A Model of Piracy

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Abstract

This paper develops a simple model of software piracy to analyze the short-run effects of piracy on software usage and the long-run effects on development incentives. We consider two types of costs associated with piracy: the *reproduction cost* that is *constant* across users and the *degradation cost* that is *proportional* to consumers' valuation of the original product. We show that the effects of piracy depend crucially on the nature of piracy costs. Policy implications concerning copyright protection are also discussed.

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1. Introduction

As the current controversy surrounding the Napster case testifies, unauthorized reproduction of intellectual property has been a serious but controversial issue for copyright holders, consumers, and policy makers alike, especially with advent of digital technology. According to a recent study by the Business Software Alliance (2001), for instance, the piracy rate in 2000 is estimated to be 37%, which can be translated into $11.75 billion dollar losses for software publishers.[1] The piracy rates in the Far East, especially China, Indonesia and Vietnam, represent in each case over 90%. The corresponding rates for Western Europe and North America are 34% and 25%, respectively. Based on these figures, copyright holders claim that piracy is a severe threat to incentives to develop new products, as well as revenue loss from the developed software.

As typical with other information goods that bear high fixed development cost and low marginal cost, software is expensive to develop but cheap to reproduce.[2] In order to "promote the progress of science and useful arts," Intellectual Property Rights (IPR) are required to protect the software publishers' interests and to give them unique rights to prohibit piracy.[3] The 'folklore' of IPR from the previous studies proposes that an increase in Intellectual Property Rights (IPR) decreases *ex post* efficiency by reducing total usage of the already developed software; whereas, it promotes *ex ante* efficiency by increasing development incentives for potential software publishers. Consequently, the optimal level of IPR needs to strike a delicate balance between the long-term incentives to innovate and the short-term dissemination of work already created.[4]

This paper reconsiders the 'folklore' by developing a simple model of software piracy to analyze the short-run effects of piracy on software usage and the long-run effects on development incentives. The premise in this paper is to distinguish two types

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[1] 'Sixth Annual BSA Global Software Piracy Study (May 2001)' conducted by International Planning and Research Corporation (IPR) for the Business Software Alliance (BSA) and its member companies.
[2] Varian (1998) defines information goods as anything that can be digitalized, such as text, images, date, audio and video.
[3] Article I, Section 8, of the U.S. Constitution. The U.S. Constitution grants the Congress the power 'to promote the progress of science and useful arts, by securing for limited times to authors and inventors the exclusive right to their respective writing and discoveries.'
[4] See, for instance, Nordhaus (1969).

of costs associated with piracy – constant and type-proportional – and to show that the effects of piracy depend crucially on the nature of piracy costs.

At this stage it is useful to comment briefly on two types of costs. The first type is *reproduction cost*, which is *constant* across all consumers. The constant reproduction cost can be thought of as the price of an illegal copy made by piracy retailers. Even with the assumption that actual reproduction cost is zero, consumers will need to exert efforts to obtain an authorized copy or may have to spend time to find the software, and then to copy or to download it for installation purposes. We assume that these costs are the same across all users or at least independently distributed with the value of consumers attached to the product.

The second type of cost is the *degradation rate* associated with unauthorized copying, which is determined by each consumer's valuation of the software. The examples of the proportional degradation rate are the following: For most cases, an authorized copy of the software is bundled with manuals, installation software, online services, as well as discount on upgrades. Users who pirate the software, in contrast, may not be able to access the entire complimentary bundle. Some software publishers strategically include some web-based functions, which require personal identification numbers (PIN), and the same PIN cannot be used on the Web at the same time.[5] With an illegal copy, a consumer can use the software off line, but is not able to use online functions, thus inducing quality degradation.[6] For the most extreme case, software publishers use a web authentication system that requires the owner of the software to have the original media, and product key.[7] This web-based anti-piracy system ensures that there will be only one running copy of the software per license. Even this technology, however, it is not perfect and can be circumvented. Furthermore, there can

[5] With development of high-speed Internet, the new components of game software are online game services, in which you can play the game with someone through the Internet. With the database for the registration keys the software publisher is able to prohibit the pirated copies with the duplicated key from running on the web.

[6] For TurboTax software, each authorized copy is designed to file tax returns electronically for only one customer. Hence, users with an unauthorized copy are only able to do tax returns except electronic filing.

[7] To reduce software piracy, Microsoft uses software-based product activation technology for Windows XP and Office XP. This technology requires that authorized users activate the software within 30 days. This process locks a product identification number assigned to each copy of the software to the PC. When the activation process is complete, the user registers the activation code with Microsoft, which stores it in a

be a counter anti-piracy program, which can bypass the anti-piracy procedure or create a fake product key. However, the web-based anti-piracy system is capable of reducing quality of the software by blocking the users with a pirated copy from receiving upgrades.[8]

As another example of policy relevance, consider the Peer to Peer Piracy Prevention Act which was proposed by Rep. Howard Berman, D-California, and is being considered in a House Judiciary subcommittee as a way to protect intellectual property against file-sharing through peer-to-peer (P2P) networks. P2P networks arose as a response to the shutdown of Napster. Unlike Napster, P2P networks do not host files on a central server; instead they list available files on individual PCs and directly connect those computers, which makes the enforcement of copyrights more difficult. The proposed bill would allow the record industry to "hack" into individuals' PCs in search of copyright violations. In our model, this type of copyright enforcement would translate into an increase in the costs of copying that are proportional to user types. Another response by record labels in face of file-sharing, is to upload the so-called "spoof" files – containing little or no music – on P2P networks to confuse downloaders. This practice would increase the expected time of downloading for a particular music file and can be considered as an increase in the uniform copy costs.

The purpose of this study is to construct a model of self-selection with heterogeneous consumers who choose among three available options: the use of a legally purchased copy, the use of an illegal copy with unauthorized reproduction, or no consumption. We conduct a two-step analysis. First, we analyze how the threat of piracy constrains the pricing behavior of the monopolist. Depending on the relative magnitudes of reproduction and degradation cost, the monopolist has to choose the optimal choice of regimes between limit pricing and accommodation to copy. We show that with the threat of piracy the monopolist's price is lowered, and usage for an authorized copy is increased

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database. Throughout this activation technology, Microsoft is able to identify conflicting activation codes to identify software that has been installed on more than one machine.

[8] Responding to piracy of Windows XP operating system, Microsoft announces that Windows XP Service Pack 1 (and possibly all future updates) will not install with pirated copies.

in the both regimes.[9] This result provides a sharp contrast to the common claims of copyright holders, in which the possibility of piracy reduces demand for a legal copy.

Second, we examine the effects of an increase in the two different margins of IPR. When an increase in IPR results from either the constant reproduction cost or the type-variant degradation rate, it has two different effects: a demand switch and a total usage change. If an increase in the copyright protection enforces consumers, who pirate the software, to purchase from the monopolist, it has a 'positive' demand switch effect on social welfare due to using more efficient technology. In contrast, increase in IPR has a 'negative' effect if the opposite demand switch happens. Since an increase in copyright protection makes piracy more costly, fewer consumers are able to pirate and, therefore, total usage change always has a negative effect on social welfare.

It is observed that with two different types of IPR, the copyright protection has different implications. In the case of an increase in the constant reproduction cost, we show that it has two opposite effects in the short run: positive demand switch and negative total usage change. Therefore, it may enhance or reduce ex ante social welfare in the short run. However, we are able to pin down the effect on the social welfare with an increase in the degradation rate. It confirms the 'folklore' of increase in IPR in short-run. It reduces ex post efficiency because of the same negative effects of demand switch and total usage change. Policy implications concerning copyright protection are also discussed.

Earlier papers concerned with the effects of increased copyright protection on social welfare include Novos and Waldman (1984) and Johnson (1985) among others. Novos and Waldman (1984) use the monopolist's quality choice model with consumers with the same valuation for product, but heterogeneous copy cost. They find that an increase in IPR has a partial effect on ex ante efficiency and no ex post effect on total usage of partially nonexcludable goods. With availability of a secondary source that makes it possible for consumers to make an unauthorized copy, increase in copyright protection may increase or decrease the monopolist's incentive to provide higher quality. Based on the assumption of more efficient monopolist's technology, higher copyright

[9] Choi and Thum (2002) provide a similar framework, if we consider purchasing an authorized copy of the software as entering the official economy, and making an illegal copy as operating in the shadow economy.

protection will bring increased ex post efficiency by forcing consumers to purchase from the monopolist.

Johnson (1985), in contrast, makes a distinction between a household reproduction model with heterogeneous marginal cost and the fixed cost model of copying technology. Both model conclude that the short run effect of copying on social welfare depends on the extent of demand switching versus total demand increase. With advent of copy, some consumers with low marginal cost change to copying, thus inducing welfare loss due to higher reproduction costs compared to the monopolist's. The long run efficiency is measured by number of work created and the social surplus per each work. Since copying technology involves marginal cost and no fixed cost in the first model all producers behave like a single-good monopolist. In the long run, consumers' surplus can fall as revenue losses reducing the number of works. Even with reduction of the number of works, the long run effect in the second model is determined by the value of variety and the elasticity of supply.

Our paper is also closely related to a recent paper by Yoon (2002), who considers a similar model in which he derives the optimal level of copyright protection. With the constant reproduction cost as the policy parameter, in the short run, the effect of increase in IPR is ambiguous; it can be positive or negative depending on the magnitude of the reproduction cost. Therefore, Yoon concludes that the optimal level of copyright protection is either no protection or full protection. For the long-run incentive for development, Yoon assumes that the development cost is fixed. The only measure for the ex ante efficiency with fixed development cost is whether the monopolist develops the new product or not; the monopolist does not introduce a new product if the development cost can not be recovered due to weak protection level. With this type of ex ante efficiency measure, the optimal protection level should be the step function because the incentive of development will be altered with the infinitesimal change of IPR. In our model, to have a continuous effect of the increase in IPR, we assume that the monopolist's long-run incentive is to choose the quality of the software.

The remainder of the paper is organized in the following way: Section 2 sets up the basic model and shows how the monopolist's pricing decision is affected by the threat of piracy. We characterize the pattern of self-selection by consumers and the optimal price for the monopolist. We then conduct a comparative statics exercise that analyzes the effects of increased copyright protection. It is shown that the effects could hinge on how it affects the two margins of the piracy costs discussed above. In Section 3, we extend the model to allow for an endogenous choice of the software quality by producer. The existence of piracy creates inefficiently low quality of the product. Furthermore, we conclude that increased copyright protection can potentially mitigate this inefficiency, whereas, the ex ante effect of increase in IPR also depends on two type of the copyright protection parameters. Section 4 contains concluding remarks.

2. The Model of Piracy: A Short-Run Analysis

Before analyzing the more complex effect of IPR protection on software usage in the short run and development incentive in the long run, we first develop the simplest feasible model of a market with specific software, which is produced by a monopolistic software publisher with IPR. There is a population of consumers whose total number is normalized to unity. Consumers are heterogeneous in their value of using the software. Let v denote a consumer's gross utility of using the software. The distribution of types is given by the *inverse* cumulative distribution function $F(v)$ with continuous density $F'(v) \leq 0$, that is, $F(v)$ denotes the proportion of consumers whose value of the software is *more* than v.

To analyze the *ex post* efficiency effects of piracy, we assume that the software is already developed and the marginal cost of production is zero. The incentives to develop new software are considered later. The copyright holder thus sets the price of the software p to maximize his revenues. As the consumer's utility v is private information, the copyright holder cannot price discriminate and charges a uniform price p.[10]

Optimal Pricing without Piracy

As a benchmark case, we first consider a situation where the option to pirate copyrighted work is not available, that is, the consumers' only choice is whether to purchase or not. The utility of buying an authorized copy is given by $U_B(v;p) = v - p$. We normalize the consumers' payoffs from not using the software to zero. Then, consumers whose value

[10] In a dynamic model, however, the monopolist can price discriminate consumers based on purchase history. See Fudenberg and Tirole (1998) for such an analysis.

of software is more than *p* will purchase the software. We assume that the marginal cost of producing software is zero without any loss of generality.

The purchase behavior of the consumers implies that the copyright holder maximizes his revenue:

$$\max_{p} \; R(p) = p \cdot F(p).$$

Since the monopolist's price *p* is uniquely determined by *v*, we will find it more convenient to treat *v* as the control variable:

$$\max_{v} \; R(v) = v \cdot F(v).$$

The marginal consumer v^* that maximizes the copyright holder's revenue is implicitly given by the first order condition:

$$F(v^*) + v^* \cdot F'(v^*) = 0. \tag{1}$$

We make the standard assumption that the distribution of types satisfies the monotone hazard rate condition, that is, $-F'/F$ is increasing:

$$-F''F + (F')^2 > 0. \tag{2}$$

This assumption ensures that the copyright holder's objective function is quasi-concave and the second order condition for the maximization problem is satisfied:

$$2 \cdot F'(v) + (v - k) \cdot F''(v) < 0.\text{[11]} \tag{3}$$

Then, the number of software users is given by $F(v^*)$. The optimal price of the software for the copyright holder is $p^* = v^*$. Note that the optimal price and the marginal consumer without piracy depends only on the distribution of consumer types $F(\cdot)$.

Needless to say, the socially optimal price for the software, once it is developed, is its marginal cost, which is assumed to be zero. Due to monopolistic pricing, consumers whose types are below v* do not use the software and the deadweight loss is given by $-\int_0^{v^*} x dF(x)$.

[11]Using the first order condition, we can rewrite the second order condition as $2 \cdot F'(v) - F''(v) \cdot F(v) / F'(v) < 0$. The second order condition holds if the distribution *F* satisfies the monotone hazard rate condition. This condition is a standard assumption in the incentive literature and is satisfied by most widely used distributions; see Fudenberg and Tirole [1991, p. 267].

Optimal Pricing with Piracy

Now we introduce the possibility of using the software through piracy without purchasing a legal copy. Piracy saves the price of the software for consumers. However, it entails potentially two types of costs. First, the unauthorized copy may not be a perfect substitute for the legal copy and typically entail some degree of quality degradation. In the case of copying with analog technology before the advent of digital technology, for instance, more iteration of additional copying meant lower quality. Even with digital copying, the unauthorized copy may lack technical supports or access to other resources offered by the manufacturer. We assume that this cost is proportional to the valuation of the consumer for the original, that is, the valuation of the type v consumer for the unauthorized copy is given by $(1-\alpha)\ v$, where is the parameter for quality degradation. Another interpretation is that α can also represent the enforcement efforts by the authority. If illegal copiers are caught with the probability of α, in which case the software is confiscated as punishment, the valuation of using an illegal copy would be given by $(1-\alpha)$. In addition, we assume that illegal copying entails reproduction cost of c, which is assumed to be the same across users. Thus, the utility of using an unauthorized copy is given by $U_{UC}(v) = (1-\alpha)\cdot v - c$. [12]

In order to have a meaningful analysis of unauthorized copying, we restrict our attention to the parameter regions in which the piracy constraint is binding, that is,

$$\frac{c}{1-\alpha} < p^* = v^* \qquad (4),$$

where v^* is defined by (1).[13] This condition is satisfied if the degree of quality degradation α and/or the cost of copying c are not too high.

When the piracy condition (4) is binding, the monopolist's problem is complicated by the fact that some consumers are better off copying the software given the monopolist's price of the software. In response to piracy, the monopolist has two

[12] If we let $w = \alpha v + c$, then $U_{UC}(v) = v - w$. Hence, we can denote w as the gross copy cost.

[13] If the possibility to copy the software does not constrain the monopolist's maximization problem, the marginal type of consumer is given by v^* as defined by (1). The marginal type v^* earns zero surplus when he makes a purchase from the monopolist. If he makes an illegal copy instead, his payoff is $U_{UC}(v^*) = (1-\alpha)v^* - c$. If $c/(1-\alpha) < p^* = v^*$, the surplus from making an illegal copy exceeds the

choices. One option is to *limit price* the software so that copying is not an attractive option. The other option is to price the software to sell only to the highest types and allow copying for intermediate types of customers.

Limit Pricing Regime without Piracy

With the piracy constraint binding ($\frac{c}{1-\alpha} < p^* = v^*$), the limit price should satisfy the following incentive constraint to eliminate the incentives to copy:

$$U_B(v;p) = v - p \geq (1-\alpha)v - c = U_{UC}(v) \text{ for any } v \geq \frac{c}{1-\alpha} \qquad (5)$$

We observe that $U_B(v;p) - U_{UC}(v)$ is increasing in v, which implies that if the constraint above holds for *v*, then it also holds for any *v′* such that *v′*>*v*. Thus, all we need is that the inequality above be satisfied for $v = \frac{c}{1-\alpha}$. This in turn implies that the limit price and the marginal type are given by $p^L = v^L = c/(1-\alpha)$. Notice that the no piracy incentive constraint (5) is always binding under the assumption $\frac{c}{1-\alpha} < p^* = v^*$.

Lemma 1. When the piracy constraint is binding, the optimal limit price that prevents the incentive to copy is given by $p^L = c/(1-\alpha)$. In this case, the monopolist's revenue is given by $R = p^L F(p^L) = \frac{c}{1-\alpha} \cdot F(\frac{c}{1-\alpha})$.

Copying Regime ($p > p^L = c/(1-\alpha)$)

If $p > p^L = c/(1-\alpha)$, the no piracy constraint is violated for some *v*'s that are higher than but close to $v^L = c/(1-\alpha)$. Each consumer has two different choices for using the software, which incur two different types of cost. First, when a consumer buys a legal copy from the monopolist, he has to pay the price (*p*) and enjoys the full quality of the software. However, with choice of making an illegal copy, his cost will be the sum of degradation of quality, which is proportional to his own valuation of the software, and

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one from other options. Thus, possibility to copy the software serves as a binding constraint for the monopolist's revenue maximization problem.

constant reproduction cost. We assume at this time that the parameters of IPR (α for the degradation rate and c for the reproduction cost) are fixed.

When consumers make their usage decision, they choose the one, which yields the highest net utility. For a given price of a legal copy p ($> p^L = c/(1-\alpha)$) and the level of IPR, consumers' optimal choices can be shown as follows:

$$\frac{p-c}{\alpha} \le v \qquad \text{purchase a legal copy}$$

$$\frac{c}{1-\alpha} \le v < \frac{p-c}{\alpha} \qquad \text{make an illegal copy}$$

$$v < \frac{c}{1-\alpha} \qquad \text{no use.}$$

Now the monopolist should take into account that potential consumers have another option to obtain the software. The monopolist, therefore, maximizes

$$Max\ \ pF(\frac{p-c}{\alpha}).$$

Once again, we treat the marginal consumer type $v = \dfrac{p-c}{\alpha}$ as the control variable:

$$Max\ (\alpha v + c)F(v).$$

The first order condition

$$(\alpha v + c)F'(v) + \alpha F(v) = 0 \qquad (6)$$

determines the marginal type of consumer $\tilde{v}$, who is indifferent from purchasing an authorized copy from the monopolist and making an unauthorized copy.[14] Therefore, with the option of making an illegal copy, consumers with low value $v < c/(1-\alpha)$ do not use the software. Those with intermediate value $c/(1-\alpha) \le v < \tilde{v}$ make illegal copies. Only consumers with high value $v < \tilde{v}$ purchase legal copies from the monopolist [see Figure 1].

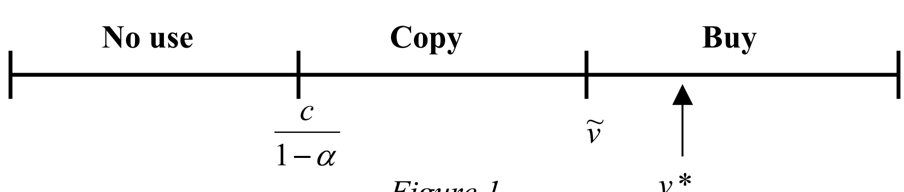


Figure 1.

We now can compare the monopolist's pricing behavior with and without piracy.
Proposition 1. With the possibility of piracy, the price of the software is lowered, thereby inducing more demand for *legal* copies. Increase in usage of both legal and illegal copies under the copying regime brings higher ex post usage for the software.

Proof. Evaluate (5) at v^* which is the marginal consumer when no copying is feasible: $\alpha[F(v^*)+v^*F'(v^*)]+cF'(v^*)=cF'(v^*)<0$. Hence, $v^*>\tilde{v}$.

Under the copying regime, there exist consumers, whose value lies between $c/(1-\alpha)$ and $\tilde{v}$, making copies. Therefore, total ex post usage for the software is unambiguously increased with piracy. Being just a threat (limit pricing regime) or an actual fact (copying regime), piracy has the same effect on the monopolist's pricing behavior: the price is lower than the one in the benchmark case. A more surprising result is that the usage of *legal* copies increases even in the presence of copying. Thus, the extent to which legal copies are used is *complementary* with the extent of the usage of unauthorized copies. The intuition for this result can be found in the monopolist's pricing behavior in response to the threat of unauthorized copying. If there were no price change, that is, at $p=p^*=v^*$ defined in (1), some of the previous purchasers of the legal copy will switch to the option of copying with the result of a lower number of legal copies being sold. Proposition 1, however, shows that the price reduction by the monopolist (from p^* to $\tilde{p}=\alpha\tilde{v}+c$) not only eliminates the incentives to switch for the previous buyers but also expands the base of buyers.

Under the copying regime consumers' willingness to pay for the software is not equal to their valuations. Since they have another option, which is piracy, consumers choose to make an illegal copy if the price exceeds the gross copy cost ($w=\alpha v+c$). Therefore, the monopolist's demand is now determined by the gross copy cost, which consists of the constant reproduction cost and the proportional degradation rate. Due to the existence of piracy the monopolist now has less market power, which results in lower price for legal copies. Proposition 2 gives an extreme but intuitive fact about the ex post social welfare.

[14] Variables under the copying regime are denoted by a tilde ($\tilde{v}$).

Proposition 2. If the cost of piracy is only of degradation, the number of legal users is the same regardless of the possibility of piracy. There is no switch. The price of the software is lower with the threat of piracy and depends on the degradation factor $p = \alpha\, p^*$. Ex post social welfare is increased with the possibility of piracy since there are illegal users who are pure addition to the number of total users.

Comparative Statics

We now analyze the effects of marginal increase in the intellectual property rights. As with the previous studies in the literature (Novos and Waldman [1984], Yoon (2002), etc), we model the increase in the intellectual property rights as an increase in the cost of piracy. It is shown that the effects can have different implications depending on which regime the monopolist is operating under and the type of costs associated with piracy.

Proposition 3. Under the limit pricing regime, both types of an increase in IPR induces higher software price and less authorized usage.

Proof. Under the limit pricing regime, we have $p^L = c/(1-\alpha)$, and $q^L = F(p^L)$. If we take partial derivatives of p^L and $F(p^L)$ with respect to c and α respectively, we have the following results:

$$\frac{\partial p^L}{\partial c} = \frac{1}{1-\alpha} > 0\,,\ \frac{\partial F(p^L)}{\partial c} = \frac{\partial F(p^L)}{\partial p^L}\frac{\partial p^L}{\partial c} < 0\,,$$

$$\frac{\partial p^L}{\partial \alpha} = \frac{c}{(1-\alpha)^2} > 0\text{, and } \frac{\partial F(p^L)}{\partial \alpha} = \frac{\partial F(p^L)}{\partial p^L}\frac{\partial p^L}{\partial \alpha} < 0\,.$$

The intuition underlying Proposition 3 is straightforward. Due to the possibility of piracy, the monopolist is not able to charge the monopoly price. However, the monopolist modifies his pricing behavior according to the relative magnitude of IPR, in which he can eliminate piracy. The maximum price he can charge under limit pricing is $p^L = c/(1-\alpha)$, which depends on the levels of the degradation rate (α) and the reproduction cost (c). The marginal increase in IPR from either the degradation rate or the reproduction cost provides the monopolist more market power, allows him to charge a higher price.

Proposition 4. Under the copying regime, as expected, the monopoly price increases with the strengthening of IPR. The effects of increase in IPR on the usage of software is ambiguous, depending on the types of costs associated with piracy. Higher degradation rate induces a higher software price and *less* authorized usage. With higher reproduction cost, the price of the software will increase and *more* authorized usage [see table 1].

Proof. Total differentiation of the first-order condition with respect to c:

$$[2\alpha F'(\tilde{v}) + \alpha \tilde{v} F''(\tilde{v}) + cF''(v)]dv = -F'(\tilde{v})dc .$$

$$\frac{d\tilde{v}}{dc} = \frac{-F'(\tilde{v})}{|H|} < 0$$

where $|H| = 2\alpha F'(\tilde{v}) + \alpha \tilde{v} F''(\tilde{v}) + cF''(v) < 0$ by the second-order condition.

Total differentiation of the first-order condition with respect to α :

$$[2\alpha F'(\tilde{v}) + \alpha \tilde{v} F''(\tilde{v}) + cF''(v)]dv = -(F(\tilde{v}) + \hat{v}F'(\tilde{v}))d\alpha .$$

$$\frac{d\tilde{v}}{d\alpha} = \frac{-(F(\tilde{v}) + \tilde{v}F'(\tilde{v}))}{|H|} = \frac{1}{|H|}\frac{cF'(\tilde{v})}{\alpha} > 0 .$$

Since the monopolist's demand is determined by the gross copy cost under the copying regime, the effects of increase in IPR depend on the types of costs associated with piracy. With higher reproduction cost, all consumers face the same increase in the gross copy cost, which is equivalent to an outward *parallel* shift in demand for legal copies. With an increased demand, the monopolist responds with a price hike. The price increase, however, does not completely offset the initial demand increase with the result of increased sales. In contrast, if an increase in IPR is derived from higher degradation rate, we observe a *pivot* change in demand that affects the *slope* of the demand curve for legal copies. Due to proportional increase in the gross copy cost, higher valuation consumers are more affected by an increase in the degradation cost. A steeper demand curve means that elasticity of consumers is lower with more market power. Thus, the monopolist is more interested in serving only the high valuation consumers.

Compared to the targeted copyright enforcement model by Harbaugh and Khemka (2001), an increase in IPR from higher degradation rate illustrates similar effects, but different results. In their model, with price discrimination, the copyright enforcement only affects the high value consumers' copy costs, which induces increase in both the monopolist's revenue and the consumer surplus of the low type. In our model, higher degradation rate imposes higher copy cost for the high type, which has the similar effects,

	Benchmark	Limit Pricing	Copying Regime
The monopolist's optimal choice	v^* p^*	v^L $(v^* > v^L)$ p^L $(p^L < p^*)$	$\tilde{v}$ $(v^* > \tilde{v})$ $\tilde{p}$ $(\tilde{p} < p^*)$
An increase in the reproduction cost	*N/C*	$\frac{\partial v^L}{\partial c} > 0$ and $\frac{\partial p^L}{\partial c} > 0$	$\frac{\partial \tilde{v}}{\partial c} < 0$ and $\frac{\partial \tilde{p}}{\partial c} > 0$
An increase in the degradation rate	*N/C*	$\frac{\partial v^L}{\partial \alpha} > 0$ and $\frac{\partial p^L}{\partial \alpha} > 0$	$\frac{\partial \tilde{v}}{\partial \alpha} > 0$ and $\frac{\partial \tilde{p}}{\partial \alpha} > 0$

Table 1

but also affects the lower value consumers without price discrimination. Therefore, it increases monopolist's revenues, but less piracy.

Welfare Effects of Increase in IPR in the Short Run

We are now in position to examine the effects of increase in IPR on the social welfare. In the benchmark case, the piracy constraint is not binding ($\frac{c}{1-\alpha} > p^* = v^*$), therefore, piracy is not feasible. Since the software publisher behaves as an unconstrained monopolist, both types of increase in IPR do not have welfare effects.

When the monopolist's optimal choice is limit pricing, it is straightforward to show the effect of increase in IPR on social welfare. As either the degradation or the reproduction cost increases, making an illegal copy becomes less attractive or more expensive. To response to this, the monopolist is able to charge a higher price and fewer consumers use a legal copy. Since the increased profit margin is only a monetary transfer from the consumers to the monopolist, decline in social welfare is resulted from less authorized usage.

Proposition 5. Under the limit pricing regime, both types of increase in IPR induces lower social welfare.

Proof. With limit pricing, the social welfare is identical to the gross consumers' surplus from authorized usage:

$$SW(p^L) = -\int_{v^L}^{\infty} v \cdot F'(v)dv = v^L F(v^L) + \int_{v^L}^{\infty} F(v)dv \, .$$

We examine the effect of increase of IPR on social welfare as

$$\frac{\partial SW}{\partial c} = \frac{c}{(1-\alpha)^2} F'(\frac{c}{1-\alpha}) < 0 \text{, and}$$

$$\frac{\partial SW}{\partial \alpha} = \frac{c^2}{(1-\alpha)^3} F'(\frac{c}{1-\alpha}) < 0 .$$

If the monopolist's optimal choice is accommodating piracy, the welfare effects of increase in IPR depend on the types of costs associated with piracy. As expected, both types of increases in IPR make piracy more costly. In addition to this effect, these costs have two different channels to affect the social welfare: demand switch and total usage change. For the increase in the type-independent reproduction cost, it is equivalent for the monopolist an outward parallel shift in demand. To response to this, his optimal reaction will be charging higher price to the extent where it still induces more sales. Additional increase to the monopolist's demand will increase the welfare, because of change to more efficient way of production [positive demand switch]. Second, it decreases the social welfare because with higher gross copy cost, the number of consumers who are making copy shrinks [total usage change]. Therefore, the welfare effects of increase in IPR by higher reproduction cost depend on the size of those two countervailing effects.

An increase in IPR with higher degradation rate shows two different channels but the same negative effects on the social welfare. First, since higher degradation rate enhances the monopolist's market power, he charges a higher price with reduction in sales. Some consumers who previously were legitimate users now are forced to choose piracy. Additional deduction from the monopolist's demand will decrease the welfare because of the change to a less efficient way of production [negative demand switch]. Second, it will also decrease the social welfare because, with higher gross copy cost, the number of consumers who are making copy contracts [total usage change].

Proposition 6. Under the copying regime, the effects on social welfare of increase in IPR depend on the types of costs associated with piracy.

Proof. The social welfare can be derived from the sum of the monopolist's revenue and the consumer's surplus:

$$SW(\tilde{p}) = R(\tilde{p}) + CS(\tilde{p})$$

$$= (\alpha\tilde{v} + c)F(\tilde{v}) - \int_{\tilde{v}}^{\infty}(v - \alpha\tilde{v} - c)F'(v)dv - \int_{\frac{c}{1-\alpha}}^{\tilde{v}}[(1-\alpha)v - c]F'(v)dv$$

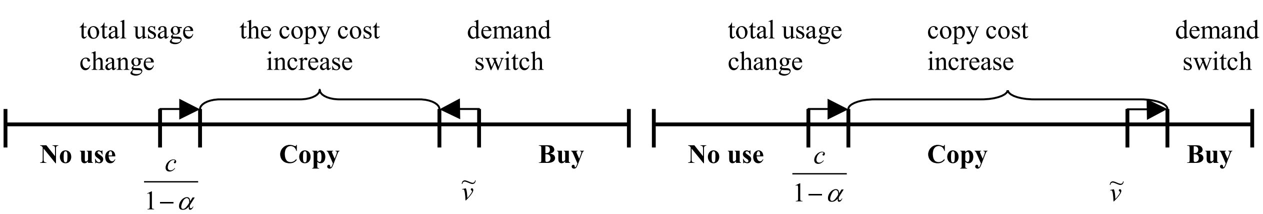
$$= \tilde{v}F(\tilde{v}) + \int_{\tilde{v}}^{\infty}F(v)dv - \int_{\frac{c}{1-\alpha}}^{\tilde{v}}[(1-\alpha)v - c]F'(v)dv\ .$$

We examine the effect of increase of IPR on social welfare as

$$\frac{\partial SW(\tilde{p})}{\partial c} = [\tilde{v} - ((1-\alpha)\tilde{v} - c)]F'(\tilde{v})\frac{\partial\tilde{v}}{\partial c} + \int_{\frac{c}{1-\alpha}}^{\tilde{v}}F'(v)dv \begin{matrix} < 0 \\ > 0 \end{matrix}, \text{ and}$$

$$\frac{\partial SW(\tilde{p})}{\partial \alpha} = [\tilde{v} - ((1-\alpha)\tilde{v} - c)]F'(\tilde{v})\frac{\partial\tilde{v}}{\partial \alpha} + \int_{\frac{c}{1-\alpha}}^{\tilde{v}}vF'(v)dv < 0\ .$$

From the above equations, we can separate two different channels. The first part of the equations represent demand switch, which induces welfare gain or loss from the change of the production methods. The second part identifies an increase in consumers gross copy cost caused by an increase in either type of cost. It decreases social welfare because consumers who pirate an official copy have to endure a higher cost. On the other hand, since the effect on the marginal consumer ($\frac{c}{1-\alpha}$), who is indifferent from no use and piracy, is the second-order, the above equations do not illustrate the effect of total usage change on social welfare [see figure 2].



(a) welfare effect of higher reproduction cost

(b) welfare effect of higher degradation rate

Figure 2

Uniform Distribution Example

We now illustrate our results using a simple uniform distribution that generates a linear demand curve for the monopolist. This example also allows a closed-form solution for welfare analysis. For example, we now assume that consumers' evaluations for the software are uniformly distributed over the unit interval, such as $v_i \in U[0, 1]$. However, other assumptions, such as universal reproduction cost and proportional degradation cost, remain the same. Then, given the monopolist's price and two different types of IPR protection, we derive two types of marginal consumers. The first marginal type, who is indifferent between making an illegal copy and no use, is denoted by $\hat{v}(copy, no\ use) = c/(1-\alpha)$. The second marginal type, who is indifferent between buying a legal copy and making an illegal copy, is denoted by $\hat{v}(copy, buy) = (p-c)/\alpha$. If $\hat{v}(no\ use, copy) > \hat{v}(copy,\ buy)$, we have the benchmark case in which piracy is not possible. In the case of $\hat{v}(no\ use, copy) = \hat{v}(copy,\ buy)$ the monopolist lowers the price enough to deter piracy. The last case is $\hat{v}(no\ use, copy) < \hat{v}(copy, buy)$, in which the monopolist accommodates piracy. Hence, the optimal choices of consumers can be summarized as follows:

Benchmark case (No piracy): with $p < c/(1-\alpha)$, only consumers whose values satisfy $p < v < 1$ buy legal copies, and no piracy is possible.

Limit pricing regime: with $p = c/(1-\alpha)$, consumers who belong to $c/(1-\alpha) < v < 1$ buy official copies, and the monopolist's price is low enough to blockade piracy.

Copying regime: with $p > c/(1-\alpha)$, consumers who belong to $(p-c)/\alpha < v < 1$ purchase authorized copies, and those who belong to $c/(1-\alpha) < v < (p-c)/\alpha$ use the software by making illegal copies.

For the benchmark case, where the option to copy the software is not feasible, consumer's choice is whether to purchase or not. With the uniformly distributed consumers' valuation, the monopolist's demand is $1 - v^*$ where the marginal consumer's valuation is $v^* = p$. The monopolist's problem is to choose the price p of the legitimate copy is as to maximize his revenue:

$$\max\ R(p) = p(1-p) = v(1-v).$$

The first order condition

$$1 - 2v = 0$$

determines the marginal consumer $v^* = p^* = 1/2$. Therefore, when $(1-\alpha)/2 < c \le 1-\alpha$, the monopolist's optimal choice is to charge the monopoly price without piracy.[15]

We now turn to the monopolist's optimal pricing problem with piracy. The first option for the monopolist is to accommodate piracy in which the monopolist sets a higher price and tolerates copying. In this case, the monopolist's objective becomes:

$$Max\ p(1-\frac{p-c}{\alpha}).$$

Once again, we treat the marginal consumer type $v = \dfrac{p-c}{\alpha}$ as the control variable:

$$Max\ (\alpha v + c)(1-v).$$

The first order condition

$$\alpha(1-v) - (\alpha v + c) = 0$$

yields $\tilde{v} = \dfrac{1}{2} - \dfrac{c}{2\alpha}$ and $\tilde{p} = \dfrac{(\alpha + c)}{2}$. Comparing v^* and $\tilde{v}$ in the uniform distribution example we can easily verify Proposition 1. As a result, when $0 < c \le \alpha(1-\alpha)/1+\alpha$, the monopolist's optimal choice is to allow piracy and charge a higher price.[16]

The second option is for the monopolist to eliminate piracy by setting the price sufficiently low. Since the monopolist should reduce the price until the piracy constraint is binding, $(1-\alpha)v - c = v - p$, the optimal price and revenues are $p^L = c/(1-\alpha)$, and $\pi^L = p^L(1-p^L)$. Therefore, when $\alpha(1-\alpha)/(1+\alpha) < c \le (1-\alpha)/2$, the monopolist's optimal choice is to lower the price until consumers do not have any incentive to make an illegal copy. For the last remark we can illustrate the monopolist's optimal regime change depending on the relative magnitude of IPR parameters [see figure 3].

[15] If we substitute $v^* = p^* = 1/2$ into the consumers' optimal behavior condition under the benchmark case $p < c/(1-\alpha)$, we have $(1-\alpha)/2 < c$. If $c > 1-\alpha$, the gross copy cost exceeds the valuation for the software, which is not a meaningful case to consider.

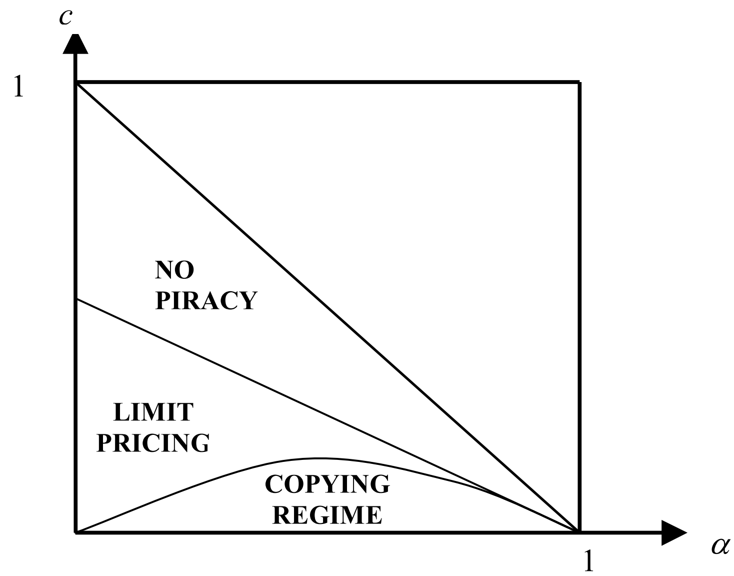


Figure 3

With liner demand and closed form solutions the effect of increased copyright protection can be shown more clearly in the uniform distribution example. Under the limit pricing regime, we can verify that both types of increase in IPR induces higher software price and less usage. With the optimal choice of price ($p^L = \dfrac{c}{1-\alpha}$) and quantity ($q^L = 1 - p^L$) under the limit pricing, we can calculate as follows:

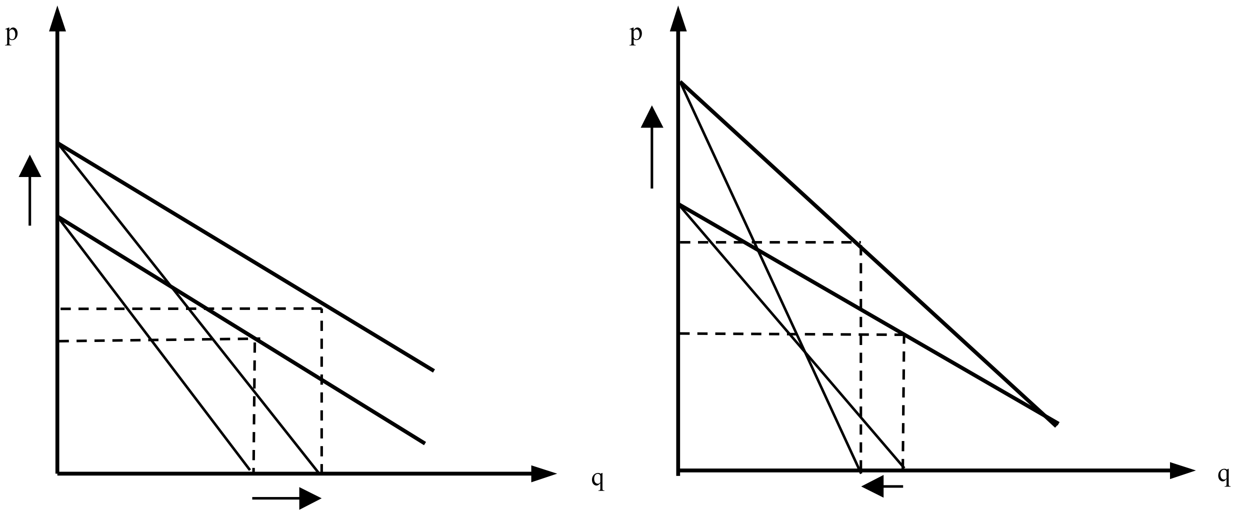
$$\frac{\partial p^L}{\partial c} = \frac{1}{1-\alpha} > 0, \quad \frac{\partial q^L}{\partial c} = \frac{-1}{1-\alpha} < 0$$

$$\frac{\partial p^L}{\partial \alpha} = \frac{c}{(1-\alpha)^2} > 0, \text{ and } \frac{\partial q^L}{\partial \alpha} = \frac{-c}{(1-\alpha)^2} < 0.$$

Under the copying regime, it is observed that the effects of increase in IPR depend on the types of costs associated with piracy [see figure 4]. As we expect, an increase in both types of costs trigger a higher price but ambiguous effect on demand switch. These effects are clearly shown as

$$\frac{\partial p}{\partial c} = \frac{1}{2} > 0, \quad \frac{\partial q}{\partial c} = \frac{1}{2\alpha} > 0, \quad \frac{\partial p}{\partial \alpha} = \frac{1}{2} > 0, \text{ and } \frac{\partial q}{\partial \alpha} = \frac{-2c}{4\alpha^2} < 0.$$

[16] If we substitute $p^* = (\alpha + c)/2$ into the consumers' optimal behavior condition under the copying regime $p > c/(1-\alpha)$, we have $0 < c \le \alpha(1-\alpha)/(1+\alpha)$.



(a) the effect of reproduction cost increase (b) the effect of the degradation rate increase

Figure 4

For the last part of the short-run analysis, we examine welfare effects of increase in IPR. If $\alpha(1-\alpha)/(1+\alpha) < c \le (1-\alpha)/2$, we identify that the monopolist prefer to eliminate piracy by lowering the price; in other words, the possibility of piracy enforce the monopolist set a lower price as $p^L = c/(1-\alpha)$. It is straightforward to show the effect of increase of IPR on social welfare. Consumer surplus and the monopolist's revenue under the limit pricing are computed as follows: $CS^L = \int_{\frac{c}{1-\alpha}}^{1} (x - \frac{c}{1-\alpha})dx$, and $\pi^L = p^L(1-p^L)$. Defining social welfare as $SW^L = CS^L + \pi^L = \frac{1-2\alpha+\alpha^2-c^2}{2(\alpha-1)^2}$ we observe that

$$\frac{\partial SW^L}{\partial c} = \frac{-c}{(\alpha-1)^2} < 0, \text{ and } \frac{\partial SW^L}{\partial \alpha} = \frac{c^2}{(\alpha-1)^3} < 0.$$

If $0 < c \le \alpha(1-\alpha)/1+\alpha$, the welfare effects of increase in IPR depend on the types of costs associated with piracy. The consumer surplus and the monopolist's revenue in the copying regime are given by

$$CS(\tilde{p}) = \int_{\frac{c}{1-\alpha}}^{\frac{p-c}{\alpha}} ((1-\alpha)x - c)dx + \int_{\frac{p-c}{\alpha}}^{1} (x-p)dx, \text{ and } \tilde{\pi} = \tilde{p}(1 - \frac{\tilde{p}-c}{\alpha}).$$

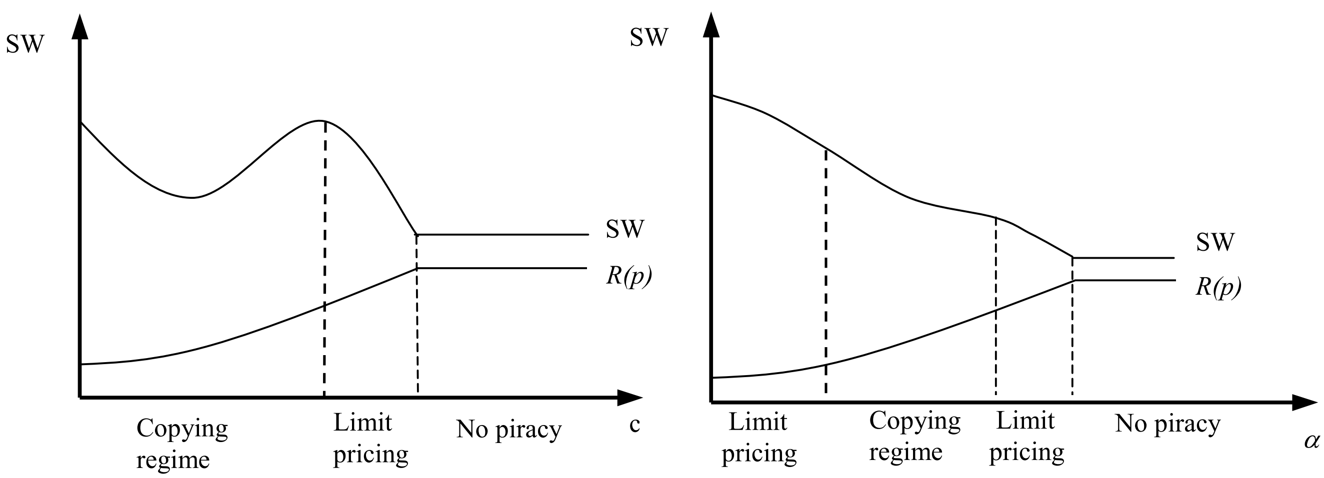
We then calculate social welfare as $SW(\tilde{p}) = CS(\tilde{p}) + \tilde{\pi} = \frac{1}{2} - \frac{\alpha}{8} - \frac{c}{4} + \frac{c^2}{2(1-\alpha)} + \frac{3c^2}{8\alpha}$.

With positive demand switch and negative total usage change, the effect of an increase in IPR with higher reproduction is uncertain. However, with negative demand switch and negative total usage change we are able to pin down the effect of an increase in IPR with higher degradation rate. These results can be illustrated by the following two partial derivatives [see figure 5].

$$\left.\frac{\partial SW}{\partial \alpha}\right|_{c \text{ is fixed}} = -\frac{1}{8} + \frac{c^2}{2(1-\alpha)^2} - \frac{3c^2}{8\alpha^2} < 0 \text{, and}$$

$$\left.\frac{\partial SW}{\partial c}\right|_{\alpha \text{ is fixed}} = -\frac{1}{4} + \frac{c}{1-\alpha} + \frac{3c}{4\alpha}.$$ [17]

To determine the sign of $\frac{\partial SW}{\partial c}$, let $\hat{c}$ be the critical value, which satisfies $\frac{\partial SW}{\partial c} = 0$ and we have $\hat{c} = \frac{\alpha(1-\alpha)}{3+\alpha}$. Hence, if $c < \hat{c}$, we have $\frac{\partial SW}{\partial c} < 0$. Otherwise, we observe $\frac{\partial SW}{\partial c} > 0$.



(a) the effect of increase in the reproduction cost

(b) the effect of increase in the degradation rate

Figure 5

[17] Since we have parameter region for the copy regime such as $c \le \frac{\alpha(1-\alpha)}{1+\alpha}$, we can verify that $\frac{\partial SW}{\partial \alpha} < 0$.

3. Copyright Protection and Incentives to Create

Up to now, we have analyzed the effects of copyright protection on pricing and the incentives to pirate once the software has been produced. In this section, we analyze the long-term effects of intellectual property rights on the incentive to create. To analyze this issue, we introduce the cost of creating the software and endogenize the quality of the software. Let θ measure the quality of software that is created at cost $C(\theta)$ [with $C'(\theta) > 0, C''(\theta) > 0$]. The quality of the good enters positively into the utility of consumers; the utility of consumer of type v is $\theta \cdot v$.

We consider now the long-term effects of piracy in which the monopolist decides not only on the pricing of the software but also on the quality of the software. In this case, the presence of piracy may lead to decreased revenue, which in turn may adversely affect the quality and quantity of the software. We continue to assume that the marginal cost of software is fixed at zero regardless of the quality of the software once it has been developed.

Software Quality without Piracy

Before analyzing the monopolist's quality provision with the possibility of piracy, we start with the benchmark case, in which consumers face a monopolistic software publisher but do not have the opportunity to make an illegal copy. Hence, the consumers' only decision is whether to purchase or not. Since the consumers' payoff from not using the software is zero, consumers with non-negative net utilities purchase an official copy: $\theta \cdot v - p \geq 0$. Given the purchase decision of consumers, the copyright holder uses the price and the level of quality to maximize his profit:[18]

$$\max_{v,\theta} \ \pi \equiv \theta \cdot v \cdot F(v) - C(\theta).$$

The first order conditions

$$\frac{\partial \pi}{\partial v} = \theta \cdot F(v) + \theta \cdot v \cdot F'(v) = 0 \tag{7}$$

$$\frac{\partial \pi}{\partial \theta} = v \cdot F(v) - C'(\theta) = 0 \tag{8}$$

determines the marginal consumer v^* and the monopolist's optimal level of quality θ^*.

[18] Again, we use v as a control instead of p.

Proposition 7. Given the number of software users $F(v^*)$, the quality of the software is sub-optimally low.

Proof. Given the number of software user $F(v^*)$, i.e. all consumers of type $v \geq v^*$ buying the software, the socially optimal quality of the software can be found by solving

$$\max_{\theta} \; -\int_{v^*}^{\infty} \theta \cdot v \cdot F'(v)dv - C(\theta).$$

The first order condition

$$-\int_{v^*}^{\infty} v \cdot F'(v)dv - C'(\theta^{opt}) = 0$$

determines the socially optimal level of quality θ^{opt}. Integration by parts of the first term on the left hand side shows $-\int_{v^*}^{\infty} v \cdot F'(v)dv \geq v^* \cdot F(v^*)$. As we have $v^* \cdot F(v^*) = C'(\theta^*)$ from (7), marginal benefits and marginal costs of the quality of the software when provided by the monopolist are below the socially optimal level: $C'(\theta^{opt}) \geq C'(\theta^*)$. Therefore, the level of the software quality is sub-optimally low: $\theta^{opt} \geq \theta^*$.

The intuition for this result is the following. The choice of θ^* by the monopolist is determined by the marginal type *v**. An increase in the benefit of the marginal consumer is captured via higher price of the software by the copyright holder. The effect on the inframarginal consumers is irrelevant for the monopolist as he cannot price discriminate between consumers. In contrast, the second-best level θ^{opt} is determined by the aggregate (or average) benefits for all consumers with values $[v^*, \infty)$. As the average consumer's valuation for the software is higher than the one of the marginal consumer, the second-best level of quality of the software exceeds the one provided by the monopolist.[19]

Software Quality with Piracy

Now we turn to the monopolist's choice of the software quality, when he faces piracy: how does the potential threat or actual piracy affect the monopolist's choice of the software quality? To answer this question, we use the previous optimal pricing framework with the monopolist's choice of the software quality. We still assume there are two different types of cost associated with piracy: the constant reproduction cost and the proportional degradation rate. The degradation rate now affects the valuation of the type v consumer for the unauthorized copy as $(1-\alpha)\cdot\theta\cdot v$. Thus, the utility of using an unauthorized copy is given by $U_{UC}(v)=(1-\alpha)\cdot\theta\cdot v-c$.

In order to have a meaningful analysis of unauthorized copying, we have the same restriction to the parameter regions, in which the piracy constraint is binding, that is,

$$\frac{c}{1-\alpha}<p^*=\theta^*v^* \qquad (9),$$

where θ^* and v^* are defined by (6) and (7). This condition is satisfied if the degree of quality degradation α and/or the cost of copying c are not too high. Therefore, with the binding constraint (8), the monopolist practices ether limit pricing or accommodation to piracy.

Software Quality under Limit Pricing Regime

With the piracy constraint (8) binding, the monopolist faces the constrained profit maximization problem as following:

$$Max\ \pi^L=p^L F(\frac{p^L}{\theta})-C(\theta)$$

Subject to

$$(1-\alpha)\theta v-c\le\theta v-p$$

Since the constraint is always binding under the assumption $\frac{c}{1-\alpha}<p^*=\theta^*v^*$, the optimal price $p^L=\frac{c}{1-\alpha}$, and revenue is $R^L=(\frac{c}{1-\alpha})F(\frac{c}{(1-\alpha)\theta})-C(\theta)$.

The monopolist now determines the optimal choice of the software quality with the optimal limit price as following:

$$\underset{\theta}{Max}\,\pi^L=(\frac{c}{1-\alpha})F(\frac{c}{(1-\alpha)\theta})-C(\theta).$$

The first order condition

[19] This point is closely related to a monopolist's choice on product quality; see Spence (1975) and Tirole (1988, pp. 100-102).

$$\frac{\partial \pi^L}{\partial \theta} = -\frac{c^2}{(1-\alpha)^2 \theta^2} F'(\frac{c}{(1-\alpha)\theta}) - C'(\theta) = 0 \qquad (10)$$

determines the monopolist's optimal choice of software quality.

Proposition 8. Under the limit pricing regime, the monopolist chooses a lower level of quality than the one without piracy.

Proof. Let us evaluate the first order condition (9) at θ* which is the level of quality without piracy.

$$\left.\frac{\partial \pi^L}{\partial \theta}\right|_{\theta^*} = -\frac{c^2}{(1-\alpha)^2 \theta^{*2}} F'(\frac{c}{(1-\alpha)\theta^*}) - C'(\theta^*)$$

We know that

$$-\frac{c^2}{(1-\alpha)^2 \theta^{*2}} F'(\frac{c}{(1-\alpha)\theta^*}) < \frac{c}{(1-\alpha)\ \theta^*} F(\frac{c}{(1-\alpha)\theta^*}) < v^* F(v^*)$$

The inequalities above follow from the fact that $vF(v)$ is a concave function which is maximized at v^* and our assumption that $\frac{c}{1-\alpha} < p^* = \theta^* v^*$. Thus, we have $\left.\frac{\partial \pi^L}{\partial \theta}\right|_{\theta^*} < v^* F(v^*) - C'(\theta^*) = 0$, which implies that $\theta^L < \theta^*$.

Software Quality under the Copying Regime

Under the copying regime consumers compare the payoffs from buying an authorized copy $[\theta \cdot v - p]$ or making an unauthorized copy $[\alpha \cdot \theta \cdot v - c]$. Given the purchase behavior of consumers, the monopolist maximizes his profit:

$$\max_{v,\theta}\ \tilde{\pi} = (\alpha \cdot \theta \cdot v + c) F(v) - C(\theta).$$

The first order conditions

$$\frac{\partial \tilde{\pi}}{\partial v} = (\alpha \theta\ v + c) F'(v) + \alpha \theta F(v) = 0 \qquad (11)$$

$$\frac{\partial \tilde{\pi}}{\partial \theta} = \alpha\, v F(v) - C'(\theta) = 0 \qquad (12)$$

again determine the marginal consumer $\tilde{v}$ and the software quality $\tilde{\theta}$. How does the existence of piracy as another option for consumers affect the outcome? There is a clear-cut answer to this question:

Proposition 9. Under the copying regime the existence of piracy leads to a further underprovision of the software quality but more authorized usage.

Proof. Evaluating (11) at $v = v^*$ yields $(\alpha\theta\, v^* + c)F'(v^*) + \alpha\theta\, F(v^*) = cF'(v^*) < 0$. Hence we have $\tilde{v} < v^*$. Also, evaluating (12) at $v = v^*$ yields $-v^* F(v^*)(1-\alpha) < 0$ and therefore $\tilde{\theta} < \theta^*$.

The intuition underlying this result is the following. The choice of $\tilde{\theta}$ by the monopolist is determined by the marginal type $\tilde{v}$. At the development stage of the software the monopolist expects that the marginal consumer is not v^* but $\tilde{v}$ when there is piracy. Given this anticipation, the optimal choice of the software quality should be lower than the one from the benchmark case. As we already identify from Proposition 9, the decision on the software quality by the monopolist already leads to an underprovision ($\theta^* < \theta^{opt}$). The existence of piracy aggravates the inefficient provision of the software quality. Our result thus lends theoretical support for the claim that piracy reduces the incentives to develop new software.

Comparative Statics

From the previous copying regime we observe that the monopolist has less incentive to provide higher quality of the software due to piracy. We now turn to analysis of the effects of marginal increase in IPR on the monopolist's development incentive.

Proposition 10. Under the limit price regime, both types of increases in IPR induce higher software quality and less authorized usage

Proof. We can rewrite (9) as $c^2 F'(v) + (1-\alpha)^2 \theta^2 C'(\theta) = 0$. By totally differentiating the first-order condition, we have

$$2cF'(v)dc + (1-\alpha)^2 \theta^2 C''(\theta) d\theta + (1-\alpha)^2 2\theta C'(\theta) d\theta = 0.$$

$$\frac{d\theta}{dc} = -\frac{2cF'(v)}{(1-\alpha)^2 \theta [\theta C''(\theta) + 2C'(\theta)]} > 0.$$

By totally differentiating the first-order condition, we have

$$-2(1-\alpha)\theta^2 C'(\theta) d\alpha + (1-\alpha)^2 \theta^2 C''(\theta) d\theta + (1-\alpha)^2 2\theta C'(\theta) d\theta = 0$$

$$\frac{d\theta}{d\alpha} = -\frac{2(1-\alpha)\theta^2 C'(\theta)}{(1-\alpha)^2 \theta [\theta C''(\theta) + 2C'(\theta)]} > 0.$$

Also we can easily verify that

$$\frac{dv}{dc}=\frac{d(\frac{c}{(1-\alpha)\theta})}{dc}=\frac{1}{(1-\alpha)\theta}>0\text{, and}$$

$$\frac{dv}{d\alpha}=\frac{d(\frac{c}{(1-\alpha)\theta})}{d\alpha}=\frac{c}{(1-\alpha)^2\theta}>0.$$

The intuition underlying Proposition 10 is straightforward. Due to the potential threat of piracy, the only option, in which monopolist can eliminate piracy, is to lower his price until $(1-\alpha)\theta v-c\le\theta v-p$ is binding. The maximum price he can charge under limit pricing is $p^L=c/(1-\alpha)$, which depends on the relative level of the degradation rate (α) and the reproduction cost (c). Increases in IPR from either the degradation rate or the reproduction cost induce less authorize usage, which is equivalent to higher valuation from the marginal consumer $\tilde{v}$. This drives monopolist has more incentive to provide higher quality.

Proposition 11. Under the copying regime the effects of increase in IPR depend on the types of costs associated with piracy. Higher degradation rate induces higher quality and less official usage. In contrast, higher reproduction cost results in lower quality and more authorized usage.

Proof. By totally differentiating (11) and (12), we have

$$\begin{bmatrix}\frac{\partial^2\pi}{\partial v^2} & \frac{\partial^2\pi}{\partial v\partial\theta}\\ \frac{\partial^2\pi}{\partial\theta\,\partial v} & \frac{\partial^2\pi}{\partial\theta^2}\end{bmatrix}\begin{bmatrix}\frac{dv}{dc}\\ \frac{d\theta}{dc}\end{bmatrix}=\begin{bmatrix}-\frac{\partial^2\pi}{\partial v\partial c}\\ -\frac{\partial^2\pi}{\partial\theta\,\partial c}\end{bmatrix}.$$

By using Cramer's rule, we have

$$\frac{dv}{dc}=\frac{1}{|H|}\begin{vmatrix}-\frac{\partial^2\pi}{\partial v\partial c} & \frac{\partial^2\pi}{\partial v\partial\theta}\\ -\frac{\partial^2\pi}{\partial\theta\,\partial c} & \frac{\partial^2\pi}{\partial\theta^2}\end{vmatrix}\text{, and }\frac{d\theta}{dc}=\frac{1}{|H|}\begin{vmatrix}\frac{\partial^2\pi}{\partial v^2} & -\frac{\partial^2\pi}{\partial v\partial c}\\ \frac{\partial^2\pi}{\partial\theta\,\partial v} & -\frac{\partial^2\pi}{\partial\theta\,\partial c}\end{vmatrix}.$$

where $\left|H\right| = \frac{\partial^2 \pi}{\partial v^2} \cdot \frac{\partial^2 \pi}{\partial \theta^2} - (\frac{\partial^2 \pi}{\partial v \partial \theta})^2$ is the determinant of the Hessian matrix with $\left|H\right| > 0$ by the second-order condition for maximization.

$$\frac{dv}{dc} = \frac{1}{\left|H\right|}(F'(v)C''(\theta)) < 0.$$ [20]

$$\frac{d\theta}{dc} = \frac{1}{\left|H\right|}(-\frac{c(F'(v))^2}{\theta}) < 0.$$ [21]

By totally differentiating the first-order conditions, we have

$$\begin{bmatrix} \frac{\partial^2 \pi}{\partial v^2} & \frac{\partial^2 \pi}{\partial v \partial \theta} \\ \frac{\partial^2 \pi}{\partial \theta \partial v} & \frac{\partial^2 \pi}{\partial \theta^2} \end{bmatrix} \begin{bmatrix} \frac{dv}{d\alpha} \\ \frac{d\theta}{d\alpha} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 \pi}{\partial v \partial \alpha} \\ -\frac{\partial^2 \pi}{\partial \theta \partial \alpha} \end{bmatrix}.$$

By using Cramer's rule, we have

$$\frac{dv}{d\alpha} = \frac{1}{\left|H\right|} \begin{vmatrix} -\frac{\partial^2 \pi}{\partial v \partial \alpha} & \frac{\partial^2 \pi}{\partial v \partial \theta} \\ -\frac{\partial^2 \pi}{\partial \theta \partial \alpha} & \frac{\partial^2 \pi}{\partial \theta^2} \end{vmatrix}, \text{ and } \frac{d\theta}{d\alpha} = \frac{1}{\left|H\right|} \begin{vmatrix} \frac{\partial^2 \pi}{\partial v^2} & -\frac{\partial^2 \pi}{\partial v \partial \alpha} \\ \frac{\partial^2 \pi}{\partial \theta \partial v} & -\frac{\partial^2 \pi}{\partial \theta \partial \alpha} \end{vmatrix}.$$

$$\frac{dv}{d\alpha} = -\frac{1}{\left|H\right|} F'(v)[\frac{c}{\alpha} C''(\theta) + \frac{cv}{\theta} F(v)] > 0.$$ [22]

[20] It can be easily verified that $\frac{\partial^2 \pi}{\partial v \partial c} = F'(v) < 0$, $\frac{\partial^2 \pi}{\partial \theta^2} = -C''(\theta) < 0$, $\frac{\partial^2 \pi}{\partial \theta \partial c} = 0$, and $\frac{\partial^2 \pi}{\partial v \partial \theta} = \alpha v F'(v) + \alpha F(v) = -\frac{cF'(v)}{\theta} > 0$.

[21] It can also easily verified that, $\frac{\partial^2 \pi}{\partial v^2} < 0$, $\frac{\partial^2 \pi}{\partial \theta \partial c} = 0$, $\frac{\partial^2 \pi}{\partial v \partial c} = F'(v) < 0$, and $\frac{\partial^2 \pi}{\partial \theta \partial v} = \alpha v F'(v) + \alpha F(v) = -\frac{cF'(v)}{\theta} > 0$.

[22] We have $\frac{\partial^2 \pi}{\partial v \partial \alpha} = \frac{-cF'(v)}{\alpha} > 0$, $\frac{\partial^2 \pi}{\partial \theta^2} = -C''(\theta) < 0$ $\frac{\partial^2 \pi}{\partial \theta \partial \alpha} = vF(v) > 0$, and $\frac{\partial^2 \pi}{\partial v \partial \theta} = \alpha v F'(v) + \alpha F(v) = -\frac{cF'(v)}{\theta} > 0$.

	Benchmark	**Limit Pricing**	**Copying Regime**
The monopolist's optimal choice	v^* p^* θ^*	v^L $(v^* > v^L)$ p^L $(p^L < p^*)$ θ^L $(\theta^L < \theta^*)$	$\tilde{v}$ $(v^* > \tilde{v})$ $\tilde{p}$ $(\tilde{p} < p^*)$ $\tilde{\theta}$ $(\tilde{\theta} < \theta^*)$
An increase in the reproduction cost	*N/C*	$\frac{\partial v^L}{\partial c} > 0$ and $\frac{d\theta^L}{dc} > 0$	$\frac{\partial \tilde{v}}{\partial c} < 0$ and $\frac{d\tilde{\theta}}{dc} < 0$
An increase in the degradation rate	*N/C*	$\frac{\partial v^L}{\partial \alpha} > 0$ and $\frac{d\theta^L}{d\alpha} > 0$	$\frac{\partial \tilde{v}}{\partial \alpha} > 0$ and $\frac{d\tilde{\theta}}{d\alpha} > 0$

Table 2

$$\frac{d\theta}{d\alpha} = \frac{1}{|H|}[\frac{\partial^2 \pi}{\partial v^2}(-vF(v)) + \frac{c^2}{\alpha^2 \theta}\{F'(v)\}^2] > 0 .$$ [23]

Since the monopolist's quality provision is determined by the marginal consumer's valuation for the software, the effects of increase in IPR depend on the change of the marginal consumer, which is different according to types of costs associated with piracy. With higher reproduction cost, all consumers face the same increase in the gross copy cost, which is equivalent to overall demand increase for the monopolist. Hence, The monopolist benefits from higher demand by charging the higher price, yet increasing sales at the same time. Facing the marginal consumer's lower valuation, the monopolist has less incentive to provide higher quality. In contrast, if an increase in IPR is derived from higher degradation rate, we observe proportional increase in the gross copy cost and comparatively more market power for the monopolist. With increase in market power, the monopolist charges a higher price by reducing quantity. Responding to the marginal consumer's higher valuation, the monopolist has more incentive to supply higher quality.

Uniform Distribution Example of the Software Quality

We now assume consumers' valuation for the software is uniformly distributed such as $v_i \in U[0, 1]$. To make our analysis more tractable, we also suppose that the monopolist's

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[23] We have $\frac{\partial^2 \pi}{\partial v^2} < 0$, $\frac{\partial^2 \pi}{\partial \theta \partial \alpha} = vF(v) > 0$, $\frac{\partial^2 \pi}{\partial v \partial \alpha} = \frac{-cF'(v)}{\alpha} > 0$, and $\frac{\partial^2 \pi}{\partial \theta \partial v} = vF'(v) + F(v) = -\frac{cF'(v)}{\alpha \theta} > 0$.

cost of developing the software, which decides the level of quality, is given by $C(\theta)=\frac{k}{2}\theta^2$.

With uniform distribution we can easily verify Proposition 7, in which given the monopolist's optimal choice of quantity, the quality provision under the benchmark case is suboptimally low. Since the consumers' only decision is whether to purchase or not, the monopolist has the marginal consumer whose valuation $v^* = p/\theta$. Given this arrangement, the monopolist maximizes his profit:

$$\underset{v,\theta}{Max}\,\pi = \theta\, v(1-v) - \frac{k}{2}\theta^2 .$$

The first order conditions

$$\frac{\partial \pi}{\partial v} = \theta - 2\theta\, v = 0 \qquad (13)$$

$$\frac{\partial \pi}{\partial \theta} = v(1-v) - k\theta = 0 \qquad (14)$$

determine $v^* = \frac{1}{2}$ and $\theta^* = \frac{1}{4k}$.

Socially optimal level of quality can be derived by maximizing the aggregate valuation of consumers between $[1/2, 1]$, which is $Max \int_{1/2}^{1} \theta\, v dv - \frac{k}{2}\theta^2$. We have $\theta^{opt} = \frac{3}{8k}$, which is $\theta^{opt} \geq \theta^*$.

Now we turn to the monopolist's optimal choice of software quality with piracy. The monopolist's choice depends on how he responds to the threat of piracy. With two margins of IPR [degradation rate (α) and copy cost (c)] the monopolist has two different strategies to choose.[24] First, we observe that the monopolist maximizes the constrained profit under the limit pricing regime as following:

$$Max\ \pi^L = p^L(1-\frac{p^L}{\theta}) - \frac{k}{2}\theta^2$$

Subject to

$$(1-\alpha)\theta\, v - c \leq \theta\, v - p$$

[24] The consumers' optimal behaviors are the same as the short run case.

The optimal price $p^L = \frac{c}{1-\alpha}$, and profit is $\pi^L = (\frac{c}{1-\alpha})(1-\frac{c}{(1-\alpha)\theta}) - \frac{k}{2}\theta^2$. The monopolist then chooses his optimal choice of quality by

$$\underset{\theta}{Max}\, \pi^L = (\frac{c}{1-\alpha})(1-\frac{c}{(1-\alpha)\theta}) - \frac{k}{2}\theta^2 .$$

The first order condition

$$\frac{\partial \pi}{\partial \theta} = \frac{c^2}{(1-\alpha)^2 \theta^2} - k\theta = 0 \qquad (15)$$

determines the optimal level of software quality for the monopolist. If we valuate the first-order condition at $\theta^* = \frac{1}{4k}$, we can verify that under the limit pricing regime, the monopolist chooses lower level of quality as the one without piracy.[25]

For the last case where the monopolist chooses to allow piracy, the marginal consumer who is indifferent between buying and copying is given by $v = \frac{p-c}{\alpha\theta}$. With facing demand of $1-\frac{p-c}{\alpha\theta}$, the monopolist's profit maximization problem is

$$Max\ (\alpha\theta\, v + c)(1-v) - \frac{k}{2}\theta^2 .$$

The first order conditions

$$\frac{\partial \pi}{\partial v} = \alpha\theta(1-v) - (\alpha\theta v + c) = 0 \qquad (16)$$

$$\frac{\partial \pi}{\partial \theta} = \alpha v(1-v) - k\theta = 0 \qquad (17)$$

determine $\tilde{v} = \frac{\alpha\theta - c}{2\alpha\theta} = \frac{1}{2} - \frac{c}{2\alpha\theta}$, which is $\tilde{v} < v^*$. Also, if we evaluate $\tilde{\theta} = \frac{\alpha v(1-v)}{k}$ at $v^* = \frac{1}{2}$, we can verify $\tilde{\theta} = \frac{\alpha}{4k} < \theta^*$.

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[25] With our assumption $\frac{c}{1-\alpha} < p^* = \theta^* v^*$ we can verify the sign of (15) at $\theta^* = \frac{1}{4k}$ such as $\frac{c^2}{(1-\alpha)^2}\frac{1}{16k^2} - \frac{1}{4} = 16k^2(\frac{c}{1+\alpha} + \frac{1}{8k})(\frac{c}{1+\alpha} - \frac{1}{8k}) < 0$.

The effect of increased copyright protection on development incentives can be shown more clearly in the uniform distribution example. By totally differentiating (15) we can show both types of increase in IPR induces higher software quality and less authorized usage as the following:

$$\frac{d\theta}{dc} = \frac{2ck}{6(1-\alpha)^2\theta^2} > 0\,,\ \frac{dv}{dc} = \frac{d(\frac{c}{(1-\alpha)\theta})}{dc} = \frac{1}{(1-\alpha)\theta} > 0\,,$$

$$\frac{d\theta}{d\alpha} = \frac{2\theta)}{3(1-\alpha)} > 0\text{, and } \frac{dv}{d\alpha} = \frac{d(\frac{c}{(1-\alpha)\theta})}{d\alpha} = \frac{c}{(1-\alpha)^2\theta} > 0\,.$$

Moreover, we can easily verify the effects of increase in IPR under the copying regime: higher quality and less official usage with increased degradation rate, and lower quality and more authorized usage with increased reproduction cost. By totally differentiating (16), (17) and using Cramer's Rule we can present a simplified version of proposition 11 as follows:

$$\frac{dv}{dc} = \frac{1}{|H|}[-\frac{2}{k}] < 0\,,\ \frac{d\theta}{dc} = -\frac{1}{|H|}[\alpha(1-2v)] < 0\,,$$

$$\frac{dv}{d\alpha} = \frac{1}{|H|}(1-2v)[\frac{2\theta}{k} + \alpha v(1-v)] > 0\text{, and}$$

$$\frac{d\theta}{d\alpha} = \frac{1}{|H|}\alpha\theta[2v(1-v) + (1-2v)^2] > 0\,.$$

4. Concluding Remarks

In this paper, we develop a simple model of piracy to analyze implications of increased intellectual property rights on the short-run and long-run resource allocations. In a model of self-selection with heterogeneous users, we show that the consumers' option to use illegal copies constrains the copyright holder's ability to charge a monopoly price. Consequently, the unauthorized copy could lead to more usage of the legal copies. In this sense, the presence of the unauthorized copies can act as a complement to use of the legal copies rather than a substitute.

To show the effects of increase in IPR more precisely, we consider two types of costs associated with piracy; the type-independent *reproduction cost* and the type-dependent *degradation cost*. We provide a theoretical framework to show that the effects of piracy depend crucially on the nature of piracy costs.

We observe that strengthening IPR in the form of an increase in the degradation cost supports the 'folklore' of IPR. An increase in IPR with higher degradation cost provides the monopolist with more market power, which results in both negative demand switch and total usage change. With less authorized usage and higher marginal consumer type, the monopolist has more incentive to provide higher quality of the software in the long run. In contrast, given an increase in the reproduction cost, which induces higher prices and more authorized usage of the software, the monopolist has less incentive to provide higher quality due to the marginal consumer's lower valuation for the software. Even though more consumers obtain the software from the monopolist with more efficient technology, an increase in IPR with higher reproduction cost also reduces the total usage of the software. Therefore, it may increase or decrease social welfare in the short run.

Finally, we would like to end this paper by suggesting possible avenues of extending our results. For analytical simplicity, we assumed that the type-independent copying costs were the same across all users. However, we can easily imagine a case in which these type-independent costs are heterogeneous. Even with more advanced figure of the digital technology like file-sharing through peer to peer network, it is still worthy to analyze a model with piracy retailers. In this framework, the reproduction cost of consumers is equivalent to the piracy retailer' s price for an illegal copy. If the retailer's expected profit is determined by the efficiency of monitoring and degree of enforcement, we are able to pin down the effect of IPR incorporating costs of copyright protection.

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