TECHNOLOGY CHOICE AND COALITION FORMATION IN STANDARDS WARS*

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Abstract. We study technology choice in standards wars and mandated standards. We show that standards wars are better than mandated standards from a welfare perspective when technological complexity is low and patent ownership is dispersed, while mandated standards are better when the uncertainty over the relative value of technologies is low. Allowing ex-ante licensing agreements between standard sponsors may decrease welfare in standards wars, unless firms' participation in standard forums is unrestricted and widespread, in which case they unambiguously improve welfare. We provide existence and characterization results relevant to the literatures of coalition formation and equal-sharing partnerships.

Keywords: Standards Wars, Technology Choice, Standard-Setting Organizations, Ex-Ante Agreements, Coalition Formation, Cooperative Game Theory, Externalities, Equal-Sharing Partnerships (JEL: C71, L15, L24, O34).

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1. Introduction

Technical standards –like the compact disk or the protocols that run the Internet– are essential for the development and adoption of new technologies. Standards often result from competition between groups of firms sponsoring different technologies. In the high-definition optical disc format war, for example, the Blu-ray standard –sponsored by Sony, Hitachi, LG, Panasonic, Pioneer, Philips, Samsung, Sharp, and Thomson– competed for adoption against the HD DVD standard –sponsored by Toshiba, NEC, Sanyo, Memory-Tech Corporation, Intel, and Microsoft. Standards wars are pervasive. Chiao et al. (2007) show there is an average of fifteen standard-setting organizations (SSOs) per technology subfield, often proposing competing standards.

Under competition, the probability that a standard is adopted depends on its technical characteristics and those of competing standards. Thus, the profit that a group of firms expects to obtain from a standard depends on the whole allocation of firms to coalitions, and the standard-setting process is a problem of coalition formation with inter-group externalities (Aumann and Peleg, 1960; Thrall and Lucas, 1963; Myerson, 1978; Bloch, 1996; Maskin, 2013).

The interdependence of coalitional values may lead to inefficient outcomes if firms act strategically when deciding what standard to sponsor. Strategic incentives may be avoided if a mandated standard is imposed on firms, but standards wars allow for experimentation, which may be valuable if the performance of alternative technologies is hard to assess before they are introduced in the market (Rosenberg, 1982; Choi, 1996).

The relative advantages and disadvantages of standards wars and mandated standards have lead to an intense policy debate over the optimal regulation of standard-setting activities (Bender and Schmidt, 2007; Cabral and Salant, 2013). In the case of second and third-generation (2G and 3G) wireless telecommunication standards, for example, the European Union mandated a single standard, while in the US several standards competed for adoption. Was the European policy of allowing only one standard reasonable? And in the case of high-definition optical discs, would it have been better to force Sony and Toshiba to create only one standard?

A related policy question is whether licensing agreements should be allowed at the standard-setting stage. Patents are becoming increasingly important for standards (Rysman and Simcoe, 2008; Baron et al., 2013). Ex-ante agreements have been proposed to alleviate the potential problems caused by hold-up and the existence *standard-essential patents* (Farrell et al., 2007). Traditionally, most SSOs have shunned discussions of licensing terms out of concern for potential antitrust exposure.[1] More recently, some SSOs have reconsidered this position and have received encouraging guidance from the Federal Trade Commission, the Department of Justice (DOJ and FTC, 2007), and the European Commission (2004). Llanes and Poblete (2014) and Lerner and Tirole (2013) show that ex-ante licensing agreements lead to better standards when firms have to agree on one standard (monopoly-standard case). However, the effects of ex-ante agreements in standards wars are largely unknown.

We develop a model of coalition formation and technology choice in standard setting to address the following questions: (i) How does competition between groups of technology sponsors affect the standard-setting process? (ii) Under what conditions is it better to have a standards war or a mandated standard? (iii) What is the effect of ex-ante agreements on technology efficiency in standards wars? The extant literature has not answered these questions when standards result from the combination of technologies of multiple firms and groups of sponsors compete in the market to have their technologies adopted.

We study the standardization of a product with multiple functionalities or components. Alternative patented technologies may be used to implement each functionality. A standard is simply a technical document specifying which technology will implement each functionality. To comply with a standard, adopters must follow its exact technical specifications. Thus, technologies selected to be part of a standard become essential for its implementation.

The value of a standard is uncertain until it is introduced in the market. In a standards war, firms form coalitions and create standards that compete for adoption in the market. Strong network effects lead to a winner-takes-all outcome (Besen and Farrell, 1994; Shapiro and Varian, 1999). Even though many standards may be proposed, only the standard with the highest value

[1]For example, the VITA Standards Organization (2009) indicates that "the negotiation or discussion of license terms among working-group members or with third parties is prohibited at all VSO and working-group meetings," the IEEE Standards Association (2010) establishes that "participants should never discuss the price at which compliant products may or will be sold, or the specific licensing fees, terms, and conditions being offered by the owner of a potential Essential Patent Claim," and ETSI (2013) establishes that "specific licensing terms and negotiations are commercial issues between the companies and shall not be addressed within ETSI."

realization is adopted. With mandated standards, a standard-setting organization with a government mandate chooses a unique standard to maximize the expected value of the technology.

Standards wars are good from a welfare perspective because they allow for greater *experimentation*. Postponing the adoption decision until after uncertainty is resolved may improve technology choice. On the other hand, standards wars may lead to the choice of suboptimal standards for two reasons. First, an *equalizing transformation* changes the relative importance of firms after standardization takes place (Llanes and Poblete, 2014). Firms with highly-substitutable patents have a small marginal contribution before a standard is defined (ex-ante), but become essential for its implementation if their technologies are included in the standard (ex-post). Without ex-ante agreements, firms cannot commit on how to share the revenues of the standard. Thus, firms with large ex-ante marginal contributions will be reluctant to join the standard-setting efforts of firms with small (but positive) ex-ante marginal contributions. Second, firms may have *strategic incentives* to select the standard's technologies to reduce the availability of patents for competing standards, which may lead to the choice of suboptimal technologies.

Our paper has three main results. First, we find that if licensing agreements are not allowed at the standard-setting stage, standards wars lead to better standards than mandated standards when patent ownership is dispersed and technologies have a monotonic effect on technical efficiency (if a technology is valuable for some standard, then it is valuable for any standard that includes it). Mandated standards, on the other hand, lead to better standards when the uncertainty about the performance of alternative standards is small.

Second, we find that allowing ex-ante agreements may decrease welfare in the case of standards wars. Even though total industry profits are larger in a standard war than in a mandated standard, welfare is not necessarily larger because profits may not be aligned with welfare.

Third, we find that if firms can sign ex-ante agreements and participate in multiple standards, and the first best allocation leads to a connected network of standard sponsors, standards wars reach the first best and unequivocally lead to higher welfare than a mandated standard. Therefore, we show that ex-ante agreements lead to higher welfare if participation in standard setting is unrestricted and widespread.

Our results contrast with the results of previous works studying monopoly standards (Llanes and Poblete, 2014; Lerner and Tirole, 2013), in which case ex-ante agreements were always found to be welfare improving. We show that this result no longer holds in the case of standards wars, unless profits and welfare are aligned, or the standard-setting process is unrestricted and widespread.

Our paper contributes to the standards literature by studying the welfare properties of alternative standard-setting rules in a model of competition between standards with multiple sponsors. We also contribute to the literatures of coalition formation and equal-sharing partnerships (Farrell and Scotchmer, 1988; Levin and Tadelis, 2005; Poblete, 2013) by providing novel existence results, characterizing stable allocations, and describing the relations between several cooperative and non-cooperative solution concepts in the presence of externalities, both with a fixed distribution of output and when the distribution of output is endogenous.

In the following section we present a model of technology choice and coalition formation in standards wars. In Section 3 we show a stable allocation exists. In Section 4 we describe technology choice in mandated standards. In Section 5 we compare the welfare performance of standards wars and mandated standards. In Sections 6 and 7 we study the effects of ex-ante agreements on technology choice and efficiency. In Section 8 we discuss related issues and potential directions for further research. In Section 9 we present the main conclusions of the paper. In Appendix A we present the proofs for the theorems in text, and in Appendix B we introduce several extensions to the basic model.

2. The model

We study the standardization of a product with M functionalities or components. Alternative patented technologies may be used to implement each functionality. Let N be the set of firms, and let P be the set of patents. A patent is a pair (i, m), where $i \in N$ indicates patent ownership and $m \in \{1, 2, \ldots M\}$ indicates patent functionality.

Firms form coalitions to propose standards, and engage in forum shopping until they find a standard-setting organization (SSO) aligned with their objectives (Lerner and Tirole, 2006), or create a new SSO to develop their standard.

A *standard* is a set of non-redundant patents implementing the product's functionalities.[2] The set of all possible standards is

$$S = \{s \subseteq P \mid |s| = M \text{ and } (i,m),(i',m') \in s \Rightarrow m \neq m'\}.$$

A firm is a *sponsor* of a standard if it owns at least one patent in the standard. Let $\mu(s)$ be the set of sponsors of standard s. In the first part of the paper, we assume that each firm may sponsor at most one standard. In Section 7 we study a model in which each firm may participate in more than one standard.

An *allocation* is a set of standards such that firms own patents in at most one standard. Each allocation implicitly defines a partition of firms into non-overlapping coalitions of sponsors. The set of all possible allocations is

$$A = \{a \subseteq S \mid \forall\, s, s' \in a,\ \mu(s) \cap \mu(s') = \varnothing\}.$$

Each functionality may be implemented by at least two patents owned by different firms. Adopters wanting to comply with a standard must follow its exact technical specifications. Thus, no patent or firm is essential *ex-ante* (before the standard is set), but technologies become essential *ex-post* (after the standard is set) if a standard that includes them is adopted.[3]

We also assume that one of the functionalities can be implemented by exactly two patents. Therefore, at most two standards may compete for adoption ($|a| \leq 2$ for all $a \in A$). Studying a standards war between two standards allows us to show the basic mechanisms at play in a simple way and is interesting in its own right since many standards wars are fought between two main standards, e.g., Blu-ray vs. HD-DVD, VHS vs. Betamax, RCA vs. Columbia in quadrophonic sound, and Sky vs. BSB in Satellite TV.[4]

The value of a standard, v, is a random variable with cumulative density function $F(v \mid s)$, and its realization is not known until all standards have been proposed and implemented. The distribution $F(v \mid s)$ is continuous and differentiable in an interval $[0, \overline{v}]$, and the values of different standards are independently distributed. Standards can be weakly ordered according to first-order stochastic dominance (FOSD). That is, for any $s, s' \in S$, either

[2]We assume all functionalities are essential. In Section 8 we discuss how our results change if functionalities are not essential and functionality choice is endogenous.

[3]We studied ex-ante essential patents and firms in Llanes and Poblete (2014).

[4]In Section B.3 of Appendix B we extend the model to allow for competition between more than two standards.

$F(v \mid s) \le F(v \mid s')$ for all $v \in [\underline{v}, \overline{v}]$, or $F(v \mid s) \ge F(v \mid s')$ for all $v \in [\underline{v}, \overline{v}]$. In what follows, let $s \succeq s'$ if $F(v \mid s) \le F(v \mid s')$ for all $v \in [\underline{v}, \overline{v}]$, and let $s \succ s'$ if $s \succeq s'$ but $s' \succeq s$ does not hold.

Markets of standardized products typically exhibit strong network effects (Farrell and Saloner, 1985; Katz and Shapiro, 1985, 1994). We capture this feature in a simple way by assuming that standardization leads to a winner-takes-all outcome: only the standard with the highest value realization is adopted by users. Besen and Farrell (1994) and Shapiro and Varian (1999) present several examples of winner-takes-all outcomes in standard setting.

The sponsors of the wining standard can appropriate total quasirents $\pi(v)$, with $\pi(0) = 0$ and $\pi'(v) \ge 0$, and firms cannot negotiate how to distribute profits when selecting the standard's technologies. Given that all patents in a standard are essential for its implementation, all sponsors have the same marginal contribution ex-post. Thus, standardization leads to an *equalizing transformation* of the marginal contributions of firms: even though firms may have a different marginal contribution *ex-ante*, all firms in a standard have the same marginal contribution *ex-post*. Consequently, we assume quasirents are divided equally among the standard's sponsors. Equal sharing is consistent with most bargaining solutions, such as the Shapley value or Nash bargaining solution (see Llanes and Poblete, 2014, for more details).[5]

Consider an allocation formed by standards s and s'. The total expected rent of standard s is given by

$$H_s(s, s') = \int_0^{\overline{v}} \pi(v)\ F(v \,|\, s')\ dF(v \,|\, s),$$

and the expected rent for each sponsor of s is

$$h_s(s, s') = \frac{H_s(s, s')}{|\mu(s)|}.$$

Given that standard sponsors share quasirents equally, we can use the tools developed in the partnerships literature (Farrell and Scotchmer, 1988; Levin

[5]Equal sharing is not essential for our results. All we need is that the bargaining power of firms changes ex-post, so that there is a redistribution of revenues from firms with small incremental value to firms with high incremental value. Similar arguments have been used to motivate the assumption of equal sharing in the partnerships literature (Levin and Tadelis, 2005; Poblete, 2013). Assuming that firms distribute revenues according to the number of patents would lead to similar conclusions than equal sharing. What is important is that revenues are distributed based on some ex-post variable which is not perfectly correlated with ex-ante marginal contributions.

and Tadelis, 2005; Poblete, 2013), which studies coalition formation in equal-sharing partnerships.

Coalition formation may be modeled using cooperative or non-cooperative game theory. Following Farrell and Scotchmer (1988), we study a cooperative game. In Section B.2 we show our results extend to a *non-cooperative* coalition-formation game based on Bloch (1996).

To the best of our knowledge, the partnerships literature has not studied coalition formation with *inter-group externalities* (i.e., when the payoff of a coalition depends on the configuration of other coalitions). In our model, externalities play an important role, because the expected profit of the sponsors of a standard depends on the value of competing standards. Therefore, when we consider deviations from a particular allocation, we need to take into account how other firms will react to the deviation.

Farrell and Scotchmer (1988) studied the core of equal-sharing partnerships. The equal-sharing assumption implies that the game is a characteristic-function game with non-transferable utility. The natural extensions of the core to coalition-formation games with externalities are given by the α-core and β-core theories of Aumann and Peleg (1960), and by the partition-function games of Thrall and Lucas (1963). We follow Thrall and Lucas (1963) and assume that firms have pessimistic beliefs. Pessimistic beliefs mean that for a deviation to be profitable, it must be profitable for any possible reaction of non-deviators.[6]

Intuitively, in cooperative games, equilibrium payoffs generally depend on the threats agents can make to each other (Myerson, 1978). Pessimistic beliefs are consistent with the worst threat that firms outside a standard can make. Moreover, pessimistic beliefs minimize the chances for a deviating coalition to be profitable. Therefore, if an allocation is stable with another belief system, it must be stable with pessimistic beliefs. Since we show our results hold for any equilibrium with pessimistic beliefs, they will also hold for equilibria with

[6]Aumann and Peleg (1960) study games with non-transferable utility, and Thrall and Lucas (1963) study games with transferable utility. Even though our game has non-transferable utility, our definition of stability is closer to Thrall and Lucas (1963), because we assume that firms only have pessimistic expectations for deviations from the stable allocation. Aumann and Peleg (1960) also assume coalitions have pessimistic expectations *at* the stable allocation.

other beliefs.[7] Formally, we study stable allocations according to the following definition.

Definition 1 (Stable allocation). *A standard s blocks allocation a if for any allocation a' that contains s, the sponsors of s are strictly better off in a' than in a. An allocation is stable if a standard blocking it does not exist.*

An allocation is stable if it is not possible to form a standard that gives higher revenues to its sponsors, for any belief that sponsors may have about the reaction of non-deviators to the creation of this standard. We consider both unilateral and multilateral deviations.

Our definition of stability is different from Farrell and Scotchmer's because the payoff of a deviating coalition depends on how the rest of the players reorganize after the deviation, but coincides to Farrell and Scotchmer's definition when there are no externalities.

Timing is as follows. First, a stable allocation is formed. Second, the values of standards are revealed. Third, the standard with the highest value is adopted, and its sponsors appropriate quasirents.

3. Existence of a stable allocation

We now study the existence of a stable allocation. We begin by showing that expected per capita profits are decreasing in the number of sponsors, increasing in the expected value of the standard, and decreasing in the expected value of rival standards. To see the last two effects, it is useful to integrate by parts,

$$\begin{aligned}
H_s(s, s') &= \int_0^{\overline{v}} \pi(v)\ F(v \mid s')\ dF(v \mid s) \\
&= -\int_0^{\overline{v}} F\left(v \mid s\right)\ \Big(\pi'(v)\ F(v \mid s') + \pi(v)\ f(v \mid s')\Big)\, dv \\
&= \int_0^{\overline{v}} \pi'(v)\, dv - \int_0^{\overline{v}} \pi'(v)\, F(v \mid s)\, F(v \mid s')\, dv \\
&\quad - \int_0^{\overline{v}} \pi(v)\ F(v \mid s)\ f(v \mid s') dv \\
&= \int_0^{\overline{v}} \pi'(v)\ \Big(1 - F(v \mid s)\ F(v \mid s')\Big)\ dv - H_{s'}(s, s'),
\end{aligned}$$

[7] In Section B.1 of Appendix B we show our results are robust to assuming firms have reactive beliefs. That is, following a deviation firms expect that non-deviators will form the most profitable standard.

from which we obtain

$$H_s(s,s') + H_{s'}(s,s') = \int_0^{\overline{v}} \pi'(v) \left(1 - F(v \mid s)\, F(v \mid s')\right) dv, \tag{1}$$

where $H_s(s,s') + H_{s'}(s,s')$ are total expected industry profits.

Consider an arbitrary standard $s \in S$. Let $R(s)$ be the set of standards that can be formed without using any of the patents of the sponsors of s,

$$R(s) = \{z \in S \mid \mu(z) \cap \mu(s) = \varnothing\}, \tag{2}$$

and let $r(s)$ be the standard that maximizes the per capita revenues of the firms that are not sponsors of s,

$$r(s) = \underset{z \in R(s)}{\operatorname{argmax}}\; h_z(s,z).$$

We will refer to $r(s)$ as the reactive standard. Note that $R(s)$ may be empty, in which case there does not exist a reactive standard. In this case, we write $r(s) = \varnothing$.

Finally, let s^* be defined as follows

$$s^* = \underset{s \in S}{\operatorname{argmax}}\; h_s(s, r(s)).$$

We refer to $a^* = \{s^*, r(s^*)\}$ as the reactive allocation.[8] The reactive allocation is akin to the equilibrium of a Stackelberg game in non-cooperative game theory. Notice, however, that the cooperative game we are studying makes no assumptions on the timing and structure of the coalition-formation process. In the non-cooperative game we study in Section B.2 of Appendix B, we make specific assumptions on the coalition-formation process, and show that the reactive allocation can be obtained as the equilibrium outcome of a dynamic game. The following proposition shows that a^* is stable. Thus, a stable allocation exists.

Proposition 1 (Existence)**.** *A stable allocation exists.*

In Section 5 we discuss several reasons why a stable allocation may be inefficient, present sufficient conditions for the uniqueness of the stable allocation,

[8]The reactive standard $r(s)$ need not be unique, but it is *generally unique*. Likewise, s^* may not be unique, but it is generally unique. A sufficient condition for uniqueness of both $r(s)$ and s^* is that the standards in S are *strongly* ordered according to FOSD. That is, for two standards $s, s' \in S$, either $s \succ s'$ or $s' \succ s$. For simplicity, in the rest of the paper we assume that $r(s)$ and s^* are unique.

and compare standards wars with mandated standards. Before doing so, we describe the standard-setting process with mandated standards.

4. Mandated standards

With a mandated standard, the creation of the standard is delegated to a single SSO, which chooses technologies in order to maximize the expected value of the standard and grants membership to any firm owning patents on standard-related technologies. Under these rules, all firms want to join the standard and the mandated standard is

$$\bar{s} = \underset{s \in S}{\operatorname{argmax}} \ \mathbb{E}(v \mid s).$$

Given our assumptions, $\bar{s}$ first-order stochastically dominates all other standards. For simplicity, we assume that the mandated standard $\bar{s}$ is unique. As we show in the following section, a mandated standard has the advantage of being the best standard that can be set ex-ante using all the available information, but it has the disadvantage of preventing independent experimentation.

The following example illustrates the model definitions.

Example 1. Consider an example with three firms, $N = \{A, B, C\}$; two functionalities, $M = 2$; and four patents $P = \{(A, 1), (A, 2), (B, 1), (C, 2)\}$.

There are four possible standards: $s_1 = \{(A, 1), (A, 2)\}$, $s_2 = \{(A, 1), (C, 2)\}$, $s_3 = \{(B, 1), (A, 2)\}$, and $s_4 = \{(B, 1), (C, 2)\}$. Likewise, there are five possible allocations: $a_1 = \{s_1, s_4\}$, $a_2 = \{s_1\}$, $a_3 = \{s_2\}$, $a_4 = \{s_3\}$, and $a_5 = \{s_4\}$.

Standard s_1 leads to a value $v = 0$ with probability 1/2 and to $v = 1$ with probability 1/2, standard s_2 leads to $v = 0$ with probability 1/2 and to $v = \overline{v} > 1$ with probability 1/2, and standards s_3 and s_4 lead to $v = 0$ with probability 1/2 and to $v = \underline{v} < 1$ with probability 1/2. Firms capture a fraction α of the social value of the standard ($\pi(v) = \alpha\, v$).

If allocation a_3 is implemented, the sponsors of s_2 obtain total quasirents $\alpha\, \overline{v}$ with probability 1/2. Thus, total expected rents are $\alpha\, \overline{v}/2$, and the expected rent of each sponsor is $\alpha\, \overline{v}/4$. The expected rents of allocations a_2, a_4 and a_5 are calculated in the same way. In the case of allocation a_1, the expected rents for the sponsors of s_1 are $\alpha/2$, and the expected rents of each sponsor are $\alpha/4$. The expected rents for the sponsors of s_2 are $\alpha\, \underline{v}/4$, and the expected rent of each sponsor is $\alpha\, \underline{v}/16$.

The stable allocation depends on the value of $\overline{v}$. If $\overline{v} > 2$, the stable allocation is $a_3 = \{s_2\}$. If $1 < \overline{v} < 2$ the stable allocation is $a_1 = \{s_1, s_4\}$. The mandated standard is $\bar{s} = s_2$. If $\overline{v} > 2$, the mandated standard coincides with the stable allocation in a standard war. If $1 < \overline{v} < 2$, the mandated standard is different from the stable allocation in a standards war.

5. Standards wars vs. mandated standards

In this section, we discuss the main trade-offs between standards wars and mandated standards. Standards wars are good from a welfare perspective because they allow for *experimentation*, but they may also lead to the choice of suboptimal standards due to the *equalizing transformation* and *strategic incentives*. Mandated standards avoid the equalizing transformation and strategic incentives, but do not allow for experimentation. We now describe these factors in detail.

Experimentation. The value of new technologies is generally hard to assess before they are introduced in the market (Rosenberg, 1982; Choi, 1996). Thus, a standard with low expected value ex-ante may turn out to have a high value after uncertainty is resolved. Thus, a standards war is valuable because it allows users to postpone the decision of which standard to adopt until after uncertainty is resolved.

Equalizing transformation. Technologies become essential when they are included in a standard, which leads to an *equalizing transformation* of the marginal contributions of firms. As a consequence, firms with large patent portfolios have an incentive to limit the number of sponsors with whom they share the revenues of a standard, which may lead to the exclusion of valuable technologies in the standard (we studied this effect in detail with a single (monopoly) standard in Llanes and Poblete (2014)). To see this effect more clearly, consider Example 1, and suppose $\overline{v} > 2$. In this case, the only stable allocation is $\{s_2\}$, which has a single standard based on the technologies of firm A. If firm A includes the patent of firm B in the standard, the expected value of the standard increases. However, the increase in expected revenues from having a better standard is not enough to compensate the decrease in revenues from having to share the revenues with another sponsor. Thus, firm A prefers to form a standard on its own.

Strategic incentives. The expected profit of the sponsors of a standard depends on the value of competing standards. Thus, standard sponsors may choose technologies to reduce the value of competing standards, instead of choosing them to increase the value of their own standard. This effect is similar to the raising rivals' cost strategy of non-cooperative games (Salop and Scheffman, 1983). To understand this effect more clearly, consider the following example.

Example 2 Consider an example with four firms, $N = \{A, B, C, D\}$; two functionalities, $M = 2$; and four patents, $P = \{(A,1), (B,1), (C,2), (D,2)\}$. There are four possible standards $s_1 = \{(A,1), (C,2)\}$, $s_2 = \{(A,1), (D,2)\}$, $s_3 = \{(B,1), (C,2)\}$, and $s_4 = \{(B,1), (D,2)\}$. Standards s_1, s_2, and s_4 lead to $v = \overline{v}$ with probability $\frac{1}{2}$ and to $v = 0$ otherwise. Standard s_3 leads to $v = \underline{v} < \overline{v}$ with probability $\frac{1}{2}$ and to $v = 0$ otherwise. Finally, let $\pi(v) = v$.

From a welfare perspective, the best allocation is $\{s_1, s_4\}$. In this allocation, total welfare is $\frac{3}{4}\overline{v}$ and each firm obtains an expected profit of $\frac{3}{8}\overline{v}$. This allocation, however, is not stable. If A and D deviate by forming s_2, they will face competition from s_3 and obtain an expected profit of $\frac{1}{2}\overline{v}$, which is larger than $\frac{3}{8}\overline{v}$. It is straightforward to check that the only stable allocation is $\{s_2, s_3\}$.

Social welfare is equal to standard value v. The expected welfare of an allocation with only one standard, s, is simply $W(s) = \mathbb{E}(v \mid s)$. The expected welfare of an allocation with two standards, s and s', is

$$W(s, s') = \mathbb{E} \max \{v, v'\},$$

where v and v' are the values of standards s and s'. To calculate this expectation, note that the distribution of the maximum is

$$G(v \mid s, s') = F(v \mid s)\, F(v \mid s'),$$

and its probability density function is

$$g(v \mid s, s') = f(v, s)\; F(v \mid s') + f(v, s')\; F(v \mid s).$$

Thus, expected welfare is

$$W(s, s') = \int_0^{\overline{v}} v\, \Big(f(v, s)\; F(v \mid s') + f(v, s')\; F(v \mid s) \Big)\, dv,$$

which integrating by parts becomes

$$W(s,s') = \int_0^{\overline{v}} \Big(1 - F(v\,|\,s)\; F(v\,|\,s')\Big)\; dv. \tag{3}$$

In Example 1, the expected welfare of the different allocations is $W(a_1) = 1/2 + \underline{v}/4$, $W(a_2) = 1/2$, $W(a_3) = \overline{v}/2$, $W(a_4) = \underline{v}/2$, and $W(a_5) = \underline{v}/2$.

Note the similarity of equations (1) and (3). If $\pi(v) = \alpha\, v$ for $\alpha \in [0,1]$, the sum of expected profits of an allocation is proportional to its expected welfare. In this case, we say that *industry profits and welfare are aligned.* The following definitions help us characterize the optimal standard-formation rule.

Definition 2 (Dispersed ownership)**.** *Patent ownership is dispersed if no firm owns more than one patent.*

Dispersed ownership implies that all standards have the same number of sponsors ($|\mu(s)| = M$ for all $s \in S$) and $R(s) = S \setminus s$.

Definition 3 (Monotonicity)**.** *Let w_1, w_2, z_1, z_2 be arbitrary sets of patents such that $w_i \cup z_j \in S$ for every i, j. Technologies are monotonic if $w_1 \cup z_1 \succeq w_1 \cup z_2$ implies that $w_2 \cup z_1 \succeq w_2 \cup z_2$.*

Monotonicity implies that the patents for a given functionality can be ranked. If we replace a patent in a standard by another patent with a higher ranking, the standard improves its value distribution in a FOSD sense.

We say that standards wars *weakly dominate* mandated standards if $W(a) \geq W(\bar{s})$ for any stable allocation a, and there exists a collection of parameters such that $W(a') > W(\bar{s})$ for a stable allocation a'. Likewise, we say that mandated standards weakly dominate standards wars if $W(\bar{s}) \geq W(a)$ for any stable allocation a, and there exists a collection of parameters such that $W(\bar{s}) > W(a')$ for a stable allocation a'.[9]

The following proposition presents a sufficient condition for standards wars to be welfare optimal. We say *technologies are simple* if patent ownership is dispersed and technologies are monotonic.

Proposition 2 (Optimal policy with simple technologies)**.** *If patent ownership is dispersed and technologies are monotonic, standards wars weakly dominate mandated standards.*

[9] A collection of parameters is particular combination of an integer M, sets N and P, and functions $\mu(s)$, $F(v|s)$ and $\pi(v)$. Sets S and A, functions $W(a)$, $H_s(a)$, and $h_s(a)$, and the set of stable allocations are endogenous.

The proof shows that with simple technologies, the stable allocation in a standards war is $\bar{a} = \{\bar{s}, r(\bar{s})\}$, while the mandated standard is simply $\bar{s}$. standards wars generate more social value, not because they lead to better technical standards from an ex-ante perspective, but because they allow for market experimentation.

With simple technologies, the *social value of experimentation* is given by $W(\bar{s}, r(\bar{s})) - W(\bar{s})$. From equation (3), we obtain

$$W\big(\bar{s}, r(\bar{s})\big) - W(\bar{s}) = \int_{\underline{v}}^{\overline{v}} F\big(v \,|\, \bar{s}\big) \left(1 - F\big(v \,|\, r(\bar{s})\big)\right) dv. \tag{4}$$

Therefore, the social value of experimentation decreases as the best technical standard improves, and increases as the best reactive standard improves.

The following lemma shows that the social value of experimentation is increasing in demand uncertainty. We say a demand generated by distribution $G(v \,|\, s)$ is more uncertain than one generated by $F(v \,|\, s)$, if $G(v \,|\, s)$ is a mean-preserving spread of $F(v \,|\, s)$ for any $s \in S$. When there is no value for experimentation, we show that standards wars are always dominated by mandated standards.

Lemma 1 (Experimentation and uncertainty)**.** *The social value of experimentation is increasing in demand uncertainty.*

A corollary of Lemma 1 is that if technologies are simple and there is a fixed cost of developing and negotiating standards, standards wars dominate mandated standards only if there is sufficient uncertainty in demand.

Let Φ be the fixed cost of developing a standard, and assume $\Phi < H_{r(\bar{s})}\big(\bar{s}, r(\bar{s})\big)$, so that two standards are developed in a standards war. The social cost of a mandated standard is Φ, and the total social cost of a standards war is $2\,\Phi$.

Corollary 1. *If technologies are simple and developing a standard is costly, standards wars weakly dominate mandated standards if and only if demand uncertainty is large enough.*

The following proposition presents a sufficient condition for mandated standards to be welfare optimal.

Proposition 3 (Optimal policy with no demand uncertainty)**.** *If there is no uncertainty in demand, mandated standards weakly dominate standards wars.*

Propositions 2 and 3, and Corollary 1 compare the welfare properties of standards wars and mandated standards. When technologies are non-monotonic, a patent that is valuable for a standard may not be valuable for other standards, and when patent ownership is concentrated, valuable technologies of firms with few patents tend to be underutilized, because firms with a large patent portfolio become more desirable for standard membership. Non-monotonic technologies and concentrated patent ownership hinder the coalition-formation process in a standards war, and may lead to an inefficient technology choice from an ex-ante perspective. A mandated standard, on the other hand, selects technologies based on ex-ante information, and may lead to inefficient technology choice from an ex-post perspective when demand uncertainty is high.

As a consequence, standards wars are preferable when technological complexity is low and demand uncertainty is high, and mandated standards are preferable when technological complexity is high and demand uncertainty is low.

Assumptions similar to those of Propositions 2 and 3 have been used extensively in the literature. Lerner and Tirole (2004) and Lerner and Tirole (2013) assume that each firm owns at most one patent, which is equivalent to dispersed ownership. Lerner and Tirole (2004) assume that the value of a pool of patents depends only on the number of patents, an assumption which is stronger than monotonicity. Llanes and Poblete (2014) assume that all firms participating in the standard-setting process have patents that are valuable for the standard, which is a stronger version of monotonicity. All these papers assume that the value of standards is certain.

Propositions 2 and 3 allow us to understand the main factors influencing technology choice in standard wars and mandated standards, and present simple conditions which can help us characterize the optimal standard-setting rules in many real-world situations. However, in situations in which standard-setting is characterized by concentrated patent ownership, non-monotonic technologies and high demand uncertainty, the above propositions will not be useful to determine whether we should favor standards wars or mandated standards.

Thus, it would be interesting to obtain results that do not depend on these assumptions. In Sections 6 and 7, we show that it is possible to obtain more general results when firms are allowed to sign ex-ante agreements.

6. Ex-ante agreements

Previous works studied the effect of allowing firms to sign enforceable contracts determining the distribution of surplus between standard sponsors at the standard-setting stage (ex-ante agreements) in the context of a single (monopoly) standard (Llanes and Poblete, 2014; Lerner and Tirole, 2013). These papers show that, in the monopoly case, ex-ante agreements improve the standard-setting process by aligning firm revenues with the marginal contributions of patents.

In this section, we study the effect of allowing ex-ante agreements in the context of a standards war. For simplicity we return to the framework of Section 2, and assume that at most two standards can compete for adoption. Formally, allowing for ex-ante agreements, the model becomes a partition function game with transferable utility (Thrall and Lucas, 1963).

An allocation is associated to a sharing rule $w = (w_i)_{i \in I}$, where w_i is the ex-ante payment of firm i. Let $H_s(a)$ be the total expected payoff for standard s in allocation a. A sharing rule w is feasible with respect to allocation a if for all $s \in a$, $\sum_{i \in \mu(s)} w_i \leq H_s(a)$. Note that we do not allow firms to make transfers to firms in other standards. The following definition explains the standard formation process.

Definition 4 (Stable allocation with ex-ante agreements)**.** *A standard s blocks allocation a with associated sharing rule w if for any allocation a' that contains s, $\sum_{i \in \mu(s)} w_i < H_s(a')$. Allocation a, with associated sharing rule w, is stable if a standard blocking it does not exist.*

The following proposition compares total industry profits in standards wars and mandated standards.

Proposition 4. *With ex-ante agreements, expected industry profits in a standards war are larger than or equal to expected profits in a mandated standard.*

Proposition 4 still holds if firms have reactive beliefs. The following Corollary shows a sufficient condition that guarantees standards wars are welfare optimal with ex-ante agreements. Recall that profits and welfare are aligned if $\pi(v) = \alpha\, v$ for a constant $\alpha \in [0, 1]$.

Corollary 2. *If profits and welfare are aligned, standard wars weakly dominate mandated standards.*

Corollary 2 shows that ex-ante agreements and standards wars are a desirable combination if the interests of industry participants and the users of the standard are aligned. Note that this result does not require dispersed ownership or monotonicity of technologies.

The following example shows that even though industry profits are always larger in a standards war, a standards war may lead to lower welfare than a mandated standard when profits and welfare are not aligned.

Example 3. Consider an example with four firms, $N = \{A, B, C, D\}$; two components, $M = 2$; and four patents $P = \{(A, 1), (B, 2), (C, 1), (D, 2)\}$. There are three standards with positive expected value, $s_1 = \{(A, 1), (B, 2)\}$, $s_2 = \{(A, 1), (D, 2)\}$, and $s_3 = \{(C, 1), (B, 2)\}$. The value distributions are as follows. s_1 leads to $v = 0$ with probability 1/2, and to $v = \bar{v} > 1$ with probability 1/2. s_2 and s_3 lead to $v = 0$ with probability 1/2, and to $v = 1$ with probability 1/2. Let $\pi(v) = v^{1/2}$.

Two allocations are of interest: $a_1 = \{s_1\}$ and $a_2 = \{s_2, s_3\}$. a_1 corresponds to the mandated standard, and its value distribution is the value distribution of s_1. a_2 leads to $v = 0$ with probability 1/4, and to $v = 1$ with probability 3/4. Note that the example satisfies monotonicity and dispersed ownership.

Welfare with a_1 is equal to $\frac{1}{2}\bar{v}$, and total industry profits are $\frac{1}{2}\bar{v}^{1/2}$. Welfare and total industry profits with a_2 are equal to $\frac{3}{4}$. If $\frac{3}{2} < \bar{v} < \frac{9}{4}$, a_1 leads to larger welfare but smaller industry profits than a_2.

It is straightforward to see that a_1 is not a stable allocation. Firms C and D, which are not part of any standard in a_1, can always compensate firms A and B to form a_2, since a_2 leads to larger industry profits. a_2 is stable if the payoffs of firms A and B are larger than or equal to $\frac{v^{1/2}}{4}$, which is always possible.

Example 3 shows that standards wars may be suboptimal with ex-ante agreements, even assuming monotonicity and dispersed ownership. Expected profits depend on $\pi(v)$, while expected welfare depends on v, and nothing guarantees that $\pi(v)$ is aligned with v.

For example, v may be related to the useful life of the standard, while patents last for a fixed amount of time. Thus, firms capture proportionally less value as v increases. In this case, $\pi''(v) < 0$ and firms will tend to choose standards with value distributions that accumulate more weight on lower values of v. On the other hand, developing a standard may involve administrative and legal costs which increase less than proportionally with v. In this case, $\pi''(v) > 0$, and firms have incentives to choose standards with value distributions that accumulate more weight on higher values of v.

The main conclusion of this section is that ex-ante agreements do not guarantee that standards wars lead to higher welfare than mandated standards, unless the interests of industry participants and users are aligned. The reason is that there exist limits to efficient bargaining. Standard sponsors cannot receive transfers from the sponsors of competing standards or from consumers. These constraints on bargaining create a misalignment between the incentives of firms and welfare.

Finally, it is interesting to compare the result of this section with that obtained in the monopoly standards models of Llanes and Poblete (2014) and Lerner and Tirole (2013). In those papers, ex-ante agreements were always welfare improving. Here, we show that this result may fail to hold when there is competition between standards.

7. Multiple standard membership

In this section we study the standard-setting process when firms can participate in more than one standard (unrestricted participation). We show that if the participation of firms in standard forums is widespread (a first-best allocation leads to a connected network of standard sponsors), ex-ante agreements unambiguously lead to better standards.

The set of allocations is $A = \wp(S)$, where $\wp(S)$ is the power set of S. Let $\mu(a)$ be the set of sponsors of the standards in allocation a, and let $B(a)$ be the set of allocations that can be formed without using the patents of the sponsors of the standards in a,

$$B(a) = \{b \in A \mid \mu(b) \cap \mu(a) = \varnothing\}.$$

Let $p(a)$ be the finest partition of allocation a in sets of standards with different sponsors. That is, for all $b, b' \in p(a)$, $\mu(b) \cap \mu(b') = \varnothing$, and for all $s, s' \in b \in p(a)$, $\mu(s) \cap \mu(s') \neq \varnothing$. Let $q(a)$ be the corresponding partition of sponsors,

$$q(a) = \{c \subseteq \mu(a) \mid c = \mu(b) \text{ for some } b \in p(a)\}.$$

We will refer to a set of firms $c \subseteq N$ as a coalition. By construction, the coalitions in $q(a)$ have empty intersection.

Two firms i, j are *linked* in allocation a if they belong to the same coalition in $q(a)$. Firms have a *direct link* if they belong to a same standard in a, and they have an *indirect link* if there is a path of direct links connecting them (for example, they belong to two different standards, but there exists a third firm which belongs to both standards).

An allocation a is *connected* if every pair of sponsors in $\mu(a)$ is linked. It is straightforward to see that if a is connected, $p(a) = \{a\}$ and $q(a) = \{\mu(a)\}$. That is, if a is connected, it is impossible to partition it into two or more groups of standards with non-overlapping sponsors.

A sharing rule is a vector $w = (w_i)_{i\in I}$ where w_i is the total expected payoff of firm i in allocation a. Let $H_s(a)$ be the total expected profit of standard s in allocation a, and let $H_b(a) = \sum_{s\in b} H_s(a)$ be the total expected profit of a subset of standards $b \subseteq a$. A sharing rule is feasible if for all $b \in p(a)$, $\sum_{i\in\mu(b)} w_i \leq H_b(a)$. That is, firms can only redistribute revenues within a coalition.[10]

Definition 5. *A connected allocation b blocks allocation a with sharing rule w if for all $b' \in B(b)$, $\sum_{i\in\mu(b)} w_i < H_b(b \cup b')$. Allocation a, with associated sharing rule w, is stable if it is not blocked by any connected allocation.*

In contrast with the previous sections, a deviation may now involve a set of standards, instead of a single standard. For this coordination to be possible, we assume the sponsors of the deviating allocation are connected.

Welfare is defined as in (12). Let A^{FB} be the set of first best allocations,

$$A^{FB} = \{a \in A \mid \nexists\, b \neq a \text{ such that } W(b) > W(a)\}$$

Proposition 5. *If there exists a connected first-best allocation, all stable allocations are first-best allocations.*

Intuitively, if an allocation is not first best, then it leads to a lower total industry profits than the first-best allocations. Firms in the connected first-best allocation can form this allocation and distribute the larger industry profits in a way that makes all the sponsors of the inefficient allocation better off.

[10]The assumption that firms can only redistribute revenues within a coalition is equivalent to the assumption that firms can only redistribute revenues within a standard. Two firms in different standards, but in the same coalition, can make indirect transfers between them through a series of transfers with other firms in the same coalition.

Proposition 5 shows that ex-ante agreements are desirable from a welfare perspective when the standard-setting process is open and collaborative. This proposition also shows it is valuable to have "umbrella" firms participating in multiple standard-setting efforts, because these firms can serve as indirect links between firms with narrower interests.

8. Discussion and further research

In Appendix B, we provide several extensions to the basic model. In this section, we discuss further extensions to our model and potential directions for further research.

First, the model can be extended to allow for an endogenous number of functionalities. We now define a standard as a set of non-redundant patents implementing *some* product functionalities. The set of all possible standards is given by

$$S = \{s \subseteq P \mid |s| \leq M \text{ and } (i,m),(i',m') \in s \Rightarrow m \neq m'\},$$

and A, $\mu(s)$, $F(v|s)$, $\pi(v)$, $W(a)$, $H_s(a)$, and $h_s(a)$ are defined as in Section 2.

Assume first that (i) one of the functionalities is essential (any standard s that does not implement the essential functionality has $F(0 \mid s) = 1$), (ii) exactly two technologies may implement the essential functionality, and (iii) each firm may sponsor only one standard. These assumptions guarantee that at most two standards may compete for adoption ($|a| \leq 2$ for all $a \in A$). Under these assumptions, it is straightforward to show that Propositions 1, 3, and 4 continue to hold. And if we assume firms can participate in more than one standard, we can also show that Proposition 5 continues to hold.

Proposition 2, on the other hand, no longer holds. The reason is that when the number of functionalities is endogenous, standards may have a different number of sponsors, even if patent ownership is dispersed. To understand this result, consider an example where only two standards are possible. Standard s_1 implements one functionality with a patent owned by firm A, and standard s_2 implements two functionalities with patents owned by firms A and B. If $\mathbb{E}(v|s_2) > \mathbb{E}(v|s_1)$, but $h_{s_2}(s_2) < h_{s_1}(s_1)$, the mandated standard is s_2 and a standards war leads to s_1. Thus, the mandated standard leads to higher welfare than a standards war, even though patent ownership is dispersed and technologies are monotonic.

Functionality choice adds another layer of complexity to the standard-setting process, which makes it harder to guarantee that standards wars are better than mandated standards *if ex-ante agreements are not allowed*. However, all our results on the effects of ex-ante agreements on technical efficiency remain unchanged. Thus, ex-ante agreements lead to more efficient standards, if profits and welfare are aligned or the participation in standard-setting bodies is unrestricted and widespread, even if functionality choice is endogenous.

Second, even though most SSOs do no allow *explicit* ex-ante licensing discussions, many of them allow (or demand) FRAND (fair, reasonable and non-discriminatory) licensing commitments. FRAND commitments have been criticized for being subjective and ambiguous, since firms may differ in the level of licensing fees they consider "fair and reasonable." Recently, some researchers and judges have interpreted FRAND licenses as the license fee that should be charged based on ex-ante marginal contributions (Swanson and Baumol, 2005; Farrell et al., 2007; Layne-Farrar et al., 2007; Dehez and Poukens, 2013). All our results hold if FRAND commitments lead to licenses based on ex-ante marginal contributions. However, it is important to note that FRAND licenses may be subject to greater uncertainty and higher litigation costs, which may affect technology choice in standard setting if firms try to avoid future disputes. Thus, explicit ex-ante licensing agreements may have different implications for the formation of standards than implicit licensing commitments.

Third, if a patent pool with standard-essential patents fails to form after the standard is set, fragmentation of intellectual property rights may lead to inefficiencies due to royalty stacking and transaction costs. We have abstracted from this problem by assuming that quasirents $\pi(v)$ do not depend on the number of sponsors of the standard. In Llanes and Poblete (2014) we studied the relation between standard-setting and patent-pool formation, and showed that ex-ante agreements may improve the stability of patent pools if firms can negotiate their participation in the patent pool at the standard-setting stage. All our results on the efficiency of ex-ante agreements in standards wars hold under this assumption.

Fourth, the standard setting process may imply bargaining costs, which may depend on the number of sponsors of the standard and on the structure of intellectual property rights (IPRs). Bargaining costs may affect our results in several ways. On one hand, firms may prefer to join standards with higher dispersion of IPRs because this guarantees that all firms have an equal footing when negotiating standards. On the other hand, firms may prefer to join standards with clear technological leaders and concentrated IPR ownership because this may reduce uncertainty and speed up the standard-setting process. Therefore, the overall effect of incorporating bargaining costs is ambiguous. We believe this is an interesting direction for further research.

Fifth, we have focused on technology choices that are difficult to reverse after the standard is defined. However, some technologies may be easy to substitute, even after the standard is set. These technologies do not impose a serious threat to efficient standard formation, because the possibility of substitution limits the bargaining power of the firms sponsoring these technologies. Our results will hold as long as some technologies become harder to substitute after they are included in a standard.

Sixth, our paper discusses optimal technology choice taking the set of existing patents as given. Recent papers have studied how standard-setting rules may affect incentives for innovation (Dequiedt and Versaevel, 2013; Cabral and Salant, 2013; Layne-Farrar et al., 2014). Incorporating incentives for innovation in a model of coalition formation and technology choice is another venue for future research.

Finally, we have assumed that standards wars are fought between coalitions in different SSOs, but competition between groups of firms sponsoring different technologies is intense even *within* SSOs. For example, the Task Group n (TGn) and the World-Wide Spectrum Efficiency Group (WWiSE) competed for control of the 802.11n Wi-Fi standard within IEEE (DeLacey et al., 2006). Our results extend directly to a model of coalition-formation within SSOs.

9. Conclusion

We develop a model of technology choice and coalition formation in standards wars to address the following questions: (i) How does competition between groups of technology sponsors affect the standard-setting process? (ii) Under what conditions is it better to have a standards war or a mandated standard? (iii) What is the effect of ex-ante agreements on technology efficiency in standards wars?

We present three main results. First, if licensing agreements are not allowed at the standard-setting stage, standards wars lead to better standards than mandated standards when patent ownership is dispersed and technologies have a monotonic effect on technical efficiency (if a technology is valuable for some standard, then it is valuable for any standard that includes it). Mandated standards, on the other hand, lead to better standards when the uncertainty about the performance of alternative standards is small.

This result has practical implications for the optimal design of standard-setting rules. For example, new technologies are generally more uncertain than generational upgrades. In the case of wireless telecommunication standards, the change from the analog first-generation to the digital second-generation standards (1G to 2G) was seen by the industry as a disruptive change, while the change from the second to the third-generation (2G to 3G) was seen more as an evolution than a discontinuity (Nokia Networks, 2003). According to our analysis, the European policy of mandating a single standard was more appropriate in the case of 3G than in the case of 2G.

Likewise, in the case of the high-definition optical-disc format war, Blu-ray was more expensive and less backward compatible than HD DVD, but had a higher storage capacity. Ex ante, there was uncertainty as to what technology was optimal. Under such conditions, our paper shows that a standards war may be an efficient way to elucidate what is the optimal technology.

Second, we find that allowing ex-ante agreements may decrease welfare in the case of standards wars. Even though total industry profits are larger in a standard war than in a mandated standard, welfare is not necessarily larger because profits may not be aligned with welfare.

Thus, it is important to interpret earlier results with caution. In particular, previous works (Llanes and Poblete, 2014; Lerner and Tirole, 2013), showed that in the case of monopoly standards, the interests of firms and society are always aligned, and thus, ex-ante agreements are welfare improving. We show that this result no longer holds in the case of standards wars, unless we impose additional restrictions on the standard-setting process.

Third, we find that ex-ante agreements lead to better technology choice in standards wars if participation in standard-setting bodies is unrestricted and widespread. This result also shows that it is valuable to have "umbrella" firms participating in multiple standard-setting efforts, because these firms can serve as indirect links between firms with narrower interests. This is the case of

HP and Sun, for example, which are involved in the development of over 150 standards at a given time (Updegrove, 2003).

Our paper contributes to the standards literature by studying the welfare properties of alternative standard-setting rules in a model of competition between standards with multiple sponsors. We also contribute to the literatures of coalition formation and equal-sharing partnerships by providing novel existence results, characterizing stable allocations, and describing the relations between several cooperative and non-cooperative solution concepts in the presence of externalities, both with a fixed distribution of output and when the distribution of output is endogenous.

Appendix A: Proofs of theorems in text

Proof of Proposition 1. We claim that a^* is stable. Suppose it is not. Then, there exists a standard $\tilde{s}$ that blocks this allocation. The blocking standard either contains sponsors of s^*, sponsors of $r(s^*)$, or both. Otherwise, three standards of positive value can be created, which violates the duopoly assumption.

Suppose first that $\tilde{s}$ contains a sponsor of s^*. Then, it must be the case that $h_{\tilde{s}}(\tilde{s}, z) > h_{s^*}(s^*, r(s^*))$ for every $z \in R(\tilde{s})$. By definition, $r(\tilde{s}) \in R(\tilde{s})$, thus $h_{\tilde{s}}(\tilde{s}, r(\tilde{s})) > h_{s^*}(s^*, r(s^*))$, which violates the definition of s^*.

Suppose now that $\tilde{s}$ contains sponsors of $r(s^*)$ but not sponsors of s^*. Then for $\tilde{s}$ to block the allocation it must be the case that $h_{\tilde{s}}(z, \tilde{s}) > h_{r(s^*)}(s^*, r(s^*))$ for every $z \in R(\tilde{s})$. We already proved that the sponsors of $\tilde{s}$ cannot be in s^*, thus it must be the case that $h_{\tilde{s}}(s^*, \tilde{s}) > h_{r(s^*)}(s^*, r(s^*))$, which violates the definition of $r(s^*)$. Therefore, a blocking standard cannot exist. ∎

Proof of Proposition 2. We will show that with simple technologies, the unique stable allocation is $\bar{a}$. Thus, standards wars weakly dominate a mandated standard, which only includes $\bar{s}$. We start by showing that any stable allocation must contain $\bar{s}$. We prove this result by contradiction. Suppose there exist s_1 and s_2 different from $\bar{s}$ such that $\{s_1, s_2\}$ is a stable allocation. Without loss of generality, assume that $h_{s_1}(s_1, s_2) \geq h_{s_2}(s_1, s_2)$. We will now show that there must exist a standard that blocks allocation $\{s_1, s_2\}$, which contradicts the hypothesis that $\{s_1, s_2\}$ is stable. We proceed in four steps.

Step 1. If $\{s_1, s_2\}$ is stable, then $h_{s_1}(s_1, s_2) \geq h_{\bar{s}}(\bar{s}, r(\bar{s}))$. Suppose not, i.e., $h_{s_1}(s_1, s_2) < h_{\bar{s}}(\bar{s}, r(\bar{s}))$. Dispersed ownership implies that the number of sponsors of any standard is constant, and that $R(s) = S \setminus s$ for any $s \in S$. Then, $r(\bar{s})$ maximizes the technical value of the standard for $r(\bar{s}) \in S \setminus \bar{s}$, and $h_{\bar{s}}(\bar{s}, z) \geq h_{\bar{s}}(\bar{s}, r(\bar{s}))$ for any $z \in S \setminus \bar{s}$. But this implies that $h_{\bar{s}}(\bar{s}, z) \geq h_{\bar{s}}(\bar{s}, r(\bar{s})) > h_{s_1}(s_1, s_2) > h_{s_2}(s_1, s_2)$ for any $z \in S \setminus \bar{s}$. Thus, $\bar{s}$ blocks the allocation $\{s_1, s_2\}$ if $h_{s_1}(s_1, s_2) < h_{\bar{s}}(\bar{s}, r(\bar{s}))$.

Step 2. If $\{s_1, s_2\}$ is stable, then $\bar{s} \succ s_1$, $r(\bar{s}) \succ s_2$, and $h_{s_2}(s_1, s_2) < h_{r(\bar{s})}(\bar{s}, r(\bar{s}))$. $\bar{s} \succ s_1$ follows from the definition of $\bar{s}$. $r(\bar{s}) \succ s_2$ follows from $\bar{s} \succ s_1$ and $hs_1(s_1, s_2) \geq h_{\bar{s}}(\bar{s}, r(\bar{s}))$. Finally, from equation (1) it follows that

$$h_{s_1}(s_1, s_2) + h_{s_2}(s_1, s_2) = \frac{1}{M} \int_{\underline{v}}^{\overline{v}} \pi'(v) \left(1 - F(v \,|\, s_1)\, F(v \,|\, s_2)\right) dv,$$

and first-order stochastic dominance implies that

$$\int_{\underline{v}}^{\overline{v}} \pi'(v) \left(1 - F(v \,|\, \bar{s})\, F(v \,|\, r(\bar{s})\right) dv > \int_{\underline{v}}^{\overline{v}} h'(v) \left(1 - F(v \,|\, s_1)\, F(v \,|\, s_2)\right) dv.$$

Thus, $h_{s_1}(s_1, s_2) \geq h_{\bar{s}}(\bar{s}, r(\bar{s}))$ implies that $h_{s_2}(s_1, s_2) < h_{r(\bar{s})}(\bar{s}, r(\bar{s}))$.

Step 3. There exists a standard $s_3 \in S \setminus s_1$ such that $s_3 \succeq r(\bar{s})$. Let $p(s, m)$ represent the patent used to implement functionality m in standard s. For each functionality $m = 1, \ldots, M$, construct s_3 as follows: (i) if $p(s_1, m) = p(\bar{s}, m)$, then $p(s_3, m) = p(r(\bar{s}), m)$, and (ii) if $p(s_1, m) \neq p(\bar{s}, m)$, then $p(s_3, m) = p(\bar{s}, m)$. By construction, $s_3 \in S \setminus s_1$, and given that $\bar{s} \succeq r(\bar{s})$, monotonicity implies that $s_3 \succeq r(\bar{s})$. To see why, observe that $s_3 = (r(\bar{s}) \setminus s_1) \cup (s_1 \cap \bar{s})$ and $\bar{s} = (\bar{s} \setminus s_1) \cup (s_1 \cap \bar{s})$, and that, by monotonicity, $(\bar{s} \setminus s_1) \cup (s_1 \cap \bar{s}) \succeq (\bar{s} \setminus s_1) \cup (r(\bar{s}) \cap s_1)$ implies $(r(\bar{s}) \setminus s_1) \cup (s_1 \cap \bar{s}) \succeq (r(\bar{s}) \setminus s_1) \cup (r(\bar{s}) \cap s_1)$.

Step 4. s_3 blocks allocation $\{s_1, s_2\}$. The definition of $\bar{s}$ implies that $h_{s_3}(z, s_3) \geq h_{s_3}(\bar{s}, s_3)$ for any standard $z \in S$. Also, $s_3 \succeq r(\bar{s})$ implies that $h_{s_3}(\bar{s}, s_3) \geq h_{r(s)}(\bar{s}, r(s))$. Finally, in step 2, we showed that $h_{s_2}(s_1, s_2) < h_{r(\bar{s})}(\bar{s}, r(\bar{s}))$. All these inequalities imply that $h_{s_3}(z, s_3) > h_{s_2}(s_1, s_2)$ for any standard $z \in S \setminus s_3$. Thus, s_3 blocks allocation $\{s_1, s_2\}$, which contradicts the original statement.

Finally, we show that the stable allocation must also include $r(\bar{s})$. Suppose not. Then, there is a stable allocation $\{\bar{s}, s_2\}$, with $s_2 \neq r(\bar{s})$. It is easy to see that $r(\bar{s})$ blocks this allocation, given that $\bar{s} = \text{argmin}_{s \in S \setminus r(\bar{s})} h_2(s, r(\bar{s}))$, so $\bar{s}$ is the worst possible reaction to a deviation, and $r(\bar{s}) = \text{argmax}_{s \in S \setminus \bar{s}} h_s(\bar{s}, s)$, which means that $r(\bar{s})$ provides maximum per capita profits to would be deviators given $\bar{s}$. Thus, $\{\bar{s}, r(\bar{s})\}$ is the unique stable allocation. ∎

Proof of Lemma 1. We need to show that (4) increases if the distribution F is replaced by G, a mean-preserving spread. Therefore, it suffices to show that

$$\int_{\underline{v}}^{\overline{v}} G(v \,|\, s_1)\, (1 - G(v \,|\, s_2))\, dv - \int_{\underline{v}}^{\overline{v}} F(v \,|\, s_1)\, (1 - F(v \,|\, s_2))\, dv \geq 0,$$

for any $s_1, s_2 \in S$. Integrating by parts and applying the definition of mean-preserving spreads, we obtain that

$$\int_{\underline{v}}^{\overline{v}} (G(v \,|\, s) - F(v \,|\, s))\, dv = \int_{\underline{v}}^{\overline{v}} (v\, f(v \,|\, s) - v\, g(v \,|\, s))\, dv = 0, \tag{5}$$

for any $s \in S$. Therefore, we only need to show that

$$\int_{\underline{v}}^{\overline{v}} (F(v \,|\, s_1)\, F(v \,|\, s_2) - G(v, \bar{s})\, G(v \,|\, s_2))\, dv \geq 0.$$

Integrating by parts, and using (5), we obtain

$$\int_{\underline{v}}^{\overline{v}} \Big(F(v \,|\, s_1)\, F(v \,|\, s_2) - G(v \,|\, \bar{s})\, G(v \,|\, s_2)\Big) dv =$$
$$\int_{\underline{v}}^{\overline{v}} \left(g(v \,|\, s_1) \int_{\underline{v}}^{v} G(x \,|\, s_2) dx - f(v \,|\, s_1) \int_{\underline{v}}^{v} F(x \,|\, s_2) dx\right) dv.$$

Given that G is a mean-preserving spread of F, $\int_{\underline{v}}^{x} G(x \,|\, s) dx \geq \int_{\underline{v}}^{x} F(x \,|\, s) dx$ for any s. Therefore,

$$\int_{\underline{v}}^{\overline{v}} g(v \,|\, s_1) \int_{\underline{v}}^{v} G(x \,|\, s_2)\, dx\, dv \quad \geq \quad \int_{\underline{v}}^{\overline{v}} g(v \,|\, s_1) \int_{\underline{v}}^{v} F(x \,|\, s_2)\, dx\, dv. \tag{6}$$

Integrating by parts, we obtain

$$\int_{\underline{v}}^{\overline{v}} g(v \,|\, s_1) \int_{\underline{v}}^{v} F(x \,|\, s_2)\, dx\, dv = \tag{7}$$
$$\int_{\underline{v}}^{\overline{v}} f(v \,|\, s_2) \int_{\underline{v}}^{v} G(x \,|\, s_1)\, dx\, dv + \int_{\underline{v}}^{\overline{v}} F(v \,|\, s_2) dv - \int_{\underline{v}}^{\overline{v}} G(v \,|\, s_1) dv.$$

Likewise, because G is a mean-preserving spread of F, we have that

$$\int_{\underline{v}}^{\overline{v}} f(v \,|\, s_2) \int_{\underline{v}}^{v} G(x \,|\, \bar{s})\, dx\, dv \geq \int_{\underline{v}}^{\overline{v}} f(v \,|\, s_2) \int_{\underline{v}}^{v} F(x \,|\, s_1)\, dx\, dv, \tag{8}$$

and integrating by parts, we obtain

$$\int_{\underline{v}}^{\overline{v}} f(v \,|\, s_2) \int_{\underline{v}}^{x} F(v \,|\, s_1)\, dx\, dv = \tag{9}$$
$$\int_{\underline{v}}^{\overline{v}} f(v \,|\, s_2) \int_{\underline{v}}^{x} G(v \,|\, s_1)\, dx\, dv - \int_{\underline{v}}^{\overline{v}} F(v \,|\, s_2) dv + \int_{\underline{v}}^{\overline{v}} G(v \,|\, s_1) dv.$$

From (6), (7), (8), and (9) it follows that

$$\int_{\underline{v}}^{\overline{v}} \left(g(v \,|\, s_1) \int_{\underline{v}}^{x} G(v \,|\, s_2) dx - f(v \,|\, s_1) \int_{\underline{v}}^{x} F(v \,|\, s_2) dx\right) dv \geq 0,$$

which proves the result. ■

Proof of Corollary 1. If $\Phi < H_{r(\bar{s})}\big(\bar{s}, r(\bar{s})\big)$, both standards yield positive profits. Thus, both standards will be formed in a stable allocation. standards wars lead to higher welfare than a mandated standard if $W(\bar{s}, r(\bar{s})) - W(\bar{s}) > \Phi$. The result follows from Lemma 1. ■

Proof of Proposition 3. Without uncertainty, the value of a standard is known ex-ante. Without loss of generality, v is deterministically given by a function $v(s)$, such that for any $s_1, s_2 \in S$, $s_1 \succ s_2$ implies that $v(s_1) > v(s_2)$.

The mandated standard maximizes the expected value of the standard, so $W_{ms} = v(\bar{s}) = \max_{s\in S} v(s)$. In a standards war, the standard with larger value is adopted. For any $s_1, s_2 \in S$, welfare in a standard war is given by $W_{sw} = \max\{v(s_1), v(s_2)\} \le \max_{s\in S} v(s) = W_{ms}$. Thus, standards wars cannot lead to larger welfare than mandated standard.

To show that welfare is not always the same, we only need to show an example in which the stable allocation in a standards war does not include $\bar{s}$.

Consider an example with two functionalities, $M = \{1, 2\}$, two firms, $N = \{A, B\}$, and three patents, $P = \{(1, A), (2, A), (2, B)\}$. Two standards are possible, $s_1 = \{(1, A), (2, B)\}$ and $s_2 = \{(1, A), (2, A)\}$. Suppose $v(s_1) > v(s_2)$. If $\pi(v(s_2)) > \frac{1}{2}\pi(v(s_1))$, the stable allocation in a standards war includes s_2 instead of s_1, which leads to lower welfare than a mandated standard. ■

Proof of Proposition 4. We have defined the mandated standard as

$$\bar{s} = \operatorname*{argmax}_{s\in S} \mathbb{E}(v|s).$$

Let $\bar{r}(s) = \operatorname{argmax}_{z\in R(s)} H_Z(s, z)$, and let $\bar{s}_2 = \bar{r}(\bar{s})$. Suppose that allocation $\tilde{a} = \{\tilde{s}_1, \tilde{s}_2\}$, with associated sharing rule w, is stable. If $\tilde{a}$ is stable, then it must not be blocked by $\bar{s}$, which implies that

$$\sum_{i\in\mu(\bar{s})} w_i \ge \min_{z\in R(\bar{s})} H_{\bar{s}}(\bar{s}, z),$$

and it must not be blocked by $\bar{s}_2$, which implies that

$$\sum_{i\in\mu(\bar{s}_2)} w_i \ge \min_{z\in R(\bar{s}_2)} H_{\bar{s}_2}(z, \bar{s}_2).$$

Feasibility of w implies that:

$$\sum_{i\in\mu(\bar{s})} w_i + \sum_{i\in\mu(\bar{s}_2)} w_i \leq H_{\tilde{s}_1}(\tilde{s}_1, \tilde{s}_2) + H_{\tilde{s}_2}(\tilde{s}_1, \tilde{s}_2),$$

and the definitions of $\bar{s}$ and $\bar{s}_2$ imply:

$$\begin{aligned} \min_{z\in R(\bar{s})} H_{\bar{s}}(\bar{s}, z) &= H_{\bar{s}}(\bar{s}, \bar{s}_2), \\ \min_{z\in R(\bar{s}_2)} H_{\bar{s}_2}(z, \bar{s}_2) &= H_{\bar{s}_2}(\bar{s}, \bar{s}_2). \end{aligned}$$

Thus,

$$H_{\tilde{s}_1}(\tilde{s}_1, \tilde{s}_2) + H\tilde{s}_2(\tilde{s}_1, \tilde{s}_2) \geq H_{\bar{s}}(\bar{s}, \bar{s}_2) + H_{\bar{s}_2}(\bar{s}, \bar{s}_2).$$

It is straightforward to see that allocation $\{\bar{s}, \bar{s}_2\}$ leads to higher industry profits than allocation $\{\bar{s}\}$. Thus, any stable allocation leads to higher profits than a mandated standard. ■

Proof of Proposition 5. Let a^* to be a first best connected allocation. Take a to be an allocation that is not first best and let w_i be the expected payment that firm i receives in such an allocation. Feasibility requires

$$\int_0^{\overline{v}} \pi'(v) \left(1 - \prod_{s\in a} F(v|s)\right) dv = \sum_{i\in\mu(a)} w_i.$$

Also notice that it must be the case that a^* includes all standards of a that creates positive value and at least one more non trivial standard. If a^* does not include a syandard of positive value it cannot maximize expected welfare. Therefore the expected industry profits under allocation a^* is

$$\Pi(a^*) = \int_0^{\overline{v}} \pi'(v) \left[1 - \prod_{s\in a^*} F(v|s)\right] dv = \int_0^{\overline{v}} \pi'(v) \left[1 - \prod_{s\in a} F(v|s) \prod_{s\in a^*\cap a^c} F(v|s)\right] dv$$

Since $\pi'(v)$ is assumed to be positive, and the standards in $a^* \cap a^c$ cannot all be trivial, it follows that industry profits are larger under a^*than a. Define $\Delta = \Pi(a^*) - \Pi(a)$.

Consider allocation a^* as a coalition (this can be done because a^* is connected, with expected payment $\widehat{w}$ defined as follows. $\widehat{w}_i = w_i + \frac{\Delta}{\#\mu(a^*)}$ if $i \in \mu(a)$ and $\widehat{w}_i = \Delta$ otherwise. This expected payments satisfy budget constraint under any belief system because all standards with positive value are included in the coalition, moreover every firm is strictly better off, therefore allocation a is blocked. ■

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Appendix B: Alternative specifications (For Online Publication)

In this online appendix, we show that our main results are robust to alternative specifications of the standard-formation mechanism. First, we consider cooperative solutions with reactive beliefs. Second, we study standard formation in a non-cooperative coalition formation game. Finally, we show that our results extend to a standards war between more than two standards.

B.1. Stability with reactive beliefs

We now study stable allocations assuming that firms have reactive beliefs. With reactive beliefs, deviating coalitions believe that non-deviators react to maximize their utility. In formal terms, an allocation is stable if it satisfies the following definition.

Definition 6 (Stable allocation with reactive standards)**.** *A standard s blocks allocation a if the sponsors of s are strictly better off in allocation $\{s, r(s)\}$ than in a. An allocation is stable if a standard blocking it does not exist.*

The following lemma shows how the above definition relates to the definition of stability with pessimistic beliefs.

Lemma 2. *Any stable allocation of the game with reactive beliefs is a stable allocation of the game with pessimistic beliefs. If patent ownership is dispersed, a stable allocation exists.*

Proof. Suppose allocation $\hat{a} = \{\hat{s}_1, \hat{s}_2\}$ is stable. Without loss of generality, suppose $h_{\hat{s}_2}(\hat{s}_1, \hat{s}_2) \geq h_{\hat{s}_2}(\hat{s}_1, \hat{s}_2)$. If $\hat{a}$ is stable with reactive beliefs, then for all $\tilde{s}_1 \in S$,

$$h_{\hat{s}_1}(\hat{s}_1, \hat{s}_2) \geq h_{\tilde{s}_1}(\tilde{s}_1, r(\tilde{s}_1)),$$

and for all $\tilde{s}_2 \in S \setminus \hat{s}_1$,

$$h_{\hat{s}_2}(\hat{s}_1, \hat{s}_2) \geq h_{\tilde{s}_2}(\tilde{s}_2, r(\tilde{s}_2)).$$

Thus, $\hat{a}$ cannot blocked with pessimistic beliefs. This proves the first part of the proposition.

For the second part, note that if patent ownership is dispersed,

$$r(s) = \operatorname*{argmax}_{z \in R(s)} \mathbb{E}(v|z). \tag{10}$$

We will now show that $a^* = \{s^*, r(s^*)\}$ is stable if patent ownership is dispersed. First, note that there does not exist a blocking standard that includes the sponsors of s^* because, by definition,

$$s^* = \underset{s \in S}{\operatorname{argmax}}\ h_s(s, r(s)).$$

Second, the best deviation that can be done by the sponsors of $r(s^*)$ is

$$\tilde{s} = \underset{s \in R(s^*)}{\operatorname{argmax}} h_1(s, r(s)).$$

The result follows by noting that $r(s^*) \succ \tilde{s}$ and $r(\tilde{s}) \succ s^*$ by (10), which imply that $h_{r(s^*)}(s^*, r(s^*)) \geq h_{\tilde{s}}(\tilde{s}, r(\tilde{s}))$. Thus, the sponsors of $r(s^*)$ cannot gain by deviating and proposing $\tilde{s}$. ■

A corollary of Lemma 2 is that Propositions 2 and 3 will hold for reactive beliefs. If technologies are simple, by Lemma 2 a stable allocation with reactive beliefs always exists (because simple technologies imply dispersed ownership), and this allocation is stable with pessimistic beliefs. By Proposition 2, the stable allocation with pessimistic beliefs is unique and dominates a mandated standard. Thus, if technologies are simple and firms have reactive beliefs, standards wars dominate mandated standards. If there is no demand uncertainty, a mandated standard weakly dominates any stable allocation. Since any stable allocation with reactive beliefs is also stable with pessimistic beliefs, it must be dominated by a mandated standard.

B.2. Non-cooperative coalition formation

In this section, we show our main results extend to a non-cooperative coalition-formation game based on Bloch (1996).

Firms take turns to propose standards and to accept proposals according to a fixed rule ρ. The game proceeds as follows. The first player in ρ proposes a standard s. Each prospective sponsor of s responds to the proposal in the order determined by ρ. If one of the players rejects the proposal, the proposal is discarded and the next player in ρ proposes a standard. If all sponsors accept, the standard is formed and its sponsors withdraw from the game. In the following stage, the next player in ρ who is not a sponsor of s proposes a standard. The first player in ρ continues play after the last player of ρ plays. The game continues in this fashion until no further standards can be formed.

History h^t at period t is a list of offers, acceptances and rejections up to period t. Let $a^t = \{s_k\}_{k=1}^t$ be the set of standards that have been formed in previous periods, where $s_k = \varnothing$ if a standard was not formed in period k. Note that $a^t \in A \cup \varnothing$. Let $T(h^t) \in S$ be the proposal received by the player moving in period t.

We need to generalize function $R(s)$, defined in (2), so that it can handle allocations, and not only standards. Let $R(a)$ be the set of standards that can be formed without using any of the patents of the sponsors of the standards in a:

$$R(a) = \{z \in S \mid \forall s \in a,\ \mu(z) \cap \mu(s) = \varnothing\}. \tag{11}$$

A strategy σ_i for player i is a mapping from the set of histories to the set of actions,

$$\begin{aligned}\sigma_i(h^t) &\in \{\text{Yes, No}\} \text{ if } T(h^t) \neq \varnothing,\\ \sigma_i(h^t) &\in R(a^t) \cup \varnothing \text{ if } T(h^t) = \varnothing.\end{aligned}$$

If $T(h^t) \neq \varnothing$, player i is a respondent to a proposal $T(h^t)$, and can choose to accept or reject it. If $T(h^t) = \varnothing$, either a standard was formed in the last period, the player of the previous period rejected a proposal, or the player of the previous period did not make a proposal. In any case, the player playing at t must propose a new standard to the set of firms that have not supported a standard yet. A player may choose to refrain from making a proposal by choosing $\sigma_i(h^t) = \varnothing$.

We focus on Markov perfect equilibria. A Markovian strategy is a strategy that conditions actions on the payoff-relevant state of the game. A Markov perfect equilibrium is a subgame perfect equilibrium in which players use Markovian strategies.

A payoff-relevant state of the game is the set of standards already formed a, an active proposal T, and a list of players who have accepted the proposal $y \subseteq \mu(T)$.[11] Specifically, a Markovian strategy is

$$\begin{aligned}\sigma_i(a, T, y) &\in \{\text{Yes, No}\} \text{ if } T \neq \varnothing,\\ \sigma_i(a, T, y) &\in R(a) \cup \varnothing \text{ if } T = \varnothing.\end{aligned}$$

[11] We depart from Bloch (1996) by assuming that the state includes the list of players who have accepted the proposal, y. This assumption is important because it allows us to rule out trivial equilibria in which a group of firms fails to coordinate on a profitable standard. See the proof of Lemma 3 for more details.

Following Bloch, we assume that players do not discount the future. In case of infinite play, players that are not part of any standard receive a payoff of zero. Bloch shows that this assumption is without loss of generality, since any equilibrium may be obtained in an equivalent game with discounting.

An outcome of the game is an allocation $a \in A$. The following proposition presents sufficient conditions for allocation $a^* = \{s^*, r(s^*)\}$ (the reactive allocation defined in Section 3) to be an equilibrium outcome of the non-cooperative game.

Lemma 3 (Non-cooperative coalition formation)**.** *There exist an order of play ρ^* and strategy profile σ^*_ρ which lead to a Markov perfect equilibrium with outcome a^*. If patent ownership is dispersed, there exists a strategy profile σ^* which leads to a Markov perfect equilibrium with outcome a^* for any order of play ρ.*

Proof. We begin by proving the second part of the proposition. Let $\mu_1 = \mu(s^*)$ and $\mu_2 = \mu(r(s^*))$, and consider the strategy profile σ^*, defined as follows. For $i \in \mu_1$,

$$\begin{aligned} \sigma_i^*(\varnothing, \varnothing, \varnothing) &= s^*, \\ \sigma_i^*(\varnothing, T, y) &= \begin{cases} \text{Yes} & \text{if } T = s^*, \\ \text{No} & \text{otherwise,} \end{cases} \end{aligned}$$

For $i \in \mu_2$,

$$\begin{aligned} \sigma_i^*(\varnothing, \varnothing, \varnothing) &= \varnothing, \\ \sigma_i^*(\varnothing, T, y) &= \begin{cases} \text{Yes} & \text{if } h_1(T, r(T)) > h_2(s^*, r(s^*)), \\ \text{No} & \text{otherwise.} \end{cases} \end{aligned}$$

For $i \in I \setminus (\mu_1 \cup \mu_2)$,

$$\begin{aligned} \sigma_i^*(\varnothing, \varnothing, \varnothing) &= s \in S \text{ such that } i \in \mu(s), \\ \sigma_i^*(\varnothing, T, y) &= \begin{cases} \text{Yes} & \text{if } i \in \mu(T), \\ \text{No} & \text{if } i \notin \mu(T). \end{cases} \end{aligned}$$

For all $i \in N$,

$$\sigma_i^*(s, \varnothing, \varnothing) = \begin{cases} r(s) \ \text{ if } i \in \mu(r(s)), \\ s \in S \text{ such that } i \in \mu(s) \text{ if } i \in \mu(r(s)), \end{cases}$$

$$\sigma_i^*(s, T, y) = \begin{cases} \text{Yes} & \text{if } i \in \mu(r(s)) \text{ and } T = r(s), \\ & \text{or } i \notin \mu(r(s)) \text{ and } i \in \mu(T), \\ \text{No} & \text{otherwise.} \end{cases}$$

Note that, by definition, $y = \varnothing$ if $T = \varnothing$. Also, note that it is important to include y in the state to rule out equilibria in which a group of firms fails to coordinate on a profitable standard. (Suppose that a group of firms have strategies that require them to reject a standard that increases their profits. If a single firm deviates an chooses to accept this standard, it cannot affect the equilibrium, because all other firms are still not accepting the standard. Including y in the state rules out these equilibria, because now strategies are required to be consistent if firms have to play in an off-the-equilibrium-path state in which all firms before them accepted the standard.)

It is straightforward to show that the outcome of the game with σ^* is a^*. Firms in μ_2 will not propose a standard until a standard is formed, and firms in μ_1 will propose s^* if no standard exists. After s^* is formed, firms in μ_2 will form $r(s^*)$. Firms in $N \setminus (\mu_1 \cup \mu_2)$ cannot propose any standard that improves the payoffs of firms in $\mu_1 \cup \mu_2$.

We now show that σ^* is a Nash equilibrium for the continuation game originating at any state (s, T, y), on or off the equilibrium path. That is, we show that no firm has incentives to choose a different strategy when it has to play on or off the equilibrium path.

We begin by studying the incentives to form standards after a standard is formed. Suppose that standard s is formed. It is easy to show that any standard in $R(s)$ must include some firm in $\mu(r(s))$. Firms in $\mu(r(s))$, however, will never accept a proposal different from $r(s)$, because $r(s)$ maximizes per capita profits in the set $R(s)$. To see this, suppose a state with standard s and proposal $T' \in R(s)$, reaches firm $i \in \mu(r(s))$, with $T' \neq r(s)$ and $i \in \mu(T')$, and all previous firms accepted the standard. Firm i will not accept this proposal, because if it rejects the proposal and waits, standard $r(s)$ will eventually be offered to her.

Therefore, if a standard s forms, the reactive standard $r(s)$ will be formed by the firms in $\mu(r(s))$. Firms in $N \setminus \mu(s \cup r(s))$ cannot take any action to change this allocation.

Now suppose that a state with standard s and proposal $T'' \in R(s)$ reaches firm $N \setminus \mu(s \cup r(s))$, such that $i \in \mu(T'')$, and all firms before i have accepted the proposal. A firm like i will always accept such an offer, because it knows that if the game continues, it will not be a part of an equilibrium standard. This concludes our analysis of the incentives to form a reactive standard.

We now study the incentives to propose a standard when a standard does not exist yet (the state has $a = \varnothing$). Firms know that if a standard s is formed, firms in $\mu(r(s))$ will react by forming standard $r(s)$. Also, they know that firms in μ_2 will never form a standard before s^*. Therefore, firms in μ_1 do not have incentives to propose or accept a standard different from s^* if no standard exists, since s^* maximizes $h_1(s, r(s))$.

Let us now look at the incentives of the firms in μ_2. We will show that a firm in μ_2 cannot gain by proposing or accepting a standard $s' \in S$ before standard s^* is formed. First, note that s' cannot include any firm in μ_1, because of the arguments of the previous paragraph. The most profitable standard that firms in $N \setminus \mu(s^*)$ can form is

$$\tilde{s} = \underset{s \in R(s^*)}{\operatorname{argmax}} \; h_s(s, r(s)).$$

In the proof of Lemma 2, we show that if patent ownership is dispersed, $h_{r(s^*)}(s^*, r(s^*)) \geq h_{\tilde{s}}(\tilde{s}, r(\tilde{s}))$. Therefore, firms in μ_2 cannot gain by proposing or accepting a standard when no standard has been formed yet.

Suppose now that a standard $T \in S$ such that $h_T(T, r(T)) > h_{r(s^*)}(s^*, r(s^*))$ is offered to firms in μ_2. The equilibrium strategy must stipulate that they accept this standard. However, this proposal is off the equilibrium path, because any such standard must include some firm in μ_1.

Given the above analysis, firms in $N \setminus (\mu_1 \cup \mu_2)$ cannot propose any standard that firms in $\mu_1 \cup \mu_2$ would accept. Therefore, it does not matter which standard they propose.

Finally, suppose that (off the equilibrium path) a state $(\varnothing, T, y)$ reaches firm $i \in I \setminus (\mu_1 \cup \mu_2)$, with a proposal T such that $i \in \mu(T)$ and all previous firms have accepted the proposal. The firm should accept the proposal because it knows that it will not be in an equilibrium standard if it passes on this opportunity.

This concludes the proof for the second part of the proposition. Actually, the second part of the proposition is true whenever $h_{r(s^*)}(s^*, r(s^*)) \geq h_{\tilde{s}}(\tilde{s}, r(\tilde{s}))$, which holds in particular if there is dispersed ownership, but may hold in other situations as well. Therefore, to prove the first part of the proposition, we only have to show there are σ_ρ^* and ρ^* that constitute a Markov perfect equilibrium with outcome a^* when $h_{r(s^*)}(s^*, r(s^*)) < h_{\tilde{s}}(\tilde{s}, r(\tilde{s}))$. Let

$$\hat{s} = \underset{s \in R(r(\tilde{s}))}{\operatorname{argmax}}\ h_s(s, r(s)),$$

and let $\mu_3 = \mu(r(\tilde{s}))$. Consider strategy profile σ_ρ^*, defined as follows.

For $i \in \mu_1 \cap \mu_3$,

$$\begin{aligned} \sigma_i^*(\varnothing, \varnothing, \varnothing) &= s^*, \\ \sigma_i^*(\varnothing, T, y) &= \begin{cases} \text{Yes} & \text{if } T = s^*, \\ \text{No} & \text{otherwise}, \end{cases} \end{aligned}$$

For $i \in \mu_1 \setminus \mu_3$,

$$\begin{aligned} \sigma_i^*(\varnothing, \varnothing, \varnothing) &= \hat{s}, \\ \sigma_i^*(\varnothing, T, y) &= \begin{cases} \text{Yes} & \text{if } T = s^* \text{ or } T = \hat{s}, \\ \text{No} & \text{otherwise}, \end{cases} \end{aligned}$$

For $i \in \mu_2$,

$$\begin{aligned} \sigma_i^*(\varnothing, \varnothing, \varnothing) &= \tilde{s}, \\ \sigma_i^*(\varnothing, T, y) &= \begin{cases} \text{Yes} & \text{if } h_T(T, r(T)) > h_{r(s^*)}(s^*, r(s^*)), \\ \text{No} & \text{otherwise}. \end{cases} \end{aligned}$$

Finally, let $\sigma_\rho^* = \sigma^*$ in all other situations. Following similar arguments than before, it can be shown that σ_ρ^* constitutes an equilibrium of the game when the rule of play ρ^* is as follows. First, all players in $\mu_1 \cap \mu_3$ play, followed by all players in $\mu_1 \setminus \mu_3$, and by all players in μ_2, to finish with all players in $N \setminus (\mu_1 \cup \mu_2)$. ■

This lemma again makes it possible to re-state our main propositions. There always exist an order of play and a strategy profile such that Propositions 2, and 3, and Lemma 1 hold with a non-cooperative game. Moreover, if patent ownership is dispersed, there exists a strategy profile for which these results hold, regardless of the order of play.

Interestingly, the condition for existence of an equilibrium with outcome a^* for any order of play is the same as the condition for the existence of a stable allocation with reactive beliefs. That is, allocation a^* is an equilibrium outcome of the non-cooperative game for any order of play if and only if a stable allocation with reactive beliefs exists. This result shows that when allocations

B.3. Standards wars with more than two standards

In this section, we relax the assumption that at most two standards may exist. Assume there exists one functionality that can be implemented by exactly $K \geq 2$ patents. All other functionalities can be implemented by K patents or more. Under these assumptions, at most K standards may compete for adoption.

The analysis of this section is technically involved because constructing the reactive allocation a^* becomes more complex. We show that in this more general case there exists a stable allocation, and that, with some qualifications, the main results of the previous sections continue to hold.

With more than two standards, the expected welfare of allocation a is

$$W(a) = \int_0^{\overline{v}} \left(1 - \prod_{s \in a} F(v \mid s)\right) \, dv, \tag{12}$$

and the expected quasirent of standard s in allocation a is

$$H_s(a) = \int_0^{\bar{v}} \pi(v) \left(\prod_{s' \in a \setminus s} F(v \mid s') \right) \, dF(v \mid s).$$

Integrating by parts, we can obtain total expected industry profits:

$$\sum_{s \in a} H_s(a) = \int_0^{\overline{v}} \pi'(v) \left(1 - \prod_{s \in a} F(v \mid s)\right) \, dv.$$

For easiness of exposition, in what follows we simplify notation by writing (a, s) to indicate $\big(a \cup \{s\}\big)$ in the arguments of functions. For example, we write $H_s(a, s)$ to indicate $H_s\big(a \cup \{s\}\big)$.

We will show that a stable allocation exists. The definition of stable allocation is still given by Definition 1. Note that, even though the largest possible allocation has K standards, a stable allocation may have fewer than K standards.

To find the reactive allocation, we need to redefine the notion of reactive standard to account for the formation of more than two standards. Suppose a standard s forms. The best response to s, which we denote by $r_2(s)$, is the most profitable standard that can be formed without the patents of the sponsors of s. To find $r_2(s)$, however, we need to forecast how the firms that are not sponsoring s and $r_2(s)$ are going to react to the creation of $r_2(s)$. That is, we need to find the best response to $\{s, r_2(s)\}$, which we denote by $r_3(s, r_2(s))$. To calculate r_3, in turn, we need to calculate r_4 and so on, until no more standards can be formed.

$$A^k = \{a \subseteq S \ \mid \ |a| = k \text{ and } \forall\, s, s' \in a,\ \mu(s) \cap \mu(s') = \varnothing\}.$$

The set of all possible allocations is $A = \cup_{k=1}^{K} A^k$. Recall we defined $R(a)$ in (11) as the set of standards that can be formed without using any of the patents of the sponsors of the standards in a.

Define function $r_K : A^{K-1} \to S$ as follows:

$$r_K(a) = \underset{s \in R(a)}{\text{argmax}}\ h_s(a, s).$$

Intuitively, suppose that firms face an allocation $a \in A^{K-1}$. Standard $r_K(a)$ is the most profitable standard that can be formed using the patents of the firms that are not sponsoring any standard in a. Loosely speaking, we can think of $r_K(a)$ as the K^{th} standard that would be formed as a response to an allocation a with $K-1$ standards.

Of course, it is possible that for some allocations in A^{K-1}, no more standards can be formed, i.e., $R(a) = \varnothing$. In that case we simply set $r_K(a) = \varnothing$. Note that although $r_K(a)$ is generally unique, for some parameter combinations it might not be unique. In such a case, simply pick one random standard that satisfies the definition.

Define function $r_{K-1} : A^{K-2} \to S$ as follows:

$$r_{K-1}(a) = \underset{s \in R(a)}{\text{argmax}}\ h_s(a, s, r_K(a, s))$$

As in the previous case, suppose that $K-2$ standards have been formed, i.e. firms face an allocation $a \in A^{K-2}$. Standard $r_{K-1}(a)$ is the most profitable standard that can be formed with the patents of the firms that are not sponsoring any standard in a, taking into account that standard $r_K(a, r_{K-1}(a))$ will be formed as a response to $r_{K-1}(a)$. Again, if more than two standards satisfy the definition choose one of them arbitrarily.

To obtain the sequence of equations $r_k : A^k \to S$, we start by defining r_K and iterate backwards using the following formula:

$$r_k(a) \;=\; \underset{s \in R(a)}{\operatorname{argmax}} \; h_s\big(a,\; s,\; r_{k+1}(a,s),\; r_{k+2}\big(a,s,r_{k+1}(a,s)\big), \dots\big).$$

Define s_1^* as follows:

$$s_1^* \;=\; \underset{s \in S}{\operatorname{argmax}}\, h_s\big(s,\; r_2(s),\; r_3\big(s, r_2(s)\big), \dots\big).$$

Standard s_1^* is simply the generalization of s^* in Section **??** to the case of K standards. Finally, define s_k^* iteratively as follows:

$$s_k^* \;=\; r_k\left(s_1^*, s_2^*, \dots, s_{k-1}^*\right).$$

Let $a^* = \{s_1^*, s_2^*, \dots\}$ be the reactive allocation. The number of standards in a reactive allocation, K^*, is the smallest integer such that $R(s_1^*, s_2^*, \dots, s_{K^*}^*) = \varnothing$. If $K^* < K$, the allocation of reactive standards has fewer than K standards.

The next proposition shows that a^* is stable. If patent ownership is dispersed and technologies are monotonic, $s_1^* = \bar{s}$, which means there exists a stable allocation that leads to larger social welfare than a mandated standard.

Lemma 4 (Standards wars with more than two standards)**.** *A stable allocation exists. If patent ownership is dispersed and technologies are monotonic, there exists a stable allocation that weakly dominates a mandated standard. If there is no demand uncertainty, mandated standards weakly dominate standards wars.*

Proof. We begin by showing that allocation a^* is stable. Suppose it is not. Then, there exists a standard $\widetilde{s}$ that blocks this allocation. The blocking standard may include patents from the sponsors of any of the standards s_k^*. We will first study the incentives of the sponsors of s_K^*, and then proceed backwards.

Suppose that $\widetilde{s}$ contains patents from the sponsors of s_K^*, but not from the sponsors of $\{s_k^*\}_{k=1}^{K-1}$. Then, it must be the case that $h_{\widetilde{s}}(\widetilde{s} \cup a) > h_{s_K^*}(s_1^*, s_2^*, \dots s_K^*)$ for any allocation $a \in B(\widetilde{s})$. Since $\{s_1^*, s_2^*, \dots s_{K-1}^*\} \in B(\widetilde{s})$, this implies

$$h_{\widetilde{s}}(s_1^*, s_2^*, \dots s_{K-1}^*, \widetilde{s}) \;>\; h_{s_K^*}(s_1^*, s_2^*, \dots s_{K-1}^*, s_K^*),$$

which violates the definition of $s_K^* = r_K(s_1^*, s_2^*, \dots s_{K-1}^*)$. Thus, $\tilde{s}$ cannot block allocation a^* if it only includes patents from the sponsors of s_K^*.

Suppose that $\widetilde{s}$ contains patents from the sponsors of s^*_{K-1} and s^*_K, but not from the sponsors of $\{s^*_k\}_{k=1}^{K-2}$. Then, by a similar argument as before, it must be the case that

$$h_{\widetilde{s}}\left(s^*_1, s^*_2, \ldots s^*_{K-2}, \widetilde{s}, r_K(s^*_1, s^*_2, \ldots s^*_{K-2}, \widetilde{s})\right) \;>\; h_{s^*_K}(s^*_1, s^*_2, \ldots s^*_{K-1}, s^*_K),$$

which violates the definition of $s^*_{K-1} = r_{K-1}(s^*_1, s^*_2, \ldots s^*_{K-2})$. Thus, $\tilde{s}$ cannot block allocation a^* if it only includes patents from the sponsors of s^*_{K-1} and s^*_K.

Repeating the argument for $s^*_{K-3}, s^*_{K-4}, \ldots s^*_1$, we can show that the standard $\tilde{s}$ cannot include patents from the sponsors of any of the standards in a^*, which proves the result.

Next, we show that if patent ownership is dispersed and technologies are monotonic, there exists a stable allocation that leads to larger welfare than a mandated standard. Given any allocation a, let $\widehat{s}(a) = \arg\min_{z\in R(a)} F_z(v)$.

Because technologies are monotonic and standards can be ordered according to FOSD, $\widehat{s}(a)$ is well defined.The standard $\widehat{s}(a)$ is the set of all the best technologies for each component available in $R(a)$. Also notice that $\widehat{s}(a)$ is generally unique.

Let $s^*_0 = \widehat{s}(\varnothing)$. Recursively define standards s^*_k as follows:

$$s_k = \widehat{s}(s_0, s_1 \ldots\ldots s_{k-1})$$

First notice that

$$r_{K-1}(s^*_0, s^*_1 \ldots\ldots s^*_{K-2}) = \text{argmax}\, h_{z\in R(s_0,s_1\ldots\ldots s_{K-2})}(s^*_0, s^*_1 \ldots\ldots s^*_{K-2}, z),$$

because we assume dispersed ownership all standards have the same number of participants and therefore

$$\begin{aligned} r_{K-1}(s^*_0, s^*_1 \ldots\ldots s^*_{K-2}) &= \underset{z\in R(s_0,s_1\ldots\ldots s_{K-2})}{\text{argmax}} h_z(s^*_0, s^*_1 \ldots\ldots s^*_{K-2}, z) \\ &= \underset{z\in R(s_0,s_1\ldots\ldots s_{K-2})}{\text{argmax}} H_z(s^*_0, s^*_1 \ldots\ldots s^*_{K-2}, z) \end{aligned}$$

Define $F_{-z}(v)$ as the distribution of the maximum realization in $(s^*_0, s^*_1 \ldots\ldots s^*_{K-2})$. Integrating by parts

$$H_z(s^*_0, s^*_1 \ldots\ldots s^*_{K-2}, z) = \int \pi(v) f_z(v) F_{-z}(v) = \pi(\overline{v}) - \int F_z(v)\pi'(v) dF_{-z}(v)$$

Because H_z is decreasing in $F_z(v)$, then $r_{K-1}(s_0, s_1 \ldots\ldots s_{K-2}) = s_{k-1}$.

Claim: We now claim that $r_{K-2}(s_0^*, s_1^*......s_{K-3}^*) = s_{K-2}^*$.

Proof: Suppose not, then there exists $z \in R(s_0^*, s_1^*......s_{K-2}^*)$ such that

$$H_z(s_0^*, s_1^*......s_{K-3}^*, z, r_{K-1}(\cup_{i=0}^{k-3} s_i^* \cup z)) > Hs_{K-2}^*(s_0^*, s_1^*......s_{K-3}^*, s_{K-2}^*, s_{K-1}^*)$$

First notice that by the definition s_{K-2}^*, it must be the case that $F_{s_{K-2}^*}(v) \leq F_z(v)$.

Moreover we claim that $r_{K-1}(\cup_{i=0}^{k-3} s_i^* \cup z) \succ s_{K-1}^*$. To avoid unnecessary notation let $r_{K-1}(\cup_{i=0}^{k-3} s_i^* \cup z) = \widetilde{r}_{K-1}$

To see why, let $p(s, m)$ represent the patent used to implement functionality m in standard s. For each functionality $m = 1, \ldots, M$, construct $\widehat{s}$ as follows: (i) if $p(s_{K-2}^*, m) = p(z, m)$, then $p(\widehat{s}, m) = p(s_{K-1}^*, m)$, and (ii) if $p(s_{K-2}^*, m) \neq p(z, m)$, then $p(\widehat{s}, m) = p(s_{K-1}^*, m)$.

By construction, $\widehat{s} \in R(\cup_{i=0}^{k-3} s_i^* \cup z)$. Notice that by definition $s_{K-2}^* \succ s_{K-1}^*$, and monotonicity implies that$\widehat{s} \succ s_{K-1}^*$. To see why, observe that $\widehat{s} = \left(s_{K-1}^* \setminus z\right) \cup \left(z \cap s_{K-2}^*\right)$ and $s_{K-2}^* = \left(s_{K-2}^* \setminus z\right) \cup \left(s_{K-2}^* \cap z\right)$, and that, by monotonicity, $s_{K-2}^* = \left(s_{K-2}^* \setminus z\right) \cup \left(s_{K-2}^* \cap z\right) \succ \left(s_{K-1}^* \setminus z\right) \cup \left(s_{K-1}^* \cap z\right) = s_{K-1}^*$ implies $\widehat{s} = \left(s_{K-1}^* \setminus z\right) \cup \left(z \cap s_{K-2}^*\right) \succ \left(s_{K-1}^* \setminus z\right) \cup \left(s_{K-1}^* \cap z\right) = s_{K-1}^*$.

Finally, notice that by definition $r_{K-1}(\cup_{i=0}^{k-3} s_i^* \cup z) \succ \widehat{s}$. This implies that

$$H_z(s_0^*, s_1^*......s_{K-3}^*, s_{K-2}^*, r_{K-1}(\cup_{i=0}^{k-3} s_i^* \cup z)) < Hs_{K-2}^*(s_0^*, s_1^*......s_{K-3}^*, s_{K-2}^*, s_{K-1}^*)$$

We already argued that $F_{s_{K-2}^*}(v) \leq F_z(v)$, which implies that

$$H_z(s_0^*, s_1^*......s_{K-3}^*, z, r_{K-1}(\cup_{i=0}^{k-3} s_i^* \cup z))s_{K-1}^* < Hs_{K-2}^*(s_0^*, s_1^*......s_{K-3}^*, s_{K-2}^*, s_{K-1}^*)$$

For every z, which is a contradiction of our original statement and therefore proves the statement of the claim.

Repeating the argument for $k = 3, 4....K - 1$ the result must hold.

The last part of the lemma follows directly from the proof of Proposition 3. ∎

Lemma 4 extends Propositions 1, 2, and 3 to the case of more than two standards. Notably, there always exists a stable allocation in standard wars that weakly dominates a mandated standard in terms of welfare. In contrast with Proposition 2, the stable allocation with simple technologies may not be unique, and there may be stable allocations that are dominated by a mandated standard.