Product Variety and Product Choice

Steven Berry

Yale Univ., Dept. of Economics and Cowles Foundation
and NBER

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Review of Demand Estimation

We discussed the problem of estimating a rich discrete-choice model of differentiated products demand. Goal is to provide realistic own- and cross-price elasticities (wrt price and characteristics.)

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- Consumer vs. Market level Data
- Unobserved Product Characteristics
- Endogeneity of Price
- Identification with weak / strong functional form assumptions
- Flexibility of Functional Form

- Talk 1: Differentiated Products Demand Estimation
- Talk 2: Product Choice and Variety
- Talk 3: Policy Applications

Endogenous Product Choice

Can think of firms choosing both horizontal and vertical product characteristics. Presumably fixed and marginal costs vary across products, not just demand. Vertical product choice may feature Sutton-style models of product quality produced via higher fixed costs.

We want to model product choice because it is an important part of competition and consumer welfare.

Anti-Trust Example

DOJ merger guidelines emphasize price effects over effects on product quality and variety. Not clear that this is correct.

Welfare loss from bad pricing is a dead-weight loss "triangle", may be small.

Welfare loss from non-optimal product location and variety can be first-order.

Welfare Problems with Product Variety

In general, no "invisible hand" theorem. Pure monopolist to some degree cares about inefficiency, although in general offers to much or too little variety. Problem is worse with oligopoly.

- Entry and "business stealing" – as market grows, extra firm may add fixed costs without adding much demand.
- Location models – Hotelling locational equilibrium can be at ends of the line – maximizes transport cost.
- Less price competition might increase variety.

Firm's Endogenous Choice of Products

Implications

- Demand estimates and "econometric" endogeneity (panel data may help this.)
- Modeling Endogeneity and it's implications.
  - Social Welfare
  - Policy / Anti-trust effects

Today's Topic

Identification and Estimation

How do we uncover the “structural” parameters that let us model the firm's choice of characteristics?

Today's Outline

Some Descriptive "Facts"

- Larger markets often have more products, sometimes linearly so and sometimes not (Bresnahan and Reiss [4], Campbell and Hopenhayn [5]).
- But concentration doesn't always fall to zero in market size, dominant firms/products can exist in very large markets (Sutton '90).
- Product choice varies with market demographics. This is perhaps obvious, but creates a kind of externality where neighbors preferences affect my welfare (Waldfogel [11]).

Quality and "Endogenous Sunk Cost"

A series of articles (e.g. Shaked and Sutton) and books (e.g. Sutton 1991) lays out a theory of endogenous product quality, starting from the theory of vertical product differentiation.

A key result of the vertical quality model is that a high quality firm may price a low quality firm out of the market if the difference in marginal costs between the two firms is relatively small.

Therefore, if the cost of increased quality is largely fixed with respect to output, some high quality firm may maintain a large share of the market even as market size increases.

Berry - Waldfogel '06

Empirical Approach

Look at two contrasting industries (Newspapers and Restaurants) across U.S. cities of different sizes.

Idea

Newspapers create quality largely though fixed costs (Sutton-style), while for a restaurant, increase in quality largely increases *mc*.

Prediction

- As market size increases, quality of newspapers should increase, but not number.
- For restaurants, number should increase and the quality space should fill in.

Contrast to earlier descriptive work:

- Unlike Sutton, not a cross-section of countries.
- Unlike others, we use direct measures of quality (not just advertising, R&D inputs.)
- Newspaper and Restaurant data are especially dramatic.

	Restaurants		Newspapers	
Variable	Log N	Log N-Equiv	Log N	Log N-Equiv
Log Pop.	0.99	0.91	0.521	0.226
	(0.014)	(0.016)	(0.029)	(0.023)
Intercept	6.613	5.564	1.455	0.733
	(0.023)	(0.026)	(0.046)	(0.036)
# obs	316	316	283	283
R^2	0.94	0.91	0.54	0.26

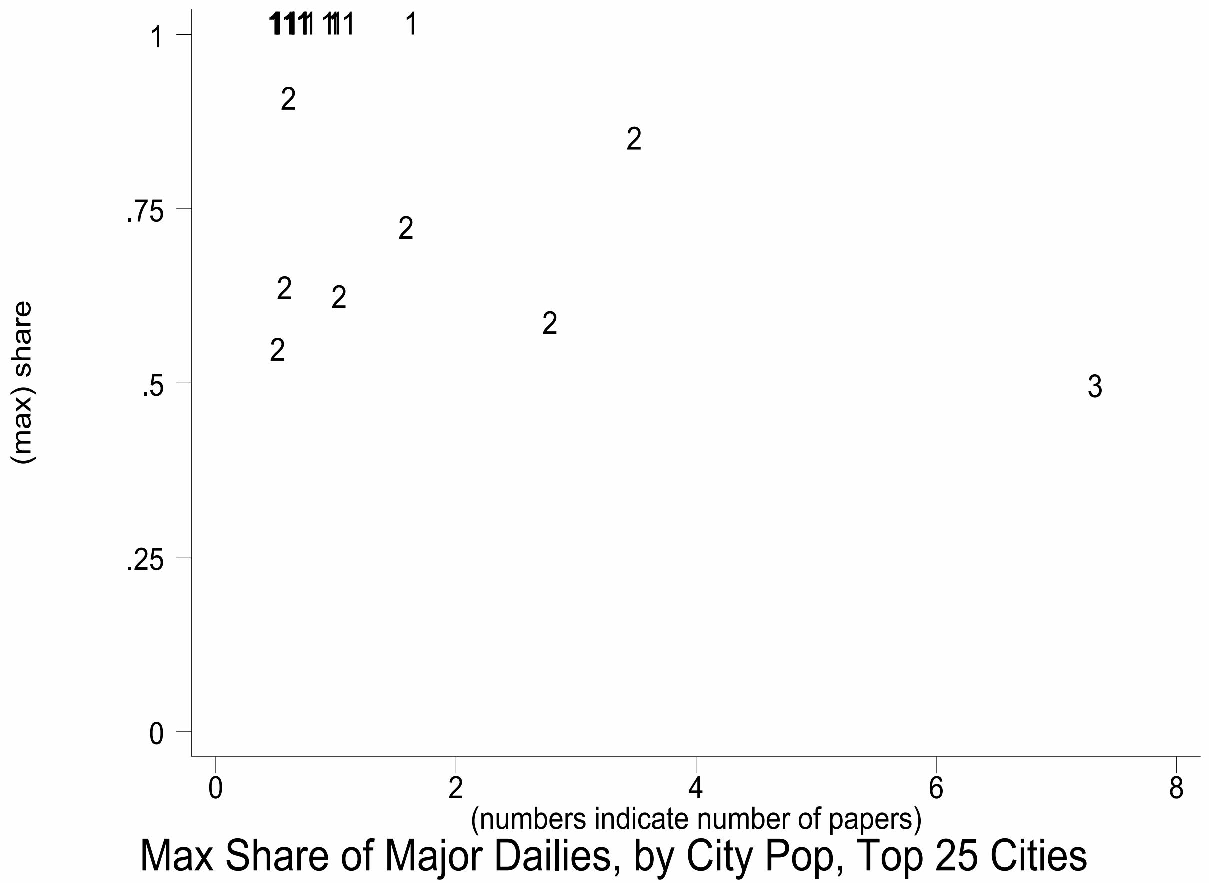
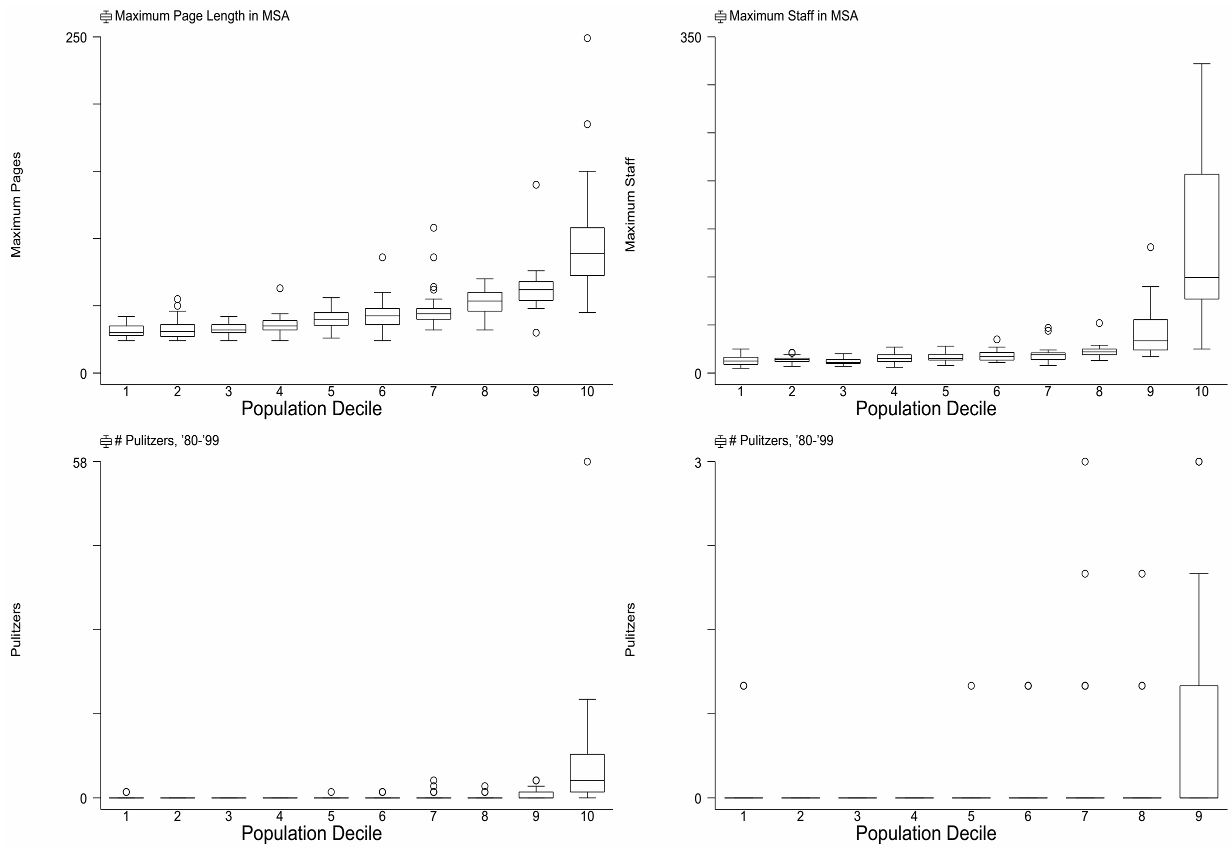
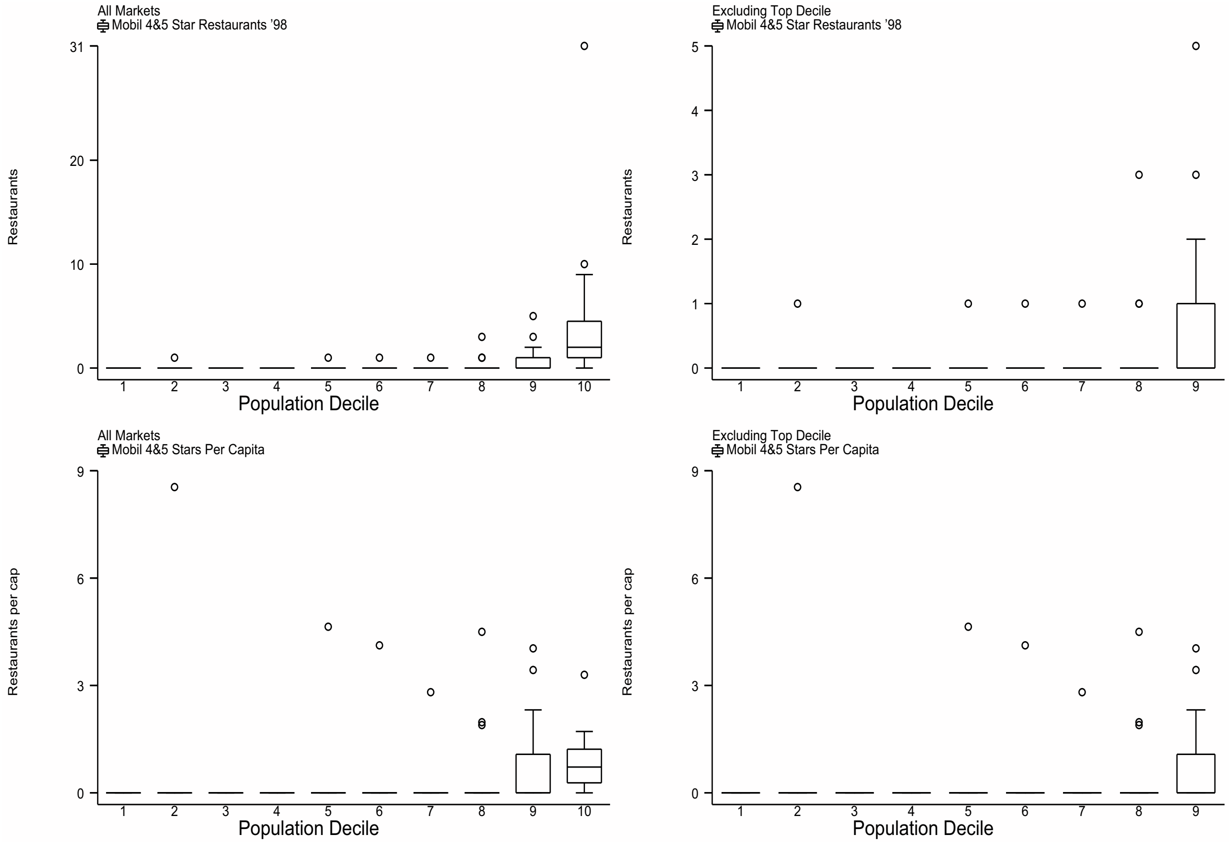


Figure 2b from Berry-Waldfogel, 2006



Newspaper Quality and Market Size



Top Restaurants and Market Size

"Tyranny of the Majority"

In a series of papers (and now a book), Waldfogel shows that the choice set in local markets varies systematically with the demogrphics of the market.

Waldfogel's conclusion: your consumer welfare then varies with the tastes of your "neighbors."

For us today: more evidence of endogenous product characteristics.

Approach I: Use a static model of continuous choice of x or quality.

Similar to the problem of endogenous prices (so we will start with that.)

Not much work here, but actually seems to be perhaps the easiest approach; maybe not believable? Have to separate marginal and fixed costs of product choice.

Approach II: Model the number of products and/or the number of products in a set of discrete categories

Many of these papers exploit regional data on market size.

These are models of entry or endogenous market structure. Discrete entry into product classes, qualities and/or locations.

Big problems here with multiple equilibria, perhaps solved recently with the use of necessary conditions and "bounds" approaches.

Approach III: Full Dynamics

- Can think of using dynamic oligopoly models (e.g. Pakes-Ericson, [6] [9]) where past decisions effect future outcomes.
- Very difficult to model, but some progress with discrete entry and strong assumptions on unobservables over time and on the choice set.
- Curse of dimensionality is a big problem.
- One example: Benkard on aircraft [1].
- Alternatively, strong assumptions on continuous choices (similar to Euler equations. Berry and Pakes '02 [3].

Approach IIIa: Simplified Dynamics

Here, use relatively simple dynamic oligopoly models with i.i.d. errors – build on Hotz-Miller to make use of simple first-stage non-parametric estimates of the equilibria transition matrix and then back out firm profit parameters from the firm's single-agent problem.

Applications: Ryan '06 on environmental costs; Roberts, et al on entry into small markets.

Continuous Choice

The choice of continuous x is similar to the choice of price, so review that first.

Further Examples: continous quality, location.

Estimation on the Supply Side

Estimation from the Supply Side can follow earlier literature in perfect competition and Cournot.

Assume, for example, that marginal cost is

$$mc_j = \mathrm{w}_j\beta + \lambda q_j + \omega_j, \tag{1}$$

where w might consist of x and of input prices, q is output and ω_j is a supply shock, unobserved to the econometrician.

The f.o.c. for a single product firm, given Nash price-setting, is

$$p_j = mc_j + \frac{q_j}{|\frac{\partial q_j}{\partial p_j}|},$$

or

$$p_j = \mathrm{w}\beta + \lambda q_j + \frac{q_j}{|\frac{\partial q_j}{\partial p_j}|} + \omega_j.$$

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Estimation of MC

- Given demand parms, markup is known
- Estimate by IV
- Instruments: demand-shifters, exog. changes in choice set (changes markup.)
- Easy to extend to multi-product firms.
- Harder: dynamic pricing.

Firm Behavior with Multi-Product Firms

Profits:

$$\pi_f = \Sigma_{j \in \mathcal{J}_f} (p_j - mc_j) M \; s_j(p, x, \xi; \theta) \; ,$$

F.O.C.:

$$s_j(p, x, \xi; \theta) + \sum_{r \in \mathcal{J}_f} (p_r - mc_r) \frac{\partial s_r(p, x, \xi; \theta)}{\partial p_j} = 0.$$

Given the demand parms, can solve this for the vector of mc's and so for markup, $b_j(p, x, \xi, \theta)$, giving the pricing equation.

In particular, write the foc as

$$s + \Delta(p - mc) = 0, \tag{2}$$

where Δ has one on the diagonal, one's on the off-diagonals of jointly owned products and zeros elsewhere. MC is then:

$$mc = p + \Delta^{-1}s. \tag{3}$$

Notes on Estimating from the F.O.C

- We do not require a unique equilibrium
- the markup depends on ξ, ω and so is econometrically endogenous
- other static equilibria are easy (*e.g.* qty-setting, collusion)

Caveats:

- Nash Pricing
- no dynamics
- no production data

Continuous Observed Product Quality

Say choose quality, x_j and price simultaneously.

$$\pi_{jf} = (p_j - mc_j(q_j, x_j))q_j - FC(x_j, \theta)$$

$$q_j = Ms_j(p, x, theta)$$

Quality changes q and both mc and FC.

F.O.C. for Continuous Quality

$$(p_j - mc_j(q_j, x_j))\frac{\partial q_j}{\partial x_j} - q_j\left(\frac{\partial mc_j}{\partial x_j} + \frac{\partial mc_j}{\partial q_j}\frac{\partial q_j}{\partial x_j}\right) - \frac{\partial FC}{\partial x_j} = 0$$

- Interesting questions: can you separate the effects of quality on *mc* and *FC*? This is central, for example, to Sutton.
- Perhaps market size helps as an exogenous shifter?
- Where to put the unobservable shock?
- Perhaps better to choose quality *before* x?

Next Approach: Cross-sectional Models of Equilibrium Market Structure

Simplest models of characteristics involve "entry" into a discrete space of possible characteristics, which might be quality, location, etc.

One-shot "static" models try to describe a cross-section of markets in equilibrium. Not really "entry" but equilibrium market structure.

How to Model the Product Space

- **"Ex-Ante" vs. "Ex-Post"** differentiation: with ex-ante, have to specify number and characteristics of *potential* competitors. For airlines (Berry '91) and Chain Stores (Jia, '06) this might might sense, but other times is quite arbitrary.

How to Model the Product Space

- **"Ex-Ante" vs. "Ex-Post"** differentiation: with ex-ante, have to specify number and characteristics of *potential* competitors. For airlines (Berry '91) and Chain Stores (Jia, '06) this might might sense, but other times is quite arbitrary.
- **Continuous v.s Discrete** product space. Easier to specify "counterfactual profits" (profits of the "next entrant") with discrete space.

Static Complete Info Nash

Static Complete Information Nash is a good assumption for work that relies on the cross-sectional nature distribution of market structure.

With no explicit dynamics, we would like firms to choose the best-response to rival's actions – otherwise why don't they move? Justification for cross-sectional would be that [i] population and demographics are strong instruments and [ii] firms are in "long-run" equilibrium.

In a dynamic model, some private info makes more sense – firms might be surprised to find themselves in a bad location and then move away.

Revealed Preference

Basic idea of cross-section market structure is revealed preference.

How does this intuition map into formal identification / estimation?

What to Estimate from "Entry"

1. The entire profit function?
2. Or, get *variable* profits from prices, quantitities, etc. and then estimate only for the distribution of fixed costs from entry model?

[2] might use demand estimation from lecture 1 and *mc* estimation from today. But: we have lost exogenous product charcteristics.

Basic Entry Model

Profits

from product choice y_j are

$$\pi(y_j, y_{-j}, x_j, \theta) + y_j \epsilon_j$$

The choice y could be binary ("entry") or else a vector (0s and 1 for "location".)

"Best-reply"

$$y_j^* = \operatorname{argmax}_{y \in Y} \pi(y, y_{-j}, x_j, \theta) + y \epsilon_j$$

Unique Equilibrium?

Often, there is a big problem with multiple equilibria – can't define prediction of model or probability of event.

Identification: The Monopoly Entry problem

See Berry and Tamer, '07.
Here we review the results of Matzkin '92 and others, using the potential monopoly entry example. Profits of an entering firm are:

$$\pi(x_i, F_i) = v(x_i) - F_i,$$

where v is the deterministic variable profit and F the random fixed cost.

(Non-)identification?

In a cross-section of markets, entry occurs when $v(x_i) > F_i$. We observe the entry probabilities $p(x_i)$.

Observed v

If we estimate v from other data (p & q), then we can directly learn the distribution of fixed costs:

$$Pr\left(F_i < v(x_i)\right)$$

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Unknown v

An immediate problem is that any monotonic transformation of both v and F results in the same entry probabilities. How bad a problem is this?

Non-Robustness to Monotonic Transformations

How bad is the problem? For many issues, not bad at all. Assume F is i.i.d. $\sim \Phi(\cdot)$.

Then $p(x_i) = \Phi(v(x_i))$ is one such monotonic transformation of v, and it does reveal

- $\partial p / \partial x$
- The sign of the effect of an x on v
- The relative effects:

$$\frac{\partial v / \partial x^1}{\partial v / \partial x^2}$$

This is the kind of problem in, e.g. Berry '92. (What is the sign and relative magnitude of "airport presence" in entry and profits?)

Qualitative Shape Restrictions

But what if we want to know the full shape of V? Matzkin '92 suggests qualitative shape restrictions, preferably derived from theory, together with an i.i.d. assumption on F.
E.g. assume constant marginal costs, then for many models variable profit is proportional to population, $v(x) = z_i \bar{v}(x_i)$ where z is population.

Sketch of Matzkin's proof:

for some x' normalize units so that $\bar{v}(x') = 1$. Then $p(z, x') = \Phi(z)$, which reveals the distribution of F, Φ, and from this get the other values of v. Done!

(Matzkin considers broader class of v's that are h.d.1 in some subset of x.)

Or, can also make conditional quantile (median) restrictions on the distribution of F can also reveal $v(x)$ up to units

Bresnahan and Reiss

The B & R model is a simple extension of the Monopoly model with identical potential entrants. Variable profits decline in the number of firms, y: $v_y(x)$.
With an i.i.d. F, we have

$$\begin{aligned} Pr(y=0|x) &= 1-\Phi(v_1(x)) \\ Pr(y=1|x) &= \Phi(v_1(x))-\Phi(v_2(x)) \\ Pr(y=2|x) &= \Phi(v_2(x))-\Phi(v_3(x)) \\ &\cdots \end{aligned} \tag{4}$$

B & R estimate by fully parametric MLE and ask: how fast does v decline in y? Question is: the "nature of competition."

The Economic Question in B & R

What is the value of $\frac{v_2(x)}{v_1(x)}$?
Think of benchmark models:

1. Fixed Prices: $\frac{v_2(x)}{v_1(x)} = 1/2$.
2. Cournot Competition: $\frac{v_2(x)}{v_1(x)} \in (0, 1/2)$?
3. Homogeneous Goods Bertrand: $\frac{v_2(x)}{v_1(x)} = 0$?

Can think of similar ratios for other y.
But these ratios are not robust to monotonic transformations, and so the economic parameter of interest is not non-parametrically identified in B & R without Matzkin-style shape restrictions.
In fact, there is a monotonic transformation that sets the ratio to anything between 0 and 1.

B & R use the Shape Restriction in Population

We can write the *B&R* model as series of threshold-crossing models:

$$\Pr(y \geq 1|x) = \Phi(v_1(x))$$

...

$$\Pr(y \geq n|x) = \Phi(v_n(x))$$

and so restricting variable profits to be proportional to population **will identify** v_y (and indeed over-identify Φ, so that we could allow $\Phi_y(F)$ as with perhaps upward-sloping supply of the fixed asset.)

Lessons from the B & R Model

Without qualitative shape restrictions, the object of interest (the "nature of competition") cannot be identified, but with one natural (though restrictive) shape assumption, the nature of competition is fully identified.

Here, the binary threshold crossing literature is enough, but it will not enough be in more complicated models.

A Model with Heterogenous Firms

The simplest two-firm entry problem:

$$\pi_y(x_{im}, F_{im}) \quad = \quad v_y(x_{im}) - F_{im}$$

A firm enters in equilibrium if $\pi_{im}(x_{im}, y_j, F_{im}) \geq 0$.
Start with pure strategies:

$$\begin{aligned} \Pr(0,0|x) &= Pr(F_1 > v_1(x_1), F_2 > v_1(x_2)) \\ \Pr(1,1|x) &= Pr(F_1 < v_2(x_1), F_2 < v_2(x_2)) \end{aligned}$$

Multiple equilibria in outcomes (0,1) and (1,0).

Identification in the Heterogeneous Firm Model

Fairly easy if you assume a parametric form for v (see, e.g., Tamer). Or think of assuming Φ. Both of these solve the "monotonic transformation" problem, but require a priori knowledge of parametric forms. (Might be more reasonble to learn v from other data.)

What to do about non-parametrics?

Can think of Heckman-style identification "at the limit" arugments;

How to Extend to Firm Heterogeneity

Strongest stylized fact about firms is that they are different (symmetry is a bad assumption.)

Start with hetrogeneity in Fixed Costs. Try to make inferences about how firm-specific variables affect at least the intercept of the profit function.

$$\pi(N, x_m, z_f, \epsilon_{mf}, \theta) = V(N, x_m, \theta) - FC(z_f, \epsilon_{mf})$$

Problem now: ordered probit doesn't work. What is Nash Equilibrium? How to estimate?

Multiple Equilibria

In the two firm case, there are multiple equilibria in the outcomes $(0, 1)$ and $(1, 0)$ and this gets worse with more potential firms.

Multiple Equilibria

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Solutions include

- Simplify to get uniqess (Berry '92 [2] assumes symmetric firms, get unique N.)
- Assume private information (smooths problem and helps, maybe not enough)
- Estimating the probability of each equilibrium (Tamer) – hard with many firms.
- using only the necessary conditions for equilibrium (Ciliberto and Tamer; Andrews, Berry and Jia).

Example

Mazzeo '02 [8] models the quality of hotels near highway exits. Finding: firms want to differentiate in quality space.

Order of moves

Mazzeo's solution to the mult. equilibria problem is to assume an order of moves. This makes the unique equlibrium outcome relatively easy to compute.

Ex-post regret

Conditional on competitor's decisions, this solution means that some firms would like to move away from equilibrium – odd for a "once-and-for-all" equilibrium.

Example: Seim

Seim '02 [10] models video store location across city districts. Once again, "competition matters".

Private Info

Seim assumption of purely private profit shocks mitigates the mult. equilibria problem.

Ex-post regret

After observing rival's decisions, firms would often like to move – perhaps better in a dynamic model,

Example with Necessary Conditions

Consider a case with two potential entrants and profits that vary across the two potential entrants (i.e. Walmart and Kmart.)

The necessary condition for equilibria is that a firm enters if it is profitable to so given the rival's action.

The joint probability of the necessary conditions is $\geq$ the probability of the observed event (and equal when there are no multiple equilibria)

Necessary Conditions in the 2 Firm Case

Let μ_j be the probability that firm 1 would be profitable in a monopoly andn δ_j be the probability that firm 2 would be profitable in a duopoly.

Event	Prob of Nec. Cond		True Prob
(0,0)	$(1-\mu_1)(1-\mu_2)$	$=$	$Pr(0,0)$
(0,1)	$\mu_2(1-\delta_1)$	$\geq$	$Pr(0,1)$
(1,0)	$\mu_1(1-\delta_2)$	$\geq$	$Pr(1,0)$
(1,1)	$\delta_1\delta_2$	$=$	$Pr(1,1)$

Here, the problem is fundamentally not point identified (unless one knows the eaquilibrium selection rule *a priori*.) But, the event probabilities still restrict the parameters to a subset of the original probability space (sometimes fairly small).

Necessary Conditions

Ciliberto and Tamer '06, Andrews-Berry-Jia '05, Pakes, Porter, Ho and Ichii, '06 all consider estimation from necessary conditions for equilibrium.

Prob. of Necessary Cond.

The probability of the necessary condition for Nash equilibrium y^* is

$$Pr\left(\epsilon_j : \pi(y_j^*, y_{-j}^*, \theta) + y_j^* \epsilon_j \geq \pi(y', y_{-j}, \theta) + y' \epsilon_j, \forall y'\right)$$

Bounds

A key insight is that

$$Pr(Nec.Cond) \geq Pr(Equilibrium)$$

Bounding Parameters

This condition is often enough to bound parameters, if not point-identify them.

Confidence regions are tough, but see Chernozukov-Hong-Tamer, etc.

Next: Simple Dynamics

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But i.i.d. shock is very strong – no serial uncorrelation in firm-specific state.

Hotz-Miller Style Models

As in Hotz-Miller '93, consider the classic single-agent dynamic discrete choice problem, defined by the Bellman equation:

$$v(x, \epsilon) = \max_{a \in A} \left[\pi_a(x) + \epsilon_a + \beta \int \sum_{x' \in X} v(x, \epsilon') f_a(x' \mid x) g(\epsilon') d\epsilon' \right].$$

This is a standard model where x is the discrete state vector, A is the discrete set of actions (a) and $f_a(x' \mid x)$ defines the transition matrix as a function of the prior state and the chosen action. There is an independent (over time and actors) vector of linear shocks, one for each action at each time: $\epsilon = (\epsilon_1, \epsilon_2, \ldots, \epsilon_{K_a})$. The ϵ's have the distribution $G(\cdot)$.

Data and Shocks

Assume that we (the econometricians) know, *a priori*, the distribution of the ϵ's (G) and the discount factor β, but that we do not observe the shocks, ϵ. Our data are a large sample of draws on states and actions, (x, a) (the draws are independent across both time and agents, so this could be a time-series or cross-section or panel data.)

Estimate directly from data [i] the transition matrix and [ii] the policy function. Then "invert" Hotz-Miller style for the single-period profit function.

Identification comes from Magnac-Thesmar '02 [7].

Sketch of Identification / Estimation

The $2K_x$ unknowns are:

1. the single-period returns: $\pi_a(x)$
2. the value functions: $v(x)$

Given v, the choice probabilities are just "logits". Can "invert" to uncover the deterministic return (just like the diff. products δ!)

Solve (Magnac-Thesmar) for unkowns from the $2K_x$ equations:

1. The choice probabilities
2. the value functions: $v(x)$

Extension to Oligopoly

With i.i.d. private shocks, current actions of rivals are not correlated with current shocks to own-profits.

The states (and so the transition matrix) now include both "nature" and "rivals", but if the same equilibrium is always played at the same states, then the Hotz-Miller Magnac-Thesmar method goes through directly. With i.i.d. private info "rival" is like "nature".
See: Aguiraberia; Schmdit-Dengler and Pesendorfer.

Bajari, Benkard and Levin

Still estimate policy function non-parametrically from data. Then, given profit function parameters can "forward simulate" future paths of industry.
Now, have one firm play some deviation from the estimated (optimal) policy function: this must decrease profits. This puts a restriction on the profit function parameters.

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Identification is an open question here – with discrete deviations, maybe set identified?

Pakes-Ostrovsky-Berry

- Hotz-Miller methods assume a distribution (normal?) for the i.i.d. shocks.
- POB argue it is more reasonable to estimate variable profits from data on price, qty, etc.
- Now use entry/exit data to estimate only the fixed cost distribution.
- Berry-Tamer show that FC distribution is non-parameterically identified.

Example: Roberts, et al looks at entry of dentists, etc., into small towns.

Empirical literature on endogenous product variety is not as methodologically well-developed as the literature on demand estimation.

However, much progess in the form of advances in dealing with

- multiple equilibrium
- identification
- dynamics

From a policy perspective, questions about product variety seem as important as quesitons seem as important as questions about pricing.

Next time: "policy applications"
Examples:

- The automobile industry: collusion, mergers, trade and environmental policy.
- "Non-traditional" (for IO) industries:
  - Schools
  - Health
- Extended Example: Competition and Welfare in U.S. Radio Market.

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