Lecture II

Dynamic Contract Theory: Efficiency in Bilateral Trade

1. Static Results
   (a) Team mechanism
   (b) Arrow/AGV/CAGV
   (c) IR & Myerson-Satterthwaite
2. Athey-Miller: i.i.d. Repetition helps
   (a) optimizing IR constraints
3. Athey-Segal: An Efficient Dynamic Mechanism
   (a) Team mechanisms
   (b) Budget balance

(c) IR constraints

4. Athey-Miller: A ex post incentive compatible dynamic mechanism with bounded budget account

Motivation: Bilateral Trade

- Two players meet in the desert *each day*
  - One of them *always* has a canteen of water; they each have some cash
  - Each one's thirst *each day* is private information
- Interpretation: A long-term vertical relationship
  - Every period a widget is traded; it can be standard or customized
  - Downstream buyer has stochastic valuation for the customization
  - Upstream seller has stochastic cost of customizing
  - Customization is not contractible ex ante
- Dynamics: serial correlation, learning-by-doing

A Model of Bilateral Trade

- Players $i \in \{b, s\}$ buyer, seller
- Time $t = 1, .., T$ (special cases: $T = 1, T = \infty$)
- Superscript/subscript notation: given $((y_{i,t})_{t=1}^{T})_{i=1}^{I}$,
$$y_t = (y_{i,t})_{i=1}^{I}, \quad y_i = (y_{i,t})_{t=1}^{T}, \; y^t = (y_{t'})_{t'=1}^{t}.$$
- Type spaces $\Theta_{i,t} \subseteq \mathbb{R}_+$, random variables $\tilde{\theta}_{i,t}$ with realizations $\theta_{i,t}$.
- Reports: $\hat{\theta}_{i,t} \in \Theta_{i,t}$
- Decisions $X_t = \{(0,1),(1,0)\}$.
- Transfer from player j to player i :
$y_{j,i,t} \geq 0$, let $y_{i,t} = \sum_j y_{j,i,t} - y_{i,j,t}$.

- Timeline in period t:
  - Types realized (θ_t)
    - History potentially affects distributions: $F_t(\theta_t; x^{t-1}, \theta^{t-1})$.
    - Types independent conditional on history
  - Players communicate $(\hat{\theta}_{i,t})$
  - Players simultaneously make decisions (x_t, seller chooses) and send transfers (y_t)
- Consider both formal mechanisms with commitment and decentralized game
  - Doesn't matter for most results

Mechanism and Strategies

- i.i.d. case: Recursive Mechanism: $\langle V, \{\gamma(v)\}_{v \in V}, v_0 \rangle$
  - $\gamma(v)$: allocation χ, transfer ψ, continuation value function w
- Allocation: $(\chi_{b,t}, \chi_{s,t}) : \Theta^t \rightarrow \{(0,1), (1,0)\}$
  - $\chi_b^*(\theta_t) = 1\{\theta_{b,t} > \theta_{s,t}\}$
- Transfers: $\psi_s, \psi_b : \Theta^t \rightarrow \mathbb{R}$
- Stationary if $w(\theta_t) = v_0$ for all θ_t.

Team Mechanism

- Transfers

$$\psi_{b,t}(\theta_t) = \theta_{s,t}\chi_s^*(\theta_t) + h_b(\theta_{s,t}),\;\; \psi_{s,t}(\theta_t) = \theta_{b,t}\chi_b^*(\theta_t) + h_s(\theta_{b,t})$$

- Each player internalizes externality on the other
- Ex post incentive compatible
- Cannot achieve EPBB without disturbing incentives
  - Sum of transfers is

$$\theta_{s,t} + \left(\theta_{b,t} - \theta_{s,t}\right)\chi_b^*(\theta_t) + h_b(\theta_{s,t}) + h_s(\theta_{b,t})$$

  - $\chi_b^*(\theta_t)$ is not additively separable

Expected Externality Mechanism Balances the Budget

- Look for Interim IC, EPBB
- Arrow; d'Aspremont, Gerard-Varet

$$\psi_{b,t}(\theta_t, \theta_{t-1}) = -\psi_{s,t}(\theta_t, \theta_{t-1}) = \gamma_{bt}\left(\theta_{b,t}, \theta_{t-1}\right) - \gamma_{st}\left(\theta_{s,t}, \theta_{t-1}\right) + K, \text{ where}$$

$$\gamma_{i,t}\left(\theta_{i,t}, \theta_{t-1}\right) = \mathbb{E}_{t,t-1}\left[\tilde{\theta}_{-i,t}\chi^*_{-i}(\tilde{\theta}_t) | \theta_{i,t}, \theta_{-i,t-1}\right].$$

- With $\delta = 0$ or independent types, this provides correct incentives and EPBB
  - Will come to more general case later!
- See also Cremer, d'Aspremont, Gerard-Varet (2003) for correlated values case

Individual Rationality Constraints (Myerson-Satterthwaite, 1983)

Proposition 3 *If $\delta = 0$, there does not exist an efficient, IC, ex ante BB, interim IR mechanism.*

- Ex Post IC: Team mechanism with constant h_i's
  - Sum of transfers is

$$\theta_{s,t} + \left(\theta_{b,t} - \theta_{s,t}\right) \chi_b^*(\theta_t) + h_b(\theta_{s,t}) + h_s(\theta_{b,t})$$

- Envelope theorem implies that sum of interim EU determined by allocation up to constant
  - Team transfers are interim IC
  - Sum of player consumption utilities and transfers is

$$2\mathbb{E}\left[\tilde{\theta}_{s,t} + \left(\tilde{\theta}_{b,t} - \tilde{\theta}_{s,t}\right) \chi_b^*(\tilde{\theta}_t)\right] + \mathbb{E}\left[h_b(\tilde{\theta}_{s,t}) + h_s(\tilde{\theta}_{b,t})\right].$$

- Ex ante BB: Expected sum of consumption utilities and transfers, less sum of consumption utilities, equals zero.

$$\mathbb{E}\left[\tilde{\theta}_{s,t} + \left(\tilde{\theta}_{b,t} - \tilde{\theta}_{s,t}\right) \chi_b^*(\tilde{\theta}_t)\right] = -\mathbb{E}\left[h_b(\tilde{\theta}_{s,t}) + h_s(\tilde{\theta}_{b,t})\right].$$

- Interim IR: highest type seller and lowest type buyer will not pay

Supporting Efficiency with Serially Uncorrelated Types

Proposition 4 *There exists $\delta < 1$ and a stationary PPE that satisfies (i) interim IC, (ii) ex post BB, (iii) ex post IR, and (iv) efficiency.*

- Monetary transfers for interim IC and ex post BB:

$$\begin{aligned}\psi_{s,t}(\theta_t) = -\psi_{b,t}(\theta_t) = \\ \mathbb{E}[\tilde{\theta}_{b,t}\chi_b^*(\tilde{\theta}_{b,t}, \theta_{s,t})] - \mathbb{E}[\tilde{\theta}_{s,t}\chi_s^*(\theta_{b,t}, \tilde{\theta}_{s,t})] \\ - \tfrac{1}{2}\mathbb{E}\left[\left(\tilde{\theta}_{b,t} - \tilde{\theta}_{s,t}\right)\chi_b^*(\tilde{\theta}_t)\right]\end{aligned}$$

- Expected gains from trade shared equally.

$$w_s(\theta_t) = v_{0,s} = \frac{1}{1-\delta}\left(\tfrac{1}{2}\mathbb{E}\left[\left(\tilde{\theta}_{b,t} - \tilde{\theta}_{s,t}\right)\chi_b^*(\tilde{\theta}_t)\right] + \mathbb{E}[\theta_s]\right)$$

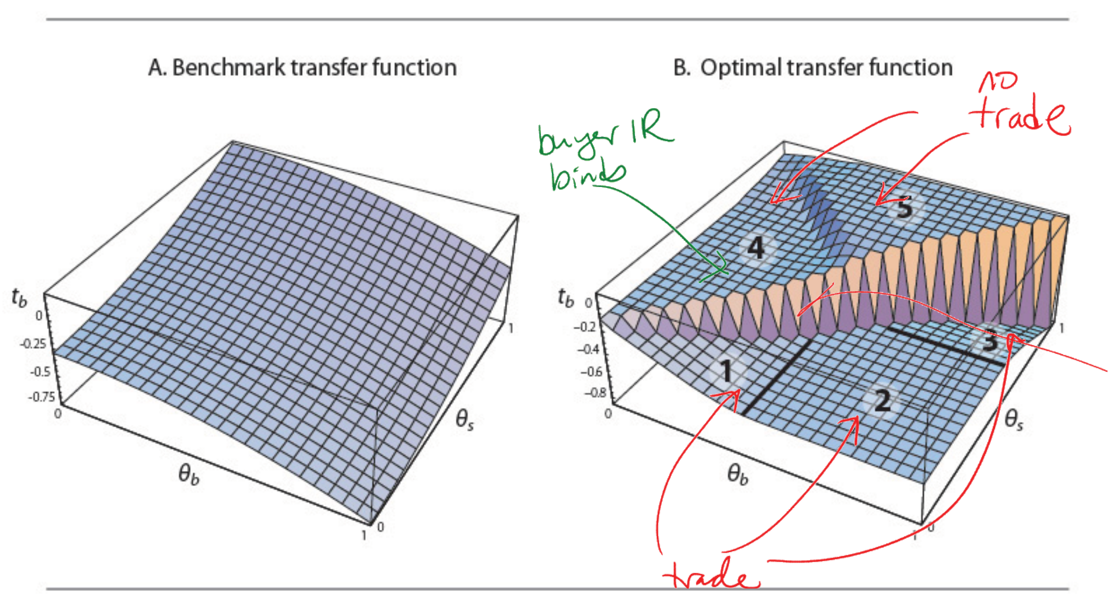
$$w_b(\theta_t) = v_{0,b} = \frac{1}{1-\delta}\tfrac{1}{2}\mathbb{E}\left[\left(\tilde{\theta}_{b,t} - \tilde{\theta}_{s,t}\right)\chi_b^*(\tilde{\theta}_t)\right]$$

- Verify interim IC, promise-keeping; solve for critical δ for ex post IR.

Making improvements

- How low can δ go?
- Split the surplus optimally, not equally (give more to seller):
  - Uniform: optimal split gives the seller $\frac{20}{21}$ of the gains from trade, for $\delta \geq \frac{6}{7}$
- Only interim expected transfers are pinned down by interim IC:
  - Employ *marginal preserving spreads* ("MPSs"; Athey & Levin, 2001) to:
    - Give add'l ex post surplus to types for whom ex post IR binds
    - Reduce ex post surplus of types for whom ex post IR is slack

Proposition 5 *The transfer function that minimizes δ is a solution to a linear program. For uniform, it achieves $\delta \gtrapprox 0.54$.*



Each graph displays the buyer's transfer function t_b on the vertical axis; the type space Θ forms the horizontal axes. The type space is discretized to a 25x25 lattice. Since t_b gives the transfer to the buyer, a lower value means a higher price for the object. The object is traded in the foreground portion of the figures (e.g., regions 1, 2, and 3 in Figure B).

A Dynamic Model

- Examples of Dynamics
  - Learning By Doing (see Athey and Segal (2006, 2007))
  - Serially Correlated Types (Focus): $F_t(\theta_t; \theta_{t-1})$.
    - $*$ Assume affiliated over time, 1st order Markov
- Strategies: $\alpha_{i,t} : \Theta^t \to \Theta_{i,t}$ specifies player i's reports in all periods t as functions of his true types and the opponents' reports.
  - Strategy is truthtelling if $\alpha_{i,t}(\theta^t) = \theta_{i,t}$ for all t and all θ^t.

Applying AGV Transfers: Problems

$$\psi_{b,t}(\theta_t, \theta_{t-1}) = -\psi_{s,t}(\theta_t, \theta_{t-1}) = \gamma_{bt}\left(\theta_{b,t}, \theta_{s,t-1}\right) - \gamma_{st}\left(\theta_{s,t}, \theta_{b,t-1}\right) + K$$

$$\text{where } \gamma_{i,t}\left(\theta_{i,t}, \theta_{-i,t-1}\right) = \mathbb{E}_{t-1}\left[\tilde{\theta}_{-i,t}\chi^*_{-i}(\theta_{i,t}, \tilde{\theta}_{-i,t}) \middle| \, \theta_{-i,t-1}\right].$$

- *Even if* you anticipate truthful reporting in future, report $\hat{\theta}_{i,t}$ in period t has the following effects:

$$\begin{aligned} \gamma_{i,t}\left(\hat{\theta}_{i,t}, \theta_{t-1}\right) - \mathbb{E}_{t-1}\left[\gamma_{-i,t+1}\left(\tilde{\theta}_{-i,t+1}, \hat{\theta}_{i,t}\right)\middle| \, \theta_{-i,t-1}\right] = \\ \mathbb{E}_{t-1}\left[\tilde{\theta}_{-i,t}\chi^*_{-i}(\hat{\theta}_{i,t}, \tilde{\theta}_{-i,t}) \middle| \, \theta_{-i,t-1}\right] \\ -\mathbb{E}_{t-1}\left[\mathbb{E}_t\left[\tilde{\theta}_{i,t+1}\chi^*_i(\tilde{\theta}_{-i,t+1}, \tilde{\theta}_{i,t+1}) \middle| \, \hat{\theta}_{i,t}\right]\middle| \, \theta_{-i,t-1}\right]. \end{aligned}$$

- Report lower-than-truthful $\hat{\theta}_{i,t}$
  - Implies that expected incentive payment to opponent in period $t+1$ is lower

Potential Fix: Corrections

- Try to add a correction term:

$$\psi_{b,t}(\theta_t, \theta_{t-1}) = -\psi_{s,t}(\theta_t, \theta_{t-1}) = \gamma_{bt}\left(\theta_{b,t}, \theta_{t-1}\right) - \gamma_{st}\left(\theta_{s,t}, \theta_{t-1}\right) + K, \text{ wh}$$

$$\begin{aligned}\gamma_{i,t}\left(\theta_{i,t}, \theta_{-i,t-1}\right) = {} & \mathbb{E}_{t-1}\left[\tilde{\theta}_{-i,t}\chi^*_{-i}(\theta_{i,t}, \tilde{\theta}_{-i,t}) \middle| \ \theta_{-i,t-1}\right] \\ & + \mathbb{E}_{t-1}\left[\mathbb{E}_t\left[\tilde{\theta}_{i,t+1}\chi^*_i(\tilde{\theta}_{-i,t+1}, \tilde{\theta}_{i,t+1}) \middle| \ \theta_{i,t}\right] \middle| \ \theta_{-i,t-1}\right.\end{aligned}$$

- Want to manipulate the corrections!
  - For an approach based on "corrections," see earlier draft of Athey-Segal (2006).

General Solution: The Balancing Mechanism

- Balanced Transfers

$$\psi_{b,t}(\theta_t, \theta_{t-1}) = -\psi_{s,t}(\theta_t, \theta_{t-1}) = \gamma_{bt}\left(\theta_{b,t}, \theta_{t-1}\right) - \gamma_{st}\left(\theta_{s,t}, \theta_{t-1}\right) + K, \text{ wh}$$

$$\begin{aligned}\gamma_{i,t}\left(\theta_{i,t}, \theta_{t-1}\right) &= \sum_{\tau=t}^{\infty} \delta^{\tau-t} \mathbb{E}_{t,t-1}\left[\tilde{\theta}_{-i,\tau}\chi^*_{-i}(\tilde{\theta}_\tau) \middle| \ \theta_{i,t}, \theta_{-i,t-1}\right] \\ &\quad - \sum_{\tau=t}^{\infty} \delta^{\tau-t} \mathbb{E}_{t-1}\left[\tilde{\theta}_{-i,\tau}\chi^*_{-i}(\tilde{\theta}_\tau) \middle| \ \theta_{t-1}\right].\end{aligned}$$

  – Player i pays change in future externality due to new information

- Needs to do two things:

  – No incentive to manipulate opponent transfers (next period)

  – No incentive to manipulate own future transfers (next period)

$$\gamma_{s,t}\left(\hat{\theta}_{s,t},\hat{\theta}_{t-1}\right)=\sum_{\tau=t}^{\infty}\delta^{\tau-t}\left(\begin{array}{c}\mathbb{E}_{t,t-1}\left[\tilde{\theta}_{b,\tau}\chi_b^*(\tilde{\theta}_\tau)|\hat{\theta}_{s,t},\hat{\theta}_{b,t-1}\right]\\ -\mathbb{E}_{t-1}\left[\tilde{\theta}_{b,\tau}\chi_b^*(\tilde{\theta}_\tau)|\hat{\theta}_{t-1}\right]\end{array}\right).$$

- First, analyze buyer's expectation of $\gamma_{s,t}$ pointwise. For every t and history of reports $\hat{\theta}_{t-1}$,

$$\mathbb{E}_{t-1}\left[\gamma_{s,t}\left(\tilde{\theta}_{s,t},\hat{\theta}_{t-1}\right)\middle|\,\hat{\theta}_{s,t-1}\right]=0. \qquad (1)$$

- Let $\tilde{\tilde{\theta}}_{s,t}|\theta_{s,t-1}=_d\tilde{\theta}_{s,t}|\theta_{s,t-1}$. (1) holds because for each $\tau\geq t$, the Law of Iterated Expectations (LIE) implies

$$\mathbb{E}_{t-1}\left[\mathbb{E}_{t,t-1}\left[\tilde{\theta}_{b,\tau}\chi_b^*(\tilde{\theta}_\tau)|\tilde{\tilde{\theta}}_{s,t},\hat{\theta}_{b,t-1}\right]\middle|\,\hat{\theta}_{s,t-1}\right]=\mathbb{E}_{t-1}\left[\tilde{\theta}_{b,\tau}\chi_b^*(\tilde{\theta}_\tau)|\hat{\theta}_{t-1}\right].$$

Holds pointwise –> holds buyer takes expectations given period t beliefs, even if the buyer does not follow a truthful reporting strategy.

- Second, show expected value of $\sum_{t=t'}^{\infty} \delta^{t-t'} \gamma_{b,t}$ provides the correct incentives. Break out period t term of the positive part:

$$\begin{aligned}
\gamma_{i,t}\left(\theta_{i,t}, \theta_{t-1}\right) = \; & \mathbb{E}_{t,t-1}\left[\tilde{\theta}_{-i,t}\chi^*_{-i}(\theta_{i,t}, \tilde{\theta}_{-i,t})| \;\; \theta_{i,t}, \theta_{-i,t-1}\right] \\
& + \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}\mathbb{E}_{t,t-1}\left[\tilde{\theta}_{-i,\tau}\chi^*_{-i}(\tilde{\theta}_{\tau})| \;\; \theta_{i,t}, \theta_{-i,t-1}\right] \\
& - \sum_{\tau=t}^{\infty} \delta^{\tau-t}\mathbb{E}_{t-1}\left[\tilde{\theta}_{-i,\tau}\chi^*_{-i}(\tilde{\theta}_{\tau})| \;\; \theta_{t-1}\right].
\end{aligned}$$

- Then, can write

$$
\begin{aligned}
&\gamma_{i,t}\left(\theta_{i,t},\theta_{t-1}\right)+\delta\gamma_{i,t+1}\left(\theta_{i,t+1},\theta_{t}\right)= \\
\gamma_{it}\ \Bigg\{\ &-\sum_{\tau=t}^{\infty}\delta^{\tau-t}\mathbb{E}_{t-1}\left[\tilde{\theta}_{-i,\tau}\chi^{*}_{-i}(\tilde{\theta}_{\tau})\middle|\ \theta_{t-1}\right] \\
&+\mathbb{E}_{t,t-1}\left[\tilde{\theta}_{-i,t}\chi^{*}_{-i}(\theta_{i,t},\tilde{\theta}_{-i,t})\middle|\ \theta_{i,t},\theta_{-i,t-1}\right] \\
&+\sum_{\tau=t+1}^{\infty}\delta^{\tau-t}\mathbb{E}_{t,t-1}\left[\tilde{\theta}_{-i,\tau}\chi^{*}_{-i}(\tilde{\theta}_{\tau})\middle|\ \theta_{i,t},\theta_{-i,t-1}\right] \\
\delta\gamma_{i,t+1}\ \Bigg\{\ &-\delta\sum_{\tau=t+1}^{\infty}\delta^{\tau-t}\mathbb{E}_{t}\left[\tilde{\theta}_{-i,\tau}\chi^{*}_{-i}(\tilde{\theta}_{\tau})\middle|\ \theta_{t}\right] \\
&+\sum_{\tau=t+1}^{\infty}\delta^{\tau-t}\mathbb{E}_{t+1,t}\left[\tilde{\theta}_{-i,\tau}\chi^{*}_{-i}(\tilde{\theta}_{\tau})\middle|\ \theta_{i,t+1},\theta_{-i,t}\right]
\end{aligned}
$$

this term depends on θ_{it}

Combine expectation of these two

- Show expected value of $\sum_{t=t'}^{\infty} \delta^{t-t'} \gamma_{b,t}$ provides the correct incentives (cont'd). Work pointwise, and fix $(\theta_{b,t}, \hat{\theta}_{b,t}, \hat{\theta}_{t-1})$.
- First term of $\gamma_{b,t}\left(\hat{\theta}_{b,t}, \hat{\theta}_{t-1}\right)$ is

$$\sum_{\tau=t}^{\infty} \delta^{\tau-t} \mathbb{E}_{t,t-1}\left[\tilde{\theta}_{s,\tau} \chi_s^*(\tilde{\theta}_\tau) | \ \hat{\theta}_{b,t}, \hat{\theta}_{s,t-1}\right]. \tag{2}$$

- Second term of $\delta\gamma_{b,t+1}\left(\tilde{\tilde{\theta}}_{b,t+1}, \left(\hat{\theta}_{b,t}, \tilde{\tilde{\theta}}_{s,t}\right)\right)$ is

$$-\sum_{\tau=t+1}^{\infty} \delta^{\tau-t} \mathbb{E}_{t'}\left[\tilde{\theta}_{s,\tau} \chi_s^*(\tilde{\theta}_\tau) | \ \hat{\theta}_{b,t}, \tilde{\tilde{\theta}}_{s,t}\right], \tag{3}$$

but does *not* depend on $\tilde{\tilde{\theta}}_{b,t+1}$, so expectation conditional on $(\theta_{b,t}, \hat{\theta}_{s,t-1})$ does not depend on $\theta_{b,t}$. Using LIE, buyer's expectation is

$$-\sum_{\tau=t+1}^{\infty} \delta^{\tau-t}\left(\mathbb{E}_{t,t-1}\left[\tilde{\theta}_{s,\tau} \chi_s^*(\tilde{\theta}_\tau) \ \middle| \ \hat{\theta}_{b,t}, \hat{\theta}_{s,t-1}\right]\right). \tag{4}$$

Adding this to (2) yields

$$\mathbb{E}_{t-1}\left[\tilde{\theta}_{s,t}\chi_s^*(\hat{\theta}_{b,t},\tilde{\theta}_{s,t}) \;\middle|\; \hat{\theta}_{s,t-1}\right]. \tag{5}$$

- In this simple model:
  - Today's report directly enters t and $t+1$ transfers
  - So (5) is the direct effect of $\hat{\theta}_{b,t}$ on transfers
  - Today's report only affects today's allocation
    - No learning by doing, longer horizon serial correlation

- More generally, extend argument to infinite horizon plan of deviations $(\alpha_{b,\tau})_{\tau=t}^{\infty}$, cases where today's deviation has long-run impact
- Summing up the incentive payments,

$$\sum_{\tau=t}^{\infty} \delta^{\tau-t}\mathbb{E}_{t,t-1}\left[\gamma_{b,\tau}\left(\alpha_{b,\tau}\left(\tilde{\theta}_b^{\tau},\tilde{\theta}_s^{\tau-1}\right),\tilde{\theta}_{s,\tau-1}\right)\middle|\,\theta_{b,t},\hat{\theta}_{s,t-1}\right]$$
$$+\mathbb{E}_{t-1}\left[\tilde{\theta}_{s,t}\chi_s^*\left(\tilde{\theta}_{s,t}\right)\middle|\,\hat{\theta}_{t-1}\right] =$$
$$\sum_{\tau=t}^{\infty} \delta^{\tau-t}\mathbb{E}_{t,t-1}\left[\mathbb{E}_{\tau-1}\left[\tilde{\theta}_{s,\tau}\chi_s^*\left(\alpha_{b,\tau}\left(\tilde{\theta}_b^{\tau},\tilde{\theta}_s^{\tau-1}\right),\tilde{\theta}_{s,\tau}\right)\middle|\,\tilde{\tilde{\theta}}_s^{\tau-1}\right]\middle|\,\theta_{b,t},\hat{\theta}_{s,t-1}\right]$$
$$=\sum_{\tau=t}^{\infty} \delta^{\tau-t}\mathbb{E}_{t,t-1}\left[\tilde{\theta}_{s,\tau}\chi_s^*(\alpha_{b,\tau}\left(\tilde{\theta}_b^{\tau},\tilde{\theta}_s^{\tau-1}\right),\tilde{\theta}_{s,\tau})\,\middle|\,\theta_{b,t},\hat{\theta}_{s,t-1}\right].$$

1st equality: substitute, for each τ, (5) evaluated at $t=\tau$ and $\hat{\theta}_{s,t-1}=\tilde{\tilde{\theta}}_s^{\tau-1}$ for the sum of the expected value of the 1st term of each $\gamma_{b,\tau}$ and 2nd term of $\delta\gamma_{b,\tau+1}$. 2nd equality: LIE.

- Buyer can achieve efficiency with truthful reporting.

IR Constraints

- IR required in every period
  - Limits on ex ante contracting, inability to post large enough bond
- "Non-participation" option
- Suppose problem has Markov structure, finite state space
  - Allow learning by doing, etc.

Condition M For the Blackwell policy χ^B, the induced Markov process over states has a unique ergodic set (with a possibily empty set of transient states).

Proposition 6 *If Condition M holds, there exists a δ large enough such that IR constraints hold in each period.*

Intuition: IR Constraints

- Transfers potentially grow without bound as δ approaches 1.

$$\gamma_{i,t}\left(\theta_{i,t},\theta_{t-1}\right) = \sum_{\tau=t}^{\infty} \delta^{\tau-t}\mathbb{E}_{t,t-1}\left[\tilde{\theta}_{-i,\tau}\chi^*_{-i}(\tilde{\theta}_\tau)|\ \ \theta_{i,t},\theta_{-i,t-1}\right] - \sum_{\tau=t}^{\infty} \delta^{\tau-t}\mathbb{E}_{t-1}\left[\tilde{\theta}_{-i,\tau}\chi^*_{-i}(\tilde{\theta}_\tau)|\ \ \theta_{t-1}\right].$$

- Transfers large when today's reports have lasting impact.
- Need to find single constant K to transfer between agents so both IR's hold.
  - Which IR binds depends on history. "Worst-case" analysis.

Proposition 7 *Suppose seller valuations are serially independent and buyer valuations are serially affiliated. A sufficient condition for IR to hold in each period is:*

$$\sum_{\tau=t+1}^{\infty} \delta^{\tau-t}\mathbb{E}_t\left[\tilde{\theta}_{b,\tau}\chi_b^*(\tilde{\theta}_\tau)\middle|\underline{\theta}_b\right] - \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}\mathbb{E}_t\left[\tilde{\theta}_{s,\tau}\chi_b^*(\tilde{\theta}_\tau)\middle|\bar{\theta}_b\right] - \bar{\theta}_s \quad (6)$$

$$\geq \bar{\varphi} - \underline{\varphi} + \sum_{\tau=t+1}^{\infty} \delta^{\tau-t}\begin{pmatrix} \mathbb{E}_{t-1}\left[\tilde{\theta}_{s,\tau}\chi_s^*(\tilde{\theta}_\tau)|\underline{\theta}_b\right] - \mathbb{E}_{t-1}\left[\tilde{\theta}_{s,\tau}\chi_s^*(\tilde{\theta}_\tau)|\bar{\theta}_b\right] \\ +\mathbb{E}_t\left[\tilde{\theta}_{s,\tau}\chi_s^*(\tilde{\theta}_\tau)|\underline{\theta}_b\right] - \mathbb{E}_t\left[\tilde{\theta}_{s,\tau}\chi_s^*(\tilde{\theta}_\tau)|\bar{\theta}_b\right] \end{pmatrix}.$$

- $\bar{\varphi}$, $\underline{\varphi}$, are worst-case period t components of incentive payments.
- If serial independence, LHS is gains from efficient trade and last term of RHS is zero.
- If strong/extreme serial correlation, LHS is negative and RHS is positive.

- Condition M: converge to ergodic distribution at geometric rate, so RHS converges to fixed constant as δ approaches 1.
- General issue: multi-stage deviations (lie and then exit)
  - Need convergence to ergodic distribution not just in equilibrium, but also from any state reachable by lies

A Decentralized Game

- Instead of centralized mechanism, decentralized game
  - Outside options–non-participation
- Sufficient patience implies any "off-schedule" deviations are not desirable
- Can also allow for privately observed actions that have no externalities

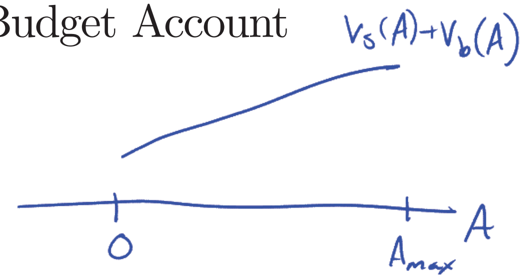
Summing Up Athey-Segal (2006)

- General dynamic model (serial correlation, decisions affect distribution of types, quality of information, etc.)
- Interim IC, EPBB dynamic mechanism exists
- Incentive payments give each player the change in expected future surplus to other agents induced by this period's report
  - Cannot manipulate opponent incentive payments: always zero expectation
  - Can redistribute among other agents without affecting incentives
  - Yields correct incentives for intended player as well
- IR constraints hold for patient players if today's information has a vanishing impact in the distant future

Dynamic Efficiency with Bounded Budget Account

- Criticisms of Interim IC
  - Simultaneous communication, Spying
  - Higher order beliefs
- What can we do with within-period ex post IC?
  - For many parameter values, optimal EPBB, EPIC mechanism for players is posted price mechanism!
    - No role for dynamics at all!
  - Give up on EPBB
- Bounded Budget Account: Interpretation
  - Players have collateral
  - Make deposits and withdrawals within bounds of account

Model of Bilateral Trade with Bounded Budget Account



- Extend Model
  - Account balance A is state variable
  - Key quantities are $v_s(A) + v_b(A)$, expected sum of discounted values, and expected sum of transfers $E[t_s(\theta; A) + t_b(\theta; A)]$.
  - Note that allocation rule and variability of transfers are the same in every period, so what varies with A is the lump sum portion of transfers
  - Net effect of withdrawal in account: change in account balance *plus* change in continuation value
  - Need to respect bounds. This affects incentives elsewhere.

- Intuition
  - At lower boundary, on average deposit a lot in order to ensure remain above the bound
  - Decreases value of being at boundary
  - If sum of values declines steeply near lower boundary, need to withdraw more just when account balance is getting low
- What happens at $A = 0$? (Efficient allocation)
  - Only deposits for all type realizations, so lump sum deposits required to preserve incentives
  - Period payoffs are low in expectation at $A = 0$
  - IR must be satisfied, so future must look sufficiently bright
    - ∗ Is this trivial as patience grows?

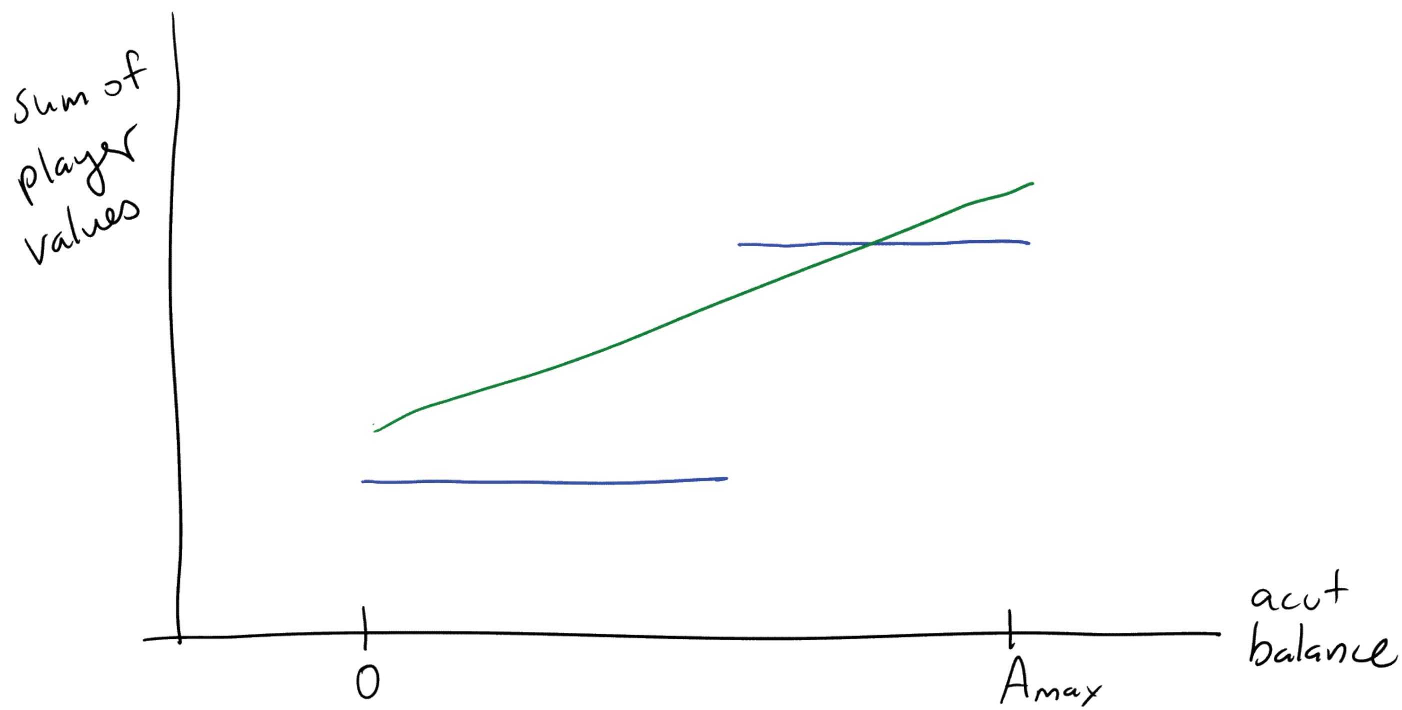
- IR not trivial at $A = 0$ even with high patience
  - Can patient players simply deposit large amounts to reach better state A' tomorrow?
    - To transition to A' tomorrow requires a deposit of A'.
    - Depositing more today to reach better state tomorrow only effective if $v_s(A) + v_b(A)$ has slope greater than $1/\delta$.
    - If it does, then what happens at A'? A *withdrawal* will *decrease* joint utility, since it hurts continuation values more than the value of the funds. Then, A' is the effective lower bound of the account.
    - If slope is less than $1/\delta$, then don't want to deposit any more than necessary

– Why would $v_s(A) + v_b(A)$ be better than $v_s(0) + v_b(0)$?

  * Lower lump sum deposits (now and in the future), which implies withdrawals in some state
  * Move away from 0 quickly only by lowering period payoffs
  * Move as slowly as possible to keep payoffs high
  * But then long-run benefit of being at A not so different from being at 0

– Implies value of future at $A = 0$ is somewhat limited even for fairly patient players

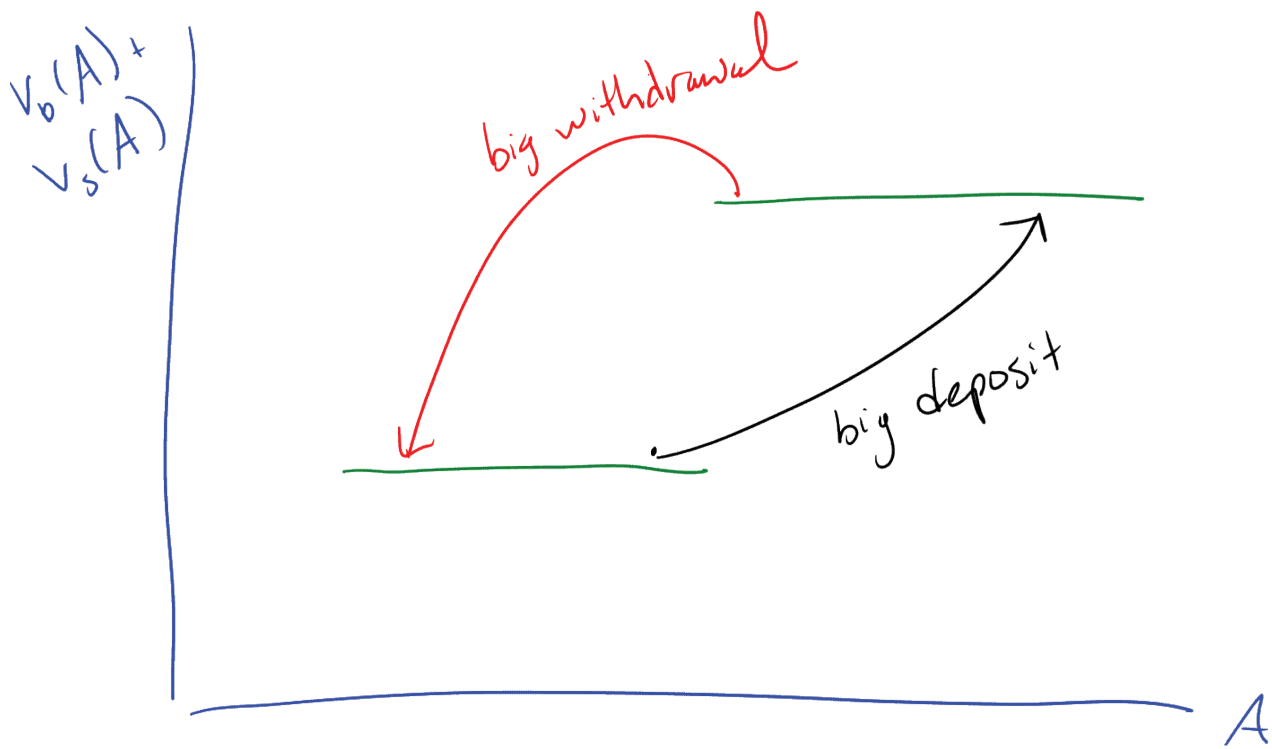
FIGURE: Sum of Values and the Account Balance



Approach: Analytical Solutions

- Trick: Make sum of values constant in account balance within two regions
  - Players are indifferent over account balances near boundaries
  - Two regimes: "always deposit" and "always withdraw"
    - Only possible for some parameter values
  - Large transfers required to compensate for changing regime
  - Can handle big transfers far away from boundary

FIGURE: Transitions in Account Balances



If Efficient Mechanism Cannot be Constructed

- Add two new regions: inefficient deposit "revenue" at bottom and inefficient withdrawal "payout" at top
  - These are of fixed size, do not change with δ, in order to guarantee that inefficiency stays small as δ grows
- Modify intermediate regions to have both upward and downward movement: "upward drift" and "downward drift"
- To guarantee that transfers at transitions do not grow with δ, need values constant across transition from revenue to upward drift, and constant across transition from downward drift to payout
- Still need upward drift and downward drift regimes to be large enough so that compensating transfers for transitions do not violate bounds

- Paper: proves that we can make upward drift and downward drift regions large enough that spend most of time there

Summing Up Athey-Miller (2006)

- Look for efficient, EPIC, IR bilateral trading mechanism with bounded budget account
- Players make deposits and withdrawals
- Trick to get tractability: construct (explicitly) mechanism with two regimes, and within each regime players are indifferent about account balance
  - "Deposit regime": always deposit, low period utility
  - "Withdrawal regime": always withdraw, high period utility
  - Constant continuation values within regime implies that value is just discounted value of period utility
  - Big change (increases with patience) in discounted utility from transitioning regimes

  - Make regime change far from boundaries, make account bounds grow with patience

- For all parameters, either efficient mechanism exists for sufficient patience, or almost-efficient

Summing up Lecture II

- Efficiency, IC and BB may not be attainable in static models due to IR
- Efficiency, IC and EPBB is attainable in general dynamic models
  - IR as well with patience, provided not too much persistence
- Efficiency, EPIC, EPBB, and IR attainable in model with patience, if players have access to bounded budget account
  - Tradeoff between robustness of model and requisite institutions