An evolutionary theory of money as a medium of exchange*

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1 Introduction

The dynamic general equilibrium framework is the fundamental analytical instrument for macroeconomics, since it has huge advantages in helping us to understand the behavior of modern economies. However, two important shortcuts have being pointed out in the past years and induced some economists to look for alternative ways of modeling. Firstly, in the Arrow-Debreu general equilibrium framework, transactions take place simultaneously at equilibrium. This fundamental property of the equilibrium concept makes it difficult to model money in modern economic theory, since simultaneous transactions do not required a medium of exchange. Another important shortcut of the equilibrium concept, particularly relevant in a dynamic framework, is that agents are supposed to know the exact description of the world and are able to do highly sophisticated computations in order to formulate their optimal strategies. This is the well-known assumption of rational expectations.

An important effort has been made in the last fifteen years to model money as a medium of exchange in the framework of search theory. Kiyotaki and Wright (1991) (1993) are key references in this literature. By using search as a device for modeling the sequential nature of transactions, and by excluding that all possible transaction are mutually beneficial, they find conditions for the existence of fiat money as a medium of exchange.[1] In this paper, the economy is specified in the spirit of the general equilibrium theory without search, and an evolutionary approach is used to model money as a medium of exchange. The paper is not addressed to understand the conditions under which money emerges as a medium of exchange, even if it may be extended in this way.

In the spirit of Clower (1967), this paper specifies a simple *general evolutionary exchange economy*,[2] where money is needed for transactions and plays the role of both a unit of account and a medium of exchange. The environment is the one of a simple general equilibrium model with cash-in-advance –see Lucas and Stokey (1987). The main result of the paper is that the long run evolutionary allocation is equal to the competitive equilibrium allocation of the cash-in-advance economy. In specifying the evolutionary game, we assume that mutants behave as in Calvo (1983). *Calvo mutants*

[1] Wright (1995), Sethi (1999) and Luo (1999) have used alternative evolutionary frameworks to show that the use of money as a medium of exchange does not require stringent assumptions regarding information and rationality.

[2] In this paper, we use the term *general evolutionary economy* in the same sense that equilibrium theorists refer to general equilibrium: An economy is fully described by its environment and the (evolutionary) institutions governing transactions.

are well-informed about the economy, in particular they know the rules of the evolutionary game, and set prices consistently with the stationary solution. Since mutants are the only agents allowed to optimally adjust prices at a given moment in time, they benefit from some market power and use it to set a markup on the unitary cost of the endowment. By doing so, they allow the economy to learn about itself and converge to a market allocation consistent with the long term competitive equilibrium. Most of the time agents repeat their past behavior; times to time they become *imitators*, observe the actions and payoffs of their competitors and follow the best local strategy; and very rarely they become *mutants* and behave rationally, in the sense that they have a deep understanding of the economy, even if it does not require perfect knowledge, and behave consistently. In this framework, the behavior of a very small number of mutants is enough to the economy learn and converge to the competitive equilibrium.

This paper makes a bridge between the dynamic general equilibrium approach to macroeconomics and evolutionary game theory, with a particular attention on the understanding of the effects of money supply on prices and quantities. In this sense, it is close to Saint-Paul (2005), who formulates an evolutionary game to understand sticky price behavior and the propagation of monetary shocks. As in Saint-Paul, price rigidity is a fundamental property of our economy. The main difference is that in our framework mutants are allowed to behave consistently with the rational expectations equilibrium, implying that in the long run the economy learns about itself and converges to an efficient allocation with a price distribution similar to the one generated by Calvo models of sticky prices. In this sense, this paper can be seen as a foundation of modern monetary theory as exposed in Woodford (2003).

As in Vega-Redondo (1997), evolutionary game theory is used to model the implicit behavior of the Adam Smith's invisible hand. This paper can be seen as a general evolutionary extension of the Vega-Redondo imitation-mutation game.[3] In an economy where the number of agents is large enough to preclude any of them to have a sizeable market power, prices are independently set by firms in an evolutionary game that puts the invisible hand at work. In the case the money supply is constant over time, the distribution of prices converges to a monomorphic solution equal to the perfectly competitive equilibrium. When the money supply is permanently increasing at a constant rate, the price index converges to the competitive equilibrium price and inflation tends to the growth rate of money supply, however the stationary distribution of prices does not collapse to a unique price. The

[3] Vega-Redondo specifies and solves a partial evolutionary economy with an exogenously given demand function.

profile of the price distribution depends crucially on the main determinants of the mutation-imitation process.

In the evolutionary game played by agents in this paper, the economy may be alternatively in an excess supply or in an excess demand situation –market clearing is a limit, rare object. Out-of-equilibrium dynamics is modeled in the spirit of quantity rationing equilibrium theory, by the main of assuming a proportional rationing scheme.[4] A fundamental difference with this approach is that we explicitly specify the price dynamics. Even if markets are competitive, firms set prices and an evolutionary dynamics is used to model price dynamics explicitly.

Section 2 describes the economy, section 3 shows the main properties of the long run dynamics and section 4 (still pending) presents some simulations showing the main properties of the short run behavior of the evolutionary economy. Section 5 concludes.

2 Description of the Economy

2.1 Environment

Following the general equilibrium tradition, let us first define the environment of this economy. Time is discrete. The economy is populated by a finite, countable and large number of both individuals and firms denoted by N and H, respectively –firms and individuals have a negligible market power.

There is one, perishable and divisible good. Firm $j \in \{1, ..., H\}$ receives every period t, as manna from heaven, an endowment of measure $q \in \mathbb{R}^+$. The total amount of goods available at any period is $Q = qH$. The representative individual has infinite life and time additive preferences represented by an instantaneous homothetic utility function with constant discount factor $\hat{\beta} \in (0, 1)$. Individuals are the owners of firms and hold a total amount of money M_t. A central bank increases the amount of money every period at the rate $\pi > 0$. The increase in the money supply is distributed across individuals as a lump-sum transfer.

2.2 A Cash-in-Advance Economy

It is well known that money do not play the role of a medium of exchange in the general equilibrium theory, and as a stock of value it is dominated by other assets –the market portfolio in our simple economy. Different theories have been formulated to render money valuable by slightly changing the

[4] See Benassy (1982).

general equilibrium framework. For example, it may be assumed that money is required for transactions, making it optimal for individuals to hold money in order to buy consumption goods.[5] Under this assumption, the so-called cash-in-advance constraint holds at equilibrium, implying the well-known *quantity theory of money* $P_t Q = M_t$, where P_t represents the *equilibrium price* of the physical good with money as the unit of account. At a steady state the market discount factor is equal to the individual discount factor $\hat{\beta}$.

2.3 Evolutionary Game

Let us interpret the cash-in-advance constraint in the following way. By assumption, transactions must always involve money –any form of barter is forbidden. In addition of playing the role of a *unit of account*, money is a *medium of exchange*. Individuals hold money at the beginning of every period and use their money holdings to buy goods. Firms collect profits in the form of money and give the money back to individuals as dividends.

Instead of using an equilibrium concept to analyze this economy, let us formulate it in the tradition of evolutionary game theory. Firm j announces price p_{jt} at the beginning of period t and engages on satisfying at this price any demand until it runs out of stock –price rules are described in a following section. Money is the unit of account and prices are supposed to be real numbers. At the end of the period, it may be that some firms keep unsold units of the perishable good, which cannot be transferred to the next period. In such a case, these firms are said to be quantity constrained by a demand shortage.

Let firms be divided into groups according to the price they announce, so that there are $K \leq H$ groups of firms, each setting a different price p_k. In the following, we will refer to price p_k as the k-strategy. Without any lost of generality, let us assume that $p_0 > p_1 > ... > p_{K-1}$. Let h_k be the fraction of firms of type k, such that $\sum_k h_k = 1$.

Individuals use their money holdings to buy the physical good. They are assumed to observe all prices every period. They order firms by prices, visit first the group of firms announcing the lowest price and buy. When firms in this group cannot satisfy the total demand, individuals move to the following group in the list. They continue visiting firms until they spend the total amount of money M or all firms run out of stock. Finally, they return home and consume. We say that individuals are quantity constrained when they cannot buy as many units as they would like, because of a shortage of supply.

[5] See Lucas and Stockey (1987) and Woodford (1994).

To complete the description of time t transactions, we must specify how agents are rationed. In order to keep a representative household, we assume that any supply shortage is equally distributed across individuals. However, the evolutionary dynamics does not depend on the particular way individuals are rationed, implying that any alternative rationing scheme would generate the same solution of the evolutionary game. How a shortage of demand is distributed across firms? Without any lost of generality, let us assume that all firms in group $k+1$ run out of stock at time t, but group k faces a demand shortage. It would be the case if the *residual demand* faced by group k is $0 < M - qH \sum_{i>k} p_i h_i < qHp_k h_k$. In such a situation, it is assumed that the residual demand is equally distributed across firms in group k.[6]

A huge literature in the seventies and eighties has developed equilibrium concepts to deal with situations where markets are not cleared by prices.[7] Typically, a rationing scheme is assumed. In this paper, we adopt proportional rationing schemes to deal with excess supply and excess demand situations.

2.4 Evolutionary Dynamics

The description of the evolutionary economy in section 2.3 is still incomplete, since the evolutionary rules telling how firms set prices have not been defined yet. In this paper, price rules rely on imitation and mutation. In particular, we apply the *imitate the best* (IB) rule.[8] Given that all firms in the same group set the same price and sell the same quantity, IB is equivalent to the *imitate the best average* rule.

Each period, a fixed number of firms $L < H$ imitates the best price strategy. Under IB, the performance of a particular price strategy is evaluated by the profits of a representative firm following this strategy. To formalize the rule, note that at each period K distinct, ordered prices are observed. Let us denote x_k at the number of goods sold by the k's representative firm, and profits by $\hat{\Pi}_k = p_k x_k$. Let us defined the *best local strategy* $r = \arg\max_k \hat{\Pi}_k$. In the case more than one strategy generate the same profits, the best local strategy corresponds to the largest price. The IB rule specifies that L randomly chosen firms set price $p_{j,t+1} = p_{rt}$ in period $t+1$.

In addition to imitators, we allow for mutants. We assume there is exactly

[6] As an alternative, the rationing scheme may be random. Under this assumption, firms in group k would have different profits, but the average profit will be the same that under proportional rationing. In this case, the imitate the best average rule would generate exactly the same solution as the IB rule in our framework.

[7] See Benassy (1982) and Dreze (1974).

[8] Cite the literature here.

one mutant every period, which is selected randomly. Like imitators, the mutant changes her price, but instead of choosing the price p_r suggested by the imitation rule, she chooses it as a factor η of the equilibrium price. The mutant knows the cash-in-advance constraint holds, and has enough information to compute the equilibrium price. She also knows she will keep her price unchanged until she will become an imitator or she will mutate again, and using this information she checks that in the long run her strategy is optimal. In the following, we refer to them as *Calvo mutants*, since they have a behavior close to the one described by Calvo (1983) and extensively used in dynamic general equilibrium macro models.[9]

At the initial period $t = 1$, firms carry a (past) price p_{j0}. Initial prices are randomly assigned on the support $[\underline{p}, \bar{p}]$, $0 < \underline{p} < \bar{p}$. In the following section, we study the properties of the imitation and mutation dynamics.

3 Stationary Solution of the Evolutionary Dynamics

At the stationary solution, mutant prices may be written as $p_{0t} = \eta P_t$, where $P_t = \frac{M_t}{qH}$ is the equilibrium price and $\eta > 0$. Note that P_t is the equilibrium value of one unit of the endowment and measures its shadow value. In this sense, we will refer to η as the *markup* set by the Calvo mutant over marginal (shadow) costs. Notice that prices p_{kt} for $k \geq 1$ have been set by previous mutants, implying that $p_{kt} = \eta P_{t-k} = \eta\left(1+\pi\right)^{-k} P_t$. Consequently, the k-strategy may be normalized by the equilibrium price P_t, such that $\tilde{p}_k = \frac{p_{kt}}{P_t} = \eta\left(1+\pi\right)^{-k}$ is time independent for all $k \geq 0$.

In a stationary situation, the *best local strategy* must be stationary. Let us refer to it as the strategy $b-1$. Imitators adopt this strategy in the following period, by playing strategy b. The random selection of imitators and mutants generates a distribution of firms across price strategies h_k, such that $\sum_{k\geq 0} h_k = 1$. In the following, we assume that $h_k = \nu$ for $k < b-1$. Under this assumption, no mutant is allowed to move to another strategy until she is playing the best local strategy. It guarantees that the best local strategy is always being played.

As stated above, the representative firm playing the k-strategy sells the amount $x_k = \min\left\{q, \frac{R_k}{p_k}\right\}$, where the residual demand is

$$R_k = \frac{M - qH\sum_{j>k} p_j h_j}{h_k H} = \frac{1-\eta\mathcal{A}_{k+1}}{\eta\left(1+\pi\right)^{-k} h_k} p_k q, \tag{1}$$

[9] See Woodford (2003).

and $\mathcal{A}_k = \sum_{j \geq k} (1+\pi)^{-j} h_j$. Normalized firm profits are $\Pi_k = \eta (1+\pi)^{-k} x_k$.

The price index at time t is defined as

$$\mathcal{P}_t \equiv \sum_{k \geq 0} p_{kt} h_k = \eta \mathcal{A}_0 P_t.$$

The ratio of the evolutionary price $\mathcal{P}_t$ to the equilibrium price P_t has two components, the mutant markup η, and the factor $\mathcal{A}_0$. In the evolutionary economy, only the Calvo mutant is adjusting her price up in order to follow the increase in money supply. The factor $\mathcal{A}_0 < 1$ measures the extent other firms follow mutant price changes. In the following, we will refer to $\mathcal{A}_0$ as the *extent of price adjustments*. Consequently, in order to the price index be close to the equilibrium price, the mutant's markup must be strictly larger than one. It is important to remind that supply is equal to demand only if the evolutionary price index is equal to the equilibrium price. It is easy to see that Calvo mutants have an incentive to charge a markup at least equal to the adjustment factor $\mathcal{A}_0$ in order to take advantage of any excess demand.

By definition of the imitators' strategy, $x_{b-1} > 0$, implying that $x_k = q$ for all $k \geq b$, $x_{b-2} \geq 0$, with $x_{b-2} = 0$ if $x_{b-1} < q$, and $x_k = 0$ for all $k < b-2$. Given that individuals visit firms following the strict order of prices, $b-1$ is the best local strategy only if all firms announcing a price smaller than p_{b-1} run out of stock. On the other side, firms pricing more than p_{b-2} face a zero residual demand. The situation of firms following strategy $b-2$ depends on their residual demand, which may be positive only if $x_{b-1} < q$. It is easy to see that normalized profits, $\Pi_k > \Pi_{k+1}$ for all $k \geq b$, $\Pi_{b-1} \geq \Pi_b$, $0 \leq \Pi_{b-2} < \Pi_{b-1}$, and $\Pi_k = 0$ for all $k < b-2$.

The expected value of strategy k is given by

$$V_k = \Pi_k + \hat{\beta} \begin{cases} V_{k+1} & \text{if } k \in \{0, \ldots, b-2\} \\ \nu V_0 + \mu V_b + (1-\nu-\mu) V_{k+1} & \text{otherwise} \end{cases}. \tag{2}$$

The value function V_k is normalized by P_t. The probability of becoming a mutant is $\nu = \frac{1}{H-b+1}$ and the probability of becoming an imitator is $\mu = \frac{L}{H-b+1}$, and both are constant at a stationary solution. Note that $L < H-b+1$ is required for these two probabilities be well defined.

The main Proposition of the paper is proved in the next, but before we need to prove some Lemmas.

Lemma 1 *In a stationary solution, $b \in \{1, 2\}$*

Proof. To prove this Lemma, we first study the value function V_k. From (2),

$$V_k - V_{k+1} = \Pi_k - \Pi_{k+1} + \begin{cases} \hat{\beta}\left(V_{k+1} - V_{k+2}\right) & \text{if } k \in \{0, \ldots, b-3\} \\ \beta\left(V_{k+1} - V_{k+2}\right) & \text{if } k \in \{b-1, \ldots\} \\ \hat{\beta}\left[\left(V_{b-1} - V_b\right) - \nu\left(V_0 - V_b\right)\right] & \text{if } k = b-2 \end{cases}$$

where $\beta = \hat{\beta}\left(1 - \nu - \mu\right)$. Then,

1. For $k \in \{b-1, b, \ldots\}$. Since $\tilde{p}_k = \eta\left(1+\pi\right)^{-k}$ for all $k \geq 0$, and $x_k = q$ for all $k \geq b$, $\Pi_k = \eta\left(1+\pi\right)^{-k} = \left(1+\pi\right)\Pi_{k+1}$, implying that

$$V_k - V_{k+1} = \sum_{j=0}^{\infty} \beta^j \left(\Pi_{k+j} - \Pi_{k+j+1}\right) = \frac{\pi}{1+\pi-\beta}\Pi_k > 0,$$

for all $k \geq b$. Since $\Pi_{b-1} \geq \Pi_b$ from the definition of the best local strategy $b-1$, $V_{b-1} > V_b$.

2. $k \in \{0, 1, \ldots, b-3\}$. As sated in point 6 above, $\Pi_k = 0$ for $k \leq b-3$. It implies that

$$V_k - V_{k+1} = \hat{\beta}^{b-3-k}\left[-\Pi_{b-2} + \hat{\beta}\left(V_{b-2} - V_{b-1}\right)\right]$$

for $k \leq b-3$. There are two possible situations:

(a) $x_{b-1} < q$, $\Pi_{b-2} = 0$. Then,

$$\begin{aligned} V_{b-2} - V_{b-1} &= -\Pi_{b-1} + \hat{\beta}\left(V_{b-1} - V_b\right) - \hat{\beta}\nu\left(V_0 - V_b\right) \\ &\leq -\Pi_{b-1} + \hat{\beta}\left(V_{b-1} - V_b\right), \end{aligned}$$

$V_0 \geq V_b$ because the mutant is playing an optimal strategy. Consequently,

$$V_{b-2} - V_{b-1} \leq -\left(1-\beta\right)\left[\Pi_{b-1} + \frac{\hat{\beta}\left(1+\pi\right)}{1+\pi-\beta}\Pi_b\right] < 0.$$

(b) $x_{b-1} = q$, $\Pi_{b-2} \geq 0$.

$$\begin{aligned} -\Pi_{b-2} + \hat{\beta}\left(V_{b-2} - V_{b-1}\right) &= -\left(1-\hat{\beta}\right)\Pi_{b-2} + \hat{\beta} \\ &\quad \left[-\Pi_{b-1} + \hat{\beta}\left(V_{b-1} - V_b\right) - \hat{\beta}\nu\left(V_0 - V_b\right)\right] \end{aligned}$$

Since $V_0 \geq V_b$

$$\begin{aligned}
-\Pi_{b-2} + \hat{\beta}\left(V_{b-2} - V_{b-1}\right) &\leq \hat{\beta}\left[\hat{\beta}\left(V_{b-1} - V_b\right) - \Pi_{b-1}\right] \\
&= -\hat{\beta}\frac{(1+\pi) - \left(\beta + \hat{\beta}\pi\right)}{1+\pi-\beta}\Pi_{b-1} < 0.
\end{aligned}$$

Since the mutant is playing the best strategy, $V_0 \geq V_k$ for all $k \geq 1$. It implies $b \in \{1, 2\}$. ■

Lemma 2 *In a stationary solution, $b = 1$ iff*

$$\mathcal{A}_0^{-1} \leq \eta \leq \hat{\eta} \equiv \left(\mathcal{A}_0 - \frac{\nu\pi}{1+\pi}\right)^{-1}$$

Proof. $b = 1$ requires

1. $k = 0$ is the best local strategy if $\Pi_0 \geq \Pi_1$. By definition of x_0 in point 4 above and some algebra, it's easy to prove that $\Pi_0 \geq \Pi_1$ iff $x_0 \geq (1+\pi)^{-1} q$ iff $\eta \leq \hat{\eta}$.
2. By definition $x_0 \leq q$, and $x_0 \leq q$ iff $\eta \geq \mathcal{A}_0^{-1}$.

This completes the proof. ■

Lemma 3 *In a stationary solution, $b = 2$ iff*

$$\hat{\eta} < \eta \leq \tilde{\eta} \equiv \left(\mathcal{A}_0 - \frac{\nu\pi}{1+\pi}\frac{1+\pi+\hat{\beta}\mu}{1+\pi-\beta}\right)^{-1}$$

Proof. $b = 2$ requires

1. $k = 0$ is the mutant's strategy if $V_0 \geq V_1$. From Lemma 1, under $b = 2$,

$$\begin{aligned}
V_0 - V_1 &= \Pi_0 - \Pi_1 + \hat{\beta}\left(V_1 - V_2\right) - \hat{\beta}\nu\left(V_0 - V_2\right) \\
&= \Pi_0 - \Pi_1 + \hat{\beta}(1-\nu)\left(V_1 - V_2\right) - \hat{\beta}\nu\left(V_0 - V_1\right) \\
&= \Pi_0 - \Pi_1 + \hat{\beta}(1-\nu)\frac{\pi}{1+\pi-\beta}\Pi_1 - \hat{\beta}\nu\left(V_0 - V_1\right) \\
&= \Pi_0 - \Pi_1\frac{(1+\pi) - \left(\beta + \hat{\beta}(1-\nu)\pi\right)}{1+\pi-\beta} - \hat{\beta}\nu\left(V_0 - V_1\right).
\end{aligned}$$

Then, under $b = 2$ $V_0 - V_1 \geq 0$ iff $\Pi_0 \geq \Pi_1 \frac{(1+\pi)-\left(\beta+\hat{\beta}(1-\nu)\pi\right)}{1+\pi-\beta}$ iff $x_0 \geq q\frac{(1+\pi)-\left(\beta+\hat{\beta}(1-\nu)\pi\right)}{(1+\pi-\beta)(1+\pi)}$. By definition of x_0 in point 4 above, and some algebra it's easy to prove that $x_0 = \frac{1-\eta\mathcal{A}_1}{\eta\nu}q$. Then, $V_0 - V_1 \geq 0$ iff $\eta \leq \tilde{\eta}$.

2. $k = 1$ is the best local strategy if $\Pi_0 < \Pi_1$, and $\Pi_0 < \Pi_1$ iff $x_0 < (1+\pi)^{-1} q$ iff $\eta > \hat{\eta}$

This completes the proof. ■

It is easy to check that $\hat{\eta} \leq \tilde{\eta}$, with strict equality only if $\pi = 0$, or $\nu = 0$, or $\hat{\beta} = 0$.

Proposition 4 *At a stationary solution the Calvo mutant's best strategy is* $\eta = \tilde{\eta}$

Proof. At a stationary solution, the Calvo mutant is restricted to play the best strategy among the stationary strategies $\eta \in \left[\mathcal{A}_0^{-1}, \tilde{\eta}\right]$. Past and future mutants play the same strategy $\breve{\eta} \in \left[\mathcal{A}_0^{-1}, \tilde{\eta}\right]$, since the economy is on a stationary solution. Let us assume that $\eta > (1+\pi)^{-1}\tilde{\eta}$, such that the Calvo mutant is playing the 0-strategy. We will prove that it is always the case. From the definition of Π_0 and x_0 in points 5 and 4 above, (2) becomes

$$V_0 = \frac{1-\breve{\eta}\mathcal{A}_1}{\nu} q + \hat{\beta} V_1,$$

and for all $k \in \{1, 2, \ldots\}$

$$V_k = \eta (1+\pi)^{-k} q + \hat{\beta}\left(\nu V_0 + \mu \breve{V}_b + (1-\nu-\mu) V_{k+1}\right).$$

$\breve{V}_b$ is the imitators' value function. It's clear that V_0 is increasing in η. Consequently, the η that maximizes V_0 is equal to the upper-bound $\tilde{\eta}$, which verifies the condition $\eta > (1+\pi)^{-1}\tilde{\eta}$. ■

Corollary 5 *At the stationary solution,* $b = 2$

Proof. It derives directly from the previous Proposition and Lemmas. ■

Under $b = 2$, the expected distribution of firms across price strategies is

$$h_k = \begin{cases} \nu & \text{if } k = 0 \\ \left[\nu(1-\nu-\mu)+\mu\right](1-\nu-\mu)^{k-1} & \text{if } k \geq 1 \end{cases},$$

implying that the extent of price adjustments is expected to be $\mathcal{A}_0 = \frac{\nu\pi+\nu+\mu}{\pi+\nu+\mu}$. In the extreme case, where all firms are mutants or imitators, i.e. $\nu + \mu = 1$, the extent of price adjustments is equal to $\frac{1+\nu\pi}{1+\pi}$, which is increasing in the frequency of mutations ν. An increase in the number of imitators has always a positive effect on the extent of price adjustments factor $\mathcal{A}_0$, since other firms follow mutants more frequently. A cut on the inflation rate increases $\mathcal{A}_0$, because the need of adjusting prices is reduced when inflation is low, making the mutation-imitation process more efficient.

Corollary 6 *At the stationary solution with Calvo mutants* $\mathcal{P}_t = \tilde{\eta}\mathcal{A}_0 P_t > P_t$

Proof. It derives directly from Proposition 1. ■

Notice that at the stationary solution of the evolutionary economy there must be a demand shortage since $\mathcal{P}_t > P_t$. Let us call $\tilde{Q} \equiv \frac{M}{\mathcal{P}} < Q$, and define the excess supply as

$$\frac{Q - \tilde{Q}}{Q} = 1 - \frac{P_t}{\mathcal{P}_t} = \frac{\mathcal{B}}{\mathcal{A}_0},$$

where $\mathcal{B} = \frac{\nu\pi}{1+\pi}\frac{1+\pi+\hat{\beta}\mu}{1+\pi-\beta}$. Since there is only one mutant, and $b = 2$ requires the mutant be selling something, $x_k = q$ for all $k > 0$. Then, only the mutant may be rationed, making the extend of an excess supply be bounded by ν, which is small under the assumption that the number of firms is large.

Remark 7 $\lim_{\nu\to 0} \mathcal{P}_t = P_t$

When the number of firms goes to infinity, the economy converges to the perfect market situation. In our framework, this is equivalent to let the frequency of mutants goes to zero. In this case, Calvo mutants cannot take any advantage of rising prices and optimally set the markup factor equal to the extent factor. Under perfect competition, the long run behavior of the economy is equal to the long run behavior of the cash-in-advance economy. In this case, $\tilde{Q} = Q$ and the economy converges to a market clearing situation.

The assumption that Calvo mutants behave rationally is enough for the evolutionary economy to be consistent with the Lucas critique. Suppose the monetary authority realizes that most firms do not react directly to changes in policy, and decides to increase the rate at which she injects money. Since prices do not follow directly the change in money supply, money would have a real effect by increasing demand and consumption. However, Calvo mutants would be informed about the change in policy, and would adjust their markup consistently, which would allow the market economy to learn about it. It is interesting to see that by increasing π, the monetary authority reduces the extent of price adjustment at the time it increases the market power of mutants. When the number of firms is large enough to make markets competitive, an increase in the growth rate of the money supply cannot affect the long run behavior of prices. Consequently, even if money is not neutral in the short run, it is in the long run.

Remark 8 *Under* $\pi = 0$, $\mathcal{A}_0 = \tilde{\eta} = 1$

In the extreme case of a zero growth rate of money supply, Calvo mutants have no incentive to rise prices. They just set as price the equilibrium price and the imitation dynamics makes the price distribution collapse to a monomorphic equilibrium with the unique price equal to the perfectly competitive price. Anagnostopoulos et al (2005) show that in such a framework mutants don't need to be of the Calvo type for the economy converge to the unique equilibrium price. It's enough that the mutant chooses a price randomly from a set of prices including the equilibrium price. At some point, a mutant will get the equilibrium price by chance and the others will eventually imitate her until the economy will converge to the unique equilibrium.

4 The Short Run Dynamics

5 Conclusions

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