Gold Rush Fever in Business Cycles

Paul Beaudry, Fabrice Collard and Franck Portier
Preliminary

September 2006

Abstract

Gold rushes are periods of economic boom, generally associated with large increases in expenditures aimed at securing claims near new found veins of gold. An interesting aspect of gold rushes is that, from a social point of view, much of the increased activity is wasteful since it contributes simply to the expansion of the stock of money. In this paper, we explore whether business cycles fluctuations may sometimes be driven by a phenomena akin to a gold rush. We first show that the business cycle is, to a large extent, the consequence of a shock that has a non-permanent effect on real quantities and that does not move consumption at all, neither in the short nor in the long run. We then propose a structural interpretation to that shock, In particular, we present a model where the opening of new market opportunities causes an economic expansion by favoring competition for market share. We call such an episode a market rush. This paper then present a simple model of a market rush that can be embedded into an otherwise standard Dynamic General Equilibrium model, and evaluate whether such a phenomena is a significant contributor to business cycle fluctuations.

Key Words: Business Cycle – Market Rush

JEL Class.: E32

Paul Beaudry
Department of Economics
University of British Columbia
997-1873 East Mall
Vancouver, B.C.
Canada, V6T 1Z1
CIAR and NBER.

paulbe@interchange.ubc.ca

Fabrice Collard
University of Toulouse
CNRS–GREMAQ and IDEI
Manufacture des tabacs,
21 allée de Brienne
31000 Toulouse
France

fabrice.collard@univ-tlse1.fr

Franck Portier
University of Toulouse
GREMAQ and IDEI
Manufacture des tabacs,
21 allée de Brienne
31000 Toulouse
France

fportier@cict.fr

Introduction

Sutter's Mill near Coloma, California. January 24, 1848. James W. Marshall, a carpenter from New Jersey, found a gold nugget in a sawmill ditch. This was the starting point of one of the most famous Gold Rush that pave economic History, the California Gold Rush of 1848-1858. More than 90,000 people made their way to California in the two years following Marshall's discovery, and more than 300,000 by 1854 —or one of about every 90 people then living in the United States. The population of San Francisco exploded from a mere 1,000 in 1848 to 20,000 full–time residents by 1850. More than a century later, the San Francisco 49ers NFL team is still named for the prospectors of the California Gold Rush. Another famous episode, from which was inspired Charlie Chaplin's movie The Gold Rush and Jack London's book The Call of the Wild, is the Klondike Gold Rush of 1896–1904. Gold prospecting took place along the Klondike River near Dawson City in the Yukon Territory, Canada. An estimated 100,000 people participated in the gold rush and about 30,000 made it to Dawson City in 1898. By 1901, when the first census was taken, the population had declined to 9,000.

Gold rushes are periods of economic boom, generally associated with large increases in expenditures aimed at securing claims near new found veins of gold. An interesting aspect of gold rushes is that, from a social point of view, much of the increased activity is wasteful since it simply contributes to the expansion of the stock of money. In this paper, we explore whether business cycles fluctuations may sometimes be driven by a phenomena akin to a gold rush. In particular, we present a dynamic general equilibrium model where the opening of new market opportunities causes an economic expansion by favoring competition for market share. We call such an episode a market rush. The market rush may simply redistribute rents with little external gain. In that case, the net social value of such a market rush can be minor as is the case in a gold rush. In the second half of the 1990s the U.S. economy exhibited historically large growth rates of output, employment and investment. Meanwhile, the "new economy" of information and communication was discovered. This period can fall into the market rush category. The long run effect on productivity of the opening of new market opportunities is *a priori* undetermined, and has often been considered as minor in the particular case of the Information Technology. Nevertheless, it is still an open question whether the IT market rush of the nineties was a gold rush. The object of this paper is to present a simple model of a market rush that can be embedded into an otherwise standard Dynamic General Equilibrium model, to evaluate whether such a phenomena is a significant contributor to business cycle fluctuations and to examine the social desirability of such fluctuations.

To capture the idea of a market rush, we present an expanding varieties model where agents compete to secure monopoly positions in new markets. However, in a first step and in contrast to standard growth models (and to some business cycles models), we do not impose that an expansion in variety induces productivity gains, and treat the growth in the potential set of varieties as technologically driven and exogenous. In this setting, when agents perceive an increase in the set of technologically feasible markets, they invest to set up a prototype firm (or product) with the hope of securing a monopoly position in the new market. It is therefore the perception of these new market opportunities that causes the market rush and the associated economic expansion. After the initial rush, there is a shake out period where one of the prototypes secures the dominant position in market. The long term effect of such a market rush depends on whether the expansion in variety has an external effect on productivity. In the case where it does not have an external effect, the induced cycle is socially wasteful as it only contributes to the redistribution of market rents. In contrast, when the expansion of variety does exert positive external effects, the induced cycle can have social value but will generally induce output fluctuations that are excessively large.

We begin the paper by presenting, in Section 1, some interesting properties of the data, that were already put forward by Cochrane [1994]. In a bivariate Output–Consumption VAR of the U.S. postwar economy, consumption is, at all horizon, almost only affected by the identified permanent shock. On the contrary, the identified temporary shock explains an important part of the short run volatility of output — i.e. the business cycle. This fairly robust feature of the data is quite challenging for business cycle models, that generally do not generate such an almost pure random walk behavior of consumption. Furthermore, the literature does not propose a structural interpretation for the temporary shock. As we think that a market rush shock is a nice candidate, in Section 2, we develop a simple model without capital accumulation, which can be solved analytically.[1] In this version, we show why current economic activity depends positively on the expectation of next period's activity and on the perceived opening of new markets. Hence, when agents believe that the economy is going through a prolonged period of market expansion, this induces an increase in investments and an associated economic expansion. In contrast, when there are no new perceived market opportunities, the economy experiences a slump. Given the tractability of the model, we can solve it in the presence of a standard technology shock, our market expansion shock and/or a preference shock. In all these cases, the model puts sufficient structure on the data to isolate the market expansion shock: it suggests that the market expansion shock is the output innovation in a Consumption–Output VAR. As shown in Section 2, this shock has a zero long run impact on output, suggesting that market rushes are socially inefficient.

Since these inferences are based on a extremely simplistic environment, in Section 3 we follow up by enriching the model to include capital accumulation and allow for potential long run impact of variety expansion. The "structural" innovations of section 1 VECM are now combinations of the true shocks. In order to better capture the short run dynamics of the economy, we also

[1] The type of models we present are ones where nominal rigidities play no role. Our interpretation of such models is that they can correspond to models with sticky prices in which monetary authorities follow rules that implement the flexible price outcomes.

introduce habit persistence in consumption and adjustment costs to investment. We then estimate the resulting (more complex) model using a minimum distance estimator, as adopting the indirect inference approach. Our findings from the larger model suggest that the market expansion shock is indeed a non negligible driving force underlying business cycle fluctuations. For instance, it explains one third of the one-quarter ahead output volatility, and more than one tenth at a one year horizon. It also explains more than two-third of hours worked volatility over the first two years. The results from the larger model qualify our initial findings regarding the long run effects of market shocks. In the larger model, we find that some market expansion shocks exert a positive long run effect on output, but the effect remains very small. This supports the idea that a large fraction of the fluctuations induced by a market expansion shocks may be excessive. Section 4 offers some concluding comments.

1 Some Interesting Properties of the Data

In this section, we present some properties of the data which were already put forward by Cochrane [1994]. More precisely, we study a Consumption–Output system with one cointegrating relation. Using a long run identification scheme *à la* Blanchard and Quah [1989], we first show that much (all) of the short run volatility of consumption is explained by the permanent shock, while this shock only account for half of that of output. We then use a short run identification *à la* Sims to show that the output innovation does not indeed explain the long run of the two variables, nor the short run of consumption. We then formally test for the identity of the temporary shock and the output innovation. Next, we discuss the implications of these results, that we consider as being properties of data, without the need to put names on the shocks. Finally, we assess the robustness of the results to alternative definitions of consumption and output and to model specification.

1.1 Long Run and Short Run Identification

We consider quarterly data for the US economy. The sample spans the period 1947Q1 to 2004Q4. Consumption, C, is defined as Real Personal Consumption Expenditures of nondurable goods and services and output, Y, is Real Gross Domestic Product. Both series are first deflated by the 16-64 U.S. population and taken in logarithm.[2] Later, we will make use of an Investment series, I, that

[2]The series are obtained from the following links:

- Real Personal Consumption Expenditures: Durable Goods, http://research.stlouisfed.org/fred2/series/PCDGCC96
- Real Personal Consumption Expenditures: Nondurable Goods, http://research.stlouisfed.org/fred2/series/PCNDGC96
- Real Personal Consumption Expenditures: Services, http://research.stlouisfed.org/fred2/series/PCESVC96
- Real Gross Private Domestic Investment, 3 Decimal, http://research.stlouisfed.org/fred2/series/GPDIC96
- Real Gross Domestic Product, 3 Decimal, http://research.stlouisfed.org/fred2/series/GDPC96

will be defined as Real Personal Consumption Expenditures of Durable Goods plus Real Gross Private Domestic.

Standard Dickey–Fuller, likelihood ratio and cointegration tests (see appendix) indicate that C and Y are $I(1)$ processes and do cointegrate. We therefore model their joint behavior with Vectorial Error Correcting Model (VECM), where the cointegrating relation coefficients are [1;-1] (meaning that the consumption to output ratio is stationary). Likelihood ratio tests suggests that the VECM should include 3 lags. Omitting constants, the joint behavior of (C,Y) admits the following Wold representation

$$\begin{pmatrix} \Delta C_t \\ \Delta Y_t \end{pmatrix} = A(L) \begin{pmatrix} \mu_{1,t} \\ \mu_{2,t} \end{pmatrix}, \tag{1}$$

where L is the lag operator, $A(L) = I + \sum_{i=1}^{\infty} A_i L^i$, and where the co-variance matrix of μ is given by Ω. As the system possesses one common stochastic trend, $A(1)$ is not full rank. More precisely, it is composed of one column of zeros and one column of real and identical numbers. Given the properties of $A(1)$, it is possible to derive a representation of the data in terms of permanent and transitory component of the form

$$\begin{pmatrix} \Delta C_t \\ \Delta Y_t \end{pmatrix} = \Gamma(L) \begin{pmatrix} \varepsilon_t^P \\ \varepsilon_t^T \end{pmatrix}, \tag{2}$$

where the covariance matrix of $(\varepsilon^P, \varepsilon^T)$ is the identity matrix and $\Gamma(L) = \sum_{i=0}^{\infty} \Gamma_i L^i$. The Γ matrices solve

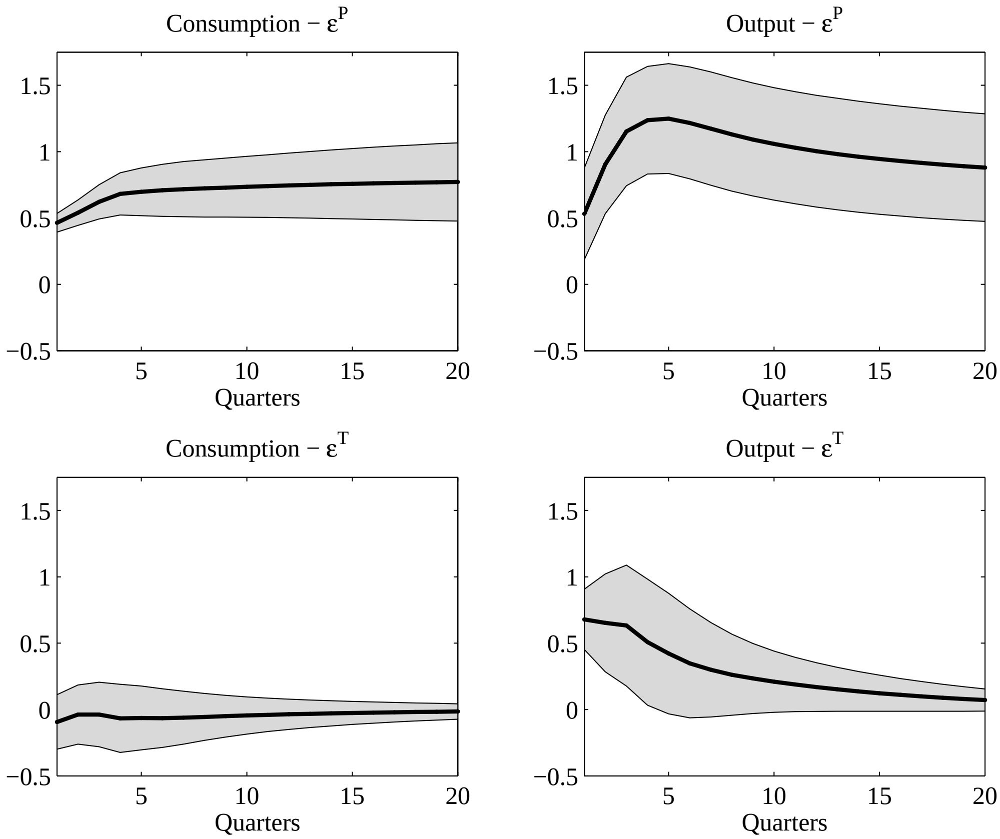
$$\begin{cases} \Gamma_0 \Gamma_0' &= \Omega \\ \Gamma_i &= A_i \Gamma_0 \quad \text{for} \quad i > 0 \end{cases} \tag{3}$$

Note that once Γ_0 is known, all Γ_i are pinned down by the second set of relations. But, due to the symmetry of the covariance matrix Ω, the first part of the system only pin down 3 parameters of Γ_0. One remains to be set. This is achieved by imposing an additional restriction. We impose that the $1,2$ element of the long run matrix $\Gamma(1) = \sum_{i=0}^{\infty} \Gamma_i$ equals zero, that is, we choose an orthogonalization where the disturbance ε^T has no long run impact on C and Y (the use of this type of orthogonalization was first proposed by Blanchard and Quah [1989]). Hence, ε^T is labeled as being a temporary shock, while ε^P is a permanent one. Figure 1 graphs the impulse response functions of C and Y to both shock as well as their associated 95% confidence band. Table 1 reports the corresponding variance decomposition of the process.

Those results provide an interesting decomposition of macroeconomic fluctuations. The lower left panel of Figure 1 clearly shows that consumption responds very little to the transitory shock, which in turn accounts for less than 4% of consumption volatility at any horizon. Conversely it is very responsive to the permanent shock and most of the adjustment dynamics takes place in less than

- Population: 15 to 64, annual, http://www.economy.com/freelunch/fl_dictionary.asp?m=34174038-A1EF-4C70-9374-59144B50A3F5&h=H00060004&f=0&c=undefined

Figure 1: Impulse Responses to ε^P and ε^T, (C, Y) Benchmark VECM.



This Figure shows the response of consumption and output to temporary ε^T and permanent ε^P one percent shocks in the long run indentification. Those impulse response functions are computed from the benchmark VECM (C, Y) estimated with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1–2004Q4. The shaded area is the 95% confidence intervals obtained from 1000 bootstraps of the VECM.

Table 1: Forecast Error Variance Decomposition, (C, Y) Benchmark VECM.

Horizon	Output		Consumption	
	ε^T	ε^Y	ε^T	ε^Y
1	62.01%	79.86%	3.90%	0.00%
4	28.10 %	46.05 %	1.16%	1.25%
8	17.20 %	32.73%	0.91%	1.26 %
20	9.79 %	22.21 %	0.42%	2.13%
∞	0 %	3.89 %	0%	3.89%

This table shows the k-period ahead share of the forecast error variance of consumption and output that is attributable to the temporary shock ε^T in the long run indentification and to the output innovation ε^Y in the short run one, for $k = 1, 4, 8, 20$ quarters and for $k \longrightarrow \infty$. Those shares are computed from the benchmark VECM (C, Y) estimated with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1–2004Q4.

one year. In other words, consumption is basically a pure random walk, that responds only to permanent shocks. On the contrary, short run fluctuations of output are mainly the consequence of temporary shocks — which explain more than 60% of output volatility on impact— whose effects are vanishing with time.

Now we consider an alternative orthogonalization that uses short run restrictions

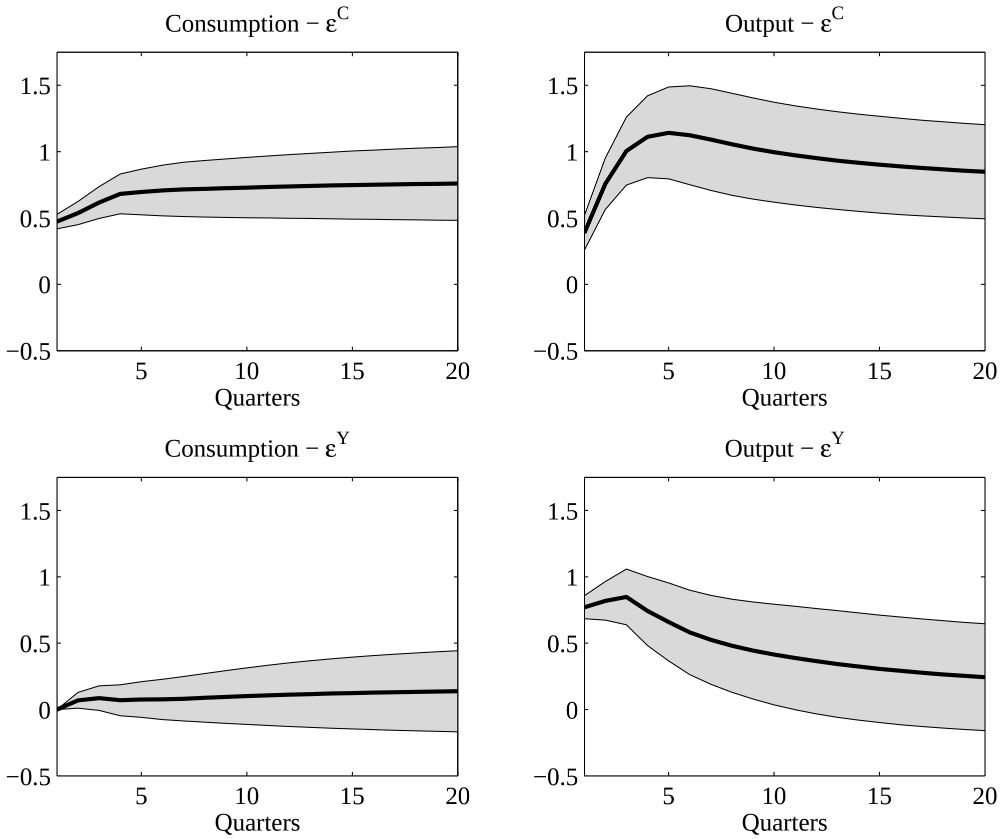
$$\begin{pmatrix} \Delta C_t \\ \Delta Y_t \end{pmatrix} = \widetilde{\Gamma}(L) \begin{pmatrix} \varepsilon_t^C \\ \varepsilon_t^Y \end{pmatrix}, \quad (4)$$

where $\widetilde{\Gamma}(L) = \sum_{i=0}^{\infty} \widetilde{\Gamma}_i L^i$ and the variance covariance matrices of $(\varepsilon^C, \varepsilon^Y)$ is the identity matrix. The $\widetilde{\Gamma}$ matrices are solution of a system of equations similar to (3). We however depart from (3) as we impose that the 1, 2 element of $\widetilde{\Gamma}_0$ be equal to zero. Therefore, ε^Y is the output innovation, and the contemporaneous response of C to ε^Y is zero.

Figure 2 graphs the impulse responses of C and Y associated to the last orthogonalization scheme. The associated variance decompositions are displayed in Table 1. The striking result from those estimations is that the consumption shock ε^C is indeed the permanent shock to consumption (ε^P in the long run identification), so that the responses and variance decompositions are very similar to those obtained using the long run identification scheme. This observation is further confirmed by Figure 3, that plots ε^P against ε^C and ε^T against ε^Y. It is then striking to observe that both shocks align along the 45° line indicating for instance that the consumption innovation is essentially identical to the permanent component.

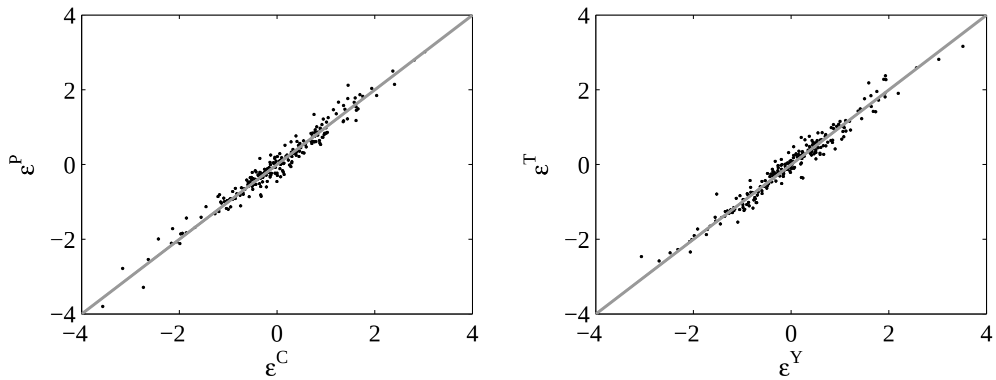
The properties of the data that we want to highlight here are the following: *(i)* the permanent shock to consumption (ε^P) is indeed the ε^C shock in an consumption–output VECM, *(ii)* there is virtually no dynamics in the consumption response to that shock, as it affects permanently and

Figure 2: Impulse Responses to ε^C and ε^Y, (C, Y) Benchmark VECM.



This Figure shows the response of consumption and output to consumption ε^C and output ε^Y one percent shocks in the short run indentification. Those impulse response functions are computed from the benchmark VECM (C, Y) estimated with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1–2004Q4. The shaded area is the 95% confidence intervals obtained from 1000 bootstraps of the VECM.

Figure 3: Plots of ε^C against ε^P and ε^Y against ε^T in the (C,Y) Benchmark VECM.



This Figure has two panels. The left panel plots the estimated permanent innovation ε^P (from the long run identification) against the consumption innovation ε^C (from the short run identification. The right panel plots the estimated temporary innovation ε^T (from the long run identification) against the output innovation ε^Y (from the short run identification. On both panels, the straight line is the 45 degre line. Those shocks are computed from the benchmark VECM (C,Y) estimated with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1–2004Q4.

almost instantaneously the level of consumption and *(iii)* the temporary shock (or the output shock in the short run identification) is responsible for a significant share of output volatility at business cycle frequencies. Therefore, much of the business cycle action seems to lie in investment, without any short or long run implications for consumption. Next, we formally test for the equality between ε^Y and ε^T.

1.2 Formal Test and Interpretation

This section proposes a formal test for the equality between ε^Y and ε^T (or equivalently between ε^C and ε^P as the shocks are pairwise orthogonal). Consider the Wold representation (1), and consider the following representation of the process:

$$\begin{pmatrix} \Delta C_t \\ \Delta Y_t \end{pmatrix} = B(L) \begin{pmatrix} \nu_{1,t} \\ \nu_{2,t} \end{pmatrix}, \tag{5}$$

where $B(L) = \sum_{i=0}^{\infty} B_i L^i$ and the variance covariance matrix of (ν_1, ν_2) is the identity matrix. We want here to perform an overidentified orthogonalization and impose at the same time that the shock ν_1 has no impact effect on C and no long run effect on C and Y. More precisely, we look for a matrix S such that $\mu = S\nu$ and $B_i = SA_i$. Imposing a zero impact effect of ν_1 on B implies

$$s_{11} = 0. \tag{6}$$

The matrix giving the long run effect of ν on both variables is given by $\widehat{B} = \widehat{A}S$, where $\widehat{A} = \sum_{i=0}^{\infty} A_i$. Imposing the long run restriction $\widetilde{b}_{11} = 0$ implies

$$\widehat{a}_{11}s_{11} + \widehat{a}_{12}s_{21} = 0 \tag{7}$$

When the two series are cointegrated, the matrix $\widehat{A}$ rewrites

$$\widehat{A} = \begin{pmatrix} \widehat{a}_{11} & k\widehat{a}_{11} \\ \widehat{a}_{21} & k\widehat{a}_{21} \end{pmatrix}$$

where k is a real number.

When $\widehat{a}_{12} \neq 0$ — which occurs when both k and $\widehat{a}_{11}$ are non zero — equations (6) and (7) imply that the first column of S is composed of zeros, meaning that S is not a full rank matrix. In other words, the two restrictions cannot hold at the same time. On the contrary, if $\widehat{a}_{12} = 0$ — when either k or $\widehat{a}_{11}$ are zero — the long run and short run constraints are simultaneously satisfied. This suggest that a convenient way of testing for the fact that both the short and long run constraints are satisfied is to test for the nullity of a particular coefficient of the long run matrix $\widehat{A}$ of the Wold representation of the process, $\widehat{a}_{12}$. The following proposition states this result in a more general case where the two series need not cointegrate.

Proposition 1 *Consider a bivariate process whose Wold decomposition is given by*

$$\begin{pmatrix} \Delta X_t^1 \\ \Delta X_t^2 \end{pmatrix} = A(L) \begin{pmatrix} \mu_{1,t} \\ \mu_{2,t} \end{pmatrix},$$

with $A(L) = I + \sum_{i=1}^{\infty} A_i L^i$, where the covariance matrix of μ is given by Ω, and where the matrix of long run effect $\widehat{A} = \sum_{i=0}^{\infty} A_i$ is not singular. Consider a structural representation

$$\begin{pmatrix} \Delta X_t^1 \\ \Delta X_t^2 \end{pmatrix} = B(L) \begin{pmatrix} \nu_{1,t} \\ \nu_{2,t} \end{pmatrix},$$

where $B(L) = \sum_{i=0}^{\infty} B_i L^i$, the covariance matrix of ν is the identity matrix and $\mu = S\nu$. Then, the two following statements are equivalent

(i) If the first structural shock ν_1 has no short run impact on C, i.e. $s_{11} = 0$, then it has no long run impact on C, i.e. $\widehat{b}_{11} = 0$, and conversely.

(ii) The $(1,2)$ element of the long run effect matrix of the Wold decomposition is zero, i.e. $\widehat{a}_{12} = 0$

Proof : We first prove that *(i)* implies *(ii)*. Assume that

$$S = \begin{pmatrix} 0 & s_{12} \\ s_{21} & s_{22} \end{pmatrix} \quad \text{and} \quad \widehat{B} = \begin{pmatrix} 0 & \widehat{b}_{12} \\ \widehat{b}_{21} & \widehat{b}_{22} \end{pmatrix}$$

Inverting S, we obtain

$$S^{-1} = \begin{pmatrix} -s_{22}/(s_{12}s_{21}) & 1/s_{21} \\ 1/s_{12} & 0 \end{pmatrix}.$$

The long run effect matrix of the Wold decomposition is $\widehat{A} = \widehat{B} \times S^{-1}$ and one can easily check that $\widehat{a}_{21} = 0$.

We then prove that *(ii)* implies *(i)*. We have the relation $\widehat{A}S = \widehat{B}$. We assume that $\widehat{a}_{12} = 0$. Then, $\widehat{b}_{12} = \widehat{a}_{11}s_{11}$. As $\widehat{A}$ is assumed to be non singular, $\widehat{a}_{11} \neq 0$, to that $\widehat{b}_{12} = 0 \Longleftrightarrow s_{11} = 0$. *Q.E.D.*

Table 2: Over–identification Test

	Benchmark	(Tot. C,Y)	(C,C+I)
Long–run Effect	0.2024	-0.3265	0.1646
95% Conf. Int.	[-0.2276,0.7912]	[-0.8134,0.2260]	[-0.2145,0.5763]

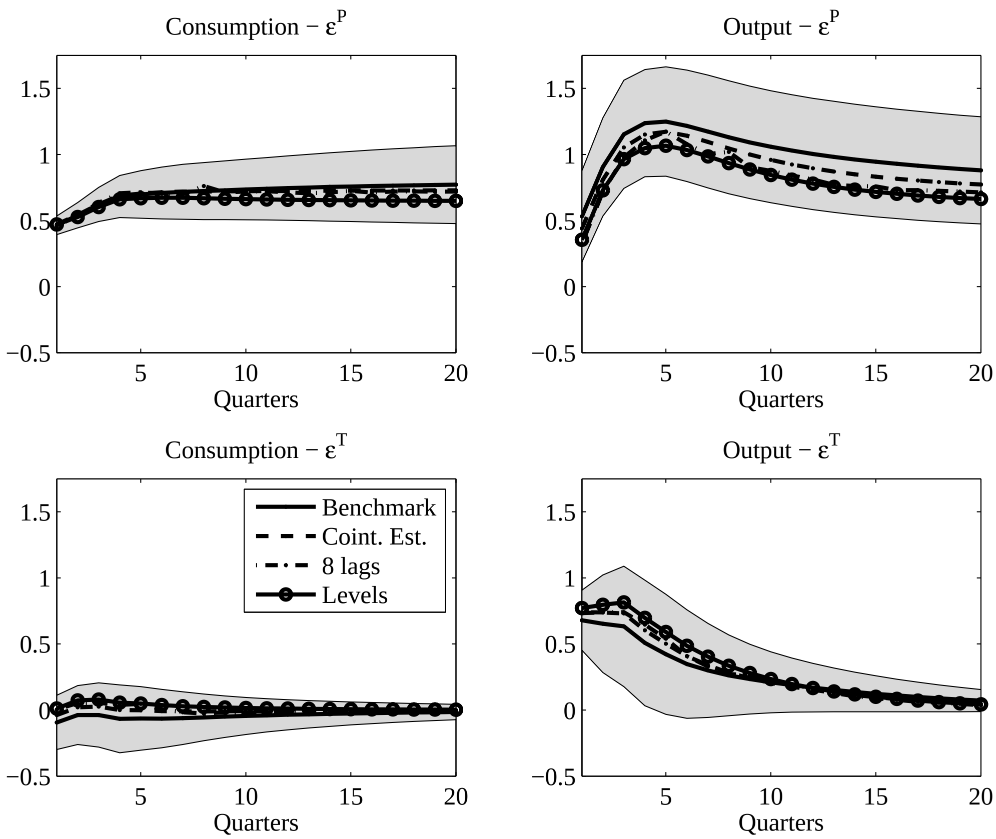
This Table shows the estimates of the Wold decomposition long run effect matrix (1,2) element, together with its 95% confidence interval, as obtained from 1000 bootstrap replications. In the first column, the estimates is computed from the benchmark VECM (C, Y) with one cointegrating relation [1;-1]. In the second column, C is measured by total consumption and the cointegrating relation is estimated. In the third column, Y is measured by consumption plus investment instead of total output and the cointegrating relation being estimated. All models are estimated with 3 lags, using quarterly per capita U.S. data over the period 1947Q1–2004Q4.

To formally test for the fact that $\varepsilon^P = \varepsilon^C$, we test for the nullity of $\widehat{a}_{12}$. The confidence intervals are obtained from 1000 bootstraps of the long run matrix. As shown in the first column of Table 2, one cannot reject the null of $\widehat{a}_{12} = 0$ at a 5% significance level. Therefore one cannot reject that the consumption shock is indeed identical to the permanent shock.

1.3 Robustness

Recall that the properties of the data that we want to highlight here are threefold: *(i)* the permanent shock to consumption (ε^P) is identical to the ε^C shock in an consumption-output VECM, *(ii)* there is virtually no dynamics in the response of consumption to that shock, as it affects permanently and almost instantaneously the level of consumption and *(iii)* the temporary shock (or the output shock in the short run identification) is responsible for a significant share of the output variance at business cycle frequencies. This section investigates the robustness of these results to various change of data and specification. First we keep the variables (C, Y), but either estimate the cointegrating relation rather than imposing a [1;-1] cointegrating vector, use eight lags in the VECM or estimate the model in levels. As shown in Figures 4 and 5, results are strikingly robust. All impulse responses lie in the confidence band of the benchmark model. Second, we use total consumption instead of consumption of nondurables and services, or consumption plus investment instead of total output. In each case, we estimate the cointegrating relation and choose the number of lags according to likelihood ratio tests. Again, as shown on Figures 6 and 7, results are robust.

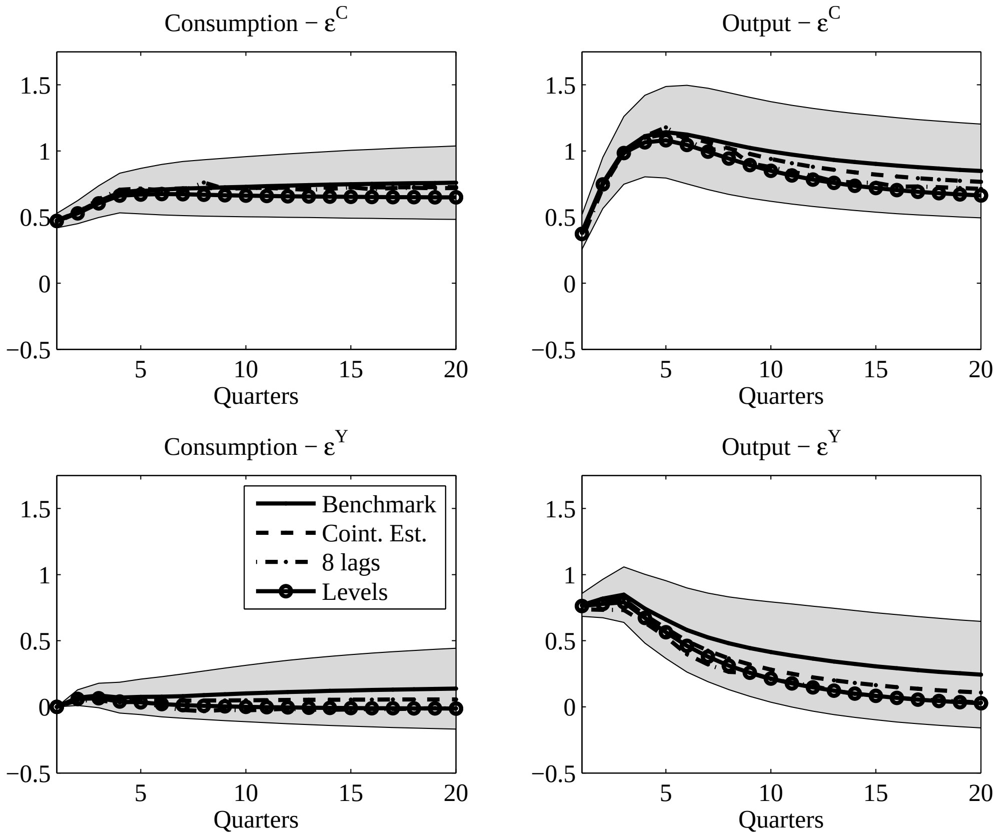
Figure 4: Robustness I (Long Run Identification)



This Figure shows the response of consumption and output to temporary ε^T and permanent ε^P one percent shocks in the long run indentification, and for different specification of the model. The bold line corresponds to the benchmark VECM (C, Y) estimated with one cointegrating relation [1;-1] with 3 lags. The dashed line corresponds to a VECM in which the cointegrating relation is estimated. The dashed-dotted line corresponds to a model with eight lags of data. The line wih circles corresponds to a VAR estimated in levels (therefore with four lags of data). All estimations are done using quarterly per capita U.S. data over the period 1947Q1–2004Q4. The shaded area is the 95% confidence intervals obtained from 1000 bootstraps of the benchmark VECM.

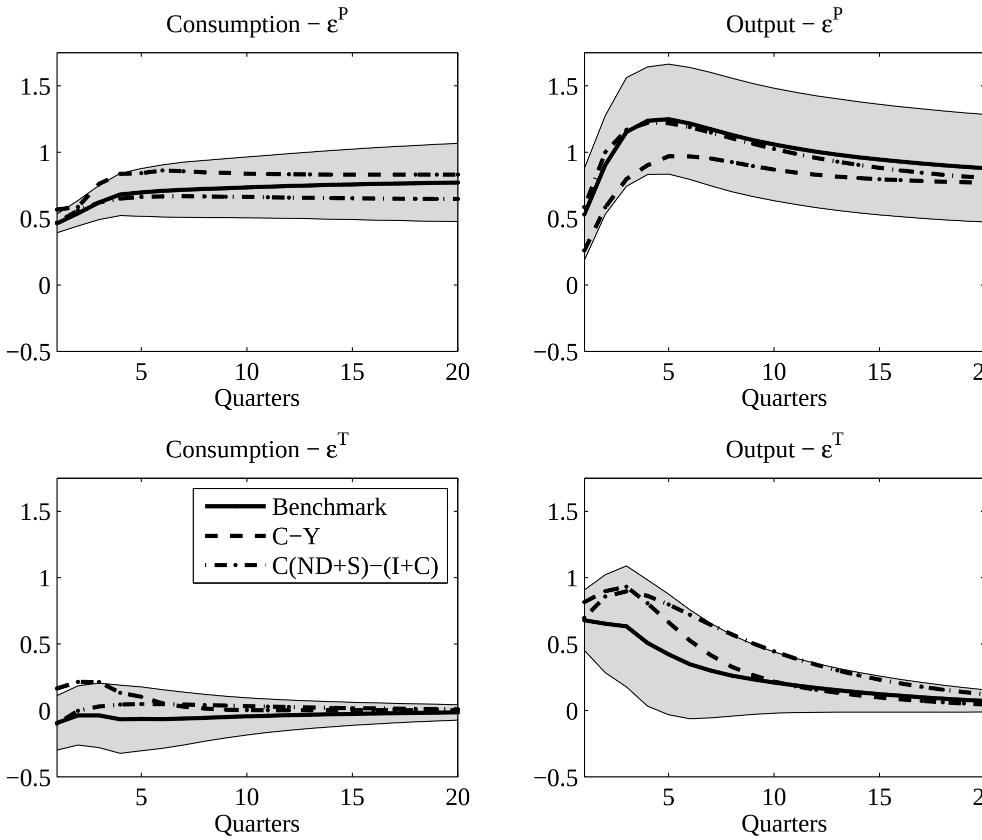
These results are confirmed by looking at the overidentification test reported in column three and four of Table 1, in both cases we find that one cannot reject that $\varepsilon^P = \varepsilon^C$ when different definitions of C or Y are used, and that it is a robust features of the data.

Figure 5: Robustness I (Short Run Identification)



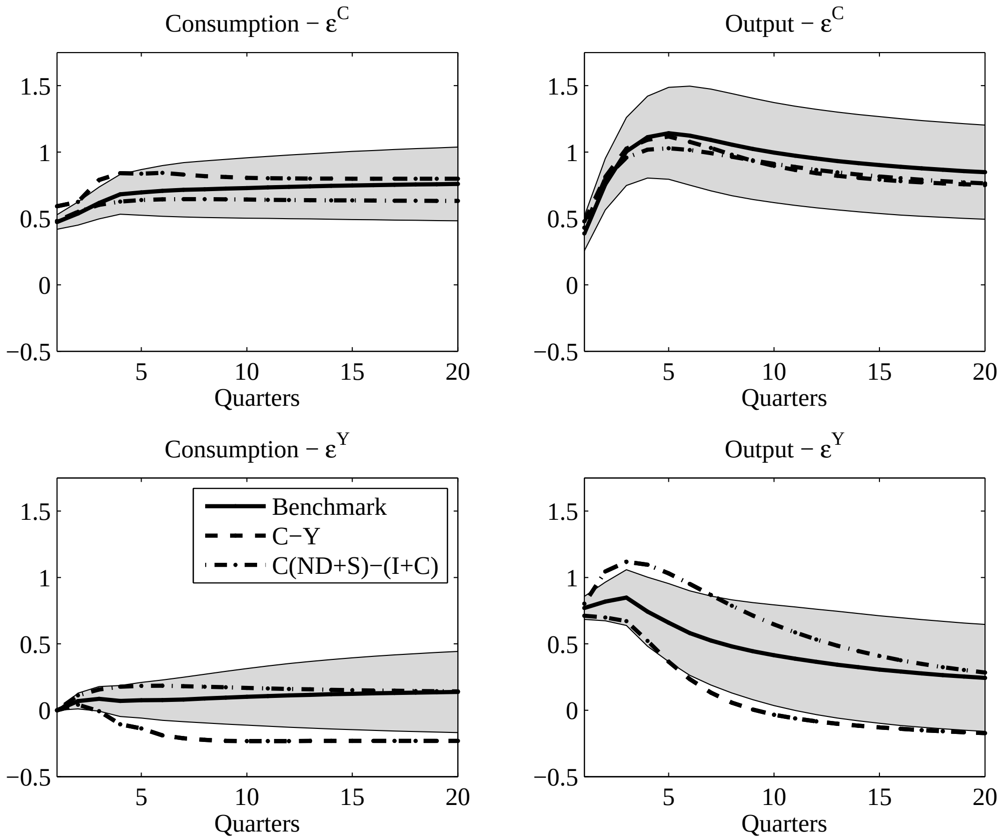
This Figure shows the response of consumption and output to consumption ε^C and output ε^Y one percent shocks in the long run indentification, and for different specification of the model. The bold line corresponds to the benchmark VECM (C, Y) estimated with one cointegrating relation [1;-1] with 3 lags. The dashed line corresponds to a VECM in which the cointegrating relation is estimated. The dashed-dotted line corresponds to a model with eight lags of data. The line wih circles corresponds to a VAR estimated in levels (therefore with four lags of data). All estimations are done using quarterly per capita U.S. data over the period 1947Q1–2004Q4. The shaded area is the 95% confidence intervals obtained from 1000 bootstraps of the benchmark VECM.

Figure 6: Robustness II (Long Run Identification)



This Figure shows the response of consumption and output to temporary ε^T *and permanent* ε^P *one percent shock in the long run indentification, and for different ways of constructing the data. The bold line corresponds to the benchmark VECM* (C, Y) *estimated with one cointegrating relation [1;-1] with 3 lags, where* C *is the consumption of nondurable goodsand services, and* Y *total output. The dashed line corresponds to a VECM in which* C *is measured by total consumption, the cointegrating relation being estimated. The dashed-dotted line corresponds to a VECM where* Y *is measured by consumption plus investment instead of total output, the cointegrating relation being estimated. All estimations are done using quarterly per capita U.S. data over the period 1947Q1–2004Q4. The shaded area is the 95% confidence intervals obtained from 1000 bootstraps of the benchmark VECM.*

Figure 7: Robustness II (Short Run Identification)



This Figure shows the response of consumption and output to consumption ε^C and output ε^Y one percent shocks in the long run indentification, and for different ways of constructing the data. The bold line corresponds to the benchmark VECM (C, Y) estimated with one cointegrating relation [1;-1] with 3 lags, where C is the consumption of nondurable goodsand services, and Y total output. The dashed line corresponds to a VECM in which C is measured by total consumption, the cointegrating relation being estimated. The dashed-dotted line corresponds to a VECM where Y is measured by consumption plus investment instead of total output, the cointegrating relation being estimated. All estimations are done using quarterly per capita U.S. data over the period 1947Q1–2004Q4. The shaded area is the 95% confidence intervals obtained from 1000 bootstraps of the benchmark VECM.

2 An Analytical Model of Market Rushes

In this section, we present a simple analytical model of market rushes. In each period, some new varieties of intermediate goods are created randomly and exogenously. Startups then spend a fixed setup cost to engage in a winner take all competition for securing the market of a newly created variety. The unique winning firm then becomes a monopoly on the market. This position may then be lost randomly at a given exogenous rate. Expansion in variety may or may not have a long run impact on productivity, so that the market rush can be or not a gold rush. We first present the model, then characterize its solution and discuss the equilibrium allocation properties. We contrast those allocations with those from the social optimum. Finally, we show that under some parameters restrictions, the model displays consumption–output dynamics that are in line with those found in the data.

2.1 Model

Firms : There exists a raw final good, denoted Q_t, produced by a representative firm using a fixed factor (say capital) K, labor h_t and a set of intermediate goods X_{jt} with mass N_t according to a constant return to scale technology represented by the production function

$$Q_t = (\Theta_t h_t)^{\alpha_h} N_t^{\xi} \left(\int_0^{N_t} X_{j,t}^{\chi} \mathrm{d}j \right)^{\frac{1-\alpha_h}{\chi}}, \tag{8}$$

where Θ_t is an index of disembodied exogenous technological progress and $\alpha_h \in (0,1)$. $\chi \leqslant 1$ drives the elasticity of substitution between intermediate goods and ξ is a parameter that determines the long run effect of variety expansion. Since this final good will also serve to produce intermediate good, we will refer to Q_t as the gross amount of final good. Also note that the raw final good will serve as the numéraire. The representative firm is price taker on the markets.

Existing intermediate goods are produced by monopolists, who may produce more that one good. Just like in the standard expanding variety model, the production of one unit of intermediate good requires one unit of the raw final output as input. Since the final good serves as a numéraire, this leads to a situation where the price of each intermediate good is given by $P_{j,t} = \frac{1}{\chi}$. Therefore, the quantity of intermediate good j, $X_{j,t}$, produced in equilibrium is given by

$$X_{j,t} = (\chi(1-\alpha_h))^{\frac{1}{\alpha_h}} \Theta_t N_t^{\frac{\xi-1+(1-\alpha_h)/\chi}{\alpha_h}} h_t. \tag{9}$$

and the profits, $\pi_{j,t}$, generated by intermediate firm j are given by

$$\pi_{j,t} = \pi_0 \Theta_t N_t^{\frac{\xi-1+(1-\alpha_h)/\chi}{\alpha_h}} h_t, \tag{10}$$

where $\pi_0 = (\frac{1-\chi}{\chi})(\chi(1-\alpha_h))^{\frac{1}{\alpha_h}}$. Equalization of the real wage with marginal product of labor implies

$$A\Theta N_t^{\frac{\xi+(1-\alpha_h)/\chi-(1-\alpha_h)}{\alpha_h}} = w_t, \tag{11}$$

where $A = (\alpha_h)(\chi(1-\alpha_h))^{\frac{(1-\alpha_h)}{\alpha_h}}$.

Value added is then given by the quantity of raw final good net of that quantity used to produce the intermediate goods. Once we substitute out for X_{jt}, and take away the amount of Q used in the production of X_{jt}s, is given by

$$\begin{aligned} Y_t &= Q_t - \int_0^{N_t} P_{j,t} X_{j,t} dj \\ &= \Theta_t N_t^{\frac{\xi+(1-\alpha_h)(1/\chi-1)}{\alpha_h}} h_t \end{aligned} \tag{12}$$

Note that π_0/A represents the share of profits in the economy, and is therefore between zero and one. This quantity will later appear a relevant parameter. Note that when $\xi = -(1-\alpha_h)(1-\chi)/\chi$, an expansion in variety exerts has no long run impact. In this case, the value–added production function reduces to

$$Y_t = A\Theta_t h_t \tag{13}$$

The net amount of raw final good can serve for consumption, C_t, and startup expenditures, S_t, purposes.

$$Y_t = C_t + S_t. \tag{14}$$

Variety Dynamics : In each period, there is an exogenous probability ε_t that a potential new variety appears in the economy.

In such a case, any entrepreneur who is willing to produce this potential new variety has to pay a fixed of one unit of the setup good to setup the new firm. In order to obtain a tractable solution, we assume that it is always optimal to exploit the whole range of intermediate goods, so that there is no difference between the potential number of varieties and the actual one. We later check that full adoption is indeed optimal. S_t will denote the total expenditures in setup costs. A time t startup will become a functioning new firms with a product monopoly at $t+1$ with the endogenous probability ρ_t. Likewise, an existing firm/monopoly becomes obsolete at an exogenous probability μ. Therefore, the dynamics for the number of products is given by

$$N_{t+1} = (1-\mu+\varepsilon_t)N_t. \tag{15}$$

In the above, μN_t represents the existing firms that are destroyed, while there will be $\varepsilon_t N_t$ openings which can be filled by startups. ε_t follows a random process, with unconditional mean μ. Note that ε is akin to a news shock, as it is bringing some information on the future value of N_t.

The S_t startups of period t compete to secure the $\varepsilon_t N_t$ new monopoly positions. We assume that in equilibrium $S_t > \varepsilon_t N_t$, which can later be verified as being satisfied. The $\varepsilon_t N_t$ successful startups are drawn randomly and equiprobably among the S_t existing ones. Therefore, the probability that a startup at time t will become a functioning firm at $t+1$ is therefore given by $\rho_t = \frac{\varepsilon_t N_t}{S_t}$.

Households : There exists an infinite number of identical households distributed over the unit interval. The preferences of the representative household are given by

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\log(c_{t+\tau}) + g(\overline{h} - h_{t+\tau})\right] \tag{16}$$

where $0 < \beta < 1$ is a constant discount factor, c_t denotes consumption in period t and h_t is the quantity of labor she supplies. Households choose how much to consume, supply labor, hold equity ($\mathcal{E}_t$) in existing firms, and invest in startups (S_t) maximizing 16 subject to the following budget constraint

$$C_t + P_t^{\mathcal{E}} \mathcal{E}_t + S_t = w_t h_t + \mathcal{E}_t \pi_t + (1-\mu) P_t^{\mathcal{E}} \mathcal{E}_{t-1} + \rho_{t-1} P_t^{\mathcal{E}} S_{t-1} \tag{17}$$

where P_t^E is the beginning of period price of equity, prior to dividend payments. Dividends per equity share are assumed to be equal to period–profits π_t.

The first order conditions imply

$$\begin{aligned} gC_t &= w_t && (18)\\ \frac{1}{C_t} &= \lambda_t && (19)\\ \lambda_t(P_t^{\mathcal{E}} - \pi_t) &= \beta \mathbb{E}_t \left[\lambda_{t+1}(1-\mu) P_{t+1}^{\mathcal{E}}\right] && (20)\\ \lambda_t &= \beta \mathbb{E}_t \left[\lambda_{t+1} \rho_t P_{t+1}^{\mathcal{E}}\right] && (21) \end{aligned}$$

2.2 Equilibrium Allocations

The three last first order conditions can be combined to give:

$$\frac{1}{\rho_t C_t} = \beta \mathbb{E}_t \left[\frac{\pi_{t+1}}{C_{t+1}}\right] + \beta \mathbb{E}_t \left[\frac{(1-\mu)}{\rho_{t+1} C_{t+1}}\right] \tag{22}$$

This condition can be interpreted as a free entry condition, whereby the cost of the startup is equal to the discounted sum of profits. Using the labor demand condition (11) and the profit equation (10), the free entry condition (22) rewrites as

$$\left(\frac{S_t}{C_t}\right) = \beta\phi \left(\frac{\varepsilon_t}{1-\mu+\varepsilon_t}\right) \mathbb{E}_t h_{t+1} + \beta \left(\frac{1-\mu}{1-\mu+\varepsilon_t}\right) \mathbb{E}_t \left[\left(\frac{\varepsilon_t}{\varepsilon_{t+1}}\right)\left(\frac{S_{t+1}}{C_{t+1}}\right)\right], \tag{23}$$

with $\phi = g\pi_0/(\widetilde{A})$. Using the labor demand condition (11) and the resource constraint (14), we get

$$\left(\frac{S_t}{C_t}\right) = \psi h_t - 1, \tag{24}$$

where $\psi = gA/(\widetilde{A})$. The free entry condition can therefore we written as:

$$\left(\frac{1-\mu+\varepsilon_t}{\varepsilon_t}\right)(\psi h_t - 1) = \beta\phi\mathbb{E}_t h_{t+1} + \beta\mathbb{E}_t\left[\frac{1-\mu}{\varepsilon_{t+1}}(\psi h_{t+1} - 1)\right] \tag{25}$$

or

$$(h_t - \psi^{-1}) = \beta\delta_t\frac{\pi_0}{A}\mathbb{E}_t h_{t+1} + \beta\delta_t\mathbb{E}_t\left[\left(\frac{1}{\delta_{t+1}} - 1\right)(h_{t+1} - \psi^{-1})\right]. \tag{26}$$

where $\delta_t = \varepsilon_t/(1-\mu+\varepsilon_t)$ is a increasing function of the fraction of newly opened markets ε_t.

Equation (26) shows that current employment h_t depends on h_{t+1}, δ_t and δ_{t+1}, and therefore indirectly depends on all the future expected δs. As δ_t is bringing news about the future, employment is purely forward looking. The reason why future employment favors current employment can be easily given an economic intuition: higher future employment reflects higher expected profits, which therefore stimulates firms entry today. Note that the model possesses a lot of neutrality, as the determination of employment does not depend on neither current nor future changes in disembodied technological change Θ_t.

By repeated substitution, the above equation can be written as a function of current and future values of δ only. Given the nonlinearity of equation (26), it is useful to compute a log–linear approximation around the deterministic steady–state value of employment h. The latter is given by:

$$h = \frac{\psi^{-1}(1-\beta(1-\delta))}{(1-\beta\delta\frac{\pi_0}{A} - \beta(1-\delta))},$$

and the log–linear approximation takes the form

$$\widehat{h}_t = \gamma\mathbb{E}_t\widehat{h}_{t+1} + \left(\frac{h-\psi^{-1}}{h}\right)\mathbb{E}_t\left[\widehat{\delta}_t - \beta\widehat{\delta}_{t+1}\right]$$

where $\widehat{h}_t$ now represents relative deviations from the steady state and $\gamma \equiv \beta\delta(\pi_0/A) + \beta(1-\delta)$ with $\gamma \in (0,1)$. Solving forward, this can be written as

$$\widehat{h}_t = \left(\frac{h-\psi^{-1}}{h}\right)\left(\widehat{\delta}_t - \delta\beta\left(\frac{A-\pi_0}{A}\right)\mathbb{E}_t\left[\sum_{i=0}^{\infty}\gamma^i\widehat{\delta}_{t+1+i}\right]\right) \tag{27}$$

Note that the model posses a unique determinate equilibrium path.

Once the equilibrium path of h is computed, output is directly obtained from equation (12). Finally, combining labor demand (11 and labor supply (18), we obtain an expression for aggregate consumption:

$$C_t = \widetilde{A}\Theta_t N_t^{\frac{\xi+(1-\alpha_h)(1/\chi-1)}{\alpha_h}} \tag{28}$$

Equation (27)reveals that a positive $\widehat{\delta}_t$, – *i.e.* an acceleration of variety expansion, causes an instantaneous increase in hours worked, output and investment in startups S. This boom arises as the result of the prospects of profits derived from securing those new monopoly positions. This occurs absent of any current change in the technology nor the number of varieties. Such an expansion is therefore akin to a "demand driven" or "investment driven" boom. In this analytical model, consumption does not increase on impact following a variety increase, but does with a lag. On the contrary, hours respond negatively to positive $\widehat{\delta}_{t+i}$ with $i > 0$. *Ceteris paribus*, the prospect of more firms creation and therefore more competitors in the future makes current startups less profitable.

2.3 Comparison to the social optimum

Optimality properties of those allocations are worth discussing, and it is useful to compute the socially optimal allocations as a benchmark. The social planner problems is given by [3]:

$$\begin{aligned} &\text{Max} \quad \mathbb{E}_t \quad \textstyle\sum_{i=0}^{\infty} \left[\log C_{t+i} + g(\overline{h} - h_{t+i})\right] \\ &\text{s.t.} \qquad C_t \leq \widehat{A}\Theta_t N_t^{\frac{\xi+(1-\alpha_h)(1/\chi-1)}{\alpha_h}} h_t - \varepsilon_t N_t, \end{aligned} \tag{29}$$

with $\widehat{A} = (1-\alpha_h)^{\frac{\alpha_h}{(1-\alpha_h)}}(\alpha_h)$ and where we have already solved for the optimal use if intermediate goods. The first order condition of the social planner program is given by

$$\frac{\widehat{A}\Theta_t N_t^{\frac{\xi+(1-\alpha_h)(1/\chi-1)}{\alpha_h}}}{\widehat{A}\Theta_t N_t^{\frac{\xi+(1-\alpha_h)(1/\chi-1)}{\alpha_h}} h_t - \varepsilon_t N_t} = g \tag{30}$$

There are many sources of inefficiency in the decentralized allocations. One obvious is the presence of imperfect competition: *ceteris paribus*, the social planner will produce more of each intermediate good. Another one is the congestion effect associated to investment in startups, because only a fraction ρ_t of startups are successful. The social planner is internalizing this congestion effect, and does not duplicate the fixed cost of startups, as the number of startups created is equal to the number of available slots.[4] Because of those imperfections, the decentralized allocation differs from the optimal allocation along a balanced growth path.

The difference between the market allocations and the socially optimal ones that we want to highlight concerns the response to expected future market shocks. It is remarkable that the social optimal allocation decision for employment (30) is static, and only depends on ε_t (positively). This

[3]We assume again here that parameters are such that it is always socially optimal to invest in new variety. One necessary condition for full adoption to be socially optimal is that the long run effect of variety expansion is positive, – *i.e* $\xi > -(1-\alpha_h)(1-\chi)/\chi$

[4]Note that we consider here that parameters are such that it is always optimal to adopt all the new varieties. Another potential source of sub-optimality would be an over or under adoption of new goods by the market. As shown in Benassy [1998] in a somewhat different setup with endogenous growth, the parameter ξ is then crucial in determining whether the decentralized allocations show too much or too little of new goods adoption.

stands in sharp contrast with the market outcome, as summarized in equation (26), in which all the future values of ε are showing up. To understand this difference, let us consider an increase at period t in the expected level of ε_{t+1}. We assume that full adoption is always optimal in both the decentralized and the socially optimal allocations. In the decentralized economy, larger ε_{t+1} means more startup investment in $t+1$ and more firms in $t+2$, and will therefore affect profits of period $t+2$ and onwards. Therefore, a period t startup will face more competitors in $t+2$, which reduces its current value, and therefore decreases startup investment and output. Such an expectation is not relevant for the social planner, that does not respond to news about future values of ε. Therefore, in that simple analytical model, part of economic fluctuations are driven by investors (rational) forecast about future profitability that are inefficient from a social point of view.[5]

2.4 A Gold Rush Configuration

We now make a set of specific assumption on some parameters of the model. The discipline is to obtain a VAR representation of that simple analytical model that provides a structural interpretation of the shocks we have recovered in Section 1. We first assume that disembodied technical change, Θ_t, follows (in log) a random walk without drift $\log \Theta_t = \log \Theta_{t-1} + \varepsilon_t^{\Theta}$. Second, we assume that variety expansion exerts no effect on productivity in the long run. This is achieved by setting that $\xi = -(1-\alpha_h)(1-\chi)/\chi$. Aggregate production function is then given by (13). Finally, we assume that variety expansion shocks ε_t are *i.i.d.* with mean μ, and denote $\varepsilon_t^N = \log(\varepsilon_t) - \log(\mu)$. Under those assumptions, the log of consumption and output is given by:

$$\log Y_t = k_y + \log \Theta_{t-1} + \varepsilon_t^{\Theta} + \log h_t \tag{31}$$

$$\log C_t = k_c + \log \Theta_{t-1} + \varepsilon_t^{\Theta} \tag{32}$$

where k_c and k_y are constants. Using equation (26) to replace h_t by its solution, it is straightforward to write the $MA(\infty)$ representation of the system. For instance, ignoring constant terms, we obtain

$$\begin{pmatrix} \Delta \log(C_t) \\ \Delta \log(Y_t) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & b(1-L) \end{pmatrix} \begin{pmatrix} \varepsilon_t^{\Theta} \\ \varepsilon_t^N \end{pmatrix} = C(L) \begin{pmatrix} \varepsilon_t^{\Theta} \\ \varepsilon_t^N \end{pmatrix}$$

with $b = \frac{(1-\mu)(h-\psi^{-1})}{h}$.

This particular version of the model shares a lot of dynamic properties with the data. First of all the system clearly shows that consumption and output do cointegrate (C(1) is not full rank) with cointegrating vector [1;-1]. Second, it shows that consumption is actually a random walk, that is only affected —in the short run as well as in the long run— by technology shocks ε^{Θ}. Output is

[5] The very result that it is socially optimal not to respond to such news is of course not general, and depends on the utility and production functions specification. The general result is not that it is socially optimal not to respond news about ε, but that the decentralized allocations are inefficient in response to news shocks.

also affected in the short run by a shock that is temporary (ε^N). Hence, computing, sequentially, our short run and long run orthogonalizations with this model would imply $\varepsilon^P = \varepsilon^C = \varepsilon^\Theta$ and $\varepsilon^T = \varepsilon^Y = \varepsilon^N$. Such a model therefore allows for a structural lecture of the results we obtained in Section 1. Permanent shocks to C and Y are indeed technology shocks. Consumption does not respond to variety expansion shocks, which however explain a lot output fluctuations and all the fluctuations of hours. Those variety expansion shocks are creating market rushes that are indeed gold rushes, therefore generating inefficient business cycles as the social planner would choose not to respond the those shocks. In effect, those shocks only trigger rent seeking activities, as startups creation allows to grab a part of the economy pure profits, without any change in the total value of those pure profits.

Although such a simple model allow for a structural interpretation of the data, it is far too simple to be considered as a credible alternative to quantitative DSGE. Furthermore, the orthogonalization we have performed need not to imply a simple mapping from the orthogonalized shocks to the structural ones. This is possible in this version of the model because the long run effect of variety expansion is *exactly* zero and the response of consumption to ε is also *exactly* zero. In the next section, we extend the model to incorporate capital, and use the estimated responses to the short and long run identification VECMs to estimate the size of the technological and variety expansion shocks, together with the long run effect of market expansion ξ. Once those parameters estimated, we will be able to use the model to decompose economic fluctuations in a meaningful way, using our model as a measurement tool.

3 Quantitative Assessment

In this section, we first present the extended model before describing the calibration and estimation procedure. Then we comment the estimated parameters and derive some implications of the estimated model.

3.1 Model, Calibration and Estimation Procedure

Our emphasis in this work is on the existence of a new type of shock, namely a market rush one. In order to gauge the quantitative importance of this shock in the business cycle, we enrich the propagation mechanisms of our baseline model, and estimate it on U.S. data.

An Extended Model: We amend the model by including capital accumulation, two type of intermediate goods and habit persistence in consumption. The final good is now produced by means of capital, K_t, labor, h_t and two sets of intermediate goods $X_{j,t}$ and $Z_{j,t}$, according to

$$Q_t = K_t^{1-\alpha_x-\alpha_z-\alpha_h}(\Theta_t h_t)^{\alpha_h} N_{x,t}^{\overline{\xi}} \left(\int_0^{N_{x,t}} X_t(i)^{\chi}\mathrm{d}i\right)^{\frac{\alpha_x}{\chi}} N_{z,t}^{\widetilde{\xi}} \left(\int_0^{N_{z,t}} Z_t(i)^{\chi}\mathrm{d}i\right)^{\frac{\alpha_z}{\chi}}, \tag{33}$$

with $\alpha_x, \alpha_z, \alpha_h \in (0,1)$, $\alpha_x + \alpha_z + \alpha_h < 1$ and $\chi \geqslant 1$. We impose that $\overline{\xi} = -\alpha_x(1-\chi)/\chi$ so that variety expansion in intermediate goods X has no long-run impact, and that $\widetilde{\xi} = (\chi(1-\alpha_x)-\alpha_z)/\chi$, so that the aggregate value added production function is linear in $N_{z,t}$, the number of intermediate goods Z. Θ_t denotes Hicks neutral technical progress that is assumed to grow at constant factor $\gamma \geqslant 1$.

As in the analytical model, the number of available varieties evolves exogenously according to

$$\begin{aligned} N_{x,t+1} &= (1-\mu+\varepsilon_t^x)N_{x,t} \\ N_{z,t+1} &= (1-\mu+\varepsilon_t^z)N_{z,t}. \end{aligned}$$

In equilibrium, full adoption will always be optimal. We assume that the stochastic processes for productivity and the number of variety are given by:

$$\begin{aligned} \log(\varepsilon_t^x) &= \rho_x \log(\varepsilon_{t-1}^x) + (1-\rho_x)\log(\overline{\varepsilon}^x) + \nu_t^x \\ \log(\varepsilon_t^z) &= \rho_z \log(\varepsilon_{t-1}^z) + (1-\rho_z)\log(\overline{\varepsilon}^z) + \nu_t^z \\ \log\Theta_t &= \log\Theta_{t-1} + \varepsilon_t^{\Theta}. \end{aligned}$$

We assume that ν^x and ν^z are independently normally distributed with zero mean and variance σ_x^2 and σ_z^2. ε^{Θ} is normally distributed with zero mean and variance σ_{Θ}^2.

Capital accumulation is governed by the law of motion

$$K_{t+1} = (1-\delta)K_t + \mathcal{F}(I_t, I_{t-1})$$

where $\delta \in (0,1)$ is the constant depreciation rate. The function $\mathcal{F}(\cdot,\cdot)$ accounts for the presence of adjustments costs in the capital accumulation. $\mathcal{F}(\cdot,\cdot)$ indeed rewrites as

$$\mathcal{F}(I_t, I_{t-1}) = \left[1 - \mathcal{S}\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$

where $\mathcal{S}(\cdot)$ reflects the presence of adjustment costs. We assume that $\mathcal{S}(\cdot)$ satisfies *(i)* $\mathcal{S}(\gamma_z) = \mathcal{S}'(\gamma_z) = 0$ and *(ii)* $\varphi = \mathcal{S}''(\gamma_z)\gamma_z^2 > 0$. It follows that the steady state of the model does not depend on the parameter φ while its dynamic properties do. Notice that following Christiano, Eichenbaum, and Evans [2005], Christiano and Fisher [2003] and Eichenbaum and Fisher [2005], we adopt the dynamic investment adjustment cost specification. In this environment, it is the growth rate of investment which is penalized when varied in the neighborhood of its steady state value. In contrast, the standard specification penalizes the investment-to-capital ratio. The dynamic specification for adjustment costs is a significant source of internal propagation mechanisms as it allows for a hump-shaped response of investment to various shocks.

Finally, we introduce some habit persistence in consumption, and the intertemporal utility function is given by

$$\mathbb{E}_t \sum_{i=0}^{\infty} \left[\log(C_{t+i} - bC_{t+i-1}) + g(\overline{h} - h_{t+i})\right]$$

Note that introducing adjustment costs to investment and habit persistence, while not affecting the main quantitative properties of the model we have presented in section 2, do improve the model ability to capture the shape of the impulse response function.

The model then solves as in the preceding section, except that no analytical solution can be found for the dynamics. Nevertheless, some ratios that are stationary along a balanced growth path can be computed. In particular, the share of profits and intermediate good in value added are given by:

$$\frac{\Pi_t}{Y_t} = \frac{(\alpha_x + \alpha_z)(1-\chi)}{1-(\alpha_x+\alpha_z)\chi} \tag{34}$$

$$\frac{P_{xt}X_t + P_{zt}Z_t}{Y_t} = \frac{\alpha_x + \alpha_z}{1-(\alpha_x+\alpha_z)\chi}. \tag{35}$$

Those ratios will be useful in the calibration phase.

Calibration : Our quantitative strategy is to calibrate those parameters for which we have estimates or that we can obtain by matching balanced growth path ratios with observed average. The time period is the quarter. The discount factor is set such that the household discounts the future at a 3% annual rate. We assume constant markups of 20%, so that $\chi = 0.833$. Depreciation rate is equal to 2.5% per quarter, as common the literature. We assume that the two set of intermediate goods differ only regarding the long run impact of a variety expansion, and therefore assume $\alpha_x = \alpha_z$. α_h and α_x are set such that the model generates a labor share and a share of intermediate goods in value added of, respectively, 60% (Cooley and Prescott [1995]) and 50% (Jorgenson, Gollop, and Fraumeni [1987]). μ is set such that the model generates a consumption share of 70%. The calibrated parameters choice is summarized in Table 3.

Estimation Procedure: We estimate the following seven parameters : the standard deviation of the technological shock innovation σ_Θ, the persistence ρ_x and ρ_z of the two numbers of variety growth rate, the standard deviations σ_x and σ_z of the rate of growth of those number of variety innovations, the habit persistence parameter b and the adjustment cost parameter φ. Those parameters are chosen in order to match the output impulse responses of the long run VECM that we have presented in section 1, and that are displayed on Figure 1. As the long run identification

Table 3: Calibrated Parameters

Preferences		
Discount factor	β	0.9926
Technology		
Elasticity of output to intermediate goods	α_x	0.3529
Elasticity of output to hours worked	α_h	0.4235
Depreciation rate	δ	0.0250
Elasticity of substitution bw intermediates	χ	0.8333
Rate of technology growth	γ	1.0060
Monopoly death rate	μ	0.0086

scheme cannot recover the model structural shocks (three shocks in the model and only two innovations in the VECM, we cannot directly simply match some model theoretical responses with the empirical ones to ε^P and ε^T. Such a procedure would be exact only in the case of the analytical model in the gold rush configuration of subsection 2.4. Therefore, we follow a simulated method of moments approach, as advocated for example by Chari, Kehoe, and McGrattan [2005]. Let $V = (\sigma_\Theta, \rho_x, \sigma_x, \rho_z, \sigma_z, b, \varphi)$ be the parameters to be estimated, M a column vector of estimated moments to match and $\mathcal{M}(V)$ a column vector of the same moments obtained from simulations of the model with parameters V. The set of estimated parameters $\widehat{V}$ is then set so as to minimize the distance $\mathcal{D}$

$$\mathcal{D} = (\mathcal{M}(V) - M)' \, W \, (\mathcal{M}(V) - M) \, ,$$

where W is a weighting matrix that has the inverse of the variance of the estimators of M on its diagonal and zeros elsewhere. The simulated moments $\mathcal{M}(V)$ are obtained as average of 20 simulations[6] of the model over 232 periods, which is the length of our data sample.

The last issue concerns the choice of moments to match. We aim at matching the output impulse responses obtained in section 1 for the long run identification, over the first twenty first quarters. We leave the reproduction of the responses of output in the short run identification and consumption in both identifications as a test for the model. In the long run identification, the output response to a permanent shock display an important hump shape [7], and for that reason, we have supplemented the model with habit persistence and adjustment costs to investment. There are therefore forty moments to match. We will have two ways of testing our model. The first one is by making use of the over-identifying restriction of the estimation procedure (seven parameters for forty moments),

[6] Michaelides and Ng [1997] have shown that efficiency gains are negligible for a number of simulations larger than 10.

[7] Cogley and Nason [1995] have also proposed the estimation of a (C, Y) VAR, and show that the response of output to the temporary shock is hump-shaped. We find in this study that it is the response to the permanent shock that is hump-shaped. This difference comes from the choice of the output variable, that is Net Domestic Product in Cogley and Nason and Gross Domestic Product in our work. We prefer the use of GDP as it is the most commonly used measure of output in the literature.

by mean of a J test, following Hansen [1982]. The second one will be to check whether or not the estimated model possesses the property of the data that we have highlighted in section 1, namely that the short and long run identification, performed on artificial data simulated from the model, are indeed displaying colinearity between ε^P and ε^C (or equivalently between ε^T and ε^Y).

3.2 Results From Estimation

Table (4) presents the estimated values of the seven parameters of interest, together with the value of the J-statistics for over-identification. A key result is that σ_z is very small compared to σ_x, and not significatively different from zero, so that one cannot reject the fact that all market rushes are indeed inefficient gold rushes.

Table 4: Estimated Parameters

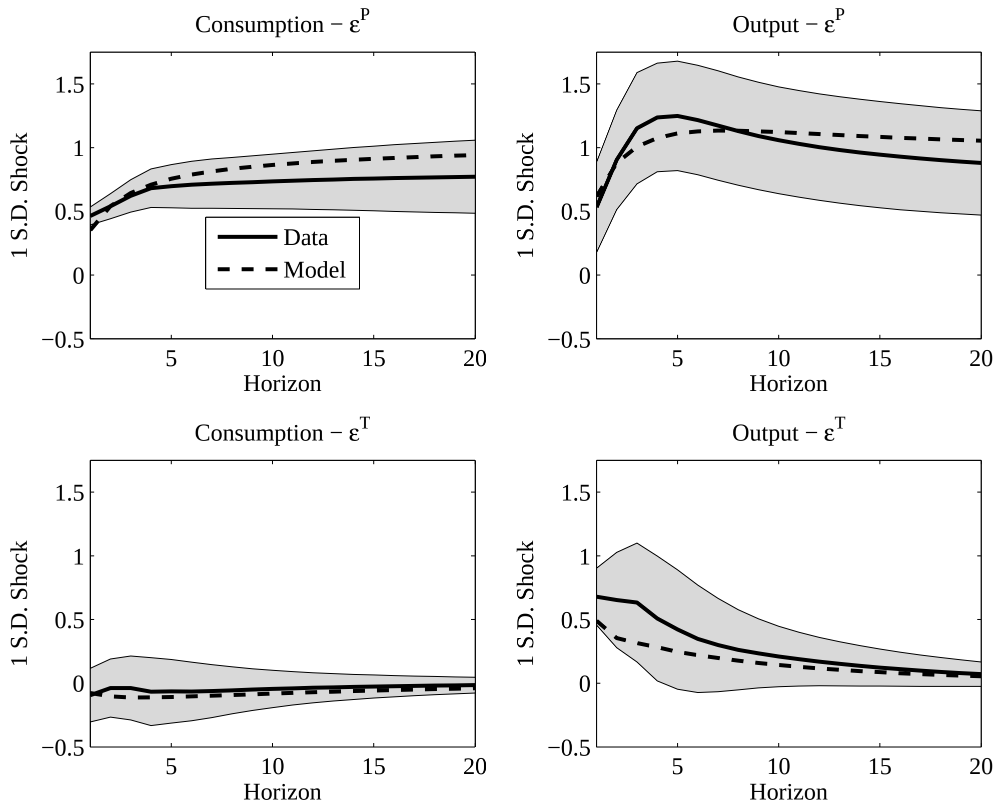
Persistence of the X Variety shocks	ρ_x	0.9166
		(0.0336)
Standard dev. of X Variety shocks	σ_x	0.2865
		(0.0317)
Persistence of the Z Variety shocks	ρ_x	0.9164
		(0.6459)
Standard dev. of Z Variety shocks	σ_z	0.0245
		(0.1534)
Standard dev. of the Technology shocks	σ_Θ	0.0131
		(0.0015)
Habit Persistence parameter	b	0.5900
		(0.1208)
Adjustment Costs parameter	φ	0.4376
		(0.3267)
J–Stat		17.00
		[99.03%]

This Table displays the estimated parameters of our extended model. Those parameters have been estimated by a simulated method of moments. Standard errors are into parenthesis, p–value into brackets.

The impulse responses of the VECM estimated on the artificial data generated with the model are presented on Figure 8, together with the ones estimated on the data. The confidence bands are the one computed from the data. Note that the models IRF are within the confidence band, which confirms that the model does a good job not only on impact and in the long run (as shown by the low level of the J–stat), but also for much of the dynamics. When one compute the distance between the consumption IRF (which is χ^2 distributed), one obtain a statistics value of 40.93, with a p-value of 16.17%: the model is not rejected when evaluated on the consumption dynamics.

The model already displays two of the three properties of the data that we put forward previously: *(ii)* there is virtually no dynamics in the consumption response to the permanent shock, as it affects permanently and almost instantaneously the level of consumption and *(iii)* the temporary shock is responsible for a significant share of output volatility at business cycle frequencies.

Figure 8: Impulse Response Functions VAR versus Model (LR identification)



It is now of interest to test whether the model also possesses the first property: *(i)* the permanent shock to consumption ε^P is indeed the ε^C shock in an consumption–output VECM. We therefore perform our test for the equality between ε^Y and ε^T in the data generated by the model. We then generate 1000 replications of the model simulations. In 87% of the cases we have the property that the $(1, 2)$ element of the long run effect matrix of the Wold decomposition of a (C, Y) VECM is not significantly different from zero. Figure 9 displays the estimated IRF of the short run identification VECM in the date and using he simulated data of the model. Again, the fit is extremely good. As shown on Figure 10, the two VECMs, as estimated on simulated data, have very similar IRF, as obtained in the data. The model is therefore able to reproduce those three salient features of the data we have put forward in section 1.

Figure 9: Impulse Response Functions VAR versus Model (SR identification)

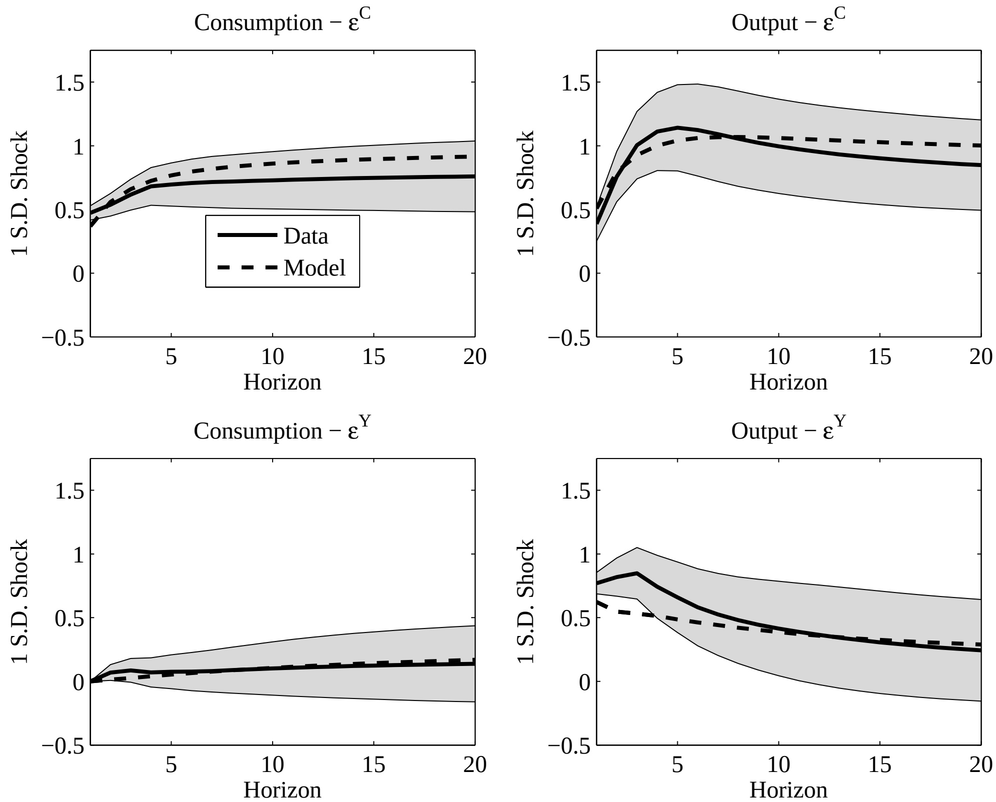
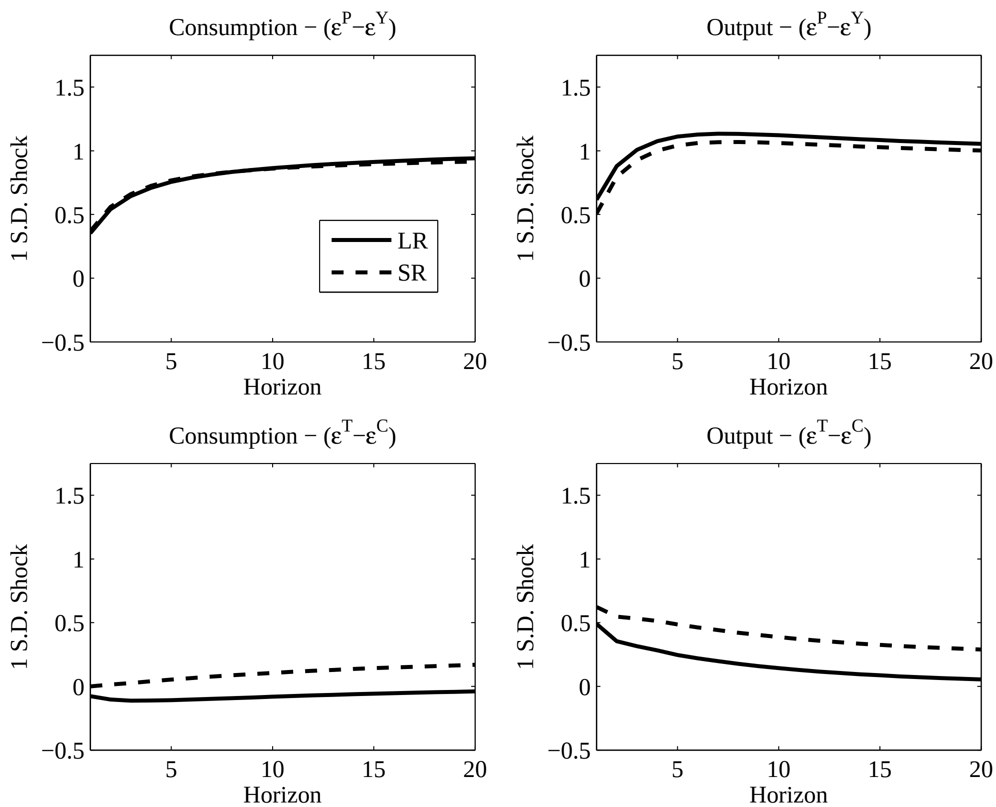


Figure 10: Impulse Response Functions (LR vs SR)



3.3 Accounting For Business Cycle

Once estimated, the model can be used to evaluate the importance of gold rush phenomena in the U.S. business cycle. In effect, the model allow for a meaningful structural variance decomposition of fluctuations. Those variance decomposition are computed on levels and on first-differences, and are displayed in Tables 5 and 6

About one-third of output level fluctuations are explained by the "gold rush" shock on impact, and at all horizons for output growth. When one considers hours, those shocks are responsible for much of the business cycle fluctuations.

Table 5: Contribution of the Variety Expansion Shock to the Business Cycle in the Estimated Model

Horizon	Output			Consumption			Hours		
	ε^{Θ}	ν^x	ν^z	ε^{Θ}	ν^x	ν^z	ε^{Θ}	ν^x	ν^z
1	64.07	35.92	0.01	94.22	5.73	0.05	14.90	85.08	0.02
4	86.36	13.63	0.00	95.09	4.70	0.21	19.34	80.46	0.20
8	92.09	7.81	0.10	96.22	3.39	0.39	32.23	67.61	0.16
20	95.72	3.37	0.91	97.74	1.39	0.87	40.32	59.01	0.67
∞	95.86	0.01	4.13	95.87	0.00	4.13	41.47	56.72	1.81

This table shows the k-period ahead share of the forecast error variance of hours, consumption and output levels that is attributable to each shock in the estimated model.

Table 6: Contribution of the Variety Expansion Shock to the Business Cycle in the Estimated Model

Horizon	Output Growth			Consumption Growth			Hours		
	ε^{Θ}	ν^x	ν^z	ε^{Θ}	ν^x	ν^z	ε^{Θ}	ν^x	ν^z
1	64.07	35.92	0.01	94.22	5.73	0.05	14.90	85.08	0.02
4	68.10	31.86	0.05	94.64	5.15	0.22	19.34	80.46	0.20
8	67.90	31.94	0.16	94.64	5.06	0.30	32.23	67.61	0.16
20	67.65	32.02	0.33	94.41	5.11	0.48	40.32	59.01	0.67
∞	67.61	32.03	0.36	94.18	5.12	0.70	41.47	56.72	1.81

This table shows the k-period ahead share of the forecast error variance of hours level, consumption and output growth that is attributable to each shock in the estimated model.

Figures 11, 12 and 13 show the theoretical responses of the model main variable to the two structural shocks, and Table 7 displays some statistics of the HP-filtered simulated series.

Figure 11: Model Impulse Response Functions to a Technology shock (Θ_t))

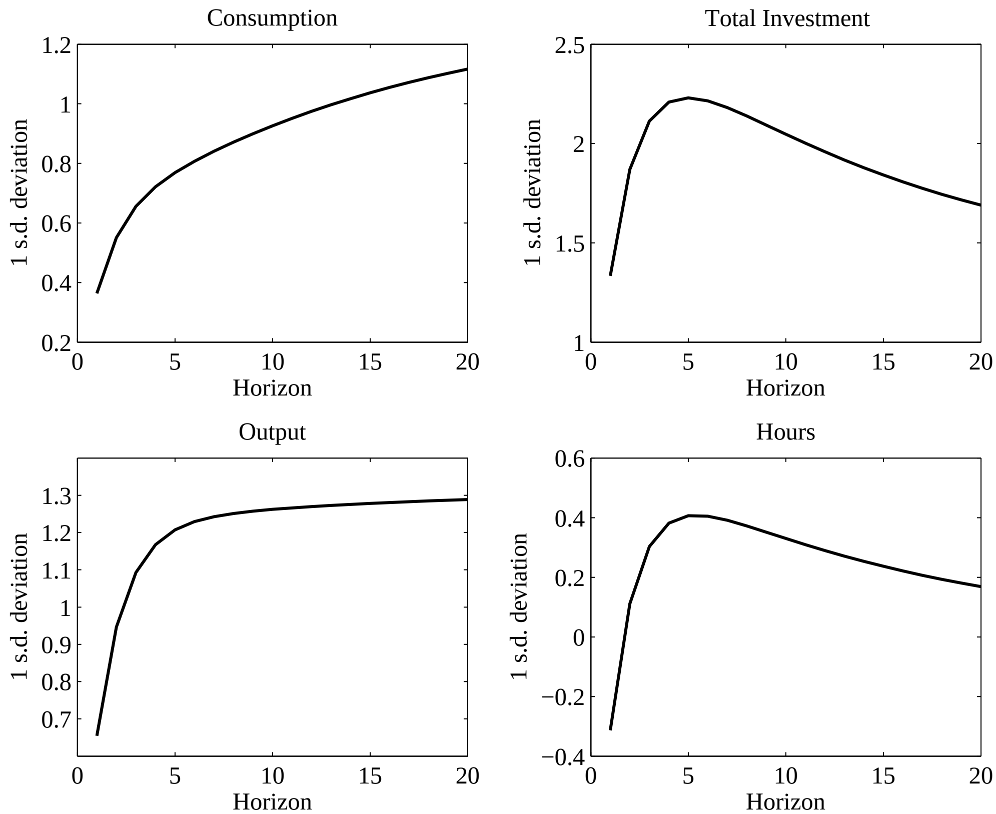


Table 7: Moments (HP–filtered)

	σ_x	$\rho(\cdot, y)$	$\rho(\cdot, h)$
y	1.44	–	–
c	0.78	0.79	–
I Total	3.56	0.95	–
h	1.12	0.66	–
y/h	1.09	0.64	-0.14

Figure 12: Model Impulse Response Functions to a Stationary Variety Expansion Shock (ε_t^x)

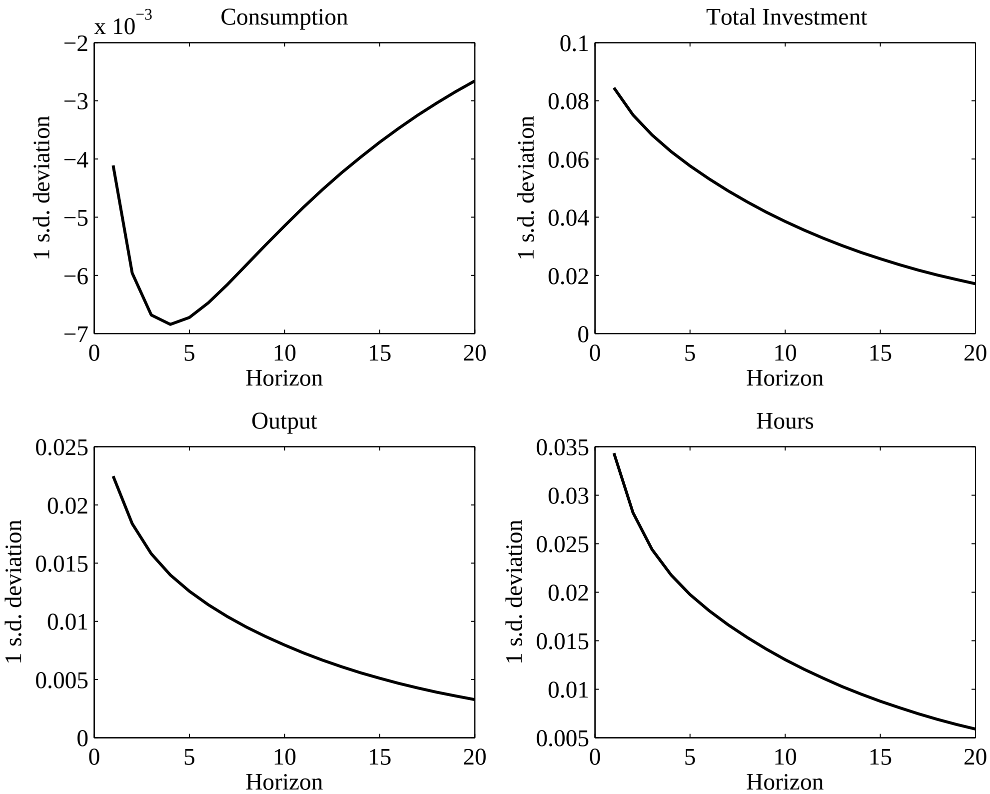
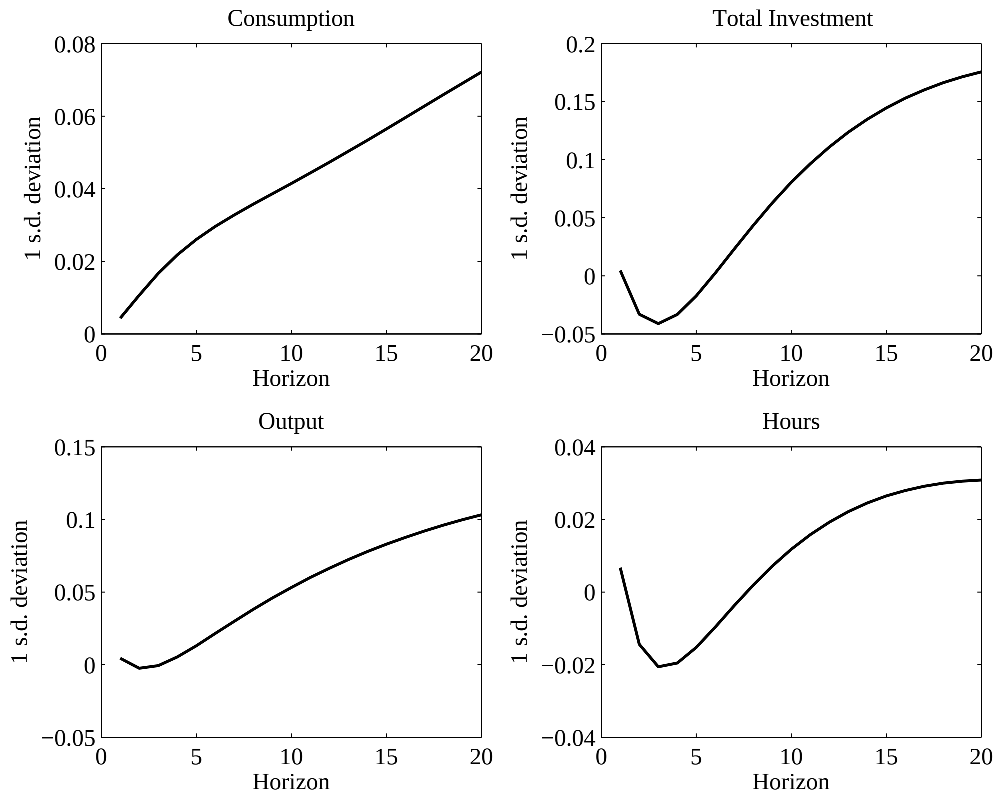


Figure 13: Model Impulse Response Functions to a Permanent Variety Expansion Shock (ε_t^z)



3.4 Alternative Models

In this subsection, we show that alternative models cannot replicate the facts we have highlighted. We consider three models. The first model is a RBC model with habit persistence, adjustment costs to investment and with a permanent and a temporary technology shock (RPC–T). The second model is a RBC model with habit persistence, adjustment costs to investment and with a permanent technology and a temporary preference shock (RBC–P). The third model is the Christiano, Eichenbaum, and Evans [2005] model augmented with a permanent technology shock.

We estimate those three models with the same simulated methods of moments that we have used for our model, matching the IRF of output in the long run identification. The estimated parameters are presented in Table 8. Table 9 then present the J-statistics for those three models, together with the distance statistics for the consumption IRF and for jointly the output and consumption IRF. We also present those statistics for our model. What do we obtain? All the models pass the J-stat, but then, the three alternative models are rejected without ambiguity when evaluated on their capacity of also fitting the consumption IRF.

Table 8: Estimation Results

	b	φ	σ_γ	ρ_χ	σ_χ	ρ_z	σ_z	J–stat(Y)
RBC–P	0.8813	0.6682	0.0143	0.5973	0.0155	–	–	30.13
	(0.0289)	(0.4305)	(0.0019)	(0.0996)	(0.0077)			[0.70])
RBC–T	0.8813	0.6683	0.0143	–	–	0.4974	0.0099	30.13
	(0.0289)	(0.4369)	(0.0019)			(0.1024)	(0.0050)	[0.70]
CEE	0.0000	0.6353	0.0129					23.06
	(0.0000)	(0.1811)	(0.0015)					[0.96]

Note: Standard errors into parenthesis, p–values into brackets.

Table 9: Chi Tests

	J–stat(Y)	Distance–stat(C)	Distance–stat(C,Y)
Our Model	17.41	42.51	92.78
	[0.99]	[0.12]	[0.06]
RBC–P	30.13	160.60	271.73
	[0.70]	[0.00]	[0.00]
RBC–T	30.13	160.58	271.70
	[0.70]	[0.00]	[0.00]
CEE	22.43	136.16	456.11
	[0.97]	[0.00]	[0.00]

Note: p–values into brackets.

4 Conclusion

TO BE WRITTEN

References

Benassy, J. (1998): "Is There Always Too Little Research in Endogenous Growth With Expanding Product Variety?," *European Economic Review*, 42(1), 61–69.

Blanchard, O., and D. Quah (1989): "The Dynamic Effects of Aggregate Supply and Demand Disturbances," *The American Economic Review*, 79(4), 655–673.

Chari, V., P. Kehoe, and E. McGrattan (2005): "A Critique of Structural VARs Using Real Business Cycle Theory," Working Paper 631, Federal Reserve Bank of Minneapolis.

Christiano, L., M. Eichenbaum, and C. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

Christiano, L., and J. Fisher (2003): "Stock Market and Investment Goods Prices: Implications for Macroeconomics," Working Paper 10031, NBER.

Cochrane, J. (1994): "Permanent and Transitory Components of GNP and Stock Prices," *The Quarterly Journal of Economics*, 109(1), 241–265.

Cogley, T., and J. Nason (1995): "Output Dynamics in Real–Business–Cycle Models," *The American Economic Review*, 85(3), 492–511.

Cooley, T., and E. Prescott (1995): "Economic Growth and Business Cycles," in *Frontiers of Business Cycle Research*, ed. by T. Cooley, chap. 1. Princeton University Press, Princeton, New–Jersey.

Eichenbaum, M., and J. Fisher (2005): "Fiscal Policy in the Aftermath of 9/11," *Journal of Money, Credit and Banking*, 37(1), 1–22.

Hansen, L. (1982): "Large Sample Properties of Generalized Method of Moment Models," *Econometrica*, 50, 1269–1286.

Jorgenson, D., F. Gollop, and B. Fraumeni (1987): *Productivity and U.S. Economic Growth*. Harvard University Press, Cambridge (MA).

Michaelides, A., and S. Ng (1997): "Estimating the Rational Expectations Model of Speculative Storage: A Monte-Carlo Comparison of Three Simulation Estimators," Technical report, Princeton University.