PrObEx and Internal Model

Calibrating dependencies among risks in Non-Life

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PrObEx and Internal Model

1	Introduction
2	SCR and risk aggregation
3	PrObEx – a Bayesian model
4	Implementation in the Internal Model
5	Conclusion

SCOR

- SCOR is the 5th largest reinsurer in the world (Premium income of EUR 9.514 billion in 2012).
- SCOR operates worldwide via its six Hubs located in Paris, Zurich, Cologne, New York and Singapore.
- Ratings:

A+	S&P	positive outlook
A	A.M.Best	stable outlook
A1	Moody's	stable outlook
A+	Fitch	stable outlook

- Priority of SCOR is the delivery of the Internal Model and its approval by the ACPR (*Autorité de Contrôle Prudentiel et de Résolution*) for purpose of use under Solvency II.
- We illustrate a key innovation in SCOR's Internal Model: PrObEx

PrObEx and Internal Model

SCR and risk aggregation

- According to Solvency II, we need to determine the Solvency Capital Requirement (SCR).

- The SCR is given by:

$$SCR = -\mathrm{VaR}_{0.5\%}(G)$$

  where G is the change in the economic value over the measurement period (one year), i.e.

$$G = vW_1 - W_0$$

  where v is the discount factor (risk-free) from the horizon date to the valuation date, and the economic value is given by:

$$W_t = \sum_{p=1}^{P} A_p(t) - \sum_{q=1}^{Q} L_q(t)$$

- W_0 and v are considered known a the valuation date, while W_1 is modeled as a random variable.

SCR and risk aggregation

- Monte-Carlo simulation methods are used to determine the stochastic value W_1 .
- The valuation of W_1 requires to calculate the distribution of the Liabilities at time 1:

$$\sum_{q=1}^{Q} L_q(1)$$

- The latest financial crisis has dramatically shown that dependence among risks can not be ignored.
- We use copula models in order to prudently account for dependence (especially in the tail!).
- Copula estimation procedures usually contain a large parameter uncertainty if data is scarce.
- We developed a Bayesian model to calibrate copula parameters → PrObEx

PrObEx and Internal Model

Copula and dependence measure

- Let (X, Y) be a bivariate random vector and assume the marginal distributions $F(x)$ and $G(y)$ are known.

- The joint cumulative distribution can be represented as
$$H(x, y) = C(F(x), G(y))$$
where C is the unique *copula* function that joins the two marginal distributions.

- There exist many copula families and some are relevant for modeling insurance risks.
We focus on the most popular families characterized by one parameter.

- We assume the copula family is already known. Our aim is to estimate the copula parameter γ_0 .

- We chose a dependence measure ρ which is familiar to insurance business experts and which can be linked to the copula parameter.
→ calculating an estimate $\widehat{\theta_0}$ of the value θ_0 of the dependence measure leads to an estimate of γ_0.

PrObEx – Combining three sources of information

- (Up to) three sources of information can be combined:

Prior A prior density $\pi(\theta)$, e.g. from previous years or from regulators.

Observation N independent observations of joint realizations from (X, Y).
The set of observation is denoted by $\mathcal{O}$

Experts K experts, each providing one point estimate φ_k of ρ.
The set of expert assessments is denoted by $\mathcal{E}$

- We replace the prior density $\pi(\theta)$ by a posterior density $\pi(\theta|\mathcal{O},\mathcal{E})$ of θ given $\mathcal{O}$ and $\mathcal{E}$.

- Bayes' Theorem leads to the relation

$$\pi(\theta|\mathcal{O},\mathcal{E})h(\mathcal{O},\mathcal{E}) = h(\mathcal{O},\mathcal{E}|\theta)\pi(\theta)$$

Our model

- We make the following assumptions:
  - The expert assessments and the observations are independent
  - The observations are independent
  - The experts form their opinion independently of each other

- Under these assumptions, the posterior distribution of the value of the dependence measure reads as:

$$\pi(\theta|\mathcal{O},\mathcal{E}) \propto \underbrace{\pi(\theta)}_{\text{Prior}} \underbrace{\prod_{n=1}^{N} c(U_n, V_n|g(\theta))}_{\text{Observation}} \underbrace{\prod_{k=1}^{K} e_k(\varphi_k|\theta)}_{\text{Experts}}$$

- Through this posterior distribution we can:
  - Estimate θ_0, e.g. via $\widehat{\theta_0} = \mathbb{E}[\theta|\mathcal{O},\mathcal{E}]$.
  - Assess the uncertainty of our estimate, e.g. via $\mathrm{var}(\theta|\mathcal{O},\mathcal{E})$.

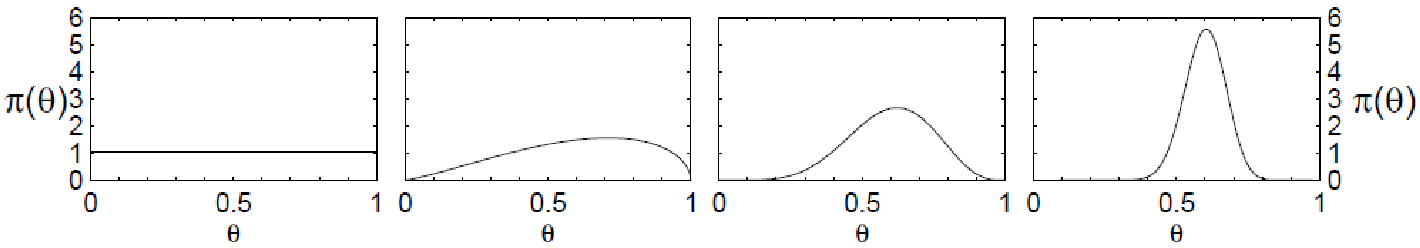
Prior information

- Suppose we can infer a point estimate $\widehat{\theta}_p$ of θ_0 from the prior source of information.
- We then model $\pi(\theta)$ with a shifted Beta distribution with mean $\mathbb{E}[\theta] = \widehat{\theta}_p$.

- If the source of information leading to $\widehat{\theta}_p$ does not specify a measure of uncertainty, we determine $\operatorname{var}(\theta)$ through a qualitative approach:

$$\operatorname{var}(\theta) = \begin{cases} 0.005(b-a)^2 & \text{for high confidence,} \\ 0.02(b-a)^2 & \text{for intermediate confidence,} \\ 0.05(b-a)^2 & \text{for low confidence.} \end{cases}$$

- If no prior belief is available then $\pi(\theta)$ can be set uninformative.
- The four mentioned qualitative approaches:



The elicitation of expert opinions

- An expert elicitation procedure needs to satisfy five principles in order to reach rational consensus, namely:
  - Reproducibility
  - Accountability
  - Empirical control
  - Neutrality
  - Fairness
- Psychological effects are involved and have to be considered carefully.
- The literature distinguishes between behavioral vs. mathematical approaches.

The modeling of expert opinions

- The conditional density of the k-th expert is modeled via a shifted Beta distribution.

- We model the expert estimates to be conditionally unbiased, i.e. $\mathbb{E}[\varphi_k|\theta] = \theta$.

- To reflect the expert uncertainty we assign each expert a variance σ_k^2, which is assumed to be independent of θ, i.e. $\mathrm{var}(\varphi_k|\theta) = \sigma_k^2$

- Three possible approaches to calculate estimates $\widehat{\sigma_k^2}$ of σ_k^2 are considered:

  - Subjective variances
  - Homogeneous experts
  - Seed variables

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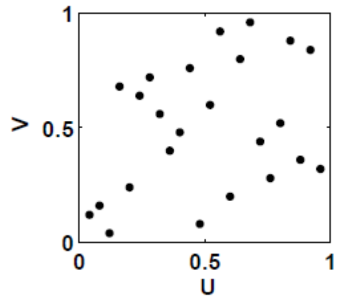
An illustrative example (1 of 2)

- Let C be a T-copula* and the dependence measure ρ be Kendall's Tau.
- Then, the dependence measure is linked to the copula parameter by the function: $g(\theta) = \sin(\theta\pi/2)$

- Suppose we have no prior information available.
- Let N=24 observations be given:



- Experts opinions: $\varphi_1 = 0.35$, $\varphi_2 = 0.5$, $\varphi_3 = 0.7$. Moreover:

	Seed variables				
True value	0.3	0.7	0.5	0.55	
	Expert assessments				Estimated variance
Expert 1	0.1	0.3	0.6	0.75	0.0625
Expert 2	0.5	0.5	0.35	0.6	0.0262
Expert 3	0.3	0.8	0.3	0.8	0.0281

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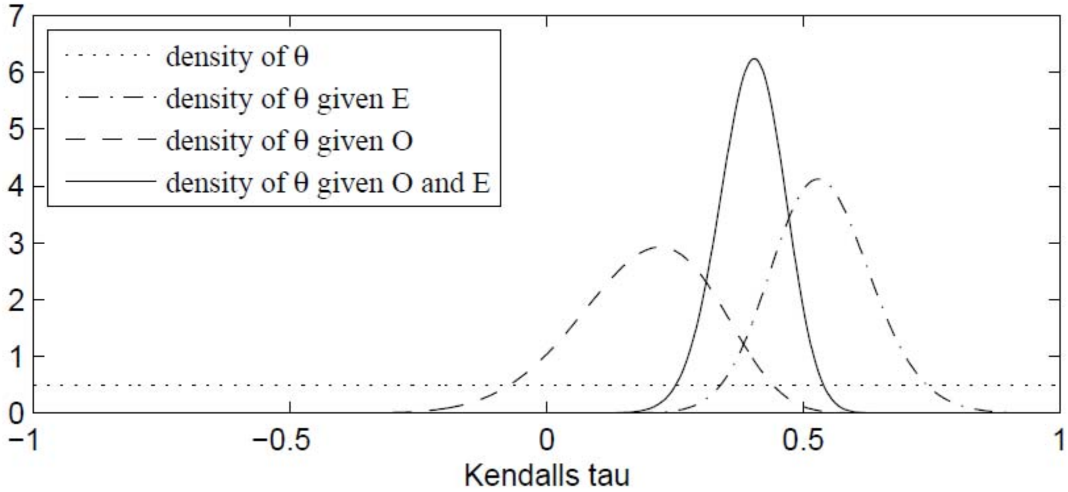
* For the purpose of this example, we consider a T-copula with 3 degrees of freedom.

An illustrative example (2 of 2)



	$\mathbb{E}[\cdot]$	$\sqrt{\mathrm{var}(\cdot)}$	90% confidence interval
θ	0.000	0.577	$[-0.900, 0.900]$
$\theta \mid \mathscr{E}$	0.537	0.098	$[0.380, 0.703]$
$\theta \mid \mathscr{O}$	0.192	0.135	$[-0.046, 0.399]$
$\theta \mid \mathscr{O}, \mathscr{E}$	0.399	0.064	$[0.291, 0.502]$

- The best estimate $\widehat{\gamma_0}$ using all information is then:

$$\widehat{\gamma_0} = g(\mathbb{E}[\theta \mid \mathscr{O}, \mathscr{E}]) = g(0.399) = 0.587$$

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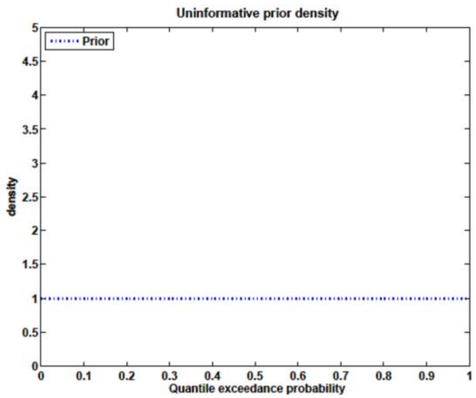
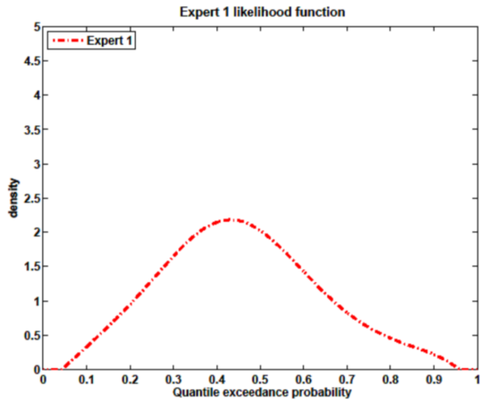
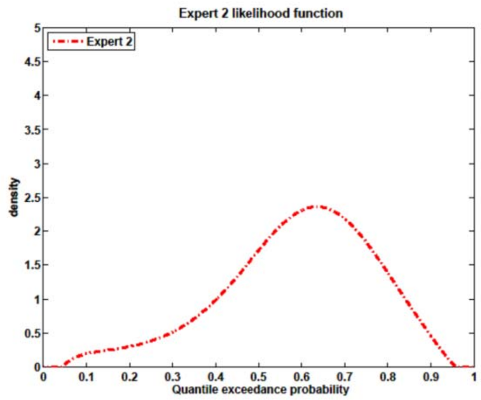
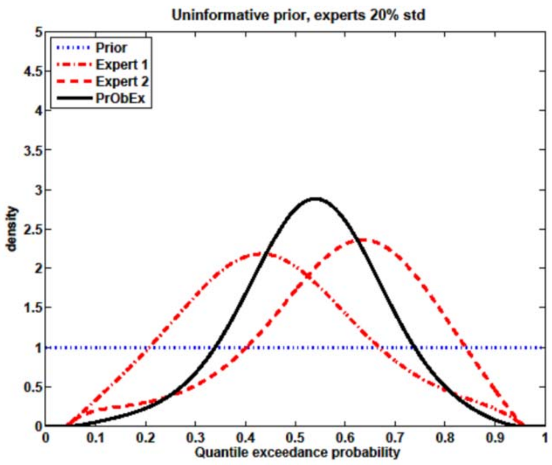
PrObEx: Two experts equally certain and no prior information…

Combining different sources of information



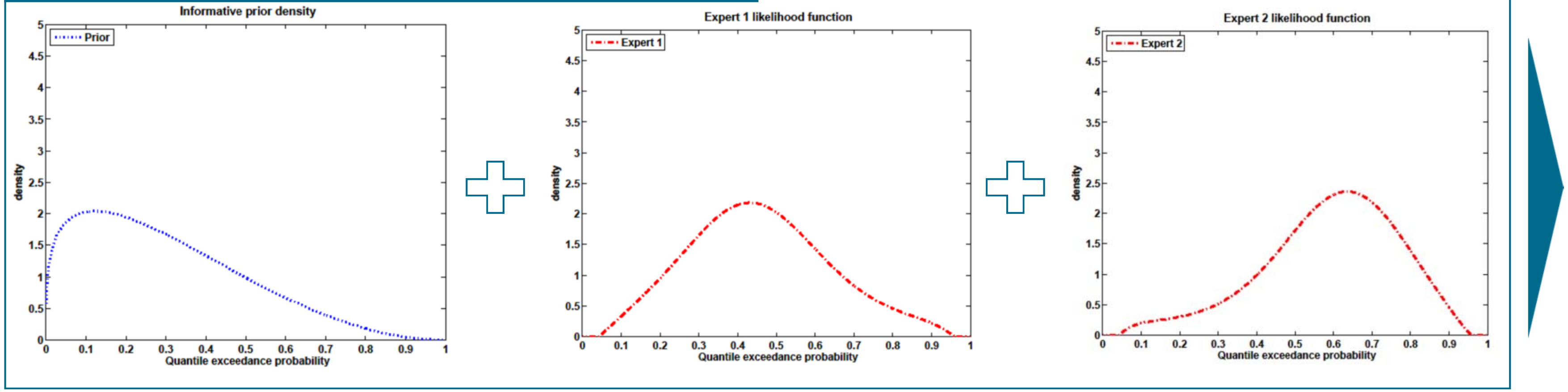


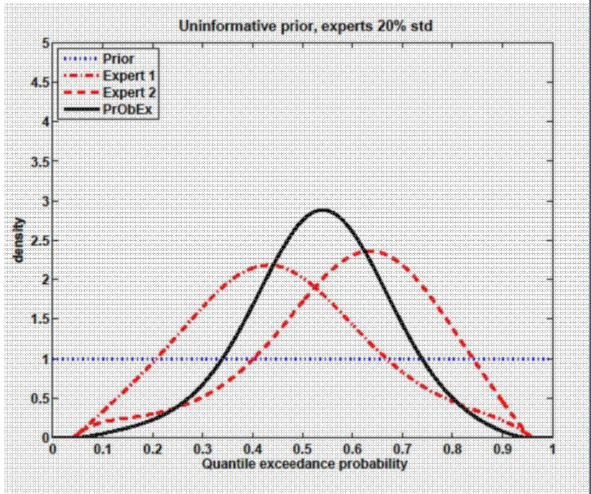


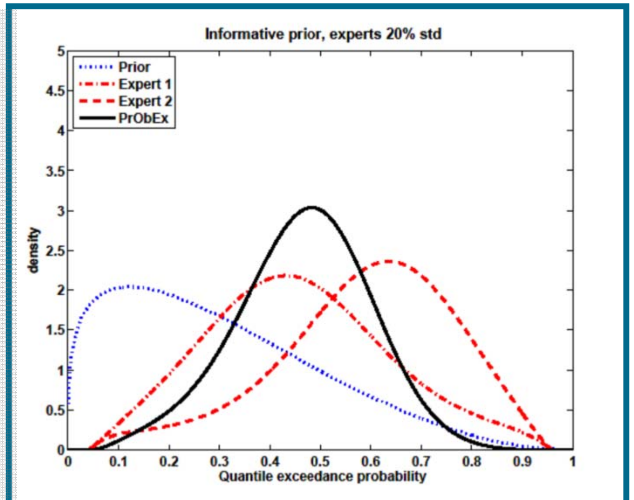


PrObEx: … what if we can use an informative prior? …

Combining different sources of information

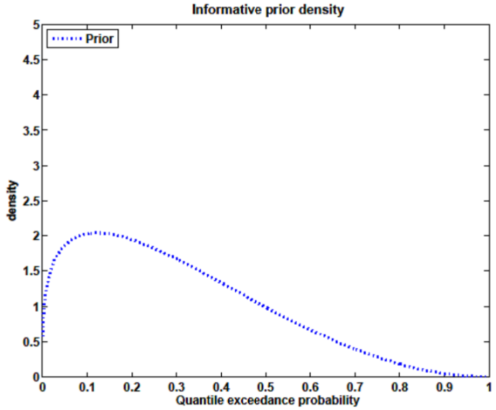


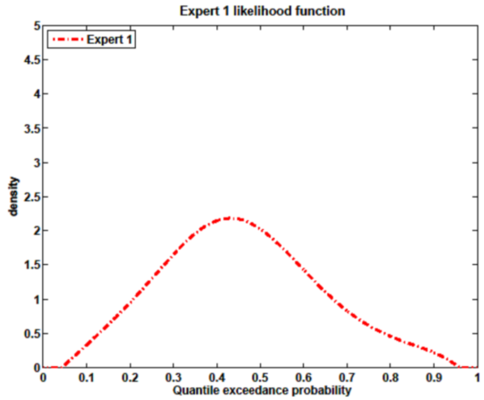


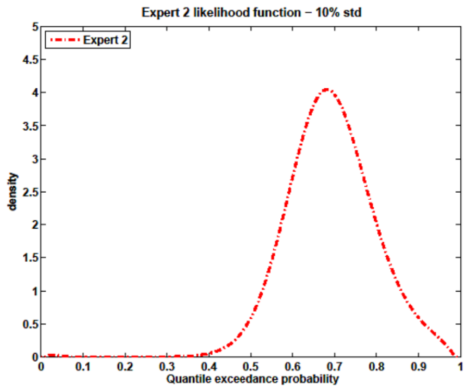


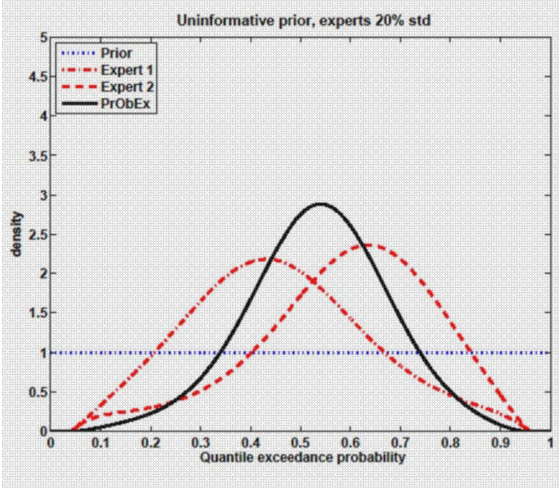
PrObEx: … confident experts increase further the precision

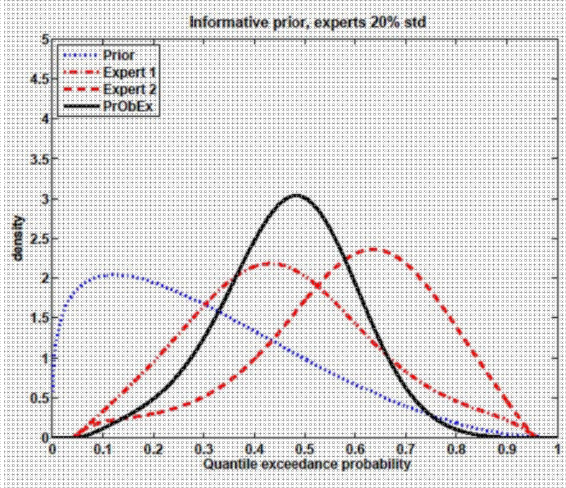
Combining different sources of information

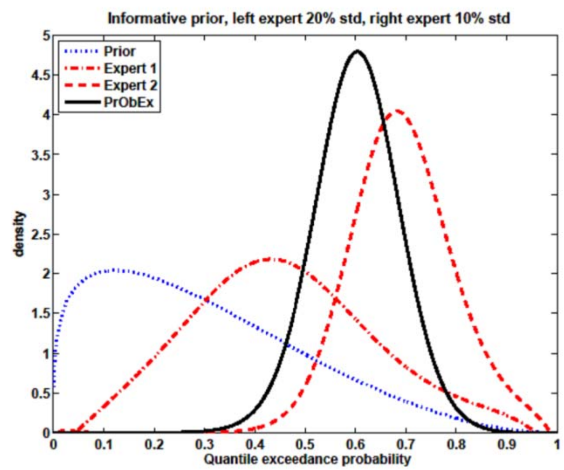






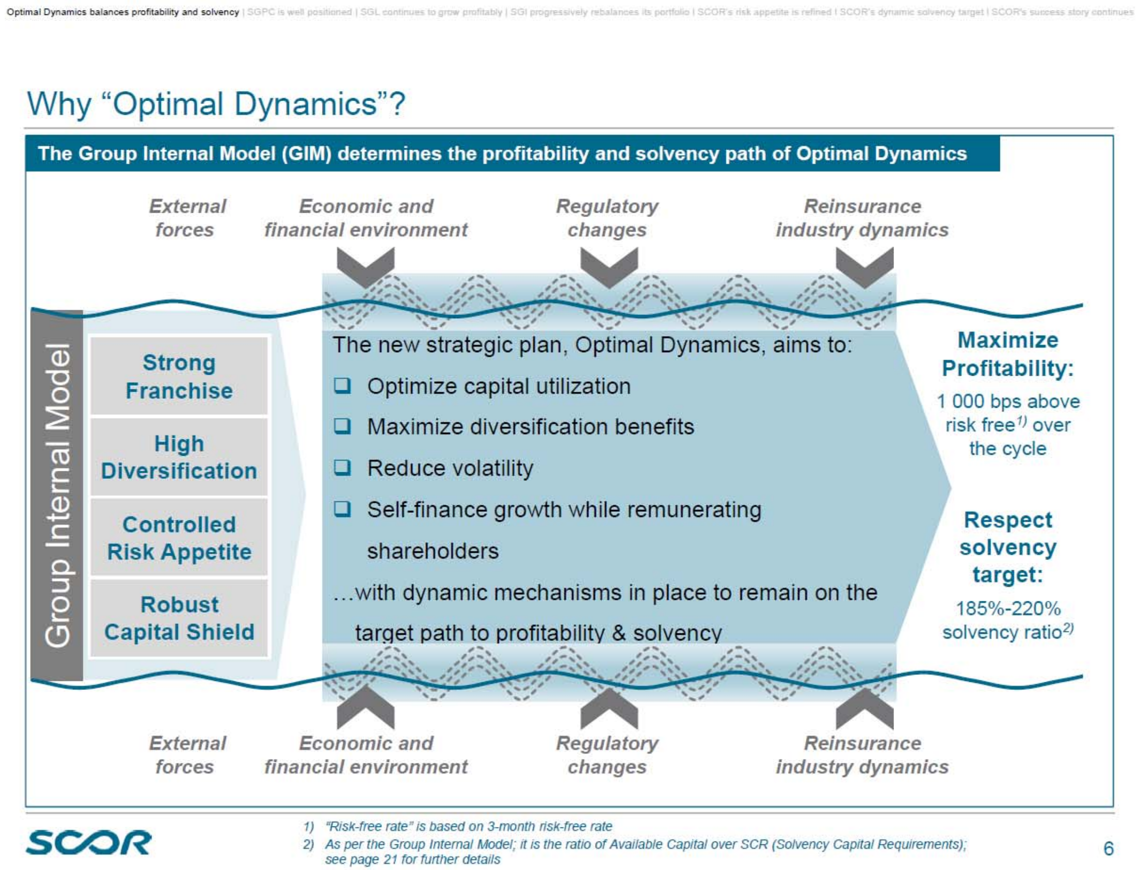






PrObEx and Internal Model

Investor’s day 2013



The relevance of the project

- As part of SCOR internal model, PrObEx contributes to the determination of the SCR → it has an impact on key areas, such as capital allocation, underwriting and investment strategies.
- In line with SCOR's strategic plan "Optimal Dynamics", PrObEx offers support for high diversification and controlled risk appetite.
- To ensure robustness of final results, the process of gathering the expert's opinion has been industrialized and fully documented.
- 33 workshops were organized and more than 100 experts, scattered in 7 different locations around the World, were involved in the project.
- Overall, more than 1'300 dependence assessments were elicited, covering 16 different Lines of Business.

The calibration process

Workshop

Overview → Training → Brainstorming → Questionnaire

Prior Information + Observations + Experts opinions

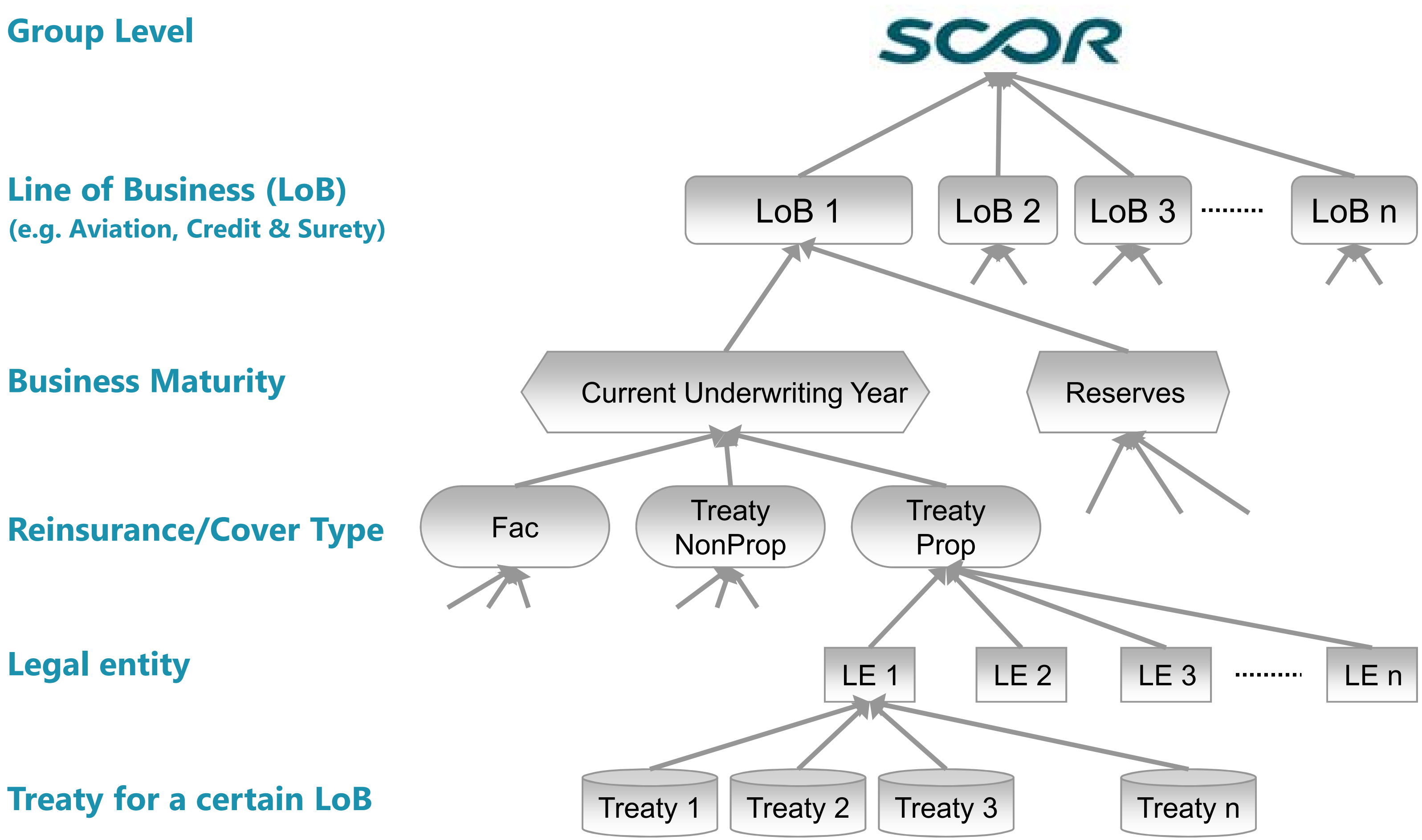
PrObEx

Dependence parameters

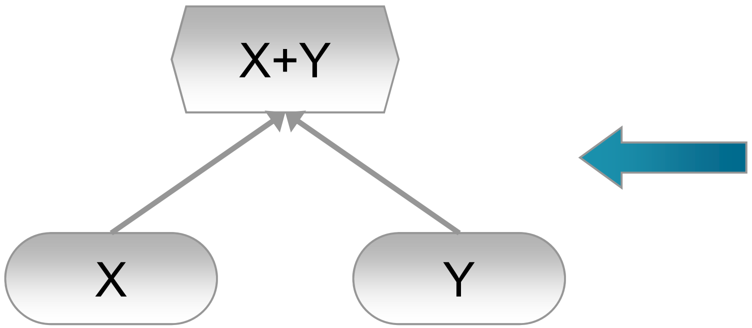
Risk aggregation

Solvency Capital Requirement (SCR)

The risk aggregation tree for **Specialty** Non-Life LoBs



Dependence measure – what we asked to SCOR experts



How to measure dependence?

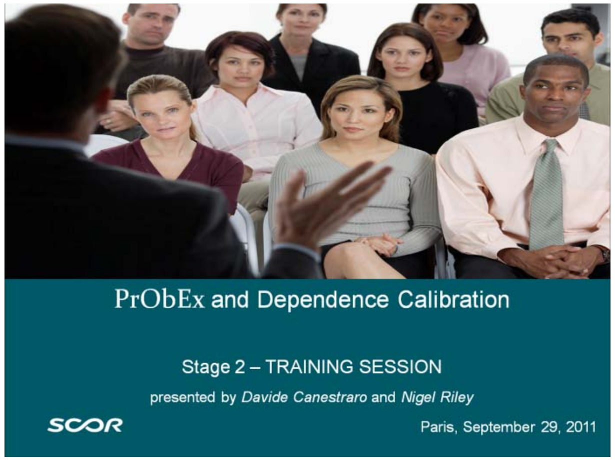
- The experts were asked to answer a question like:

> “Suppose Y exceeds the 1-in-100 year threshold.
> What is the probability that also X exceeds its 1-in-100 year threshold?”

- This is equivalent to quantify the so called Quantile Exceedance Probability:

$$P\left[X > VaR_{0.99}(X) \mid Y > VaR_{0.99}(Y)\right]$$

Workshop agenda



Expert judgment and heuristics (1)

- Representativeness (1)

Linda is 31 years old, single, outspoken and very bright. She majored in philosopy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in antinuclear demostrations.

Is it more likely that:

(A) Linda is a bank teller?

(B) Linda is a bank teller and active in the feminist movement?

Expert judgment and heuristics (2)

- Representativeness (2)

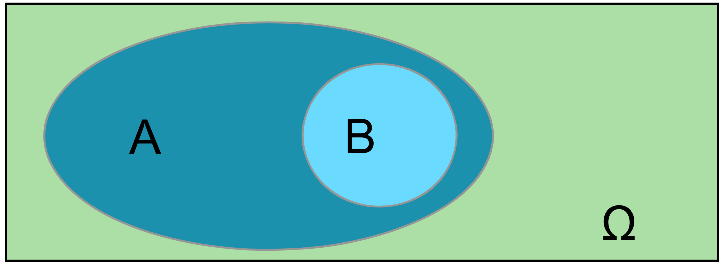
Linda is 31 years old, single, outspoken and very bright. She majored in philosopy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in antinuclear demostrations.

There are 100 people who fit the description above. How many of them are:

(A) bank tellers?

(B) bank tellers and active in the feminist movement?

Answer:



Expert judgment and heuristics (3)

- Availability

Are there more words in the English language that begin with R or have R as their third letter?

Which hazard claims more lives in the United States: lightning or tornadoes?

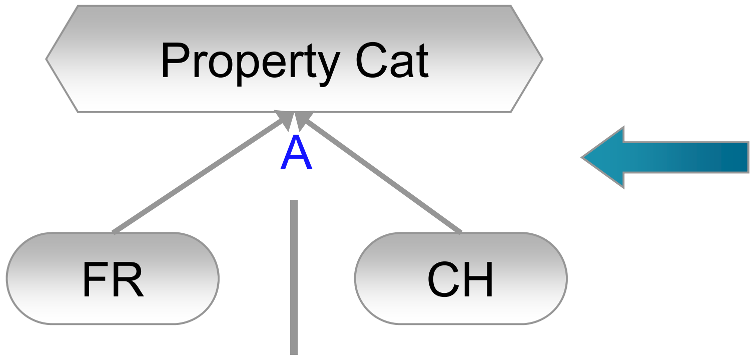
Expert judgment and heuristics (3)

- Anchoring

Is the population of Chicago more or less than 200,000?
Estimate the population.

Is the population of Chicago more or less than 5 million?
Estimate the population.

Questionnaire (example) (1 of 2)



Given that an extremely bad outcome is observed in the legal entity Switzerland, what is your estimate of the probability that the legal entity France will experience an extremely bad outcome?

Which are the risk drivers which can cause such a bad outcome in the legal entity Switzerland?

Assume they are:

- Eurowind
- European Earthquake
- North American Tropical Cyclone

Questionnaire (example) (2 of 2)

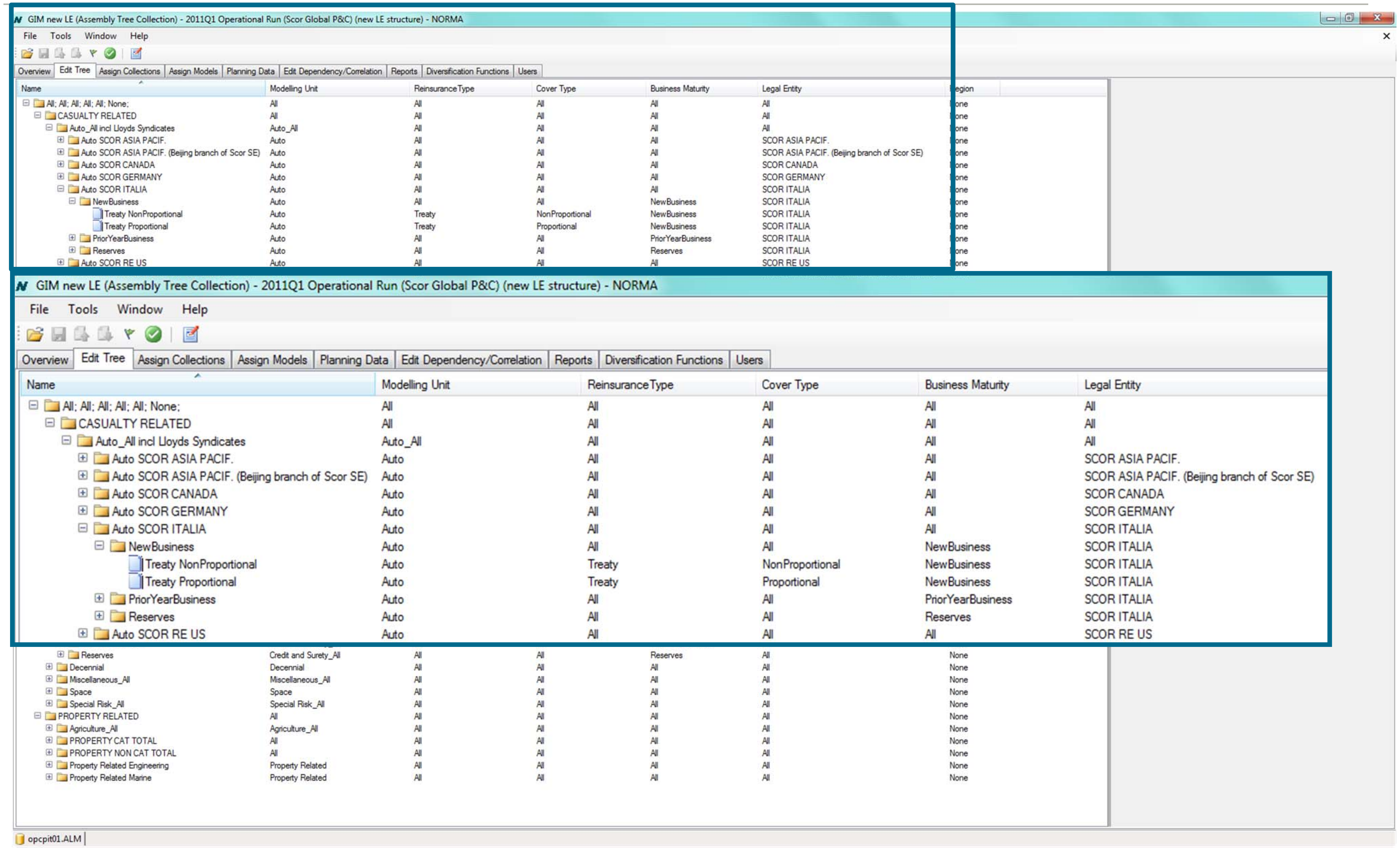
Section A.1
Given that an extremely bad outcome is observed in the Legal Entity Switzerland, list some of the risk drivers for which ALSO Legal Entity France will experience an extremely bad outcome.

Probability	Risk driver	Weight
100%	Eurowind	0.3
50%	European Earthquake	0.3

Section A.2
Given that an extremely bad outcome is observed in the Legal Entity Switzerland, list some of the risk drivers for which Legal Entity France will NOT experience an extremely bad outcome.

Probability	Risk driver	Weight
0%	North American Tropical Cyclone	0.4

The aggregation tree for Non-Life



GIM new LE (Assembly Tree Collection) - 2011Q1 Operational Run (Scor Global P&C) (new LE structure) - NORMA

File Tools Window Help

Overview | Edit Tree | Assign Collections | Assign Models | Planning Data | Edit Dependency/Correlation | Reports | Diversification Functions | Users

Name	Modelling Unit	Reinsurance Type	Cover Type	Business Maturity	Legal Entity	Region
All; All; All; All; All; None;	All	All	All	All	All	None
CASUALTY RELATED	All	All	All	All	All	None
Auto_All incl Lloyds Syndicates	Auto_All	All	All	All	All	None
Auto SCOR ASIA PACIF.	Auto	All	All	All	SCOR ASIA PACIF.	None
Auto SCOR ASIA PACIF. (Beijing branch of Scor SE)	Auto	All	All	All	SCOR ASIA PACIF. (Beijing branch of Scor SE)	None
Auto SCOR CANADA	Auto	All	All	All	SCOR CANADA	None
Auto SCOR GERMANY	Auto	All	All	All	SCOR GERMANY	None
Auto SCOR ITALIA	Auto	All	All	All	SCOR ITALIA	None
NewBusiness	Auto	All	All	NewBusiness	SCOR ITALIA	None
Treaty NonProportional	Auto	Treaty	NonProportional	NewBusiness	SCOR ITALIA	None
Treaty Proportional	Auto	Treaty	Proportional	NewBusiness	SCOR ITALIA	None
PriorYearBusiness	Auto	All	All	PriorYearBusiness	SCOR ITALIA	None
Reserves	Auto	All	All	Reserves	SCOR ITALIA	None
Auto SCOR RE US	Auto	All	All	All	SCOR RE US	None
Reserves	Credit and Surety_All	All	All	Reserves	All	None
Decennial	Decennial	All	All	All	All	None
Miscellaneous_All	Miscellaneous_All	All	All	All	All	None
Space	Space	All	All	All	All	None
Special Risk_All	Special Risk_All	All	All	All	All	None
PROPERTY RELATED	All	All	All	All	All	None
Agriculture_All	Agriculture_All	All	All	All	All	None
PROPERTY CAT TOTAL	All	All	All	All	All	None
PROPERTY NON CAT TOTAL	All	All	All	All	All	None
Property Related Engineering	Property Related	All	All	All	All	None
Property Related Marine	Property Related	All	All	All	All	None

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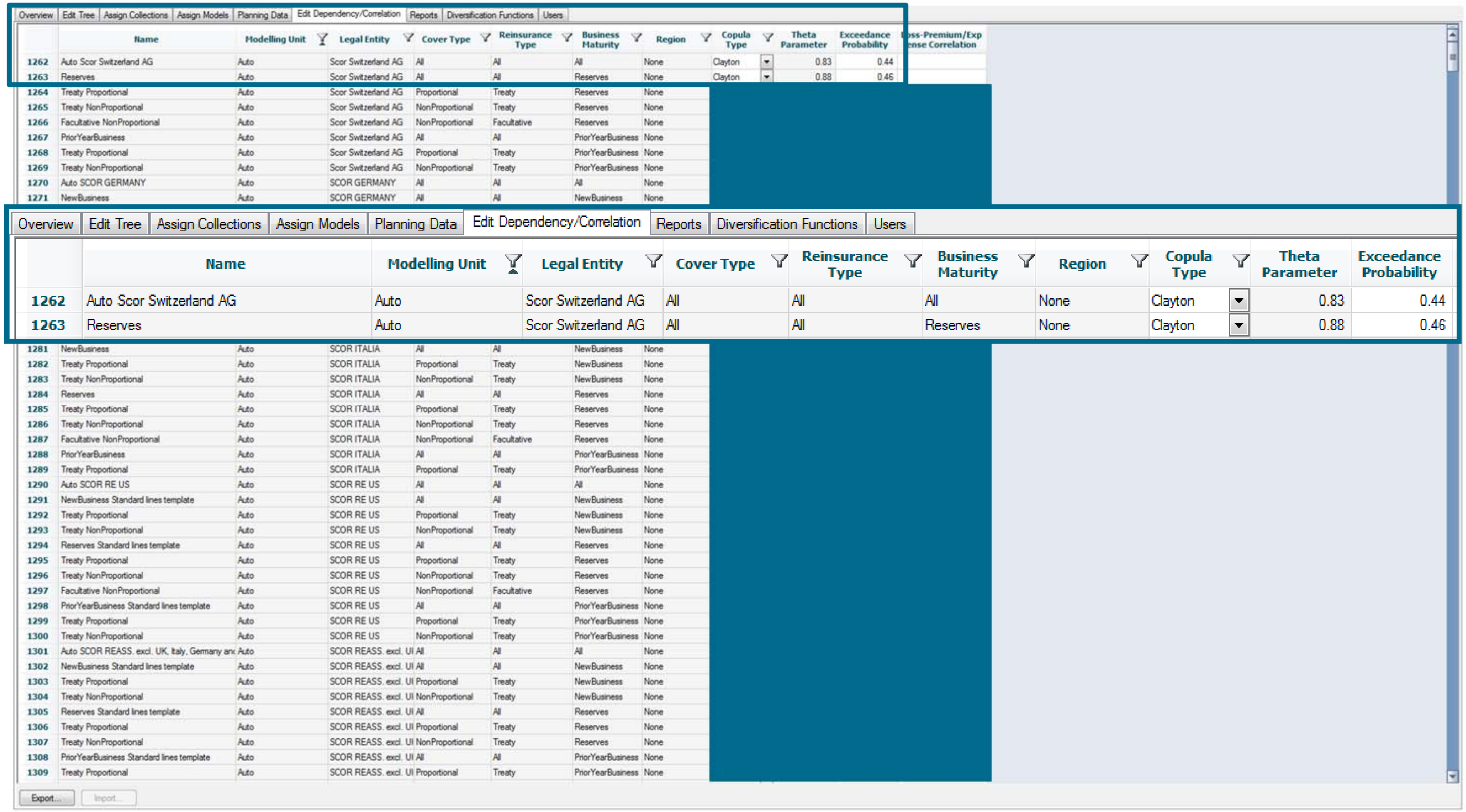
Dependence parameters



PrObEx and Internal Model

Conclusion

- PrObEx provides a sound mathematical framework for estimating copula parameters.

- PrObEx allows to reduce the parameter uncertainty when estimating copula parameters.

- A statistical analysis conducted from Professor Sebastien Van Bellegem (Toulouse School of Economics) has demonstrated the robustness and the absence of bias in the results.

- PrObEx can be used to calibrate dependencies also in other contexts (e.g. Life, Economy, etc.).

- A scientific paper on PrObEx has been published in the ASTIN Bulletin.

References

- Arbenz, P. and Canestraro, D. (2010): PrObEx - *A new method for the calibration of copula parameters from prior information, observations and expert opinions*. SCOR Paper n. 10

- Arbenz, P. and Canestraro, D. (2012): *Estimating copula for insurance from scarce observations, expert opinion and prior information: a Bayesian approach*. ASTIN Bulletin, 42 (1): 271-290

- Dacorogna, M.D. and Canestraro, D. (2010): *The Influence of Risk Measures and Tail Dependencies on Capital Allocation*. SCOR Paper n. 7

Q&A

Thank you for your attention !

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Q&A

Thank you for your attention !

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Appendix

Copula

Let $(X, Y) \in \mathbb{R}^2$ be a bivariate random vector.
Assume the margins $F(x) = \mathbb{P}[X \leq x]$ and $G(y) = \mathbb{P}[Y \leq y]$ of (X, Y) are continuous.

We can represent the joint cdf $H(x, y) = \mathbb{P}[X \leq x, Y \leq y]$ as:

$$H(x, y) = C(F(x), G(y)), \qquad \text{for all} \qquad x, y \in \mathbb{R},$$

where $C : [0,1]^2 \to [0,1]$ is the unique copula function.

The copula C is also the cdf of the random vector $(U, V) = (F(X), G(Y)) \in [0,1]^2$, denoted with $(U, V) \sim C$.

Four popular copula families

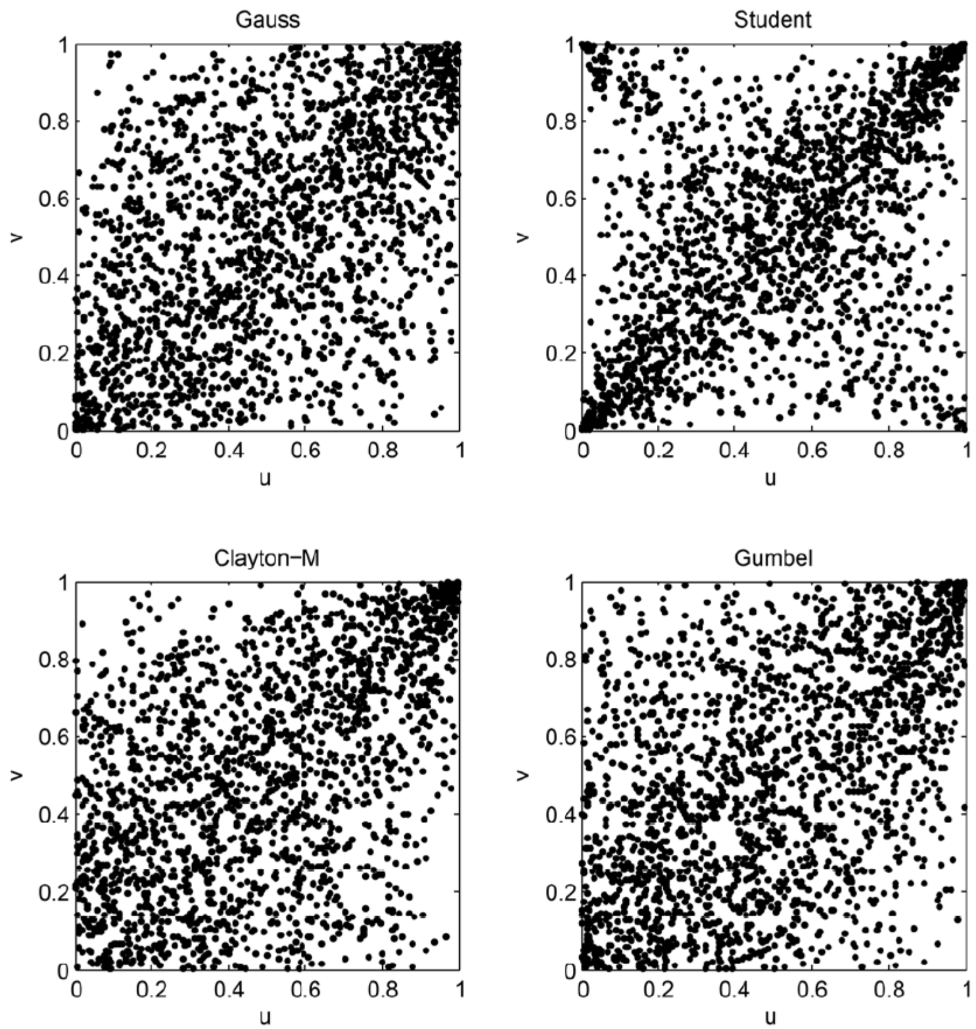
Let $\mathscr{C}_0 = \{C_\gamma : \gamma \in \Gamma\}$ denote a family of bivariate copulas, with parameter set Γ and density $c(\cdot|\gamma)$.
We assume that

$$C = C_{\gamma_0} \in \mathscr{C}_0$$

where γ_0 is an unknown but fixed parameter. Our aim is to estimate γ_0.

Copula family	Definition	Parameter range Γ
Gaussian	$C_\gamma^{Ga}(u,v) = \Phi_\gamma\left(\Phi^{-1}(u), \Phi^{-1}(v)\right)$	$\gamma \in (-1,1)$
t	$C_{\nu,\gamma}^{t}(u,v) = \mathbf{t}_{\nu,\gamma}\left(t_\nu^{-1}(u), t_\nu^{-1}(v)\right)$	$\gamma \in (-1,1)$
Clayton	$C_\gamma^{Cl}(u,v) = (u^{-\gamma} + v^{-\gamma} - 1)^{-1/\gamma}$	$\gamma \in (0,\infty)$
Gumbel	$C_\gamma^{Gu}(u,v) = \exp\left(-((-\ln(u))^\gamma + (-\ln(v))^\gamma)^{1/\gamma}\right)$	$\gamma \in [1,\infty)$

Four popular copula families – rank scatter plots



Dependence measure

Let $\rho(\cdot,\cdot)$ denote a fixed dependence measure. Define the set of attainable values of ρ for copulas in $\mathscr{C}_0$ by

$$\Theta = \{\rho(U^*, V^*) : (U^*, V^*) \sim C^* \in \mathscr{C}_0\}.$$

We assume that Θ is an interval, i.e. $\Theta = [a, b] \subset \mathbb{R}$

We focus on $\rho(\cdot,\cdot)$ which satisfy $\rho(U, V) = \rho(X, Y)$

We assume there exists $g : [a, b] \to \Gamma$ a bijective link function s.t.
$g(\rho(U^*, V^*)) = \gamma^* \quad \text{for all} \quad (U^*, V^*) \sim C_{\gamma^*}$

Calculating an estimate $\widehat{\theta_0}$ of θ_0 leads to an estimate $\widehat{\gamma_0} = g(\widehat{\theta_0})$ of γ_0.

PrObEx – Combining three sources of information

(Up to) three sources of information can be combined:

Prior A prior density $\pi(\theta) : [a, b] \to [0, \infty)$ e.g. from previous years or from regulators.

Observations N independent observations (U_n, V_n), $n = 1, \ldots, N$, of $(U, V) \sim C_{\gamma_0}$. The set of observations is denoted by $\mathscr{O} = \{(U_n, V_n) : n = 1, \ldots, N\}$.

Experts K experts, each providing one point estimate φ_k, $k = 1, \ldots, K$, of $\rho(U, V)$. The set of expert assessments is denoted by $\mathscr{E} = \{\varphi_k : k = 1, \ldots, K\}$.

We replace the prior density $\pi(\theta)$ by a posterior density $\pi(\theta|\mathscr{O}, \mathscr{E})$ of θ given $\mathscr{O}$ and $\mathscr{E}$.
Bayes' Theorem leads to the relation

$$\pi(\theta|\mathscr{O}, \mathscr{E})h(\mathscr{O}, \mathscr{E}) = h(\mathscr{O}, \mathscr{E}|\theta)\pi(\theta),$$

Bayesian inference

We assume that the expert assessments and the observations are independent, thus:

$$h(\mathscr{O},\mathscr{E}|\theta) = h_O(\mathscr{O}|\theta)h_E(\mathscr{E}|\theta),$$

where h_O and h_E are the conditional densities, given θ, of $\mathscr{O}$ and $\mathscr{E}$, respectively.

As the observations (U_n, V_n), $n = 1, \ldots, N$, are independent,

$$h_O(\mathscr{O}|\theta) = \prod_{n=1}^{N} c\left(U_n, V_n|g(\theta)\right),$$

where $c(u, v|\gamma) = \frac{\partial}{\partial u}\frac{\partial}{\partial v} C_\gamma(u, v)$.

Assuming the experts form their opinions independently of each other, we have:

$$h_E(\mathscr{E}|\theta) = \prod_{k=1}^{K} e_k(\varphi_k|\theta),$$

where $e_k(\cdot|\theta)$ is the conditional density, given θ, of the k-th expert assessment.

Our model

Since

$$\pi(\theta|\mathscr{O},\mathscr{E}) \propto \pi(\theta)h(\mathscr{O},\mathscr{E}|\theta)$$

we get:

$$\pi(\theta|\mathscr{O},\mathscr{E}) \propto \pi(\theta)\prod_{n=1}^{N} c(U_n, V_n|g(\theta))\prod_{k=1}^{K} e_k(\varphi_k|\theta)$$

For sensitivity analysis or in case no expert opinions or observations are available, we can also compute:

$$\pi(\theta|\mathscr{O}) \propto \pi(\theta)\prod_{n=1}^{N} c(U_n, V_n|g(\theta)), \qquad \pi(\theta|\mathscr{E}) \propto \pi(\theta)\prod_{k=1}^{K} e_k(\varphi_k|\theta).$$

Through this posterior distribution we can:

- estimate θ_0, e.g. via $\widehat{\theta_0} = \mathbb{E}[\theta|\mathscr{O},\mathscr{E}]$.
- assess the uncertainty of our estimate, e.g. via $\mathrm{var}(\theta|\mathscr{O},\mathscr{E})$.

The modeling of expert opinions (1 of 2)

The conditional density $e_k(\cdot|\theta)$ is modeled via a shifted Beta distribution. We model the expert estimates to be conditionally unbiased, i.e.

$$\mathbb{E}[\varphi_k|\theta] = \theta \quad \text{for all} \quad \theta \in [a, b],$$

To reflect the expert uncertainty we assign each expert a variance σ_k^2, $k = 1, \dots, K$, which is assumed to be independent of θ:

$$\mathrm{var}(\varphi_k|\theta) = \sigma_k^2 \quad \text{for all} \quad \theta \in [a, b].$$

Three possible approaches to calculate estimates $\widehat{\sigma_k^2}$ of σ_k^2:

- Subjective variances
- Homogeneous experts

$$\widehat{\sigma^2} = \frac{1}{K-1} \sum_{k=1}^{K} (\varphi_k - \overline{\varphi})^2,$$

where $\overline{\varphi} = \frac{1}{K} \sum_{k=1}^{K} \varphi_k$.

The modeling of expert opinions (2 of 2)

- Seed variables

	Seed variables			
True value	$\psi_0^{(1)}$	$\cdots$	$\psi_0^{(H)}$	
	Experts' estimates			Estimated variance
Expert 1	$\psi_1^{(1)}$	$\cdots$	$\psi_1^{(H)}$	$\widehat{\sigma_1^2} = \frac{1}{H}\sum_{h=1}^{H}(\psi_1^{(h)} - \psi_0^{(h)})^2$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
Expert M	$\psi_M^{(1)}$	$\cdots$	$\psi_M^{(H)}$	$\widehat{\sigma_M^2} = \frac{1}{H}\sum_{h=1}^{H}(\psi_M^{(h)} - \psi_0^{(h)})^2$

Investor’s day 2011

Strong Momentum V1.1 is consistent with the Group’s four strategic cornerstones

	SCOR consistent execution of its strategic cornerstones
Strong franchise	❑ Is reaching a **new perimeter** ❑ Is deepening its **global franchise** ❑ Is pursuing the announced **“Strong Momentum” growth initiatives**
Controlled risk appetite	❑ Sticks to the “Strong Momentum” **mid level risk appetite**
High diversification	❑ Adds additional **diversification benefits**
Robust capital shield	❑ Proves the relevance of its **capital shield policy** ❑ Pursues **active capital management**

The risk aggregation tree for **Standard** Non-Life LoBs