Les 4èmes

« TOULOUSE LECTURES IN ECONOMICS »

16-17-18 JANVIER 2007

Toulouse Lectures

Dynamic Games and Contracts with Hidden Information

Susan Athey

Professor of Economics

Harvard University

January 16-18, 2007

INSTITUT D'ECONOMIE INDUSTRIELLE

PARTENAIRE MAIRIE DE TOULOUSE www.toulouse.fr

PRINCETON

Overview

- Focus on Hidden Information
  - Hidden actions impt, techniques and applications often different
  - Auctions, collusion, bilateral or multilateral trade, public good provision, resource allocation, favor-trading in relationships, mutual insurance
- Contracts, Games, and Games as Contracts
- Mechanism Design with Commitment
  - When can you get efficiency?
  - What do optimal contracts look like?
  - What are tools for analyzing them?

- Dynamic Games
  - When can efficiency be sustained as an eqm?
  - What do equilibria look like for different discount factors?
- Mechanism Design Approach to Dynamic Games
  - In static theory, we are familiar with mechanism design approach to analyzing games such as auctions
  - Use tools such as envelope theorem, revenue equivalence, etc. to characterize equilibria
  - Analyze constraints
  - Take this approach to dynamic games
  - Combine dynamic programming and mechanism design tools
  - Current research: fully dynamic games (not repeated)

Lecture I: A Toolkit for Analyzing Dynamic Games and Contracts

1. Mechanism design preliminaries
   (a) Set-up
   (b) Solution concepts
2. Repeated Games
   (a) Abreu-Pearce-Stacchetti and dynamic programming
   (b) The mechanism design approach to repeated games with hidden information
   (c) Sustaining efficiency with transfers
   (d) The folk theorem without transfers
3. Dynamic Programming for Dynamic Games

Lecture II: Efficiency in Dynamic Contracts
Application to Bilateral Trade

1. Static Results
   (a) Team Mechanisms and Vickrey-Groves-Clark
   (b) Expected Externality Mechanisms (Arrow/AGV/CAGV)
   (c) Individual Rationality Constraints (Myerson-Satterthwaite)
2. Dynamics with i.i.d. types
   (a) Relaxes IR constraints
   (b) Modify transfers to optimize IR constraints (Athey and Miller, 2006a)

3. An Efficient Dynamic Mechanism (Athey and Segal, 2006)
   (a) Team mechanisms
   (b) Budget balance
   (c) IR constraints
4. A ex post IC dynamic mechanism with bounded budget account (Athey and Miller, 2006b)

Lecture III Outline: Dynamic Games with Hidden Information

1. Games with "Wasteful" Transfers
   (a) Theme: Pooling and Rules rather than Discretion
   (b) Collusion and Price Rigidity (Athey, Bagwell, and Sanchirico, 2004)
   (c) Monetary Policy (Athey, Atkeson, and Kehoe, 2005)
2. Games with Asymmetric Equilibria
   (a) Collusion (Athey and Bagwell, 2001)
   (b) Trading Favors (Hopenhayn, Abdulkadiroglu & Bagwell)
3. Persistent types (Athey and Bagwell, 2004)

Analyzing Repeated and Dynamic Games with Hidden Information

- Model the game/contract in extensive form
  - Dynamic games–see Battiglini (2005), Athey and Segal (2006)
  - Cumbersome to specify full strategy space and optimize over it
- Use APS dynamic programming "classic"
  - Assumptions don't apply directly
    - Continuum of types, actions, period strategies are functions
  - May need to solve differential equation, e.g. first-price auction
- Use APS/Mechanism Design combination
  - Applicability of results with the right assumptions
  - Can apply body of knowledge for hidden info games

A Dynamic Game with Time-Varying Hidden Information

- Players $i = 1, .., I$
- Time $t = 1, .., T$ (special cases: $T = 1, T = \infty$)
- Superscript/subscript notation: given $((y_{i,t})_{t=1}^{T})_{i=1}^{I}$,

$$y_t = (y_{i,t})_{i=1}^{I}, \quad y_i = (y_{i,t})_{t=1}^{T}, \; y^t = (y_{t'})_{t'=1}^{t}.$$

- Type spaces $\Theta_{i,t} \subseteq \mathbf{R}^n$, random variables $\tilde{\theta}_{i,t}$ with realizations $\theta_{i,t}$.
- Communication amoung players: $m_{i,t} \in \mathcal{M}_{i,t}$
- Decisions $X_{i,t} \subseteq \mathbf{R}^n$.
- Transfer from player j to player i :
  $y_{j,i,t} \geq 0$, let $y_{i,t} = \sum_j y_{j,i,t} - y_{i,j,t}$.
  - Some models rule out transfers, e.g. collusion

- History has two components:
  - Public history $h^{t-1} = (x^{t-1}, m^{t-1}, y^{t-1})$, private histories θ^{t-1}
- Timeline in period t:
  - Types realized (θ_t)
    - History potentially affects distributions: $F_t(\theta_t; x^{t-1}, \theta^{t-1})$.
  - Players communicate (m_t)
  - Players simultaneously make decisions (x_t) and send transfers (y_t)
- Note: can consider models without communication in this framework
  - Messages can be contentless
  - Athey-Bagwell (2001) show this can relax incentive constraints

Solution Concept

- Static: Bayesian Nash versus Ex Post
  - Best response given beliefs about opponent types and strategies
  - Ex Post
    - Best response given realization of opponent types
    - Has desirable robustness properties
    - Equivalent to dominant strategy in static private values models
      - NOT in dynamic models
- Dynamic games and mechanisms
  - Perfect Public Equilibrium (PPE) for repeated games
  - Bayesian Nash equilibrium in history-contingent strategies

  - ∗ BNE satisfies refinements (e.g. PBE) when mechanism only allows announcements in support, given history of announcements
  - ∗ Strategies are history-contingent, subgame perfection is imposed
  - ∗ Refinements have additional role in decentralized games
- *Ex post*: best response to opponents' actual future types?
  - ∗ Far too strong, rules out any risky investment
- *Within-period ex post*
  - ∗ Captures some types of robustness (spying, non-simultaneous communication, higher order beliefs within period)
  - ∗ Not others: still need beliefs about future types
  - ∗ Ex Post Perfect Public Equilibrium (EPPE) for repeated games

Approach: Model Game with Mechanism Design Tools

- Define a recursive (direct revelation) mechanism
  - Replace mapping from types to actions with reporting strategy
  - Many games of interest have single crossing property, already restricted to monotone strategies
- Specify appropriate constraints
  - "On-schedule" and "off-schedule" deviations
  - Comparison between decentralized game and recursive mechanism
    - Game has add'l constraints, action space unrestricted
    - With patience, these can be satisfied
    - Game without transfers must deal with restrictions on continuation values

- The role of patience
  - Static mechanism that satisfies BIC, EPBB, IR may *not* be eqm in game with low patience
  - Static mechanism that satisfies BIC, EPBB, *fails* IR may be eqm in game with high patience
- Independent (over time) types or perfectly persistent types
  - Use static tools
- Later: more general dynamics
  - Contingent, multi-stage deviations
  - Transfers and continuation equilibria not perfect substitutes

Dynamic Programming for Games: Abreu, Pearce and Stacchetti

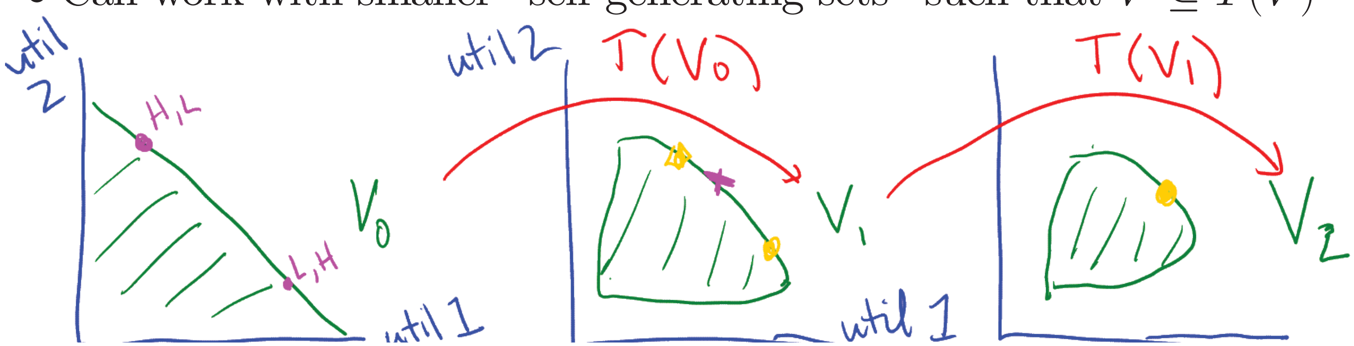
- Take a repeated game (ignore transfers for simplicity)
- Define V^* to be the set of equilibrium values ("promised utilities")
- Use $v \in V$ as the "state variable," index strategies and "continuation values" by elements of V
- Continuation value function specifies payoffs tomorrow as a function of public outcomes today

- $T(V)$ is set of $v \in \mathbb{R}^I$ for which there exist period choices x_t and continuation value functions $w : X_t \to \mathbb{R}^I$ whereby

1. *Promise-keeping*: $\pi_i(x_t) + \delta w_i(x_t) = v_i$.
2. *Coherence*: $w : X_t \to V$.
3. *Best response*:

$$x_{i,t} = \arg\max_{\hat{x}_i} \pi_i(\hat{x}_i, x_{-i,t}) + \delta w_i(\hat{x}_i, x_{-i,t})$$

- V^* can be computed as the largest fixed point of $T(V)$
- Can work with smaller "self-generating sets" such that $V \subseteq T(V)$



Approach Here: Recursive Mechanisms

- Athey and Bagwell (2001), Athey, Bagwell, and Sanchirico (2004)
  - Miller (2005) sets out approach for general model
- Idea: use APS approach together with mechanism design tools
- Start by focusing on stationary (repeated) games
  - For appropriately selected constraints, a "self-generating" recursive mechanism will be a PPE
  - A PPE can be written as a recursive mechanism
- Apply tools from static mechanism design theory

The Recursive Mechanism

- Stage Mechanism
  - Action plan for each player: $\chi : \Theta_t \to X$
  - Transfer plan from i to j, $\psi_{i,j} : \Theta_t \to \mathbf{R}^+$, $\psi_i = \sum_j \psi_{j,i} - \psi_{i,j}$
  - Continuation value function $w : \Theta_t \to \mathbf{R}^I$.
  - Let $\gamma = (\chi, \psi, w)$

  Ex post utility: $u_i(\hat{\theta}_t, \theta_{i,t}; \gamma) = \pi_i(\chi(\hat{\theta}_t), \theta_{i,t}) + \psi_i(\hat{\theta}_t) + \delta w_i(\hat{\theta}_t)$

  Interim utility: $\bar{u}_i(\hat{\theta}_{i,t}, \theta_{i,t}; \gamma) = \mathbb{E}_{\tilde{\theta}_{-i,t}}[u_i((\hat{\theta}_{i,t}, \tilde{\theta}_{-i,t}), \theta_{i,t}; \gamma)]$

- Recursive Mechanism: $\langle V, \{\gamma(v)\}_{v \in V}, v_0 \rangle$
  - A set V An initial condition $v_0 \in V$
  - A set of stage mechanisms $\{\gamma(v)\}_{v \in V}$

Constraints

- (Bayesian, Interim) IC:

$$\bar{u}_i(\theta_{i,t}, \theta_{i,t}; \gamma) \geq \ \bar{u}_i(\hat{\theta}_{i,t}, \theta_{i,t}; \gamma) \text{ for all } \hat{\theta}_{i,t} \in \Theta_{i,t}$$

- IR(p_0)
  - "Outside option": punishment equilibrium with payoffs p_0.
  - Could be static Nash, "Nonparticipation."
  - For simplicity, assume informative communication.

$$\bar{u}_i(\theta_{i,t}, \theta_{i,t}; \gamma) \geq \sup_{\hat{\theta}_{i,t}} \left\{ \sup_{x_i} \left(\pi_i(x_i, \chi_{-i}(\hat{\theta}_t), \theta_{i,t}) + \sum_j \psi_{j,i}(\hat{\theta}_t) \right) \right\} + \delta p_{0,i}.$$

    * More generally, take expectations given messages. See Athey and Bagwell (2001) for more discussion of alternative IRs.
    * Note assn about transfers and actions simultaneous.

Self-Generating Recursive Mechanism

- Define the set of attainable payoffs to be

$$\mathcal{V} = co\left\{ v \in \mathbf{R}^I : \exists \gamma \text{ s.t. } \sum_i v_i = \sum_i \frac{\mathbb{E}_{\tilde{\theta}_t}\left[u_i(\tilde{\theta}_t, \tilde{\theta}_{i,t}; \gamma)\right]}{1-\delta} \right\}.$$

- For $V \subset \mathcal{V}$, $p_0 \in \mathbf{R}^I$, define $T(V; p_0)$ to be the set of $v \in \mathbb{R}^I$ for which there exist stage mechanisms $\gamma(v) = (\chi, \psi, w)(v)$ whereby

1. *Promise-keeping*: $\mathbb{E}_{\tilde{\theta}_t}\left[u_i(\tilde{\theta}_t, \tilde{\theta}_{i,t}; \gamma(v))\right] = v_i$.
2. *Coherence*: $w(v) : \Theta_t \to V$.
3. *Best response*: $\gamma(v)$ satisfies IC and IR(p_0).

- V is *self-generating relative to* p_0 if $V \subseteq T(V; p_0)$.
  - Note: full set is $V \cup p_0$. Worst eqm not our focus; can extend to address this.
- $\langle V, \{\gamma(v)\}_{v \in V}, v_0 \rangle$ is *self-generating relative to* p_0 (SGRM(p_0)) if:
  V is self-generating relative to p_0 and,
  for each $v \in V$, (1)-(3) hold for $\gamma(v)$ and p_0.

Recursive Mechanism as a Tool for Analyzing PPE

Proposition 1 *Fix δ. Suppose p_0 is a PPE and consider $V >> p_0$.*
(i) If V is a set of PPE payoffs with informative communication, then there exists $v_0 \in V$, $\{\gamma(v)\}_{v \in V}$ such that $\langle V, \{\gamma(v)\}_{v \in V}, v_0 \rangle$ is a SGRM(p_0).
(ii) Suppose that $\langle V, \{\gamma(v)\}_{v \in V}, v_0 \rangle$ is a SGRM(p_0). Then V is in the set of PPE payoffs.

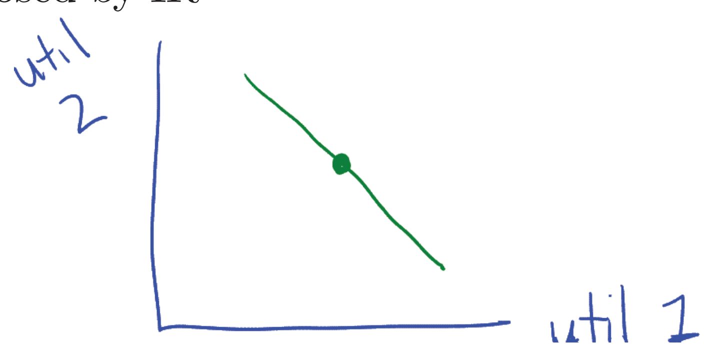
- Proof: See Miller (2005) (does folk theorem; adapt arguments). Analogous to APS. Have to verify that constraints deter relevant deviations.
- If interested in set V of PPE payoffs w/o informative communication, modify IRs to get corresponding result.
- IR constraints imply that deviating "off-schedule" is not desirable.

Transforming to a Static Problem: The Case with Transfers

- Recall

$$u_i(\hat{\theta}_t, \theta_{i,t}; \gamma) = \pi_i(\chi(\hat{\theta}_t), \theta_{i,t}) + \psi_i(\hat{\theta}_t) + \delta w_i(\hat{\theta}_t).$$

  - With independent types, value for future play is the same for all types
  - Transfers and continuation values completely fungible
- WLOG, can consider stationary mechanisms (Levin, 2003)
- Then, consider static mechanism design problem with bounds on transfers imposed by IR



Folk Theorem with Transfers

Proposition 2 *Given χ, suppose there exist EPBB, uniformly bounded, IC transfers for χ, and that*

$$\sum_i \mathbb{E}[\pi_i(\chi(\tilde{\theta}_t), \tilde{\theta}_{i,t})] > \sum p_{0,i}.$$

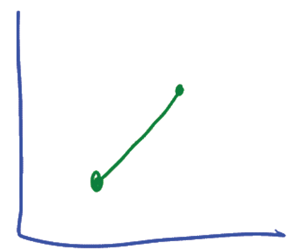
Then for δ sufficiently large, there exists a SGRM(p), $\langle V, \{\gamma(v)\}_{v \in V}, v_0 \rangle$ that is stationary, where

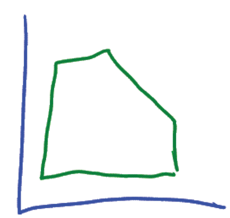
$$\sum_i v_{0,i} = \sum_i \mathbb{E}[\pi_i(\chi(\tilde{\theta}_t), \tilde{\theta}_{i,t})].$$

- See Cremer, d'Aspremont, Gerard-Varet (2003) for sufficient conditions; see also Miller (2005).
- As δ grows, value of future eventually outweighs transfers. Independent future key.

Transforming to a Static Problem: The Case without Transfers

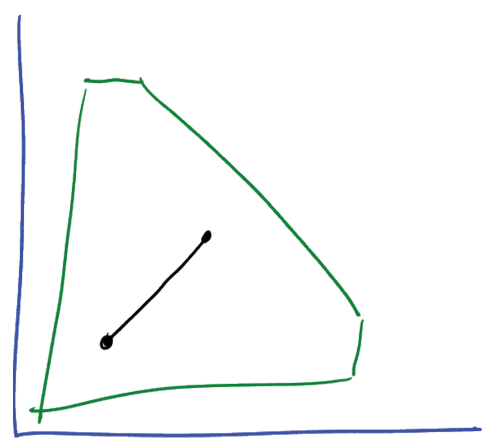
- Continuation values can mimic role of transfers, but for fixed δ, Pareto frontier of V is not in general linear
- Tradeoff between using variation in continuation values to provide incentives, and Pareto efficient continuation values
  - "Efficiency today v. efficiency tomorrow"
  - Finding: Sacrifice efficiency today
- Details of model determine shape of frontier of V
  - Strong symmetry
    - ∗ Observability
    - ∗ Policy games
  - Multiplicity of efficient outcomes: partial linearity





- Approach (see Athey and Bagwell (2001)): start with large V, characterize $T(V)$
  - Analogous to static problem with restricted transfers

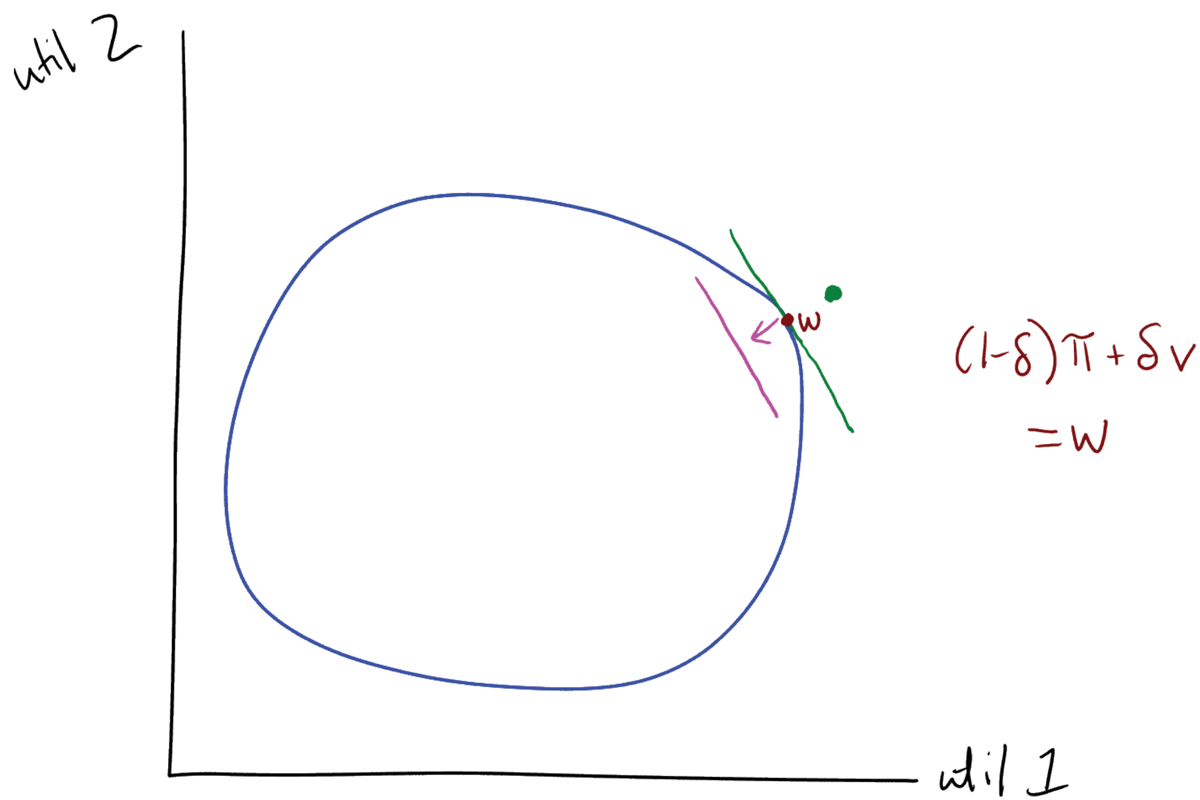
FIGURE: Possible Pareto Frontiers of Continuation Value Sets



Folk Theorem without Transfers

- Fudenberg, Levine and Maskin (1994), Miller (2005)
  - Small changes in future per-period utility mimic transfers
  - FLM make unnecessary assumptions: independent, finite types
    - They focus on hidden action models and so don't look for most general conditions
  - Miller (2005) generalizes to continuous types, correlated values
- Key elements of argument
  - Angle of supporting hyperplanes doesn't matter generically
  - Average period payoffs (outside set) and hyperplane (inside set)
  - As $\delta \to 1$, length of hyperplane shrinks fast enough
  - Nothing about what to do for fixed δ

FIGURE: Supporting Hyperplanes



Recursive Mechanisms in Dynamic Games with Hidden Information

- Persistent types
  - Labor or input contracts
  - Knowledge/capabilities
- Learning by Doing
- Markov Structure
  - Private states $\tilde{\theta}_{i,t}|\theta_{i,t-1}, x_{t-1} = g_i(\theta_{i,t-1}, x_{t-1}, \tilde{\varepsilon}_{i,t-1})$
    - For simplicity: types, states independent across players conditional on history
  - Payoffs, probability distributions depend on states
  - For simplicity, focus on "informative" equilibria
  - Public states: messages $\hat{\theta}_t$

- For simplicity, suppose there is a punishment equilibrium p_0 that does not depend on beliefs
  - "Nonparticipation"
  - A pooling eqm (see Athey and Bagwell (2004))

Extending Recursive Mechanism Approach to Dynamic Games

- Adapted for Hidden Information Games from Cole and Kocherlakota (2001); see Athey and Bagwell (2004)
- Let $\mathcal{V}$ be the set of functions $\breve{v}=(\breve{v}_1, .., \breve{v}_I)$ such that $\breve{v}_i : \Theta_i \rightarrow \mathbb{R}$.
  - Set of possible "type-contingent payoff functions."
- Let $\mathcal{P} = \mathcal{V} \times \Delta\Theta$. Set of PPBE is a subset P of $\mathcal{P}$.
- Each eqm is described by beliefs about opponents, $\mu \in \Delta\Theta$, $\breve{\mathbf{v}} \in \mathcal{V}$
  - State-contingent payoff functions allow player to compute expectations about future payoffs using private beliefs, can be different than reports
  - Use Bayes' rule to update beliefs, need to specify off-path

- Stage Mechanism
  - Initial belief $\mu_t \in \Delta\Theta$
  - Action plan for each player: $\chi : \Theta_t \to X$
  - Transfer plan, $\psi_i : \Theta_t \to \mathbf{R}$
  - Continuation value and belief functions $(w, b) : \Theta_t \to \mathcal{P}$.
  - Let $\gamma = (\chi, \psi, w, b)$,

Ex post utility:
$$u_i(\hat{\theta}_t, \theta_{i,t}; \gamma) = \pi_i(\chi(\hat{\theta}_t), \theta_{i,t}) + \psi_i(\hat{\theta}_t) + \delta \mathbb{E}_{\tilde{\theta}_{t+1}} \left[w_i(\tilde{\theta}_{t+1}) \middle| \theta_{i,t}, b_{-i}(\hat{\theta}_{-i,t}) \right]$$

Interim utility:
$$\bar{u}_i(\hat{\theta}_{i,t}, \theta_{i,t}; \gamma, \mu_t) = \mathbb{E}_{\tilde{\theta}_{-i,t}} [u_i((\hat{\theta}_{i,t}, \tilde{\theta}_{-i,t}), \theta_{i,t}; \gamma) \Big| \mu_{-i,t}]$$

- Recursive Mechanism: $\langle P, \{\gamma(p)\}_{p \in P}, p' \rangle$
  - A set P An initial condition $p' \in P$
  - A set of stage mechanisms $\{\gamma(p)\}_{p \in V}$
  - For each $p \in \mathcal{P}$, there are associated "continuation value and belief functions" (w, b) mapping message space M_t to $\mathcal{P}$.
    * $v_i = w_i(\hat{\theta}_t)$ is type-contingent payoff function if announcements were $\hat{\theta}_t$
    * $v_i(\theta_{t+1}) = w_i(\hat{\theta}_t)(\theta_{t+1})$ is payoff if state turns out to be θ_{t+1}
    * $\mu_{t+1} = b(\hat{\theta}_t)$ is set of "beliefs about opponents"

- For $P \subset \mathcal{P}$, we can now define $T(P)$ as the set of $(\mathbf{v}, \mu) \in \mathcal{P}$ for which there exists $\gamma = (\chi, \psi, w, b)$ such that:

1. *Promise-keeping*: $\mathbb{E}_{\tilde{\theta}_t}\left[u_i((\theta_{i,t}, \tilde{\theta}_{-i,t}), \theta_{i,t}; \gamma)\middle| \mu\right] = v_i(\theta_{i,t})$.
2. *Coherence*:
   (a) $(w, b) : \Theta_t \to P$.
   (b) $b(\hat{\theta}_t)$ formed by Bayes' rule given μ and $\hat{\theta}_t$
3. *Best response*: γ satisfies IC and IR(p_0).

- Note that 2 (a) and (b) jointly restrict what payoffs can be promised to coincide with payoffs that are feasible given new beliefs
- Note that the constraints account for multi-stage deviations, since they must hold for each player's true type, which may differ from the announced type

Lecture I: Conclusions

- Analyzing decentralized game with moderate patience
- Tools of dynamic programming apply
- Can transform problem into static mechanism design problem
- Constraints somewhat different in some cases
  - IR constraints
  - Constraints on transfers
- Folk theorem: static BIC & EPBB –> PPE for patient players
- Techniques can be used for moderate patience
- Dynamic programming extends to dynamic games with Markov structure