Labor supply elasticities and borrowing constraints*

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Abstract

The labor-supply elasticity used in typical macroeconomic models is higher than the elasticity that labor economists estimate from microdata. We argue that assumptions underlying previous econometric estimates of the intertemporal labor supply elasticity are not consistent with heterogeneous agents economies with incomplete markets. In particular, if the econometrician ignores borrowing constraints, the elasticity will be biased downwards. Our preliminary finding is that this bias may be up to 50 percent.

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1 Introduction

Because intertemporal substitution of labor is a central element in most models used by macroeconomists today (some version of the permanent-income hypothesis model with capital market imperfections), much research has been devoted to examining whether it is an important determinant of labor supply.\footnote{The classical article emphasizing the role of intertemporal labor supply is Lucas & Rapping (1969)} If the intertemporal substitution is important for labor-leisure choices, we would observe that individuals expecting increases in the real wage work little today and more in the future. Using microdata on wages, hours worked and various household characteristics, most studies however find that expected changes in the real wage only lead to small changes in hours worked. For men, most estimates of the intertemporal labor supply elasticity are in the range between 0 and 0.45 (see for example Heckman & MaCurdy 1980, MaCurdy 1981, Altonji 1986, Blundell & MaCurdy 1999).

Most macromodels on the other hand require a higher substitutability in order to explain for example business cycles movements. In this paper we try to reconcile the low empirical values found using microdata with larger substitutability required in typical macromodels.

We argue that assumptions underlying previous econometric estimates of the intertemporal labor supply elasticity are not consistent with heterogeneous agents economies with incomplete markets. In particular, if the econometrician ignores borrowing constraints, the elasticity will be biased downwards. The goal of this paper is to quantitatively assess this bias. We do this by applying standard econometric methods on synthetic data generated from a macroeconomic model in which we know the true labor-supply elasticity.

In the next section, we introduce a model, developed by Bewley (1986), Huggett (1993), and Aiyagari (1994). This model has become the workhorse tool for analyzing economies with incomplete markets and household heterogeneity. The economy is populated by a large number of infinitely-lived households that face uninsurable idiosyncratic labor productivity risk, supply labor elastically and trade a single asset. Each household can achieve a path for consumption that is smoother than its path for labor income by
adjusting its asset holdings and its labor supply in response to income shocks.

Households have an incentive to accumulate a buffer stock of savings when their labor income is above average, since borrowing is ruled out by assumption. Because earnings shocks are uncorrelated across households, the distributions of income and wealth in the model are endogenous.

Section 3 outlines the most important estimation procedures that have been applied in the empirical literature. We also demonstrate how the presence of borrowing constraints affects the equations to be estimated, and why estimates of the elasticity may be downward biased if borrowing constraints are ignored.

In Section 4, we report the results of using these econometric methods to estimate the labor supply elasticity from data generated by the model. Our preliminary findings are that the downward bias is around 50 percent. That is if the true labor supply elasticity is 1, the econometrician would find a value of 0.5.

2 The model

Our economy is populated by a continuum of infinitely lived households. These households supply labor elastically and maximize the expected discounted utility from consumption. In aggregate, household savings decisions determine the evolution of the aggregate capital stock, which in turn determines aggregate output and the return to saving.

Households are endowed with one unit of time which is divided between labor, \( h_t \), and leisure, \( l_t \). A household’s effective labor supply depends both on the hours it works and on its labor productivity, which is stochastic. Let \( e_t \) denote an individual’s labor productivity at date \( t \), which can take one of \( l < \infty \) values in the set \( P \). Productivity evolves through time according to a first-order Markov chain with transition probabilities defined by the \( l \times l \) matrix \( \Pi \). We normalize productivity to unity so that \( \mathbb{E}(e_t) = 1 \). We assume that there is no aggregate uncertainty.

We now describe the household’s problem. The timing convention is that \( e_t \) is observed before decisions are made in period \( t \). Let \( c_t \) and \( a_t \) denote individual a household’s consumption in period \( t \) and asset in the beginning of period \( t \), respectively. We assume
that a household’s wealth at date zero, \( a_0 \), is non-negative and that households are unable to borrow, i.e.,

\[
a_t \geq 0.
\]

Households maximize their expected discounted lifetime utility,

\[
U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t),
\]

where \( \beta \) is the subjective discount factor. Let \( r_t \) denote the real return at \( t \) to one unit of the asset purchased at \( t - 1 \) and let \( w_t \) denote the real return to supplying one unit of effective labor at date \( t \). The household’s budget constraint in period \( t \) is then given by

\[
c_t + a_{t+1} = (1 + r_t) a_t + w_t e_t h_t.
\]

Aggregate per capita output at \( t \), \( Y_t \), is produced according to a Cobb-Douglas technology from aggregate per capita capital at date \( t \), \( K_t \), and aggregate per capita labor supply \( H_t \):

\[
Y_t = K_t^{\theta} H_t^{1-\theta} \quad t \geq 0
\]

where \( \theta \in [0,1] \).

Output can be transformed into future capital and consumption according to

\[
C_t + K_{t+1} - (1 - \delta) K_t = Y_t
\]

where \( \delta \in [0,1] \) is the rate of depreciation. Product and factor markets are assumed to be competitive, which implies that the real return to saving and the wage rate are given by

\[
r_t = \theta K_t^{\theta-1} H_t^{1-\theta} - \delta
\]

\[
w_t = (1 - \theta) K_t^{\theta} H_t^{-\theta}
\]
respectively.

We focus on stationary equilibria only. See e.g. Aiyagari (1994) for the precise definition of such equilibria. Note that, since there is no aggregate uncertainty in the model, factor prices will be constant in equilibrium. Further, decision rules for consumption and labor supply are time-invariant functions of the household’s asset holdings and productivity.

2.1 Parameterization

The model period is one year. All parameter values used are reported in yearly terms in table I. The parameters relating to aggregate production and preferences are set to standard values. Capital’s share in the Cobb-Douglas production function is 0.3 and the depreciation rate is 0.075. The discount factor $\beta$ is 0.95.

We assume that logged productivity follows an AR(1) process with fixed effects

$$\log e_t = \psi + z_t$$

$$z_t = \rho z_{t-1} + \varepsilon_t.$$  

Various authors have estimated similar stochastic processes for logged labor productivity using data from the PSID. These processes can be summarized by $\rho$, $\sigma_\varepsilon$, and $\sigma_\psi$ – the serial correlation coefficient, the standard deviation of the innovation term $\varepsilon$, and the standard deviation of fixed effects $\psi$, respectively. Allowing for the presence of measurement error and the effects of observable characteristics such as education and age, work by Card (1991), Flodén & Lindé (1999), Hubbard, Skinner & Zeldes (1995) and Storesletten, Telmer & Yaron (1999) indicates a $\rho$ in the range 0.88 to 0.96, and a $\sigma_\varepsilon$ in the range 0.12 to 0.25.\(^2\) We adopt the the estimates of Flodén and Lindé and use the parameter values $\{\rho, \sigma_\varepsilon, \sigma_\psi\} = \{0.90, 0.21, 0.34\}.\(^3\)

The utility function is specified below.

\(^2\) Heaton and Lucas (1996) allow for permanent but unobservable household-specific effects, and find a much lower $\rho$ of 0.53, and a $\sigma$ of 0.25.

\(^3\)When solving the model, we approximate the productivity process by a discrete Markov process using seven grid points for $z$ and two grid points for $\psi$. \(\)
3 Estimation procedure

In this section we describe the standard estimation procedures used in the literature. Typically, first order conditions for household optimization are derived. Additional assumptions such as separability of the utility function or an explicit functional form for the utility are then added in order to obtain equations that can be estimated.

The first order conditions for the household’s utility maximization in this framework are

\[ u_{ct} = \lambda_t \]  
\[ \lambda_t w e_t = -u_{ht} \]  
\[ \lambda_t - \phi_t = rE_t \lambda_{t+1} \]

where \( \lambda_t \) is the marginal utility of wealth in period \( t \) and \( \phi_t \) is the marginal utility of borrowing in period \( t \). In principle we can use these conditions to estimate the household’s willingness to intertemporally substitute. In practise, this may only be done if consumption and hours worked enter the instantaneous utility function in a tractable way.

Before turning to the details of the estimation procedures, it is important to specify exactly which labor-supply elasticity we intend to estimate. The Frisch (or constant marginal utility of wealth) labor-supply elasticity is defined as

\[ \eta^\lambda \equiv \frac{dh}{dw} \frac{w}{h} \bigg|_{\lambda} . \]

From the first order conditions (7) and (8), we get

\[ \eta^\lambda = \frac{u_h}{hu_{hh} - hu_{hc}} \]  

This elasticity shows how labor supply responds to an intertemporal reallocation of wages that leaves marginal utility of wealth unaffected. As Blundell and MaCurdy (1999, p. 1595) point out, this ‘is the correct elasticity for assessing the impact of wage changes through time on labor supply’.

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4See Blundell & MaCurdy 1999 for a more elaborate discussion.
3.1 Estimation with separable utility

Let the instantaneous utility function be given by

\[ u(c, h) = \frac{c^{1-\mu}}{1-\mu} - \zeta \frac{h^{1+1/\gamma}}{1+1/\gamma}, \quad (11) \]

where \( \mu > 0 \) is the coefficient of relative risk aversion, \( \zeta \) is a parameter determining the level of labor supply, and \( \gamma \) is the Frisch intertemporal elasticity of substitution, which we aim to estimate.

In an economy without borrowing constraints (i.e. where \( \phi = 0 \) always), the first order conditions can be rewritten into the following log-linear equations

\[ \ln h_t = \text{constant} + \gamma [\ln w + \ln e_t + \ln \lambda_t], \quad (12) \]

\[ \ln c_t = \text{constant} - \frac{1}{\mu} \ln \lambda_t, \quad (13) \]

\[ \ln \lambda_t = \ln \lambda_{t+1} + \ln r + \xi_{t+1}, \quad (14) \]

where \( \xi_{t+1} \) is the forecast error. Let \( \Delta x_{t+1} = x_{t+1} - x_t \). Equation (12) can then be written in first differences,

\[ \Delta \ln h_{t+1} = \text{constant} + \gamma \Delta \ln e_{t+1} - \xi_{t+1}. \quad (15) \]

On the basis of this equation, the Frisch intertemporal labor supply elasticity is estimated by regressing the difference on log-wages on the difference in log-hours worked. This approach is pursued in MaCurdy (1981) and Altonji (1986).

The error term \( \xi \) in equation (15) is clearly correlated with the explanatory variable \( \Delta \ln e \).\(^5\) Equation (15) is therefore estimated with instruments for the productivity changes.\(^6\) The instruments used by Altonji are lagged wages and lagged wage changes. We use two different approaches to instrument for \( \Delta \ln e_{t+1} \).

\(^5\)An exception is when households know their wage one period ahead, which is assumed by MaCurdy and in some of Altonji’s specifications. In our model the only innovation between periods is the shock to household productivity.

\(^6\)Another reason to instrument for \( \Delta \ln e \) is the occurrence of measurement errors. MaCurdy use year dummies and individual specific information such as age and education as instruments. Altonji uses two different wage series for each household.
The first approach is to regress $\Delta \ln e_{t+1}$ on $\ln e_{t}$, which follows Altonji. Note that with our productivity process,

$$\Delta \ln e_{t+1} = z_{t+1} - z_t = (\rho - 1) z_t + \varepsilon_{t+1} = (\rho - 1) (\ln e_t - \psi) + \varepsilon_{t+1}$$  \hspace{1cm} (16)$$

This equation shows that $\ln e_t$ is correlated with the productivity change. Obvioulsy $\ln e_t$ is uncorrelated with the error term $\xi_{t+1}$. A potential problem when using $\ln e_t$ as the instrument is that it is correlated with the fixed effect $\psi$.\(^7\) This motivates, as is sometimes done in the empirical literature, using additional household variables when estimating (16).

Rather than trying to improve on the estimates in (16) our second approach is to use the mathematical expectation of $\Delta \ln e_{t+1}$ at time $t$, $E_t \Delta \ln e_{t+1}$. This approach is available in our synthetic framework but not to the econometrician who does not know the true productivity process and who does not observe exact productivity levels, fixed effects, etc. We will henceforth call this instrument the theoretically best, since it contains all information about the productivity change except what is contained in $\xi_{t+1}$.

In an economy where borrowing constraints cannot be assumed away, equation (15) should be replaced by (as a first order approximation)

$$\Delta \ln h_{t+1} = \text{constant} + \gamma \Delta \ln e_{t+1} - \frac{\gamma \phi_t}{\lambda_t} - \xi_{t+1}.$$  \hspace{1cm} (17)$$

Note here that expected future wage increases are positively correlated with the marginal utility of borrowing, $\phi$, and uncorrelated with the marginal utility of wealth, $\lambda$, as long as $\phi$ is non-zero. Consequently, equation (17) shows that the estimate of $\gamma$ will be biased downwards if the term $-\gamma \phi_t / \lambda_t$ is ignored.

An alternative procedure for estimating the elasticity is pursued in Altonji (1986). He uses (13) to rewrite equation (12) in log-levels,

$$\ln h_t = \text{constant} + \gamma \ln e_t - \frac{\gamma}{\mu} \ln c_t,$$  \hspace{1cm} (18)$$

and uses data on hours worked, wages, and food consumption to estimate $\gamma$. The advantage with this procedure is that borrowing constraints do not enter this equation.

\(^7\)The estimated coefficient on $\ln e_t$ in Table III below is thus not $\rho - 1$.  

7
However, the use of consumption data is problematic. It is difficult to find good micro-data containing both total consumption and labor supply. The PSID, used by Altonji, contains food consumption and income data but using food consumption as a proxy for total consumption requires that food consumption is sufficiently separable from other consumption goods in the utility function.

To handle participation constraints, some authors (e.g. Browning et al. 1985, and Blundell et al. 1993, 1998) have estimated semi-log equations of the form

$$\Delta h_{t+1} = \text{constant} + \gamma \Delta \ln e_{t+1} + \epsilon_{t+1}. \quad (19)$$

Again, a correlation between $\Delta \ln e_{t+1}$ and the error term is likely. We therefore instrument for $\Delta \ln e_{t+1}$ using its mathematical expectation. Blundell and MaCurdy (1999) point out that this equation cannot be derived from any standard utility function. Furthermore, we find (see below) that this formulation performs poorly on the synthetic data.

### 3.2 Non-separable utility

The assumption that preferences are separable in consumption and leisure is generally regarded as restrictive and possibly unrealistic. Quite naturally, therefore, attempts have been made to allow for more general preferences in the empirical literature. Altonji (1986) argues that adding cross-substitution terms to equations (12) and (13) results in an approximation of the log-linearized first order conditions. As long as measurement errors are negligible, the elasticity can still be estimated from the difference form (as in equations 15 and 17). The log-level form (18) will, however, result in a biased estimate of the labor-supply elasticity.

It is not clear what restrictions on preferences are needed for Altonji’s approximation to be valid. Blundell and MaCurdy (1999) indeed argue that estimating the Frisch elasticity is not possible unless preferences are separable.

Unless a specific functional form for preferences is assumed, it will be difficult to further assess this issue. Let us therefore also consider the Cobb-Douglas utility functions

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8 The participation constraint is never binding with the utility function in equation (11).
that has been used frequently in the macroeconomic literature,
\[ u(c, h) = \frac{[c^{\alpha} (1 - h)^{1-\alpha}]^{1-\mu}}{1-\mu}. \]

For this utility function, we can derive the intertemporal elasticity of leisure as
\[ \eta^l = \frac{1 - \alpha (1 - \mu)}{\mu}. \]

The Frisch elasticity, \( \eta^\lambda = \frac{1-h}{h} \eta^l \), then depends on each household’s labor supply. If participation constraints and borrowing constraints are not binding, one can derive the following relationships
\[ \Delta \ln (1 - h_{t+1}) = \text{constant} - \eta^l \Delta \ln e_{t+1} + \xi_{t+1} \tag{20} \]
and
\[ \ln (1 - h_t) = \text{constant} - \ln e_t + \ln c_t. \tag{21} \]

If \( \Delta \ln h_{t+1} \) is used on the right hand side of (20), there is no reason to expect that the Frisch elasticity for labor supply will be estimated. First, the change in log hours is not identical to the negative of changes in log leisure. Further, even if that were the case, the elasticity for leisure is different from the Frisch elasticity. Equation (21) shows that the log-level equation, even if estimated on leisure, is not related to the intertemporal elasticity.

The straightforward approach for obtaining an estimate of the average labor supply elasticity is thus to estimate \( \eta^l \) from (20) and multiply by average \( (1 - h) / h \). One problem with this approach is to measure leisure in the data. Moreover, there will be a downward bias in the estimate of \( \eta^l \) if borrowing constraints are binding but ignored. Finally, with the Cobb Douglas utility function, participation constraints will bind for households with much wealth and low wages. The typical procedure would then be to exclude all households with no or low labor supply. Since \( h_{t+1} \) is correlated with the wage change, that procedure may, however, induce a bias in the estimates. Alternatively, participation constraints can be handled using the semi-log specification (19). As noted before, this equation cannot be derived from the model. Further, with the Cobb Douglas utility function, the Frisch elasticity is not defined for households with binding participation constraints.
4 Results

Tables II to V report results based on the utility function specified in (11). As the benchmark, we have used the parameter values $\mu = 2$ and $\gamma = 1$ for the risk aversion and labor-supply elasticity, respectively.

Table II shows results from estimations of (15) on data generated by the model when we use the theoretical instrument for productivity. The first column reports regression results for the full sample, thus ignoring borrowing constraints captured by $\phi$. The estimated elasticity is then 0.55 (recall that the true elasticity is unity). Households with little wealth (less than ten percent of average wealth) were excluded in the estimation reported in the second column. The estimate is then much closer to unity. A further confirmation that the borrowing constrained households cause the downward bias in the estimate is evident from the third column. Only the households with little wealth were included in that regression, which shows a negative labor-supply response to wage increases.

Figure 1 is useful for understanding this bias. The figure shows labor supply decisions as a function of the current wage (productivity) and wealth. In a model with no borrowing constraints, households would choose to work hard in periods with high wages and to consume more leisure when wages are low. Here, however, the top lines in the figure show that labor supply is falling in the wage rate for households with little wealth and low wages. These households are (or are close to being) borrowing constrained. They would consequently have to reduce consumption drastically if they did not increase labor supply in response to falling wages. This kind of behavior is only quantitatively important for our estimations if it is displayed by a substantial fraction of households in the economy. Figure 2 plots the wealth-productivity distribution of households in the model economy. It is clear that many households are in regions where labor supply is flat or even falling in the wage rate.

It is worth noting that even though our model is simplistic, it fairly well captures the correlations between earnings and wealth, and between disposable income and wealth.

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9 The figure only shows decision rules for households with the high fixed effect. Decision rules for households with the low fixed effect are similar.
(0.15 and 0.29 in the model and 0.23 and 0.32 in the data). However, the model does not exactly match the wealth distribution observed in the data. In particular, the model generates too little wealth inequality. The Gini coefficient for wealth is 0.58 in the model and 0.78 in U.S. data, the bottom 40 percent in the wealth distribution hold 4.8 percent of assets in the model and 1.4 percent in data, and the top one percent hold 6 percent in the model and 29 percent in the data (U.S. data based on Diaz-Gimenez et al., 1997). The number of borrowing constrained households thus appears to be underpredicted by the model, indicating that the bias we find is not overstated.

The choice of instrument does not seem important for the results we obtain. Table III reports the results for the regressions when lagged wages are used to instrument for future wage changes. Throughout, the estimated elasticities are similar to those reported in Table II.

Table IV confirms that the estimates using the log-level formulation perform well even in the presence of borrowing constraints.

Table V reports estimates of the semi-log specification in equation (19). As expected, the labor-supply elasticity is poorly estimated even when the borrowing constrained households are excluded.

The results with the Cobb-Douglas utility function (see Tables VI and VII) reconfirm the results above. With our specification, the true elasticity of leisure is 0.561, while our estimate on the full sample is 0.416. If households with little wealth and households with low labor supply are excluded, we obtain estimates close to the true elasticity.10 We have also ran regressions like (15), (18), and (19) on the data generated by this model. When households with labor supply less than ten percent of average are excluded, the average of the Frisch elasticities in the data is 1.55. As noted in the previous section, these regressions have no foundation in the model. The latter two regressions consequently yield estimates far from the true elasticity.11 The first regression yields estimates that appear

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10 Instrumenting $\Delta e_{t+1}$ with lagged wages is now superior to using $E\Delta e_{t+1}|e_t$, since the prediction of $\Delta e_{t+1}$ then can be conditioned on $h_{t+1}$ being positive.

11 As pointed out in Altonji (1986), cross-section terms should be introduced in equation (18) when the utility function is non-separable. The equation is then underidentified.
related to the true elasticity, albeit clearly biased downwards.

5 Concluding remarks

[To be written]
References


Table I
Parameter values
\[
\begin{array}{lcl}
\beta & = & 0.95 \\
\delta & = & 0.075 \\
\theta & = & 0.30 \\
\mu & = & 2 \\
\gamma & = & 1 \\
\zeta & = & 10 \\
\rho & = & 0.90 \\
\sigma_{\varepsilon} & = & 0.21 \\
\sigma_{\psi} & = & 0.34 \\
\end{array}
\]

Table II
Estimation of \( \Delta \ln h_{t+1} = \varphi + \gamma \Delta \ln e_{t+1} - \xi_{t+1} \), using \( \mathbb{E}_t \Delta \ln e_{t+1} \) as instrument

\[
\begin{array}{lcl}
\text{sample: households with assets, } a, \text{ belonging to} \\
[0, \infty) & | & [0.1\bar{a}, \infty) & | & [0,0.1\bar{a}) & | & [0.1\bar{a}, \bar{a}) & | & [\bar{a}, \infty) \\
\varphi & = & 0.00 & | & 0.02 & | & 0.01 & | & 0.01 & | & 0.02 \\
& & (0.00) & | & (0.00) & | & (0.00) & | & (0.00) & | & (0.00) \\
\Delta \ln e & = & 0.55 & | & 0.94 & | & -0.12 & | & 0.94 & | & 1.00 \\
& & (0.00) & | & (0.01) & | & (0.01) & | & (0.01) & | & (0.01) \\
R^2 & = & 0.10 & | & 0.18 & | & 0.01 & | & 0.26 & | & 0.13 \\
\# \text{ obs.} & = & 199400 & | & 144765 & | & 54635 & | & 78313 & | & 66452 \\
\end{array}
\]
Table III

Estimation of $\Delta \ln h_{t+1} = \varphi + \gamma \Delta \ln e_{t+1} - \xi_{t+1}$, using 2SLS with $\ln e_t$ as instrument

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Second Stage

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### Table IV

**Estimation of** \( \Delta \ln h_t = \varphi + \gamma \ln e_t - \frac{\gamma}{\mu} \ln c_t + \varepsilon_t \)

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<td>( R^2 )</td>
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### Table V

**Estimation of** \( \Delta h_{t+1} = \varphi + \gamma \Delta \ln e_{t+1} + \varepsilon_{t+1} \), **using** \( E_t \Delta \ln e_{t+1} \) **as instrument**

<table>
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<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>( \Delta \ln e )</td>
<td>0.16</td>
<td>0.27</td>
<td>-0.07</td>
<td>0.31</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.08</td>
<td>0.19</td>
<td>0.01</td>
<td>0.24</td>
<td>0.15</td>
</tr>
<tr>
<td># obs.</td>
<td>199400</td>
<td>144765</td>
<td>54635</td>
<td>78313</td>
<td>66452</td>
</tr>
</tbody>
</table>
Table VI
Cobb Douglas utility, estimates of $\eta'$

<table>
<thead>
<tr>
<th>sample: households with assets, $a$, belonging to</th>
<th>$[0, \infty)$</th>
<th>$(0.1\bar{a}, \infty)$</th>
<th>$(0, 0.1\bar{a})$</th>
<th>$(0.1\bar{a}, \bar{a})$</th>
<th>$(\bar{a}, \infty)$</th>
</tr>
</thead>
</table>

*Estimation from eqn. (15)*

| instrument $= E_t \Delta \ln e_{t+1}$ | 0.43 | 0.60 | 0.34 | 0.63 | 0.72 |
| instrument $= \ln e_t$ | 0.42 | 0.51 | 0.20 | 0.51 | 0.53 |

*Estimation from eqn. (19)*

| 0.29 | 0.40 | 0.22 | 0.44 | 0.47 |

Note: True $\eta'$ is 0.561. Non-working households have been excluded in all regressions.

Table VII
Cobb Douglas utility, estimates of $\eta^\lambda$

<table>
<thead>
<tr>
<th>sample: households with assets, $a$, belonging to</th>
<th>$[0, \infty)$</th>
<th>$(0.1\bar{a}, \infty)$</th>
<th>$(0, 0.1\bar{a})$</th>
<th>$(0.1\bar{a}, \bar{a})$</th>
<th>$(\bar{a}, \infty)$</th>
</tr>
</thead>
</table>

*Estimation from eqn. (15)*

| instrument $= E_t \Delta \ln e_{t+1}$ | 1.01 | 1.49 | 0.70 | 1.68 | 1.80 |
| instrument $= \ln e_t$ | 0.97 | 1.23 | 0.43 | 1.35 | 1.26 |
| true average $\eta^\lambda$ | 1.27 | 1.37 | 0.97 | 1.26 | 1.50 |

*Estimation from eqn. (19)*

| no exclusion of low $h$’s | 0.28 | 0.36 | 0.22 | 0.41 | 0.37 |
| true average $\eta^\lambda$ | $\infty$ | $\infty$ | 0.97 | $\infty$ | $\infty$ |
| low $h$ excluded | 0.27 | 0.37 | 0.22 | 0.40 | 0.41 |
| true average $\eta^\lambda$ | 1.30 | 1.40 | 0.97 | 1.28 | 1.55 |

Note: Households with labor supply less than ten percent of average are excluded in equations including $\log h$. 

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Figure 1: Selected decision rules for labor supply

The different lines represent different wealth levels, and wealth increases from top to bottom. The top line shows labor supply decisions as a function of the idiosyncratic wage for a household with no wealth. The bottom line shows decision rules for a household with asset holdings three times as high as average asset holdings.
Figure 2: Decision rules and distribution of households

Each dot indicates one household in a subsample of the simulated data.