Neighborhood Informational Effects and the Organization of the City

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January 2004

1We would like to thank the participants of the ESF/SCSS workshop on “Local Public Goods, Politics and Multijurisdictional Economies” (Université Paris-1, July 2002) and especially Francisco Martinez Mora, as well as the participants of the seminars held at the Université Aix-Marseille (GREQAM) and Université de Toulouse (GREMAQ) for their comments. We are also grateful to Patrick Fève and Jean-Marc Tallon.

A previous version of this paper was presented at the 17th Annual Congress of the European Economic Association (Venice, August 2002).

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Abstract

In this paper, we provide a theoretical framework for exploring the consequences of neighborhood informational effects - identified as role models - so as to deduce the urban configuration. In particular, we have examined whether this role model effect is a source of urban segregation. With this aim, we have developed an overlapping generations model of community formation. When young, an individual must choose whether to invest in education or not. The crucial feature of our framework is that children assess the economic pay-off of education by observing the experience of the older generation residing in their neighborhood. When an adult, an individual who cares about her offspring’s income must choose the family’s location. We show that there exist three urban configurations. (i) An efficient-neutral city may occur where the socio-economic composition of each neighborhood makes its inhabitants well informed and therefore willing to invest in education. (ii) An inefficient-regressive city may emerge where socio-economic segregation makes the inhabitants of poor communities be misinformed about the benefits of education. Thus an inefficient-regressive city displays high-poverty communities that are deprived of any opportunity for youths to invest in education. (iii) An inefficient-neutral city may arise where the misperception of returns on schooling spreads across all neighborhoods and none of the young people exerts any educational effort.

KEYWORDS: Beliefs, Informational Effects, Inequality, Social Segmentation.

JEL CLASSIFICATION: D30, D82, I2, J24, R1.
1 Introduction

Residential location greatly affects children’s access to opportunity through neighborhood effects (see for instance Case and Katz [1991], Katz, Kling and Liebman [2001]). The literature offers three main reasons why spatial neighborhood characteristics influence children’s school achievement: peer effects, community resources and adult influences. In particular, the last causal effect refers to the fact that outcomes of the older inhabitants may affect dramatically decisions of younger residents. For instance, in his influential study on American ghettos, Wilson [1987] argues that ghettos, which are deprived of role models, may harm the aspirations and motivations of children who grow up in these communities and may affect their educational achievements and professional success. Thus, one way of understanding adult influences is to analyse how children form their expectations based on the experiences of the adults in their neighborhood.

However, the literature is more interested in peer effects and community resources rather than in role model effects. On the empirical side, most studies estimate reduced form models where individual outcomes are regressed on the local average outcome (see Ginther et alii [2000] for a survey). On the theoretical side, most frameworks consider neighborhood effects as a black box. A common way of formalizing such effects is to consider that youths’ educational achievement depends in a monotonic way on the parents’ local average level of human capital (de Bartolome [1990], Bénabou [1996a,b], Durlauf [1996]) or on the children’s local average level of educational effort (Bénabou [1993]). Despite the fact that informational mechanisms are not explicitly introduced\textsuperscript{1}, the common view is to consider that this modelling strategy captures role model effects.

Our paper provides a theoretical framework that explores the consequences of informational neighborhood effects, identified as role models, on urban configurations. In particular, we have examined whether this role-model effect is a source of urban segregation.

The role-model proposed here is a local informational externality defined as follows: an agent updates her beliefs based on the observation of the experiences of the older members of her group. More precisely, we have considered the case of a family composed of an adult and a child. The latter does not know the true returns on education captured by the probabilities of becoming rich or poor. She estimates such returns on the basis of the trajectories of the parents living in the same neighborhood. Hence, the social composition of a neighborhood plays a crucial role in children’s educational choice. For instance, imagine a child observing a large fraction of poor parents in her neighborhood, some of them having exerted effort. From these observations, she is likely to decide not to invest in schooling because her observations lead her to believe that the chances of ending

\textsuperscript{1}Exceptions are Roemer and Wets [1995] and Streufert [2001].
up poor are high no matter what educational decision she makes. Therefore, she forms pessimistic beliefs about the probability of becoming rich and concludes that education is useless. Similarly, a child living in an area populated mostly by rich parents with some of them not having exerted any effort is also induced not to invest in education as she thinks her chances of ending up rich are high whatever the educational effort. Instead, in a mixed area constituted by a more representative sample of the population with some poor and some rich parents having exerted an effort, a child is likely to perceive the usefulness of education since she becomes aware of the fact that educational effort improves the chances of becoming rich. Thus, we claim that the relevant local variable is the number of rich agents relative to poor agents. As discussed above, this ratio, which we call the “social composition index”, acts on the educational decision in a non-monotonic way. This non-monotonic influence departs from the common reduced form traditionally considered in the literature where the educational effort is always positively influenced by the local average level of the parents’ human capital.

Some empirical findings are consistent with our role-model effect. Corroborating Wilson’s view, Brooks-Gunn et alii [1993] find that the presence of affluent families in a neighborhood has a significant impact on children’s educational achievement even after controlling for family background. According to the authors, this result suggests that role models may act on youth’s educational success. Examining neighborhood effects on the drop-out decisions of Australian teenagers, Overman [2002] finds that a high proportion of vocationally qualified adults in a small neighborhood encourages students to stay in school. More anecdotal evidence of the fundamental role played by role models on educational decisions can be found in the French debate over the recruitment reform in the Political Science Institute (Institut d’Etudes Politiques), one of the most prestigious French elite schools. To tackle its social class-biased recruitment, this reform allows for the elite school to facilitate the recruitment of students from secondary schools of poor neighborhoods. Following the proponents of this affirmative action policy, many students from disadvantaged environments think that elite schools are not for them and hence do not apply for their entrance examinations. Thus, opening access to the Political Sciences Institute of Paris is an example of a communications policy devoted to showing that elite schools recruit good students even from low social backgrounds (Le Monde, 03/27/01). This communication policy clearly stresses the importance of information mechanisms as it aims at improving children’s perception of the returns on educational effort exerted in secondary schools. Using the 1989 NBER Boston Youth Survey, Case and Katz [1991] also find that children in poor neighborhoods misperceive returns on educational effort as only 50% of them consider that “there is a strong support for people who try to get ahead through schooling and getting a job”.

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Furthermore, the fact that empirical studies are not consistent with a significant impact of the local level of parents’ income on school achievement (see Ginther et alii [2000]) may suggest that socio-economic diversity rather than the absolute level of wealth in the neighborhood is the key influential variable. More interestingly, some empirical work provides support for the existence of a non-monotonic relationship between the neighborhood socio-economic composition and individual success. For instance, examining the Metropolitan Council for Educational Opportunities desegregation program in Boston (Metco program), which sends black students out of the Boston public school district to attend school in more affluent suburban districts, Angrist and Lang [2002] find that those black students, while improving their educational achievement, do not affect average scores in the sample of all non-Metco classmates. At a more aggregated level, that is across cities, Cutler and Glaeser [1997] obtain a similar result. Studying the effects of spatial separation of racial groups, they find that blacks in more segregated areas have significantly worse outcomes than blacks in less segregated areas while segregation has only a small effect on outcomes for whites.

Our urban setting is based on four key assumptions. First, we assume that each agent cares for the income of her offspring. Second, we assume that only parents know the true returns on schooling. Third, the parents decide where to live. Fourth, children differ with respect to their educational costs, which can be high, middle or low, and which is not observable by parents. For those with a high (respectively low) cost, their educational choice consists in exerting a low (respectively high) level of effort whatever their expectations on the returns of education. For those children with a middle cost, however, their decision crucially depends on their information set. These assumptions lead adults to try to live in a neighborhood that provides the best information to their offspring. The location choice gives rise to non-trivial strategic interactions as the benefits of individual location choices, which correspond to the quality of the information available in the neighborhood, depend on the location decisions made in the rest of the population. In an urban context, this location game is mediated by the land market; the land rent does not necessarily reflect the social value of individual’s location choice. A key point of this analysis is that parents’ and children’s beliefs on educational return differ. Such discrepancy is due to the fact that children believe that their neighborhood is an unbiased sample of the whole society. Yet, our assumptions allow the young individual to explain why she may have contradicting sources of information. In particular, children can find reasons for why their parents may transmit their opinions on the efficiency of schooling which potentially refutes what children observe within the neighborhood. Indeed, she can argue that a parent wants her child to exert an educational effort and increase her chances of becoming rich whatever her information set. That is because her parents are sensitive to her expected income. Moreover, as individuals differ with respect to their individual educational cost, a child is also able
to justify why members of the previous generation did not make identical educational choices.

Being the place where social interactions occur, the city determines both the number of children with a middle cost who exert an educational effort, and dynastic inequality, depending on the urban organisation that shapes the socio-economic composition of each neighborhood. Hence, a city can give rise to more-or-less efficient outcomes and to more-or-less unequal opportunities among children. Three types of equilibrium may emerge. (i) The first one is an “efficient-neutral” equilibrium. This is where all children, except those with a high educational cost, exert effort at school. In such a city almost all neighborhoods are populated by rich and poor parents. The socioeconomic composition of each local community provides information that makes the inhabitants willing to invest in education. According to the efficiency aspect, it is “efficient” in the sense that all children exert educational effort; for the social mobility opportunities, this equilibrium is “neutral” in the sense that it displays no persistent inequalities since educational choice does not depend on parents’ income. (ii) The second type is an “inefficient-regressive” equilibrium. In this case, all children from rich families exert effort while only a fraction of the children from poor families decide to pursue an education. The city is called “inefficient” because there are some homogeneously poor neighborhoods where children conclude that education is useless. There are also some neighborhoods composed of both rich and poor types of agents where children exert effort. The rent differential between mixed neighborhoods and homogeneously poor communities is determined by the willingness of parents to pay to live in communities that provide good information about the efficacy of education. The city is called “regressive” because social mobility opportunities depend on parents’ income. Unlike the poor families, the rich families always locate in high educational effort areas as they are willing to outbid poor families for land in such “good” communities. This equilibrium thus leads to persistent inequality as now, a child from a poor family has fewer chances of ending up rich. (iii) The third type of equilibrium is called the “inefficient-neutral” equilibrium. The city exhibits homogeneously rich and poor neighborhoods where it is impossible for children to be optimistic about the returns on education. This urban configuration is “inefficient” as the fraction of young people who pursue education is minimal. That means it only concerns children with a low educational cost. It is “neutral” because social mobility opportunities do not exhibit any inter-generational link. Moreover, we show that these three equilibria are stable. From a normative point of view, we have found that the efficient-neutral equilibrium is Pareto-optimal and dominates both the inefficient-regressive and inefficient-neutral equilibria.

Our paper is related to theoretical works that analyze peer effects as informational externalities (see, for instance, Banerjee and Besley [1990], Battaglini, Bénabou and Tirole [2001], Heavner and Lochner [2002]). The common idea is that individuals are not fully informed about their
choices and gather information about outcomes and choices of a few others who have already made these decisions. In particular, Battaglini, Bénabou and Tirole [2001] stress that membership in organizations like Alcoholic Anonymous or Gamblers Anonymous as a way for individuals to share experience and to seek information about their own capacity to self control. However, these works only consider the effects of group composition on individuals’ choice. They do not provide a full analysis of endogenous group formation. We depart from these works as our framework allows individuals to choose their companions.

Our paper directly belongs to the strand of literature on neighborhood effects (see for instance, Miyao [1978], de Bartolome [1990], Bénabou [1993, 1996a,b], Durlauf [1996], Fernandez and Roger-son [1996]). According to these papers, the inefficient-regressive city is the only stable equilibrium while the efficient-neutral city is never a stable equilibrium. This result comes from the very nature of the local externalities which leads any individual to seek the company of the richest people, thus generating an incentive to segregate. Instead, in our framework, in the presence of informational externalities, it is the socio-economic diversity of the neighborhood that matters. Hence, as long as the proportion of poor parents is not too excessive, the poor have a positive impact since they convey useful information. Thus, our model does not involve any segregation force. The inefficient-regressive city is only due to coordination failures. Similar results can be found in racial segregation model à la Schelling [1978] where tastes for racial diversity are allowed. Moreover, we differ from the traditional literature which shows that social segregation is likely to be efficient (see Bénabou [1993, 1996a,b]). On the contrary, we stress that there is no trade-off between efficiency and equity as the city where neighborhoods are populated by rich and poor parents leads to the higher level of education.

Finally, the papers of Roemer and Wets [1995] and Streufert [2000], which study the role of informational group-level influences on inequality, are the closest to our point of view. We differ from them on the following point: rather than assuming a priori neighborhood segregation by income, we combine an endogenous formation of communities with informational group effects. Roemer and Wets show that neighborhood segregation by income leads individuals to misperceive the true returns on education. More specifically, youths form beliefs about the economic pay-offs of education by observing the experience of neighborhood adults and by making linear extrapolations. As the true relationship between education and earnings is assumed to be convex, this makes individuals in rich (poor) neighborhoods overestimate (underestimate) the returns on education. Roemer and Wets thus obtain a monotonic impact of neighborhood wealth on individual outcome. Considering a different information gathering process (i.e. a perfect nonparametric regression of income on schooling), Streufert studies the effects of underclass social isolation modeled as the
elimination of high-income observations at each level of schooling. Interestingly, he shows that, under some conditions, social isolation needs not depress schooling. The intuition behind this result is that social isolation causes young person to underestimate the income she forgoes while attending school, thus making education more attractive.

The rest of the paper is organized as follows. Section 2 presents the model and the equilibrium concept we have used. In section 3, we characterize the equilibrium patterns of residential location in communities. Section 4 presents our conclusions. Most of the technical proofs are in the Appendix.

2 The Model

We consider an overlapping generations model of neighborhood formation. The essential features of the model are such that (i) during their lives individuals make educational decisions and location choices, (ii) and that information on the returns of education is affected by neighborhood affiliation.

2.1 Beliefs and Neighborhood Effects

Locations are indexed by $j$. The number of residential areas is constant and equal to $n$. We consider that landowners are absent and without loss of generality we normalize the opportunity cost of building a house to 0. Houses are identical across the city. The inelastic supply of houses within a community is of mass $L$. This land-market model is a closed-city model where the population of the city is a continuum of families of constant mass $N$. Each family lives in one and only one house. The city can accommodate the entire population $N < nL$ but the whole population cannot fit within $n - 1$ areas: $N > (n - 1)L$.

Agents live two periods. When child, an agent faces a discrete educational choice: either she exerts effort ($e = \tau$) and incurs a disutility cost $c$ ($c(e = \tau) = c > 0$) or she does not exert effort ($e = \xi$) at cost 0 ($c(e = \xi) = 0$). The educational cost $c$ depends on the child’s ability. We assume that there are three levels of cost, $\xi, \hat{c}$ and $\tau$, such that $\xi < \hat{c} < \tau$. There is a fraction $\beta$ of both low and high costs and thus a fraction $1 - 2\beta$ of children incurs cost $\hat{c}$. The child’s cost is independent from the parent’s cost. A child knows her educational cost and the existence of a cost heterogeneity. She does not, however, know the cost distribution ($\beta$) and does not observe the parents’ educational cost. When an adult, any individual can enjoy a high income $w_h$ (respectively low $w_l$), with probability $p(e)$ (respectively $1 - p(e)$). The effort supplied at school increases the probability of becoming rich as:

$$0 < p(e) = \underline{p} < \overline{p} = p(\tau) < 1$$
The model does not introduce any of the neighborhood effects traditionally considered in the literature since neither the probabilities of success nor the cost of effort are influenced by neighborhood characteristics. We do not claim here that standard local externalities are not relevant. We rather think that children benefit from various types of externalities (including, among others, local human capital externalities, informational externalities) that empirical studies have difficulties to disentangle (see Manki [1993]). However, our purpose here is to shed light on a particular type of local externalities in order to better understand its consequences on residential choices.

A key assumption is that, while all parents know the right values of the probabilities $p(e)$, all children only hold beliefs about such probabilities. A child’s a priori beliefs are given by a distribution on $p(e)$ with a density function $f_e(p)$ ($f_e(p) > 0$, $\forall p \in [0,1]$) such that the average inferences are correct ($\int_0^1 f_e(p)pdp = p(e)$). If such a priori beliefs are equal to the parent’s beliefs, a child does not make any mistakes, since her parent knows the right probability (i.e. $f_e(p) = 1$ if $p = p(e)$ and $f_e(p) = 0$ otherwise). However, a child does not attach full confidence to parental opinion and gives more importance to her observations of the older generation in her neighborhood. We will justify this assumption later on in section 2.2.2. We suppose that a child can observe some of the characteristics of the parents living in her neighborhood. She can observe their level of income. She also knows their educational effort exerted during childhood as it is revealed by their diplomas. However, the child is not able to know the parents’ educational costs.

Hence, a child can distinguish four categories of parents:

1- rich parent who did not make an effort as a child ($w_h, \xi$),
2- rich parent who made an effort as a child ($w_h, \tau$),
3- poor parent who did not make an effort as a child ($w_l, \xi$),
4- poor parent who made an effort as a child ($w_l, \tau$).

Therefore, a neighborhood $j$ is characterized by the number of each type of individual denoted by $N_j(w, e)$. The child updates her beliefs about returns on education, using the Bayesian rule, depending on the characteristics of her community. Since we suppose that the population in a neighborhood is large (the order of the continuum), whatever the prior $f_e(p)$, the beliefs will coincide with the real frequencies observed, that is:

$$\tilde{p}_j(e) = \frac{\text{Number of rich in } j \text{ who exerted effort } e}{\text{Number of agents in } j \text{ who exerted effort } e}$$

(2)

$$\tilde{p}_j(e) = \frac{N_j(w_h,e)}{N_j(w_h,e) + N_j(w_l,e)}$$

(3)

Let us express these inferred probabilities in a more simple way. First, we introduce the following condition.
Condition 1: \(\forall j, k \in \{1, \ldots, n\}, \forall i = h, l\), if \(N_j(w_i, \varepsilon) + N_j(w_i, \bar{\varepsilon}) \neq 0\) and \(N_k(w_i, \varepsilon) + N_k(w_i, \bar{\varepsilon}) \neq 0\) then

\[
\frac{N_j(w_i, \varepsilon)}{N_j(w_i, \varepsilon) + N_j(w_i, \bar{\varepsilon})} = \frac{N_k(w_i, \varepsilon)}{N_k(w_i, \varepsilon) + N_k(w_i, \bar{\varepsilon})}
\] (4)

Condition 1 states that the fraction of the population \(w_i\) with a given effort \(e\) in neighborhood \(j\) is the same as the fraction of the population \(w_i\) with a given effort \(e\) in the whole population. This condition is not crucial but allows us to avoid useless technical complications. Furthermore, we will see that there is no reason why parents should choose different locations according to their past-effort, thus justifying condition 1.

Let us now introduce \(\theta_j\) the social composition index of community \(j\) which is defined by the following expression: if \(N_j(w_h, \bar{\varepsilon}) + N_j(w_h, \varepsilon) \neq 0\)

\[
\theta_j = \left(\frac{N_j(w_h, \bar{\varepsilon}) + N_j(w_h, \varepsilon)}{\sum_{i=1}^n N_i(w_h, \bar{\varepsilon}) + N_i(w_h, \varepsilon)}\right)^{-1}\left(\frac{N_j(w_h, \bar{\varepsilon}) + N_j(w_h, \varepsilon)}{\sum_{i=1}^n N_i(w_h, \bar{\varepsilon}) + N_i(w_h, \varepsilon)}\right)\] (5)

By convention, if \(N_j(w_h, \bar{\varepsilon}) + N_j(w_h, \varepsilon) = 0\) and \(N_j(w_l, \bar{\varepsilon}) + N_j(w_l, \varepsilon) \neq 0\) we set that \(\theta_j = +\infty\).

In other words, the index \(\theta_j\) is the ratio between the share of poor parents living in the community \(j\) over the share of rich parents living in the same community \(j\). This ratio captures the relevant social composition of the community \(j\). Thus it is possible to check (see Appendix 1) that if the children have observations about the effect of effort \(e\), the inferred probability is

\[
\tilde{p}_j(e) = \frac{p(e)}{p(e) + (1-p(e)) \theta_j}
\] (6)

with the convention that \(\tilde{p}_j(e) = 0\) if \(\theta_j = +\infty\).

If there is no observation about \(e\), children hold on to their prior belief and thus \(\tilde{p}_j(e) = p(e)\).

Thus, if no poor people reside in the community \(j\), \((\theta_j = 0)\), children are certain to become rich whatever the exerted effort \((\tilde{p}_j(e) = 1)\). Otherwise, when no rich people reside in the community \(j\), \((\theta_j = +\infty)\), children are certain to become poor whatever the exerted effort \((\tilde{p}_j(e) = 0)\). If community \(j\) is representative of the whole population \((\theta_j = 1)\), the neighborhood is a non-biased sample of the whole population and leads children to infer the correct values of the probability \((\tilde{p}_j(e) = p(e))\).

We previously assumed that children form their beliefs based on local observations and do not adopt the opinion of their parents. In the following section, which presents agents’ preferences, we will justify why a child dismisses her parent’s beliefs.
2.2 Agent’s Preferences

An agent maximizes her expected utility which consists of her present utility and an altruistic component given by the expected offspring’s income. An agent has to decide, when young, on her educational effort and, when adult, on her location.

Even though a child and a parent have the same preferences, they do not have the same information set. Therefore, we must examine separately the child’s educational decision and the parent’s residential choice.

2.2.1 Educational Choice

A child chooses her educational effort knowing that it will determine her future wealth. Given her beliefs, a child solves the following simple program with respect to her educational effort $\tilde{e}$:

$$
\max_{\tilde{e}} [\tilde{p}_j(\tilde{e})u(w_h - \rho_j) + (1 - \tilde{p}_j(\tilde{e}))u(w_l - \rho_j) - c(\tilde{e})] + a\tilde{E}_c(\tilde{p}(e(c)))w_h + (1 - \tilde{p}(e(c)))w_l)
$$ (7)

where $u(.)$ is the utility, $\rho_j$ the rent paid in the child’s neighborhood $j$, $\tilde{E}_c$ her expectation of her future offspring’s educational cost and $e(c)$ the educational effort of her future offspring. We suppose that $u$ is a strictly concave, increasing, and unbounded function defined on $\mathbb{R}^+$.

Two remarks are worth making here. First, according to her program, the child perceives no impact of her present educational choice on her offspring’s decision. Indeed, she considers that she has inferred the right value of probabilities and that her offspring will also have the same beliefs. Second, we assume that the child only observes the rent in her neighborhood ($\rho_j$) and considers that the current positive level of local rent is the competitive one, which explains why she thinks it will persist in the next period.

Furthermore, the following two assumptions will be considered throughout the rest of the paper:

**Condition 2** When the children are indifferent between the two levels of effort, they choose the high level of effort.

This assumption is introduced in order to exclude mixed strategies in cases of indifference.

**Condition 3**

(i) The cost $\overline{c}$ is so high that a child with $\overline{c}$ never exerts educational effort. (ii) The cost $\underline{c}$ is so low ($\leq 0$) that a child with $\underline{c}$ always exerts educational effort. (iii) Educational effort is efficient for $\hat{c}$: $(\overline{p} - \underline{p})(u(w_h) - u(w_l)) > \hat{c}$.

We assume that the social composition of the neighborhood has no impact on the educational effort chosen by individuals with either a high or a low educational cost. Thus, Condition 1 and items (i) and (ii) of Condition 3 guarantee that in any neighborhood there are always people who have made both types of educational decisions.
Concerning item (iii), we have not considered the other case where it would be efficient for the children with a middle cost to not get education, knowing the true probabilities.

Thus, denoting $N(w_i, e)$ the number of parents of type $w_i$ who exerted the level of effort $e$ for $e = e, \bar{e}$ and $w_i = w_h, w_l$, we immediately derive the child’s educational choice in the following lemma:

**Lemma 1** There exist functions $\theta'(\rho)$ and $\theta''(\rho)$ such that $\theta'(\rho) \leq 1 \leq \theta''(\rho)$ whatever $\rho \geq 0$, if $N(w_i, e)$ and $N(w_i, \bar{e})$ are both different from 0 whatever $i = h, m$, a child with cost $\tilde{c}$ located in area $j$ exerts effort iff $\theta'(\rho) \leq \theta_j \leq \theta''(\rho)$ i.e.:

$$\arg\max_e [\tilde{p}_j(e)u(w_h - \rho) + (1 - \tilde{p}_j(e))u(w_l - \rho) - c(e)] = \bar{e}$$

$$iff \theta'(\rho) \leq \theta_j \leq \theta''(\rho)$$

**Proof.** Proofs are in Appendix.

According to this Lemma, the socio-economic composition only influences the decision of the middle-cost children. Moreover, this Lemma stresses the main differences between peer effects or fiscal effects and informational effects. Indeed, one should note that the social composition has a non-monotonic impact on the exerted effort. Indeed when neighborhood $j$ is biased towards the rich $\theta_j < \theta'$ or towards the poor $\theta_j > \theta''$, children do not exert effort. It is only when the neighborhood is relatively mixed, $\theta_j \approx 1$, that an efficient effort is exerted since children infer the correct probabilities and decide to invest in education. If instead the social composition of the neighborhood is relatively homogeneous, neither poor nor rich people decide to exert any effort.

This non-monotonic impact has interesting empirical implications. Indeed, we think that it can be estimated without the identification problem usually encountered in econometric estimations that try to disentangle group level influences. Following Manski [1993], three types of causal effects are often advanced to explain group-level influences. First, “correlated individual characteristics” correspond to similar individual-level influences for members of the same group. Second, “contextual effects” are related to the fact that individuals within the same group are exposed to common influences. Third, there are “endogenous effects” when an individual’s choice is influenced by the behavior of other members of the group. Our causal mechanism falls under the “contextual effects” category, as the individuals within a group are exposed to the group’s social composition. Identification problems arise when, for instance, the economist tries to separately estimate the “contextual effects”, which correspond to local public goods, and the “endogenous effects” such as peer effects which are positively correlated to local public goods. Therefore, if a positive monotonic link is found between $\theta_j$ and the individual outcomes, it is impossible to disentangle the two causal effects. We suggest testing a non-monotonic link between $\theta_j$ and the individual’s outcomes. We claim that even
if we try to test simultaneously monotonic and non-monotonic links, the identification problems do not arise.\(^2\)

### 2.2.2 Residential Choice

As a parent, the agent receives her income, gets the true values of both probabilities \(\bar{p}\) and \(p\) and knows the cost distribution over the whole population. However, she is not able to know her child’s educational cost\(^3\). Thus, she must choose a location knowing that it will determine her child’s beliefs and educational behavior. Thus, a parent solves the following program:

\[
\text{Max}_j u(w - \rho_j) + aE_c(p(e_j(c))w_h + (1 - p(e_j(c)))w_l)
\]

with \(e_j(c)\) being the solution to

\[
\max_{\tilde{e}} \left[ \tilde{p}_j(\tilde{e})u(w_h - \rho_j) + (1 - \tilde{p}_j(\tilde{e}))u(w_l - \rho_j) - c(\tilde{e}) \right] + a\tilde{E}_c(\tilde{p}(e(c)))w_h + (1 - \tilde{p}(e(c)))w_l \quad (8)
\]

\(E_c()\) denoting the true cost distribution over the whole population.

Our assumption that a parent takes into account the expected income of her child has two key implications. First, parents will like their children to make an effort at school and will try to locate in a relatively mixed neighborhood in order to provide their offspring with the best information on the returns of schooling. Second, a child who decides not to invest in education can justify why her beliefs differ from her parent’s opinion. Indeed, she can argue that her parent wants her to exert effort not because her parent has the right information but because her parent wants her to become rich whatever the cost of education.

Let us note that even though children do not attach full confidence to their parent’s information, this does not mean that the parent does not have any influence on her children’s educational choices. In our set-up, parental influence comes from the parent’s location choices, which determines her child’s expectations.

Finally, eventhough she thinks that her beliefs are correct, a child is able to explain differences in parents’ educational choices by arguing that a cost heterogeneity exists. Moreover, since she does not know the distribution of educational choice, she cannot observe that parents with the same cost have exerted different efforts.

\(^2\)Indeed, a way to proceed is to define two variables that are not correlated. For instance, the average wealth of neighbors is a proxy for \(\theta_j\) while the variance in the neighborhood income distribution captures the socioeconomic composition.

\(^3\)This assumption seems reasonable when we consider that residential choices are made before primary schooling. Nevertheless, we could levy this assumption. In this case where only the parents of middle cost children would care where they live, our results would be maintained but at the cost of more complicated computations.
2.3 Equilibrium Definition

Let us now introduce the definition of an equilibrium. We denote $N_j$ the size of the population in community $j$.

**Definition 1** Under condition 1 and given $N(w_h,e)$ and $N(w_l,e)$, we can say that

$$[\{\rho_j\}_{j=1,...,n}, \{\theta_j\}_{j=1,...,n}, \{N_j\}_{j=1,...,n}]$$

is a urban equilibrium if it satisfies:

(i) housing demand does not exceed housing supply i.e $\forall j = 1,...,n$,

$$N_j \leq L \tag{9}$$

(ii) the housing market is at equilibrium i.e $\forall j = 1,...,n$,

if $N_j < L$ then $\rho_j = 0 \tag{10}$

(iii) No one wishes to move.

In words, an equilibrium is defined both by the level of rent in all communities and by a spatial distribution of rich and poor adults across the different residential areas such that no one is induced to move from one area to another (condition (iii) of Definition 1). Condition (ii) of Definition 1 assumes that whenever vacant houses remain in an area, the rent is at its opportunity cost. The next section aims at studying the possible equilibria.

3 Urban Equilibria

A city is mainly characterized by the social composition of the different neighborhoods. Moreover, the specific local externality that we consider here has a dramatic impact on such urban organization. Thus, in order to compare our result with what happens in the presence of standard local externalities, we have focused on the aggregate level of educational effort and on the inequality of expected income between children.

Consider a child with a cost $\hat{c}$ born in a family whose income is equal to $w_i$.\footnote{We have only focused on a child $\hat{c}$ since, for other types of children, the effort exerted does not depend on the residential location (see Condition 3).} Denoting $f_i$ the probability of a parent $i$ living in a neighborhood where the social composition leads to educational effort, her expected income is given by

$$f_i (pw_h + (1 - p) w_l) + (1 - f_i) \left( pw_h + (1 - p) w_l \right). \tag{11}$$
Thus, in terms of efficiency, the higher the $f_i$, the more efficient the city. Two main cases can therefore be identified:

**Definition 2** A city is efficient whenever $f_h = f_l = 1$ and inefficient otherwise.

In words, the efficient city grants the right incentives to all children whereas in the inefficient city a fraction of either the rich or poor population is likely to live in a neighborhood where children do not exert effort.

From an equity point of view, we focus on the difference between $f_h$ and $f_l$ which measures the degree of social mobility between the rich and the poor populations from one period to another. Three cases can thus be identified:

**Definition 3** A city is (i) progressive whenever $f_l > f_h$, (ii) regressive whenever $f_h > f_l$ and (iii) neutral otherwise.

In the case of the progressive city, this definition means that the expected income of a child born to a poor family is higher than for a child born to a rich family. The regressive city corresponds to the opposite case. A city is “neutral” when children have the same social mobility opportunities whatever the income of their parents.

Hence, what is crucial for the value of $f_i$ at equilibrium is the social composition within each area. In addition, the type of local externality considered has a priori a dramatic impact on the equilibrium social composition of the neighborhood. In the presence of informational externalities, one may expect that there is an equilibrium when a good social composition is ensured in all neighborhoods. Nonetheless, the decentralized organization of the city can also lead to equilibria with poor neighborhoods where children do not exert effort and partially mixed neighborhoods where there is a positive rent and where children exert effort. We have proved below that there are only three kinds of urban equilibria.
The following proposition provides a formal description of the efficient-neutral equilibrium.

**Proposition 1** Under conditions 1, 2 and 3, for any city (n neighborhoods of size L) an efficient-neutral equilibrium always exists and is such that, \( \forall j = 1, \ldots, n, \)

\[
\{\rho_j\}_{j=1}^{n}, \{\theta_j\}_{j=1}^{n}, \{N_j\}_{j=1}^{n}
\]

satisfies

(a) \( \theta'(0) \leq \theta_j \leq \theta''(0) \),
(b) \( \rho_j = 0. \)

An efficient-neutral equilibrium corresponds to a city where the youths with low and middle costs invest in education and the social mobility opportunities are the same whatever the parents’ income.

This city is composed of neighborhoods populated by rich and poor people such that the socioeconomic composition within each neighborhood makes young inhabitants infer that returns to education are high and therefore decide to exert effort. Furthermore, since every neighborhood provides the same benefits to its inhabitants as information about the usefulness of education is the same across the city, it turns out that there is no differential rent between neighborhoods. The rent is therefore set at its opportunity cost.

The city is neutral. The location and hence the income of the parent do not matter as children exert effort in every area. More precisely, we can display the following mobility matrix which reveals no difference among income types in the transition probabilities\(^5\):

<table>
<thead>
<tr>
<th>Social mobility matrix in the efficient-neutral city:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rich when adult</td>
</tr>
<tr>
<td>Rich parent</td>
</tr>
<tr>
<td>Poor parent</td>
</tr>
</tbody>
</table>

with \( \Pi = (1 - \beta)\bar{p} + \beta \bar{p}. \)

In the presence of standard local externalities, the existence of the efficient-neutral city is not ensured. Indeed, in the presence of such externalities, the concentration of rich parents within the same neighborhood can lead to a more efficient city depending on the curvature of the aggregation function of the local externalities. As a result, even if the presence of both poor and rich parents within each area grants more-or-less the same level of human capital to every child (a neutral city) one cannot exclude that the concentration of rich parents in one area (a regressive city) does not lead to a more efficient city.

---

\(^5\)The social mobility matrices that we build describe the trajectories of the high, middle and low cost children.
Finally, not only is the efficient-neutral city a theoretical result, but it has strong implications on the empirical side. Indeed, we can find some empirical evidence that supports the existence of cities exhibiting some social heterogeneity. For instance, empirical sociological studies on French urban segregation have emphasized that French cities are not stratified. Focusing on Paris region data, Prétèceille et alii [2003] find that in areas that are dominated by high income agents, the proportion of those high income agents amounts to only 50% of those areas’ inhabitants.

The following proposition formally describes another type of urban equilibrium called the “inefficient-regressive” equilibrium.

**Proposition 2** Under conditions 1, 2 and 3, there is a city (n neighborhoods of size L) such that there always exists an inefficient-regressive equilibrium, and it is such that $\forall j = 1, ..., n,$

\[
\left\{\rho_j\right\}_{j=1}^{n}, \left\{\theta_j\right\}_{j=1}^{n}, \left\{N_j\right\}_{j=1}^{n}
\]

satisfies the following:

(a) $\forall j \in J_E, \, \rho_j = 0$ and $\forall j \in J_T, \, \rho_j = \rho > 0$,
(b) $\forall j \in J_E, \, \theta_j = +\infty$ and $\forall j \in J_T, \, \theta'(\rho) \leq \theta_j \leq \theta''(\rho)$,
(c) $f_h = 1 > f_l$.

An inefficient-regressive equilibrium corresponds to a city where a fraction of the young population with a middle cost does not invest in education and upward mobility is harder for a poor family child.\(^6\)

An inefficient-regressive equilibrium is characterized by two types of communities. On the one hand, there are communities inhabited by rich and poor agents such that the social composition of such neighborhoods provides information about the usefulness of education. It then turns out that every young individual exerts effort. On the other hand, there are homogeneously poor communities where inhabitants make very pessimistic inferences about the returns on education and do not exert any effort. This social polarization arises because of the land market mechanism. Due to their higher propensity of paying land rents, rich agents are able to bid higher rents than poor agents for land in a high-effort community. The land rent equilibrium is such that poor agents are indifferent between living in the two types of communities. Moreover, the land rent does not depend on the

\(^6\)Let us remark that there are cities for which the inefficient-regressive city does not exist. For instance, one can imagine a city with two areas such that $N \simeq 2L$. In such a case, it may be impossible, for some values of $\pi$ and $p$, to have one homogenously poor neighborhood and the other inhabited by rich and poor agents such that $\theta'(\rho) \leq \theta \leq \theta''(\rho)$. 

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number of high-effort communities. Let us note that the higher the number of mixed communities,
the bigger the fraction of the society that exerts educational effort, thus the less inefficient the city.

The inefficient-regressive equilibrium generates persistent inequality in the sense that the loca-
tion, which crucially depends on the parents’ income, affects the child’s social mobility. For instance,
children from poor families stuck in inefficient communities, have a lower chance of becoming rich.

More precisely, we can display the following mobility matrix with differing probability transitions
across income types.

<table>
<thead>
<tr>
<th></th>
<th>Rich when adult</th>
<th>Poor when adult</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rich parent</td>
<td>$\Pi$</td>
<td>$1 - \Pi$</td>
</tr>
<tr>
<td>Poor parent</td>
<td>$f_i \Pi + (1 - f_i)\Gamma$</td>
<td>$f_i (1 - \Pi) + (1 - f_i)(1 - \Gamma)$</td>
</tr>
</tbody>
</table>

with $\Gamma = \beta p + (1 - \beta)\mu < \Pi$

We can see that the probability of becoming rich is higher for a child born to a rich family. On the other hand the probability of becoming poor is higher for a young individual from a poor family. Moreover, comparing this matrix to the one obtained in the efficient-neutral equilibrium, we can see that the social opportunities of a child from a rich family remain unchanged while the social mobility of a child from a poor family worsens.

We can also note that the lower the number of neighborhoods with high educational effort, the
less mobile the society.

Finally, a third type of equilibrium may arise which is given in the following proposition:

**Proposition 3** Under conditions 1, 2 and 3, there is a city (n neighborhoods of size L) such that
there always exists an inefficient-neutral equilibrium, and it is such that $\forall j = 1, ..., n,

$$\left[ \{\rho_j\}_{j=1}^{n}, \{\theta_j\}_{j=1}^{n}, \{\theta_j''\}_{j=1}^{n} \right]$$

satisfies:

(a) $\theta'(0) > \theta_j$ or $\theta_j > \theta''(0)$,

(b) $\rho_j = 0$,

(c) $f_h = f_l = 0$.

In every neighborhood of such a city, the socio-economic composition is too biased and makes
children misperceive the returns on education. Thus, this city produces the lowest aggregate level
of education, that is, only those children with a low educational cost invest in education. As for
the land rent, it is set at its opportunity cost all over the city, as no neighborhood provides “good” information on the efficiency of schooling.\(^7\)

<table>
<thead>
<tr>
<th></th>
<th>Rich when adult</th>
<th>Poor when adult</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rich parent</td>
<td>Γ</td>
<td>1 − Γ</td>
</tr>
<tr>
<td>Poor parent</td>
<td>Γ</td>
<td>1 − Γ</td>
</tr>
</tbody>
</table>

With respect to the social mobility prospect, such an urban configuration performs the worst as it leads to the highest, respectively lowest, probability of ending up poor, respectively rich. Furthermore, this city is neutral as the social mobility opportunities display no inter-generational link. Of course, we are aware of the fact that children from an all rich or all poor neighborhoods are equally likely to become rich is an extreme result. Adding standard local externalities might indeed weaken the impact of informational externalities but would not cancel the harmful effect of social homogeneity.

The following proposition tells us that these three urban configurations are the only possible urban equilibria:

**Proposition 4** (i) The presence of local informational externalities gives rise to only three equilibria: the efficient-neutral equilibrium, the inefficient-regressive equilibrium and the inefficient-neutral equilibrium.

(ii) There is a city \( (n \text{ neighborhoods of size } L) \) such that the efficient-neutral equilibrium, the inefficient-regressive equilibrium and the inefficient-neutral equilibrium coexist and are stable.

The proposition stresses that the progressive city does not exist. In other words, the location of all poor agents within high-effort areas combined with the location of some rich agents within low-effort areas fails to be an equilibrium. The basic reason lies in the land market mechanism. The rent reflects the willingness to pay in order to live in a high-effort area and it is such that rich agents who locate in both types of communities are indifferent to these two alternatives. As a consequence, poor agents would prefer living in low-effort communities rather than paying a higher rent in order to reside in high-effort communities.

Furthermore, these urban equilibria are stable. The stability condition refers to the fact that after small perturbations of the equilibrium distribution of individuals between neighborhoods, some agents wish to move back to their original community. More precisely, this proposition holds

---

\(^7\)Note that, as well as for the inefficient-regressive equilibrium, the inefficient-neutral equilibrium does not exist for all possible cities. If we consider, for instance, a city with two areas such that \( N \simeq 2L \), it may be impossible, for some values of \( p \) and \( p' \), to accomodate rich and poor agents in the neighborhoods such \( \theta'(0) > \theta_1 \) and \( \theta_2 > \theta''(\rho) \).
if the mixed communities in the efficient-neutral and inefficient-regressive equilibria are such that
the social composition is not too close to the limits of the interval $[\theta'(\rho), \theta''(\rho)]$. Otherwise, small
perturbations in the equilibrium distribution of individuals may lead to a radical change in the
educational choices of its young inhabitants as beliefs in the social mobility become pessimistic (i.e.
the new $\theta_j$ does no longer belong to $[\theta'(\rho), \theta''(\rho)]$).8

The stability result departs from the findings of the standard literature which stresses that the
only stable urban equilibrium is the inefficient-regressive city. Indeed, the neutral city turns out
to be unstable since a slight migration of, for instance, rich parents would increase the level of
human capital of the community where they choose to migrate to. This area would then become
more attractive and the migrants would have no incentive to go back to their original community.
This instability result of the efficient-neutral city holds also in the case of local taxation as the
level of the neighborhood tax rate and thus the school success of children depends directly on the
average income of the parents (Fernandez and Rogerson [1996]). The stability result thus restores
the importance of efficient-neutral and inefficient-neutral equilibria in the analysis.

Furthermore, the multiple urban equilibria we have obtained can be Pareto-ranked. For in-
stance, it is clear that inefficient-regressive equilibrium Pareto-dominates the inefficient-neutral
equilibria. Indeed, rich agents are better off in the inefficient-regressive equilibrium (they pay a
positive rent which is smaller than their willingness to pay to live in a high-effort community). Poor
agents are indifferent to both equilibria since their rent is equal to their willingness to pay. As for
landowners, they become poorer in the inefficient-neutral equilibrium, as no parent pays a positive
rent. Finally, if in the efficient-neutral equilibrium we introduce a uniform lump sum taxation that
levies the value of the rent paid in the inefficient-regressive equilibrium, it becomes easy to deduce
that rich and poor agents are indifferent between those equilibria. The taxation proceeds, however,
exclude the total amount of rent paid to the landowners in the inefficient-regressive equilibrium.

In our framework, segregation is due to coordination failures rather than to basic conflicts be-
tween the rich and the poor. Indeed, unlike the standard literature on local externalities, rich and
poor people are not reluctant to live together in the same areas, rather, they appreciate social diver-
sity because of the nature of informational externalities. Thus, this provides new argument in favor
of public interventions that promote socio-economic heterogeneity. The “Moving To Opportunity
Program” is a well-known illustration of public policies that consist in encouraging poor individuals
to reside in rich neighborhoods. In the same spirit, the Gayssot-Besson law introduced in 2000 in
France aims at impeding the agglomeration of rich people within a low number of communities by
imposing a minimum threshold of 20% of total houses dedicated to low-income housing for every

---
8This stability result is true whenever parameters are such that $\theta''(\rho)$ is sufficiently larger than $\theta'(\rho)$.
city.

4 Conclusion

This paper provides a theoretical framework that allows us to address the issue of persistent inequality generated by social interactions when those social interactions correspond to local informational externalities.

We have shown that local informational externalities in the city give rise to three types of urban equilibria. First, there exist efficient-neutral equilibria where the entire young generation of the city exerts a high educational effort. This type of equilibria is characterized by no persistent inequalities as the educational choice of the child and the subsequent social mobility opportunities do not depend on the parent’s income. Second, there are inefficient-regressive equilibria that describe a city comprised of mixed neighborhoods where children exert educational effort and poor neighborhoods where children do not exert effort, as their expectations on the returns of education are too pessimistic in such areas. Third, the equilibrium city can be inefficient-neutral where all children misperceive the returns on education and no middle-cost child, whatever her social background, exerts a great effort.

Moreover, the presence of local informational externalities restores the relevance of efficient-neutral equilibria as we have shown that they are stable and Pareto-optimal. We have thus departed from the standard literature which concludes that the efficient-neutral city is not always Pareto-optimal and is never sustained as a stable outcome.

Finally, our setting suggests that indices measuring social segregation should be introduced as exogenous variables in regressions in order to capture role-model effects and improve their estimation on individual educational outcomes.

5 Appendix

Proof. Computation of $\tilde{p}_j(e)$.

Let us start with the definition of $\tilde{p}_j(e)$

$$\tilde{p}_j(e) = \frac{N_j(w_h, e)}{N_j(w_h, e) + N_j(w_l, e)}$$

Given condition 1 and denoting $N(w_i, e)$ the number of parents of type $w_i$ who exerted the level of effort $e$ for $e = h, l$ and $w_i = w_h, w_l$, we can rewrite $\tilde{p}_j(e)$ as follows:

$$\tilde{p}_j(e) = \frac{N(w_h, e)\frac{N_j(w_h, e)+N_j(w_h, l)}{N(w_h, h)+N(w_h, l)}}{N(w_h, e)\frac{N_j(w_h, e)+N_j(w_h, l)}{N(w_h, h)+N(w_h, l)} + N(w_l, e)\frac{N_j(w_l, e)+N_j(w_l, h)}{N(w_l, h)+N(w_l, l)}}.$$
Given the definition of $\theta_{j,t}$, we have:

$$\tilde{p}_j(e) = \frac{N(w_h,e)}{N(w_h,e) + \theta_j N(w_l,e)}$$

Let us denote $\alpha$ the proportion of parents who exerted effort $e$. Thus, as the probabilities of becoming rich or poor with effort $e$ are assumed to be the same for parents when they were themselves young, we can deduce that $N(w_h,e) = \alpha p(e) N$ and $N(w_l,e) = \alpha (1-p(e)) N$ which leads to

$$\tilde{p}_j(e) = \frac{p(e)}{p(e) + \theta_j (1-p(e))}$$

\[ \Box \]

**Proof of Lemma 1.**

Let us consider the following function $\Delta(\theta)$:

$$\Delta(\theta) = \frac{\overline{p}}{\overline{p} + (1-\overline{p}) \theta_j} - \frac{\overline{p}}{\overline{p} + (1-\overline{p}) \theta_j}$$

(12)

$\Delta(\theta)$ has a maximum for $\theta^* = \sqrt{\frac{\overline{p} \overline{p}}{(1-\overline{p})(1-\overline{p})}}$. It is increasing when $\theta \leq \theta^*$ and decreasing when $\theta \geq \theta^*$. Furthermore, we have $\Delta(0) = \Delta(+\infty) = 0$ and $\Delta(1) = \overline{p} - \frac{\overline{p}}{1-\overline{p}}$. 

(i) Thus, for any given $\rho \geq 0$, there exists $\theta'(\rho)$ and $\theta''(\rho)$ such that $\theta'(\rho) < 1 < \theta''(\rho)$, $\Delta(\theta'(\rho)) = \Delta(\theta''(\rho)) = (u(w_{h\rho}) - u(w_{l\rho}))$ and $\Delta(\theta) \geq (u(w_{h\rho}) - u(w_{l\rho}))$ if $\theta'(\rho) \leq \theta \leq \theta''(\rho)$.

(ii) According to Condition 3, $\bar{c}$ is such that for any $\rho > 0$

$$\Delta(\theta^*) < \frac{\bar{c}}{u(w_{h\rho}) - u(w_{l\rho})}$$

and $\bar{c}$ is such that $\Delta(0) = \Delta(+\infty) = \frac{\bar{c}}{u(w_{h\rho}) - u(w_{l\rho})}$.

\[ \Box \]

**Proof of Proposition 1.**

In an efficient-neutral equilibrium we have by definition $f_l = f_h = 1$. Hence for any $j$, $e_j(\bar{c}) = \bar{c}$.

Hence by lemma 1, we have $\theta'(\rho_j) \leq \theta_j \leq \theta''(\rho_j)$.

Now, let us show that for any $j$, $\rho_j = 0$.

We know that a $k$ such that $\rho_k = 0$ exists. Let us suppose that $j$ such that $\rho_j > 0$ exists. Then, by condition (ii) of definition 1, it implies that $N_j = L > 0$. Condition (iii) of definition 1 implies that

$$u(w_i - \rho_j) + aE_c(p(e_j(c))w_h + (1-p(e_j(c)))w_l)$$

$$= \max_{j'} u(w_i - \rho_{j'}) + aE_c(p(e_{j'}(c))w_h + (1-p(e_{j'}(c)))w_l)$$

with $\forall j'$, 

$$e_{j'}(c) = \arg \max_{\tilde{p}_j(\tilde{c})} [\tilde{p}_j(\tilde{e})u(w_h - \rho_j) + (1 - \tilde{p}_j(\tilde{e}))u(w_l - \rho_j) - c(\tilde{e})]$$

$$+ aE_c(\tilde{p}(c))w_h + (1-\tilde{p}(c))w_l$$

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Yet, \( e_j = e_k \) and thus

\[
\begin{align*}
&u(w_i - \rho_j) + aEc(p(e_j(c))w_h + (1 - p(e_j(c)))w_l) \\
&< u(w_i) + aEc(p(e_k(c))w_h + (1 - p(e_k(c)))w_l)
\end{align*}
\]

which shows that we cannot have

\[
\begin{align*}
&u(w_i - \rho_{j'}) + aEc(p(e_{j'}(c))w_h + (1 - p(e_{j'}(c)))w_l) \\
&= \max_{j'} u(w_i - \rho_{j'}) + aEc(p(e_{j'}(c))w_h + (1 - p(e_{j'}(c)))w_l)
\end{align*}
\]

yielding a contradiction.

Thus in an efficient-neutral equilibrium we have for any \( j \)

\[
\rho_j = 0 \quad \text{and} \quad \theta_j'(0) \leq \theta_j \leq \theta_j''(0).
\]

The existence of the efficient-neutral equilibrium is ensured since we can always have for any \( j \), \( \theta_j = 1 \) which satisfies the condition \( N_j \leq L \).

**Proof of Proposition 2.**

Let us show that an inefficient-regressive equilibrium exists and is characterized by:

(a) \( \forall j \in J_e, \rho_j = 0 \) and \( \forall j \in J_r, \rho_j = \rho > 0 \),

(b) \( \forall j \in J_e, \theta_j = +\infty \) and \( \forall j \in J_r, \theta_j'(\rho) \leq \theta_j \leq \theta_j''(\rho) \),

(c) \( f_h = 1 > f_l \)

By definition, in an inefficient-regressive equilibrium we have \( f_h > f_l > 0 \).

Let us define the set \( J_e, J_r \) by

\[
J_e = \{ j \in \{1, \ldots, n\} \mid e_j(\hat{c}) = \hat{e} \} \quad \text{and} \quad J_r = \{ j \in \{1, \ldots, n\} \mid e_j(\hat{c}) = \tau \}.
\]

**Fact 1** If \( j, j' \) both belong to the same set \( J_e \), then \( \rho_j = \rho_{j'} \) (the rent is the same when the effort is the same).

Let us suppose the contrary, then w.l.o.g. let us suppose that \( \rho_j > \rho_{j'} \geq 0 \). Then by condition (ii) of definition 1, we have that \( N_j = L > 0 \) and condition (iii) of definition 1 implies that \( i \) exists such that

\[
\begin{align*}
&u(w_i - \rho_j) + aEc(p(e_j(c))w_h + (1 - p(e_j(c)))w_l) \\
&= \max_k u(w_i - \rho_k) + aEc(p(e_k(c))w_h + (1 - p(e_k(c)))w_l) \\
&\quad \text{with} \ \forall k,
\end{align*}
\]

\[
e_k = \arg \max_{\tilde{e}} \tilde{p}_k(\tilde{e})u(w_h - \rho_j) + (1 - \tilde{p}_k(\tilde{e}))u(w_l - \rho_j) - c(\tilde{e}) \\
+ \tilde{a}E_c(\tilde{p}(e(c)))w_h + (1 - \tilde{p}(e(c)))w_l
\]

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We have that

\[ u(w_i - \rho_j) + aE_c(p(e_j(c))w_h + (1 - p(e_j(c)))w_l) \]
\[ < u(w_i - \rho_j') + aE_c(p(e_j'(c))w_h + (1 - p(e_j'(c)))w_l) \]
\[ \leq \max_k [u(w_i - \rho_k) + aE_c(p(e_k(c))w_h + (1 - p(e_k(c)))w_l)] \]
\[ \text{with } \forall k, \]
\[ e_k = \arg \max_c \tilde{p}_{k,\ell}(\tilde{c})u(w_h - \rho_j) + (1 - \tilde{p}_{k,\ell}(\tilde{c}))u(w_l - \rho_j) - c(\tilde{c}) \]
\[ + aE_c(\tilde{p}(e(c))w_h + (1 - \tilde{p}(e(c)))w_l) \]

yielding a contradiction.

**Fact 2** \( \forall j \in J_{L}, \rho_j = 0. \)

Suppose on the contrary that \( \forall j \in J_{L}, \rho_j > 0 \) and \( \forall j \in J_{\bar{L}}, \rho_j = 0. \) Then for \( j \in J_{L}, \) we have that \( N_j = L > 0 \) and condition (iii) of definition 1 implies that there exists \( i \) such that

\[ u(w_i - \rho) + aN((1 - \beta)(\bar{p}w_h + (1 - \bar{p})w_l)) + \beta(\bar{p}w_h + (1 - \bar{p})w_l)) \]
\[ \geq u(w_i) + aN(\beta(\bar{p}w_h + (1 - \bar{p})w_l)) + (1 - \beta)(\bar{p}w_h + (1 - \bar{p})w_l)) \]

Since \( u \) is a strictly concave function and \( \beta(\bar{p}w_h + (1 - \bar{p})w_l) + (1 - \beta)(\bar{p}w_h + (1 - \bar{p})w_l) \)
\[ > (1 - \beta)(\bar{p}w_h + (1 - \bar{p})w_l) \]
\[ \beta(\bar{p}w_h + (1 - \bar{p})w_l), \] the last inequality does not hold, yielding a contradiction.

**Fact 3** \( \forall j \in J_{\bar{L}}, \text{if } J_{L} \neq \emptyset \text{ then } \rho_j > 0. \)

Suppose on the contrary that \( \forall j \in J_{\bar{L}}, \rho_j = 0. \) Therefore, the rent is nil everywhere and parents can choose locations where the effort is either \( \bar{L} \) or \( \bar{R}. \) Then for \( i = h, l \)

\[ u(w_i) + aN(\beta(\bar{p}w_h + (1 - \bar{p})w_l)) + (1 - \beta)(\bar{p}w_h + (1 - \bar{p})w_l)) \]
\[ > u(w_i) + aN((1 - \beta)(\bar{p}w_h + (1 - \bar{p})w_l)) + \beta(\bar{p}w_h + (1 - \bar{p})w_l)) \]

and thus, no one will choose a location where the effort is \( \bar{L} \), which contradicts the fact that \( J_{L} \neq \emptyset. \)

**Fact 4** If \( f_l > 0 \) then \( f_h = 1. \)

In other words, fact 4 means that if there is at least one neighborhood where \( e_j(\bar{c}) = \bar{R}, \) the rich locate in that neighborhood. Indeed, fact 3 implies that \( \forall j \in J_{\bar{L}}, N_j = L \) and since the effort is \( \bar{R}, \) \( 0 < \theta'(\rho) \leq \theta_j \leq \theta''(\rho) < +\infty \) and thus

\[ u(w_h - \rho) + aN(\beta(\bar{p}w_h + (1 - \bar{p})w_l)) + (1 - \beta)(\bar{p}w_h + (1 - \bar{p})w_l)) \]
\[ \geq u(w_h) + aN((1 - \beta)(\bar{p}w_h + (1 - \bar{p})w_l)) + \beta(\bar{p}w_h + (1 - \bar{p})w_l)) \]
\[ u(w_l - \rho) + aN(\beta(\bar{p}w_h + (1 - \bar{p})w_l)) + (1 - \beta)(\bar{p}w_h + (1 - \bar{p})w_l)) \]
\[ \geq u(w_l) + aN((1 - \beta)(\bar{p}w_h + (1 - \bar{p})w_l)) + \beta(\bar{p}w_h + (1 - \bar{p})w_l)) \]
Since \( u \) is a strictly concave function, the last inequality implies that the previous one is strict, that is:

\[
\begin{align*}
&u(wh - \rho) + aN(\beta(\overline{p}wh + (1 - \overline{p})w_l) + (1 - \beta)(\overline{p}wh + (1 - \overline{p})w_l)) \\
&> u(wh) + aN((1 - \beta)(\overline{p}wh + (1 - \overline{p})w_l) + \beta(\overline{p}wh + (1 - \overline{p})w_l))
\end{align*}
\]

which proves fact 4.

To ensure the existence of an inefficient-regressive equilibrium, let us consider for instance that \( \theta_j = \theta \) for any \( j \in J_\pi \) with \( |J_\pi| = n_1 \). Then, given \( N(wh) = N(wh, \pi) + N(wh, \varepsilon) \), we have:

\[
\theta = \frac{1}{\frac{n_1 L - N(wh)}{n_1}} = \frac{N - N(wh)}{n_1 L - N(wh)}
\]

Given that \( n_1 L < nL < N \), it is obvious that \( \theta \) always exceeds 1. Now let us show that there is a city with for instance two neighborhoods of size \( N - \varepsilon \) (with \( \varepsilon \) small enough) such that \( \theta < \theta''(\rho) \) for any \( N(wh) \).

We thus have

\[
\theta = \frac{N - N(wh)}{N - \varepsilon - N(wh)} \simeq 1 < \theta''(\rho).
\]

Proof of Proposition 3.

By definition, an inefficient-neutral equilibrium is a urban configuration where \( f_l = f_h = 0 \). Hence for any \( j \), by lemma 1, we have \( \theta_j \leq \theta'(0) \) or \( \theta_j \geq \theta''(0) \). Moreover, by fact 2, we have, for any \( j \), \( \rho_j = 0 \).

Thus, so as to ensure the existence, let us consider a city with two neighborhoods with \( L = N - \varepsilon \). We can always locate all the poor in the first neighborhood and the rich in the other one. Consequently, we would have \( \theta_1 = +\infty \) and \( \theta_2 = 0 \), thus completing the proof of proposition 3.

Proof of Proposition 4.

The other possible equilibria not examined are the progressive equilibria, the efficient-regressive equilibrium and the inefficient-regressive equilibrium. By definition, a efficient equilibrium is never regressive.

Let us prove that progressive equilibria never exist i.e. we never have an equilibrium with \( f_l > f_h \). Assume by contradiction that we have either (i) \( f_l > f_h = 0 \) or (ii) \( f_l > f_h > 0 \).

Case (i) is impossible, because if \( f_h = 0 \), \( N_j(wh, \pi) + N_j(wh, \varepsilon) = 0 \) for any \( j \) where \( e_j(\tilde{c}) = \varepsilon \). Thus, in these neighborhoods, \( \theta_j = +\infty \) which contradicts lemma 1.

Case (ii) is impossible, because by fact 4 if \( f_l > 0 \), we have \( f_h = 1 \).
Let us prove that the inefficient-regressive equilibrium does not exist i.e. \( f_h > f_l = 0 \). This case is impossible because if \( f_l = 0 \), \( N_j(w_l, \bar{z}) + N_j(w_l, \bar{c}) = 0 \) for any \( j \) where \( \epsilon_j(\tilde{c}) = \bar{z} \). Thus, in these neighborhoods, \( \theta_j = 0 \) contradicting lemma 1.

As for our stability results, let us start with some definitions.

**Definition 4** Given two distinct communities \( j \) and \( k \), we call perturbation-(\( \epsilon_{jk}, \epsilon_{kj} \)) a movement of \( \epsilon_{jk} \) rich agents in community \( j \) to \( k \) joined by a reverse movement of \( \epsilon_{kj} \) poor agents.

**Definition 5** For two distinct communities \( j \) and \( k \), denoting \( N_j(w_l) = N_j(w_l, \bar{z}) + N_j(w_l, \bar{c}) \), a perturbation-(\( \epsilon_{jk}, \epsilon_{kj} \)) is said to be feasible if,

(i) There are at least \( \epsilon_{jk} \) rich agents in community \( j \), i.e. \( \epsilon_{jk} \leq N_j(w_h) \),

(ii) There are at least \( \epsilon_{kj} \) poor agents in community \( k \), i.e. \( \epsilon_{kj} \leq N_k(w_l) \) and,

(iii) After the perturbation-(\( \epsilon_{jk}, \epsilon_{kj} \)), the housing demand does not exceed the housing supply, i.e. \( N_j + \epsilon_{kj} - \epsilon_{jk} \leq L \) and \( N_k + \epsilon_{jk} - \epsilon_{kj} \leq L \).

**Definition 6** An equilibrium is robust to a feasible perturbation-(\( \epsilon_{jk}, \epsilon_{kj} \)) if,

(i) Rich agents who have moved from \( j \) to \( k \) are at least willing to pay the equilibrium rent in order to go back to community \( j \), i.e. \( \tilde{\rho}_{jk} \geq \rho_j \), with \( \rho_j \) being the equilibrium rent and \( \tilde{\rho}_{kj} \) defined by

\[
\begin{align*}
u(w_h - \rho_k) + aE_c(p(e_k(c, \epsilon_{jk}, \epsilon_{kj})))w_h + (1 - p(e_k(c, \epsilon_{jk}, \epsilon_{kj})))w_l \\
= u(w_h - \tilde{\rho}_{hj}) + aE_c(p(e_j(c, \epsilon_{jk}, \epsilon_{kj})))w_h + (1 - p(e_j(c, \epsilon_{jk}, \epsilon_{kj})))w_l
\end{align*}
\]

with for \( z = j, k \), \( e_z(c, \epsilon_{jk}, \epsilon_{kj}) = \arg \max_{\epsilon} \left[ \tilde{p}_j(\tilde{c})u(w_h - \rho_j) + (1 - \tilde{p}_j(\tilde{c}))u(w_l - \rho_j) - c(\tilde{c}) \right] + aE_c(\tilde{p}(e(c)))w_h + (1 - \tilde{p}(e(c)))w_l \) and \( \tilde{p}(e) = \frac{p(e)}{p(e) + (1 - p(e))\theta_j(e, \epsilon_{jk}, \epsilon_{kj})} \), \( \theta_j(e, \epsilon_{jk}, \epsilon_{kj}) \) which denotes the new social composition in community \( z \) after perturbation-(\( \epsilon_{jk}, \epsilon_{kj} \)) and,

(ii) Rich agents outbid poor ones for land in community \( j \), i.e. \( \tilde{\rho}_{hj} \geq \tilde{\rho}_{l} \) with \( \tilde{\rho}_{l} \) defined by

\[
\begin{align*}
u(w_l - \rho_k) + aE_c(p(e_k(c, \epsilon_{jk}, \epsilon_{kj})))w_h + (1 - p(e_k(c, \epsilon_{jk}, \epsilon_{kj})))w_l \\
= u(w_l - \tilde{\rho}_{lj}) + aE_c(p(e_j(c, \epsilon_{jk}, \epsilon_{kj})))w_h + (1 - p(e_j(c, \epsilon_{jk}, \epsilon_{kj})))w_l
\end{align*}
\]

**Definition 7** An equilibrium is stable if there exist \( \bar{z}_{kj} > 0 \) and \( \bar{z}_{jk} > 0 \) such that, for all \( \epsilon_{jk} \) and \( \epsilon_{kj} \) that satisfy \( 0 < \epsilon_{jk} \leq \bar{z}_{jk} \) and \( 0 < \epsilon_{kj} \leq \bar{z}_{kj} \), it is robust to any feasible perturbation-(\( \epsilon_{jk}, \epsilon_{kj} \)) with \( j = 1, \ldots, n \), \( k = 1, \ldots, n \) and \( j \neq k \).

(i) There always exists an efficient-neutral equilibrium that is stable.

Let us consider an efficient-neutral equilibrium which is such that \( \forall j = 1, \ldots, n \), \( \theta_j = 1 \). Given Proposition 1, we know that it always exists.
The proof consists in showing that this efficient-neutral equilibrium is stable. This amounts to say that for any \(j = 1, \ldots, n, k = 1, \ldots, n,\) and \(j \neq k,\) we can easily find \(\varepsilon_{kj} > 0\) and \(\varepsilon_{jk} > 0\) such that, for all \(\varepsilon_{jk}\) and \(\varepsilon_{kj}\) that satisfy \(0 < \varepsilon_{jk} \leq \varepsilon_{kj}\) and \(0 < \varepsilon_{kj} \leq \varepsilon_{kj},\) we still have after the feasible perturbation-(\(\varepsilon_{jk}, \varepsilon_{kj}\)) the same educational effort in both communities after migrations, i.e. \(e_z(\hat{c}, \varepsilon_{jk}, \varepsilon_{kj}) = e_z(\hat{c}, 0, 0) = 1\) for \(z = j, k.\) \(\tilde{\rho}_{hj}\) and \(\tilde{\rho}_{ij}\) are then obtained by the following equations:

\[
u(w_h) + aN(\beta(pw_h + (1 - \beta)(\bar{w}_h + (1 - \beta)w_l))) = u(w_h - \tilde{\rho}_{hj}) + aN(\beta(pw_h + (1 - \beta)(\bar{w}_h + (1 - \beta)w_l))) \]

and

\[
u(w_l) + aN(\beta(pw_h + (1 - \beta)(\bar{w}_h + (1 - \beta)w_l))) = u(w_l - \tilde{\rho}_{lj}) + aN(\beta(pw_h + (1 - \beta)(\bar{w}_h + (1 - \beta)w_l))) \]

which yield \(\tilde{\rho}_{hj} = \tilde{\rho}_{lj} = 0.\)

As a consequence, given definition 6, the efficient-neutral equilibrium is robust to the feasible perturbation-(\(\varepsilon_{jk}, \varepsilon_{kj}\)). As this is true for any \(j = 1, \ldots, n, k = 1, \ldots, n,\) and \(j \neq k,\) given definition 7, the efficient-neutral equilibrium is stable.

(ii) There always exists a stable inefficient-regressive equilibrium.

Let us consider an inefficient-regressive equilibrium which is such that \(\forall j = 1, \ldots, n, \theta_j = \hat{\theta} = \frac{N - N(w_h)}{N1 - L - N(w_h)}.\) We know that we can find a city with \(n\) neighborhoods of size \(L\) such that it always exists.

The proof then proceeds in showing that this inefficient-regressive equilibrium is stable. This amounts to say that for any \(j = 1, \ldots, n, k = 1, \ldots, n,\) and \(j \neq k,\) knowing that \(\hat{\theta} < \theta^\prime,\) we can easily find \(\varepsilon_{kj} > 0\) and \(\varepsilon_{jk} > 0\) such that, for all \(\varepsilon_{jk}\) and \(\varepsilon_{kj}\) that satisfy \(0 < \varepsilon_{jk} \leq \varepsilon_{kj}\) and \(0 < \varepsilon_{kj} \leq \varepsilon_{kj},\) we still have after the feasible perturbation-(\(\varepsilon_{jk}, \varepsilon_{kj}\)) the same educational effort in both communities after migrations, i.e. \(e_j(\hat{c}, \varepsilon_{jk}, \varepsilon_{kj}) = e_j(\hat{c}, 0, 0) = 1\) and \(e_k(\hat{c}, \varepsilon_{jk}, \varepsilon_{kj}) = e_k(\hat{c}, 0, 0)^9.\) \(\tilde{\rho}_{hj}\) and \(\tilde{\rho}_{ij}\) are then obtained by the following equations:

When \(e_k(\hat{c}, \varepsilon_{jk}, \varepsilon_{kj}) = 0,\)

\[
u(w_h) + aN((1 - \beta)(pw_h + (1 - \beta)w_l)) + \beta(pw_h + (1 - \beta)w_l)) = u(w_h - \tilde{\rho}_{hj}) + aN((1 - \beta)(pw_h + (1 - \beta)w_l)) + \beta(pw_h + (1 - \beta)w_l)) \]

9Remember that in a inefficient-regressive equilibrium, all rich agents live in communities \(j\) such that \(j \in J_r\) and poor people live in either communities \(z \in J_r\) or \(z \in J_r^c.\)

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and
\[
    u(w_l) + aN((1 - \beta)(\bar{pw}_h + (1 - \bar{p})w_l) + \beta(\bar{pw}_h + (1 - \bar{p})w_l)) = u(w_l - \tilde{\rho}_{ij}) + aN(\beta(\bar{pw}_h + (1 - \bar{p})w_l) + (1 - \beta)(\bar{pw}_h + (1 - \bar{p})w_l))
\]

which yield \( \tilde{\rho}_{ij} > \tilde{\rho}_{ij} = \rho_j^{10} \).

When \( e_k(\hat{c}, \varepsilon_{jk}, \varepsilon_{kj}) = 1 \),
\[
    u(w_h - \rho_k) + aN(\beta(\bar{pw}_h + (1 - \bar{p})w_l) + (1 - \beta)(\bar{pw}_h + (1 - \bar{p})w_l)) = u(w_h - \tilde{\rho}_{ij}) + aN(\beta(\bar{pw}_h + (1 - \bar{p})w_l) + (1 - \beta)(\bar{pw}_h + (1 - \bar{p})w_l))
\]
and
\[
    u(w_l - \rho_k) + aN(\beta(\bar{pw}_h + (1 - \bar{p})w_l) + (1 - \beta)(\bar{pw}_h + (1 - \bar{p})w_l)) = u(w_l - \tilde{\rho}_{ij}) + aN(\beta(\bar{pw}_h + (1 - \bar{p})w_l) + (1 - \beta)(\bar{pw}_h + (1 - \bar{p})w_l))
\]

which yield \( \tilde{\rho}_{ij} = \tilde{\rho}_{ij} = \rho_k \).

As a consequence, given definition 6, the inefficient-regressive equilibrium is robust to the feasible perturbation-(\( \varepsilon_{jk}, \varepsilon_{kj} \)) whatever \( e_k(\hat{c}, \varepsilon_{jk}, \varepsilon_{kj}) = 0, 1 \). As this is true for any \( j = 1, \ldots, n, k = 1, \ldots, n \), and \( j \neq k \), given definition 7, the inefficient-regressive equilibrium is stable.

(iii) There always exists a stable inefficient-neutral equilibrium. Considering the inefficient-neutral equilibrium characterized in the proof of proposition 3, it is straightforward to deduce that it is stable. ■

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\(^{10}\)Given Proposition 2, the equilibrium-rent \( \rho_j \) is such that poor agents are indifferent between living in community \( j \in J_\tau \) and residing in community \( z \in J_{\bar{\tau}} \).
References


