by contrast, since the seller's profit is equal to social welfare net of transaction costs, the seller sets a zero reserve price and always sells the asset. (We could modify the model to generate the more realistic result that the sale rate is less than 100 percent in live auctions by supposing the seller has a positive value for the item unsold. The seller's value might, for example, be given as a draw from the distribution F^{off}minus, perhaps, some constant to represent the seller's need for liquidity. In such a model the sale rate would be less than 100 percent live, but still lower online because of the reserve-price effect.)

An alternative explanation is about the cost of resale. Holding another auction is costly for offline auctions but costless for online markets. As a result, the seller should be eager to trade the asset more in a live auction.

Can a seller design an auction to combine the benefits of live and online auctions? An alternative auction form, in between live and online auctions, is offered by both Sotheby's and Christie's: in live auctions, bidders may participate from a distance, by telephone, fax, or internet. The auction-house web sites offer the catalogues for live auctions and allow advance bids, which an auction-house employee will submit during the course of the live auction. There is even a provision in some live auctions for real-time online bidding. Christie's offers live streaming videos, so that people can bid in live auctions from their computers.

From the seller's point of view, however, this in-between alternative offers none of the saving in transaction costs of true online bidding. Even from a bidder's point of view, our analysis implies, this is strictly inferior. Bidders would do better participating either directly in a live auction or in a true online auction. Distant bidders, who have not incurred the transaction costs entailed in seeing the piece of art for themselves, are competing against the live bidders, who have seen it. Less-informed bidders, in common-value auctions in general, are handicapped when competing against better-informed ones, and end up with little or no net gain (Engelbrecht-Wiggans, Milgrom, and Weber, 1983). When the artwork is of high value, they will be outbid by the better-informed live bidders; when it is of low value, they might win the auction but overpay. Online bidders in a live auction are in a no-win situation. They can overcome this disadvantage by either inspecting the item themselves at the presale exhibition or hiring an agent to do the inspection; but this means incurring the same transaction costs as the live bidders.

4. Art Auction Data

We now examine data from online and live art auctions in light of our theory. We collected data from the Sotheby's and eBay websites using Perl programs. The live-auction data are from Sotheby's New York sales between June 1 and June 30, 2002 (representing 1,890 auctioned items, with 1,213 successful sales and a total of $23,572,639 raised). The online auctions are transactions on the Sotheby's site on eBay between June 26 and July 23, 2002 (representing 1,300 auctioned items, with 517 successful sales and a total of $68,2845 raised).

Prices are much lower online than live. The mean selling price in the live auctions is $18,801 and in the online auctions $1,483. The highest live-auction price in our sample is a George Graham timepiece, at $1,219,500; the highest online price is a Frank Lloyd Wright copper weed holder (that is, a vase) at $83,750. The lowest live-auction price is a Cartier watch, at $358; the lowest online price is a drawing by a British artist, Nicole Hornby, at $11.50. Histograms of sales prices are given in Chart 1. Notice how skewed the distributions are: in both online and live auctions the vast majority of items sold are of relatively low value.

The types of items auctioned are compared in Table 1. For the live auctions, this categorization is based on the auction titles. For the online auctions, we use eBay's categorization.

	Live	Online
Jewelry	635	73
Paintings and sculpture	621	113
Clocks and watches	562	41
Antiques and furniture	286	233
Books, prints	283	37
Silver and ceramics	0	259
Photos, stamps, coins	0	272
Collectibles	0	356

Table1: Number of auctions by category

A higher percentage of successful sales in live auctions than online is predicted by our theory. The data are in accord with this. Overall, the sale rate is 64.1% in live auctions and 39.7% in online markets. Table 2 shows the breakdown of sale rates by category. These sale rates are roughly consistent with data from elsewhere. For example, eBay's online art auctions in 2000 (in the Great Collections auctions, eBay's predecessor to its Sotheby's joint venture), the sale rate was 48% (Tully, 2000). At Christies London live auctions in 1995 and 1996, the sale rate was in the 70% to 80% range for paintings of various kinds, 61% for photographs, 88% for clocks, and 86% for jewelry (Ashenfelter and Graddy, 2003).

	Live	Online
Paintings & prints	83%	44%
Watches & clocks	82%	63%
Jewelry	54%	n.a.
Furniture	n.a.	66%
Coins	n.a.	80%
Silver	n.a.	80%
Books	57%	n.a.

Table2: Sale rates compared (*n.a.* means no data available)

In auctions run under eBay's rules, because of the fixed end-time, bidders usually submit bids very close to the closing time (Roth and Ockenfels, 2002). In our online-auction data, the mean timing of a bid is made 78% through the period of auction; the median is at the 96% point, and the 75th percentile is at the 99.7% point. In other words, half of the bids, typically, are placed during the last 4% of the time of the auction, and 25% in the last 0.3 percent of the auction period. (See Chart 2.) Last-minute bidding, therefore, is prevalent not only in lower-value eBay auctions but also in the art and antiques auctions on Sothebys.com.

Are the auctioneer's presale estimates done honestly or strategically? The existing evidence for live auctions is mixed. Ashenfelter (1989) found that auction houses are generally truthful, in that the mean of the high and low estimate is well correlated with the winning bid. Beggs and Graddy (1997) and Bauwens and Ginsburgh (2000), however, found a tendency to systematically underestimate for some kinds of goods and overestimate for others. Mei and Moses (2002) offered evidence that estimates are strategically manipulated, expensive paintings showing an upward bias.

The main result of our theory is that the decision to sell online rather than live reflects the dispersion of bidder value estimates but not their mean. It also reflects the extent to which the dispersion is reduced if the bidders can see the item live. We run a simple regression to test this proposition. We use the auctioneer's presale announcement of high and low price estimates to get a measure of the mean and dispersion. We take the mean of these two numbers to represent the mean of the estimate distribution, and the difference between them to represent the dispersion. Histograms of estimate means and dispersions, online and live, are given in Charts 3 and 4.

The highest mean estimate in our live auction data is a Pierre Frederich Ingold timepiece, at $375,000 (it was sold); the highest in our online data is a Marilyn Monroe wedding gown, at $60,000 (unsold). The lowest mean estimate in our live auctions is a 1995 Cartier watch, at $600 (sold); the lowest in our online data is a Lee Tanner photograph of John Coltrane, at $15 (unsold).

The highest estimate dispersion in live auctions is the same Ingold timepiece, at $250,000; the highest online also is the same item, the Marilyn Monroe wedding gown, at $20,000. The lowest estimate dispersion in the live auctions is a Fouga wristwatch, at $100 (sold); online, the lowest is the same Tanner photograph, at $10.

These cases suggest that the pre-auction estimate dispersion does indeed reflect the degree of valuation uncertainty. A watch or photo is not unique, so the history of prices of identical or similar items can be used to assess its value. Moreover, its value can be reasonably judged from a written description plus a web-page reproduction. The value of a deceased actress's wedding dress, by contrast is harder to assess. It is unique, so its future resale value is highly uncertain. Moreover, it may be hard to judge its quality from a web-page photograph: is it still in a good condition? Seeing it live may give a bidder significantly more precise information.

These cases further suggest, as we would expect, that items with a higher value show a higher dispersion of value estimates. This is the case in our data. The correlation between the mean of the high and low estimates and their difference is 0.9. The result of a probit regression of the choice of online or live auction regressed on both mean and dispersion of estimates is given in Table 3. Despite the strong positive correlation between the two independent varioable, each is significantly different from zero. Both the mean and the dispersion of estmates matter. Items with a high variance of value *or* a high mean value are likely to be sold live rather than online.

Off	Coefficient	Standard Error	z	P>|z|
EstAvg	0.0000686	0.0000135	5.09	0
EstDiff	0.0000336	0.00000461	7.3	0
Const	-0.493068	0.0325403	-15.15	0
Obs	3180			
LR	1207.75			
Prob>chi2	0			
PseudoR2	0.2801			

Table3: Regression of online vs. liveauction

5. Simulation

In this section, we start the estimation of transaction costs and information revelation by formulating qualitative response models.

5.1. **A qualitative response model.** We consider the decision of the seller regarding whether to sell the asset in online auctions or in live auctions. We assume that the seller's utility associated with the choice of the auction is the expected profit π_S^{off} and π_S^{on} plus an additive error term ϵ^{off} and ϵ^{on}. The data is the upper and the lower bound of the estimate $x = [\underline{x}, \overline{x}]$. The parameters we are interested in are transaction costs c_s and c_b, and efficiency improvement k. Let $\theta = (c, k)$. Let U^{off} and U^{on} be the seller's expected utility: $U^{off} = \pi_S^{off}(x, \theta) + \epsilon^{off}$ and $U^{on} = \pi_S^{on}(x) + \epsilon^{on}$.

The basic assumption is that the seller sells the asset in a live auction if $U^{off} \geq U^{on}$. Thus defining $Off = 1$ if the seller sells the asset in a live auction, we have

$$P(Off=1)=P(U^{off}\geq U^{on})=F(\pi_S^{off}(\underline{x},\overline{x})-\pi_S^{on}(\underline{x},\overline{x},\theta))$$

where F is the distribution function of $\epsilon^{off}-\epsilon^{on}$. The log likelihood function is

$$\begin{aligned}\log L &= \sum_{i=1}^{n} Off_i \log F(\pi_S^{on}(x)-\pi_S^{off}(x,\theta)) \\ &+\sum_{i=1}^{n}(1-Off_i)\log(1-F(\pi_S^{on}(x)-\pi_S^{off}(x,\theta))).\end{aligned}$$

The maximum likelihood estimator $\hat{\theta}$ is defined by $\frac{\partial \log L}{\partial\theta}|_{\theta=\hat{\theta}}=0$.

5.2. **A simulation.** *The following simulation is based on very rough assumptions.*

First, in this draft, we use a very simple parametrization. Z=uniform $[-0.5,0.5]$, $X=\mu+\sigma Z$, $u_i(s,x)=\sum x_i/n$ where n is the number of bidders.

Second, compute the functional form of the discrete choice model give above. In online auction[12], since $H(w,m)=F(w)^{n-1}$,

$$\begin{aligned}\pi_B &= \frac{1}{n}(EY_{1,n}-EY_{2,n})=\frac{\sigma}{n}(EY_{1,n}^Z-EY_{2,n}^Z)=\frac{\sigma}{n(n+1)} \\ \pi_S &= \mu-n\pi_B=\mu-\frac{\sigma}{(n+1)}.\end{aligned}$$

Note the simple comparative statics result: the seller's expected payoff is decreasing in the dispersion and increasing in the number of bidders. Given the smoothness, the consistency and asymptotic normality of maximum likelihood estimator is standard (Amemiya (1985), Section 9.2.2.) The number of bidders is determined by $k\sigma/n(n+1)=c_b$. For simplicity, we approximate[13] the solution of this equation by $n=(k\sigma/c_b)^{0.5}$ to make a model linear in parameters. We set the number of bidders in the online market be 2. Note 1860/1300=1.43 was an mean number of bidders for online auctions. Thus we obtain

[12] We do not consider reserve prices in this estimation and simulation. It is easy to compute Bulow and Klemperer (1996) bounds.

[13] The next version of the draft will give estimateion based on analytical solution of this equation. Meanwhile, for reasonably large k, the differences between the solution of $x^2=k$ and $x^2+x=k$ is small, for $k=10$, the solution is 3.16228 and 2.70156.

$$\pi_S^{on}(x) - \pi_S^{off}(x,\theta) = c_b^{0.5} k^{0.5} \sigma^{0.5} + c_s - \frac{\sigma}{3}.$$

Note that in this model, we cannot separately estimate k and c_b.

Third, we report the result of an estimation. For the probit model of 3190 samples, assuming $\epsilon^{off} - \epsilon^{on}\tilde{}N(0, 400^2)$,

off	Coefficient	Standard Error	z	P>|z|
$c_b^{1/2} k^{1/2}$	3.011	.7443	4.05	0.000
c_s	222.2	22.80	9.74	0.000

A possible value of c is 90 for $k = 0.1$. There are 200~300 auctions in one day for live auctions, so if each bidder bids for 10 assets, the total bidding cost will be around \$1000. The threshold value where the bidder's rent from selling online is equal to that from selling live is \$942.30.

6. Conclusion

The seller's choice between online and live auctions involves a trade-off between information generation, favoring a live auction, and transaction costs, favoring an online auction. Bidders do better online, even though they are relatively uninformed about the item's true value, because their competitors are equally uninformed and so, from fear of the winner's curse, the bidding competition is less fierce. Sellers, also, in some cases do better online because of the saving in transaction costs.

The model could be extended in various ways: to incorporate more general assumptions about bidders' valuations (affiliation rather than common value: Milgrom and Weber, 1962a); to derive endogenous participation costs in the presence of bidder asymmetry; and to estimate participation costs and information revelation rates.

Online auctions may not up to now have been used to their full potential. If our model is correct, the use of online auctions in the first few years of their existence may have been unduly cautious. Sellers have been reluctant to opt for online auctions for high-value items. According to our model, however, what determines whether the seller does better selling live than online is not the expected price. What matters is the dispersion of valuations, and the extent to which this dispersion is reduced if bidders can see the item for themselves. A high-value item can be successfully sold online if the bidders' estimates of that value are tightly bunched. It seems that online sellers do not fully recognize this, and so the limits of online auctions may not yet have been reached.

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7. **Appendix: Proofs of the Propositions**

7.1. **Proof of Lemma 1.** First, compute the expected profit of the bidder and the seller in a standard ascending auction of the general symmetric model with n bidders and a signal distribution f. In an ascending auction, there exists a symmetric monotone equilibrium where each bidder drops out at a price equal to the expected value of the asset given the signal of bidders who have already dropped out and assuming that the bidders remaining in the auction have the same signal with the bidder (Milgrom and Weber, 1982a). Compute an expected payoff $\pi_B(x_i)$ of a bidder i with a signal x_i. By symmetry, π_B is independent of the identity of the bidder. Suppose bidder i with signal x_i bids as if her signal is y_i. Let the expected profit be $\pi_B(x_i, y_i)$. Then

$$\pi_B(x_i, y_i) = \int_{X_{-i}} (v(x_i, x_{-i}) - v(y_{1,-i}, x_{-i}))f(x_{-i}|y_{1,-i} \leq y_i)dx_{-i}$$

where $v(x) = E[s|X = x]$.and $y_{1,-i}$ is the highest signal among bidders other than i. This is because bidder i wins if and only if bidder i's claimed type y_i is higher than the other bidders. If bidder i wins, the value is $E[s|X = x]$.

$$\frac{d\pi_B(x_i)}{dx_i} = \frac{d\pi_B(x_i, x_i)}{dx_i} = \int \frac{\partial \pi_B(w, y)}{\partial w}|_{y=w} dw.$$

This is because, since v and f are smooth and bounded, we can apply the envelope theorem. The bidder with the lowest type never wins, so her expected profit is zero. Then,

$$\begin{aligned}
\pi_B(x_i) &= \int_{-\infty}^{x_i} \int_{X_{-i}} \frac{\partial v_i(w, x_{-i})}{\partial w} f(x_{-i}|y_{1,-i} \le w) dx_{-i} dw \\
&= \int_{-\infty}^{x_i} H(w, n) dw \\
\text{(where } H(w,n) &\cong \int_{X_{-i}} \frac{\partial v_i(w, x_{-i})}{\partial w} f(x_{-i}|y_{1,-i} \le w) dx_{-i}.)
\end{aligned}$$

Then the bidder's ex ante payoff is:

$$\begin{aligned}
\pi_B &= \int \int_{-\infty}^{x_i} H(w, n) dw f(x_i) dx_i \\
&= \{\int_{-\infty}^{x_i} H(w, n) dw F(x_i)\ \}_{-\infty}^{+\infty} - \int H_i(x_i, n) F(x_i) dx_i \\
&= \int H(w, n) dw - \int H_i(x_i, n) F(x_i) dx_i \\
&= \int H(x_i, n) dx_i - \int H_i(x_i, n) F(x_i) dx_i \\
&= \int H(x_i, n)(1 - F(x_i)) dx_i.
\end{aligned}$$

The first line is by taking the expectation of $\pi_B(x_i)$ and changing the order of integration. The second line is by integration by parts. The third line is by integration by parts using the formula $\int ab = [ab] - \int a'b$ with $a = \int H(w)dx_i$ and $b = F(x)$. The seller's ex ante profit is the difference between the item's expected value and the bidders' expected profits:

$$\pi_S = Es - n\pi_B$$

Second, we compute expected profits in an online auction without a reserve price. From the previous formula, but with N bidders,

$$\pi_B^{on} = \int H^{on}(x_i, N)(1 - F^{on}(x_i)) dx_i.$$

$$\pi_S^{on} = Es - N \int H^{on}(x_i, N)(1 - F^{on}(x_i)) dx_i.$$

The number of bidders in the live auction is determined by the zero-profit condition (with bidders entering until profits are competed away, ignoring the integer constraint):

$$\pi_B^{off} = \int H^{off}(x_i, n^{off})(1 - F^{off}(x_i))dx_i = c_b$$

The seller's ex ante expected live-auction profit is

$$\begin{aligned}
\pi_S^{off} &= Es - n^{off}c_b - c_s \\
&= Es - n^{off}\int H^{off}(x_i, n^{off})(1 - F^{off}(x_i))dx_i - c_s.
\end{aligned}$$

7.2. **Proof of Proposition 1.** First, we claim that it is suffice to show that bidder's expected profit is lower in an auction with σ_1 than one with σ_2. This is because the seller's expected profit is the difference between the value of the asset and the bidders' expected profit. (By the assumption of absolute common value, the value of the asset is independent of σ.)

Second, we compute the lower and upper bound of the bidder's expected profit. Recall from Lemma 1, the bidder's expected profit is

$$\pi_B = \int H(x_i, n)(1 - F^{on}(x_i))dx_i.$$

Thus, from the definition of H and the assumption (*), π_B is bounded:

$$m\int F(x_i)^{n-1}(1 - F(x_i))dx_i \leq \pi_B \leq M\int F(x_i)^{n-1}(1 - F^{on}(x_i))dx_i.$$

The distributions and densities of the order statistics are (David, 1981):

$$\begin{aligned}
F_{1,n}(x) &= F(x)^n \\
F_{2,n}(x) &= F(x)^n + nF(x)^{n-1}(1 - F(x)) \\
f_{2,n}(x) &= n(n-1)(1 - F(x))F(x)^{n-1}f(x)
\end{aligned}$$

and therefore a bidder's expected profit can be written as:

$$\begin{aligned}\int F(x_i)^{n-1}(1-F(x_i))dx_i &= (1/n)\int n(F(x_i)^{n-1}-F(x_i)^n)dx_i \\ &= (1/n)\int (F_{2,n}(x_i)-F_{1,n}(x_i))dx_i \\ &= (1/n)\int x_i(f_{1,n}(x_i)-f(x_{2,n}(x_i))dx_i \\ &= \frac{1}{n}(EY_{1,n}-EY_{2,n}) \\ &= \frac{\sigma}{n}(EZ_{1,n}-EZ_{2,n}).\end{aligned}$$

where the third equality uses integration by parts, the fouth is by definition, and the last comes from the properties of mean-dispersion families (where Z is the base random variable as defined above).

Third, we show that the bidder's profit is lower in auctions with dispersion parameter σ_1. The upper bound of the bidder's profit from an auction with dispersion parameter σ_1 is $\sigma_1 M/n(EZ_{1,n}-EZ_{2,n})$. The lower bound is $\sigma_2 m/n(EZ_{1,n}-EZ_{2.n})$. Thus the bidder's profit from an auction with σ_1 is lower if

$$\sigma_1 M/n(EZ_{1,n}-EZ_{2,n}) \leq \sigma_2 m/n(EZ_{1,n}-EZ_{2.n})$$

which implies

$$\sigma_1 < (m/M)\sigma_2. \tag{7.1}$$

7.3. **Proof of Proposition 2.** From Lemma 1, the bidder's ex ante payoff is $\frac{1}{n}(EY_{1,n}-EY_{2,n}) = \frac{\sigma}{n}(EZ_{1,n}-EZ_{2,n})$, which is independent of μ.

7.4. **Proof of Proposition 3.** First, we claim π_B is monotone decreasing in n. The seller's profit is the highest marginal revenue among bidders. Then the addition of one bidder weakly increases the seller's profit. This implies that the total surplus for the bidders decreases. Thus the bidder's ex ante profit decreases.

Second, we claim that n is monotone increasing in σ.The number of bidders n is determined by the zero profit condition: $\pi_B(n,\sigma) = c$. From the previous discussion, $\pi_B(n,\sigma)$ is monotone increasing in σ and monotone decreasing in n. Thus n is monotone increasing in σ.

Third, we claim that the seller's expected profit is monotone decreasing in σ. From Lemma 1, the seller's expected payoff is the value of the asset minus the total payment of entry costs. By the absolute common value assumption, the value of the asset is constant and the total payment of entry cost is monotone decreasing in σ.

Fourth, consider the impact of the shift in μ for mean common value model. This is a direct consequence of Proposition 2.

7.5. **Proof of Proposition 4.** First, we compute an upper bound of the seller's profit from online auctions with reserve prices. By Bulow and Klemperer (1996), the seller's expected profit from an auction with n bidders with optimally a dynamically set reserve price is less than the expected profit from $n+1$ bidders without a reserve price. Thus,

$$\begin{aligned}
\overline{\pi}_S^{on} &= V-(N+1)\int H^{on}(x_i,N+1)(1-F^{on}(x_i))dx_i. \\
&\leq V-(N+1)m\int F^{on}(x_i)^N(1-F^{on}(x_i))dx_i \\
&= V-m(EY^{on}_{1,N+1}-EY^{on}_{2,N+1}) \\
&= V-m\sigma(EY^{Z}_{1,N+1}-EY^{Z}_{2,N+1}).
\end{aligned}$$

Second, we compute a lower bound of the seller's profit in a live auction with N bidders. By the assumption (*), there exists M such that $\partial v(x)/\partial x_i \leq M$. Then,

$$\pi_S^{off} \geq V-k\sigma M(EY^Z_{1,N}-EY^Z_{2,n})-(N+1)c_b-c_s.$$

Third, assuming there are N bidders in the live auction, we compute k' and σ' such that for $k\leq k'$ and $\sigma'\geq\sigma$,a bidder's rent is smaller in live auctions. This holds if

$V-k\sigma M(EY^Z_{1,N}-EY^Z_{2,n})-(N+1)c_b-c_s \geq V-m\sigma(EY^Z_{1,N+1}-EY^Z_{2,N+1})$

$$k\sigma M(EY^Z_{1,N}-EY^Z_{2,n})+(N+1)c_b+c_s \leq m\sigma(EY^Z_{1,N+1}-EY^Z_{2,N+1})$$

$$\sigma(m(EY^Z_{1,N+1}-EY^Z_{2,N+1})-kM(EY^Z_{1,N}-EY^Z_{2,n}))\geq(N+1)c_b+c_s$$

Thus for

$$k\leq m(EY^Z_{1,N+1}-EY^Z_{2,N+1})/M(EY^Z_{1,N}-EY^Z_{2,n})$$

if

$$\sigma\geq[(N+1)c_b+c_s]/[m(EY^Z_{1,N+1}-EY^Z_{2,N+1})-kM(EY^Z_{1,N}-EY^Z_{2,n})],$$

the seller's payoff is higher in live auctions.

Fourth, we compute the conditions where there will be at least N bidders in the live auction. This occurs if the bidder's ex ante payoff with N bidders is higher than the entry cost:

$\pi_B^{off} \geq m \int F^{on}(x_i)^N(1 - F^{on}(x_i))dx_i = \frac{m}{N} k\sigma(EY^Z_{1,N+1} - EY^Z_{2,N+1}) \geq c_b$, That is,

$$\sigma \geq \frac{Nc_b}{mk(EY^Z_{1,N+1} - EY^Z_{2,N+1})}$$

Fifth, we compute the condition when both conditions are satisfied. For each k which satisfies $k \leq m(EY^Z_{1,N+1} - EY^Z_{2,N+1})/M(EY^Z_{1,N} - EY^Z_{2,n})$, there exists

$$\sigma \geq \max\left[\frac{Nc_b}{mk(EY^Z_{1,N+1} - EY^Z_{2,N+1})}, \frac{(N+1)c_b + c_s}{m(EY^Z_{1,N+1} - EY^Z_{2,N+1}) - kM(EY^Z_{1,N} - EY^Z_{2,n})}\right]$$

where the seller's profit is higher in a live auction. And if μ is high enough so that expected value of the allocation $V(\mu)$ is higher than $c_b N + c_s$,the seller will get a positive profit.

7.6. **Proof of Proposition 5.** First, compute the reserve price in an online auction. In online auctions, by lemma 2 of Bulow and Klemperer (1996), the seller's take-it-or-leave-it price is the price which makes marginal revenue equal to zero. Thus, if the bidder with the highest marginal revenue is less than zero, the seller does not trade.

Second, compute the reserve price in a live auction. If the zero profit condition is binding, the seller's expected profit is equal to the expected social surplus, thus the seller, who has the value zero for the asset, does not trade. If zero profit condition is not binding, then each bidder's marginal revenue is strictly higher than that in an online auction, thus the probability of exclusion is strictly lower.

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