Endogenous information and Self-Insurance in Insurance Markets: a Welfare Analysis

Francesca Barigozzi∗ and Dominique Henriet†

December 2007

Abstract

We analyze a model where decision-makers are initially uninformed of their risk type and can obtain such information by performing a costless test before insurance policy purchase. Information status can be concealed or revealed to insurers at the discretion of decision-makers. Moreover, information has decision-making value since it allows to optimally choose a self-insurance action (secondary prevention). First insurers propose contracts to decision-makers. Then, decision-makers decide whether to perform the test and, possibly, whether to show it to insurers. Then decision-makers accept a contract and, finally, they choose prevention.

We focus, in particular, on the welfare properties of equilibria and provide a simple graphical analysis. The case of genetic testing serves as an illustration.

Keywords: adverse selection; information gathering, classification risk, self-insurance.

JEL classification: D82; D83; G22.

1 Introduction

The standard assumption in insurance models is that consumers perfectly observe their risk type, while insurers do not. In many situations, however, consumers have only a vague perception of their probability of incurring a loss: they do not have ex-ante superior information. This is the case, for example, of health related risk. Nevertheless, recent developments in medical science makes genetic and other diagnostic tests for many diseases available to consumers: whenever consumers choose to undertake a test, they decide to acquire more precise information about their risk. This means that individuals can learn information about their risk of illness before purchasing the insurance contract: information is endogenous.

∗Department of Economics, University of Bologna, P.zza Scaravilli 2, 40126 Bologna (Italy). E-mail: francesca.barigozzi@unibo.it
†Department of Economics, Ecole Centrale Marseille, GREQAM and IDEP. E-mail: dominique.henriet@ec-marseille.fr
Consumers’ decision to learn information on their risk is influenced by the reaction of insurance market to such information. Market response to endogenous information is clearly essential in understanding consumers’ incentives to search for information and crucially depends on whether consumers’ information status is observable by insurers. Despite its importance, very few papers investigate the issue. Crocker and Snow (1992) first showed that, if insurers can observe consumers’ information status and test result and if consumers have no private prior information, then the private value of information is negative and consumers prefer to remain ignorant. The reason is that, when uninformed, consumers have access to a full insurance contract based on the average probability of loss in the population. On the contrary, if consumers decide to acquire information on their types, insurers can write contracts that depend on the consumers’ risk. Ex-ante risk-averse consumers obviously prefer the first scenario.

In the same vein Doherty and Thistle (1996) develop a model where some consumers are initially informed on their risk type and other are uninformed. They show that information has positive private value only when insurers cannot observe consumers’ information status, or if consumers can conceal that they performed the test. Both when information provided by the test is not verifiable and when the test result is certifiable, on the equilibrium all consumers learn their type. Doherty and Thistle (1996) analyzed the existence and characterization of equilibria under different configurations of information costs and benefits; however they focused on the case in which information has no decision-making value. In other words, consumers only choose whether to become perfectly informed on their risk or to stay ignorant: information does not create new opportunity and no (preventative) action can be taken.

However, information provided by genetic and other diagnostic tests allows consumers to take more efficient decisions: primary and secondary prevention measures are often available and their efficacy is higher the higher the precision of information about the individuals’ characteristics.

When information has decision-making value, consumers choose whether to become informed not only evaluating the consequences of information on the insurance premium but also taking into account the benefit of information in terms of more efficient actions. We consider in particular a self-insurance measure that reduces the loss when the negative outcome occurs. Self-insurance has been defined in opposition to self-protection; the latter being an action that reduces the probability of the loss. In medical terms self-insurance corresponds to secondary prevention. As an example, let us consider the BRCA1 and BRCA2 genetic mutations which are implicated in many hereditary breast cancer cases and the genetic mutation responsible for hereditary non-polyposis colorectal cancer (HNPCC). An individual who is positive to one of the mentioned tests can undertake effective preventive measures to detect the illness at an early stage, in fact screening tests as mammography and colonoscopy are available.

In our analysis consumers face two different risks: the first risk is standard and is related to the monetary loss in the bad-outcome. The second one is associated to the risk of being a high-risk and, thus, it corresponds to the risk of paying a high premium (classification risk). The market is not able to provide
insurance policies that cover the classification risk, despite the latter would obviously increase consumers’ welfare. Investigating the welfare losses due to such a market failure seems important to design efficient regulation as for genetic testing in insurance markets.

In our model information raises the following trade-off. On the one hand information allows to avoid under- or over-prevention. On the other hand, since no coverage for the classification risk is available in the market, consumers can be worse off when they gather information. The timing of actions in the model is the following: first insurance firms propose contracts to decision-makers. Second, decision-makers decide whether to perform the test and, possibly, whether to show it to insurers. Then decision-makers accept a contract and, given the insurance policy, they choose the preferred action.

Our results show that: (i) when insurers observe consumers’ information status and test result, the private and social value of information is almost always negative: information is acquired at the equilibrium only if the classification risk is low and/or the benefit of prevention is high and, however, even in this case consumers learn their type uniquely for intermediate values of prevention costs. In this specific case prevention choices are efficient, in all the other situations over- or under-prevention occurs. (ii) When consumers’ information status is not observable and the insurees can conceal the test result, at the equilibrium information is always gathered and low-risk consumers show the test result to insurance firms. In the equilibrium the private value of information is positive and prevention choices are always efficient. However the social value of information is negative since social welfare when consumers stay uninformed weakly dominates social welfare in the equilibrium allocation.

A model closely related to our is Doherty and Posey (1998). Also in their paper information has decision-making value, however these authors analyze the case where self-protection is possible for the high-risks; we instead consider the case where self-insurance is available to all decision-makers. Moreover, these authors study a model where the information provided by the test cannot be credibly transmitted to insurers. Finally, our simple model allows welfare analysis to be performed: we investigate, also graphically, the welfare losses due to endogenous information gathering when information has decision-making value and insurance against classification risk is missing.

The paper is organized as follows. Section 2 introduces the model set-up and analyses the decision-maker’s problem without insurance. Subsection 2.2 describes how insurance coverage affects consumers’ choice of prevention and defines the interim efficient allocation. Subsection 2.3 shows the first-best of the model and discuss how to decentralize such allocation in the market. In section 3 the equilibrium in the insurance market is obtained and characterized allowing for different informational structures: first the case where information is symmetric and then the case where insurers do not observe decision-makers’ information status and test result are analyzed. The private and social welfare of

---

1 Policies covering the classification risk have been called "genetic insurance" by Tabarrok (1994). We will discuss them in subsection 2.3.1.
information in the different equilibrium allocations are also investigated. Section 4 provides some final remarks.

2 The model

Decision-makers are endowed with a fixed amount of wealth $w$, and are characterized by the von Neumann-Morgenstern utility function $u(w)$, increasing and concave. They face the risk of a monetary loss $L(a)$, where $0 < L(a) < w$. The action $a$ is a self-insurance measure and we assumed it is observable. If we interpret $L(.)$ as the monetary equivalent of a negative health shock, the action $a$ refers to secondary prevention or early detection of disease: in the real world screening tests are generally observable and certifiable. The action can take only two values, 0 and 1 (decision-makers perform the screening test or not) and makes the loss decrease such that $L(1) = l < L(0) = L$. Moreover, the action $a$ is taken before the realization of the risk and implies a utility cost $\Psi(a)$, with $\Psi(0) = 0$ and $\Psi(1) = \Psi$.

We consider two decision-makers’ types, the high- and the low-risks, respectively characterized by the probabilities $p_L$ and $p_H$, with $0 < p_L < p_H < 1$. We assume that the probabilities $p_L$ and $p_H$ are fixed, so that no ex-ante moral hazard problem exists. The proportion of high- and low-risk types in the population is $\lambda$ and $(1-\lambda)$ respectively. These parameters are assumed to be common knowledge.

Consumers do not know their type ex-ante. The loss probability of uninformed individuals is $p_U = \lambda p_H + (1-\lambda) p_L$. Information can be gathered without cost by performing a genetic test. Risk neutral insurance companies can propose insurance contracts to consumers.

2.1 The decision-maker’s problem without insurance

Let us first examine the case where no insurance is available. We focus on the decision whether to acquire information or not when the risk of the loss $L(a)$ is not covered in the insurance market. Since no insurance is available, the classification risk is not an issue here.

The decision-maker chooses whether to gather information or not by anticipating that, in the subsequent stage, he will choose the optimal action given the information possibly acquired.

Proceeding backward, let us consider the second stage, that is the choice of the preventative action. An individual characterized by loss probability $p_i \in \{p_L, p_U, p_H\}$ who chooses action $a$, achieves the following expected utility level:

$$V(p_i, a) = p_i u(w - L(a)) + (1 - p_i) u(w) - \Psi(a)$$

The decision-maker chooses a positive amount of prevention if $V(p_i, 1) \geq V(p_i, 0)$, that is if $p_i u(w - l) + (1 - p_i) u(w) - \Psi \geq p_i u(w - L) + (1 - p_i) u(w)$,
Remark 1, their expected utility becomes:

\[ p_l \geq \frac{\Psi}{u(w-l)-u(w-L)} = \frac{\Psi}{\Delta_0} \]  

(1)

The term \( \Delta_0 \) is positive and measures the benefit from prevention. When the benefit from prevention is large and its cost \( \Psi \) is low, inequality (1) is easily verified. Put differently, inequality (1) shows that decision-makers choose prevention when their loss probability is sufficiently high.

**Remark 1** The uninsured decision-makers choose prevention if inequality (1) holds. This implies that incentives to perform prevention are increasing in the risk.

Let us define \( \hat{a}(p_l) \) the action chosen by an individual characterized by probability of loss \( p_l \) and \( \hat{V}(p_l) \) the individual’s indirect expected utility when the probability is \( p_l \) and the chosen action is \( \hat{a}(p_l) \).

In the first stage, uninsured decision-makers compare expected utility when they stay uninformed to expected utility when they gather information, that is \( \hat{V}(p_U) \) to \( \lambda \hat{V}(p_H) + (1-\lambda)\hat{V}(p_L) \). The following remark can be stated:

**Remark 2** Without insurance, (i) when prevention is optimal for low-risks \( (p_L \geq \frac{\Psi}{\Delta_0}) \) and when no-prevention is optimal for high-risks \( (p_H \leq \frac{\Psi}{\Delta_0}) \), decision-makers are indifferent between staying uninformed and gathering information. Information has zero private and social value (ii) When \( p_L \leq \frac{\Psi}{\Delta_0} \leq p_H \) uninsured decision-makers acquire information on their risk-type. Information has a positive private and social value.

**Proof.** (i) When \( p_L \geq \frac{\Psi}{\Delta_0} \) the optimal action for low-type decision-makers corresponds to a positive level of prevention: \( \hat{a}(p_L) = 1 \). Given Remark 1, this implies: \( \hat{a}(p_U) = \hat{a}(p_H) = 1 \), and, \( \hat{V}(p_U) = \lambda \hat{V}(p_H) + (1-\lambda)\hat{V}(p_L) \). Whereas when \( p_H \leq \frac{\Psi}{\Delta_0} \) the optimal action for high-type decision-makers corresponds to no-prevention: \( \hat{a}(p_H) = 0 \). Thus, \( \hat{a}(p_U) = \hat{a}(p_L) = 0 \) and, again, \( \hat{V}(p_U) = \lambda \hat{V}(p_H) + (1-\lambda)\hat{V}(p_L) \). (ii) Suppose first that \( p_L \leq p_U \leq \frac{\Psi}{\Delta_0} \leq p_H \). When uninsured, decision-makers do not exert prevention such that \( \hat{V}(p_U) = p_Uu(w-l)+(1-p_U)u(w) \). If decision-makers acquire information, given again Remark 1, their expected utility becomes:

\[ \lambda \hat{V}(p_H) + (1-\lambda)\hat{V}(p_L) = \lambda (p_Hu(w-l)+(1-p_H)u(w)-\Psi) \\
\quad + (1-\lambda)(p_Lu(w-L)+(1-p_L)u(w)) \\
\quad = \lambda p_Hu(w-l)+(1-\lambda)p_Lu(w-L)+(1-p_U)u(w)-\lambda \Psi \]

It is easy to verify that \( \hat{V}(p_U) < \lambda \hat{V}(p_H) + (1-\lambda)\hat{V}(p_L) \). Suppose now that \( p_L \leq \frac{\Psi}{\Delta_0} \leq p_U \leq p_H \). Here uninsured consumers choose prevention and \( \hat{V}(p_U) = p_Uu(w-l)+(1-p_U)u(w)-\Psi \). By comparing \( \hat{V}(p_U) \) and \( \lambda \hat{V}(p_H) + (1-\lambda)\hat{V}(p_L) \) it is easy to show that, again, \( \hat{V}(p_U) < \lambda \hat{V}(p_H) + (1-\lambda)\hat{V}(p_L) \). ■

In words: when the optimal action for low-risk decision-makers is a positive level of prevention, prevention is optimal also for uninformed and high-risk
decision-makers: uninformed individuals are indifferent between acquiring and not acquiring information. Whereas, when high-risks choose no-prevention, the same action is optimal also for uninformed and high-risk decision-makers: again uninformed individuals are indifferent between acquiring and not acquiring information.

Note that, when the action $a$ is not available, uninsured decision-makers are always indifferent between staying uninformed and learning their type. In other words, when the decision-maker is uniquely concerned with information gathering and insurance coverage is not available, $V(p_U) = \lambda V(p_H) + (1 - \lambda)V(p_L)$ always holds.

More interesting is the case where $p_L \leq \frac{\Psi}{\Delta_0} \leq p_H$, or positive prevention is optimal for high-risks whereas no-prevention is the optimal choice for low-risks. Here information is useful for appropriate prevention decisions. The previous remark shows that, in such a case, uninformed consumers acquire information.\footnote{Such result is robust to the introduction of a cost for the test, provided the cost is sufficiently low.}

This is not surprising: since no insurance is available, when deciding to acquire information individuals do not face any classification risk, they simply anticipate the positive effect of information in terms of better prevention choices.

Let us assume that, when indifferent between gathering and not gathering information, decision-makers do not learn information. Social welfare when insurance is not available $W^*_0$ is described in the following remark and represented in Figure 1 as a function of prevention cost $\Psi$:

\textbf{Remark 3 (Social welfare without insurance)} When insurance is not available social welfare $W^*_0$ is:

- $p_Uu(w - l) + (1 - p_U)u(w) - \Psi$ for $0 \leq \Psi \leq \Delta_0p_L$: decision-makers stay uninformed and perform prevention.
- $\lambda p_Hu(w - L) + (1 - \lambda)p_Lu(w - L) + (1 - p_U)u(w) - \lambda\Psi$ for $\Delta_0p_L < \Psi \leq \Delta_0p_H$: decision-makers learn information and high-types perform prevention.
- $p_Uu(w - l) + (1 - p_U)u(w)$ for $\Psi > \Delta_0p_H$: decision-makers stay uninformed and do not prevent.

We conclude this section by observing that, without insurance, information has positive private value only when $\Delta_0p_L < \Psi \leq \Delta_0p_H$. In all the other cases information has zero private value and decision-makers stay uninformed.

\subsection*{2.2 Interim optimal insurance}

We analyze here the optimal insurance contract from an \textit{interim} perspective, that is when decision-makers learnt their risk and insurers observe it.

In the following $P_i$ indicates the insurance premium and $I_i$ the indemnity reimbursed by insurers when the negative shock realizes. Assuming risk-neutral
Prevention is positive if \( p \) corresponds to a predictable action and, thus, it does not properly represent an insurable all for prevention costs. However, since prevention is performed before the risk realization, inequality can be rewritten as characterized by risk \( p \) expected indemnity. Competition brings insurance pro

Remark 4

Under full insurance (i) prevention is performed if inequality (2) holds. (ii) Incentives to perform prevention are increasing in the decision-maker’s risk. (iii) Given a risk \( p \), incentives to perform prevention are lower than without insurance.

Proof. (i) It comes directly from discussion above. (ii) It is easy to prove that \( \Delta(p_i) \) is an increasing function. In fact, \( \frac{\partial \Delta(p_i)}{\partial p_i} = -p_i u'(w - p_i l) + p_i w'(w - p_i L) \) \( = p_i [u'(w - p_i L) - u'(w - p_i l)] > 0 \). (iii) Inequality (1) is the condition for positive prevention choice without insurance and can be written as \( p_i \Delta_0 \geq \Psi \). We compare inequality (1) with (2), and we prove that \( \Delta(p_i) \leq p_i \Delta_0 \). The latter inequality can be rewritten as \( u(w - p_i l) - u(w - p_i L) \leq p_i \left[ u(w - l) - u(w - L) \right] \) \( = p_i \left[ u(w - l) - u(w - L) + (1 - p_i) [u(w) - u(w)] \right] \) or \( u(w - p_i l) - u(w - p_i L) \leq \) \( p_i \left[ u(w - l) + (1 - p_i) u(w) \right] - p_i [u(w - L) + (1 - p_i) u(w)] \) which is true given that \( l < L \) and \( u(\cdot) \) concave.

Note that, when \( \Delta(p_i) < \Psi \leq p_i \Delta_0 \), the fully insured decision-maker characterized by risk \( p_i \) does not exert prevention although the uninsured one does. As we expected, insurance reduces the benefits from prevention and conditions for positive prevention become stronger: insurance discourages prevention for a given risk.

Total welfare in the interim optimal allocation is:

\[
W_i^* = \lambda u(w - p_i L(\tilde{h}(p_i))) + (1 - \lambda) u(w - p_i L(\tilde{h}(p_i))) - \lambda \Psi(\tilde{h}(p_i)) - (1 - \lambda) \Psi(\tilde{h}(p_i))
\]

where \( \tilde{h}(p_i) \) is one if inequality (2) holds and zero otherwise. From Remark 4:

\footnote{Note that prevention imposes a utility cost that is not covered by the insurance policy. Thus, decision-makers obtain full insurance for the monetary loss \( L(a) \) and no-insurance at all for prevention costs. However, since prevention is performed before the risk realization, it corresponds to a predictable action and, thus, it does not properly represent an insurable cost.}
Definition 1. The interim optimal allocation $W^*_I$ is such that all decision-makers are informed and fully insured. Premium is type-dependent and equal to $P_i = p_i L(\hat{a}(p_i))$. Moreover:

- when $0 \leq \Psi \leq \Delta(p_L)$ both types perform prevention.
- when $\Delta(p_L) < \Psi \leq \Delta(p_H)$ only high-types perform prevention.
- when $\Psi > \Delta(p_H)$ none prevents.

Figure 1 below describes total welfare under interim optimal insurance as a function of the cost of prevention $\Psi$ and offers a graphical representation of 3.

We will show in Section 3.2 that the interim optimal allocation is obtained as an equilibrium when the decision-makers’ information status is not observable by insurers. This means that, with endogenous information, the insurance market provides incentives for information acquisition and prevention choices turn out to be efficient.

2.3 Ex-ante optimal insurance (the first-best)

We now define the ex-ante optimal allocation as the allocation maximizing ex-ante expected utility under the feasibility constraint and such that decision-makers acquire information after the contract is offered. Both coverages for the classification risk and for the risk of the loss are thus available. It is as if the social planner designs the contract “under the veil of ignorance”. Everything is observable and contractible. However, as before, the cost of the action $a$ is not insurable.

The first-best contract solves:

$$\max_{P_H, I_H, P_L, I_L, a_H, a_L} \left\{ \lambda(P_H u(w - P_H - L(a_H) + I_H) + (1 - p_H)u(w - P_H) - \Psi(a_H)) + (1 - \lambda)(p_L u(w - P_L - L(a_L) + I_L) + (1 - p_L)u(w - P_L) - \Psi(a_L)) \right\}$$

s.t.: $\lambda P_H + (1 - \lambda)P_L \geq \lambda p_H I_H + (1 - \lambda)p_L I_L$

Obviously the first-best implies full insurance: $I_i = L(\hat{a}_i), i = H, L$. Moreover, the optimal premium is uniform and equal to $P^* = \lambda p_H L(\hat{a}_H) + (1 - \lambda)p_L L(\hat{a}_L)$. Since both types pay the same premium irrespective of their loss $L(\hat{a}_i)$ and get utility $u(w - P^*)$, when the action performed by the two types is different the ones performing prevention suffer the disutility loss $\Psi$ and, thus, are characterized by a lower utility. In this case the social planner may want to introduce two transfers aiming at redistributing between the two groups the monetary equivalent of the disutility loss.

The optimal values of $\hat{a}_i$ are the solutions of:

$$\max_{a_H, a_L} W_{EA}(a_H, a_L) = u(w - \lambda p_H L(a_H) - (1 - \lambda)p_L L(a_L)) - \lambda \Psi(a_H) - (1 - \lambda)\Psi(a_L)$$

(4)
Note that, according to which group performs prevention, four possible values of the welfare function are possible:

\[ W_{1}^{*} = u(w - p_U) - \Psi \]  
(5)  
\[ W_{2}^{*} = u(w - \lambda p_H l - (1 - \lambda)p_L L) - \lambda \Psi \]  
(6)  
\[ W_{3}^{*} = u(w - \lambda p_H L - (1 - \lambda)p_L L) - (1 - \lambda)\Psi \]  
(7)  
\[ W_{4}^{*} = u(w - p_U L) \]  
(8)  

Welfare is \( W_{1}^{*} \) or \( W_{4}^{*} \) when both types (no type) perform prevention. Whereas \( W_{2}^{*} \) and \( W_{3}^{*} \) correspond to the cases where only high-types and only low-types respectively make prevention. Between \( W_{2}^{*} \) and \( W_{3}^{*} \) probably the most natural case to analyze is the one where prevention is performed by high-types, as in the interim optimal allocation. Thus, we assume that \( \forall \Psi, W_{2}^{*} \geq W_{3}^{*} \).

It can be easily checked that this happens when the following assumption holds:

**Assumption 1:**  
\[ \begin{align*}  
\text{a)} & \quad \lambda p_H \geq (1 - \lambda)p_L  
\text{b)} & \quad \lambda \leq (1 - \lambda)  
\end{align*} \]

Inequalities 1a and 1b are sufficient conditions such that it is socially optimal that only high-risk decision-makers perform prevention in the interval of prevention cost \( \Psi \) specified below. Note that, according to assumption 1b, the proportion of high-risks in the population must be lower than that of low-risks: \( \lambda \leq 1/2 \). Assumption 1a and 1b together indicate that the loss probability \( p_H \) must be sufficiently higher than \( p_L \), in particular \( p_H \geq \frac{1-\lambda}{\lambda} p_L \) where \( \frac{1-\lambda}{\lambda} \geq 1 \).

**Definition 2** The first-best \( W_{EA}^{*} \) is the allocation such that decision-makers are informed and fully insured. They pay the uniform premium \( P^* = \lambda p_H L(\hat{a}_L) + (1 - \lambda)p_L L(\hat{a}_L) \) and, under assumption 1:

- **when** \( 0 \leq \Psi \leq \frac{u(w-p_U l) - u(w-\lambda p_H l -(1-\lambda)p_L L)}{1-\lambda} = \Psi_1 \) **both types perform prevention.**
- **when** \( \Psi_1 \leq \Psi \leq \frac{u(w-\lambda p_H l -(1-\lambda)p_L L) - u(w-p_U L)}{\lambda} = \Psi_2 \) **only high-types perform prevention.**
- **when** \( \Psi \geq \Psi_2 \) **none prevent.**

Definition 2 shows that, as in the interim optimal allocation, when the cost of prevention is low, both types perform prevention; as the cost of prevention increases, only high-types make prevention; finally, when the cost is sufficiently high, no prevention is performed. Figure 2 below describes social welfare in first-best \( W_{EA}^{*} \) as a function of the cost of prevention \( \Psi \).

Obviously, since the ex-ante optimal allocation covers the classification risk whether the interim one does not, the ex-ante optimal allocation dominates the interim one.
2.3.1 How to implement the first-best: insurance against classification risk

Tabarrok (1994), discussing the issue of genetic testing, proposes to decentralize the \textit{ex-ante} optimal allocation by creating a market selling insurance against the classification risk (he calls it "genetic insurance"). Such a policy should be mandatory for those who decide to gather information: information acquisition is possible only after insurance against classification risk has been purchased. That can be enforced by making it illegal for physicians and laboratories to run tests without proof that genetic insurance has been bought. In this way welfare losses due to adverse-selection problems can be avoided.

Let us consider, in our model, compulsory insurance against classification risk for those who want to learn information. If decision-makers purchase insurance against classification risk, as in the \textit{ex-ante} optimal allocation they pay the premium $P^* = \lambda p_H L (\hat{a}_H) + (1 - \lambda) p_L L (\hat{a}_L)$. Once insurance against classification risk has been bought, decision-makers perform the test, acquire information and exhibit their test result to the insurer. Informed high-types receive reimbursement $p_H L (\hat{a}_H)$ and, with that amount, purchase fair insurance in the market; informed low-types receive reimbursement $p_L L (\hat{a}_L)$ and purchase fair insurance as well. Thus, decision-makers must choose whether staying uninformed and receiving utility $W^*_{U} = u (w - p_U L (\hat{a}_U)) - \Psi (\hat{a}_U)$ or purchasing insurance against classification risk, performing the test and receiving expected utility (4). Note that when $\Psi \leq \Psi_1$ and $\Psi \geq \Psi_2$ both types choose the same action, then $P^* = p_U L (\hat{a}_U)$ and decision-makers are indifferent between learning information and staying uninformed. On the contrary, when $\Psi_1 \leq \Psi \leq \Psi_2$, $P^* \neq p_U L (\hat{a}_U)$ and $W^*_U$ is dominated by $W^*_E_A$. Obviously this happens since, with information acquisition, prevention is targeted to the decision-makers’ risk. In Figure 2 utility $W^*_U$ is represented by the dotted line: according to Remark (4) fully insured, uninformed individuals make positive prevention for $\Psi \leq \Delta (p_U)$ and do not prevent for $\Psi > \Delta (p_U)$.

We can conclude that compulsory insurance against classification risk for those who want to gather information does allow the utilitarian optimum to be decentralized.

Note that insurance against classification risk presents some similarities with Cochrane’s (1995) "time-consistent insurance". As Cochrane writes, time-consistent insurance provides insurance against classification risk as well as insurance against the uncertain component of one period health expenditures. Moreover, the key feature for time-consistent insurance contracts is a severance payment: a person whose premium increases (for example because a long-term illness is diagnosed) receives a lump sum equals to the increased present value of his premium. The severance payment scheme compensates for changes in premium and allows every consumer to purchase insurance at his actuarially fair premium.

What is different with respect to time-consistent insurance is that, in our context, decision-makers decide whether to gather private information on their risk, and insurance firms, when designing insurance policies, must anticipate
consumers’ information choice. Adverse selection can be a crucial issue.

3 The insurance market

*Ex-ante* all decision-makers are uninformed; they can remain uninformed or they can decide to perform a test. As in the real world, insurance against classification risk is not available here; insurance firms offer coverage only for the monetary loss $L(a)$. The insurance market is assumed to be competitive. The timing of actions is the following: first, insurance companies propose contracts which can depend on decision-makers’ information status and type according to their observability; then insurees decide whether to perform the test, accept a contract and choose their level of prevention.

We first consider the case with symmetric information between decision-makers and insurance firms and then we analyze the case with asymmetric information.

3.1 Insurance firms observe decision-makers’ information status and test result

In this subsection insurance firms observe both the decision-makers’ informational status and the test result. Thus, insurance contracts can be contingent on informational status and risk, as well as on preventative action. Competition brings profits to zero and full-insurance is provided. Insurance firms can offer three different types of contract: the full coverage contract for uninformed, for high-risk and for low-risk decision-makers.

If decision-makers choose to remain uninformed, they obtain with certainty the full coverage contract for uninformed and achieve the level of utility $W^*_U$ represented in Figure 2 and described in the previous subsection. When, on the contrary, decision-makers choose to perform the test, they obtain the full coverage contract for high-risks with probability $\lambda$ and the full coverage contract for low-risks with probability $1 - \lambda$. That is, they obtain the interim optimal allocation $W^*_I$ defined in section 2.2 and represented in Figure 1. Thus, information gathering depends on the comparison between utility when decision-makers are uninformed $W^*_U$ and expected utility defined by the interim optimal allocation $W^*_I$.

Two antagonistic effects are at stake to determine the relative positions of $W^*_U$ and $W^*_I$. On the one hand, information gathering amounts in taking the classification risk and hence has a negative effect on welfare; on the other hand, it allows a more efficient choice of prevention which is beneficial. Intuitively, when the classification risk is not too large $(p_H - p_L)$ and/or when consumers are not too risk-averse, they prefer to perform the test: the gain due to information is larger than the loss due to increased risk.

In our graphical analysis two cases are possible: either $W^*_U$ always dominates $W^*_I$; we call it Equilibrium of type 1. Or $W^*_U$ and $W^*_I$ cross each other inside
the interval \([\Delta(p_L), \Delta(p_H)]\) and values of \(\Psi\) for which \(W^*_I\) dominates \(W^*_U\) exist; we call it Equilibrium of type 2 (see Figure 3). In particular:

**Lemma 1** When insurance firms observe decision-makers’ informational status and the test result, if any, decision-makers’ expected utility with the test dominates utility without the test in the interval \(\Psi_3 < \Psi < \Psi_4\), where:

\[
\Psi_3 = \frac{1}{1 - \lambda} \left[ u(w - p_U L) - \lambda u(w - p_H L) - (1 - \lambda) u(w - p_L L) \right]
\]

\[
\Psi_4 = \frac{1}{\lambda} \left[ \lambda u(w - p_H L) + (1 - \lambda) u(w - p_L L) - u(w - p_U L) \right],
\]

if and only if the following condition is satisfied:

\[
\frac{\left[ u(w - p_U L) - u(w - p_H L) \right]}{\left[ u(w - p_L L) - u(w - p_U L) \right]} < \frac{L}{T} \tag{9}
\]

**Proof.** Functions \(W^*_I\) and \(W^*_U\) cross each other twice if \(W^*_I\) calculated in \(\Psi = \Delta(p_U)\) is larger than \(W^*_U = u(w - p_U L)\) (see Figure 3); this writes:

\[
\lambda u(w - p_H L) + (1 - \lambda) u(w - p_L L) - \lambda \Delta(p_U) > u(w - p_U L) \tag{10}
\]

Substituting \(\Delta(p_U) = u(w - p_U L) - u(w - p_L L)\) and rearranging inequality (10), condition (9) can be easily found. \(\Psi_3\) is the value on the left of \(\Delta(p_U)\) such that \(W^*_I = u(w - p_U L) - \Psi\), whereas \(\Psi_4\) is the value on the right of \(\Delta(p_U)\) such that \(W^*_I = u(w - p_U L)\). □

Insert figure 3 here

**Remark 5** Condition (9) is verified if \(p_H - p_L\) is smaller than a threshold \(\alpha \left( \frac{L}{T} \right)\) which is increasing with \(L/T\).

**Proof.** See Appendix 5.1. □

The previous remark shows that, when the classification risk is low and/or when the potential gain of prevention is high, decision-makers learn information since, in a sense, its benefits are higher than its costs.

From Lemma 1 and from the previous discussion:

**Proposition 1** When insurance firms observe consumers’ informational status and the test result, if any, (i) decision-makers always remain uninformed (Equilibrium of type 1) if the opposite of inequality (9) holds. (ii) Decision-makers perform the test for \(\Psi_3 < \Psi < \Psi_4\) and remain uninformed elsewhere (Equilibrium of type 2) if inequality (9) holds.

Proposition 1 extend Croker and Snow’s result to the case of secondary prevention. Croker and Snow (1992) show that, when insurance against the classification risk is not available and under symmetric information between insurance firms and decision-makers, the private value of information is negative.
In fact risk-averse decision-makers prefer to stay uninformed than to face the classification risk.

However, when information has decision-making value, for intermediate values of prevention cost consumers may prefer to acquire information. In fact, when prevention cost is close to $\Delta(p_U)$ ignorance can make decision-makers’ prevention choices very inefficient: for $\Psi_3 \leq \Psi \leq \Delta(p_U)$ uninformed low-types perform prevention even if prevention cost is too high given their risk and, for $\Delta(p_U) \leq \Psi \leq \Psi_4$, uninformed high-types do not perform prevention even if prevention cost is sufficiently low given their risk. Moreover, as Remark 5 shows, this is the case when the classification risk is low and/or the benefit from prevention is high.

Moreover, the lower the proportion of high-risk in the population, the less negative the slope of the line $\lambda u(w - p_H l) + (1 - \lambda)u(w - p_L l) - \lambda \Psi$ (see Figure 1) and the higher the probability that $W^*_I$ and $W^*_U$ cross each other inside the interval $[\Delta(p_L) < \Psi < \Delta(p_U)]$. In fact a low $\lambda$ implies that social welfare $W^*_I$ decreases slowly with $\Psi$ when only high-risks perform prevention: close to $\Delta(p_U)$ the social cost of prevention $\lambda \Psi$ is low.

The following corollary summarizes the welfare properties of the equilibrium allocations described in Proposition 1.

**Corollary 1 (Welfare properties of the equilibrium allocation)** When insurance against classification risk is not available and information is symmetric: (i) in type 1 Equilibrium social welfare is $W^*_U$: over-prevention arises for $\Psi_1 < \Psi \leq \Delta(p_U)$ whereas under-prevention arises for $\Delta(p_U) < \Psi < \Psi_2$. (ii) in type 2 Equilibrium, in the interval $\Psi_3 \leq \Psi \leq \Psi_4$ prevention choices are efficient and the interim optimal allocation is reached; whereas over-prevention arises for $\Psi_1 < \Psi < \Psi_3$ and under-prevention arises for $\Psi_4 < \Psi < \Psi_2$. (iii) Welfare losses with respect to the first-best are lower in type 2 Equilibrium than in that of type 1. In both equilibria first-best is reached for $\Psi \leq \Psi_1$ and $\Psi \geq \Psi_2$.

**Proof.** (i) The welfare comparison between Equilibrium of type 1 $W^*_U$ and first-best $W^*_{EA}$ can be easily verified observing Figure 2. (ii) The welfare comparison between Equilibrium of type 2 and first-best can be verified by comparing $W^*_{EA}$ in Figure 2 with the bold line in Figure 3 and noting that $\Psi_1 < \Psi_3 < \Psi_4 < \Psi_2$.

As we expected, decision-makers are better off when $p_H - p_L$ is low and $L - l$ is high since type 2 Equilibrium prevails and prevention choices are more efficient.

**3.2 Insurance firms do not observe decision-makers’ information status and test result**

In the previous subsection information was symmetric. Here, we assume that decision-makers can secretly take the test before insurance purchase and are then free to show the test result or to conceal it. As in the previous subsection, if decision-makers show the test result to insurers, the latter can offer contracts
contingent on such information. Moreover, as before insurance contracts will be contingent on decision-makers’ action. Figure 4 shows the decision-makers’ decision tree.

As in the free entry equilibrium analyzed by Rothschild and Stiglitz (1976) only "fair contracts" are sustainable at the equilibrium and firms make zero profits; otherwise new insurers would enter the market and make positive profits.

Obviously informed individuals who received good news have incentives to show the test result to insurers so that they can buy the policy at a low premium. On the contrary, individuals who received bad news prefer to conceal the test result pretending to be uninformed. Put differently, informed high-risks can mimic uninformed individuals. This implies that insurers must offer the same contract to uninformed decision-makers and to individuals that do not show the test result (see Figure 4).

In this situation, if decision-makers choose to perform the test, insurance firms can easily screen consumers’ types by offering full-insurance at a fair premium to low-risks showing the test result. Decision-makers pretending to be uninformed are necessarily high-risks and, at the equilibrium, they also receive full-insurance at a fair premium. This screening mechanism works only if performing the test is a dominant strategy for uninformed consumers. We show in the following proposition that decision-makers prefer to acquire information if firms offer to uninformed and to informed high-risk individuals a full or partial insurance policy such that with this policy high-risks receive the same utility they would reach showing the test result. The intuition is that, by performing the test, decision-makers can always obtain the same policy as uninformed (when they learn to be high-risk) or they may be able to choose a policy that is strictly preferred (when they learn to be low-risk).

The described equilibrium is unique. In fact, no other fully revealing equilibrium exists, nor an equilibrium where decision-makers stay uninformed. To see this recall that the same policy must be offered to informed consumers not showing the test result and to the uninformed, while fair full insurance contracts are offered to people showing the test result (otherwise other insurers would enter the market and make positive profits on low-risks). This implies that staying uninformed can never be a dominant strategy for decision-makers.

**Proposition 2** When insurance firms cannot observe decision-makers’ information status and the test result can be concealed, at the equilibrium decision-makers perform the test and show the test-result to insurers when they learn to be low-risk. Both types receive full-insurance at a fair premium.

**Proof.** See the Appendix 5.2.

Proposition 2 extends Doherty and Thistle (1996)’s Proposition 2 to the case of secondary prevention: when insurers do not observe consumers’ information

---

4Our proof is however different since in the present model all consumers are *ex-ante* uninformed whereas in Doherty and Thistle (1996)’s a part of consumers knows the risk type *ex-ante.*
status and the test result can be concealed, decision-makers always acquire information. Thus, with asymmetric information between decision-makers and insurers, the insurance market provides good incentives for information acquisition.

**Corollary 2** *(Welfare properties of the equilibrium allocation)* The equilibrium allocation corresponds to the interim optimal allocation $W_I^*$. Since decision-makers learn their risk, prevention choices are always optimal. In the equilibrium allocation welfare losses are exclusively due to the lack of insurance against classification risk.

The following corollary compares social welfare in the equilibrium allocation with symmetric and with asymmetric information:

**Corollary 3** *(Welfare comparison under different informational structures)* When inequality (9) holds and $\Psi_3 \leq \Psi \leq \Psi_4$, the equilibrium allocation is the same under symmetric and under asymmetric information. In all the other cases decision-makers are better off under symmetric information: private information is detrimental to decision-makers.

In standard models with adverse-selection, the Rothschild-Stiglitz equilibrium is such that good-risks receive partial- and bad-risks full-insurance. Here, with endogenous private information and verifiable test results, both types receive full-insurance at a fair premium and efficiency is restored. However, from an *ex-ante* perspective, both good- and bad-risks are worse off with respect to the case where information is equally shared between insurance firms and decision-makers. In fact, almost always decision-makers would reach a higher welfare by staying uninformed. We can conclude that, when the information status is not observable and consumers choose whether to show the test result or not, insurance firms are still able to screen consumers’ risks but information disclosure makes decision-makers worse off.

Note that, when on the contrary insurers observe decision-makers’ information status, a different equilibrium allocation arises. In fact, different contracts can be offered to informed high-risks and to uninformed decision-makers. In particular both uninformed and informed high-risks receive full-coverage at a fair premium (respectively $p_U^l/p_U^L$ and $p_H^l/p_H^L$). Here, even if the test result can be concealed, since informed low-risks always show the test result and insurers know whether decision-makers are informed or not, insurers can always unmask informed high-risks. Thus, when deciding whether to learn their type, decision-makers must choose between full coverage at premium $p_U^l/p_U^L$ with certainty and the lottery assigning full coverage at a high premium with probability $\lambda$ and full coverage at a low premium with probability $1 - \lambda$. So that, at the equilibrium, consumers stay uninformed.5

---

5 We could also consider the situation where the information status is observable but the information provided by the test is not verifiable such that high-risks cannot show the test result to insurers. In this case insurers offer self-selective (Rothschild-Stiglitz) contracts to...
3.3 Private and social value of information

In this subsection we investigate the private and social value of information under the two different assumptions on decision-makers’ information status observability.

First of all note that, under symmetric information, when deciding whether to gather information consumers compare their expected utility when the test is performed with their utility without information and, for the Law of Large Numbers, consumers’ expected utility and social welfare are the same. This implies that the private and social value of information are equivalent. On the contrary, under asymmetric information, we showed that acquiring information is a dominant strategy for decision-makers, and this implies that the private value of information is positive. However, decision-makers would almost always be better off without information: social welfare when consumers stay uninformed $W^*_U$ dominates social welfare when they get information $W^*_I$ except when condition (9) holds.\(^6\)

The previous considerations are summarized below.

Corollary 4 (Private and social value of information) (i) When insurance firms observe decision-makers’ information status and test result: the private and social value of information are the same and correspond to $W^*_I - W^*_U$. When Equilibrium of type 1 arises the value of information is always negative. When Equilibrium of type 2 arises the value of information is positive for $\Psi_3 \leq \Psi \leq \Psi_4$ and negative elsewhere. (ii) When insurance firms do not observe decision-makers’ information status and test result: the private value of information is positive and depends on the self-selecting contracts offered by insurers. The social value of information corresponds to $W^*_I - W^*_U$. When the opposite of condition (9) holds the social value of information is always negative. When condition (9) holds the social value of information is positive for $\Psi_3 \leq \Psi \leq \Psi_4$ and negative elsewhere.

4 Conclusion

We analyzed the welfare properties of equilibria allocations when consumers are uninformed but may gather information on their risk type before insurance policy purchase, insurance firms offer policies covering the risk of the loss but informed decision-makers. Thus, when deciding whether to learn their type, decision-makers must choose between full coverage at premium $pU/pL$ with certainty and the lottery assigning full coverage at a high premium with probability $\lambda$ and partial coverage at a low premium with probability $1 - \lambda$ (such a lottery corresponds to the Rothschild-Stiglitz equilibrium). So that, at the equilibrium, consumers stay uninformed (See Doherty and Thistle 1996).

\(^6\)Contrary to us, Doherty and Posey (1998) find that the social value of information is positive. This difference essentially depends on the fact that in their model a part of consumers is ex-ante informed. Thus, to evaluate the social value of information under asymmetric information, they compare the equilibrium allocation that fully discloses information with the self-selective allocation that would arise without information gathering. We instead compare the equilibrium allocation $W^*_I$ with the allocation that would arise when consumers stay uninformed, that is $W^*_U$.\(^6\)
not the classification risk, and information allows consumers to take efficient self-insurance measures. We put ourselves in the context which seems more natural when considering information provided by genetic testing: we assumed that all consumers are ex-ante uninformed and that insurance firms do not observe information status and test result but that information can be credibly revealed to insurers if consumers desire to.

Concerning the important debate on the regulation of genetic information in insurance market (see, for example, Hoy and Ruse 2005) our results contribute in the following way. First, when risk-rating of consumers on the basis of genetic testing is forbidden by specific market regulation, insurance firms should not be worried about their ability to estimate future losses and costs since information is fully disclosed at the equilibrium; as already stated in different models by Doherty and Thistle (1996) and Doherty and Posey (1998), no adverse selection problems arise. Second, even if information has positive private value, consumers would be better off without information gathering: because of the lack of insurance against classification risk, information disclosure is detrimental to consumers except when the difference in the type risk is low and/or prevention effectiveness is high. Moreover, even in such a case, consumers are not worse off uniquely for intermediate values of prevention cost. Finally, in the model the welfare loss due to the lack of classification risk is explicitly addressed and the importance of "genetic insurance" provision, as proposed by Tabarrok (1994), clearly stated.

5 Appendix

5.1 Proof of Remark 5.

Let us substitute \( p_H - p_U = (1 - \lambda)(p_H - p_L) \) and \( p_U - p_L = \lambda(p_H - p_L) \) in (9) and call \( p_H - p_L = x \). Thus, (9) can be rewritten as a function of \( x \):

\[
\Gamma_u(x) = \frac{\left[u(w - p_U I) - u(w - p_H I - (1 - \lambda)Ix)\right]}{\left[u(w - p_H I + \lambda Lx) - u(w - p_U I)\right]}.
\]

Because of the concavity of \( u \), \( \Gamma \) is an increasing function such that:

\[
\Gamma_u(0) = \frac{u'(w - p_U I)}{u'(w - p_H I)} \leq 1.
\]

The condition (9) can be rewritten like that:

\[
\Gamma_u(p_H - p_L) \leq \frac{L}{I}
\]

That is:

\[
0 \leq p_H - p_L \leq \Gamma_u^{-1}\left(\frac{L}{I}\right)
\]
5.2 Proof of Proposition 2

The proof is organized in two steps. First we show that, at the equilibrium, decision-makers perform the test when insurance firms offer full-insurance contracts; then we show that the result does not change when firms are free to offer partial-insurance contracts.

(i) Full-insurance contracts. Suppose that firms are constrained to offer full-insurance contracts. Since prevention is contractible, insurance firms ex-ante propose 4 full-insurance contracts contingent on the decision-maker’s action and on the test result in the case decision-makers decide to show it; and 2 full-insurance contracts only contingent on the preventative action in the case decision-makers do not show the test result. The insurance premiums are:

<table>
<thead>
<tr>
<th>Show result</th>
<th>with prevention</th>
<th>without prevention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Show result L</td>
<td>$\pi_{L1} = p_L L$</td>
<td>$\pi_{L0} = p_L L$</td>
</tr>
<tr>
<td>Show result H</td>
<td>$\pi_{H1} = p_H L$</td>
<td>$\pi_{H0} = p_H L$</td>
</tr>
<tr>
<td>Don’t show</td>
<td>$\pi_{N1}$</td>
<td>$\pi_{N0}$</td>
</tr>
</tbody>
</table>

We are looking for an equilibrium where decision-makers perform the test and show the test-result to insurers when the test reveals good news. Thus, for this equilibrium to exist, we have necessarily

$$p_{H L} \geq \pi_{N1} \geq p_{L L}$$

and

$$p_{H L} \geq \pi_{N0} \geq p_{L L}.$$

So that when individuals learn that they are good-type (respectively bad-type) it is optimal to show (respectively conceal) the test result.

When deciding whether to perform the test or not, decision-makers must compare:

$$\lambda \max \{ u(w - \pi_{N1}) - \Psi, u(w - \pi_{N0}) \} + (1 - \lambda) \max_{a_L} (u(w - p_L L(a_L)) - \Psi(a_L))$$

(11)

with:

$$\max \{ u(w - \pi_{N1}) - \Psi, u(w - \pi_{N0}) \},$$

(12)

where (11) is expected utility when the test is performed: with probability $\lambda$ the decision maker is high-risk, does not show the test result, and chooses the maximum between full-insurance with prevention and full-insurance without prevention; with probability $1 - \lambda$ the decision maker is low-risk, shows the test result, and maximizes his (full-insurance) utility with respect to the action. Whereas (12) is utility when decision-makers stay uninformed.

We now show that expected utility with the test (12) is higher than expected utility without the test (11). Suppose it is not. Then it must necessarily be:

$$\max_{a_L} (u(w - p_L L(a_L)) - \Psi(a_L)) \leq \max_{a_U} (u(w - p_U L(a_U)) - \Psi(a_U))$$

then nobody performs the test and $\pi_{N1} = p_U l$ , $\pi_{N0} = p_U L$. However this is impossible since:

$$\max_{a_L} (u(w - p_L L(a_L)) - \Psi(a_L)) > \max_{a_U} (u(w - p_U L(a_U)) - \Psi(a_U))$$
We proved that it must be:

$$\max(u(w - \pi_{N1}) - \Psi, u(w - \pi_{N0})) \leq \max_{aL}(u(w - p_{L}L(a_{L}))) - \Psi(a_{L})$$

This implies that expected utility with the test (11) dominates utility without the test (12) and decision-makers prefer to gather information.

We can conclude that the allocation where uninformed decision-makers perform the test and show it to insurers only when the result is $L$ is an equilibrium. Thus, at the equilibrium, all decision-makers not showing the test are high-risk such that $\pi_{N1} = p_{H}l$ and $\pi_{N0} = p_{H}L$; both high- and low-risks receive full-insurance at a fair premium.

(ii) **Partial-insurance contracts.** Suppose now that insurance companies can propose *ex-ante* self-selective contracts with partial coverage. In this case, firms will offer full-insurance contracts for those who show the test result, and a contract with partial coverage for those who announce to be uninformed. In such a contract the fair premium is calculated using probability $p_{U}$. Let us call the partial insurance coverage $y$ or $Y$ according to whether decision-makers choose prevention or not. We obtain the set of contracts depicted in the following table:

<table>
<thead>
<tr>
<th></th>
<th>with prevention</th>
<th>without prevention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Show result L</td>
<td>$\pi_{L1} = p_{L}L$, full coverage</td>
<td>$\pi_{L0} = p_{L}L$, full coverage</td>
</tr>
<tr>
<td>Show result H</td>
<td>$\pi_{H1} = p_{H}l$, full coverage</td>
<td>$\pi_{H0} = p_{H}L$, full coverage</td>
</tr>
<tr>
<td>Don’t show</td>
<td>$p_{U}y$, partial coverage</td>
<td>$p_{U}Y$, partial coverage</td>
</tr>
</tbody>
</table>

To be self-selecting the proposed partial-insurance contracts must be such that high-types are indifferent between showing the test result and thus obtaining full-insurance at a fair premium and pretending to be uninformed and thus obtaining partial-insurance. Or:

$$u(w - p_{H}L) = p_{H}u(w - p_{U}Y + Y - L) + (1 - p_{H})u(w - p_{U}Y) = U_{H}(Y)$$

$$u(w - p_{H}l) = p_{H}u(w - p_{U}y + y - l) + (1 - p_{H})u(w - p_{U}y) = U_{H}(y)$$

where $U_{H}(Y)$ is expected utility for high-risks under partial insurance and without prevention, and $U_{H}(y)$ is expected utility gross of prevention cost under partial insurance and with prevention.

Suppose now that uninformed consumers perform the test. When the test result is $p_{L}$ decision-makers show it to insurers and receive full-insurance at a fair premium. When the test result is $p_{H}$ decision-makers are indifferent between showing the test result and pretending to be uninformed. Assume that, when indifferent, high-risks show the test result to insurers and receive full-insurance at a fair premium too.

As a consequence, performing the test gives:

$$W^{*}_{L} = \lambda u(w - p_{H}L(\hat{a}_{H})) + (1 - \lambda)u(w - p_{L}L(\hat{a}_{L})) - \lambda\Psi(\hat{a}_{H}) - (1 - \lambda)\Psi(\hat{a}_{L})$$

$$= \lambda W^{*}_{H} + (1 - \lambda)W^{*}_{L}$$

where:

$$W^{*}_{L} = \max(u(w - p_{L}L), u(w - p_{L}l) - \Psi)$$

19
and, for construction:

\[ W^*_H = \max(U_H(Y), U_H(y) - \Psi) = \max(u(w - p_H L), u(w - p_H l) - \Psi) \] (15)

Let us compute now expected utility, gross of prevention cost, obtained by low-risks when they stay uninformed, perform prevention and receive the partial insurance contract:

\[ U_L(y) = p_L u(w - p_U y + y - \lambda) + (1 - p_L) u(w - p_U y) \]
\[ < u(p_L (w - p_U y + y - \lambda) + (1 - p_L) (w - p_U y)) \]
\[ = u(w - p_L l - (p_U - p_L) y) \]
\[ < u(w - p_L l) \] (16)

In the same way let us consider expected utility obtained by low-risks when they stay uninformed, do not perform prevention and receive the partial insurance contract. The following holds:

\[ U_L(Y) < u(w - p_L l) \] (17)

Inequalities (16) and (17) imply that low-risks receive a larger utility when they obtain full insurance at a fair premium then then when they stay uninformed and obtain the partial insurance contract. This writes:

\[ W^*_L > \max(U_L(Y), U_L(y) - \Psi) \] (18)

Finally, by staying uninformed decision-makers get:

\[ \max(U_U(Y), U_U(y) - \Psi) = \max(\lambda U_H(Y) + (1 - \lambda) U_L(Y), \lambda U_H(y) + (1 - \lambda) U_L(y) - \Psi) \] (19)

Now we can compare (13) and (19) taking into account (15) and (18). We find:

\[ \lambda W^*_H + (1 - \lambda) W^*_L > \max(U_U(Y), U_U(y) - \Psi) \]

so that uninformed decision-makers strictly prefer to take the test.

References


Figure 1: social welfare without insurance $W_0^*$ and the 
interim optimal allocation $W_1^*$. 

Figure 2: social welfare with full insurance and no 
information $W_U^*$ and the ex-ante optimal allocation $W_{EA}^*$. 
Figure 3: equilibrium allocation with full information when \( W_U^* \) and \( W_1^* \) cross each other.

Figure 4: consumers’ decision-tree under asymmetric information.