Presented at the

# A Contestable Market Model of the Delivery of Commercial Mail 

Edward S. Pearsall and Charles L. Trozzo*


#### Abstract

In this paper we explore the effects of liberalizations in which the delivery monopoly of the U.S. Postal Service (USPS) is relaxed to permit the competitive delivery of commercial mail. Liberalization is assumed to leave USPS facing a single potential entrant offering an imperfectly substitutable delivery service for commercial mail in an array of contestable local markets. USPS is allowed to respond with differentiated prices for the delivery of commercial mail. The local markets are analyzed as non-zero-sum non-cooperative two-person games with Nash equilibriums corresponding to one of several models of market equilibrium. We find that USPS retains sufficient market power after liberalization to secure its finances without a subsidy, but that liberalization is likely to cause significantly higher average rates across all markets for commercial mail and a loss in total welfare.


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## 1. Introduction

In this paper we explore the effects of liberalizations of postal markets in which the delivery monopoly of an incumbent post is relaxed to permit the competitive delivery of commercial mail. The incumbent post is the U. S. Postal Service (USPS). In our analysis USPS is assumed to retain a monopoly on the delivery of non-commercial mail and to remain under a universal service obligation to provide delivery service for all mail in all local markets, but USPS would be permitted to respond to potential competitors by setting locally differentiated rates for delivering commercial mail.

Most previous analyses of postal liberalization have relied on the assumption that competition would take the form of a competitive fringe. Notably these studies include models by Crew and Kleindorfer (2007) and by De Donder et al (2007 and 2008). These models were conceived to analyze the effects of the upcoming full market opening of postal markets among the countries of the European Union.

In our view competition in a geographically delineated local postal market is unlikely to be a competitive fringe. Instead, potential competitors will possess cost functions exhibiting economies of scope and scale in delivery similar to that of the incumbent post. Such economies are a firmly established characteristic of mail delivery in the U.S. (Bradley et al 2007) and elsewhere (Farsi et al 2007, Casals et al 2005a and Casals et al 2005b). Competition to USPS under these conditions is most likely to take the form of a single potential entrant offering a more-or-less substitutable delivery service for commercial mail. Competition to the incumbent post in Sweden (Posten) came in the form of a single entrant (CityMail), not a competitive fringe, following liberalization of the Swedish postal market in 1993 (Cohen et al 2007).

Liberalized postal markets have been treated as duopolies in just a few previous studies. Recent research that has employed such models to analyze the opening of European postal markets includes Gautier (2007), d’Alcantara and Gautier (2008) and Bloch and Gautier (2008). These studies rely on assumptions that postal markets will become duopolies or remain monopolies under pre-specified conditions. In effect, entry and exit by potential entrants is treated as determinate. In our contestable market model entry and exit is treated as stochastic.

The effects of liberalization on rates, demand, competition, net revenue and welfare are analyzed using a model of market behavior derived from game theory. The local market for delivery services is analyzed as a non-zero-sum non-cooperative twoperson game with a Nash equilibrium. The incumbent's strategies are his rates, which are set to maximize his economic objective (such as profit, revenue or welfare) and are then left unchanged. The potential entrant takes the incumbent's rates as given and sets his own rate for commercial mail to maximize his profit if he chooses to enter. The entrant's pure strategies in the game are to enter the market if he can make a profit, and, not to enter if the result would be a loss.

In Appendix A it is shown that the game's Nash equilibrium corresponds to one of three market models. These are: 1) Monopoly - the potential entrant stays out of the market; 2) Stackelberg Duopoly - the potential entrant enters and remains in the market; and 3) Stochastic Equilibrium - the incumbent's rates are set to deter entry by leaving the potential entrant with a zero profit if he enters. Stochastic Equilibrium resembles the market model proposed by conventional contestable market theory. However, in a

Stochastic Equilibrium the entrant's equilibrium strategy is a mixed strategy of entry and non-entry parameterized by a state probability of entry. At equilibrium the state probability of entry leaves the incumbent with no expected gain from changing his price.

Calculations of equilibriums with the model are made using calibrated demand and cost functions to determine if USPS could continue to offer delivery services for all mail in all markets following liberalization without requiring a subsidy. We also explore the possibility that rates for non-commercial mail, and for commercial mail in markets that are unattractive to a potential entrant, would rise sharply. Calculations of consumer and producer surpluses are made to see if liberalization would reduce welfare. A welfare loss can occur because economies of scope and scale in delivery make one producer more efficient than two, and because the scope for efficient (Ramsey) pricing is restricted by liberalization. However, these losses may be offset by gains from the realignment of commercial mail rates to better reflect local delivery costs, from the diversification of commercial mail delivery services that occurs with entry, from the entry of a competitor who may be a lower-cost producer than USPS and from the profit that such a competitor would earn in an imperfectly competitive market.

## 2. How a Contestable Market Works

Our analysis employs a theory of equilibrium in a simple contestable market with an incumbent and one potential entrant. In later sections this theory is applied to U.S. postal markets that are separable geographically by post office. The incumbent is the USPS and the potential entrant is assumed to be a competitor offering a substitute delivery service for commercial mail in the local markets. USPS is always present in every market and offers delivery service according to an established tariff. The tariff may differ from market to market with respect to the price charged for the delivery of commercial mail. The potential entrant is present in or absent from each market according to a state probability of entry that depends upon his profit or loss on entry. Entering and exiting the market is assumed to be costless for the entrant. Among the models that have previously been proposed for analyzing liberalization, our model most closely resembles the pure bypass model of Gautier (2007).

The potential entrant in every local market may be a single firm, such as United Parcel Service (UPS), or may be a collection of smaller local delivery services, such as the deliverers of the morning newspapers. However, only one potential competitor to USPS is assumed to exist in any single contestable market. This assumption is justified by conditions that would undoubtedly characterize any market for delivery services following liberalization. First, the familiar economies of scope and scale in delivery will always present a strong incentive to multiple competitors to combine, or will make entry by more than one of them unattractive. Second, USPS is assumed to compete rather than collude with any potential entrant. In effect, USPS would be subject to all of the usual law and regulation that deters anticompetitive behavior in U.S. markets. In the language of game theory both side payments and cooperative strategies between USPS and any potential entrant would be prohibited.

The theory makes three basic assumptions about the behavior of the entrant and the incumbent in a contestable market. First, if the potential entrant enters the market, we assume that he is able to take the incumbent's price as given and set his own price to
maximize his profit. Thus the entrant's price is a function of the incumbent's price. This function is the entrant's reaction function. Second, we assume that the potential entrant's positions in or out of the market can be described by his state probability of entry. The state probability increases up to a limit of one if the entrant can make a profit from entry, and decreases down to a limit of zero if the entrant would take a loss. The incumbent is able to observe or estimate the state probability of entry and regards it as fixed. Third, the incumbent sets his own price to maximize his expected profit given the entrant's reaction function and the state probability of entry. The incumbent chooses his price without knowing if the potential entrant will be in or out of the market, and does not change his price in response to entry or exit by the potential entrant.

In Appendix A it is shown that a contestable market under these assumptions is a non-zero-sum two-person game with a Nash equilibrium. ${ }^{1}$ The Nash equilibrium takes one of three forms. These are:

- Monopoly - the entrant never enters, the state probability of entry is zero, and the incumbent acts as a monopolist. The incumbent's price is the monopoly price.
- Stackelberg Duopoly - the entrant never leaves, the state probability of entry is one, and the incumbent acts as the price leader. The incumbent sets a price that maximizes his profit in the expectation that the entrant will be present and will set a price according to his reaction function.
- Stochastic Equilibrium - the entrant is present in or absent from the market according to a state probability of entry that is greater than zero but less than one. The incumbent sets a price that leaves the entrant with a zero profit if he enters. The entrant's profit is also zero if he remains out. So the entrant is indifferent about being in or out of the market. The incumbent's price also maximizes his expected profit given the equilibrium state probability of entry. Stochastic Equilibrium is the result of price behavior by the incumbent that is usually associated with contestable market theory. However, contestable market theory generally overlooks the fact that the incumbent's pricing behavior leaves the potential entrant's status indeterminate. An economic profit of zero neither encourages nor discourages entry. Consequently, the potential entrant's profit or loss cannot tell us if he is in or out of the market. Unfortunately, many of the calculations that we would like to make of profits, net revenue, consumer surplus and welfare for a contestable market all depend on whether or not the potential entrant has chosen to enter.

Game theory provides the means to resolve the indeterminacy in the entrant's status. The entrant's equilibrium state probability of entry and the incumbent's contestable market price constitute an equilibrium pair of strategies. They are optimal against each other. Given the incumbent's price the entrant maximizes his profit whether he enters, leaves or executes a mixed strategy according to a state a probability of entry. On the other hand the incumbent's price only maximizes his expected profit when the state probability of entry assumes the value that leaves a Stochastic Equilibrium. This probability is unique. When the incumbent takes it as given and maximizes his profit he

[^1]finds that he has no incentive at the margin to change the price that deters the potential entrant.

The form that the Nash equilibrium takes for a contestable market depends upon the details of the demand and cost functions of the two participants. All three forms arise in different contexts in the cases we have developed for analysis in later sections of this paper. Furthermore, a monopoly can arise in two distinct ways. All of these forms are illustrated in the figures that follow. In these figures the incumbent's net revenue, defined as revenue minus variable cost, is plotted versus the incumbent's price under different assumptions regarding the entrant's behavior. The incumbent's profit only differs from his net revenue by a fixed amount, his fixed costs, because the incumbent is always present in the market. The blue curve in the figures describes the incumbent's net revenue when the entrant is in the market. The red curve describes net revenue when the entrant is out. The black curve follows the red line when the entrant's profit is negative (a loss) and follows the blue curve when his profit is positive. The black line drops vertically from the red to the blue curve at the incumbent price that leaves the entrant with no economic profit or loss. A star marks the equilibrium point in the figures.

Figure 1


Figure 1 depicts a "natural" monopoly. Equilibrium occurs at the point of maximum net revenue for the incumbent with the potential entrant out of the market. This is an equilibrium because the entrant suffers a loss if he is in the market at the incumbent's monopoly price. The potential entrant cannot earn a profit until the incumbent's price reaches the higher price where the black curve drops vertically from
the red to the blue curve. A natural monopoly typically results when the incumbent has a cost advantage over the potential entrant.

Figure 2 also depicts a monopoly. However, the reason for the monopoly is different from a natural monopoly so we have designated this equilibrium an "entrance" monopoly. An entrance monopoly occurs when the incumbent lacks the market power to drive his price up to the level where the potential entrant would earn a profit from entering. The blue curve shows what happens as the incumbent raises his price with the entrant in the market. When the blue net revenue curve reaches the horizontal axis, the incumbent has been driven entirely out of the market. This is the highest price (about 0.40 in Figure 2) that the incumbent can make effective. However, it is not high enough to produce a positive profit for the entrant because the black curve's vertical drop occurs at a price that is above the point where the blue curve crosses the horizontal axis. At this point the black curve is entirely hypothetical since the incumbent is effectively out of the market at a lower price. The incumbent's demand would have to be negative to stimulate entry at the price shown in Figure 2. So the entrant remains out of the market and the incumbent is able to set a monopoly price.

Figure 2


Figure 3 shows a Stackelberg Duopoly. The drop of the black curve from the red to the blue curve occurs at a price that is below the price that maximizes the incumbent's profit with the entrant in the market. Equilibrium occurs at the price that maximizes the incumbent's net revenue with the entrant in the market. This is the point labeled with a star in Figure 3. The lower price corresponding to the entry point of the potential entrant is not a Nash equilibrium because both the blue and red curves have positive slopes at
this point. Any state probability of entry between zero and one, when taken as given, would lead the incumbent to raise his price to reach the point designated as the Stackelberg Duopoly equilibrium.

Figure 3


Figure 4 displays a Stochastic Equilibrium. The incumbent's price is set to eliminate the entrant's profit leaving him with no incentive to either enter or leave the market. The incumbent's equilibrium price corresponds to the price where the black curve drops vertically. At this price the entrant's profit is zero if he chooses to be in the market. Since entry and exit are costless the entrant's profit is also zero if he chooses to be out of the market.

Any mixed strategy of entry and exit yields a zero profit for the entrant. Nevertheless, the contestable market depicted in Figure 4 is in equilibrium for only one state probability of entry. The Stochastic Equilibrium is found at the maximum of the dashed green curve that represents the incumbent's expected net revenue when he takes the state probability of entry as given. The dashed green line is the probability weighted average of the incumbent's net revenue functions with the potential entrant in and out of the market. When the state probability of entry is close to zero the dashed green curve approaches the red curve representing the incumbent's net revenue with the entrant out. At the other extreme a state probability of entry close to one produces a dashed green curve that lies close to the blue curve depicting net revenue with the entrant in the market. The equilibrium value for the state probability of entry is the probability that produces a dashed green curve with a maximum that occurs at the incumbent's equilibrium price.

Figure 4


The state probability of entry for the contestable market depicted in Figure 4 is 0.427 . This is the only value that leaves the market in equilibrium. To see why this is so we may consider what would happen within this market when the state probability is below and above 0.427 . A state probability less than 0.427 moves the dashed green curve up and to the right towards the red curve. The price that maximizes the incumbent's expected net revenue moves with the dashed green curve and is higher than the price that deters entry. When the incumbent raises his price to maximize his expected net revenue he sets off a chain of events that will eventually raise the state probability back to its equilibrium value. At the higher incumbent price the entrant is rewarded with a positive profit if he enters. He will enter more frequently and eventually just stay in the market at the higher price. This behavior raises the state probability of entry.

A state probability of entry greater than 0.427 moves the dashed green line downwards and to the left towards the blue curve. The incumbent's expected net revenue is maximized at a lower price. At the lower price the entrant takes a loss and responds by entering less frequently until he eventually just stays out of the market. This lowers the state probability of entry. Therefore, a state probability of entry higher than 0.427 also sets off a chain of reactions that moves the market back to the Stochastic Equilibrium.

The Monopoly and Stackelberg Duopoly forms of the Nash equilibrium conform to well-known economic models. The Stochastic Equilibrium form also conforms to a previously known model with respect to the determination of the entrant's and incumbent's prices - the contestable market model. However, the state probability of
entry and the role that we have described for it in bringing a contestable market to a Stochastic Equilibrium are new.

In our version of contestable market theory the price set by the incumbent and the state probability of entry are the signals that describe conditions in the market to its participants. The role of the incumbent's price is no different from that of a seller's announced price in any kind of market. In a contestable market the incumbent's price is taken as given not only by buyers, but also by the potential entrant. Our theory of contestable markets ascribes a similar role to the state probability of entry. It is taken as given by the incumbent when he sets his price. The result of this assumption, as described above and shown in more detail in Appendix A, is a classic but novel model of market equilibrium.

In its more conventional form, contestable market theory makes a far stronger, and much less realistic, assumption about the information available to the incumbent when he sets his price. In the conventional form of the theory the incumbent is able to calculate the price that yields a zero profit for the entrant. To do this the incumbent would have to know the potential entrant's demand function, his cost function and his reaction function. The only item on this list that an incumbent might actually be expected to learn from observing the market is the entrant's reaction function. The incumbent must also know the entrant's objective such as profit and be able to rely on the entrant to set his price to maximize the objective.

Even if the entrant and the incumbent produce very similar products under nearly identical conditions, it is difficult to see how an incumbent could have this much information about a potential entrant. In practice most firms are unable to predict their own sales and costs accurately, let alone those of a potential entrant who may not yet have appeared in the market. Estimating an entrant's demand and cost functions would be difficult, even after entry, because firms do not ordinarily share information regarding sales and costs with their competitors. Just how problematic the assumption of the conventional theory really is becomes clear when we realize that it is the entrant's own estimate of his objective, and not necessarily the entrant's profit itself, that must actually be discovered by the incumbent in order to successfully predict the entrant's point of entry. As a rule, this kind of prediction of individual behavior is impossible to make.

Most models of economic markets do not assume that participating firms have a need to project their competitors' objective in order to set their own prices. Instead, the pricing behavior of the firms in the market is derived from assumptions about the way they apply the information they might reasonably be expected to obtain from analyzing theirs own demands and costs, and observing the behavior of their customers and competitors in the market. In our view the information from a contestable market would be sufficient to permit an incumbent to estimate an entrant's reaction function after entry and his state probability of entry, but would not be sufficient to enable an incumbent to correctly predict when and where the entrances and exits of a potential competitor would actually occur.

A Stochastic Equilibrium is most easily envisioned for a market that is divided geographically or temporally into many identical segments with the same incumbent in each segment. The incumbent does not engage in price discrimination based upon entry because he does not respond to entry by changing his price. Therefore, the prices and state probability of entry for a Stochastic Equilibrium would be the same in every
segment. An entrant is either in or out at any place or time as the result of a random trial with a probability of "success" that is equal to the state probability of entry. The state probability is a frequency that may be observed more-or-less accurately, depending upon the number of segments, simply by calculating the proportion of the segments in which a potential entrant has actually entered.

The idea of statistically estimating the state probability of entry may be extended to a market with dissimilar segments caused by systematic variations over time or space in the cost and demand functions of the entrant and incumbent. Here, all three forms of equilibriums may occur in different segments. Nevertheless, the state probability of entry may be estimated econometrically as an hedonic function of exogenous descriptors of the market segments, for example, by fitting a logit or probit.

## 3. Calibrations for U.S. Delivery Markets

USPS has always held a monopoly on the delivery of most kinds of U.S. mail. The current exceptions are Express mail, Priority mail and single-piece Parcel Post. Very little of the mail in any of these competitive classes is comprised of commercial mail so our research required the creation of an hypothetical collection of local delivery markets for markets that do not currently exist. To keep the analysis simple we have also aggregated the mail stream into three broad categories consisting of non-commercial mail, commercial mail and packages for USPS; and one class of commercial mail for potential entrants. We assume that USPS retains its monopoly of the delivery of noncommercial mail consisting of all categories of First-Class Letters and Cards, Penalty mail, Free mail and all International mail. All categories of Periodicals and Standard Mail, as well as Bound Printed Matter and Media mail are combined as commercial mail. In our analysis it is assumed that these are the existing categories of mail for which a potential entrant would compete. Priority mail and Parcel Post are aggregated into the single category of packages.

Demand and cost functions for the incumbent and the potential entrant were calibrated to USPS data using two recent econometric studies of USPS mail volumes and detailed regulatory cost data for FY 2005. Most of this information is taken from the testimony, work papers and library references of USPS witnesses and the work papers and library references supporting the PRC's Recommended Decision for the R2006-1 omnibus rate case. We can provide here only an outline of the essential data, assumptions and mathematics of our calibration method. To examine the details readers are directed, first, to Appendix B for a complete and annotated example of our calculations for a single postal market, and, second, to Appendix C for a roadmap to the pair of Lotus 1-2-3 worksheets we constructed to make the calculations. These worksheets reproduce all of the data and estimates that have been extracted from other sources. They are available on request from the authors.

The U.S. was divided into postal markets on the basis of quarterly USPS records for 368 USPS mail processing plants over a seven-year period from the start of FY 1999 to the end of FY $2005^{2}$. These records were compiled by USPS in Library References and in responses to interrogatories during the R2006-1 postal rate proceeding. The reported statistics for the processing plants included delivery volumes by mail category,

[^2]numbers of delivery points by type of delivery, and the numbers of offices and stations. Up to four representative markets were formed for each processing plant. These markets correspond to offices and stations serving high density city carrier delivery points, low density city carrier delivery points (including most suburban deliveries), rural carrier delivery points including highway contracts, and post office boxes.

Delivery mail through the plants was aggregated into delivery streams for noncommercial mail, commercial mail and packages using USPS unit delivery cost data for FY 2005. The method was to weight the pieces for each category of mail using weights based upon their average system-wide USPS unit delivery cost. The weights were normalized to yield the number of pieces for each of the three aggregated categories as reported in the USPS Revenue, Pieces and Weights Report (RPW) for FY 2005. The weighted volumes in each mail category consist of piece equivalents that are all of about equal average cost to deliver.

The delivery points served by each processing plant were aggregated into highdensity city carrier delivery points, low density city carrier-delivery points and rural carrier delivery points. PO box deliveries were allocated to these categories proportionately. The averages of the volume streams and delivery point counts for all years were used to regress volumes on delivery points without an intercept. The estimated coefficients were then used to apportion volumes to the high density city carrier, low density city carrier and rural carrier delivery points. Volumes were apportioned to PO boxes based upon the ratio of PO box delivery points to the total. The apportionments were adjusted so that the average volume for the aggregate categories of mail matched the totals for the processing plant.

The offices and stations served by a processing center are each assumed to serve only one kind of delivery point. While this is clearly not the case in practice, the data for the processing plants affords no alternative to this assumption. In any case most USPS offices and stations tend to have delivery points that concentrate heavily in only one of the categories that we have defined. The offices and stations served by each processing plant are allocated by volume to each type of delivery point. Volumes and delivery points are divided by the number of offices and stations to obtain the average volumes and number of delivery points for the offices. These offices are treated as geographically distinct contestable markets. Each of these representative markets has delivery points of only one kind and serves as the proxy for a number of identical markets within the region served by the mail processing plant.

The demand functions for mail delivered to each representative market are calibrated to pieces adjusted to FY 2005 RPW totals and to volume elasticities derived from two econometric studies. These are a study done by one of the authors (Pearsall 2005) for an analysis of the effects of unbundling, other service innovations and the 9-11 attack on postal volumes; and a study by Thress (2006) to forecast volumes for the R2006-1 postal rate proceeding. Elasticities for subclasses and worksharing categories of mail were extracted from each of these studies and combined to obtain elasticities for non-commercial mail, commercial mail and packages. The elasticity estimates were combined by treating the volume proportions exhibited by the RPW data in FY 2005 as fixed.

Four sets of own-price elasticities were derived from the studies. These are: 1) Thress' elasticities with his cross-price elasticities treated as zero, 2) Thress' elasticities
adjusted by adding his cross-price elasticities to his own-price elasticities, 3) Pearsall's elasticities corresponding to fixed-weight index (FWI) prices, and 4) Pearsall's elasticities when average revenues per piece are used as prices. The elasticity estimates 2,3 and 4 are quite similar. The own-price elasticities in estimate 1 are somewhat higher absolutely that the others. The elasticities (2) obtained by adjusting the Thress estimates used during the R2006-1 rate proceeding are shown in the following table.

| R2006-1 Elasticity Matrix Adjusted |  | Non-Comm. | Commercial | Packages |
| :---: | :---: | :---: | :---: | :---: |
| Non-Commercial |  | -0.2032 | 0.0000 | 0.0000 |
| Commercial |  | 0.0000 | -0.3876 | 0.0000 |
| Packages | 0.0000 | 0.0000 | -0.9214 |  |

Linear demand functions were calibrated to the estimated elasticities, FY 2005 FWI prices and allocated average annual volumes from FY 1999 to FY 2005 for the representative postal markets. The idea here was to calibrate the demand functions to generally represent demand conditions from FY 1999 to FY 2005. The postal prices in place in FY 2005 were mostly installed in FY 2002. The calibrations were performed along the lines of a scheme found in DeDonder et al (2007) and (2008). An example for a single market is shown in Appendix B.

Calibrations were made using just the demand elasticities, volumes and prices for non-commercial mail, commercial mail with the entrant out of the market, and packages. The calibrations were made by linearizing demand functions of the form $X=K P^{e}$, where $e$ is the elasticity of demand, $X$ is volume and $P$ is the price at the point where the model is calibrated. $K$ is a scale parameter that is determined by solving the demand function for $K$ at the calibration point.

Two additional parameters were defined to calibrate the demand functions for commercial mail for the incumbent and the entrant when the potential entrant is in the market. These additional parameters are the displacement ratio, $s$, defined as the share of the volume gained or lost that produces a symmetric loss or gain by the competitor, and the entrant's market share, $w$, when he enters the market at the same price charged by USPS. Slutsky-Schultz symmetry is also assumed for the calibration.

Commercial mail for the incumbent and entrant are designated as products " 2 " and " 3 ". The demand equations with volumes displaced symmetrically are $X_{2}=K_{2} P_{2}^{e_{2}}-s X_{3}$ and $X_{3}=K_{3} P_{3}^{e_{3}}-s X_{2}$. The direct price elasticities for the two products are $e_{2}$ and $e_{3}$, volumes are $X_{2}$ and $X_{3}$, the prices are $P_{2}$ and $P_{3}$, and $K_{2}$ and $K_{3}$ are the scale parameters. The demand functions are obtained by solving these equations for $X_{2}$ and $X_{3}$ as follows: $X_{2}=\left\lfloor K_{2} P_{2}^{e_{2}}-s K_{3} P_{3}^{e_{3}}\right\rfloor /\left(1-s^{2}\right)$ and $X_{2}=\left[-s K_{2} P_{2}^{e_{2}}+K_{3} P_{3}^{e_{3}}\right] /\left(1-s^{2}\right)$.

The demand functions cannot yet be calibrated because they include five elements with unknown values. These unknowns are $X_{3}, P_{3}, e_{3}, K_{2}$ and $K_{3}$. Therefore, three additional relationships must be found to calibrate the demand model when the entrant is present in the market.

The first of the added relationships is obtained by calibrating the demand model for values that arise when the entrant appears with the same price as the incumbent. The first added relationship is just $P_{3}=P_{2}$.

The second of the added relationships is the Slutsky-Schultz symmetry condition $\partial X_{2} / \partial P_{3}=\partial X_{3} / \partial P_{2}$. Slutsky-Schultz symmetry is assumed to hold if the entrant enters the market at the same price as the incumbent. For the demand functions of our model the Slutsky-Schultz symmetry condition is $\frac{-e_{3} s K_{3} P_{3}^{e_{3}}}{\left(1-s^{2}\right) P_{3}}=\frac{-e_{2} s K_{2} P_{2}^{e_{2}}}{\left(1-s^{2}\right) P_{2}}$. At $P_{3}=P_{2}$ this simplifies to $K_{3}=\left(e_{2} / e_{3}\right) K_{2} P_{2}^{e_{2}-e_{3}}$.

The third added relationship is derived from the entrant's market share which is assumed to be known if the entrant enters at the same price as the incumbent. This assumption leads directly to $w=\frac{X_{3}}{X_{2}+X_{3}}=\frac{e_{2}-s e_{3}}{(1-s)\left(e_{2}+e_{3}\right)}$.

The two demand functions and the three added relationships allow us to calibrate the demand model using just the observed values for $X_{2}$ and $P_{2}$, and the estimate of $e_{2}$ as shown in Appendix B.

The cost functions used in our work are linear functions of pieces, delivery points and offices. The cost accounting system used by the PRC for rate decisions divides USPS costs into "attributable" costs and "institutional" costs. For most practical purposes the PRC's attributable costs per piece can be regarded as marginal costs for postal volumes that do not differ greatly from volumes during the year the cost data were collected. In the PRC's cost system (and also USPS') costs are built up from postal data broadly by segments and in more detail by components within the segments. The attributable costs within each segment and component are further decomposed according to the classes and subclasses of mail and special services.

Our USPS cost equations are simply the mundane result of extracting and categorizing cost elements from the PRC's cost accounting for FY 2005 which served as the base year for the R2006-1 rate proceeding. The data that was lifted for this purpose from the PRC's cost system is reproduced in one of the worksheets described in Appendix C.

In the table below the unit variable delivery costs were obtained by summing all of the PRC's attributable costs for city carriers, rural carriers and PO boxes that are identifiable as delivery costs. These attributable costs were also summed over the subclasses included in our definitions of non-commercial mail, commercial mail and packages. The summed attributable costs were then divided by pieces from the FY 2005 RPW to obtain the unit variable delivery costs shown in the table.
$\left.\begin{array}{lcccc} & \begin{array}{c}\text { Unit Variable Delivery Cost } \\ \\ \text { City Carrier }\end{array} & \text { Rural Carrier } & & \text { UO Box }\end{array}\right)$

Unit variable non-delivery costs were found by summing attributable costs that are not identifiable as delivery costs. These are mostly costs of processing and transporting the mail.

The fixed delivery point costs shown in the table are the institutional costs identifiable as delivery costs from the PRC's cost system. We assumed that these costs would vary directly with the numbers of the various kinds of delivery points served by city and rural carriers, and by the number of PO boxes that existed in FY 2005. The fixed delivery costs were estimated by dividing the institutional delivery costs by delivery points.

Finally, the PRC's cost system includes institutional costs that are not identifiably associated with deliveries. These costs are the remainder of all USPS costs that are not accounted for as variable costs or fixed delivery costs. These costs have been divided by the total number of offices and stations in FY 2005 to get the non-delivery fixed cost for offices in the table.

The cost function for a potential entrant is calibrated by applying a set of proportions to the USPS cost function for commercial mail. Proportions are individually defined for unit variable delivery costs, unit variable non-delivery costs, fixed delivery point costs and non-delivery fixed (office) costs. The entrant's cost function for deliveries to post office boxes is assumed to be the same as his cost function for high density city delivery points.

## 4. Numerical Investigations

Our research strategy was to use the worksheets described in Appendix C to construct cases that could be directly compared to FY 2005. For this reason the cases that are described and analyzed in the following sections all have identical fiscal consequences for USPS. They all leave USPS with a fiscal deficit of about 3,396,000 (\$thousands) from delivering mail. This deficit occurs primarily because revenues from Express Mail, Special Services and some government use of the mail (which is paid for by appropriations) are not included in the revenues that we compute by applying FY 2005 rates to volumes. When these revenues are included and other USPS revenues and costs are added, USPS actually had a small but positive net revenue in FY 2005. However, to reproduce the outcome of FY 2005 with our model we must reproduce the deficit that occurs when the demand and cost models are applied with the prices used to calibrate the demand functions without entry.

To produce the cases we assumed Ramsey pricing for non-commercial mail and packages. Under Ramsey pricing the USPS sets the prices for non-commercial mail and packages to maximize welfare, defined as the total of consumer and producer surplus, subject to a constraint on USPS net revenue. When all products and services are included in the maximization, the customary level for net revenue for a Ramsey pricing exercise is zero. But with the omissions noted above, Ramsey pricing will leave cases that are directly comparable to the calibration case if the net revenue that is left to USPS from mail deliveries is minus $3,396,000$ ( $\$$ thousands).

Our demand model has no cross elasticities that involve either non-commercial mail or packages, so the Ramsey prices for these categories can be computed according to the well-known inverse-elasticity rule. Let product " 1 " be non-commercial mail and
product "4" be packages. The inverse elasticity rule for product 1 is $\frac{P_{1}-M_{1}}{P_{1}}=\frac{K_{c}}{e_{1}}$ and for product $4 \frac{P_{4}-M_{4}}{P_{4}}=\frac{K_{c}}{e_{4}} . P, M$ and $e$ are the price, marginal cost and demand elasticity for the products (subscripts omitted). $K_{c}$ is a Lagrange multiplier associated with the net revenue constraint. Ramsey prices for cases with different net revenues can be generated by changing the value of $K_{c}$ used in the inverse elasticity rule. In all of our cases values for $K_{c}$ were chosen to leave a net revenue of minus 3,396,000.

The adopted model of market behavior has been used to explore the effects of liberalization on several indicators of postal performance. The discussion of the numerical results below is organized according to a set of hypotheses about how liberalization might affect USPS behavior and performance. These deal with 1. whether a Graveyard Spiral might result from liberalization, 2. whether liberalization may constrain the market power of the universal service provider, 3 . what types of conditions might be inferred to be necessary to facilitate entry, 4. how may the entrant's differentiation of his product affect pricing discipline and consumer welfare, and 5 . what appears to be the overall affects of liberalization on welfare.

To obtain insights into these issues, computations were carried out for five different "cases" each of which represent different relative costs of supply between the incumbent and the entrant and different demand entry conditions. These are shown in the following table.

| Case 1: | Base Case <br> Entrant with no cost advantage. <br> Identical product. |
| :--- | :--- |
| Case 2: | Cost Advantage <br> Entrant with cost advantage suggested by City Mail experience. <br> Identical product. |
| Case 3: | Cost Advantage and Product Differentiation <br> Entrant with cost advantage suggested by City Mail experience. <br> Case 4: <br> Product differentiation compatible with cost reductions. <br> Cost Advantage and Higher Product Differentiation <br> Entrant with cost advantage suggested by City Mail experience. <br> Dissimilar product. <br> Large Cost Advantage and Higher Product Differentiation <br> Entrant with large cost advantage. <br> Dissimilar product. |

## 5. The Graveyard Spiral

Model computations give no hint that the regulatory authorities should be concerned that liberalization will lead to a more difficult financial situation for the incumbent. In fact, they suggest that by setting its rates as Ramsey prices within the given demand and entry conditions, it is always feasible for the incumbent's net revenues to satisfy its regulatory constraint. This occurs for the contestable market equilibrium found in every case.

While the net revenue totals for the different case analyses are virtually equal, the same is not true across the different market types within the different cases. That is, given the different demand conditions across the markets, the incumbent picks up different amounts of its revenue from the different market types across the cases. This "flexibility" is shown clearly in the table below, which shows that PO boxes earn a striking positive net revenue while net revenues decrease in the other market types as density decreases.

|  |  | Market Type |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | High Density | Low Density | $\underline{\text { Rural }}$ | $\underline{\text { PO Boxes }}$ | Total |
| Case 1 | $(843,593)$ | $(795,154)$ | $(3,309,535)$ | $1,552,267$ | $(3,396,016)$ |
| Case 2 | $(304,389)$ | $(2,531,768)$ | $(2,238,457)$ | $1,678,603$ | $(3,396,010)$ |
| Case 3 | $(459,068)$ | $(1,950,860)$ | $(2,712,449)$ | $1,726,364$ | $(3,396,013)$ |
| Case 4 | $(697,235)$ | $(1,146,175)$ | $(3,168,251)$ | $1,615,644$ | $(3,396,017)$ |
| Case 5 | $(708,777)$ | $(1,001,776)$ | $(3,234,916)$ | $1,549,469$ | $(3,396,000)$ |

## 6. Market Power of the Incumbent

A principal indicator of whether the incumbent retains market power is the extent to which "price discipline" is exercised over it by the presence of actual or potential entrants. The model computations indicate that any pricing discipline that might result from Contestable Market equilibriums depends upon a complex of factors, including the extent of entry, the entrants' costs of supplying the delivery of Product 3, the extent to which Product 2 and Product 3 are differentiated, and the resultant cross-elasticities of demand.

For Case 1, where the entrant's Product 3 and costs are the same as the incumbent's, the results indicate that, except for one token, all of the equilibriums are of monopoly form. The average price for Product 1 is .3113 , for Product 2, .4700, and for Product 4, 4.0809. The bottom rows of the table show that the average for Product 2 consists of individual equilibriums ranging interestingly from .3160 to .4751 . Corresponding volumes are 108.0 million for Product 1, 48.6 million for Product 2, and 1.6 million for Product 4.

|  | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Equilbrium Frequencies | Total | Total | Total | Total | Total |
| Natural Monopolies | 782 | 8 | 326 | 1067 | 394 |
| Stackelberg Duopolies | 0 | 135 | 175 | 150 | 823 |
| Stochastic Equilibria | 1 | 453 | 700 | 44 | 44 |
| Entrance Monopolies | 478 | 665 | 60 | 0 | 0 |
| Total Equilibria | 1261 | 1261 | 1261 | 1261 | 1261 |
| Expected Equilibrium Volume | Total | Total | Total | Total | Total |
| Product 1 | 108,047,711 | 102,945,449 | 104,605,354 | 107,532,247 | 106,644,189 |
| Product 2 | 48,650,695 | 32,767,317 | 41,379,905 | 47,236,247 | 44,749,475 |
| Product 3 | 32,824 | 49,820,229 | 33,921,727 | 12,007,612 | 32,188,217 |
| Product 4 | 1,609,534 | 1,562,751 | 1,575,680 | 1,603,720 | 1,594,380 |
| Expected Equilibrium Price | Total | Total | Total | Total | Total |
| Product 1 | 0.3113 | 0.4090 | 0.3772 | 0.3211 | 0.3382 |
| Product 2 Weighted Average | 0.4700 | 0.3158 | 0.3593 | 0.4569 | 0.4336 |
| Product 3 Weighted Average | 0.2650 | 0.1861 | 0.2364 | 0.2864 | 0.2647 |
| Product 4 Price | 4.0809 | 4.2467 | 4.2009 | 4.1015 | 4.1346 |
| Equilibrium Price Range | Total | Total | Total | Total | Total |
| Product 2 High | 0.4751 | 0.4751 | 0.4751 | 0.4751 | 0.4751 |
| Product 2 Low | 0.3160 | 0.1886 | 0.2853 | 0.4108 | 0.4077 |
| Product 3 High | 0.2650 | 0.2039 | 0.2848 | 0.2921 | 0.2722 |
| Product 3 Low | 0.2650 | 0.1689 | 0.2190 | 0.2778 | 0.2574 |

In Case 2 the entrant's costs are lower but he is assumed again to supply Product 3 which is identical to the incumbent's Product 2. In this case, the Stackelberg and Stochastic Equilibriums indicate that entry would occur in several Product 2 and 3 markets. The incumbent's average price for Product 2 is .3158 , (down from .4700 in Case 1) whereas the entrant's price for Product 3 is .1861 . It should be pointed out, however, that the incumbent's price for Product 2 in individual markets ranges from a low of .1886 to a high of .4751 . The quantity results are more striking. The incumbent's volume of Product 2 is now 32.8 million while the entrants' volume of Product 3 is 49.8 million for a total for both of 82.6 million. In its markets for Products 1 and 4, the incumbent follows constrained Ramsey pricing with the results that both have higher prices and slightly lower volumes than in Case 1.

In Case 3, the entrant retains the same cost advantage over the incumbent, however, the component cost advantages are related to the differentiation of his Product 3 from the incumbent's Product 2. Again, the extent of the Stackelberg and Stochastic Equilibriums indicates entry in the Product 2-Product 3 markets. However, the incumbent's average price for Product 2 increases to .3593 and the entrants' Product 3 average price increases to .2364 . Moreover, the incumbent's volume of Product 2 increases to 41.4 million while the entrants' volume of this Product 3 declines to 33.9 million along with total volume of these which falls to 75.3 million.

Case 4 involves the entrant having the same cost advantages as he had in Case 3 but in this instance, his product is much more highly differentiated from that of the incumbent. The latter results in a much lower cross elasticity of demand between Products 2 and 3. Stackelberg Duopolies and Stochastic Equilibriums result but not nearly to the extent as in Cases 2 and 3. The resulting average prices of both Product 2 and Product 3 are higher but those of incumbent's Products 1 and 4 are lower in this instance. Note that the ranges of the prices of both Products 2 and 3 are much narrower
than in the previously discussed cases. Product differentiation appears to result in higher margins for the entrants but they lose a substantial volume in the process. The incumbent's volume of Product 2 rises to 47.2 million while the volume of Product 3 falls quite substantially to 12.0 million.

In Case 5, the entrant continues the same product differentiation strategy as in Case 4 , but in this example, his costs are substantially lower than those of the incumbent. These lower costs possibly stem from mailers taking on more sorting or other nondelivery tasks and from employing lower cost processes or resources in the fixed route and office activities. These factors lead to a much larger number of Stackelberg Duopolies emerging in the total equilibriums picture. The average price of Product 2 decreases slightly to .4336 while the average price for entrants' Product 3 also decreases slightly to .2647 from their levels in Case 4 . Entrant volume of Product 3 increases to 32.2 million while Product 2 volume falls slightly to 44.8 million.

These results do not point uniformly to contestable markets limiting the market power of the incumbent. In fact, it is clear that the "forces" engendered by entrants having lower costs will effectively be offset by the same entrants following a strategy to establish niches for themselves through sharper differentiation of their commercial mail delivery product from that of the incumbent.

## 7. Entrant Costs, Product Differentiation, and Entry

If one focuses strictly on promoting the entry of suppliers of commercial mail Product 3, it would appear that the entrant's having significantly lower costs than the incumbent is strongly supportive, if not necessary, for substantial entry to take place. The entrants' volume increase in Case 2 over Case 1 and in Case 5 over Case 4 illustrate this most clearly. In both cases, the average equilibrium price of Product 2 declines but by more in Case 2 over Case 1.

Differentiation of the entrant's product from that of the incumbent's also appears to promote entry, as can be seen from the numbers of Stackelberg Duopolies and Stochastic Equilibriums in Cases 2 and 3. However, that is somewhat countered by the apparent entries in Case 4 relative to Case 3. The principal difference between those two cases is the greater extent of product differentiation in Case 4. This may well indicate that it is possible to have too much product differentiation from this perspective. And, in fact, the average prices of both Products 2 and 3 increase with this greater differentiation. However, there is more to say about the differentiation issue below.

## 8. Consumer and Producer Surplus

The model computations included calculations of the welfare levels, as the sum of consumer and producer surpluses, that result from the different liberalization conditions of the various cases explored.

|  | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Total Consumer and Producer Surplu: | Total | Total | Total | Total | Total |
| Contestable Market Equilibrium | 120,612,583 | 125,325,773 | 123,270,580 | 121,743,110 | 126,267,946 |
| Contestable Market Consumer Surplu | Total | Total | Total | Total | Total |
| Product 1 | 111,794,865 | 101,485,738 | 104,784,859 | 110,730,728 | 108,909,335 |
| Products 2 \& 3 | 7,624,382 | 22,526,445 | 17,005,709 | 9,486,162 | 12,944,504 |
| Product 4 | 4,589,353 | 4,326,441 | 4,398,323 | 4,556,261 | 4,503,340 |
| Total Consumer Surplus | 124,008,599 | 128,338,625 | 126,188,892 | 124,773,151 | 126,357,180 |
| Contestable Market Producer Surplus | Total | Total | Total | Total | Total |
| Incumbent's Net Revenue | $(3,396,016)$ | $(3,396,010)$ | $(3,396,013)$ | $(3,396,017)$ | $(3,396,000)$ |
| Entrant's Profit | 0 | 383,159 | 477,702 | 365,976 | 3,306,766 |
| Total Producer Surplus | $(3,396,016)$ | $(3,012,851)$ | $(2,918,311)$ | $(3,030,041)$ | $(89,233)$ |
| Total Consumer and Producer Surplus |  |  |  |  |  |
| Benchmarks |  |  | Total |  |  |
| Calibration Prices |  |  | 127,246,223 |  |  |
| Product 2 Price Uniform |  |  | 127,269,251 |  |  |
| Product 2 Price Variable |  |  | 127,288,295 |  |  |

The calculations indicate that liberalization is not single dimensional in which its welfare effects can be readily attributable. This, of course, has been anticipated in the results discussed above.

In terms of total consumer and producer surplus (the first row in the table), it is clear that entry is associated with an increase in welfare, as shown in Cases 2 and 5. It is not so clear that the degree of product differentiation contributes substantially to total consumer and producer surplus. This question arises from the decreases in total surplus experienced from Case 2 to Case 3 and from Case 3 to Case 4 where the principal underlying differences are in the extents to which entrant Product 3 is differentiated from incumbent Product 2.

Lower entrant costs, whether his Product 3 is identical to the incumbent's commercial mail product or differentiated from it, are associated most directly with total welfare gains. This can be seen both in the instances of surplus increases from Case 1 to Case 2 and from Case 4 to Case 5 where the principal differences in the underlying market conditions are the lower costs of the entrant.

The changes in the consumer and producer components of the total surplus generated under the different liberalization conditions are quite informative. The largest increase in consumer surplus occurs between Cases 1 and 2, where the entrant and incumbent commercial mail products are identical but the entrant has significantly lower costs. This is brought about by the tripling of the consumer surplus generated by the increase in total commercial mail volumes. ${ }^{3}$ But note should be taken that this increase in consumer surplus is a tradeoff with the combination of (1) a nearly ten percent decrease in the consumer surplus derived from incumbent's Product 1 and (2) about a five percent loss in the consumer surplus generated by incumbent Product 4. The gain in producer surplus is solely due to the profits earned by the entrants.

[^3]Through Cases 3 and 4, where the principal differences are in the extents of the differentiation between the incumbent and entrant products, the consumer surplus generated by commercial mail decreases quite substantially. However, the consumer surplus generated by incumbent's Products 1 and 4 somewhat but not sufficiently offset that decrease. While entrants earn profits in Cases 3 and 4, total producer surplus does not change substantially.

Case 5, which differs from Case 4 primarily by the much lower costs the entrant must incur, generates, by a thin margin over Case 2 , the greatest aggregate consumer and producer surplus of all the cases explored. But as can be seen in the separate consumer and producer components of this surplus, it involves a significant transfer of benefits from consumers to producers in the form of entrant profits.

Finally, and most importantly, the last three lines of the table present the most problematic indicators of the benefits that might be gained from liberalization's relying upon contestable markets. These lines show measures of the total consumer and producer surplus generated by prices that result from non-contestable market conditions. The first of these "benchmarks" is based upon the rates that were in effect during FY 2005. The second benchmark is generated by assuming that the Postal Service is constrained to set the rate for its commercial mail product at one uniform Ramsey price for all markets. The third benchmark reflects the total consumer and producer surplus that would be generated if the Postal Service were permitted to set Ramsey prices for its product 2 that could be different in the various markets. That is, the last benchmark lifts the restriction that the rate must be the same in all markets.

As can be seen, the benchmark aggregate surplus does increase as USPS might apply Ramsey pricing with greater flexibility; but the increases in welfare benefits are quite small. However, the benchmark aggregate consumer and producer surpluses are all greater than the aggregate consumer and producer surpluses generated in the contestable market equilibriums shown for the entire range of cases explored in this study!

## 9. Conclusion

Our results show generally that USPS would retain a great deal of market power following a liberalization of the delivery of commercial mail. This fact underlies all of our more specific findings.

Following liberalization local markets that are small and less densely populated tend to remain USPS monopolies. The price for commercial mail delivery is always well above the Ramsey price wherever USPS can ignore the threat of a potential entrant. In the markets that become Stackelberg Duopolies, USPS' competition remains imperfect, however, the USPS price in these markets is much closer to the Ramsey price without a contestable market. In the markets where Stochastic Equilibriums occur, the contestable market price occurs in a range between the Monopoly price and the Stackelberg price. However, in many of these markets the entrant is deterred at a USPS price that is not much below the monopoly price. Overall our cases suggest that contestable markets are a fairly weak form of competition for an incumbent post..

In every case we have examined, USPS is left with more than enough market power to secure its finances. In fact, USPS enjoys so much market power that the rates for non-commercial mail and packages are often lower than they would be without
liberalization while the USPS average price for commercial mail soars. This kind of outcome would invite a regulator to place a cap on the USPS price for delivering commercial mail.

Our numerical results respond to changes in the assumed demand and cost functions as expected for imperfect markets. When potential entrants enjoy a greater cost advantage over USPS they enter more markets and at a lower price. This, in turn, drives the average USPS price for commercial mail down but still leaves USPS with a decreased share of the total volume. As the entrants differentiate their product offerings from USPS' commercial mail service, the entrants' competition has less effect on the incumbent's price and volume.

Finally, our results show that liberalization of commercial mail delivery in the U.S. would cause a welfare loss. Ramsey prices, even when allowed to vary locally for commercial mail, produce a gain in consumer and producer surplus of only about $\$ 42$ million over the welfare that results from the FY 2005 prices. In all of our cases this small gain is completely overtaken by the welfare losses that accompany the abandonment of Ramsey pricing for commercial mail and the loss of economies of scope and scale that occurs with the entry of postal competitors. Among the cases, the total welfare losses range from $\$ 978$ million (Case 5) up to $\$ 6,664$ million (Case 1). These losses arise even though we have assumed, for all but Case 1, that potential entrants would enjoy a cost advantage over USPS and would offer a delivery service that is an imperfect substitute. The losses are smaller when the entrant enjoys a greater cost advantage largely because of the higher profits earned by the entrant.

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## Appendix A: Equilibrium in a Contestable Market

## 1. Introduction

In this Appendix we describe mathematically how equilibrium occurs in a simple contestable market with an incumbent and one potential entrant, both producing single more-or-less substitutable products when entry and exit are costless. Under these conditions a contestable market is a non-zero-sum non-cooperative two-person game for which it has long been known that there exists a Nash equilibrium ${ }^{4}$. The Nash equilibrium is unique under the assumptions that are commonly made for a duopoly's demand and cost functions.

The Nash equilibrium corresponds to one of three market models. These are: 1) Monopoly - the potential entrant stays out of the market; 2) Stackelberg Duopoly - the incumbent is the price leader and the potential entrant always enters; and 3) Stochastic Equilibrium - the equilibrium consists of a price for the incumbent that leaves the entrant with a zero profit, and a mixed strategy of entry and exit for the potential entrant according to a state probability of entry. When the incumbent takes the equilibrium state probability of entry as a fixed characteristic of the contestable market, he has no incentive to change his price.

## 2. Stochastic Entry and Exit

Stochastic entry and exit may be described formally by introducing a variable $\mu$ in the range $[0,1]$ for the entrant's state probability of entry. When $\mu=1$, the entrant is in the market; when $\mu=0$, he is out of the market; and, when $0<\mu<1$, the probability that the entrant is in the market is $\mu$, and the probability that he is out is $1-\mu$.

The state probability of entry may be interpreted as either a statistical frequency, a subjective probability, or, the probability for a mixed strategy of entry and exit for a twoperson game. If a contestable market is segmented over space or time, then $\mu$ is the frequency that determines the proportion of the segments where the incumbent finds the entrant as an actual competitor. If entry or exit occurs infrequently or only once, as might be the case following the deregulation of a monopoly, then $\mu$ is the subjective probability that the incumbent uses to calculate his expected profit and set his price as the deregulation occurs. Finally, the state probability may be viewed as an entrant's mixed strategy for a non-zero-sum non-cooperative two-person game in which the entrant enjoys the advantage of the last move ${ }^{5}$. To preserve this advantage the entrant randomizes his entries and exits using $\mu$ over repeated plays of the game.

[^4]The state probability of entry $\mu$ is used to state the assumptions of the contestable market model as follows.

A1: If the entrant chooses to enter the market or to remain in, he takes the incumbent's price, $P_{I}$, as given and sets his own price, $P_{E}$, to maximize his profit function, $f_{E}\left(P_{E} \mid P_{I}\right)$. Therefore, the entrant sets his price according to his reaction function, $P_{E}\left(P_{I}\right)=\operatorname{ArgMax}\left\{f_{E}\left(P_{E} \mid P_{I}\right)\right\}$.
A2: The state probability of entry increases up to a limit of $\mu=1$ if the entrant's profit under A1 is positive; decreases down to a limit of $\mu=0$ if the entrant's profit under A1 is negative; and, remains unchanged in the range $[0,1]$ if the entrant's profit under A1 is zero.
A3: The incumbent sets his price to maximize his expected profit ${ }^{6}$ given the entrant's reaction function, $P_{E}\left(P_{I}\right)$, and the state probability of entry, $\mu$, and he does not change his price in response to the entrant's actions. Therefore, the incumbent chooses $P_{I}=\operatorname{ArgMax}\left\{\mu f_{I}\left(P_{I} \mid P_{E}\left(P_{I}\right)\right)_{\text {in }}+(1-\mu) f_{I}\left(P_{I}\right)_{\text {out }}\right\}$.

The entrant's entries and exits are stochastic and the state probability of entry and not the state itself remains unchanged if the entrant's profit is zero. Also, the entrant's and the incumbent's proximate objective is to maximize expected profit. This is the usual objective attributed to a firm by conventional microeconomic theory. However, the mathematics is essentially unaltered for other objective functions so long as they are convex functions of the incumbent's and entrant's prices.

The incumbent calculates his expected profit based upon an estimate of the state probability of entry. This estimate of $\mu$ may be derived from any information that the incumbent might reasonably be expected to collect by observing the market and/or the potential entrant's behavior. The incumbent regards $\mu$ as a fixed value describing a characteristic of the market that will be unaffected by his own choice of a price.

## 3. Equilibrium in a Contestable Market

Three kinds of equilibrium are possible under A1-A3. These possibilities correspond to the sub-ranges $\mu=0, \mu=1$ and $\mu=[0,1]$ for the state probability of entry.

E1: Monopoly $P_{E}\left(P_{I}\right)=\operatorname{ArgMax}\left\{f_{E}\left(P_{E} \mid P_{I}\right)\right\}, f_{E}\left(P_{E}\left(P_{I}\right) \mid P_{I}\right)<0$ and $P_{I}=\operatorname{ArgMax}\left\{f_{I}\left(P_{I}\right)_{\text {out }}\right\} . \mu=0$, the entrant is out of the market.

[^5]E2: Stackelberg Duopoly $P_{E}\left(P_{I}\right)=\operatorname{ArgMax}\left\{f_{E}\left(P_{E} \mid P_{I}\right)\right\}, f_{E}\left(P_{E}\left(P_{I}\right) \mid P_{I}\right)>0$ and $P_{I}=\operatorname{ArgMax}\left\{f_{I}\left(P_{I} \mid P_{E}\left(P_{I}\right)\right)_{i n}\right\} . \mu=1$, the entrant is in the market.
E3: Stochastic Equilibrium $P_{E}\left(P_{I}\right)=\operatorname{ArgMax}\left(f_{E}\left(P_{E} \mid P_{I}\right)\right), f_{E}\left(P_{E}\left(P_{I}\right) \mid P_{I}\right)=0$ and $P_{I}=\operatorname{ArgMax}\left\{\mu f_{I}\left(P_{I} \mid P_{E}\left(P_{I}\right)\right)_{\text {in }}+(1-\mu) f_{I}\left(P_{I}\right)_{\text {out }}\right\} . \mu=[0,1]$, the entrant enters, exits and remains in-or-out of the market stochastically according to the state probability of entry $\mu$ which remains constant.

The Monopoly E1 results when the potential entrant does not make a profit from entering the market even when the incumbent sets a monopoly price. Since there is no effective threat of entry, the incumbent is free to behave as a monopolist.

E2 is a Stackelberg Duopoly with the entrant acting as the follower and the incumbent acting as the leader ${ }^{7}$. The entrant takes the incumbent's price as fixed but the incumbent takes account of the entrant's reaction function when he sets his own price. The equilibrium E2 occurs when the incumbent's profit-maximizing price, with the entrant in, leaves the entrant with a positive profit. The Stackelberg Duopoly equilibrium is then stable with the entrant permanently in the market ${ }^{8}$.

The Stackelberg Duopoly E2 depends upon the incumbent treating the state probability of entry as fixed. When the incumbent sets his price to maximize his profit function with the entrant in, the maximum occurs at a price that is higher than the price that leaves the entrant with a zero profit. However, the incumbent will not lower his price to deter entry because lowering $P_{I}$ reduces his expected profit with $\mu=1$ in the vicinity of E2.

The Stochastic Equilibrium E3 corresponds to the equilibrium of contestable market theory. In a contestable market the incumbent's price is set to deter entry by the potential entrant. E3 includes the entrant's reaction function $P_{E}\left(P_{I}\right)=\operatorname{ArgMax}\left(f_{E}\left(P_{E} \mid P_{I}\right)\right)$ and the zero-profit condition $f_{E}\left(P_{E}\left(P_{I}\right) \mid P_{I}\right)=0$ among the equations that determine the equilibrium. These two equations alone are sufficient to establish the entrant and incumbent's prices, $P_{E}$ and $P_{I}$, at equilibrium.

However, E3 includes a third equation that was not anticipated by the authors of contestable market theory, $P_{I}=\operatorname{ArgMax}\left\{\mu f_{I}\left(P_{I} \mid P_{E}\left(P_{I}\right)\right)_{\text {in }}+(1-\mu) f_{I}\left(P_{I}\right)_{\text {out }}\right\}$. The solution to this equation establishes the value of the state probability of entry, $\mu$, at equilibrium. This equation must be included in E3 when the incumbent takes the state probability of entry as fixed as he maximizes his expected profit, even though the equation is not needed to find the equilibrium prices $P_{E}$ and $P_{I}$.

A simple formula for the state probability of entry at a Stochastic Equilibrium E3 may be obtained when the incumbent's profit functions have continuous first derivatives. We maximize the incumbent's expected profit by differentiating

[^6]$\mu f_{I}\left(P_{I} \mid P_{E}\left(P_{I}\right)\right)_{\text {in }}+(1-\mu) f_{I}\left(P_{I}\right)_{\text {out }}$ with respect to $P_{I}$, setting the result equal to zero, and solving for $\mu$. The resulting formula for the equilibrium state probability of entry is $\mu=\frac{d f_{I}(.)_{\text {out }}}{d P_{I}} /\left[\frac{d f_{I}(.)_{\text {out }}}{d P_{I}}-\frac{d f_{I}(.)_{\text {in }}}{d P_{I}}\right]$, with the derivatives evaluated at E3.

This formula ultimately relates the state probability of entry, through the incumbent's profit function and the entrant's reaction function, to the incumbent's and entrant's demand and cost functions. If the formula yields a state probability of entry in the range $[0,1]$, then the market has the stochastic equilibrium E3. If the formula implies $\mu<0$, then the contestable market has the monopoly equilibrium E1. If the formula implies $\mu>1$, then the market has the Stackelberg equilibrium E2. Therefore, the formula describes how the characteristics of demand and cost determine the form of equilibrium in a contestable market as well as the state probability of entry for a stochastic equilibrium E3.

## 4. Nash Equilibrium

The connection of the equilibriums, E1, E2 and E3, to the theory of two-person games is straightforward. These equilibriums are alternative kinds of outcomes to a non-zero-sum non-cooperative two-person game with equilibrium pairs of strategies. The game is non-zero-sum because the profit functions for the two players do not divide a fixed total amount. It is non-cooperative because the two players are not permitted to set their prices cooperatively or to make side payments.

The assumption A1 substitutes a simple decision rule, the reaction function $P_{E}\left(P_{I}\right)$, for what would otherwise be a complex tactical choice of the price $P_{E}$ by the entrant. This reduces the number of pure strategies for the entrant to two: to enter or not to enter. The incumbent's pure strategies are the possible choices for his price $P_{I}$. The payoff for the entrant is $f_{E}\left(P_{E}\left(P_{I}\right) \mid P_{I}\right)$ with entry, and zero (0) without entry. The payoff for the incumbent is $f_{I}\left(P_{I} \mid P_{E}\left(P_{I}\right)\right)_{\text {in }}$ with entry, and $f_{I}\left(P_{I}\right)_{\text {out }}$ without entry.

A pair of strategies, including mixed strategies, is a Nash equilibrium if the strategies are optimal against each other. E1 is an equilibrium pair of pure strategies. The incumbent's price $P_{I}$ maximizes his profit $f_{I}\left(P_{I}\right)_{\text {out }}$ with the entrant out. The entrant's strategy, not to enter or to stay out, is optimal against $P_{I}$ because $f_{E}\left(P_{E}\left(P_{I}\right) \mid P_{I}\right)<0$.

E2 is also an equilibrium pair of pure strategies. The incumbent's price $P_{I}$ maximizes his profit $f_{I}\left(P_{I} \mid P_{E}\left(P_{I}\right)\right)_{\text {in }}$ with the entrant in. The entrant's strategy, to enter or to stay in, is optimal against $P_{I}$ because $f_{E}\left(P_{E}\left(P_{I}\right) \mid P_{I}\right)>0$.

E3 arises when the equilibrium pair is a mixed strategy for the entrant and a pure strategy for the incumbent. The incumbent's price $P_{I}$ maximizes his expected profit $\mu f_{I}\left(P_{I} \mid P_{E}\left(P_{I}\right)\right)_{i n}+(1-\mu) f_{I}\left(P_{I}\right)_{\text {out }}$ against an entrant's mixed strategy of entry with probability $\mu$, and no entry with probability $1-\mu$. The entrant's mixed strategy
maximizes his profit against $P_{I}$ because $f_{E}\left(P_{E}\left(P_{I}\right) \mid P_{I}\right)=0$. No other choice of $\mu$ in the range $[0,1]$ will improve the entrant's profit.

An equilibrium pair never involves a mixed strategy for the incumbent because the incumbent's profit functions, $f_{I}\left(P_{I}\right)_{\text {out }}$ and $f_{I}\left(P_{I} \mid P_{E}\left(P_{I}\right)\right)_{\text {in }}$, are strictly concave and have finite maximums in $P_{I} \geq 0$. So the incumbent's expected profit is also a strictly concave function with a finite maximum in $P_{I} \geq 0$ for any $\mu$ in [0,1]. This means that any incumbent mixed strategy with two or more non-zero probabilities is always dominated by a single-price pure strategy. This property eliminates any reason the incumbent may have for concealment, so the game can be analyzed as though both players choose their strategies without knowing the other's choice. At least one equilibrium pair of mixed strategies must exist for a non-cooperative two-person game with a finite set of pure strategies. See Nash (1950 and 1951) and Luce and Raiffa (1957), pp.106-109 and pp. 391-393).

The existence of an equilibrium E1, E2 or E3 under the assumptions A1-A3 can be proved specifically for a contestable market using the Brouwer Fixed-Point Theorem along the lines of Nash's proof of the Minimax Theorem. A point-to-point mapping of a pair of strategies $\left(P_{I}^{t}, \mu^{t}\right)$ into another pair of strategies $\left(P_{I}^{t+1}, \mu^{t+1}\right)$ is defined so that a fixed-point (a point that maps into itself) corresponds to an equilibrium pair as follows. Let $D^{+}=\left(1-\mu^{t}\right) f_{E}\left(P_{E}\left(P_{I}^{t}\right) \mid P_{I}^{t}\right)$ if $f_{E}\left(P_{E}\left(P_{I}^{t}\right) \mid P_{I}^{t}\right)>0$, and $D^{+}=0$ otherwise; let $D^{-}=-\mu^{t} f_{E}\left(P_{E}\left(P_{I}^{t}\right) \mid P_{I}^{t}\right)$ if $f_{E}\left(P_{E}\left(P_{I}^{t}\right) \mid P_{I}^{t}\right)<0$, and $D^{-}=0$ otherwise. The mapping is: $\left(P_{I}^{t+1}, \mu^{t+1}\right)=\left(\operatorname{ArgMax}\left\{\mu^{t} f_{I}\left(P_{I}, P_{E}\left(P_{I}\right)\right)_{\text {in }}+\left(1-\mu^{t}\right) f_{I}\left(P_{I}\right)_{\text {out }}\right\},\left(\mu^{t}+D^{+}\right) /\left(1+D^{+}+D^{-}\right)\right)$. This mapping is a continuous mapping from a compact and convex subset $S \subset R^{2}$ back into itself. The subset S is defined as $S=\left\{0 \leq P_{I} \leq \operatorname{ArgMax}\left(f_{I}\left(P_{I}\right)_{\text {out }}\right), 0 \leq \mu \leq 1\right\}$ At least one fixed-point must exist according to the Brouwer Fixed-Point Theorem.

A contestable market will always have at least one of the equilibriums E1, E2 or E3. Since these equilibriums are mutually exclusive, one and only one of them must exist for a contestable market.

## 5. A Graphical Interpretation

In the figure below the incumbent's profit function $f_{I}\left(P_{I} \mid P_{E}\left(P_{I}\right)\right)_{i n}$, with the entrant in the market, is the parabola at the bottom of the figure. The profit function $f_{I}\left(P_{I}\right)_{\text {out }}$, with the entrant out of the market, is the parabola at the top of the figure. Both functions are strictly concave and have finite maximums in $P_{I} \geq 0$. The point E 1 is shown at the maximum of $f_{I}\left(P_{I}\right)_{\text {out }}$. It is the monopoly equilibrium E1 if the entrant remains out of the market. The point E2 is at the maximum of $f_{I}\left(P_{I} \mid P_{E}\left(P_{I}\right)\right)_{i n}$. This is the Stackelberg duopoly equilibrium when the entrant is certain to enter the market.

The vertical dashed line connecting the two profit functions is drawn at the incumbent price that leaves the entrant with a zero profit, i.e., at the price $P_{I}$ determined by the zero-profit condition $f_{E}\left(P_{E}\left(P_{I}\right) \mid P_{I}\right)=0$. An incumbent price below the location
of the dashed line leaves the potential entrant with a loss if he enters. This outcome reduces the state probability of entry towards $\mu=0$ under assumption A2. An incumbent price above the dashed line gives the entrant a positive profit if he enters. This increases the state probability of entry towards the limit $\mu=1$ under assumption A2.

Neither E1 nor E2 can be the equilibrium of the contestable market as the figure has been drawn. At E1 the entrant's profit is positive so he will not remain permanently out of the market as required for the monopoly outcome E1. At E2 the entrant takes a loss so he will not remain in the market as required for the Stackelberg duopoly E2.

The stochastic equilibrium E3 occurs at an intermediate point on the dashed line that is determined by the state probability of entry at equilibrium. Curves such as $\mathrm{a}-\mathrm{a}, \mathrm{b}$ b' and c-c' describe the incumbent's expected profit as a function of $P_{I}$ for different fixed values of $\mu$. They are weighted averages of the incumbent's profit functions with the entrant in and out of the market. Each of the curves reaches a maximum at a different price $P_{I}$. The equilibrium E3 occurs along the curve b-b' where the price $P_{I}$ that maximizes the incumbent's expected profit coincides with the price that leaves the entrant with no profit or loss. Therefore, the equilibrium state probability of entry is the value of $\mu$ that produces the curve b-b' with a maximum on the dashed vertical line.

The stochastic equilibrium E3 occurs at this point along the curve b-b' because, under assumption A 3 , the incumbent takes $\mu$ as given when he maximizes his expected profit. E3 does not occur where $\mathrm{a}-\mathrm{a}$ ' or $\mathrm{c}-\mathrm{c}$ ' reach their maxima because only an incumbent price corresponding to the vertical dashed line will leave $\mu$ unchanged under assumption A2. Prices to the right of the dashed line cause $\mu$ to increase; prices to the left cause $\mu$ to decrease.

Maximizing expected profit along either a-a' or c-c' sets off a chain of actions and reactions that shifts these curves towards b-b'. If the incumbent maximizes his expected profit along $a-a^{\prime}$, the resultant price is higher than his price at the dashed vertical line. This higher price causes a positive profit when the entrant takes the price as given and maximizes his own profit as per A1. The entrant's positive profit then increases the state probability of entry according to A2. The increase in $\mu$ shifts the curve a-a' down towards the incumbent's profit function with the entrant in. Increases in $\mu$ continue to shift a-a' downward until it coincides with b-b' and the entrant's positive profit is eliminated. If the incumbent maximizes his expected profit along $\mathrm{c}-\mathrm{c}$ ', then the price that maximizes his expected profit is below the price at the dashed line. The low price leads to a negative profit for the potential entrant, $\mu$ decreases, and c-c' shifts upward toward the incumbent's profit function with the entrant out. The upward shifts continue until c-c' coincides with $\mathrm{b}-\mathrm{b}$ '. Thus the curve $\mathrm{b}-\mathrm{b}$ ' is the only curve along which the incumbent can maximize his expected profit while the market remains in equilibrium.


## Appendix B: A Numerical Example with Formulas

## 1. Introduction

This Appendix contains a fully worked numerical example with equations of a Stochastic Equilibrium for a single contestable market. The market is depicted in the figure below. The calculations that were made along with the formulas are shown in the tables that follow the figure. The first table identifies the market as corresponding to an office serving low density city-carrier delivery points attached to the processing center with the median number of delivery points per office. This office and its market are the proxy for 33.73 offices in the USPS delivery network. The demand functions were calibrated using elasticities that are calculated and adjusted from estimates found in the R2006-1 testimony of USPS witness Thomas Thress before the PRC. The entrant's cost function was calibrated to USPS FY2005 costs using an equal-price market share of 30 percent and a displacement ratio of 50 percent. These parameters are compatible with a moderate amount of product differentiation between the delivery services of USPS and an entrant. The example uses adjustments to USPS marginal and fixed costs that give the entrant a substantial cost advantage over USPS. The adjustments are all downward and correspond to lower labor rates ( 80 percent of USPS unit labor costs), less frequent deliveries ( 3 days per week), and reduced processing costs for unaddressed mail (50 percent of USPS processing and other costs).


## 2. Tables

In the tables non-commercial mail is product " 1 ", the incumbent's commercial mail is product " 2 ", the entrant's commercial mail is product " 3 " and packages are product " 4 ". The subscripts " $i$ " and " $e$ " denote the incumbent and the entrant.

Controls and parameter values that may be selected by the user are shown in blue.

Contestable Market Equilibrium

| Case Selection | Symbol/Formula | Value |
| :---: | :---: | :---: |
| Case Identification |  |  |
| Mail Processing Sequence No. | Range 0 to 320 (0=Median Office) | 0 |
| Delivery Point Category | 1 - High Density City Delivery Points 2 - Low Density City Delivery Points 3 - Rural Delivery Points 4- Post Office Boxes | 2 |
| Demand Model | 1 - R2006-1 Unadjusted 2 - R2006-1 Adjusted 3 - Pearsall (2005) FWI Prices 4 - Pearsall (2005) Rev. per Pc. | 2 |
| Case Parameters |  |  |
| Number of Delivery Offices | N | 33.73 |
| Delivery Points per Office | Di | 3,713 |
| Non-Commercial Mail per Office | X1 | 3,082,332 |
| Commercial Mail per Office | X2 | 3,033,646 |
| Packages Per Office | X4 | 38,928 |
| Demand Calibration Parameters |  |  |
| Entrant Market Share with P3=P2 | w | 0.3000 |
| Displacement Ratio | s | 0.5000 |
| Entrant Cost Adjustments |  |  |
| Delivery Marginal Cost Ratio | Md3/Md2 | 0.80 |
| Non-Delivery Marginal Cost Ratio | Mn3/Mn2 | 0.40 |
| Delivery Point Fixed Cost Ratio | Fed/Fid | 0.40 |
| Office Fixed Cost Ratio | Feo/Fio | 0.20 |

Solution Summary
Equilibrium Category

State Probability of Entry
Product 1
Quantity of Product 1
Ramsey Price of Product 1
Marginal Cost of Product 1 Net Revenue from Product 1
$\frac{\text { Product } 2 \text { without Entry }}{\text { Quantity of Product } 2}$ Price of Product 2
Marginal Cost of Product 2 Net Revenue from Product 2

Product 2 with Entry
Quantity of Product 2
Price of Product 2
Marginal Cost of Product 2
Net Revenue from Product 2
Product 3 with Entry
Quantity of Product 3
Price of Product 3
Marginal Cost of Product 3
Net Revenue from Product 3
Product 4
Quantity of Product 4
Ramsey Price of Product 4
Marginal Cost of Product 4
Net Revenue from Product 4
Incumbent
Net Revenue from Product 1 Exp. Net Revenue from Product 2

Fixed Cost
Expected Profit
Entrant with Entry
Net Revenue from Product 3
Fixed Cost
Profit

Symbol/Formula
1=Natural Monopoly 2=Stackelberg Duopoly 3=Stochastic Equilibrium 4=Entrance Monopoly

| M | 0.5769 |
| :---: | :---: |
| X1 | 3,167,267 |
| P1 | 0.3475 |
| M1 | 0.2533 |
| (P1-M1)* ${ }^{\text {P1 }}$ | 298,369 |
| X2 | 1,917,744 |
| P2 | 0.4263 |
| M2 | 0.1672 |
| [(P2-M2)*X2]out | 496,946 |
| X2 | 1,240,292 |
| P2 | 0.4263 |
| M2 | 0.1672 |
| [(P2-M2)* X 2 ]in | 321,398 |
| X3 | 1,354,905 |
| P3 | 0.2829 |
| M3 | 0.0938 |
| (P3-M3)* ${ }^{\text { }}$ | 256,130 |
| X4 | 43,057 |
| P4 | 4.1516 |
| M4 | 3.9999 |
| (P4-M4)* X 4 | 6,534 |
| (P1-M1)* ${ }^{\text {¢ }}$ | 298,369 |
| $\mathrm{M}^{\star}\left[(\mathrm{P} 2-\mathrm{M} 2)^{\star} \mathrm{X} 2\right] \mathrm{in}+(1-\mathrm{M})^{\star}\left[(\mathrm{P} 2-\mathrm{M} 2)^{\star} \mathrm{X} 2\right]$ out | 395,665 |
| Fi | 879,749 |
| $M^{*}($.$) in +(1-M)^{*}($.$) out$ | $(179,181)$ |
| (P3-M3)* ${ }^{\text { }}$ | 256,130 |
| Fe | 256,130 |
| $(\mathrm{P} 3-\mathrm{M} 3)^{*} \times 3-\mathrm{Fe}$ | 0 |

Value
3


3,167,267
0.3475
0.2533

298,369

1,917,744
0.4263
0.1672

496,946

1,240,292
0.4263
0.1672

321,398

1,354,905
0.2829
0.0938

256,130

43,057
4.1516
3.9999

298,369
395,665
879,749
$(179,181)$

256,130
256,130
0

| Welfare Summary | Symbol/Formula | Value |
| :---: | :---: | :---: |
| Product 2 Price Uniform | Kf | -0.098279 |
| Ramsey Inverse Elasticity Multiplier | P 1 | 0.4161 |
| Ramsey Price Product | P 2 | 0.2094 |
| Ramsey Price Product 2 | P 4 | 4.2562 |
| Ramsey Price Product 4 | Sf | $3,766,957$ |
| Total Welfare w/Ramsey Pricing |  |  |
|  | Kv | -0.097987 |
| Product 2 Price Variable | P 1 | 0.4150 |
| Ramsey Inverse Elasticity Multiplier | P 2 | 0.2238 |
| Ramsey Price Product 1 | P 4 | 4.2546 |
| Ramsey Price Product 2 | Sv | $3,763,443$ |
| Ramsey Price Product 4 |  | -0.077555 |
| Total Welfare w/Ramsey Pricing | Kc | 0.3475 |
| with Contestable Market | P 1 | 0.4263 |
| Ramsey Inverse Elasticity Multiplier | P 2 | 0.2829 |
| Ramsey Price Product 1 | P 3 | 4.1516 |
| Equilibrium Price Product 2 | P 4 | $3,605,179$ |
| Equilibrium Price Product 3 | Sn | $3,557,696$ |
| Ramsey Price Product 4 | Se | $3,577,784$ |

## Calibrations of the Cost Functions

|  | Symbol/Formula | Value |
| :---: | :---: | :---: |
| Incumbent Sha Value |  |  |
| Incumbent Cost Equation Components |  |  |
| Aarginal Cost of Non-Commercial Mail (Product 1) | M1 | 0.2533 |
| Marginal Cost of Commercial Mail (Product 2) | M2=Md2+Mn2 | 0.1672 |
| Delivery Marginal Cost of Product 2 | Md2 | 0.0674 |
| Non-Delivery Marginal Cost of Product 2 | Mn2 | 0.0998 |
| Marginal Cost of Packages (Product 4) | M4 | 3.9999 |
| Fixed Cost per Delivery Point | Fid | 107.9658 |
| Number of Delivery Points | Di | 3,713 |
| Fixed Cost per Office | Fio | 478,847 |
| Incumbent's Fixed Cost | $\mathrm{Fi}=\mathrm{Fid}{ }^{*} \mathrm{Di}+\mathrm{Fio}$ | 879,749 |
| Incumbent's Linear Cost Function | $\mathrm{Ci}=\mathrm{M} 1^{*} \mathrm{X} 1+\mathrm{M} 2^{*} \mathrm{X} 2+\mathrm{M} 4^{*} \mathrm{X} 4+\mathrm{Fi}$ |  |
| Marginal Cost of Product 1 | M1 | 0.2533 |
| Marginal Cost of Product 2 | M2 | 0.1672 |
| Marginal Cost of Product 4 | M4 | 3.9999 |
| Incumbent's Fixed Cost | Fi | 879,749 |

## Potential Entrant

Entrant Cost Equation Components
Marginal Cost of Product 3 Delivery Marginal Cost of Product 3
Delivery Marginal Cost of Product 3 (Carrier)
Delivery Marginal Cost of Product 3 (PO Box) Non-Delivery Marginal Cost of Product 3

Fixed Cost per Delivery Point Fixed Cost per Delivery Point (Carrier)
Fixed Cost per Delivery Point (PO Box)
Number of Delivery Points
Fixed Cost per Office
Entrant's Fixed Cost
Entrant's Linear Cost Function
Marginal Cost of Product 3
Entrant's Fixed Cost

## Ramsey Pricing

| System Marginal Cost of Product 1 | M1 | 0.2149 |
| :--- | :--- | :--- |
| System Marginal Cost of Product 2 | M2 | 0.1563 |
| System Marginal Cost of Product 4 | M4 | 3.8022 |


|  | Symbol/Formula | Value |
| :---: | :---: | :---: |
| Non-Commercial Mail (Product 1) Sy |  |  |
| Incumbent Product 1 Demand Function | $\mathrm{X} 1=\mathrm{K} 1^{*}\left(\mathrm{P} 1^{\wedge} \mathrm{e} 1\right)$ |  |
| Quantity of Product 1 | X1 | 3,082,332 |
| Callibration Price of Product 1 | P1 | 0.4020 |
| Direct Price Elasticity of Product 1 | e1 | -0.2032 |
| Scale Product 1 | K1 | 2,561,353 |
| Product 1 Linearized Demand Function | $\mathrm{X} 1=\mathrm{A} 1+\mathrm{B} 11 * \mathrm{P} 1$ |  |
| Intercept | $\mathrm{A} 1=\mathrm{X1}-\mathrm{B} 11 * \mathrm{P} 1$ | 3,708,601 |
| Slope w/r Own Price | $\mathrm{B} 11=\mathrm{e}{ }^{*} \mathrm{X} 1 / \mathrm{P} 1$ | $(1,557,823)$ |
| Commercial Mail (Product 2) without Entry |  |  |
| Incumbent Product 2 Demand Function | X2 $=$ K2 ${ }^{*}$ ( $\left.\mathrm{P}^{\wedge} \mathrm{e} 2\right)$ |  |
| Quantity of Product 2 | X2 | 3,033,646 |
| Callibration Price of Product 2 | P2 | 0.2187 |
| Direct Price Elasticity of Product 2 | e2 | -0.3876 |
| Scale Product 2 | $\mathrm{K} 2=\mathrm{X} 2^{*}\left(\mathrm{P}^{\wedge}(-\mathrm{e} 2)\right)$ | 1,683,145 |
| Product 2 Linearized Demand Function | X2=A2+B22*P2 |  |
| Intercept | A2=X2-B22*P2 | 4,209,494 |
| Slope w/r Own Price | B22=e2* 2/P2 $^{\text {P }}$ | $(5,375,490)$ |
| Packages (Product 4) |  |  |
| Incumbent Product 4 Demand Function | $\mathrm{X} 4=\mathrm{K} 4^{*}$ ( $\left.\mathrm{P} 4^{\wedge} \mathrm{e} 4\right)$ |  |
| Quantity of Product 4 | X4 | 38,928 |
| Callibration Price of Product 4 | P4 | 4.6918 |
| Direct Price Elasticity of Product 4 | e4 | -0.9214 |
| Scale Product 4 | $\mathrm{K} 4=\mathrm{X4}{ }^{\star}(\mathrm{P} 4 \wedge(-\mathrm{e} 4))$ | 161,754 |
| Product 4 Linearized Demand Function | $\mathrm{X} 4=\mathrm{A} 4+\mathrm{B44*P4}$ |  |
| Intercept | A4 $=$ X4-B44*P4 | 74,797 |
| Slope w/r Own Price | $B 44=e 4^{*} \times 4 / \mathrm{P} 4$ | $(7,645)$ |
| Commercial Mail (Products 2 and 3) with Entry |  |  |
| Demand Calibration Parameters |  |  |
| Entrant Market Share with P3=P2 | w | 0.3000 |
| Incumbent Market Share with P3=P2 | 1-w | 0.7000 |
| Displacement Ratio | s | 0.5000 |
| Calabrated Parameters |  |  |
| Direct Price Elasticity of Product 3 | e3=((1-(1-s) ${ }^{*}$ w)/(s+(1-s)w)**e2 | -0.5069 |
| Scale Product 3 | $\mathrm{K} 3=(\mathrm{e} 2 / \mathrm{e} 3)^{*} \mathrm{~K} 2^{*}\left(\mathrm{P}^{\wedge}(\mathrm{e} 2-\mathrm{e} 3)\right)$ | 1,073,729 |
| Price of Product 2 | $\mathrm{P} 3=\mathrm{P} 2$ | 0.2187 |
| Incumbent Product 2 Demand Function | $\mathrm{X} 2=\left(\mathrm{K} 2^{*}\left(\mathrm{P}^{\wedge} \mathrm{e} 2\right)-\mathrm{s}^{*} \mathrm{~K} 3\left(\mathrm{P} 3^{\wedge} \mathrm{e} 3\right) /\right.$ /(1-s^2) |  |
| Quantity of Product 2 | X2 | 2,498,297 |
| Entrant Product 3 Demand Function | $\mathrm{X} 3=\left(-\mathrm{s}^{*} \mathrm{~K} 2^{\star}\left(\mathrm{P}^{\wedge} \mathrm{e} 2\right)+\mathrm{K} 3^{*}\left(\mathrm{P} 3^{\wedge} \mathrm{e} 3\right)\right.$ )/(1-s^2) |  |
| Quantity of Product 3 | X3 | 1,070,699 |
| Product 2 Linearized Demand Function | X2=A2+B22*P2+B23*P3 |  |
| Intercept | A2=X2-B22*P2-B23*P3 | 2,498,297 |
| Slope w/r Incumbent Price P2 | $\mathrm{B} 22=\mathrm{e}^{*} \mathrm{~K} 2^{*}\left(\mathrm{P}^{\wedge} \mathrm{e} 2\right) /\left(\left(1-\mathrm{s}^{\wedge} 2\right)^{*} \mathrm{P} 2\right)$ | (7,167,320) |
| Slope w/r Entrant Price P3 | $\mathrm{B} 23=-\mathrm{e} 3^{*} \mathrm{~s}^{\star} \mathrm{K} 3^{*}\left(\mathrm{P}^{\wedge} \mathrm{e} 3\right) /\left(\left(1-\mathrm{s}^{\wedge}\right)^{*} \mathrm{P} 3\right)$ | 3,583,660 |
| Product 3 Linearized Demand Function | X3=A3+B32*P2+B33*P3 |  |
| Intercept | A3=X3-B32*P2-B33*P3 | 1,070,699 |
| Slope w/r Incumbent Price P2 | B32 $=-\mathrm{e} 3^{*} \mathrm{~s}^{*} \mathrm{~K} 2^{*}\left(\mathrm{P}^{\wedge} \mathrm{e} 2\right) /\left(\left(1-\mathrm{s}^{\wedge} 2\right)^{*} \mathrm{P} 2\right)$ | 3,583,660 |
| Slope w/r Entrant Price P3 | $\mathrm{B} 33=\mathrm{e}^{*} \mathrm{~K} 3^{*}\left(\mathrm{P}^{\wedge} \mathrm{e} 3\right) /\left(\left(1-\mathrm{s}^{\wedge} 2\right)^{*} \mathrm{P} 3\right)$ | $(7,167,320)$ |

## Contestable Market Possible Solutions

Cost Functions
$\frac{\text { Incumbent's Linear Cost Function }}{\text { Marginal Cost of Product } 1}$
Marginal Cost of Product 2
Marginal Cost of Product 4
Incumbent's Fixed Cost
$\frac{\text { Entrant's Linear Cost Function }}{\text { Marginal Cost of Product 3 }}$ Entrant's Fixed Cost

Demand Functions
\(\left.\begin{array}{c}\frac{Demand Functions}{\frac{Product 1 Demand Function}{Intercept}} <br>

Slope w / r Own Price\end{array}\right\}\)| Product 2 Demand Function (Entrant Out) |
| :---: |
| Intercept |
| Slope w/r Own Price |
| Product 2 Demand Function (Entrant In) |
| Intercept |
| Slope w/r Incumbent Price |
| Slope $w / r$ Entrant Price |
| Product 3 Demand Function (Entrant In) |
| Intercept |
| Slope w/r Incumbent Price |
| Slope $w / r$ Entrant Price |
| $\frac{\text { Product } 4 \text { Demand Function }}{\text { Intercept }}$ |
| Slope $w / r$ Own Price |

## Inverse Demand Functions

Inverse Demand Coefficients (Entrant Out)

> | Product 1 Slope $w / r \times 1$ |
| :--- |
| Product 2 Slope $w / r \times 2$ |
| Product 4 Slope $w / r \times 4$ |

Inverse Demand Coefficients (Entrant In)
Product 1 Slope w/r X1
Product 2 Slope $w / r$ X2
Product 3 Slope $w / r$ X3
CrossProduct Slope w/r X2, X3
Product 4 Slope w/r X4

## Ramsey Pricing

System Marginal Cost of Product 1
System Marginal Cost of Product 2 System Marginal Cost of Product 4
Ramsey Inverse Elasticity Multiplier P2 Fixed Ramsey Inverse Elasticity Multiplier P2 Variable

| $\mathrm{C} 11=1 / \mathrm{B} 11$ | $-6.42 \mathrm{E}-007$ |
| :--- | :--- |
| $\mathrm{C} 22=1 / \mathrm{B} 22$ | $-1.86 \mathrm{E}-007$ |
| $\mathrm{C} 44=1 / \mathrm{B} 44$ | $-1.31 \mathrm{E}-004$ |


| C11=1/B11 | $-6.42 \mathrm{E}-007$ |
| :---: | :---: |
| C22 $=\mathrm{B} 33 /\left(\mathrm{B} 22^{*} \mathrm{~B} 33-\mathrm{B} 23^{*} \mathrm{~B} 32\right)$ | $-1.86 \mathrm{E}-007$ |
| $\mathrm{C} 33=\mathrm{B} 22 /\left(\mathrm{B} 22^{*} \mathrm{~B} 33-\mathrm{B} 23^{*} \mathrm{~B} 32\right)$ | $-1.86 \mathrm{E}-007$ |
| $\mathrm{C} 23=\mathrm{C} 32-\mathrm{B} 23 /\left(\mathrm{B} 22^{*} \mathrm{~B} 33-\mathrm{B} 23^{*} 32\right)$ | $-9.30 \mathrm{E}-008$ |
| $\mathrm{C} 44=1 / \mathrm{B} 44$ | $-1.31 \mathrm{E}-004$ |

Ramsey IE Multiplier Contestable Market
Direct Price Elasticity of Product 1 Direct Price Elasticity of Product 2 Direct Price Elasticity of Product 4

| M1 | 0.2149 |
| :--- | ---: |
| M2 | 0.1563 |
| M4 | 3.8022 |
| Kf | -0.0983 |
| Kv | -0.0980 |
| Kc | -0.0776 |
| e1 | -0.2032 |
| e2 | -0.3876 |
| e4 | -0.9214 |


|  | Symbol/Formula | Value |
| :---: | :---: | :---: |
| Incumbent Monopoly |  |  |
| Incumbent Product 1 |  |  |
| Quantity of Product 1 | X1=A1+B11*P1 | 3,167,267 |
| Ramsey Price of Product 1 | P1=M1 (System)/(1-Kc/e1) | 0.3475 |
| Marginal Cost of Product 1 | M1 | 0.2533 |
| Net Revenue from Product 1 | (P1-M1)**1 | 298,369 |
| Incumbent Product 2 (Entrant Out) |  |  |
| Quantity of Product 2 | X2=A2+B22*P2 | 1,655,349 |
| Price of Product 2 | P2=-(A2-M2*B22)/(2*B22) | 0.4751 |
| Marginal Cost of Product 2 | M2 | 0.1672 |
| Net Revenue from Product 2 | (P2-M2)*X2 | 509,754 |
| Entrant Product 3 (with Entry) |  |  |
| Quantity of Product 3 | X3=A3+B23*P2+B33*P3 | 1,442,370 |
| Price of Product 3 | $\mathrm{P} 3=-\left(\mathrm{A} 3+\mathrm{B} 23^{*} \mathrm{P} 2-\mathrm{B} 33^{*} \mathrm{M} 3\right) /\left(2^{*} \mathrm{~B} 33\right)$ | 0.2951 |
| Marginal Cost of Product 3 | M3 | 0.0938 |
| Net Revenue from Product 3 | $(\mathrm{P} 3-\mathrm{M} 3)^{*} \times 3$ | 290,266 |
| Incumbent Product 4 |  |  |
| Quantity of Product 4 | X4=A4+B44*P4 | 43,057 |
| Ramsey Price of Product 4 | P4=M4(System)/(1-Kv/e4) | 4.1516 |
| Marginal Cost of Product 4 | M4 | 3.9999 |
| Net Revenue from Product 4 | (P4-M4)* ${ }^{*} 4$ | 6,534 |
| Incumbent Profit w/Entrant Out | (P1-M1)* ${ }^{*} 1+(\mathrm{P} 2-\mathrm{M} 2)^{*} \mathrm{X} 2+(\mathrm{P} 4-\mathrm{M} 4)^{*} \mathrm{X} 4-\mathrm{Fi}$ | $(65,091)$ |
| Entrant Profit with Entry | $(\mathrm{P} 3-\mathrm{M} 3)^{*} \times 3-\mathrm{Fe}$ | 34,136 |
| Consumers' Surplus (Entrant Out) | $-\left(\mathrm{X1*}{ }^{*} 1^{*} \mathrm{C} 11+\mathrm{X} \mathrm{I}^{*} \mathrm{X} \mathbf{2}^{*} \mathrm{C} 22+\mathrm{X} 4^{*} \mathrm{X} 4^{*} \mathrm{C} 44\right) / 2$ | 3,595,872 |
| Producers' Surplus (Entrant Out) | (P1-M1)* $\mathrm{X} 1+(\mathrm{P} 2-\mathrm{M} 2)^{*} \mathrm{X} 2+(\mathrm{P} 4-\mathrm{M} 4)^{*} \mathrm{X} 4-\mathrm{Fi}$ | $(65,091)$ |
| Total Welfare Surplus (Entrant Out) | $\mathrm{Sn}=$ Consumers' plus Producers' Surplus | 3,530,780 |
| Stackelberg Duopoly |  |  |
| Incumbent Product 2 (Entrant In) |  |  |
| Quantity of Product 2 | X2=A2+B22*P2+B23*P3 | 1,432,702 |
| Price of Product 2 | P2=(A2+B23*M3/2-M2* ${ }^{*}$ (B22-B23*B23/(2*B33)) | 0.3957 |
|  | - $\mathrm{A}{ }^{*} \mathrm{~B} 23 /\left(2^{*} \mathrm{~B} 33\right)$ )/(B23*B23/B33-2*B22) |  |
| Marginal Cost of Product 2 | M2 | 0.1672 |
| Net Revenue from Product 2 | (P2-M2)*X2 | 327,301 |
| Entrant Product 3 (with Entry) |  |  |
| Quantity of Product 3 | X3=A3+B23*P2+B33*P3 | 1,299,930 |
| Price of Product 3 | $\mathrm{P} 3=-\left(\mathrm{A} 3+\mathrm{B} 23^{*} \mathrm{P} 2-\mathrm{B} 33^{*} \mathrm{M} 3\right) /\left(2^{*} \mathrm{~B} 33\right)$ | 0.2752 |
| Marginal Cost of Product 3 | M3 | 0.0938 |
| Net Revenue from Product 3 | $(\mathrm{P} 3-\mathrm{M} 3)^{*}{ }^{\text {K }}$ | 235,767 |
| Incumbent Profit w/Entrant In | (P1-M1)* ${ }^{*} 1+(\mathrm{P} 2-\mathrm{M} 2)^{*} \mathrm{X} 2+(\mathrm{P} 4-\mathrm{M} 4)^{*} \mathrm{X} 4-\mathrm{Fi}$ | $(247,545)$ |
| Entrant Profit with Entry | (P3-M3)* ${ }^{*} 3-\mathrm{Fe}$ | $(20,363)$ |
| Consumers' Surplus (Entrant In) | -(X1* ${ }^{1}{ }^{*} \mathrm{C} 11+\mathrm{X} 2^{*} \mathrm{X} 2^{*} \mathrm{C} 22$ | 3,862,330 |
|  | $\left.+X 3^{*} X 3^{*} C 33+2^{*} X 2^{*} \times 3^{*} C 23+X 4^{*} X 4^{*} C 44\right) / 2$ |  |
| Producers' Surplus (Entrant In) | (P1-M1)* ${ }^{*} 1+(\mathrm{P} 2-\mathrm{M} 2)^{*} \mathrm{X} 2+(\mathrm{P} 3-\mathrm{M} 3)^{*} \mathrm{X} 3+(\mathrm{P} 4-\mathrm{M} 4)^{*} \mathrm{X} 4-\mathrm{Fi}-\mathrm{Fe}$ | $(267,908)$ |
| Total Welfare Surplus (Entrant In) | $\mathrm{Se}=$ Consumers' plus Producers' Surplus | 3,594,422 |

Stochastic Equilibrium
Incumbent Product 2 (Entrant Out)
Quantity of Product 2
Price of Product 2
Marginal Cost of Product 2
Net Revenue from Product 2
Incumbent Product 2 (Entrant In)
Quantity of Product 2
Price of Product 2
Marginal Cost of Product 2
Net Revenue from Product 2
Entrant Product 3 (with Entry)
Quantity of Product 3
Price of Product 3
Marginal Cost of Product 3
Net Revenue from Product 3
Incumbent Profit w/Entrant Out
Incumbent Profit w/Entrant In
Entrant Profit with Entry
State Probability of Entry
Incumbent Marginal Profit w/Entrant Out
Incumbent Marginal Profit w/Entrant In
State Probability of Entry
Expected Net Revenue from Product 2
Expected Incumbent Profit
Consumers' Surplus (Entrant Out)
Producers' Surplus (Entrant Out)
Total Welfare Surplus (Entrant Out)
Consumers' Surplus (Entrant In)
Producers' Surplus (Entrant In)
Total Welfare Surplus (Entrant In)
Expected Consumers' Surplus
Expected Producers' Surplus
Expected Total Welfare Surplus

| Symbol/Formula | Value |
| :---: | :---: |
| X2 $=$ A2 + B22* ${ }^{\text {P }}$ 2 | 1,917,744 |
| P2=(SQRT(-4*B33*Fe)-A3-B33*M3)/B23 | 0.4263 |
| M2 | 0.1672 |
| [(P2-M2)*X2]out | 496,946 |
| X2=A2+B22*P2+B23*P3 | 1,240,292 |
| P2=(SQRT(-4*B33*Fe)-A3-B33*M3)/B23 | 0.4263 |
| M2 | 0.1672 |
| [(P2-M2)*X2]in | 321,398 |
| X3=A3+B23*P2+B33*P3 | 1,354,905 |
| $\mathrm{P} 3=-\left(\mathrm{A} 3+\mathrm{B} 23^{*} \mathrm{P} 2-\mathrm{B} 33^{*} \mathrm{M} 3\right) /\left(2^{*} \mathrm{~B} 33\right)$ | 0.2829 |
| M3 | 0.0938 |
| (P3-M3)* ${ }^{*} 3$ | 256,130 |
| $\mathrm{f}($.$) )out =(\mathrm{P} 1-\mathrm{M} 1)^{*} \mathrm{X} 1+\left[(\mathrm{P} 2-\mathrm{M} 2)^{*} \mathrm{X} 2\right]$ out+(P4-M4)*X4-Fi | $(77,900)$ |
| $\mathrm{f}($ ) $) \mathrm{in}=(\mathrm{P} 1-\mathrm{M} 1)^{*} \mathrm{X} 1+\left[(\mathrm{P} 2-\mathrm{M} 2)^{*} \mathrm{X} 2\right] \mathrm{in}+(\mathrm{P} 4-\mathrm{M} 4)^{*} \mathrm{X} 4-\mathrm{Fi}$ | $(253,448)$ |
| $(\mathrm{P} 3-\mathrm{M} 3)^{*} \times 3-\mathrm{Fe}$ | 0 |
| df(.)out/dP2=A2+(2*P2-M2)*B22 | 524,791 |
| $\mathrm{df}() .\mathrm{in} / \mathrm{dP} 2=\mathrm{A} 2+\mathrm{B} 23^{*}\left(\mathrm{~B} 33^{*} \mathrm{M} 3-\mathrm{A} 3\right) /\left(2^{*} \mathrm{~B} 33\right)$ | $(384,821)$ |
| +(2*P2-M2)**(B22-B23*B23/(2*B33)) |  |
|  | 0.5769 |
| $\mathrm{E}=\mathrm{M}^{*}\left[(\mathrm{P} 2-\mathrm{M} 2)^{*} \mathrm{X} 2\right] \mathrm{in}+(1-\mathrm{M})\left[(\mathrm{P} 2-\mathrm{M} 2)^{*} \mathrm{X} 2\right]$ out | 395,665 |
| $M^{*}(\mathrm{f}$.$) in +(1-M)^{* *}($.$) out$ | $(179,181)$ |
| $-\left(\mathrm{X} 1^{*} \mathrm{X} 1^{*} \mathrm{C} 11+\mathrm{X} 2^{*} \mathrm{X} 2^{*} \mathrm{C} 22+\mathrm{X} 4^{*} \mathrm{X} 4^{*} \mathrm{C} 44\right) / 2$ | 3,683,079 |
| (P1-M1)* ${ }^{*} 1+(\mathrm{P} 2-\mathrm{M} 2)^{*} \mathrm{X} 2+(\mathrm{P} 4-\mathrm{M} 4)^{*} \mathrm{X} 4-\mathrm{Fi}$ | $(77,900)$ |
| Consumers' plus Producers' Surplus | 3,605,179 |
| $-\left(\mathrm{X} 1^{*} \mathrm{X} 1^{*} \mathrm{C} 11+\mathrm{X} 2^{*} \mathrm{X} 2^{*} \mathrm{C} 22\right.$ | 3,811,144 |
| $\left.+X 3^{*} X 3^{*} C 33+2^{*} X 2^{*} X 3^{*} C 23+X 4^{*} X 4^{*} \mathrm{C} 44\right) / 2$ |  |
| $(\mathrm{P} 1-\mathrm{M} 1)^{*} \mathrm{X} 1+(\mathrm{P} 2-\mathrm{M} 2)^{*} \mathrm{X} 2-\mathrm{Fi}+(\mathrm{P} 3-\mathrm{M} 3)^{*} \mathrm{X} 3+(\mathrm{P} 4-\mathrm{M} 4)^{*} \mathrm{X} 4-\mathrm{Fe}$ | $(253,448)$ |
| Consumers' plus Producers' Surplus | 3,557,696 |
| $\mathrm{M}^{*}$ (Con. Surplus)in+(1-M)* ${ }^{\text {(Con. Surplus }) \text { out }}$ | 3,756,965 |
| $M^{*}$ (Prod. Surplus)in+(1-M)* (Prod. Surplus)out | $(179,181)$ |
| Consumers' plus Producers' Surplus | 3,577,784 |



| Symbol/Formula | Value |
| :---: | :---: |
| $\mathrm{X} 2=\mathrm{A} 2+\mathrm{B} 22^{*} \mathrm{P} 2$ | 4,209,494 |
| P2 | 0.4751 |
| M2 | 0.1672 |
| (P2-M2)*X2 | 1,296,288 |
| $\mathrm{X} 2=\mathrm{A} 2+\mathrm{B} 22^{*} \mathrm{P} 2+\mathrm{B} 23 * \mathrm{P} 3$ | 3,282,195 |
| $\mathrm{P} 2=-\left(2^{*} \mathrm{~B} 33^{*} \mathrm{~A} 2-\mathrm{B} 23^{*} \mathrm{~A} 3+\mathrm{B} 23^{*} \mathrm{~B} 33 \mathrm{M} 3\right)$ | 0.5973 |
| /(2*B22*B33-B23*B23) |  |
| M2 | 0.1672 |
| (P2-M2)**2 | 1,411,629 |
| X3=A3+B23*P2+B33*P3 | 3,995,080 |
| $\mathrm{P} 3=-\left(\mathrm{A} 3+\mathrm{B} 23^{*} \mathrm{P} 2-\mathrm{B} 33^{*} \mathrm{M} 3\right) /\left(2^{*} \mathrm{~B} 33\right)$ | 0.2787 |
| M3 | 0.0938 |
| $(\mathrm{P} 3-\mathrm{M} 3)^{*} \mathrm{X}^{3}$ | 738,513 |
| (P1-M1)* ${ }^{*} 1+\left[(\mathrm{P} 2-\mathrm{M} 2)^{*} \mathrm{X} 2\right] \mathrm{in}+(\mathrm{P} 4-\mathrm{M} 4)^{*} \mathrm{X} 4-\mathrm{Fi}$ | 721,442 |
| (P3-M3)* ${ }^{*} 3-\mathrm{Fe}$ | 482,383 |
| $-\left(\mathrm{X1*}{ }^{*} 1^{*} \mathrm{C} 11+\mathrm{X} 2^{*} \mathrm{X} 2^{*} \mathrm{C} 22+\mathrm{X} 4^{*} \mathrm{X} 4^{*} \mathrm{C} 44\right) / 2$ | 4,989,201 |
| (P1-M1)* ${ }^{\text {² }} 1+(\mathrm{P} 2-\mathrm{M} 2)^{*} \mathrm{X} 2+(\mathrm{P} 4-\mathrm{M} 4)^{*} \mathrm{X} 4-\mathrm{Fi}$ | 721,442 |
| Consumers' plus Producers' Surplus | 5,710,643 |


| Welfare | Symbol/Formula | Value |
| :---: | :---: | :---: |
| Entrant Out |  |  |
| Quantity of Product 1 | $\mathrm{X} 1=\mathrm{A} 1+\mathrm{B} 11^{*} \mathrm{P} 1$ | 3,167,267 |
| Price of Product 1 | P1 | 0.3475 |
| Marginal Cost of Product 1 | M1 | 0.2533 |
| Net Revenue from Product 1 | (P1-M1)* ${ }^{*} 1$ | 298,369 |
| Quantity of Product 2 | $\mathrm{X} 2=\mathrm{A} 2+\mathrm{B} 22^{*} \mathrm{P} 2$ | 3,006,606 |
| Price of Product 2 | $\mathrm{P} 2=\mathrm{M} 2 /(1-\mathrm{Kc} / \mathrm{e} 2)$ | 0.2238 |
| Marginal Cost of Product 2 | M2 | 0.1672 |
| Net Revenue from Product 2 | (P2-M2)*X2 | 170,084 |
| Quantity of Product 4 | $\mathrm{X} 4=\mathrm{A} 4+\mathrm{B44*}{ }^{*} 4$ | 43,057 |
| Price of Product 4 | P4 | 4.1516 |
| Marginal Cost of Product 4 | M4 | 3.9999 |
| Net Revenue from Product 4 | (P4-M4)* ${ }^{*} 4$ | 6,534 |
| Consumers' Surplus (Entrant Out) | $-\left(\mathrm{X} 1^{*} \mathrm{X} 1^{*} \mathrm{C} 11+\mathrm{X} \mathrm{S}^{*} \mathrm{X} \mathbf{2}^{*} \mathrm{C} 22+\mathrm{X} 4^{*} \mathrm{X} 4^{*} \mathrm{C} 44\right) / 2$ | 4,181,818 |
| Producers' Surplus (Entrant Out) | (P1-M1)* ${ }^{\text {² }} 1+(\mathrm{P} 2-\mathrm{M} 2)^{*} \mathrm{X} 2+(\mathrm{P} 4-\mathrm{M} 4)^{*} \mathrm{X} 4-\mathrm{Fi}$ | $(404,762)$ |
| Total Welfare Surplus (Entrant Out) | $\mathrm{Sn}=$ Consumers' plus Producers' Surplus | 3,777,056 |
| Product 2 Price Uniform |  |  |
| Quantity of Product 1 | X1=A1+B11*P1 | 3,060,323 |
| Price of Product 1 | P1=M1 (System)/(1-Kt/e1) | 0.4161 |
| Marginal Cost of Product 1 | M1 | 0.2533 |
| Net Revenue from Product 1 | $(\mathrm{P} 1-\mathrm{M} 1)^{*} \mathrm{X} 1$ | 498,386 |
| Quantity of Product 2 | X2=A2+B22*P2 | 3,083,711 |
| Price of Product 2 | P2=M2(System)/(1-Kt/e2) | 0.2094 |
| Marginal Cost of Product 2 | M2 | 0.1672 |
| Net Revenue from Product 2 | (P2-M2)*X2 | 130,214 |
| Quantity of Product 4 | X4=A4+B44*P4 | 42,258 |
| Price of Product 4 | P4=M4(System)/(1-Kf/e4) | 4.2562 |
| Marginal Cost of Product 4 | M4 | 3.9999 |
| Net Revenue from Product 4 | (P4-M4)* ${ }^{*} 4$ | 10,830 |
| Consumers' Surplus (Entrant Out) | $-\left(\mathrm{X1}{ }^{*} \mathrm{X} 1^{*} \mathrm{C} 11+\mathrm{X} \mathbf{2}^{*} \mathrm{X} \mathbf{2}^{*} \mathrm{C} 22+\mathrm{X} 4^{*} \mathrm{X} 4^{*} \mathrm{C} 44\right) / 2$ | 4,007,276 |
| Producers' Surplus (Entrant Out) | (P1-M1)* ${ }^{*} 1+(\mathrm{P} 2-\mathrm{M} 2)^{*} \mathrm{X} 2+(\mathrm{P} 4-\mathrm{M} 4)^{*} \mathrm{X} 4-\mathrm{Fi}$ | $(240,319)$ |
| Total Welfare Surplus (Entrant Out) | $\mathrm{Sf}=$ Consumers' plus Producers' Surplus | 3,766,957 |
| Product 2 Price Variable |  |  |
| Quantity of Product 1 | X1=A1+B11*P1 | 3,062,125 |
| Price of Product 1 | P1=M1 (System)/(1-Kv/e1) | 0.4150 |
| Marginal Cost of Product 1 | M1 | 0.2533 |
| Net Revenue from Product 1 | (P1-M1)**1 | 495,136 |
| Quantity of Product 2 | X2=A2+B22*P2 | 3,006,606 |
| Price of Product 2 | P2=M2(Office)/(1-Kv/e2) | 0.2238 |
| Marginal Cost of Product 2 | M2 | 0.1672 |
| Net Revenue from Product 2 | (P2-M2)*X2 | 170,084 |
| Quantity of Product 4 | X4=A4+B44*P4 | 42,270 |
| Price of Product 4 | P4=M4(System)/(1-Kv/e4) | 4.2546 |
| Marginal Cost of Product 4 | M4 | 3.9999 |
| Net Revenue from Product 4 | (P4-M4)* ${ }^{*} 4$ | 10,769 |
| Consumers' Surplus (Entrant Out) | $-\left(\mathrm{X} 1^{*} \mathrm{X} 1^{*} \mathrm{C} 11+\mathrm{X} 2^{*} \mathrm{X} \mathbf{2}^{*} \mathrm{C} 22+\mathrm{X} 4^{*} \mathrm{X} 4^{*} \mathrm{C} 44\right) / 2$ | 3,967,203 |
| Producers' Surplus (Entrant Out) | (P1-M1)* ${ }^{\text {2 }} 1+(\mathrm{P} 2-\mathrm{M} 2)^{*} \mathrm{X} 2+(\mathrm{P} 4-\mathrm{M} 4)^{*} \mathrm{X} 4-\mathrm{Fi}$ | $(203,760)$ |
| Total Welfare Surplus (Entrant Out) | $\mathrm{Sv}=$ Consumers' plus Producers' Surplus | 3,763,443 |

## Appendix C: A Roadmap to the Worksheets

The calibrations and calculations of equilibria described in the text and shown in Appendix B were performed with two linked Lotus 1-2-3 worksheets. These worksheets are available on request from the authors.

The worksheet ProcessingCenters 4.123 holds folders containing all of the data used to calibrate the model. In this worksheet the data describing local USPS delivery markets is assembled. The worksheet computes the piece weights based on unit delivery costs and performs the aggregations of USPS volumes into categories of non-commercial mail, commercial mail and packages. The demand elasticities and coefficients of the cost functions of the model are also derived from the source data in this worksheet.

The worksheet CaseExample7.123 performs the computations for a single case. The computations are made in two ways. First, they are made for a single market selected by the user. These computations are found in annotated tables as shown in Appendix B. They are also used to draw the graphs depicting solutions in the text and in Appendix B. Second, the worksheet performs un-annotated solution calculations for every markets in the USPS network. These solutions are used to compute the various summary statistics we cite in the text for the entire USPS network.
The folders in ProcessingCenters 4.123 and their contents are:

- Volumes - This folder holds the data for the average annual delivery volumes passing through 368 USPS processing plants for the period FY 1999 to FY 2005 and for FY 2005 alone. The data is shown by class, subclass and major worksharing category, and also by shape (letters, cards and packages). The delivery volumes are combined into the broader categories of non-commercial mail, commercial mail and packages using weights derived from system-wide unit delivery costs. Elasticities and USPS cost coefficients for FY 2005 are also derived for these mail categories.
- Stops Offices - This folder holds the data for the average numbers of delivery points and offices served by the 368 processing plants for the period FY 1999 to FY 2005 and for FY 2005 alone. The delivery point data has been aggregated into high density city carrier delivery points, low density city carrier delivery points, rural carrier and highway delivery points and post office boxes. The folder contains the regressions of average volumes on average delivery points (with PO boxes distributes) used to allocate volumes to delivery points. The allocation of FY 2005 volumes is also made in this worksheet. Finally, the data for volumes per office, delivery points per office and the number of such offices in the region served by the processing plant are assembled and sorted by delivery points per office. These are the representative local markets for the USPS delivery system.
- Elasticities - Own-price and cross-price elasticities are extracted from the econometric research of Thress (2006) and Pearsall (2005). Some of the price elasticities are derived from discount elasticities using FY 2005 RPW volumes.
- UDC Model - This folder holds a copy of the Unit Delivery Cost Model used by the PRC for the R2006-1 rate proceeding (PRC 2007). The system-wide unit delivery costs from this folder are used to form the weights for aggregating pieces into non-commercial mail, commercial mail and packages.
- BY 2005 - This folder holds selected cost component tables from the PRC's cost roll forward worksheets for the R2006-1 rate proceeding (PRC 2007). The data shown is for the base year used in R2006-1 which was FY 2005. The cost coefficients of the USPS cost model are derived as the sums of various elements in these tables. Unit variable delivery costs for city carriers and Rural carriers are the sums of the unit attributable costs for these carriers and carrier support components divided by FY 2005 volume. Unit costs per city and rural delivery point are the non-volume variable carrier and carrier support costs divided by the average numbers of city and rural delivery points in FY 2005. Unit variable nondelivery costs are the remainder of all attributable costs after the deduction of unit variable delivery costs divided by FY 2005 volume. Finally, the non-delivery cost per office is all of the remaining un-attributed cost divided by the number of delivery offices in FY 2005.
- Prices - The folder holds Fixed-Weight Index Prices from Thress (2006). These prices are from 1998 CQ1 through 2005 CQ4 and for mail by RPW class, subclass and discount category. They are used to compute FY 2005 prices for non-commercial mail, commercial mail and packages. These prices are used to calibrate the demand model.
The folders in ProcessingCenters 4.123 and their contents are:
- Summary - The tables in this folder summarize the solution equilibrium for a selected local market as seen in Appendix B and for the entire USPS distribution system. The appropriate equilibrium for the selected market is identified and displayed. The folder also contains a summary of the various calculations of consumer and producer surplus made for the selected market and for USPS. The folder contains the cell locations for inserting assumed values for the parameters that define the cases and for the Lagrange multipliers for obtaining three kinds of Ramsey rates. The locations of the cells for user inputs are shown in blue. The worksheet is set to recalculate manually when the F9 key is pressed. All of the calculations are performed automatically and a graph of the solution is drawn and displayed in the folder. Users must find the Lagrange multiplier values that leave an aggregate net surplus of minus $3,396,009$ by experimenting with different values for the Ramsey K.
- Cost - This folder holds the annotated tables as shown in Appendix B for the calibration of the cost functions for the selected local market.
- Demand - This folder holds the annotated tables as shown in Appendix B for the calibration of the demand functions for the selected local market.
- Case - This folder holds the annotated tables as shown in Appendix B for the four types of equilibriums for the selected local market. The folder also does the welfare calculations for the case and three benchmarks as shown in Appendix B.
- High Density, Low Density, Rural and PO Boxes - These are four identically structured folders. Each folder holds the un-annotated computations for the representative markets for all processing centers for a single class of delivery points. The computations reproduce the annotated computations shown in Appendix B and in the Cost, Demand and Case folders. The form of equilibrium is identified for each market and results are collected to facilitate the computation of the aggregate USPS results displayed in Summary.
- Cases - This folder is just a storage area for the input values and solution summary tables of previously computed cases.


[^0]:    * The authors are not now, nor have we ever been, members of the faculties of Northwestern University or the University of Auckland. Edward S. Pearsall can be reached at Pearsall@cox.net and Charles L. Trozzo at Mrtrozzo@comcast.net.

[^1]:    ${ }^{1}$ The descriptions here of a non-zero-sum non-cooperative two-person game and Nash equilibrium follow Luce and Raiffa's (1957) definition of an equilibrium pair of strategies. The equilibrium pair is commonly known as a Nash equilibrium after Nash (1950 and 1951).

[^2]:    ${ }^{2}$ Positive volumes were reported for only 320 of these plants over this time span.

[^3]:    ${ }^{3}$ The consumer surplus generated by commercial mail Products 2 and 3, are combined because of the cross elasticities of demand between these two products.

[^4]:    ${ }^{4}$ The descriptions here of a non-zero-sum non-cooperative two-person game and an equilibrium pair follow Luce and Raiffa (1957) 88-113. The equilibrium pair is also known as a Nash equilibrium after Nash (1950 and 1951).
    ${ }^{5}$ This mirrors the approach used by Pearsall (1976) to solve a constrained min-max problem with applications to anti-ballistic missile (ABM) defense. The attacker is in roughly the same role as the entrant setting his price in a contestable market since he sees the defense before assigning his warheads to possible targets. The defender acts as the incumbent setting his own price when he assigns ABMs to defend the targets. An optimal defense usually leaves the attacker indifferent

[^5]:    between targeting and not targeting some of the possible targets. A solution may be found by introducing probabilities for attacking these targets that are analogous to the state probability of entry for a contestable market.
    ${ }^{6}$ This is the same as maximizing expected net revenue (revenue minus variable costs) since the incumbent is always in the market and, consequently, always pays any fixed costs.

[^6]:    ${ }^{7}$ Baumol, Panzar and Willig (1988) cite the Stackelberg equilibrium while discussing the Bertrand-Nash assumption at 493. For a general discussion of Stackelberg solutions and disequilibriums in a duopoly setting see Henderson and Quandt (1958) 180-2.
    ${ }^{8}$ The stability of the Stackelberg equilibrium is dependent upon the assumption that there is no more than one entrant. If there are additional potential entrants with positive profits at E 2 , then these additional entrants will be encouraged to enter.

