Is the Optimal Auction a Beauty Contest?
The Interaction of Market Allocation and Supervision

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Abstract*

Deregulation has often faltered at the monopolistic nature of network industries. One way around this problem has been to tender the monopoly right amongst firms. The interaction of these allocation procedures and the ex-post industry regulation has not been fully analysed within economic literature. This paper introduces an auction stage into a model of regulatory audits. Three surprising results follow; firstly, the optimal auction is identical to a beauty contest as equilibrium bids are independent of firms’ types. Secondly, even with an auction, ex-post audits are still necessary, as the use of the bids to reveal cost types conflicts directly with the auction winning rule. Thirdly a service contest, in which the auction winner is not the highest monetary bidder, strictly increases welfare over the optimal auction when the cost of audit is significant. Lastly we extend the analysis to show that results are robust even when governments value the bid revenues significantly as a means to secure tax revenue.

Key Words: Regulation, Auctions, Licenses

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Introduction

Over the last ten years there has been a strong trend towards both deregulation and competition within formerly regulated markets. Whilst this has been straightforward in some markets, for others, their natural monopoly characteristics have forced governments to rely on ex-ante competitive licensing for the market rather than (ex-post) in the market. Thus the allocation of production rights has become increasingly important. Recently, auctions have been the favored allocation procedure in many monopoly industries including rail train routes, regional television and gas storage. Their most notable use has been in the telecommunications industry, where they have been proposed for universal service obligations, and adopted for local loop and mobile spectrum licenses. Although deregulation is a relatively recent phenomenon, the creation of competition for the market where competition in a market is not possible has been around some time. In France the auctioning of medium to long term contracts to run monopolies such as the highways and local water companies has been commonplace for many decades.

Relying on the principle of sunk cost theory, auction proponents argue that license costs have no impact on consumers. As such, auctions have too often been seen as a means to raise or transfer revenue between firms and the government. However, this simple view does not take account of perhaps the key reason for auctions; their ability to provide information and perhaps bypass the need for explicit price regulation within the product market via efficiently allocating the license. The paper considers several different types of allocation procedures in the search for means to bypass the costly ex-post regulation by efficient allocation of the licenses. Firstly a random allocation mechanism where the
license is allocated randomly to a single firm for a single determined price. Secondly a beauty contest in which all firms pay the same amount and the license is allocated to the firm providing the best service. Thirdly an auction in which the regulator uses all available information and allocates the license to the highest bidder or highest service if bids are identical. Finally a ‘service contest’ where the regulator considers both bids and level of services and allocates the license to the firm offering the highest level of service. Although each of these allocation mechanisms involve less restrictions on the instruments available to the regulator, the paper shows that the interaction between auctions and subsequent product market regulation provides surprising policy conclusion to the question of how a monopoly market should be allocated and regulated. This interaction between allocation and regulation is an area which has received relatively little attention despite the increased use of auctions, a gap this paper is intended to fill.

This paper is a synthesis of two ideas; the use of contracts to regulate firms, and the use of auctions to dictate market structure. One of the earliest papers to deal with this first idea was Baron and Myerson’s (1982) paper on regulating a monopoly with unknown costs. Their main result is now standard in many models; efficient firms receive an information rent and inefficient firms must reduce production in order to reduce the rent paid to the more efficient firm. One criticism of this literature was the use of transfers within the regulatory contracts being unrealistic. A more realistic probability of audit was first incorporated into a model with transfers by Baron and Besanko (1984), with the transfers being dropped by Besanko and Spulber (1989). This last paper shows the optimal auditing policy trades off the benefit of reducing an efficient firms incentive to overstate its cost, against the authorities cost of auditing. Throughout the asymmetric information
literature, the models have taken industry structure as given, whilst in reality the ability to dictate structure gives a valuable additional instrument with which to regulate firms.

The use of auctions to determine market structure was first contemplated by Demsetz (1968) who advocated competitive bidding as a means to award a monopoly license. This idea was developed formally by Riordan and Sappington (1987) in the context of allocating franchises optimally when costs are unknown. Klemperer (2001) and Binmore and Klemperer (2002) give both a general overview of the auction literature and a very readable account of the practical pitfalls of auction design. In particular the latter paper stresses the advantages of an auction over a beauty contest, a question our paper addresses in the case of a single license. In the context of the spectrum auctions, Jehiel and Moldovanu (2000) looked at the possibility that an auction will select firms which are best able to collude, thus increasing prices through higher collusive activity in the second period. Whilst linking the auction outcome and the market outcome, this does not consider the impact of subsequent market regulation. Bennett (2000) has also studied this link within the context of regulatory uncertainty. Firms bid upon licenses as a function of the expected type of the regulator thus higher bids translate to higher degrees of collusion and hence higher prices. Whilst reflecting the political lobbying seen after the 3G auctions, this earlier paper models the regulator’s interaction as largely exogenous.2

In combining auctions and regulation, Laffont and Tirole (1993) extended the asymmetric information model using fixed transfers to include the allocation of market production rights. However the absence of ex-post audits does not allow a comparison between

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2 It is worth noting that since the completion of the third generation spectrum auctions, firms have had some degree of success in lobbying the government for looser regulatory arrangements including the sharing of antennas and the extension of license periods.
the use of audits and auctions as a means to extract information. With ex-ante (allocation) and ex-post (market) competition the regulator now has the choice of information revelation during the allocation procedure, regulating the market, or a combination of both. It is this interesting relationship that this paper considers, providing a logical next step in combining both the ex-post monitoring markets with the ex-ante allocation production rights.

The remainder of this paper is structured as follows. Section one outlines a model of firms with unknown costs bidding for a production license which is subject to market regulation. Section two analyses a sealed bid first price auction in which the regulator does not want to acquire information in the auction stage. The optimal policy is identical to a random allocation of licenses (as considered by Besanko and Spulber). Section three gives a positive analysis of an auction which the regulator uses to acquire information. The unexpected main result is that the optimal auction has the same characteristics as a beauty contest in which firms pay license amounts independent of their cost types. Section four compares welfare results and shows that even though the optimal auction still uses ex-post audits, welfare is strictly higher compared to a random allocation of licences. Section five provides a normative analysis of the optimal mechanism. For low audit costs this optimal mechanism is identical to the optimal auction/beauty contest. However when audit costs are significant a ‘service contest’ which separates the auction winning decision and the bid amounts, strictly dominates the use of a standard auction. The last section extends the analysis to include a cost of public funds term within the regulatory welfare function, showing that for a low to medium costs of public funds, the results remain robust. Only when the cost of public funds becomes sufficiently high that
the government facilitates firms in maximising profits by removing regulation, do the results differ from the main section. Finally the paper concludes.

1 Model Assumptions and Set-Up

Consider two risk neutral firms competing in an auction for a monopoly market with an inverse demand function $P(q)$. Each firm has a constant marginal cost $\theta_i$ which can take one of two values; $\theta_H$ for high, or $\theta_L < \theta_H$ for low cost firms. The probability of a firm being low cost is equal to $\gamma \in (0, 1)$. Only the firm knows its own cost. The winning firm picks quantity $q$ to be supplied in the market.\(^3\) If the firm winning the auction has a cost $\theta_i$, its profit in the product market is $\pi(q, \theta_i) = P(q)q - \theta_iq$, which is assumed to be twice continuously differentiable and strictly concave in $q_i$ with a unique interior maximum at the monopoly level of quantity denoted by $q_i^m$. We say that the firm of cost $\theta_i$ is competitive if it produces $q^c_i$ and prices at marginal cost $\theta_i$ thus making zero profit.\(^4\)

The license is allocated using a first price sealed bid auction which is used both for its reality and simplicity.\(^5\) Each firm submits a sealed bid $b$. Both the winning bid $b_1$ and the losing bid $b_2$ are observed by the regulator who awards the license to the highest bidder for the amount bid (the losing firm pays and receives nothing). Where bids are identical and the regulator is indifferent to the winner, the license is allocated with a

\(^3\)Although this model considers quantity as the strategic variable, it is straightforward to think of this model in terms of service or quality levels. In such a model $\theta_i$ is the cost of producing the service/quality level. Like the main model, a low cost of service firm has an incentive to pretend to be a high cost of service firm to secure higher prices.

\(^4\)Baniak and Philips (1996) have extended Besanko and Spulber’s basic analysis to allow for Cournot profits rather than Bertand. Similarly we could think of a regulator that sets maximum firm profits to greater than 0, without any significant changes to our results.

\(^5\)With extreme punishments for out of equilibrium bids, other types of auctions such as the ascending can also be modeled in the same framework.
Regulator announces credible audit policy
\[ \beta(b_1, b_2, q), F \]

Nature picks two firms to participate in auction

Highest bid wins.
If bids are same, regulator decides which firm wins.

Audit may be initiated, firm fined if \( q < q_i^c \)

Winning firm picks quantity output \( q \)

Figure 1: Information and Timing Structure

probability of 1/2 to each firm. The product market is regulated by an institution that observes firms’ bids and the market quantity produced. Before the auction the regulator commits itself to auditing the firms with probability \( \beta(b_1, b_2, q) \in [0, 1] \).

Auditing a firm incurs a cost of \( K \) to the regulator but reveals the cost of the firm. If the audit reveals a firm producing its competitive output nothing happens otherwise a fine \( F \) is imposed on the firm, thus \( F = \tilde{F}(q_i, \theta_i) \) where \( \tilde{F}(q_i, \theta_i) = 0 \) if \( P(q_i) = \theta_i \), and \( \tilde{F}(q_i, \theta_i) > \max \pi(q_i, \theta_i) \) otherwise. The regulator maximises total welfare net of the audit cost, \( W = V(q) - \theta_i q - \beta(b_1, b_2, q) K \), where \( V(q) = \int_0^q P(t) dt \) denotes consumer surplus. The timing of the game is summarised in Figure 1:

There are two types of bidding equilibria. A revealing auction equilibrium (denoted by capital \( R \) superscript) is a separating equilibrium in which both firms bid according to their type. A non-revealing auction equilibrium (denoted by capital \( N \) superscript)

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\[^6\text{Note that precommitment by the regulatory institution is essential for this model. In the absence of commitment, there would be the potential to renegotiate, an issue this paper does not consider.}\]

\[^7\text{It is always optimal for a regulator to set the level of fine to } F_{\text{max}} \text{ for offending firms. Increasing } F \text{ is a costless deterrent to firms pricing above cost, whilst increasing } \beta \text{ has a direct welfare cost via } K.\]

\[^8\text{In the final section we consider the extension of an opportunity cost to government spending. An additional term } \alpha b_i \text{ is included in the welfare function to capture the use of revenues as a means of financing government spending. A high } \alpha \text{ translates to a high cost of public capital.}\]
is an equilibrium in which both types bid common amounts irrespective of their types and separation occurs in the market stage. Within the market stage, we define an outcome ‘competitive’ where both firm types would produce their competitive quantities \(q_i^c\) if they won the auction, ‘partially competitive’ when only one type would produce the competitive quantity and ‘non-competitive’ if neither type would produce the competitive quantity. We denote the optimums by the small superscript \(p\) and \(n\) for partially competitive and non-competitive outcomes respectively. We can restrict analysis to only one partially competitive equilibrium by the following:

**Lemma 1** There is never an equilibrium in which a low cost firm would price at its ‘competitive’ level and the high cost firm would price above its ‘competitive’ level.

**Proof.** If the high cost firm is non-competitive then its participation constraint implies \(\pi(q_H, \theta_H) - \beta(b_H, b_2, q_H)F - b_H \geq 0\). For the low cost firm to be pricing at the competitive level in equilibrium, all other levels must be loss making; \(\pi(q_H, \theta_L) - \beta(b_H, b_2, q_H)F \leq 0\). However by mimicking the high-cost firm, a low cost firm could get \(\pi(q_H, \theta_H) - \beta(b_H, b_2, q_H)F + (\theta_H - \theta_L)q_H \geq (\theta_H - \theta_L)q_H > 0\), a contradiction.

The three remaining market equilibria are the competitive (C), partially competitive with the high cost firm pricing at its competitive level (PC), and non-competitive (NC). The following sections study the non-revealing and revealing bidding equilibria respectively. After discounting for strictly dominated equilibria, the final section compares the best bidding equilibria of each type in order to determine the optimal regulatory contract and market outcome.
2 Non-Revealing Auctions

In a non-revealing auction both types of firms must bid identical amounts and denote this non-revealing level as $b^N$; $b_H = b_L = b^N$. A non-revealing auction can easily be implemented by auditing with maximum probability for any $b \neq b^N$. Expected welfare in a non-revealing bidding equilibria is;

$$E(W) = \gamma [V(q_L) - \theta_L q_L - \beta_L K] + (1 - \gamma) [V(q_H) - \theta_H q_H - \beta_H K]$$ (1)

Where $\beta_i$ is the audit probability if a quantity ($q_i$) corresponding to type $i$ is produced. Any equilibrium must be compatible with the firm’s bidding participation constraint (BP):

$$\pi(q_i, \theta_i) - \beta_i F - b^N \geq 0.$$ (BP)

Secondly, for a winning firm of type $i$ to produce $q_i$ the net benefit must be greater or equal than the net benefit of producing any other quantity $q_j$, where $j \neq i$. We term this the market incentive constraint (MI);

$$\pi(q_i, \theta_i) - \beta_i F \geq \pi(q_j, \theta_i) - \beta_j F.$$ (MI)

As the bid only enters expected welfare indirectly via BP, increasing it only reduces the ability for an optimal solution to hold. For this reason the simplest means to satisfy BP is to set $b_{NR} = 0$. This reduced regulatory problem is identical to that studied by Besanko and Spulber, where only the quantity can be used to condition the audit probability on.
Proposition 1 Non-Revealing equilibria randomly allocate the licenses, and regulate ex-post in an identical manner to that characterised by Besanko and Spulber (1989), such that the optimal bid $b^N = 0$ and:

- The optimal policy in a partially competitive market outcome is as follows:
  1. $\beta_L = 0$, $\beta_H = \frac{\pi(q^N_H, \theta_L) - \pi(q^N_L, \theta_L)}{\pi(q^N_L, \theta_L)}$
  2. $q^{Np}_L = \max\{\bar{q}_L, \bar{q}_L\}$, where $\bar{q}_L$ is determined by the condition:
     $$\gamma [P(\bar{q}_L) - \theta_L] + (1 - \gamma) \frac{K}{F} \left[ P(\bar{q}_L) + \frac{\partial P(\bar{q}_L)}{\partial \bar{q}_L} \bar{q}_L - \theta_L \right] = 0$$
     and $\bar{q}_L = \sup\{q_L : \pi(q_L, \theta_L) = \pi(q^N_H, \theta_L)\}$
  3. $q^m_L < q^{Np}_L < q^c_L$

- The optimal policy in a non-competitive market outcome is as follows:
  1. $\beta_L = 0$, $\beta_H = \frac{\pi(q^{Nn}_H, \theta_L) - \pi(q^{Nn}_L, \theta_L)}{\pi(q^{Nn}_L, \theta_L)}$
     If $\beta_H > 0$:
  2. $q^{Nn}_L \leq \bar{q}_L$ where $\bar{q}_L$ is defined within the partially competitive outcome and $q^m_L < q^{Nn}_L \leq q^c_L$
     If in addition, $\pi(q^{Nn}_H, \theta_H) - \beta_H F > 0$
  3. $q^{Nn}_H$ is characterised by:
     $$[P(q^{Nn}_H) - \theta_H] = \frac{K}{F} \left[ P(q^{Nn}_H) + \frac{\partial P(q^{Nn}_H)}{\partial q_H} q^{Nn}_H - \theta_L \right]$$
     where $q^{Nn}_H < q^m_L$

- When $\theta_i$ is such that $q^m_L \leq q^c_H$ the partially competitive solution dominates the non-competitive solution.
Proof. See Besanko and Spulber (1989). ■

From proposition 1 several conclusions can be drawn. Firstly it shows that above cost prices by the low cost firm should be tolerated in equilibrium. This derives from the fact that a small reduction in the maximum probability of audit gives a first order increase in regulatory welfare via $\beta K$, whilst producing only a second order negative impact on welfare via the increase in price through relaxing MI. Thus because auditing has a direct cost, and it is optimal to tolerate limited price deviations from cost by the low cost industry, a low cost firm should never be audited. Secondly only high prices trigger an audit, this is not too ensure the high cost firm is not pricing above cost, but to ensure the low cost firm never finds it profitable to pretend to be a high cost firm.

3 Revealing Auctions

Unlike the non-revealing, a revealing equilibrium uses the bid as an extra regulatory instrument to identify the cost of the firms without the costly audit. In a revealing equilibrium the two types of firms must submit different bids; however we allow for a revealing policy with identical monetary bids through the use of voluntary ‘hand-waving’. If the auction ends with identical bid amounts, the regulator allows any low types present to ‘wave’ to signal their type and win the license. Such a framework is equivalent to an auction in which the highest bid wins, but if there is a tie the regulator looks at some secondary attribute (in this case market prices) to decide how to allocate the license. This hand-waving is not hard information and must therefore be incentive compatible.9

9An implementation of a strict auction framework without hand waving would be to allow the low type to bid $b_H + \epsilon$, where $\epsilon$ is a very small amount. This allows the low firm to win, although reduces welfare by an order of $\epsilon$ relative to hand waving.
We assume non extreme levels of $K$ such that it is always optimal for the regulator to have a low cost firm win over a high cost firm.\textsuperscript{10} This necessity creates an additional 'auction rule' constraint such that $b_L \geq b_H$. Expected welfare within a revealing equilibrium where low types win the auction can be expressed as:

$$E(W) = \gamma^2[V(q_{LL}) - \theta_L q_{LL} - \beta_{LL} K] + 2\gamma(1 - \gamma)[V(q_{LH}) - \theta_L q_{LH} - \beta_{LH} K]$$

$$+(1 - \gamma)^2[V(q_{HH}) - \theta_H q_{HH} - \beta_{HH} K]$$

where the first subscript denotes the winning bidder’s bid type and the second denotes the losing bidder’s bid type. Expected welfare is maximised with respect to $q$, and $\beta$ subject to the two types’ bid participation constraints (BPs);\textsuperscript{11}

$$\frac{1 - \gamma}{2}[\pi(q_{HH}, \theta_H) - \beta_{HH} F - b_H] \geq 0, \quad \text{(HBP)}$$

$$\frac{\gamma}{2}[\pi(q_{LL}, \theta_L) - \beta_{LL} F - b_L] + (1 - \gamma)[\pi(q_{LH}, \theta_L) - \beta_{LH} F - b_L] \geq 0. \quad \text{(LBP)}$$

and the two bidding incentive constraints (BIs);

$$\frac{\gamma}{2}[\pi(q_{LL}, \theta_L) - \beta_{LL} F - b_L] + (1 - \gamma)[\pi(q_{LH}, \theta_L) - \beta_{LH} F - b_L] \geq 0 \quad \text{(LBI)}$$

$$\frac{1 - \gamma}{2}[\pi(q_{HH}, \theta_L) - \beta_{HH} F - b_H],$$

\textsuperscript{10} The reader is referred to appendix ?? after having read this section to see why the regulator normally wants the low type firm to win.

\textsuperscript{11} The reader should recall that for a PC market outcome both the participation and incentive constraints are simpler as $\bar{F}(\theta_H, q_{HH}) = 0$ and hence $\pi(q_{HH}, \theta_H) - b_H = 0$. 

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\[
\frac{1 - \gamma}{2} [\pi (q_{HH}, \theta_H) - \beta_{HH} F - b_H] \geq \\
\frac{\gamma}{2} [\pi (q_{LL}, \theta_H) - \beta_{LL} F - b_L] + (1 - \gamma) [\pi (q_{LH}, \theta_H) - \beta_{LH} F - b_L].
\]

(HBI)

Only the incentive constraints at the bidding stage are relevant, as any deviation from the firm’s revealed type within the market stage can be simply ruled out in equilibrium by maximum punishments such that \( \beta(b_1, b_2, q_j) = 1 \). The problem is further simplified as both the optimal quantity and audit probability are independent of the loser’s bid: From expected welfare, the audit terms conditional on a low type winning can be rearranged to:

\[-K \gamma \left( \frac{\gamma}{2} \beta_{LL} + (1 - \gamma) \beta_{LH} \right) \].

Likewise, taking the audit terms within the low firm’s profit conditional on winning the auction yields:

\[-F \left( \frac{\gamma}{2} \beta_{LL} + (1 - \gamma) \beta_{LH} \right) \].

Because the \( \beta \)s enter in the exact same form, there is no gain to the regulator in adjusting the audit probability with the losing bid. Quantities are also independent of the losing bid as both profit and consumer surplus are strictly concave in quantity, consequently when offered the choice between a lottery of two bundles or a single bundle, both the firm and regulator prefer the single bundle. Given these two results we use \( \beta_L, \beta_H (q_L, q_H) \) to denote the equilibrium audit probabilities (quantities) as a function of the type revealed at the auction stage.

Lastly, to make comparisons, we define a beauty contest as a license allocation procedure in which the regulator sets a single price for the license independent of which type buys it. Firms submit proposals to buy the license at the prescribed price and the regulator picks which of the firms win.

**Proposition 2** *In the optimal revealing auction, bids are identical and equal to zero, thus the optimal auction is a ‘beauty contest’. Firms producing a quantity compatible with a*
low type are never audited.

Proof. If LBI does not bind, one equilibrium consistent with the other constraints, is for both types to produce their competitive quantities with no audits and bids equal to 0. However, this clearly violates LBI which thus must bind. Starting from a binding LBI with $\beta_L > 0$, reducing $\beta_L$ allows the regulator to increase $q_L$ which strictly increases welfare but does not alter the other constraints, hence $\beta_L = 0$. Similarly starting from a $b_L > b_H > 0$ and reducing $b_L$ allows the regulator to increase $q_L$ holding constant the other constraints, thus $b_L$ is decreased until the auction winning constraint binds at $b_L = b_H$.\footnote{For a PC market outcome, HBP implies that $b_H = 0$ and consequently from the auction constraint $b_L \geq b_H$, it follows directly that $b_L = 0$.} Starting from a positive $b_H = b_L$, and a binding LBI reducing $b_H$ and $b_L$ by equal amounts still ensures all constraints are satisfied, but allows a further increase in welfare via $q_L$ until welfare is maximised at the point $b_H = b_L = 0$.\footnote{The fact that the optimal bids are 0 stems from the assumption that the competitive profit is 0, i.e. $\hat{b} = \pi_{c0}^H = 0$. More generally, as long as the bids are identical, the regulator could allow positive competitive profits (consistent with a declining average costs curve; economies of scale) which would translate into higher optimal bids.} In comparing this and the beauty contest, both policies set a single type-independent price for the license and allow the regulator to pick the winner. As the allocation procedure and outcome are identical, the optimal auction is a beauty contest. 

To separate types, the regulator must create a cost for the low type to pretend to be a high cost type, thus increasing $b_H$ relative to $b_L$. For a PC market outcome, any $b_H > 0$ violates HBP. Hence without a negative $b_L$, the use of an auction to costlessly obtain a PC market outcome is not possible. For a NC market outcome, the problem is not within the incompatibility of competition and costless separation as previously, but within the winning rule of the auction. Setting a $b_H$ greater than $b_L$ such that LBI holds, would
allow the regulator to reduce $\beta_H$ and increase welfare. However, such a level violates the auction winning constraint, $b_L \geq b_H$, which ensures a low type always wins against a high type.\textsuperscript{14} The fact that both the auction constraint and LBI binds forces identical bids.

This strong result gives a simple answer to a question much studied in economics and business, ‘which is better, an auction or a beauty contest?’ When considering a monopoly market subject to regulation, the answer is it doesn’t matter as both are equivalent.\textsuperscript{15} Even though the set of possible equilibria under a beauty contest (where the bids are type fixed) is a subset of the auction equilibria (where bids can differ with type), the auction winning rule constrains the optimal bids to be identical and hence the two mechanisms are equivalent.

In characterising the optimal solution, the regulator’s problem is simplified by examining only the relevant constraints. From proposition 2, LBI must bind and $b_i = \beta_L = 0$. Moreover HBP and LBI imply that LBP is satisfied. Like usual in this literature, HBI does not bind. The regulator’s reduced form problem is:

$$\max_{q_i, \beta_H} \gamma (2 - \gamma) [V (q_L) - \theta_L q_L] + (1 - \gamma)^2 [V (q_H) - \theta_H q_H - \beta_H K]$$

\textsuperscript{14}Note the assumption that the regulator wants to ensure the low type wins is critical. As previously mentioned, one strategy is to allow the high type to win if bidding against a low type, thus freeing the regulator to set $b_H > b_L$ such that the LBIC is satisfied and separation is ensured costlessly. This strategy is considered, but ruled out, in appendix ??

\textsuperscript{15}Beauty contest have been criticised for their ability to allow governments the ability to distort the choice of firms, (see McAfee and McMillan (1994) for evidence). However, in this model because the auction results in identical bids, the regulator chooses a firm, in the same manner as a beauty contest. Thus, if there is an incentive to distort the choice within the beauty contest the identical incentive exists within the auction.
subject to LBI and HBP;

\[(1 - \frac{\gamma}{2})[\pi(q_L, \theta_L)] \geq \frac{1 - \gamma}{2}[\pi(q_H, \theta_L) - \beta_H F] \]  \hspace{1cm} \text{(LBI)}

\[\pi(q_H, \theta_H) - \beta_H F \geq 0 \]  \hspace{1cm} \text{(HBP)}

In a PC market outcome, HBP holds trivially as competitive firms are never fined. However in a NC market outcome, for low \(K\) HBP also binds. The intuition is as follows; As the high firm is not competitive, it is always fined when audited. When \(K \to 0\), auditing becomes cheaper and thus the regulator increases the probability of audit, \(\beta_H\). This brings the high firm’s quantity closer to the competitive \(q^c_H\) and 0 profits but still incurs the high expected fine, thus HBP must bind.

**Proposition 3** Within revealing equilibria,

- The optimal partially competitive outcome is as follows:

  1. \(b_L = b_H = 0\), \(\beta_L = 0\)
  2. If the fundamentals are such that \(\pi(q^c_H, \theta_L) > \frac{(2-\gamma)}{(1-\gamma)}\pi(q_L, \theta_L)\):
     \[\beta_H = \frac{\pi(q^c_H, \theta_L)}{F} - \frac{(2-\gamma)\pi(q_L, \theta_L)}{F (1-\gamma)}, q^R_L = \tilde{q}_L = q^N_L\]
  3. If the fundamentals are such that \(\pi(q^c_H, \theta_L) \leq \frac{(2-\gamma)}{(1-\gamma)}\pi(q_L, \theta_L)\):
     \[\beta_H = 0, q^R_L = \sup\{q_L : \frac{(2-\gamma)}{(1-\gamma)}\pi(q_L, \theta_L) \geq \pi(q^c_H, \theta_L)\} > q^N_L\]
  4. \(q^m_L < q^R_L < q^c_L\)

- The optimal non-competitive outcome is as follows:
1. \( b_L = 0, b_H = 0, \beta_L = 0 \).

2. \( \beta_H = \begin{cases} \frac{\pi(q_{Rn}^H, \theta_L)}{F} - \frac{(2-\gamma)(\pi(q_{Rn}^L, \theta_L))}{F(1-\gamma)} & \text{if } \frac{(2-\gamma)(\pi(q_{Rn}^L, \theta_L))}{F(1-\gamma)} < \pi(q_{Rn}^H, \theta_L) \\ 0 & \text{if } \frac{(2-\gamma)(\pi(q_{Rn}^L, \theta_L))}{F(1-\gamma)} \geq \pi(q_{Rn}^H, \theta_L) \end{cases} \)

If \( \beta_H > 0 \):

3. \( q_{L}^{Rn} \geq q_{L}^{Nn} \) and \( q_{L}^{m} < q_{L}^{Rn} \leq q_{L}^{c} \).

If in addition, \( \pi(q_{Rn}^H, \theta_H) - \beta_H F > 0 \):

4. \( q_{H}^{Rn} \) is characterised by:

\[
[P(q_{Rn}^H) - \theta_H] = \frac{K}{F} \left( P(q_{Rn}^H) + \frac{\partial P(q_{Rn}^H)}{\partial q_H} q_{Rn}^H - \theta_L \right)
\]

where \( q_{Rn}^H < q_{L}^{m} \).

- When \( \theta_i \) is such that \( q_{L}^{m} \leq q_{H}^{m} \), the partially competitive solution dominates the non-competitive solution.

**Corollary 1** In the optimal auction, costly ex-ante audits are still necessary but with lower probability compared to a random license allocation.

The proof is within the appendix, whilst the corollary follows from a straightforward comparison with proposition 1. Proposition 3 implies that separating types at the allocation stage (even if this still requires an audit) provides some benefit to the regulator via the lower probability of audit needed to separate types.

To implement this beauty contest the simplest mechanism is to use maximum probabilities of audit for all quantities and bids off the equilibrium path and only use the
possibility of audit when the quantity produced is high.

\[
\beta(b_i, q) = \begin{cases} 
0 & \text{if } q \in [q^R_L, \infty), \ b_i = 0 \\
\frac{\pi(q^R_L, \theta_L)}{F} - \frac{(2-\gamma)\pi(q^R_H, \theta_L)}{F(1-\gamma)} & \text{if } q \in [q^R_H, q^R_L), \ b_i = 0 \\
1 & \text{if } q \in [0, q^R_H) \text{ or } b_i \neq 0 
\end{cases}
\]

The optimal response for a high cost firm is to bid 0. If it wins the auction it produces \(q^R_H\) and is audited with a probability equal to \(\pi(q^R_H, \theta_L)\frac{(2-\gamma)\pi(q^R_L, \theta_L)}{F(1-\gamma)}\) but never fined.

The low type’s optimal response is to bid 0, and ‘hand wave’ at the end of the auction to signal its true type. At the market stage the low cost type produces \(q^R_L\) and is never audited.

4 Welfare Analysis

Within proposition 3, values of \(\theta_i\) such that \(q^m_L > q^M_H\), give an ambiguous comparison between the market equilibria within a given bidding equilibrium. For this reason numerical solutions are used to illustrate the welfare results. For easy comparison we use the same parameter values as B&S where \(P(q) = 100 - q_i\), \(\theta_H = 70\), \(\theta_L = 20\), \(\gamma = 0.5\), \(F = 2000\) and \(K\) is the varying parameter. Table 1 shows the PC and NC market outcomes for the revealing auction.

\[\text{At levels of } K \text{ greater than } 196.16, \beta_H = 0. \text{ Note that even though quantities are higher than the monopoly levels, they can be sustained by maximal punishments off the equilibrium path. Because } \beta_H = 0, \text{ a separating equilibrium is not implementable with the optimum being a pooling equilibrium where } q_L = \sup \{q_L : \pi(q_L, \theta_L) = \pi(q_H, \theta_L)\}.\]

\[\text{At levels of } K \text{ higher than } 206.04, \beta_H = 0. \text{ At this point, quantities } q_i > q^M_i \text{ are sustained by using maximum punishments off the equilibrium path. Like the non-competitive equilibrium, the optimum is a pooling equilibrium.}\]
Reveal, Partially Competitive. | Reveal, Non-Competitive - $b_L > b_H$

<table>
<thead>
<tr>
<th>$K$</th>
<th>$q_L$</th>
<th>$q_H^*$</th>
<th>$\beta_H$</th>
<th>$\pi_L$</th>
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<td>0</td>
<td>481.7</td>
<td>67.3</td>
<td>2495.6</td>
</tr>
</tbody>
</table>

Table 1: Numerical Solutions for Revealing Equilibria

In this example at low levels of $K$ when the cost of enforcement is small, the optimal market outcome is PC. As $K$ increases relative to $F$ the cost of separating types within a PC market increases and the welfare difference between the two market outcomes fall. At large levels of $K$ it is optimal to allow both firms to price above cost rather than trying to enforce the costly separation required by the PC outcome.

In comparing the revealing auction with the non-revealing auction, it is straightforward to see why the revealing auction always yields higher welfare. Consider an equilibrium with the exact same quantities, audit probabilities and bids amounts as the optimal non-revealing. Now change the allocation rule to allow hand waving so that the low cost firm wins when it comes against a high cost firm. HBP remains the same, whilst LBI and JBIC are relaxed due to the increased allocative efficiency. As all constraints are satisfied, but the low cost firm now receives the license with higher frequency, welfare must be strictly higher within such an equilibrium.

**Proposition 4** The use of a beauty contest strictly increases welfare compared to a random allocation of licenses (such as the non-revealing mechanism).
A beauty contest is superior to the allocatively efficient equilibrium constructed above for two reasons. Not only does it increase allocative efficiency, but the relaxation of LBI through this allocative efficiency delivers the same quantities at a lower probability of audit. This reduction in the probability of audit further increases welfare via the cost of audit. Secondly, because the beauty contest has a lower probability of audit, HBP is also relaxed, thus allowing higher quantities and welfare when it binds. In summary, to answer the question of whether a regulator should use an auction/beauty contest or just randomly allocate the licenses, the auction/beauty contest increases welfare. Intuitively this choice is equivalent to the choice between chance or a welfare maximising regulator picking the firm and adjusting the probability of audit accordingly.

5 Service Contest

The winning rule of a standard auction restricts the regulator’s ability to separating firm types at the allocation stage. Consequently a logical extension to improve welfare is to divorce the winning allocation rule from the bid amounts. We term such an allocation mechanism a ‘service contest’. In such a contest the regulator looks primarily at the service promised by the firms (in the context of the model \( q_i \)) and determines the price each firm will have to pay contingent upon the service promised. In the context of our model, the regulators choice is to allocate the license to the low cost (high service) firm even if it is not paying the most for the license. This enables the regulator to simultaneously use the license amounts (and setting \( b_H > b_L \)) as a regulatory tool to ensure separation and still ensure the low cost firm wins.

We note that if the regulator wants to achieve a partially competitive market outcome,
$b_H$ is constrained to be 0 by HBP. As subsidies are not allowed, separation via the bids alone is not possible. For this reason the solution to the service contest is identical to that of a beauty contest.\(^\text{18}\)

In solving for the service contest all constraints remain identical to that of the revealing auction. Thus by the same argument we need only consider LBI and HBP. Without the winning auction constraint, the regulator’s problem is to choose $\beta_i$, $b_i$ and $q_i$ to maximise:

$$E(W) = \gamma (2-\gamma) [V(q_{L}) - \theta_L q_{L} - \beta_L K]$$

$$+ (1-\gamma)^2 [V(q_{H}) - \theta_H q_{H} - \beta_H K]$$

subject to the reduced form HBP, LBI, JBIC within the revealing auction section, and the no-subsidy constraint ($b_L \geq 0$, $b_H \geq 0$).

**Proposition 5** If the antitrust authority desires a non-competitive market within a revealing equilibrium, the optimal service contest has the following features:

1. $b_L = 0$
2. $b_H = \pi (q_{H}^S, \theta_H) > 0$
3. $\beta_i = 0 \ \forall \ i$
4. $q_{L}^S$ is characterised by:

$$\gamma [P(q_{L}^S) - \theta_L] + (1-\gamma) \left( \frac{P(q_{H}^S) - \theta_H}{\theta_H - \theta_L} \right) \left[ P(q_{L}^S) + \frac{\partial P(q_{L}^S)}{\partial q_{L}} q_{L}^S - \theta_L \right] = 0$$

and $q_{L}^S \geq q_{L}^{Rn} \geq q_{L}^m$

\(^{18}\)Relaxing the no-subsidy constraint to allow negative bids would be consistent with a two way transfer system. This would allow a service contest within a PC market outcome that would welfare dominate the auction solution.
5. $q^S_H$ is given by:

$$q^S_H = \frac{2 - \gamma \left(P(q^S_L) - \theta_L\right) q^S_L}{1 - \gamma \theta_H - \theta_L}$$

and $q^S_H \geq q^R_H$

**Corollary 2** The service contest does not require costly ex-post audits.

See appendix for the proof. Using the service contest, the regulator is freed from the winning rule that constrains the license sale to the highest bidder. This enables the regulator to utilise both the license amount to reveal cost types and ensure that the low firm wins the license when a low and a high firm both apply. In effect, the use of the bids within the unrestricted mechanism is equivalent to a one way transfer from firms to the regulator. Without the constraint of the auctions winning rule, this instrument is sufficient to allow the regulator to separate firms types without an audit.

Implementation of such a contest is simple. The regulator offers the two firms a choice of two different packages the firm can either pay a high price for the license and it is then allowed to produce a low quantity/service, or it can have the license for free, but it must produce a high quantity/service. Market production/service levels consistent with the license amount paid is never audited, but productions inconsistent with the license amount paid is audited with a probability of one. When a low and a high type enter the contest, the regulator favors the low cost/high service type as it does not value bid outside of there ability to separate firms.

To complete the analysis, we compare the welfare from a service contest and the auction/beauty contest.
Proposition 6 For all $K$ greater than $\hat{K}$, the service contest strictly welfare dominates the auction mechanism.

Proof. If within both the service and auction mechanism a NC market outcome is optimal, the only difference between the two mechanisms is the auction winning constraint $b_L \geq b_H$. As the optimal bids in the service contest are such that $b_H > b_L$, whilst the winning constraint of the auction binds, the service contest is less constrained. Thus within a NC market outcome, welfare from the service contest is strictly greater than the auction.

Secondly, from proposition 5, the level of welfare for a NC outcome using the service is independent of $K$ due to the absence of audits. Within the PC market outcome, welfare is strictly declining in $K$, thus there exists some $\hat{K}$ at which point the welfare in a NC market outcome using a service contest is equal to welfare in a PC market outcome using an auction mechanism. The point $\hat{K}$ is characterised by:

\[
W^{Rp} - W^{Sn} = 0 = \gamma(2 - \gamma) \left[ V(q_L^p) - \theta_L q_L^p \right] \\
+ (1 - \gamma)^2 \left[ V(q_H^c) - \theta_H q_H^c - \frac{\hat{K}}{F} \left[ \pi(q_H^c, \theta_L) - \frac{2 - \gamma}{1 - \gamma} \pi(q_L^p, \theta_L) \right] \right] \\
- \left\{ \gamma(2 - \gamma) \left[ V(q_L^o) - \theta_L q_L^o \right] + (1 - \gamma)^2 \left[ V(q_H^o) - \theta_H q_H^o \right] \right\}
\]

Proposition 6 answers the question; ‘can one do better than an optimal auction?’ For markets such that $K \leq \hat{K}$, the optimal market outcome is PC, and the service contest is equivalent to the optimal auction. For markets in which the costs of audit are significant such that $K > \hat{K}$, the answer is yes. Free from the auction winning constraint, the
regulator can design a contract that allows separation of types without having to audit
the firm. Welfare in the value contest welfare is higher than the auction as separating
by the bids still allocates the licence efficiency but has no direct cost unlike the audit
necessary within the auction.

6 Other Issues

6.1 Bidding Revenue

One of the arguments used for selling licenses is that taxation is distortionary and that
the revenues raised by the bids are lump sum and hence are less distortionary. Along
these lines much of the new regulation literature considers a cost of transfers or funds,
that results in the optimal quantities being a function of this cost. To consider such an
argument an explicit value on the money raised from the auction is included within the
welfare function. This section briefly considers and outlines the most interesting elements
of the optimal regulatory policy under such a cost of funds. With a cost of public funds,
regulatory welfare when firm $i$ produces is;

$$W_i = V(q_i) - \theta_i q_i - \beta_i K + \alpha b_i$$

As $\alpha$ increases the share of welfare due to the bid increases. Intuitively such a welfare
function generates two effects with respect to the bid. Firstly there is the positive direct
increase in welfare via $\alpha b_i$, secondly, there is a negative influence due to the link between
$V(q_i)$ and $b_i$ via the firms BPCs. Higher bids have to be financed via higher profits and
consequently lower consumer surplus. This relationship is illustrated in the example of a
non-competitive market within a revealing equilibrium;\textsuperscript{19}

**Proposition 7** With a cost of public funds, the optimal antitrust policy using an auction in a non-competitive market outcome has the following features:

1. $\beta_L = 0$

<table>
<thead>
<tr>
<th>Level of $\alpha$</th>
<th>Bid Level</th>
<th>High Audit Probability $\beta_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘Low’</td>
<td>$b_L = b_H = 0$</td>
<td>$\beta_H = \beta_H^B$</td>
</tr>
<tr>
<td>‘Medium’</td>
<td>$b_L = b_H = \pi(q_H, \theta_H) - \beta_H F$</td>
<td>$\beta_H &lt; \beta_H^B$</td>
</tr>
<tr>
<td>‘High’</td>
<td>$b_L &gt; b_H = \pi(q_H, \theta_H)$</td>
<td>0</td>
</tr>
</tbody>
</table>

where $\beta_H^B$ is the level of audit under a beauty contest.

The proof is within the appendix. Whilst we only characterise the optimal policy that involves a NC market outcome, the findings are consistent with the other market outcomes. At low levels of $\alpha$ the optimal bids are negative and thus both the no-subsidy and auction winning constraint bind. This is the solution characterised within the main sections. At medium levels of $\alpha$ the auction winning constraint continues to bind but the regulator maximises the bids where HBP binds. In both the low and medium cases because the auction winning constraint binds, the paper’s main results hold; an optimal auction remains a beauty contest and the unrestricted mechanism increases welfare. Only when $\alpha$ is large enough that the regulator effectively maximises firm’s profits instead of consumer welfare, does the auction become the optimal mechanism. In this case, the desire to maximise bid revenue and hence firms’ profits, allows the optimal low bid to be greater than the high bid which relaxes the auction winning constraint.

\textsuperscript{19}This equilibrium was chosen as it allows the use of both bids to obtain separation (unlike the partial). Because the primary interest for this paper is the bids, solutions for quantities are not characterised and are left for the interested reader.
This result implies that countries with relatively low costs of public funds, i.e. developed countries, where consumer surplus is of primary importance, an auction is not the optimum allocation method.\textsuperscript{20} Alternatively in lower developed countries where the cost of public funds is very high, the auction provides a way to maximise the rent extracted from firms albeit at the cost of consumers. Thus studying the 3G auctions in this light, one may conclude that the design of the UK auction followed the erroneous assumption of a high cost of public funds and consequently whilst successful in maximising revenue, was unsuccessful in maximising a correct measure of welfare.

\textbf{Conclusion}

This paper has presented a simple but relevant model of the allocation of a monopoly license with subsequent regulation. The findings cast doubt on two accepted wisdoms. Firstly that auctions as opposed to beauty contests are the best means to allocate licenses. Secondly that the use of an auction has no impact upon the price of the final product.

When the regulator rules out information gained via the auction; preferring a non-revealing outcome, there are no gains to using an auction relative to a random license allocation. Although this equivalence to the random allocation appears trivial, it has important policy implications. If the government operates the auction and the regulator pays no attention to information received within the auction (as one might argue is often the case) the auction is equivalent to a lottery mechanism and thus does not increase efficiency per se.

\textsuperscript{20}In simulations using the previous estimates, this includes countries with levels of \( \alpha \) below 0.12 are medium. As the cost types become closer (\( \theta_L = 50, \theta_H = 60 \)) this level increases to 0.30.
When the regulator uses the auction information; preferring a revealing outcome, several important and surprising results are derived. Firstly, in the optimal auction, the bids are not used to separate types, and the costly ex-ante audit necessary within a random allocation is still required, although at a lower level. Secondly, the optimal auction is a beauty contest. The license is sold for the same amount independent of the cost type of the bidder and hence the auction is no different from a beauty contest where the government charges a single price for the license and picks the most efficient firm. The intuition behind these two results is simple; to separate the cost types in the auction, the regulator must create an opportunity cost for a cheating low type by increasing a high cost type’s bid above that of a low type’s bid. However in an auction, the highest bidder wins, thus separating firm types ex-ante via the bids allows the high cost inefficient firm to win the license and produce. The best the regulator can do is to set both bids equal and separate types through the ex-post audit probability. However, even though ex-ante separation is not possible, welfare using a revealing auction is strictly higher than randomly allocating the licenses due to the increased frequency of winning for the low cost firm.

To solve the above conundrum of ex-ante separation via bids, we put forward the use of a ‘service’ contest. This combines the bid instrument used in an auction, with the quality selection procedure used in the beauty contest. The regulator offers two prices for licenses, a low price (0) for firms offering high service levels (low cost firms) and a high price for firms offering low service levels. Firms submit levels of market service (quantity in our model) and the regulator picks the highest service offering. This allows the regulator to set a high license price to the high cost type, using the license price to
separate types, but still allowing the low cost firm to win. In this equilibrium audits are not required as firms are perfectly separated by their bids before entering the market. Such a service contest strictly increases welfare when audit costs are significant enough that a non-competitive market outcome is preferred.

One touted advantage of auctions has been that they are an important source of revenues for the government. As such we extended the model to illustrate the impact of adding a cost of public funds into the welfare function. The result derived is both intuitive and relevant. In countries where the opportunity costs of funds is low, the consumer surplus element of the welfare equation is strongest and the main results of the paper remain unchanged. Only when the opportunity cost of funds is so high that governments in effect maximise firm profits in order to extract the maximum bid revenues, does the results of this paper change.

In conclusion, this paper shows that auction design should not be merely about securing the highest revenues for government coffers. The use of auctions must be taken in conjunction with the regulatory environment in the market stage. Whilst auctions generally bring welfare benefits if the derived information is used in the implementation of a market outcome, in some cases it may be that auctions are not optimal and other mechanisms (such as the service contest) would provide at least same benefits of an auction without the restrictions. Thus when Paul Klemperer (2002) writes his caution on auction design ‘one size [of auction] does not fit all’, this paper would adds the proviso that sometimes a simple auction just does not fit.
References


7 Appendices

7.1 Proof to Proposition 3

Proof. From proposition 2, LBI must bind. To simplify, for both the PC and NC market outcomes, we can combine LBI and HBI to yield a joint bidding incentive constraint (JBI): \((\frac{2-\gamma}{1-\gamma})q_L \geq q_H\) (note this is simply \(q_L \geq q_H^c\) in the PC market). As LBI binds, JBI implies HBI, and for now we ‘relax’ HBI and verify JBI holds at both optimums.

Analysing the PC market outcome, from proposition 2, \(b_L = b_H = 0\) and \(\beta_L = 0\). Solving LBI for the minimum audit probability to ensure separation yields; \(\beta_H \geq \frac{\pi(q_H^c, \theta_L)}{F} - \frac{(2-\gamma)\pi(q_L, \theta_L)}{F(1-\gamma)}\). Under the policy \([q_i, \beta_i, b_i] = \left\{[q_L, 0, 0], \left[q_H^c, \frac{\pi(q_H^c, \theta_L)}{F} - \frac{(2-\gamma)\pi(q_L, \theta_L)}{F(1-\gamma)}, 0\right]\right\}\), the relaxed regulatory problem is to maximise;

\[
E(W) = \gamma (2 - \gamma) [V(q_L) - \theta_L q_L] + (1 - \gamma)^2 \left[ V(q_H^c) - \theta_H q_H^c - \frac{\pi(q_H^c, \theta_L) K}{F} + \frac{K(2 - \gamma)\pi(q_L, \theta_L)}{F(1 - \gamma)} \right]
\]
with respect to $q_L$, subject to HBP. Maximising welfare with respect to $q_L$ yields;

$$
\frac{\partial W}{\partial q_L} = \gamma [P(q_L) - \theta_L] + (1 - \gamma) \frac{K}{F} \left[ P(q_L) + \frac{\partial P(q_L)}{\partial q_L} q_L - \theta_L \right] = 0
$$

resulting in the optimum $q_L^\text{lp} = q_L^\text{np}$ proving part 2. When $\pi(q_H^*, \theta_L) \leq \frac{(2-\gamma)}{(1-\gamma)} \pi(q_L^*, \theta_L)$, such that $\beta_H \leq 0$, it is strictly welfare increasing to set $\beta_H = 0$, allowing the low type to produce $q_L^\text{lp} = \sup \left\{ q_L : \frac{2-\gamma}{1-\gamma} \pi(q_L, \theta_L) = \pi(q_H^*, \theta_L) \right\}$ which when compared with proposition 1 proves part 3 and generates a minimum level of $q_L$ that ensures JBI holds. Part 4 is proved in the identical manner to Besanko and Spulber 1989.

For the NC market outcome, part 1 is derived from proposition 2. Solving the binding LBI for $\beta_H$ yields;

$$
\beta_H \geq \frac{\pi(q_H^*, \theta_L)}{P(q_H)} - \frac{2 - \gamma}{1 - \gamma} \frac{\pi(q_L^*, \theta_L)}{p}
$$

proving part 2. Under the policy $[q_i, \beta_i, b_i] = \left\{ [q_L, 0, 0], [q_H, \frac{\pi(q_H^*, \theta_L)}{p} - \frac{2 - \gamma}{1 - \gamma} \frac{\pi(q_L^*, \theta_L)}{p}, 0] \right\}$, the relaxed welfare problem is to maximise expected welfare;

$$
E(W) = \gamma (2 - \gamma) [V(q_L) - \theta_L q_L]
+ (1 - \gamma)^2 \left[ V(q_H) - \theta_H q_H - \frac{\pi(q_H, \theta_L) K}{F} + \frac{2 - \gamma}{1 - \gamma} \frac{\pi(q_L, \theta_L) K}{F} \right]
$$

with respect to $q_i$ subject to HBP yielding equation 6 and;

$$
\frac{\partial W}{\partial q_H} = P(q_H) - \theta_H - \frac{K}{F} \left[ P(q_H) + \frac{\partial P(q_H)}{\partial q_H} q_H - \theta_L \right] = 0
$$

which proves part 4 when HBP does not bind. If $\beta_H < 0$, it is strictly welfare increasing to set $\beta_H = 0$, allowing the low type to produce $q_L^\text{rn} = \sup \left\{ q_L : \frac{2 - \gamma}{1 - \gamma} \pi(q_L, \theta_L) = \pi(q_H, \theta_L) \right\}$. This generates a minimum level of $q_L = \frac{1 - \gamma}{2 - \gamma} q_H$, thus ensuring the relaxed problem satisfies
the JBIC. When HBP binds, the revealing audit probability is strictly lower than the non-
revealing, thus the non-revealing HBP binds tighter and \( q_{Rn}^L > q_{Ln}^R \) which proves part 3.

Finally to facilitate comparisons between the two equilibria we denote \( q_{Rp}^L, q_{H}^R, (W_{Rp}) \) and \( q_{Rn}^L, q_{H}^R, (W_{Rn}) \) the optimal quantities (welfare) in the PC and NC market outcomes respectively. The welfare difference can be expressed as:

\[
W_{Rp} - W_{Rn} = (2 - \gamma) \left\{ \gamma \left[ V \left( q_{Rp}^L \right) - \theta_L q_{Rp}^L \right] + (1 - \gamma) \frac{K}{T} \pi \left( q_{Rp}^L, \theta_L \right) - \right\} \\
\gamma \left[ V \left( q_{Rn}^L \right) - \theta_L q_{Rn}^L \right] + (1 - \gamma) \frac{K}{T} \pi \left( q_{Rn}^L, \theta_L \right) \\
+ (1 - \gamma)^2 \left\{ \left[ V \left( q_{H}^R \right) - \theta_H q_{H}^R - \frac{K}{T} \pi \left( q_{H}^R, \theta_L \right) \right] - \right\} \\
\left[ V \left( q_{H}^R \right) - \theta_H q_{H}^R - \frac{K}{T} \pi \left( q_{H}^R, \theta_L \right) \right] \\
\right\} 
\]  

The first term of equation 8 is positive as \( q_{L}^R \) is the unique maximiser of \( \gamma \left[ V \left( q_{L}^R \right) - \theta_L q_{L}^R \right] + (1 - \gamma) \frac{K}{T} \pi \left( q_{L}^R, \theta_L \right) \). The second term is positive when \( q_{L}^R \leq q_{H}^R \) (thus \( V \left( q_{H}^R \right) - \theta_H q_{H}^R - \frac{K}{T} \pi \left( q_{H}^R, \theta_L \right) \) is increasing in \( q_{H} \)) which yields the sufficient condition for \( W_{Rp} > W_{Rn} \).

7.2 Proof of Proposition 5

Proof. From proposition 2, \( b_L = 0 \). With no constraint on which firm wins, the regulator can use \( b_H \) to satisfy LBI subject to HBP. Starting from a relaxed HBP and a binding LBI, increasing \( b_H \) allows a corresponding increase in \( q_L \) and hence an increase in welfare until the point at which HBP binds. Consequently both LBI and HBP bind at the optimum.

\[21\] The same term denotes low quantity within revealing and non-revealing as they are identical.
The two constraints combined are such that;

\[(2 − \gamma) \pi (q_L, \theta_L) \geq (1 − \gamma) q_H (\theta_H − \theta_L)\]

As the audit probability cancels, the audit is of no use (as discussed in appendix ??) and \(\beta_H = 0\). Under the policy \([q_i, \beta_i, b_i] = \{[q_L, 0, 0], [q_H, 0, \pi (q_H, \theta_H)]\}\), the regulatory problem is to maximise;

\[
E(W) = \gamma (2 − \gamma) [V(q_L) − \theta_L q_L] + (1 − \gamma)^2 [V(q_H) − \theta_H q_H] \\
+ \lambda_1 [(2 − \gamma) \pi (q_L, \theta_L) − (1 − \gamma) q_H (\theta_H − \theta_L)]
\]

with respect to \(q_i\) which yields the two first order conditions;

\[
\frac{\partial W}{\partial q_L} = \gamma [P(q_L) - \theta_L] + \lambda_1 \left[ P(q_L) + \frac{\partial P(q_L)}{\partial q_L} q_L - \theta_L \right]
\]

\[
\frac{\partial W}{\partial q_H} = (1 − \gamma) [P(q_H) − \theta_H] − \lambda_1 [\theta_H − \theta_L]
\]

Solving equation 11 for \(\lambda_1\) and substituting back into equation 10 proves part 4. The combined LBI and HBP constraints provides the last equation;

\[
\frac{2 − \gamma}{1 − \gamma} (P(q_L) − \theta_L) q_L = q_H (\theta_H − \theta_L)
\]

which when solved for \(q_H\) proves part five.
7.3 Revealing auctions where the high cost firm wins

Creating an equilibrium in which \( b_H > b_L \) allows the regulator to use \( b_H \) to separate firms ex-ante without the use of the costly ex-post audit. This case is ruled out in the following lemma.

**Lemma 2** For non-extreme levels such that \( K < F/2 \) designing a regulatory policy in which the inefficient type wins the auction is never the optimal regulatory policy.

**Proof.** This proof is done in two stages, firstly deriving the high winning solution, secondly comparing this to the low winning case where HBP binds. If the latter dominates the former, then the optimum low winning case without a binding HBP must also dominate.

As both the participation constraints remain unchanged it is simple to show that if HBP holds LBP is also satisfied. With the high type winning in a tie break, the two incentive constraints are;

\[
\frac{\gamma}{2} [\pi(q_L, \theta_L) - \beta_{LL} F - b_L] \geq \frac{1-\gamma}{2} [\pi(q_H, \theta_L) - \beta_{HH} F - b_H] + \gamma [\pi(q_H, \theta_L) - \beta_{HL} F - b_H] \quad \text{(LBIC)}
\]

\[
\frac{1-\gamma}{2} [\pi(q_H, \theta_H) - \beta_{HH} F - b_H] + \gamma [\pi(q_H, \theta_H) - \beta_{HL} F - b_H] \geq \frac{\gamma}{2} [\pi(q_L, \theta_H) - \beta_{LL} F - b_L] \quad \text{(HBI)}
\]

Combining LBI and HBI yields the a joint constraint (JBI): \( q_L \geq \frac{1+\gamma}{\gamma} q_H \). As LBI binds, starting from positive levels and reducing \( b_L \) and \( \beta(b_L, q_L) \) to 0 whilst increasing \( b_H \) allows
an increase in \( q_L \) and hence welfare. However, because \( b_H \) is limited by HBP, the optimum \( b_H \) is where both HBP and LBI bind. Substituting HBP into LBI yields:

\[
(1 + \gamma) \pi (q_L, \theta_L) \geq q_H (\theta_H - \theta_L).
\]

Under the regulatory policy \([q_i, \beta_i, b_i] = \{(q_L, 0, 0), [q_H, 0, \pi (q_H, \theta_H)]\}\), the Lagrangian for welfare is:

\[
L(W) = \gamma^2 [V(q_L) - \theta_L q_L] + (1 - \gamma^2) [V(q_H) - \theta_H q_H] + \lambda_H [\gamma \pi (q_L, \theta_L) - (1 + \gamma) q_H (\theta_H - \theta_L)]
\]

subject to JBI and bid constraint \( b_H \geq b_L \geq 0 \). The FOC for \( q_L, q_H \) are:

\[
\frac{\partial W}{\partial q_L} = \gamma (P(q_L) - \theta_L) + \lambda_H \left[ P(q_L) + \frac{\partial P(q_L)}{\partial q_L} q_L - \theta_L \right] = 0 \tag{12}
\]

\[
\frac{\partial W}{\partial q_H} = (1 - \gamma) [P(q_H) - \theta_H] - \lambda_H [\theta_H - \theta_L] = 0 \tag{13}
\]

The constraint provides the last equation and the solution for \( q_H \) in terms of \( q_L \):

\[
q_H = \frac{\gamma}{1 + \gamma} \frac{(P(q_L) - \theta_L) q_L}{(\theta_H - \theta_L)} \tag{14}
\]

and also ensure that JBI is satisfied as \( P(q_L) \geq \theta_H \).

As proved in proposition 3, the ‘normal’ case where the low type wins with a binding HBP has \( b_L = b_H = \beta_L = 0 \). Combining HBP and LBI, the regulatory problem under

\[22\] Notice the audit probability cancels, this is because given the HBPC binds, increasing the expected fine means having to reduce the level of bid demanded. As increasing the audit probability only reduces the ability to use \( b_H \) to satisfy the LBIC, the optimum is simply to set \( \beta_H = 0 \).
the policy $[q_i, \beta_i(q_i), b_i] = \{[q_L, 0, 0], [q_H, 0, 0]\}$, is to maximise;

$$L(W) = \gamma (2 - \gamma) [V(q_L) - \theta_L q_L] + (1 - \gamma)^2 \left[ V(q_H) - \theta_H q_H - \frac{\pi(q_H, \theta_H) K}{F} \right]$$

$$+ \lambda_L [(2 - \gamma) \pi(q_L, \theta_L) - (1 - \gamma) q_H (\theta_H - \theta_L)]$$

subject to JBI, and the auction constraint: $b_L \geq b_H \geq 0$. Maximising with respect to $q_i$ yields the FOCs;

$$\frac{\partial W}{\partial q_L} = \gamma [P(q_L) - \theta_L] + \lambda_L \left[ P(q_L) + \frac{\partial P(q_L)}{\partial q_L} q_L - \theta_L \right] = 0$$ \hspace{1cm} (15)

$$\frac{\partial W}{\partial q_H} = (1 - \gamma) [P(q_H) - \theta_H] - (1 - \gamma) \frac{K}{F} \left[ P(q_H) + \frac{\partial P(q_H)}{\partial q_H} q_H - \theta_H \right] - \lambda_L [\theta_H - \theta_L] = 0$$ \hspace{1cm} (16)

with the constraint yielding the solution for $q_H$

$$q_H = \frac{(2 - \gamma) (P(q_L) - \theta_L) q_L}{(1 - \gamma) \theta_H - \theta_L}$$ \hspace{1cm} (17)

Comparing equations 14 and 17, as $\frac{2 - \gamma}{1 - \gamma} > \frac{\gamma}{1 + \gamma}$, at a set level of $q_L$ the ‘normal’ $q_H$ is always greater. To determine the impact on $q_L$ we examine $\lambda$. Comparing equations 16 and 13 there is an additional, positive, distortionary audit term in the ‘normal’ case. As $q_H$ increases, the first term of equation 16 decreases whilst the second audit term increases. Thus whether $\lambda_L$ or $\lambda_H$ is smaller depends upon the relative magnitude of
these distortions and hence the second derivative of welfare with respect to $q_H$:

$$\frac{\partial^2 W}{\partial q_H^2} = \frac{\partial P(q_H)}{\partial q_H} \left( 1 - 2\frac{K}{F} \right) - \frac{K}{F} \frac{\partial^2 P(q_H)}{\partial q_H^2} q_H$$

As profit is concave in $q_H$ the second term is positive. With non-extreme levels of audit cost defined such that $K < \frac{F}{2}$, the second derivative w.r.t. welfare is positive and hence $\lambda_L < \lambda_H$, values of $K$ greater than this are ambiguous. Finally in examining $q_L$, from equations 15 and 12 it is straight forward to see that $q_L$ is greater when $\lambda_L < \lambda_H$. Consequently for non-extreme levels of $K$ designing a regulatory framework such that the inefficient high cost type wins is never optimal. 

7.4 Proof of Proposition 7

**Proof.** Starting by illustrating the optimum when the no-subsidy and auction constraints are relaxed; for any positive $\alpha$, unconstrained welfare is increasing in $b_H$, thus HBP must bind. This yields an unconstrained optimum such that; $b_H^* \leq \pi(q_H, \theta_H)$. Combining this with LBI yields a condition on $b_L$ such that; $b_L^* \leq \pi(q_L, \theta_L) - \frac{1-\gamma}{2-\gamma} q_H (\theta_H - \theta_L)$.

To determine how the equilibria evolve with $\alpha$ it is necessary to determine how the optimum bids and hence quantities change with $\alpha$. Welfare without the no-subsidy and auction constraint is simply:

$$E(W) = \gamma (2 - \gamma) \left[ V(q_L) - \theta_L q_L + \alpha \left[ \pi(q_L, \theta_L) - \frac{1-\gamma}{2-\gamma} q_H (\theta_H - \theta_L) \right] \right] + (1 - \gamma)^2 \left[ V(q_H) - \theta_H q_H + \alpha \pi(q_H, \theta_H) \right]$$

Using the implicit function theorem, $\frac{\partial q_i^*}{\partial \alpha} = -\left( \frac{\partial^2 W}{\partial q_i^2} - \frac{\partial^2 W}{\partial q_i \partial q_j} \right) / \det[H]$ (where $H$
is the Hessian matrix of $E(W)$. As $\frac{\partial^2 W}{\partial q_i \partial \alpha} < 0$ so $\frac{\partial q^*_i}{\partial \alpha} < 0$.\footnote{Note the concavity of the welfare function in $q_i$ and the lack of cross derivative between $q_L, q_H$ greatly simplifies the problem.} Returning to the optimal bids, as $q^*_i < q^m_i$, $\frac{\partial b^*_L}{\partial \alpha} > 0$. Furthermore, when $\alpha = 0$, $b^*_H = 0$, and a revealing auction is only possible through $b^*_L < 0$.\footnote{Note that if $\gamma$ is very high the LBIC will always bind independent of the parameter levels, however this would render the problem trivial and thus is not considered.} Lastly as $\alpha \to \infty$, $q^*_L \to q^m_L$ and thus for LBI to bind, $q^*_H \to q^m_H - \frac{\gamma}{1-\gamma} (\theta_H - \theta_L)$, and thus $b^*_L > b^*_H$. The relationship leads to three equilibria.

For low levels of $\alpha$ such that $b^*_L(\alpha) < 0$, $b^*_H = 0$, the auction constraint forces $b_L = 0$. Given $b_L = b^*_H = 0$, the optimal low $\alpha$ solution is identical to proposition 3. For medium levels of $\alpha$ such that $0 < b^*_L(\alpha) = \pi(q_L, \theta_L) - \frac{1-\gamma}{2-\gamma} q_H (\theta_L - \theta_H) < \pi(q_H, \theta_H)$, the auction constraint remains binding and hence $b_H$ remains constrained to the level of $b_L$. Using $b^*_L = b^*_H$ and solving LBI for the optimal audit policy derives the $\beta_H$ less than that of a beauty contest. Finally, high levels of $\alpha$ are defined as the point at which $b^*_L \geq b^*_H$ that is $\pi(q_L, \theta_L) - \frac{1-\gamma}{2-\gamma} q_H (\theta_L - \theta_H) \geq \pi(q_H, \theta_H)$ and hence the auction constraint no longer binds. In this case the regulator sets HBP to binding, and uses $b_L$ to ensure LBI holds. As LBI holds through the bids, the optimal $\beta_H$ is simply 0. \vspace{3pt}

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\footnote{23 Note the concavity of the welfare function in $q_i$ and the lack of cross derivative between $q_L, q_H$ greatly simplifies the problem.}

\footnote{24 Note that if $\gamma$ is very high the LBIC will always bind independent of the parameter levels, however this would render the problem trivial and thus is not considered.

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