Lecture III Outline: Dynamic Games with Hidden Information

1. Games with "Wasteful" Transfers
   (a) Theme: Pooling and Rules rather than Discretion
   (b) Collusion and Price Rigidity (Athey, Bagwell, and Sanchirico, 2004)
   (c) Monetary Policy (Athey, Atkeson, and Kehoe, 2005)
2. Games with Asymmetric Equilibria
   (a) Collusion (Athey and Bagwell, 2001)
   (b) Trading Favors (Hopenhayn, Abdulkadiroglu. & Bagwell)
3. Persistent types (Athey and Bagwell, 2004)

Motivation

- Ongoing Relationships
  - Time-varying individual costs and benefits to acting, i.i.d. private information
  - Restrictions on monetary transfers
- Examples
  - Colluding firms, i.i.d. cost/inventory shocks
  - Public good provision
    - ∗ Families/villages — Organizations
    - ∗ Legislatures — Academic departments
  - Policy games (government is privately informed)

- Questions about Collusion
  - Response of collusive behavior to institutional setting
  - Effects of anti-trust policy (Restrictions on communication, side-payments)
  - Market design: info. about indiv. bids and identities
  - Institutional design: industry assoc., smoke-filled rooms
- Central Tradeoffs
  - Productive efficiency requires low-cost firm serves market
  - Firms like market-share, incentive to mimic low-cost firm
  - Need low prices or future "punishment" with high market-share
  - Future price wars v. "future market-share favors"

The Game

- Stage Game
  - I firms produce perfect substitutes
  - Unit mass of consumers, reservation price r
  - No transfers among firms
  - Cost: $\theta_i \tilde{} F(\cdot)$, support $[\underline{\theta}, \overline{\theta}]$, density $f > 0$
  - Per-period profit from serving market at price p_i: $p_i - \theta_i$
  - Bertrand market-share allocation function $m_i(\mathbf{p})$:
    - $m_i(\mathbf{p}) = 1$ if $p_i < \min_{j \neq i} p_j$, $m_i(\mathbf{p}) = 0$ if $p_i > \min_{j \neq i} p_j$, and $m_i(p) = 1/L$ if there are $L - 1$ other firms that tie firm i for the lowest price.

- Stage Game strategies and interim profits
  - Pricing strategy: $\rho_i(\theta_i)$
  - Interim profits: $\bar{\pi}(p_i, \theta_i; \rho_{-i}) \equiv (p_i - \theta_i)\, E_{\boldsymbol{\theta}_{-i}}[m_i\left(p_i, \boldsymbol{\rho}_{-i}(\boldsymbol{\theta}_{-i})\right)]$
  - Define $\Pi(\hat{\theta}_{i,t}, \theta_{i,t}; \boldsymbol{\rho}) = \bar{\pi}(\rho_i(\hat{\theta}_{i,t}), \theta_i; \boldsymbol{\rho}_{-i})$
  - Single crossing property.
- The Repeated Game
  - Repeat stage game over infinite horizon
  - Environment is stationary, i.i.d. types

Strongly Symmetric v. Asymmetric Equilibrium

- Strongly Symmetric (SPPE)
  - At the beginning of each period, before learning types, all firms treated symmetrically
  - Either all in "price war," or all in "cooperative phase," etc.
  - Requires V^s, the SPPE equilibrium set, is a 45 degree line. Just one Pareto optimal point, and any "punishment" hurts all firms equally.
  - Punishments wasteful, transfer surplus from cartel to consumers.
  - Motivation
    - ∗ Observability: suppose market price observable, but not individual market shares, or only winning bid in auction

* Ability to track individual firms: firms may enter and exit or change names in "anonymous" markets
* Simplicity: may be difficult to single out firms when there are many

- Asymmetric (PPE)
  - Individual firms can be singled out for rewards and punishments
  - I win today, I lose market share tomorrow
  - Allows V^*, the equilibrium set, to (in principle) have frontier with slope -1.
    - Transfers of utility from one firm to another are theoretically possible, and may not be wasteful.
  - Plausible with small numbers of individuals who can track one another over time

"Collusion and Price Rigidity"

- Athey, Bagwell, and Sanchirico (2004)
- Focus on SPPE
- Focus on Inelastic demand: $\pi(p_i, \theta_i) \equiv p_i - \theta_i$
  - Some, but not all, results extend to downward-sloping demand

The Recursive Mechanism

- Stage Mechanism
  - No communication (show doesn't help)
  - Pricing function for each player: $\rho : \Theta_i \to \mathbf{R}^+$
  - Continuation value function $w : \Theta_i \to \mathbf{R}^I$.
  - Let $\gamma = (\rho, w)$

  Ex post utility: $u_i(\hat{\theta}_t, \theta_{i,t}; \gamma) = \left(\rho_i(\hat{\theta}_{i,t}) - \theta_{i,t}\right) \cdot m_i(\boldsymbol{\rho}(\hat{\theta}_t)) + \delta w_i(\hat{\theta}_t)$

  Interim utility: $\bar{u}_i(\hat{\theta}_{i,t}, \theta_{i,t}; \gamma) = \Pi(\hat{\theta}_{i,t}, \theta_{i,t}; \boldsymbol{\rho}) + \delta \mathbb{E}_{\tilde{\theta}_{-i,t}}[w_i(\hat{\theta}_{i,t}, \tilde{\theta}_{-i,t})]$
- Recursive Mechanism: $\langle V, \{\gamma(v)\}_{v \in V}, v_0 \rangle$

Constraints

- (Bayesian, Interim) IC:

$$\bar{u}_i(\theta_{i,t}, \theta_{i,t}; \gamma) \geq \bar{u}_i(\hat{\theta}_{i,t}, \theta_{i,t}; \gamma) \text{ for all } \hat{\theta}_{i,t} \in \Theta_{i,t}$$

- IR(π^{NE})

  – Static Nash has payoffs π^{NE}.s

$$\bar{u}_i(\theta_{i,t}, \theta_{i,t}; \gamma) \geq \sup_{p_{i,t}} \left(\bar{\pi}(p_{i,t}, \theta_i; \boldsymbol{\rho}_{-i}) \right) + \frac{\delta}{1-\delta} \pi^{NE}.$$

SPPE: Alternative Pricing Schemes that Satisfy Truth-Telling

- Many possible pricing-continuation value schemes satisfy IC:
  - Rigid-price scheme: $\rho_i(\theta_i) \equiv r$
  - Static Nash equilibrium: $\rho^{NE}(\theta_i)$, strictly increasing, $\rho^{NE}(\bar{\theta}) = \bar{\theta}$
  - 1-step schemes with two prices, r and p' and a cutoff, θ', where

$$\rho_i(\theta_i) = \begin{cases} p' & \theta_i < \theta'_i \\ r & \theta_i \geq \theta'_i \end{cases}.$$

If we always return to the same continuation equilibrium irrespective of price, IC-On requires

$$(p' - \theta'_i)\bar{m}\left(p_i; \boldsymbol{\rho}_{-i}\right) = (r - \theta'_i)\bar{m}\left(r; \boldsymbol{\rho}_{-i}\right).$$

For example, with two firms,

$$(p' - \theta'_i)\left(1 - \frac{1}{2}F(\theta'_i)\right) = (r - \theta'_i)\frac{1}{2}\left(1 - F(\theta'_i)\right).$$

This doesn't require future punishments.

– 1-step pricing scheme with $p'_i > p$ and $\bar{w}_i(\underline{\theta}_i) < \sup V_s$. That is, some punishment following price of p', none following price of r. In the 2-firm case, IC-On requires

$$(p' - \theta'_i)\left(1 - \frac{1}{2}F(\theta')\right) + \delta\bar{w}_i(\underline{\theta}_i) = (r - \theta')\frac{1}{2}\left(1 - F(\theta')\right) + \delta\bar{w}_i(\bar{\theta}_i)$$

The Optimal Collusive SPPE

The Mechanism Design Program: Choose symmetric price schedule ρ and a continuation value function w_i to maximize

$$\mathbb{E}[\Pi(\hat{\theta}_{i,t}, \tilde{\theta}_{i,t}; \rho) + w_i(\tilde{\theta}_{i,t})]$$

$$\text{subject to : For all } \theta_i,\ w_i(\theta_i) \leq \bar{w};$$

$$\text{(IC-onM) } \forall \hat{\theta}_i, \theta_i,\ \Pi(\theta_i, \theta_i; \rho) + w_i(\theta_i) \geq \Pi(\hat{\theta}_i, \theta_i; \rho) + w_i(\hat{\theta}_i).$$

- Note: symmetry implies can maximize utility of representative firm

Consequences of On-Schedule Incentive Compatibility

- Define

$$M(\theta_i;\rho) = \mathbb{E}_{\tilde{\boldsymbol{\theta}}_{-i}}[m_i\left(\rho(\tilde{\theta}_i), \boldsymbol{\rho}_{-i}\left(\tilde{\boldsymbol{\theta}}_{-i}\right)\right)].$$

Lemma 8 *(Envelope Theorem)* *(ρ, w_i) satisfies (IC-onM) if and only if (ρ, w_i) also satisfies:*
(i). $\rho(\theta_i)$ is weakly increasing, and
(ii).

$$\begin{aligned}\Pi(\theta_i,\theta_i;\rho) - \psi_i(\theta_i) &= \Pi(\overline{\theta}_i,\overline{\theta}_i;\rho) + w_i(\overline{\theta}_i) - \int_{\theta_i}^{\overline{\theta}_i} \Pi_\theta(t,t;\rho)dt \\ &= \Pi(\overline{\theta}_i,\overline{\theta}_i;\rho) + w_i(\overline{\theta}_i) + \int_{\theta_i}^{\overline{\theta}_i} M(t;\rho)dt\end{aligned}$$

.

Consequences of On-Schedule Incentive Compatibility

Lemma 9 ***(Revenue Equivalence Theorem)*** *Consider any* (ρ, w_i) *which satisfies (IC-onM). Then any other* (ρ, W_i) *which satisfies (IC-onM),* $M(\theta_i;\rho) = M(\theta_i;\rho)$ *and* $\Pi(\bar{\theta}_i, \bar{\theta}_i;\rho) + w_i(\bar{\theta}_i) = \Pi(\bar{\theta}_i, \bar{\theta}_i;\rho) + w_i(\bar{\theta}_i)$ *must also satisfy* $\Pi(\theta_i, \theta_i;\rho) + w_i(\theta_i) = \Pi(\theta_i, \theta_i;\rho) + w_i(\theta_i)$ *for all* θ_i.

Corollary 10 *A PPE that is fully sorting in each period has payoffs equal to the repeated play of the static Nash.*

No Wars on the Equilibrium Path

Proposition 11 *If* (ρ^*, w_i^*) *is a solution to the Mechanism Design Program, then there exists as well a solution* $(\widetilde{\rho}, \widetilde{w}_i)$ *with* $\widetilde{\rho}(\theta_i) \leq \rho(\theta_i)$ *and* $\widetilde{w}_i(\theta_i) \equiv \bar{w}$*. Thus, there exists an optimal SPPE that is stationary.*

Proof: Low prices today and punishments tomorrow are substitute instruments. Construct new pricing scheme by raising w_i and lowering ρ at each step to respect incentive compatibility.

- Note: In games with transfers we got stationarity because transfers were substitute for continuation values. Here, the result is that low prices (which burn money) are a substitute for wasteful continuation values.
- Prices interact with market share but not with true type.

Optimal Pricing

Proposition 12 *For δ sufficiently large:*
(i) If either (a) F is log-concave, or (b) r is sufficiently large, then the optimal SPPE is characterized by price rigidity ($\rho^*(\theta_i) \equiv r$) and stationarity.
(ii) In any optimal SPPE, there exists an open interval of cost types where pricing is rigid, and per-period profits above the static Nash equilibrium are attained: $\overline{\mathcal{V}}_s > \pi^{NE}/(1-\delta)$.

Proof that optimum is rigid pricing (pooling):

$$\mathbb{E}[\Pi(\tilde{\theta}_i, \tilde{\theta}_i; \rho)] = \mathbb{E}[(\rho(\overline{\theta}) - \overline{\theta}) \cdot M(\overline{\theta}; \rho) + \int_{\theta_i}^{\overline{\theta}} M(t; \rho) dt].$$

Integrate by parts and rewrite our objective function as

$$\mathbb{E}[\Pi(\tilde{\theta}_i, \tilde{\theta}_i; \rho)] = \mathbb{E}\left[(\rho(\overline{\theta}) - \overline{\theta}) \cdot M(\overline{\theta}; \rho) + \frac{F}{f}(\tilde{\theta}_i) \cdot M(\tilde{\theta}_i; \rho)\right]. \qquad (7)$$

$\mathbb{E}[M(\tilde{\theta}_i; \rho)] = 1/I$. Thus, $I \cdot M(\theta_i; \rho) \cdot f(\theta_i)$ is a probability density. If $M(\theta_i; \rho) = 1/I$ for all θ_i (a rigid pricing policy), $I \cdot M(\theta_i; \rho) \cdot f(\theta_i) = f(\theta_i)$. For any $M(\theta_i; \rho)$ that is non-increasing (corresponding to any ρ nondecreasing), $f(\theta_i)$ dominates $I \cdot M(\theta_i; \rho) \cdot f(\theta_i)$ by FOSD. Since $\frac{F}{f}(\theta_i)$ is nondecreasing, expected profits are improved by a FOSD shift in the distribution, and $M(\theta_i; \rho) = 1/I$ must be optimal.

Off-Schedule Constraints

- What is worst punishment?
  - Following similar logic to optimal pricing program, scheme that minimizes informational rent is strictly increasing when F is log-concave.
  - On-schedule constraint plus "adding up" imply that:

$$(\rho(\bar{\theta}) - \bar{\theta})M(\bar{\theta};\rho) + \delta\overline{w}(\bar{\theta}) + \mathbb{E}\left[\frac{F}{f}(\tilde{\theta}_i)M(\tilde{\theta}_i;\rho)\right] = \underline{V}_s$$

  - Off-schedule (IR) constraint implies that:

$$(\rho(\bar{\theta}) - \bar{\theta})M(\bar{\theta};\rho) + \delta\overline{w}(\bar{\theta}) \geq \delta\underline{V}_s$$

  - Substitute: $\mathbb{E}[\frac{F}{f}(\tilde{\theta}_i)M(\tilde{\theta}_i;\rho)] \leq (1-\delta)\underline{V}_s$. This is minimized with $M(\theta_i;\rho)$ strictly decreasing since $\frac{F}{f}$ is nondecreasing.

Proposition 13 *If F is log-concave, then for all discount factors δ, $\underline{V}_s = \pi^{NE}/(1-\delta)$.*

- – Intuition: you can't take away firms' information rents.

"Escape Clause"

- What if $\delta < \delta^*$?
  - Show in paper: optimal pricing scheme is step function
  - Relative to rigidity, type most tempted to cheat is low-cost type
  - Allow an "escape valve" if low-cost type is drawn
  - Must be incentive-compatible: no one wants to mimic this type!
  - Issue: does lowering price for some types reduce future benefit to cooperation more than it reduces incentive to cheat today?
  - Answer: not if density is small enough for low type.

Extension: Observable Fluctuations in Demand

- Example
  - Distribution $F(\theta_i; \mu, z)$, mean is μ, support is parameterized by z, so that $\underline{\theta} = \mu - z$ and $\overline{\theta} = \mu + z$. Suppose $F(\theta_i; \mu, z)$ is log-concave.
    - Example: "triangle" distribution, density $f({}_i\theta; z)$ is symmetric about μ, and $f(\theta; z) = \frac{1}{z^2}(\theta - (\mu - z))$ on $[\mu - z, z]$.
  - When z is small, can support rigidity, not when z is large
  - Optimal to have two-step scheme for large z for some δ
- Application: price variability and inflation (Simon, 1999)

Conclusions for SPPE

1. Optimal symmetric collusion can be achieved without equilibrium-path price wars.
2. If firms are sufficiently patient and the distribution of costs is log-concave, optimal symmetric collusion is characterized by price rigidity.
3. If firms are less patient, optimal symmetric collusion may be characterized by price rigidity over intervals of costs (a step function), where the price of a lower-cost firms is distorted downward to diminish the incentive that such a firm has to cheat.
4. If firms are less patient and the model is modified to include i.i.d. public demand shocks, under optimal symmetric collusion, the downward pricing distortion that accompanies a firm's lower-cost realiza-

tion may occur only when current demand is high.

5. Overall: symmetry implies that informational costs outweigh efficiency benefits, since "transfers" are wasteful for colluding firms

Broader Theme: Rules Versus Discretion

- Wasteful instruments for transferring utility: sacrifice productive efficiency
  - Cross-type externalities: providing incentives isn't worth it
- Other applications
  - "Policy Games": single informed agent
    - ∗ Monetary policy
  - Parent-child

Monetary Policy

- The problem
  - Central bank maximizes social welfare
  - Privately informed about benefits of inflation θ_t
  - Short-term incentive for "surprise" inflation
- Special case of social welfare function:

$$S(p_t, g_t, \theta_t) = -\frac{1}{2}\left[(L + p_t - g_t)^2 + (g_t - \theta_t)^2\right], \qquad (8)$$

where L is unemployment, p_t is wage growth, g_t is money growth rate (inflation), θ_t is private information.

- Constraint on central bank: wage growth equals expected inflation

$$p_t = \mathbb{E}[g_t(\tilde{\theta}_t)]$$

  - Externalities across states of the world
  - Analogous to market shares –more to one type, less to another for a fixed level of inflation
- Use a recursive mechanism
  - Only incentive instrument: reduced continuation values tomorrow

$$u(\hat{\theta}_t, \theta_t; g, w) = S(p_t, g(\hat{\theta}_t), \theta_t) + \delta w(\hat{\theta}_t)$$

- Optimal recursive mechanism:

$$\max_{g} \mathbb{E}[S(p_t, g(\tilde{\theta}_t), \tilde{\theta}_t) + \delta w(\tilde{\theta}_t)]$$

s.t. IC, $w(\hat{\theta}_t) \leq \bar{w}$, $p_t = \mathbb{E}[g_t(\tilde{\theta}_t)]$.

- Envelope theorem:

$$\begin{aligned} &\mathbb{E}[S(p_t, g(\tilde{\theta}_t), \tilde{\theta}_t) + \delta w(\tilde{\theta}_t)] \\ = &\ S(p_t, g(\underline{\theta}_t), \underline{\theta}_t) + \delta w(\underline{\theta}_t) + \mathbb{E}\left[\frac{1 - F(\tilde{\theta}_t)}{f(\tilde{\theta}_t)}(g(\tilde{\theta}_t) - \tilde{\theta}_t)\right] \end{aligned}$$

- Decreasing hazard rate implies we can improve second term by keeping expected inflation constant, but "flattening" inflation schedule (which must be nondecreasing by SCP).

- If $S(p_t, g, \underline{\theta}_t)$ is increasing in g at best rigid rule, then use rules, otherwise, use an "inflation cap."

Monetary Policy: Conclusions

- Only way to provide incentives is through wasteful transfers
- Cross-type externalities: through incentive constraints, aggregate wage growth
- If log-concave, then optimal policy is rules (inflation cap), not discretion
- If log-concavity fails, some discretion is optimal
- More general welfare function requires more subtle proof

Asymmetric Collusion: A Modified Stage Game

- Setup
  - 2 firms produce perfect substitutes
  - Unit mass of consumers, reservation price r
  - 2 cost types: $\theta^i \in \{\theta_L, \theta_H\}$, $\Pr(\theta^i = \theta_j) = \eta_j$.
    Case: $\eta_L > 1/2$.
- Firms...
  - may split the market unevenly; details not imp't.
  - may not charge different prices to different consumers.
  - communicate prior to producing (see Athey and Bagwell (2001) for analysis of communication)

Summary of Ideas for Asymmetric Eq'a

- A first best scheme, always price at r
  - Eqm described by two "states"
  - Each period, announce types
  - State x: low cost firm serves market, but firm 2 serves most of market if firms have same cost
    - If (H, L), switch to state y, oth. return to x
  - State y: low cost firm serves market, but firm 1 serves most of market if firms have same cost
    - If (L, H), switch to state x, oth. return to y
- Paper: shows that first-best scheme can work if patient enough that diff. betw. x and y provides suff. incentives; if less patient shows similar schemes with partial prod. eff. are optimal.

A Linear Self-Generating Set with First-Best Profits

- Goal: Compute a critical discount factor above which first-best profits can be attained in every period.
  - Requires linear, "self-generating" set with slope -1 :

$$[(x, y), (y, x)]$$

  - Two parts.
  - "Adding Up": First, ignore IC-Off. Is it possible to have linear self-generating set with full efficiency?
    - Need to implement (x, y) using $v_{jk} \in [(x, y), (y, x)]$.

  * Future looks brighter than today for firm 1, and enough brighter when firm 1 has high cost to satisfy IC-On.
  * Does it all "add up"?
- Second, when are IC-Off's cleared?.

Proposition 14 *Suppose that* $r - \theta_H < \theta_H - \theta_L$. *Then, for all* $\delta \in (\delta^{FB}, 1]$, *there exist values* $y > x > 0$ *such that* $x + y = 2\pi^{FB}/(1-\delta)$, *and the line segment* $[(x, y), (y, x)]$ *is "self-generating" and in the set of PPE values,* V^*.

A Linear Portion of the Pareto Frontier

- For less patient firms: There is a linear portion of pareto frontier
  - Allows the "free" transfer of utility.
  - Implies optimal equilibria always non-stationary, since can use these free transfers to provide incentives for productive efficiency
  - Thus, even if you had access to monetary transfers (bribes), would not need to use them, at least not small ones

Persistent Types

- Two-period sophisticated rotation
  - Produce today, give up market share tomorrow
  - Not very effective with persistent types
- First-best example
  - Extends to persistent types
  - Keep track of beliefs as state variables
- As persistence grows relative to patience, rigid pricing approximately optimal with log-concavity
  - Cannot do efficient transfers, so pooling is optimal

- Rigid pricing not optimal if log-concavity fails
  - Static solution not equilibrium in dynamic game
  - Negotiate over market shares
  - Price war in initial period to signal types
  - Low cost type gets higher market share, high cost gets just enough to keep from deviating

Summary

- Legalized Cartel with Transfers
  - Stationary, productive efficiency, high prices
- Optimal Asymmetric PPE: "Market-Share Favors"
  - Non-stationary, high prices, efficient except when implementing asymmetric equilibria.
  - Communication (see paper): allows asymmetric market-share favors, but tightens incentive to deviate off-schedule.
  - Bribes: never replace market-share favors completely, but increase efficiency for firms of moderate patience
  - Works with independent or not-too-persistent types

- Price Rigidity
  - Stationary, productive inefficiency, high prices
  - Optimal if can only see market price, no individual rewards, i.i.d.
  - Optimal if costs perfectly persistent
- Static NE
  - Stationary, productive efficiency, price variability, low prices.

Broader Theme: Rules Versus Discretion

- Wasteful instruments for transferring utility: sacrifice productive efficiency
  - Cross-type externalities: providing incentives isn't worth it
- When efficient means exist for transferring utility, firms use them by trading favors over time

Conclusions of Lectures

- Bring together mechanism design and dynamic programming to analyze repeated and dynamic games
- Apply tools from static literature
- Generalize to incorporate interesting dynamics
  - Serial correlation
  - Learning-by-doing
  - Maintaining budget account
- Efficiency possible in wide range of circumstances
- Pooling is optimal for agents when limited instruments for providing incentives