Subsidy Design and Asymmetric Information: Wealth versus Benefits

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Public Provision Of A Private Good

- **Education**: scholarships based on merit or family wealth?
- **Health care**: treatment subsidies based on illness severity or income?
- **Family assistance**: day care subsidies based on family composition or income?

Literature: Huge, Many Papers

Typical problem: given Social Welfare Function and information assumptions about wealth or ability, derive optimal policy

Focus here: Implementation of ANY policy, not just optimal ones; no need for Social Welfare Function

- K. Arrow, "An Utilitarian Approach to the Concept of Equality in Public Expenditure," QJE, 1971: people differ in ability
- Blackorby C. and D. Donaldson, "Cash versus Kind. Self Selection and Efficient Transfers," AER, 1988: unobservable ability
- Besley T. and S. Coate, "Public Provision of Private Goods and the Redistribution of Income," AER, 1991: wealth unobservable
- DeFraja G., "The Design of Optimal Education Policies," RES, 2002: unobservable ability but known wealth

Subsidies When Information Is Incomplete

Subsidies based on benefits

Subsidies based on wealth

Benefits and Wealth together determine "willingness to pay"

Wealth-based allocations = benefit based allocations?

Wealth information = benefits information?

Missing information benefits or wealth means different costs?

Key Concept $\dashrightarrow$ ASSIGNMENT

Should type (w, ℓ) get the good?

Answers:

Wealth-based allocation $\neq$ benefit-based allocation

Wealth-based assignment $=$ benefit-based assignment

With general tax instruments, both kinds of subsidies require same cost

The Model

- **regulator allocates private good with limited budget** B
- **unit mass of consumers**
- **consumer gets either 0 or 1 unit**
- **cost of one unit of the good:** $c > 0$
- $0 \leq B < c$ **: budget not enough to cover all consumers**

- **Consumers: heterogeneous in two dimensions**
- **Consumer type:** (w, ℓ)
- **wealth** w, **and benefit** ℓ
- **Consumer getting 1 unit at price** p **:** $U(w - p) + \ell$
- U **increasing and concave:** $U' > 0$, $U'' < 0$
- **Consumer not getting the good:** $U(w)$
- $w \sim F$ **and** f **on** $[\underline{w}, \overline{w}]$; $\ell \sim G$ **and** g **on** $[\underline{\ell}, \overline{\ell}]$. **Independent**
- **Independence is unimportant; paper not about inferring one information from another**

Consumer's WILLINGNESS TO PAY: depends on wealth and benefit

Type (w, ℓ) willing to pay p

$$U(w - p) + \ell \geq U(w)$$

Monotonicity

Suppose (w, ℓ) is willing to pay, so is $(w', \ell') > (w, \ell)$

$w' > w \quad \ell' > \ell \Rightarrow$

$$\begin{aligned} U(w' - p) + \ell &> U(w') \\ U(w - p) + \ell' &> U(w) \\ U(w' - p) + \ell' &> U(w') \end{aligned}$$

Information:

costly; regulator observes either w or ℓ

Presentation here only on unknown ℓ; unknown w in the paper

w known; ℓ unkown: payment policy $t(w)$ based on w

Assignment:

the set of consumers getting the good

$$\alpha(t) = \{(w, \ell) : U(w - t(w)) + \ell \geq U(w)\}$$

Revenue Collected:

$$\int_{\alpha(t)} t(w) \, dF \, dG \equiv R(t)$$

Wealth Observable: Given Policy $t(w)$

THE INDIFFERENCE BOUNDARY

$$U(w - t(w)) + \ell = U(w) \Longrightarrow$$

$$\ell = \theta(w) \equiv U(w) - U(w - t(w))$$

For any w, find $\ell = \theta(w)$ s.t. type $(w, \theta(w))$ is indifferent

Special case: $t(w)$ differentiable

$\frac{\mathrm{d}\ell}{\mathrm{d}w} = U'(w) - U'(w - t(w))(1 - t'(w)) < 0$ if $t(w)$ is constant

Examples of indifference boundaries:

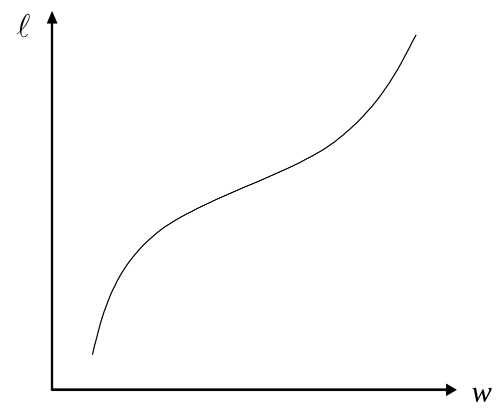

Figure 1. **Increasing Indifference Boundary**

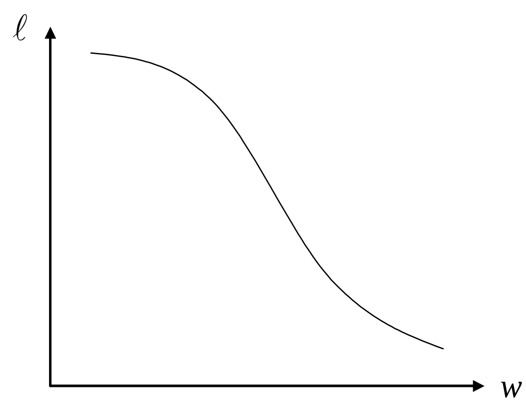

Figure 2. **Decreasing Indifference Boundary**

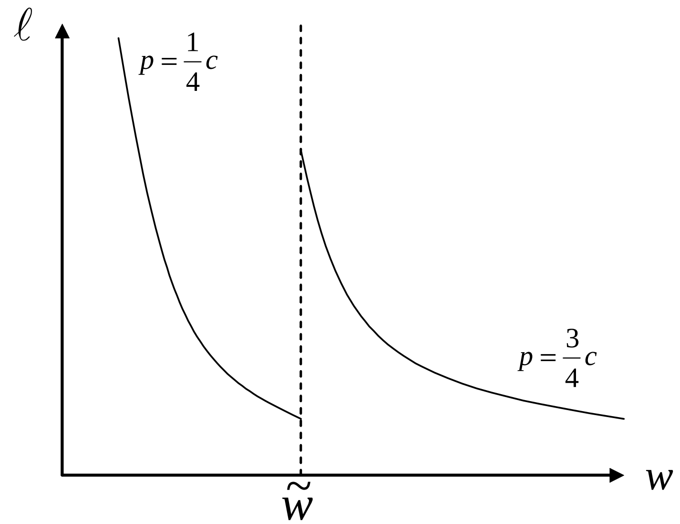

Figure 3. **Discontinuous Indifference Boundary**

$$t(w) = \frac{1}{4}c \qquad \text{for} \quad \underline{w} \leq w \leq \tilde{w}$$

$$t(w) = \frac{3}{4}c \qquad \text{for} \quad \tilde{w} < w \leq \overline{w}$$

Condition 1: Decreasing Indifference Boundary

- $t(w)$ continuous ($\Leftrightarrow \theta(w)$ continuous)
- $\theta(w)$ strictly decreasing

  ($\Longrightarrow \ell = \theta(w)$ has an inverse $w = \phi(\ell)$)

TRANSLATION: Given $t(w)$, construct $s(\ell)$ for same indifference boundary

Replace all w by $\phi(\ell)$: $\quad U(\phi(\ell) - s) + \ell = U(\phi(\ell))$

SUBSTITUTION $\dashrightarrow$ find *equivalent* $s(\ell)$

Example:

$U(w) = \ln w;$

from $t(w) = a + bw$ to $s(\ell) = \dfrac{a(e^\ell - 1)}{(1-b)e^\ell - 1}$

Proposition 1: Under Condition 1 (*Decreasing Indifference Boundary*)

Identical assignment sets under $t(w)$ and equivalent $s(\ell)$:

$$\alpha(t) = \beta(s)$$

Type (w, ℓ) almost never pays the same

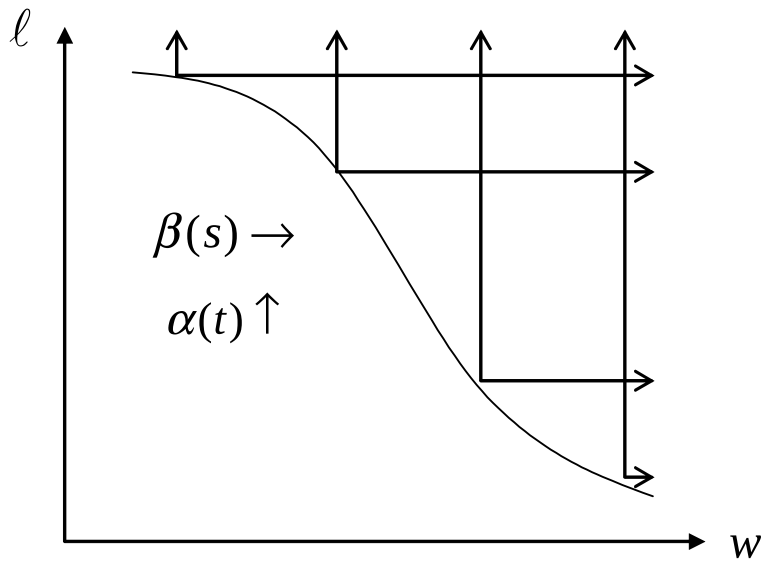

Figure 4. Downward Sloping Boundary: Direction of Preferences

Condition 2: Increasing Indifference Boundary

- $t(w)$ **continuous (**$\Leftrightarrow \theta(w)$ **continuous)**
- $\theta(w)$ **strictly increasing**

  ($\Longrightarrow \ell = \theta(w)$ **has an inverse** $w = \phi(\ell)$**)**

Corollay 1: Under Condition 2 (*Increasing Indifference Boundary*)

Assignment sets $\alpha(t)$ and $\beta(s)$ (almost) complements

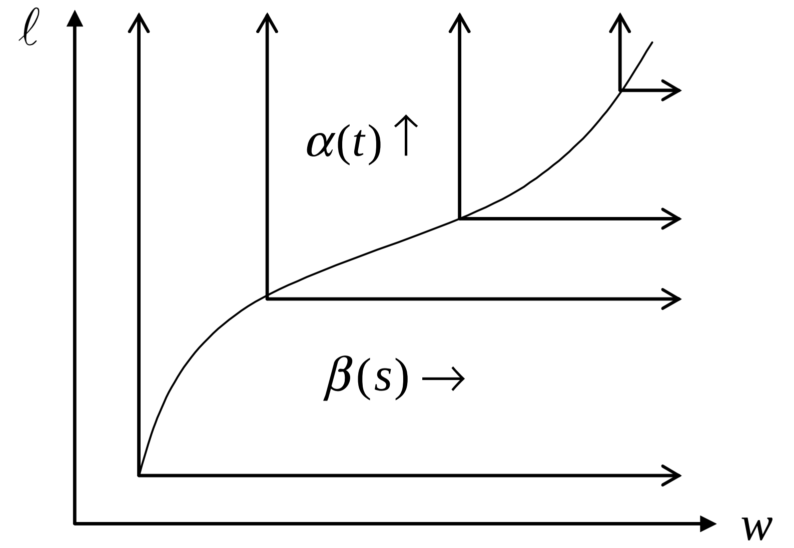

Figure 5. Upward Sloping Boundary: Direction of Preferences

Implementable Assignment Set

Regulator wants to implement an assignment $\Omega \subset [\overline{w}, \underline{w}] \times [\underline{\ell}, \overline{\ell}]$

- Ω **implementable by wealth-based policy if** $\exists\ t(w) : [\underline{w}, \overline{w}] \to \mathbb{R}^+$ **s.t.** $\Omega = \{(w, \ell) : U(w - t(w)) + \ell \geq U(w)\}$
- **Analogous definition for** Ω **implementable by benefit-based policy** $s(\ell)$
- Ω **implementable SIMULTANEOUSLY by wealth-based and benefit-based policies:**

$$\Omega = \{(w, \ell) : U(w - t(w)) + \ell \geq U(w)\} = \{(w, \ell) : U(w - s(\ell)) + \ell \geq U(w)\}$$

Example of an assignment set implementable by $t(w)$ but not by $s(\ell)$:

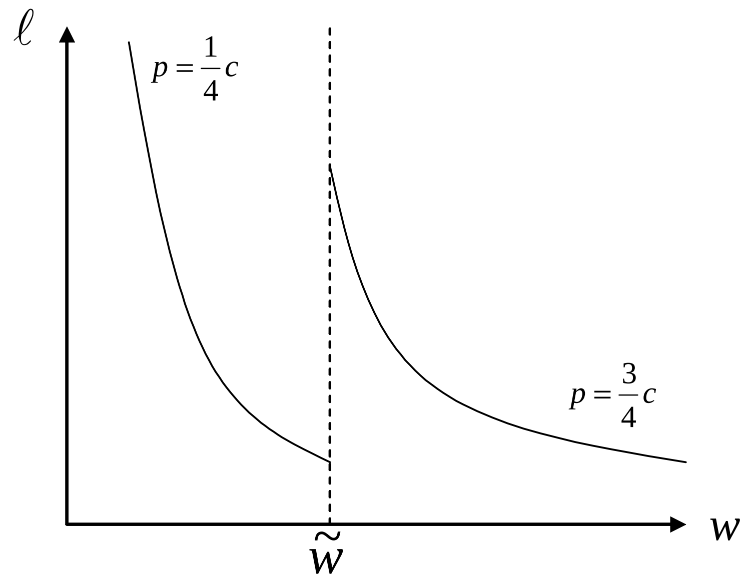

Figure 6. Assignment Set

Ω is subset above the two downward sloping curves,

Ω implemented by $t(w)$—but never by an $s(\ell)$

Proposition 3: Simultaneous Implementation

If $\Omega = \{(w,\ell) : U(w - t(w)) + \ell \geq U(w)\} = \{(w,\ell) : U(w - s(\ell)) + \ell \geq U(w)\}$**, for some** t **and** s

then $t(w)$ **and** $s(\ell)$ **must be**

- **continuous and**
- **induce a decreasing indifference boundary.**

Conditions 1 and 3 are necessary and sufficient for simultaneous assignment implementation

Intuition:

- If Ω is implementable by $t(w)$ and $s(\ell)$, it must be closed
- A closed set has a continuous boundary
- If the boundary is continuous, $t(w)$ and $s(\ell)$ are continuous
- Indifference boundary must be strictly downward sloping

Revenue

Proposition 4: Unless $t(w) = s(\ell) = k$, a constant, $\alpha(t) = \beta(s) \Longrightarrow R(t) \neq R(s)$ for generic distributions F, G

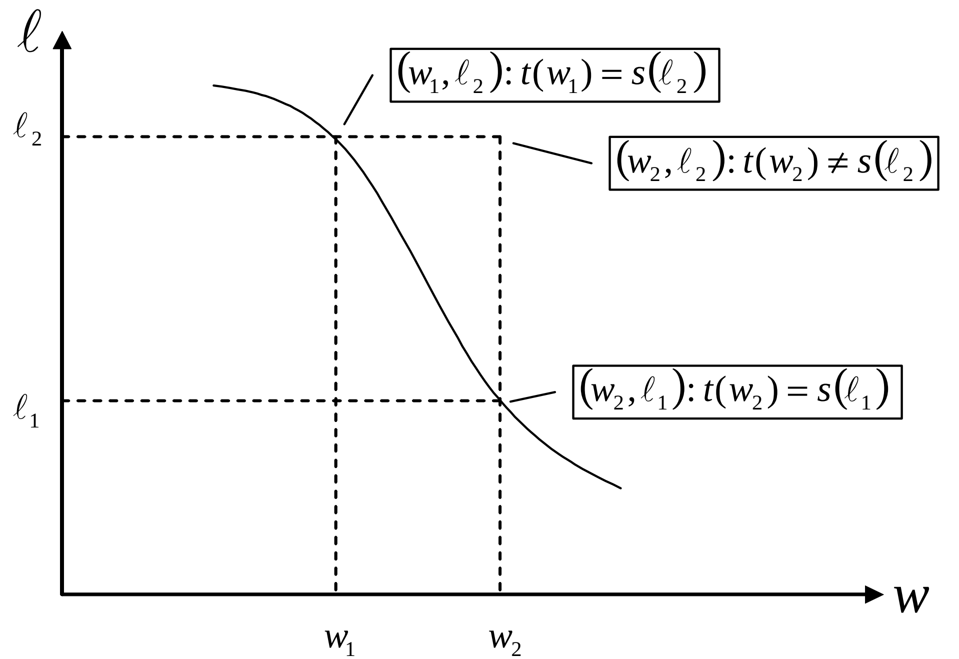

Figure 7. Nonequivalent Revenue

All inframarginal consumers pay different amounts according to t or s

Equivalent Revenue and General Subsidy

Two payments:

- $t_1(w)$ **when the consumer chooses not to buy the good**
- $t_2(w)$ **when the consumer chooses to buy the good**

If the boundary is strictly decreasing, can translate $t_1(w)$ $t_2(w)$ to equivalent $s_1(\ell)$ $s_2(\ell)$

Such s_1 and s_2 are NOT unique (one equation, two unknowns)

When general taxation or subsidy is possible, assignment and expected cost can be identical (two equations, two unknowns)

Proposition 6:

- **Suppose that a regulator sets a wealth-based policy $t(w)$, or equivalently, a budget allocation policy $B(w)$ for consumers with wealth w, to maximize a social welfare function.**
- **Suppose that the optimal budget allocation policy is increasing in w**
- **Then the optimal wealth-based policy must give rise to a strictly decreasing indifference boundary.**

Conclusions

- **Obviously, wealth-based subsidies generally must be different from benefit-based subsidies**
- **But we show that they CAN be ASSIGNMENT-EQUIVALENT**
- **The strength of the analysis: not based on OPTIMAL policies**
- **Relate different kinds of information**
- **Translate one type of policy to another for similar allocations**
- **Which information to collect?**
- **Collection cost and implementation cost can be considered separately**
- **Indivisible good assumption is critical, but seems natural for us to focus on assignments**