Relationship Formation Under Asymmetric Information: The Role of Contractual Commitment

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Abstract

This paper studies contract design by an informed principal in a principal-agent relationship. We analyze the role of the principal’s contractual commitment to the relationship both in signalling information to extract surplus and in providing incentives for the agent to invest in the relationship. When the role of contracting is one of surplus extraction only, symmetric information outcomes are obtained through commitments of future trade (contracted quantity) or of exclusivity. When investment is considered, signalling and efficiency of investment can conflict if only quantity is contractible. This conflict generates inefficient equilibria. However, contracting on exclusivity in addition to quantity can resolve the conflict.

Keywords: Principal-agent, asymmetric information, hold-up, contractual commitment

JEL Classification: D82, L14, L40, K21

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1 Introduction

Many relationships are formed under asymmetric information. When two or more parties meet to agree on the terms of a future relationship, some of them may have relevant private information about how successful the relationship will be. For example, in vertical relationships, a final good producer contracting with a specific supplier about future trade may have private information about her future value of trading with the supplier. In labor market relationships, an employer offering a job position to a new worker may have private information about the fit of the worker to the job position. In franchising, a manufacturer offering a franchising agreement to a retailer may be better informed than the retailer about the ability of the retailer to sell his product. As was emphasized by Myerson (1983) and Maskin and Tirole (1992), in these cases, if the parties with private information participate in the design of the contract (or the terms of the relationship if established in an informal way), the contract’s terms may reveal some of their private information to the other parties.

In this paper, we study contract design by an informed principal in a context of relationship formation with an agent. Specifically, we analyze how two forms of contractual commitment to a relationship, a promise to trade in the future (contracted quantity) and a promise not to trade with anyone else (contracted exclusivity), play a role both in signalling information to extract surplus and in providing incentives to invest in the relationship.

In a symmetric information environment with transferable utilities, surplus extraction by a principal with full bargaining power can be achieved in an obvious way; a simple up-front transfer leaving the agent with his reservation value allows the principal to appropriate the entire expected surplus generated by the relationship. This up-front transfer can always be implemented, even when the contract regulating the relationship serves other purposes in addition to surplus extraction (e.g., inducing efficient investment in situations of hold-up or efficient risk allocation). However, when the principal has better information about the value of the relationship at the contracting stage (and this value affects the payoff of the agent), a transfer alone may not be sufficient to allow the principal to appropriate all the surplus. When demanded to pay the transfer, the agent may draw negative inferences about the value of the relationship and decline the principal’s offer. Hence, in this case, surplus extraction depends on the ability of the principal to signal information through the contract. Moreover, signalling information through the contract to extract surplus may interfere with other roles of contracting, such as providing efficient investment incentives.

We develop a model in which a principal and an agent first meet and contract about future transactions, while knowing that later on the principal may wish to trade with an external party instead. At the initial contracting stage, both parties are uncertain about the value of the relationship, but the principal has better information about how successful it will be. This is either because she is better informed about how much she values trade with the agent (private internal information) or because she is better informed about the
value of her future outside options (*private external information*). We consider both cases of internal and external information because, as we show, the source of asymmetry of information affects which forms of commitment can be used to signal information. Once both parties agree on a contract, a relationship is formed and the agent has the opportunity to invest in it. At a later date, uncertainty is realized and both the principal and the agent observe the value of trading with each other and the value of the principal’s trade with others. Finally, after observing the trade valuations, the principal and agent renegotiate the initial contract whenever it is inefficient.\(^1\) Although renegotiated, the initial contract still matters because it determines the status quo positions of both parties (disagreement point) during renegotiation.

By studying contractual signalling by an informed party and in the presence of relationship-specific investment, this paper is inherently related to two strands of the literature: the literature on contract design by an informed party and the literature on the hold-up problem. The former can be divided into two groups. The first group focuses on the characterization (in a general way) of the equilibrium contract proposal by an informed principal in a principal-agent relationship (e.g., Myerson (1983), Maskin and Tirole (1990) and Maskin and Tirole (1992), Beaudry and Poitevin (1993)). In the first part of the present paper, where we focus on surplus extraction only, our framework is close to the one in Maskin and Tirole (1992). In the second part of the paper, where the agent can invest in the relationship, we extend their results to the case in which the agent makes a noncontractible investment decision.\(^2\) The second group of this literature has studied contract design by an informed party in more concrete settings (e.g., Aghion and Bolton (1987), Aghion and Hermalin (1990) and Spier (1992)).\(^3\) This paper contributes to this literature by explicitly studying the signalling role of the *ex-ante* contractual commitments of future trade and of exclusivity. Although these types of agreement have been extensively studied in the literature on contracting under symmetric information, not much attention has been devoted to their role in conveying information when an informed party takes part in the design of the contract.\(^4\)

\(^1\)Considering that parties observe valuations of trade and subsequently renegotiate the initial contract allows us to capture important features of many relationships. First, parties to a relationship often learn its real value (only) after the relationship has started. Second, in those cases parties tend to renegotiate initial contracts that are inefficient *ex-post*. Beaudry and Poitevin (1993) study equilibrium contracting by an informed party in a setting with renegotiation. However, they consider the case in which the asymmetry of information persists during the renegotiation stage.

\(^2\)Maskin and Tirole (1992) assume that all payoff relevant variables are contractible.

\(^3\)For example, Aghion and Hermalin (1990) use a contract signalling model to show that imposing legal restrictions on private contracts can enhance efficiency. Spier (1992) identifies a reason for contractual incompleteness by showing that an informed principal can signal information by deliberately proposing an incomplete contract to an agent.

\(^4\)There are some exceptions. In the second part of Aghion and Bolton (1987), the authors suggest that contracted liquidated damages, i.e., a penalty paid by the buyer to the incumbent seller in case he buys from an entrant, can signal information about the probability of entry of another seller in the market. These penalties can be interpreted as an exclusive agreement. Our analysis differs from theirs in several instances. First, we consider both quantity and exclusive contracts. Second, we analyze both internal and external private information. Finally, we allow for relationship-specific investment. Bishop (1984) and Rowthorn (2002) have emphasized the idea of marriage (which can also be interpreted as an exclusive contract) as a signal.
The existing literature on the hold-up problem assumes symmetric information at the initial contracting stage (e.g., Grossman and Hart (1986), Hart and Moore (1990), Chung (1991), Rogerson (1992), MacLeod and Malcomson (1993), Aghion et al. (1994), Edlin and Reichelstein (1996), Che and Hausch (1999), Whinston and Segal (2000)). By studying contract design by an informed party in a relationship with specific investments, this paper extends the literature on the hold-up problem to the case in which there is asymmetric information at the contracting stage. In the hold-up problem literature with symmetric information at the contracting stage, the contract is typically designed with one goal: to provide the right incentives to invest. The presence of asymmetric information at the contracting stage introduces a new role for the contract: signalling information to extract surplus. This enables us to study the interaction between these two roles of contracting.

The paper is structured as follows. After presenting the model in Section 2, we start the analysis by establishing an irrelevance result in Section 3: exclusivity does not affect the set of equilibrium outcomes when the principal has only private internal information. The result holds regardless of whether or not the agent can invest in the relationship. This result implies that, when the asymmetry of information is about internal valuations only, commitment to the relationship through exclusivity cannot be used to signal information and is irrelevant both in terms of surplus extraction and investment in the relationship.

We then focus on the environment without an investment decision by the agent in Section 4. In this case, the role of contracting is one of surplus extraction only. We analyze separately the cases of private internal information and private external information. We show that when private information is internal, contracted quantity (but not exclusivity) allows the principal to signal information and always extract all the surplus. In contrast, when private information is external, both quantity and/or exclusivity contracts allow the principal to signal information and to always extract all the surplus. In both cases, the principal signals to the agent that the relationship is more likely to be successful by *ex-ante* committing more to it (committing to trade a higher quantity or to a higher level of exclusivity, or both). In a simplistic way, the intuition is that from an *ex-ante* point of view it is much less costly to a principal who expects a high valuation of trade with the agent or low outside options in the future to commit to a contract specifying high trade or high exclusivity than it is for a principal who expects low value of trade with the agent or high future outside options. This is true even when the contract is renegotiated later on: increasing contractual commitment affects less positively the disagreement point for renegotiation (disagreement payoff) of the latter principal than the former.

Next, we allow the agent to make a non-contractible investment decision that affects the value of the relationship (Section 5). We find that surplus extraction and efficiency of investment can conflict with one another if the contract specifies only quantity. This conflict generates inefficient equilibria, as the principal may choose a quantity that distorts the agent’s investment decision relative to its socially efficient level to
signal information so as extract surplus from the agent. We then show that if the principal can use both quantity and exclusivity in the contract, this conflict can be resolved when the asymmetry of information concerns the value of the outside option. In this case there is full efficiency in terms of investment. Thus, we identify a situation in which the use of exclusive agreements enhances investment efficiency. This result is important for two reasons. First, in contrast to Segal and Whinston (2000), it rationalizes the use of exclusive contracts in situations of hold-up with pure relationship-specific investments. Motivated by informal discussions (in anti-trust and exclusive contracts) on whether exclusive provisions foster relationship-specific investments, Segal and Whinston (2000) show that exclusivity does not affect whatsoever investments that are fully relationship-specific, when information is symmetric at the contracting stage. Second, it contributes to the unsettled debate on whether exclusive agreements should be contractually allowed by courts or not. In this specific matter, a long-standing concern of courts is that exclusive contracts serve anticompetitive purposes and consequently prevent efficiency.

2 The Model

Consider a principal and an agent who can initially contract, knowing that the principal may later wish to deal with an external party instead. If they agree on a contract, a relationship is formed. If not, the two parties obtain their reservation payoffs and the game ends.

Once a relationship is formed, it evolves in three stages: an investment stage, in which the agent has the opportunity to make a relationship specific investment \((i_A \in I_A)\); a renegotiation stage, which occurs after uncertainty is realized and where the principal and the agent renegotiate the terms of the initial contract; and a trading stage. Trading between the principal and the agent may never occur if the parties decide, at the renegotiation stage, not to deal with each other and end the relationship. In this case, the principal can trade with an available external party.

The parties’ utilities are quasilinear in money. The agent’s utility is additive in the investment cost, which we denote by \(\psi(i_A)\) and assume is an increasing function of investment satisfying \(\psi(0) = 0\). In addition to money transfers, if the principal and the agent trade with each other they obtain values of \(v_p\) and \(v_a\), respectively. For future convenience, the value of trade between the principal and the agent is denoted \(V \equiv v_p + v_a\). The value of trade between the principal and the external party, on the other hand, is denoted

\[V \equiv v_p + v_a.\]

\[\text{Selvaggi and de Meza (2005) have a model in which exclusivity affects specific investments. They consider a different bargaining game from the one in Segal and Whinston (2000).}
\]

\[\text{For example, consider a model in which the principal is the buyer, the agent is the seller and the buyer needs at most one unit of the good. In this case, } v_p \text{ corresponds to the utility the buyer derives from consuming one unit of the good and } v_a = -c, \text{ where } c \text{ denotes the cost of the seller from producing one unit of this same good. The value created by the buyer and the seller is given by } V = v_p + v_a = v_p - c.\]
by \( V \). We assume that \( V \) is always nonnegative.

Information. During the contracting and investment stages (\textit{ex-ante}), the principal is better informed than the agent about the probability of success of the relationship. Specifically, let \( \theta \in \Theta = \{ \theta_L, \theta_H \} \) denote the parameter over which there is private information, i.e., the principal knows the true \( \theta \) while the agent only knows its distribution \( p(\theta) \). We assume that the distribution of the valuations \( v_p, v_a \) and \( V \) depends on the state \( \theta \), i.e., \((v_p, v_a, V) \sim G(\cdot | i_A, \theta)\) where \( G(\cdot | .) \) is the joint distribution. Throughout the analysis, we will focus on two specific forms of asymmetry of information: (i) better information about \( v_p \) and \( v_a \) (\textit{internal valuations}); or (ii) better information about the principal’s outside option \( V \) (\textit{external valuation}).

After the investment stage, but before renegotiation, uncertainty is realized and both the principal and the agent observe the realization of valuations \( v_a, v_p \) and \( V \). Hence, renegotiation and trading (\textit{ex-post}) occur under symmetric information. Although observable by the parties, valuations cannot be verified by a court. Therefore, contracts that are directly contingent on valuations are not feasible.

Contracting. At the contracting stage, the principal has the opportunity to make a “take it or leave it” offer of a finite menu of contracts. If the agent accepts the menu, the principal herself then chooses a contract from the menu (throughout, we use feminine pronouns for the principal and masculine pronouns for the agent). This is the contract that governs the relationship between the principal and the agent. By allowing the principal to propose menus of contracts, we follow the approach of Maskin and Tirole (1992) in analyzing the problem of mechanism design by an informed principal.

The potentially contractible variables are: an up-front transfer from the agent to the principal \( T_u \), a quantity \( q \), and a level of exclusivity \( e \). The up-front transfer \( T_u \) can take any real value, \( T_u \in \mathbb{R} \). The quantity and exclusivity variables are modeled as probabilities. Quantity \( q \) denotes the probability that the principal and the agent must trade. The set of allowable quantities is denoted by \( Q \). When quantity is contractible, \( Q = [0,1] \). When it is not, \( Q = \{ 0 \} \). The exclusivity variable \( e \) denotes the probability that the agreement is exclusive; i.e., that the principal cannot trade with outsiders. We denote the set of allowable exclusivity levels by \( E \). When exclusivity is contractible, \( E = [0,1] \). Noncontractible exclusivity is modelled by imposing \( E = \{ 0 \} \). Thus, a contract is an object of the form \((T_u, q, e) \in C, \) where \( C = \mathbb{R} \times Q \times E \).

Our analysis considers both cases with and without an investment decision by the agent. In the latter case, we assume that \( I_A = \{ 0 \} \). The agent’s investment decision is not verifiable and therefore cannot be contractually specified.\(^9\)

\(^7\) \( V \) can be interpreted as the value of the best outside option of the principal at the renegotiation stage.

\(^8\) The quantity and exclusivity variables can be interpreted as proportions. Suppose that the principal has a trade capacity. In this case, quantity \( q \) represents the proportion of the trade capacity of the principal that is contractually allocated to the agent. Similarly, exclusivity \( e \) represents the proportion of the remaining \((1 - q)\) of the trade capacity of the principal which is not contractually allocated to the agent, that cannot be traded with an external party.

\(^9\) Relationship-specific investments often take a nonmonetary, intangible form, such as human capital investment. In these cases, it is difficult to contract on investment-related information. The existence of relationship-specific investments that are
Renegotiation. After observing the realization of the valuations, if the initial contract prescribes an inefficient level of trade, the principal and the agent renegotiate trade to the efficient level. The agent’s investment decision is irreversible at this stage. As in Edlin and Reichelstein (1996), Che and Hausch (1999), Segal and Whinston (2000) and Segal and Whinston (2002), we assume that the bargaining shares of the principal and the agent during renegotiation are exogenously specified. More specifically, it is assumed that the principal and the agent split equally their renegotiation surplus over the disagreement point, which is determined by the original contract. Thus, despite renegotiation the original contract still matters because it affects the distribution of ex-post surplus, which in turn is important for surplus extraction and investment incentives. Finally, we suppose that the external party with whom the principal can alternatively deal receives no surplus. This would be consistent, for instance, with a case of competition among many external parties who are willing to deal with the principal in case she does not trade with the agent.

Efficiency of ex-post renegotiation implies that whenever dealing with third parties creates more value than dealing with each other, the principal and the agent agree to end the relationship. Hence, from an ex-ante point of view, the probability of success of their relationship is the probability that the value of trade $V$ is bigger than $\bar{V}$. We denote this probability by $P_s(i_A, \theta)$ and assume it is strictly positive for all $i_A \in I_A$ and $\theta \in \Theta$.

Throughout the analysis, the equilibrium concept used is the Perfect Bayesian Equilibrium (PBE).

2.1 Ex-ante Utility and Agent’s Investment Decision

At the renegotiation stage, the principal and the agent receive a fixed share (one half) of the renegotiation surplus in addition to their disagreement payoffs, which are the payoffs in case they do not reach a renegotiation agreement and the contract is executed. Since the disagreement payoffs (ignoring sunk investment costs) are $qv_a - T_u$ for the agent and $qv_p + (1 - q)(1 - e)V + T_u$ for the principal, and the efficient total noncontractible has been the basis assumption on the hold-up problem literature. As in most of that literature, we follow an incomplete contracting approach by assuming that investments are not contractible.

10 Some papers in the literature have considered contracts incorporating schemes that ex-ante manipulate parties’ future bargaining power (e.g., Chung (1991) and Aghion et al. (1994)). The implementation of such schemes (which can be quite elaborate) may not be feasible in environments like ours that render contracting incomplete. Moreover, those schemes may fail if parties always renegotiate inefficient outcomes or face financial constraints. For a more thorough discussion of this issue, see Che and Hausch (1999).

11 The assumption that the principal and the agent have equal bargaining shares is not crucial. All the results remain unchanged if instead of $\frac{1}{2}$ we consider that the agent’s bargaining share is any number in the interval $(0, 1)$. The important assumption is that the agent has some (strictly positive) bargaining power at the renegotiation stage. Otherwise, in general, his payoff would not depend on the private information of the principal. Also, if the agent had all the bargaining power, we would not be able to study efficiency implications of investments. In that case, if investments affect only the agent’s valuation $v_a$, the agent would always invest efficiently.

12 See Fudenberg and Tirole (1991) for a precise definition of a PBE.
surplus (also ignoring sunk investment costs) is \( \max\{V, \nabla\} \), the agent’s post renegotiation payoff given state \( \alpha \equiv (v_a, v_p, \nabla, i_A) \) and the outcome contractually prescribed \((T_u, q, e)\) is given by

\[
u^A(\alpha; T_u, q, e) = (qv_a - T_u) + \frac{1}{2}\max\{V, \nabla\} - (qv_a - T_u) - (qv_p + (1 - q)(1 - e)\nabla + T_u) - \psi_A(i_A)
\]

\[
= \frac{1}{2}\max\{V, \nabla\} - \frac{1}{2}[q(v_p - v_a) + (1 - q)(1 - e)\nabla] - T_u - \psi_A(i_A).
\] (2.1)

Similarly, the principal’s post renegotiation payoff can be written as

\[
u^P(\alpha; T_u, q, e) = \frac{1}{2}\max\{V, \nabla\} + \frac{1}{2}[q(v_p - v_a) + (1 - q)(1 - e)\nabla] + T_u.
\] (2.2)

Efficient renegotiation implies that the sum of the payoffs to principal and agent are always equal to efficient total surplus (henceforth, total surplus) given state \( \alpha \), i.e.,

\[
u^A(\alpha; T_u, q, e) + \nu^P(\alpha; T_u, q, e) = TS(\alpha), \ \text{all} \ \alpha, \ \text{all} \ (T_u, q, e),
\] (2.3)

where \(TS(\alpha) = \max\{V, \nabla\} - \psi_A(i_A)\).

Since the analysis will focus on the equilibrium of the contracting game between the principal and the agent, which occurs before uncertainty is resolved, we are ultimately interested in the ex-ante expected payoffs. The expected payoffs of the principal and the agent are both functions of the contract \( c = (T_u, q, e) \) and the agent’s investment level \( i_A \). However, since at the contracting and investment stages the principal knows state \( \theta \) and the agent does not, the expected payoff of the principal is a direct function of \( \theta \), while the expected payoff of the agent is a function of his beliefs about \( \theta \). More specifically, expected payoffs are given by

\[
U^A(c; i_A | b_L) = b_LE[u^A \mid i_A, \theta_L] + (1 - b_L)E[u^A \mid i_A, \theta_H]
\]

and

\[
U^P(c; i_A | \theta) = E[u^P \mid i_A, \theta], \ \theta \in \{\theta_L, \theta_H\},
\]

where \( b_L \in [0, 1] \) represents the agent’s belief that \( \theta = \theta_L \). With a slight abuse of notation, \(U^A(c; i_A | 1)\) and \(U^A(c; i_A | 0)\) will be denoted frequently by \(U^A(c; i_A | \theta_L)\) and \(U^A(c; i_A | \theta_H)\), respectively.

The expected total surplus given investment \( i_A \) and state \( \theta \) will be denoted by \(TS(i_A, \theta) \equiv E[TS(\alpha) \mid i_A, \theta]\). From (2.3), it follows that for any contract \( c \in C \),

\[
U^A(c; i_A | \theta) + U^P(c; i_A | \theta) = TS(i_A, \theta), \ \text{all} \ \theta \in \{\theta_L, \theta_H\}, \ \text{all} \ i_A \in I_A.
\] (2.4)

The first-best level of investment, given state \( \theta \), is the investment that maximizes total expected surplus, i.e.,

\[
i^0_A(\theta) \equiv \arg \max_{i_A \in I_A} TS(i_A, \theta).
\]
The agent’s investment decision, on the other hand, is given by the investment level that maximizes his expected payoff, given beliefs $b_L$ and contract $c$, i.e.,

$$i^*_A(c, b_L) \equiv \arg \max_{i_A \in I_A} U^A(c; i_A \mid b_L). \quad (2.5)$$

### 3 Internal Information and Exclusives: An Irrelevance Result

In this section, we begin by showing that if the principal does not have private information about the value of her outside option $V$, then exclusivity does not expand the set of equilibrium outcomes. More specifically, we show that for any equilibrium when the contract space is $C = R \times Q \times E$, there is an equilibrium with identical expected payoffs and identical agent investment when the contract space is $C' = R \times Q \times \{0\}$.

For future reference, equilibria satisfying this property are called *outcome equivalent*. Because this outcome equivalence result holds independently of whether the agent has an investment decision, it will prove useful in both of the following sections.

This result also holds independent of the way in which the state $\theta$ affects the distributions of internal values. In what follows, let $F_V(\cdot \mid \theta)$ denote the marginal c.d.f. of $V$ given state $\theta$.

**Proposition 1 (Irrelevance Result)** Suppose that the principal’s private information does not include information about her outside option, i.e., $F_V(\cdot \mid \theta) = F_V(\cdot)$ for all $\theta \in \{\theta_L, \theta_H\}$, and that investment affects only the internal values $v_p$ and $v_a$. Then exclusivity has no effect on the set of equilibrium outcomes: for any equilibrium when the contract space is $C = R \times Q \times E$, there is an outcome equivalent equilibrium when the contract space is $C' = R \times Q \times \{0\}$.

**Proof.** Taking expectations of (2.1) and (2.2), and using the facts that investment and information affect only internal values, the expected payoffs of the principal and the agent can be written as:

$$U^P(T_u, q, e; i^*_A(c, b_L) \mid \theta) = \frac{1}{2} E[\max\{V, V\} + q(v_p - v_a) \mid \theta, i^*_A(c, b_L)] + \frac{1}{2}(1 - q)(1 - e)E[V] + T_u, \quad \theta \in \Theta$$

and

$$U^A(T_u, q, e; i^*_A(c, b_L) \mid b_L) = b_L \frac{1}{2} E[\max\{V, V\} - q(v_p - v_a) \mid \theta_L, i^*_A(c, b_L)] + (1 - b_L) \frac{1}{2} E[\max\{V, V\} - q(v_p - v_a) \mid \theta_H, i^*_A(c, b_L)]$$

$$- \frac{1}{2}(1 - q)(1 - e)E[V] - \psi(i^*_A(c, b_L)) - T_u,$$

where $i^*_A(c, b_L)$ is as defined in (2.5). Notice that the agent’s payoff from investment is additively separable from both the exclusivity term $e$ and the transfer $T_u$. Thus, the agent’s investment choice does not depend on $e$ and $T_u$, so instead of $i^*_A(c, b_L)$ we can simply write $i^*_A(q, b_L)$. 

9
Given a contract \( c = (T_u, q, e) \in C \) and any beliefs \( b_L \), an outcome equivalent contract \( c' = (T_u', q', e' = 0) \in C' \) can be constructed in the following way:

\[
c'(c) = \begin{cases} 
  e' = 0 \\
  q' = q \\
  T_u' = T_u - \frac{1}{2}(1 - q)eE[V]
\end{cases}
\]

The fact that \( q' = q \) implies that for beliefs \( b_L \), the investment decision of the agent is the same under \( c \) and \( c' \). Moreover, from direct inspection of (3.1) and (3.2), we obtain that \( T_u' \) is such that expected payoffs of the principal (in both states \( \theta_L \) and \( \theta_H \)) and agent remain the same as under contract \( c \).

Finally, we show for each equilibrium when the contract space is \( C \) how to construct an outcome equivalent equilibrium when the contract space is \( C' \). Consider an equilibrium when the contract space is \( C \). For each menu of contracts \( m = \{c_1, ..., c_n(m)\} \) chosen with positive probability by the principal, construct a corresponding menu \( m'(m) \) of contracts in \( C' \) in the following way: \( m'(m) = \{c'(c_1), ..., c'(c_n(m))\} \). Let \( \mathcal{M}' \) denote the set of menus constructed in this way.\(^{13}\) Now, consider the following strategies when the contract space is \( C' \). In each of the states \( \theta \), the principal chooses menu \( m'(m) \in \mathcal{M}' \) with the same probability that she chooses menu \( m \) in the equilibrium when \( C \) is the contract space. Moreover, in each of the states \( \theta \), and for each menu \( m'(m) \in \mathcal{M}' \), the principal chooses contract \( c'(c_j) \) with the same probability that she chooses contract \( c_j \) in the equilibrium when \( C \) is the contract space. In a similar way, upon observing menu \( m'(m) \in \mathcal{M}' \) proposal and contract \( c'(c_j) \) choice by the principal, the agent’s actions and beliefs are the same as after observing menu \( m \) proposal and contract \( c_j \) choice by the principal in the equilibrium when \( C \) is the contract space. Menus \( m \notin \mathcal{M}' \) are not chosen by the principal and therefore are off-the-equilibrium path. For all menus off-the-equilibrium path, we let the actions of the principal and the agent and the beliefs of the agent be the same as in the initial equilibrium in when \( C \) is the contract space. Outcome equivalence together with the fact that \( C' \subseteq C \) implies that if the initial strategies constitute an equilibrium when \( C \) is the contract space then these strategies also constitute an equilibrium when \( C' \) is the contract space. \( \blacksquare \)

There are two crucial points in understanding this result. First, since investment affects internal values only, there is no cross effect between investment and exclusivity in the payoff functions. This implies that exclusivity does not affect the agent’s investment decision directly.\(^{14}\) Second, since information is only about internal values, there is also no cross effect between exclusivity and private information \( \theta \) on expected utilities of the principal and the agent. In other words, the expected utilities of the principal and the agent do not

\(^{13}\) We implicitly consider that the specific placement of contracts within a menu distinguishes contracts that are otherwise identical, and that when constructing \( \mathcal{M}' \) an irrelevant contract (e.g., a contract specifying a very low transfer such that is never chosen by the principal) is introduced in menus whenever necessary to differentiate between identical menus.

\(^{14}\) This would not be the case if the distribution of the value of the principal’s outside option \( V \) was a function of the agent’s investment.
satisfy the single crossing property with respect to exclusivity. Thus, for given beliefs, the principal can exchange exclusivity by up-front transfer in the contract and still generate the same expected utilities and the same agent’s investment decision.

The irrelevance result in this section extends the one in Segal and Whinston (2000) to a setting with asymmetric information. In that paper, the authors show that exclusivity has no effect on investment decisions when investment affects only internal values. Proposition 1 implies not only that exclusivity has no effect on investment decisions when investment affects only internal values, but also that exclusivity has no signalling effect when private information \( \theta \) is only about internal values to the relationship.

4 Contractual Signalling and Surplus Extraction

In this section, we focus solely on the principal’s task of extracting surplus. We do so by assuming that the agent cannot invest \( (I_A = \{0\}) \). In this case, the role of contracting is one of surplus sharing only (recall that renegotiation implies that the \( \text{ex-post} \) trade decisions are always efficient). Therefore, the only concern of the principal at the contract proposal stage is to extract surplus from the agent.

In settings in which the principal has private information at the contract proposal stage that directly affects the payoffs of the agent, surplus extraction may depend on the ability of the principal to signal her information. To emphasize the role of commitment to the relationship through quantity and exclusivity in signalling information to extract surplus, we first study in Section 4.1 the case in which only up-front transfers are contractible. Next, in Sections 4.2 and 4.3 we allow quantity and exclusivity to be contractible and analyze the cases of internal and external private information, respectively.

4.1 When Only Transfers are Contractible

When only up-front transfers are contractible one can restrict the analysis, without loss of generality, to menus with a single contract. This is the case because for each menu, only the contract with the highest transfer is relevant since both types of principal always select this contract from the menu. From Maskin and Tirole (1992) we know that any equilibrium outcome can be supported as a pooling equilibrium where all types of principal propose the same menu. In our context, this implies that it is without loss of generality to focus on pooling equilibria where both types of principal propose the same transfer.

The agent always accepts any contract in which he pays an up-front transfer no greater than his minimum (across states \( \theta \)) expected payoff. Therefore, only transfers no less than \( \min\{E(u_A | \theta_L), E(u_A | \theta_H)\} \) can be an equilibrium. Moreover, in a pooling equilibrium the transfer cannot exceed the expected payoff of the agent (where the agent’s beliefs coincide with the prior), otherwise the agent would reject the contract. In
fact, any transfer

\[ T_u \in [\min\{\mathbb{E}(u_A | \theta_L), \mathbb{E}(u_A | \theta_H)\}, \mathbb{E}(u_A | \theta_L) + (1 - p(\theta_L))\mathbb{E}(u_A | \theta_H)], \quad (4.1) \]

where

\[ u_A = \frac{1}{2} \max\{V, \bar{V}\} - \frac{1}{2} \bar{V} \]

can be an equilibrium. The following off-the-equilibrium path beliefs sustain any of these transfers as an equilibrium: \( b(\theta = \theta_i \mid t) = 1 \) for all \( t \neq T_u \), where \( \theta_i \) is the state under which \( \mathbb{E}(u_A | \theta) \) is the lowest. With these beliefs, any deviation by the principal to a contract with a higher transfer is not profitable because the agent would reject it.

The important feature of the equilibria presented here is that the principal is unable to signal her information and therefore the symmetric information outcome cannot be replicated. In particular, observe that in the state in which the value of \( \mathbb{E}(u_A | \theta) \) is the greatest, when proposing an equilibrium contract the principal already knows (since she knows the state \( \theta \)) that the agent will have a strictly positive expected payoff that she would like to extract. However, this is not possible because at that stage the agent still does not know the state \( \theta \) and may draw bad inferences about it after observing the principal’s contract proposal specifying a higher transfer. The next sections investigate whether this feature of the equilibria holds when the principal can contract on exclusivity and quantity in addition to transfers.

### 4.2 Internal Information

In many situations, better ex-ante information about a relationship stems only from superior information about internal valuations to the relationship. In this section, we focus on the case in which the principal is better informed about her valuation of the relationship \( v_p \).\(^{15}\) For example, in a buyer-seller transaction, this corresponds to the case of a better informed buyer about her valuation of the good offered by the seller.

Let \( F_{v_p}[\cdot \mid \theta] \) denote the marginal distribution of the valuation of the principal’s outside option \( V \) given state \( \theta \). In a similar way, let \( F_{v_a}[\cdot \mid \theta] \) and \( F_{v_p}[\cdot \mid \theta] \) denote the state \( \theta \) marginal distributions of the internal valuations \( v_a \) and \( v_p \), respectively. We model the case of private information about \( v_p \) by assuming that only the distribution of \( v_p \) depends on state \( \theta \) (i.e., \( F_{v_p}[\cdot \mid \theta_L] = F_{v_p}[\cdot \mid \theta_H] \)) and that \( F_{v_p}[\cdot \mid \theta_H] \) strictly first order stochastically dominates \( F_{v_p}[\cdot \mid \theta_L] \).\(^{16}\) With this specification, state \( \theta_H (\theta_L) \) corresponds to the state in which the principal expects to have a high (low) valuation of trading with the agent. This specification also implies that \( P_s(\theta) \), the probability of success of the relationship (i.e.,

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\(^{15}\)Below, we discuss briefly the case when the principal is better informed about the agent’s valuation \( v_a \).

\(^{16}\)A distribution \( F(x) \) strictly first order stochastically dominates a distribution \( G(x) \) if \( F(x) < G(x) \) for all \( x \) such that \( G(x) \neq 0 \) and \( F(x) \neq 1 \).
\[ P_s(\theta) \equiv P_r[V > \bar{V} \mid \theta], \] is high (low) in state \( \theta_H (\theta_L) \), i.e.,
\[ P_s(\theta_L) \leq P_s(\theta_H). \]

Since private information is only about internal values, by proposition 1 (Irrelevance Result) we can restrict attention to non-exclusive contracts (contracts in which \( e = 0 \)). Therefore, in this section a contract \( c \) is a transfer-quantity pair \((T_u, q)\). We thus write \( U^A(T_u, q \mid b_L) \) and \( U^P(T_u, q \mid \theta) \). The main question then is to know the extent to which the principal can use quantity and transfers to extract surplus from the agent.

**Proposition 2** Suppose \( Q = [0, 1] \) and that the principal has private information only about her own valuation of the relationship \( v_p \). Then the principal always extracts all the surplus, i.e., in any equilibrium the principal’s payoffs are
\[ \overline{U}^P(\theta) = TS(\theta) \text{ for all } \theta \in \{\theta_L, \theta_H\}. \]
Moreover, the principal of type \( \theta_H \) always commits ex-ante to a higher quantity than the principal of type \( \theta_L \). Formally, suppose \( q^H \) is the contracted quantity chosen in an equilibrium by the principal of type \( \theta_H \), while \( q^L \) is the contracted quantity chosen in an equilibrium by the principal of type \( \theta_L \) (not necessarily the same equilibrium). Then, there exists \( \tilde{q} \in [0, 1] \) such that \( q^L \leq \tilde{q} \leq q^H \).

**Proof.** See Appendix A. \( \blacksquare \)

Proposition 2 establishes that the principal extracts all the surplus in both states \( \theta_L \) and \( \theta_H \). This implies that when quantity is contractible, the symmetric information outcomes are replicated. This is the case because the principal is able to construct a menu of contracts (which is always accepted by the agent) that simultaneously satisfies incentive compatibility (IC) and allows her to extract the entire expected surplus in both states.

To illustrate that IC and full surplus extraction are compatible when quantity is contractible, consider a menu of contracts \( m = \{c^L, c^H\} \).\(^{17}\) Full surplus extraction means that each contract in \( m \) must leave the principal with the entire expected total surplus, i.e., for \( i = L, H \),
\[ c^i \in \{c : U^P(c \mid \theta_i) = TS(\theta_i)\} \equiv I(\theta_i), \]
where the expected payoff of the principal in state \( \theta \in \{\theta_L, \theta_H\} \) is given by
\[ U^P(c \mid \theta) = \frac{1}{2}TS(\theta) + \frac{1}{2}(q(E[v_p \mid \theta] - E[v_a]) + (1 - q)E[V]) + T_u. \]

\(^{17}\)By the Principle of Inscrutability (see Myerson (1983)) we can restrict, without loss of generality, to equilibria in which both types of principal propose the same menu and then choose their favorite contract from the menu.
The question is then whether contracts \( \{c_L, c_H\} \) can also be incentive compatible for the principal. Note that, for each \( \theta \in \{\theta_L, \theta_H\} \), the full surplus extraction set \( I(\theta) \) is the indifference curve of the principal of type \( \theta \) with associated payoff \( TS(\theta) \). Moreover, when private information is about \( v_p \), the expected payoff function of the principal exhibits the single crossing property: the marginal rate of substitution between transfer and quantity given by

\[
\frac{\partial T_u}{\partial q} \bigg|_{U^P(c|\theta)=\overline{U}} = -\frac{1}{2}(E[v_p|\theta] - E[v_a] - E[V])
\]

is bigger in state \( \theta_L \) than in state \( \theta_H \), since \( E[v_p|\theta_H] \geq E[v_p|\theta_L] \). We plot in Figure 1 the indifference curves \( I(\theta), \theta \in \{\theta_L, \theta_H\} \), which by (2.4) (efficiency of ex-post renegotiation) are also indifference curves of the agent giving him zero payoff, i.e., they are the agent’s individual rationality constraints (IR(\( \theta \))), \( \theta \in \{\theta_L, \theta_H\} \).

Suppose now that contract \( c_L \) is in the segment AB and contract \( c_H \) is in the segment BD in Figure 1. Then menu \( m = \{c_L, c_H\} \) is clearly IC: the principal of type \( \theta_L (\theta_H) \) prefers contract \( c_L (c_H) \) to contract \( c_H (c_L) \) (as contract \( c_L (c_H) \) lies in an indifference curve with higher payoff than contract \( c_H (c_L) \)). Therefore, menu \( m \) is IC and allows full surplus extraction by the principal. Finally, note that the agent always accepts menu \( m \) regardless of his beliefs about the principal: by IC of menu \( m \), the agent knows that a principal of type \( \theta_L \) chooses contract \( c_L \) from \( m \), in which case he gets his reservation payoff, and a principal of type \( \theta_H \) chooses contract \( c_H \) from \( m \), in which case he also gets his reservation payoff. Thus, independently of the principal’s type the agent always obtains at least his reservation payoff if he accepts menu \( m \). The menus for which \( c_L \) is in the segment AB and \( c_H \) is in the segment BD are the menus that can be offered by the principal in an equilibrium.

18 For concreteness, we plot the case in which both curves are upward slopping. The fact that the indifference curves \( I(\theta_L) \) and \( I(\theta_H) \) intersect at \( q = \hat{q} \in [0,1] \) is a property of the model. For a formal derivation of this property, see the proof of proposition 2 in appendix A.
In this case, quantity plays a crucial role in surplus extraction because it has a strong signalling effect. By increasing quantity at the contracting stage, the principal increases the probability that her disagreement payoff (in a future renegotiation) is her valuation of trade with the agent. Therefore, the marginal gain of increasing contracted quantity is bigger for a principal who expects to have a high valuation of trade with the agent than it is for a principal who expects to have a low valuation of trade with the agent. This allows a principal who believes more in the success of the relationship (because she expects to have a high valuation of trade with the agent) to signal her information to the agent by promising \textit{ex-ante} to trade with him in the future with higher probability (committing \textit{ex-ante} more to the relationship).

4.3 External Information

In this section, we analyze the case in which the principal is better informed than the agent about her future value of trading with an external party \( \bar{V} \). We model the case of \textit{private information about the external valuation} \( \bar{V} \) by assuming that the distributions of internal valuations, both of \( v_a \) and of \( v_p \), do not depend on state \( \theta \) (i.e., \( F_{v_j}[\cdot | \theta_L] = F_{v_j}[\cdot | \theta_H] \) for \( j = a, p \)), and that \( F_{\bar{V}}[\cdot | \theta_L] \) strictly first order stochastically dominates \( F_{\bar{V}}[\cdot | \theta_H] \). Under these conditions, state \( \theta_H (\theta_L) \) corresponds to the state in which the principal has a high (low) outside option. Therefore, like in the previous section, state \( \theta_H (\theta_L) \) is associated with a high (low) probability of success of the relationship, i.e.,

\[ P_s(\theta_L) \leq P_s(\theta_H). \]

Since exclusivity may play a role in the equilibrium of the contracting game when private information is external, this variable is reconsidered. Hence, a contract is a triple \( c = (T_u, q, e) \in C \). The following proposition establishes the main results of this section.

\textbf{Proposition 3} \textit{Suppose that the principal has private information only about the external valuation \( \bar{V} \). If} \( Q = [0, 1] \) (and \( 0 \in E \)) \textit{or} \( E = [0, 1] \) (and \( 0 \in Q \)) \textit{then the principal always extracts all the surplus, i.e., in}

\[ P_s(\theta_L) \leq P_s(\theta_H). \]

\[ 15 \]
any equilibrium the principal’s payoffs are

\[ \bar{U}^P(\theta) = \bar{T}\bar{S}(\theta) \text{ for all } \theta \in \{\theta_L, \theta_H\}. \]

Moreover, the principal always commits more to the relationship in state \( \theta_H \) than in state \( \theta_L \). Formally, suppose \((q^L, e^L)\) is the contracted quantity-exclusivity pair chosen in an equilibrium by the principal of type \( \theta_L \), while \((q^H, e^H)\) is the contracted quantity-exclusivity pair chosen in an equilibrium by the principal of type \( \theta_H \) (not necessarily the same equilibrium). Then, there exists \( \bar{x} \in [0,1] \) such that \((1 - q^H)(1 - e^H) \leq \bar{x} \leq (1 - q^L)(1 - e^L)\).

**Proof.** See Appendix A. ■

Propositions 2 and 3 establish that the principal extracts all the surplus in each of the states \( \theta \in \{\theta_L, \theta_H\} \). In both cases, the result stems from the principal’s ability to construct a menu of contracts (which is always accepted by the agent) that simultaneously satisfies incentive compatibility (IC) and allows her to extract the entire expected surplus in both states. The only difference between the two cases is that, when the principal’s private information is external (proposition 3), the principal can use not only quantity but also exclusivity to design such menus.

When private information is about the external valuation \( V \), we can write the expected payoff of the principal given state \( \theta \) and contract \( c \) as:

\[ U^P(c \mid \theta) = \frac{1}{2} \bar{T}\bar{S}(\theta) + \frac{1}{2} \{q(\mathbb{E}[v_p] - \mathbb{E}[v_a]) + (1 - q)(1 - e)\mathbb{E}[V \mid \theta]\} + T_u. \]

In this case, \( U^P(c \mid \theta) \) exhibits the single crossing property with respect to both variables \( q \) and \( e \). In particular, the marginal rate of substitution between \( T_u \) and \( q \):

\[ \left. \frac{\partial T_u}{\partial q} \right|_{U^P(c \mid \theta) = \bar{U}} = -\frac{1}{2} (\mathbb{E}[v_p] - \mathbb{E}[v_a] - (1 - e)\mathbb{E}[V \mid \theta]) \]

and the one between \( T_u \) and \( e \):

\[ \left. \frac{\partial T_u}{\partial e} \right|_{U^P(c \mid \theta) = \bar{U}} = \frac{1}{2} (1 - q)\mathbb{E}[V \mid \theta], \]

are bigger in state \( \theta_L \) than in state \( \theta_H \), since \( \mathbb{E}[V \mid \theta_L] > \mathbb{E}[V \mid \theta_H] \). We depict in Figure 3 the full surplus extraction curves \( I(\theta_L)|_{q=0} \) and \( I(\theta_H)|_{q=0} \) for the case in which \( q = 0 \) and \( E = [0,1] \).

\[ ^{20} \text{The case of } Q = [0,1] \text{ and } e = 0 \text{ is graphically identical to the one in Figure 1. When we impose } q = 0, \text{ we denote the full surplus extraction sets by } I(\theta_L)|_{q=0} \text{ and } I(\theta_H)|_{q=0} \text{ in order to distinguish from the general full surplus extraction sets } I(\theta) \equiv \{(T_u, q, e) : U^P(T_u, q, e \mid \theta) = \bar{T}\bar{S}(\theta)\}. \text{ For concreteness, we plot the case in which both curves are upward sloping. The fact that the indifference curves } I(\theta_L)|_{q=0} \text{ and } I(\theta_H)|_{q=0} \text{ intersect at } e = \bar{e} \in [0,1] \text{ is a property of the model. For a formal derivation of this property, see the proof of proposition 3 in appendix A.} \]
All the menus $m = \{c^L, c^H\}$ for which contracts $c^L$ and $c^H$ belong to the full surplus extraction sets $I(\theta_L)$ and $I(\theta_H)$ respectively, and satisfy condition $(1 - q^H)(1 - e^H) \leq \hat{\alpha} \leq (1 - q^L)(1 - e^L)$ are IC and can be offered by the principal in equilibrium. Menus consisting of a contract $c^L$ in segment AB and a contract $c^H$ in segment BD in Figure 2 are just examples of such menus.

Let us focus on the intuition for the result that $q$ and $e$ can be used as signalling instruments when private information is external. By increasing $e$ or $q$ at the contracting stage, the principal reduces the possibility of dealing with external parties in the future, which lowers her disagreement payoff during an eventual renegotiation of the initial contract. Giving away the possibility of dealing with a third party in the future is more costly to a principal who expects to have a high outside option (type $\theta_L$) than it is to a principal who expects to have a low outside option (type $\theta_H$). Therefore, a principal who expects to have a low outside option in the future can signal this information to the agent by proposing contracts specifying a high quantity and/or a high exclusivity level.

Proposition 3 has two important implications. First, the more a principal “believes” in the relationship (because she expects a future low outside option), the more she is willing to commit to it, \textit{ex-ante}, in order to signal her type. Note that the value $(1 - q)(1 - e)$ can be used as a measure of \textit{ex-ante} (non)commitment to the relationship. More specifically, increasing $(1 - q)(1 - e)$ implies decreasing either $q$ or $e$. This means less commitment to the relationship, either by directly decreasing the contracted probability of dealing with each other or by decreasing the probability of not dealing with a third party. Therefore, signalling through commitment follows directly from the fact that in any equilibrium $(1 - q^H)(1 - e^H) \leq (1 - q^L)(1 - e^L)$.

Second, the principal can use either quantity or exclusivity to extract surplus. In this sense, when private information is external, there is perfect substitutability between these two variables as signalling instruments. Hence, while adding exclusivity as a contractible variable to up-front transfers changes the nature of the equilibria and the set of equilibrium payoffs, these remain unchanged when adding exclusivity as a contractible variable when quantity is already contractible. This suggests that the ability to commit to
a relationship through exclusivity has no incremental effect in terms of surplus extraction when commitment through quantity is possible.

The next section investigates whether these conclusions also hold in the more general setting where the agent has the opportunity to invest in the relationship with the principal.

5 Contractual Signalling and Agent’s Investment

In this section, we consider that the agent has the opportunity to make a noncontractible investment decision that affects the value of the relationship with the principal. This decision is made after the agent learns the contract chosen by the principal but before the realization of uncertainty.

Allowing the agent to make a noncontractible investment decision that affects the value of the relationship with the principal is appealing for at least two reasons. First, for its realism. In many circumstances the value of a relationship between two or more parties depends on how much effort they put in the relationship, or on how much they invested in getting prepared to it. Moreover, certain forms of effort and investment cannot be easily verified by a court, which renders them as noncontractible. Second, because it enables us to study efficiency implications. As a consequence of ex-post renegotiation, efficiency is always achieved in the version of the model without investment. When we consider investment in the model, this is not necessarily the case. In contrast to the trading decision, the agent’s investment decision is irreversible at the renegotiation stage. Hence, efficiency of investment is not ensured by renegotiation.

The possible inefficiency associated with the level of non-contractible investment has two sources: the agent’s uncertainty about the state $\theta$, and a problem of hold-up that emerges due to ex-post renegotiation. Since exclusivity and up-front transfer do not affect directly the investment decision of the agent, the investment inefficiency with hold-up cannot be reduced if these two variables are the only ones contractible. In contrast, contracted-upon quantity does affect directly the investment decision of the agent. Hence, contracted quantity will be used by the principal not only to signal private information to the agent and increase surplus extraction (as in Section 4), but also to provide the right incentives to invest and potentially eliminate the hold-up inefficiency. The main question then is to investigate whether these two roles of contracted quantity conflict with one another and to explore the resolution of their conflict if they do.

Agent’s Investment and Information Specifications

We consider the case of selfish investment, i.e., the agent’s investment affects only his own valuation of the relationship. Specifically, we assume that this valuation is given by the non-stochastic increasing function

\[ v(\theta) = f(\theta) \]

21 The existence of noncontractible investments that are relationship-specific is the central assumption of the literature on the hold-up problem (see for example Edlin and Reichelstein (1996), Macleod and Malcomson (1993), Rogerson (1992), Che and Hausch (1999)).
\( v_a(i_A) \).\(^{22}\)

As before, only the distribution of the variable that is the source of private information depends on \( \theta \). However, for simplicity, it is further assumed that only this variable is a random variable.

As in Section 4, we focus on two sources of the principal’s private information:

(i) **Private information about** \( v_p \) ("internal information"): valuations \( V \) and \( v_a(i_A) \) do not depend on state \( \theta \) and \( F_{v_p}[\cdot | \theta_H] \) strictly first order stochastically dominates \( F_{v_p}[\cdot | \theta_L] \);

(ii) **Private information about** \( V \) ("external information"): valuations \( v_p \) and \( v_a(i_A) \) do not depend on state \( \theta \) and \( F_{\theta[V]}[\cdot | \theta_H] \) strictly first order stochastically dominates \( F_{\theta[V]}[\cdot | \theta_H] \).

When the agent makes an investment decision, the probability that the relationship between the principal and the agent succeeds depends on this level of investment, rather than only on the state \( \theta \). Recall that, due to *ex-post* renegotiation, the probability of success of the relationship \( P_s(i_A, \theta) \) is the probability that the value of trade \( V(i_A) \equiv v_a(i_A) + v_p \) between the principal and the agent is bigger than the value of trade \( V \) between the principal and the external party. Given the above specifications, this probability is always bigger in state \( \theta_H \) than it is in state \( \theta_L \), for each level of investment \( i_A \), i.e.,

\[
P_s(i_A, \theta_H) \geq P_s(i_A, \theta_L) \text{ for all } i_A \in I_A.
\]

Moreover, from the fact that \( v_a(i_A) \) is an increasing function of \( i_A \), it follows that \( P_s(i_A, \theta) \) is increasing in the level \( i_A \) for all \( \theta \in \{ \theta_L, \theta_H \} \).

The agent’s investment decision maximizes his expected payoff following the observation of the contract \( c \) proposed by the principal, i.e.,

\[
i^*_A(c, b_L) \equiv \arg \max_{i_A \in I_A} b_L U^A(c; i_A | \theta_L) + (1 - b_L) U^A(c; i_A | \theta_H),
\]

where

\[
U^A(c; i_A | \theta) = \frac{1}{2} \mathbb{E}[\max\{V(i_A), V\} | \theta] - \frac{1}{2} \mathbb{E}[q(v_p - v_a(i_A)) + (1 - q)(1 - e)\theta - \psi(i_A) - T_u]. \tag{5.1}
\]

Throughout, we assume that \( U^A(c; i_A | \theta) \) is twice continuously differentiable and strictly concave in investment \( i_A \), for any contract \( c \) and state \( \theta \).

Therefore, given beliefs \( b_L \) and a contract \( c \), the agent’s optimal investment decision is unique and satisfies the following first-order condition:

\[
\frac{1}{2} \psi'(i_A)[b_L P_s(i_A, \theta_L) + (1 - b_L) P_s(i_A, \theta_H) + q] = \psi'(i_A). \tag{5.2}
\]

\(^{22}\text{Concentrating on the case of selfish investment, as opposed to the case of cooperative investment (i.e., } i_A \text{ affects both the principal and the agent’s valuations), allows us to better assess the effect of asymmetry of information at the contracting stage on efficiency of investment. Note that, contrarily to the case of selfish investment, a contract ensuring efficient cooperative investment may not exist even when information is symmetric at the contracting stage (see for example Che and Hausch(1999)).} \)}
From (5.2), we obtain that the agent’s investment decision depends on contract \( c \) only through the quantity \( q \). Thus, henceforth we use \( i_A^*(c; b_L) \) and \( i_A^*(q; b_L) \) interchangeably. Moreover, since \( P_s(i_A, \theta_H) \geq P_s(i_A, \theta_L) \) for all \( i_A \in I_A \), the agent’s investment is decreasing in his belief \( b_L \). Intuitively, agents who believe that the relationship with the principal is less likely to succeed are willing to invest less on it. Finally, note that the agent’s investment decision increases with the contracted quantity \( q \). Intuitively, when facing a contract specifying a high quantity level, the agent knows that with high probability his disagreement payoff at the renegotiation stage will be his valuation of the relationship \( v_a(i_A) \). Therefore, in that case, his incentives to invest are also high in order to protect his disagreement payoff.

We summarize the above results in the following Lemma.

**Lemma 1** The agent’s investment decision \( i_A^*(q; b_L) \) is increasing with quantity \( q \) and decreasing with beliefs \( b_L \).

Although the agent’s investment \( i_A \) does not affect the principal’s valuation of the relationship \( v_p \), it affects her expected payoff:

\[
U^P(c; i_A | \theta) = \frac{1}{2} \mathbb{E}[\max\{V(i_A), V\} | \theta] + \frac{1}{2} \mathbb{E}[q(v_p - v_a(i_A)) + (1 - q)(1 - e)V | \theta] + T_u. \tag{5.3}
\]

In fact, it does so through two different ways: by affecting the total surplus (the first term in (5.3)), which is a positive effect; and by affecting the agent’s disagreement payoff \((-q v_a(i_A))\) in the second term in (5.3)), which is a negative effect. Differentiating \( U^P(c; i_A | \theta) \) with respect to \( i_A \), we obtain

\[
\frac{1}{2} \frac{d}{dt} v_a(i_A)[P_s(i_A, \theta) - q]. \tag{5.4}
\]

Hence, which effect is the dominant one, depends on the relative values of the contracted quantity \( q \) and the probability of success of the relationship \( P_s(i_A, \theta) \). In particular, note that when contracted quantity is zero, the “total surplus effect” dominates and therefore the principal’s payoff is increasing with investment.

When contracted quantity is one, the reverse happens. Finally, note from (5.4) that the principal’s payoff responds more positively to investment when the probability of success of the relationship is high. For future convenience, we state this result in the following Lemma.

**Lemma 2** The expected payoff of the principal of type \( \theta_H \) responds more positively to the agent’s investment than the expected payoff of the principal of type \( \theta_L \), i.e., for all \( i_1, i_2 \in I_A \) such that \( i_2 \geq i_1 \) and for all \( c \in C \),

\[
U^P(c; i_2 | \theta_H) - U^P(c; i_1 | \theta_H) \geq U^P(c; i_2 | \theta_L) - U^P(c; i_1 | \theta_L).
\]

**A Benchmark Case: Symmetric Information Contracting**

20
Suppose that the agent also knows the state $\theta$ at the contracting and investment stages. Then, given contract $c = (T_u, q, e)$ and state $\theta$, the agent’s investment decision solves:

$$\max_{i_A \in I_A} U^A(c; i_A \mid \theta).$$

(5.5)

In a symmetric information environment, the contracting problem faced by the principal of type $\theta$ consists of choosing the contract that maximizes her expected payoff taking into account the individual rationality constraint and the investment decision of the agent, i.e.,

$$\max_{c = (T_u, q, e) \in C} U^P(0, q, e; i_A \mid \theta) + T_u$$

s.t. (i) $U^A(0, q, e; i_A \mid \theta) - T_u \geq 0$

(ii) $i_A \in \arg\max_{x \in I_A} U^A(c; x \mid \theta)$

It is easy to see that in any solution to this problem, constraint (i) must bind. Therefore, using the fact that ex-post renegotiation implies that $U^P(0, q, e; i_A \mid \theta) + U^A(0, q, e; i_A \mid \theta) = TS(i_A, \theta)$, the problem can be rewritten as:

$$\max_{\{q, e, i_A\}} TS(i_A, \theta)$$

(5.6)

s.t. $i_A \in \arg\max_{x \in I_A} U^A(0, q, e; x \mid \theta)$

Hence, the principal always proposes the contract that induces the agent to invest as efficiently as possible and uses the transfer $T_u$ to extract all the surplus. We shall see that this contrasts with the asymmetric information environment where, when choosing the contract, the principal also takes into account the need to signal her type in order to extract surplus.

In the case of symmetric information, the first-best level of investment given state $\theta$ is always implementable. Note that, the first-best investment $i^*_A(\theta)$ solves the problem

$$\max_{i_A \in I_A} TS(i_A, \theta) = \max_{i_A \in I_A} \mathbb{E} [\max\{V(i_A), V\} \mid \theta] - \psi(i_A),$$

for which the first-order condition is

$$v'_a(i_A) P_s(i_A, \theta) = \psi'(i_A).$$

(5.7)

We assume that total expected surplus $TS(i_A, \theta)$ is a strictly concave function of investment $i_A$ in each state $\theta \in \{\theta_L, \theta_H\}$.

In a similar way, from (5.2) we obtain that the agent’s investment decision when he knows state $\theta$ with certainty has to satisfy:

$$\frac{1}{2} v'_a(i_A) [P_s(i_A, \theta) + q] = \psi'(i_A).$$

(5.8)
Comparing (5.7) and (5.8), we obtain that the principal can induce the agent to invest the first-best investment in state $\theta$ by setting

$$q = P_s(i_A^0(\theta), \theta) \equiv q^0(\theta),$$

i.e., by setting the contracted quantity equal to the probability of success of the relationship evaluated at the first-best investment $i_A^0(\theta)$. Therefore, when information is symmetric, the principal always receives the first-best total expected surplus $TS(i_A^0(\theta), \theta)$.

### 5.1 When Only Transfers are Possible

We begin the analysis of the asymmetric information environment by presenting the equilibrium when only transfers are contractible. Thus, a contract is denoted by $c = (T_u)$ and, with a slight abuse of notation, the agent’s investment decision is denoted by $i_A^*(b_L)$ instead of $i_A(q = 0, b_L)$. In order to facilitate comparison with the case without investment (Section 4.1), we allow the principal to propose only menus with a single contract in this subsection.\(^{23}\)

**Proposition 4** Suppose that neither quantity nor exclusivity is contractible ($Q = E = \{0\}$) and that the principal proposes only menus with a single contract. Then, there are both pooling and separating equilibria. In any pooling equilibrium the principal proposes a transfer

$$T_u \in [TS(i_A^*(1), \theta_L) - U^P(0; i_A^*(p(\theta_L)) | \theta_L), U^A(0; i_A^*(p(\theta_L)) | p(\theta_L))],$$

where $p(\theta_L)$ is the (prior) probability that the principal is of type $\theta_L$. In any separating equilibrium the principal of type $\theta_L$ proposes transfer $T_u^L = T_u^L = U^A(0; i_A^*(1) | \theta_L)$ and the principal of type $\theta_H$ proposes a transfer

$$T_u^H \in [T_u^L - \{U^P(0; i_A^*(0) | \theta_H) - U^P(0; i_A^*(1) | \theta_H)\},$$

$$T_u^L - \{U^P(0; i_A^*(0) | \theta_L) - U^P(0; i_A^*(1) | \theta_L)\}].$$

Moreover, there is no equilibrium in which the principal of type $\theta_H$ extracts all the surplus (generated in state $\theta_H$) and no equilibrium in which investment is efficient in both states $\theta_L$ and $\theta_H$.

**Proof.** See Appendix B. $\blacksquare$

The result in the proposition is important for two reasons. First, because it shows that there exist separating equilibria even when the principal can use only up-front transfers in the contract and restricts

\(^{23}\)All the qualitative results about the equilibrium that are emphasized in this subsection, also hold if the principal proposes menus with more than one contract; specifically, the result that there are both separating and pooling equilibria and the results regarding surplus extraction and investment efficiency.
herself to single contract menus, which contrasts with the setting without investment. This is the case because the agent’s investment affects differently the expected payoffs of the principal in state $\theta_L$ and in state $\theta_H$, and because the agent’s investment decision depends on his beliefs about the state $\theta$. In the state in which the payoff of the principal is more sensitive to investment (state $\theta_H$), she is willing to give up some surplus extraction by proposing a lower up-front transfer in order to signal to the agent that the relationship is successful and induce him to invest more. This identifies another reason for the principal to wish to signal her type, in addition to pure surplus extraction.

Second, the result is important because it establishes that when the principal can only contract on up-front transfers, neither efficiency in terms of investment nor full surplus extraction occur in equilibrium. This motivates the next two sections, where we investigate the roles of quantity and exclusivity in situations in which the principal uses the contract not only to extract surplus but also to provide incentives for the agent to invest efficiently. We do this by allowing first quantity to be contractible and then both quantity and exclusivity.

5.2 Quantity Contracts

When the agent cannot invest in the relationship, we have already shown in the previous section that contractibility of quantity solves the surplus extraction problem, both in the cases of internal private information (about $v_p$) and external private information (about $V$). Thus, the remaining question is whether surplus extraction and investment efficiency can be achieved simultaneously when the principal can use only contracted quantity to commit to the relationship.

In this section, a contract is a transfer-quantity pair, i.e., $c = (T, q)$. We consider the cases of private information about $v_p$ and private information about $V$. All the analysis in this section holds for both cases.

We proceed as follows. We first define a specific allocation that will be used to characterize the equilibria of the game. We then derive it, and finally we characterize equilibria.

To characterize the set of equilibria of the game, we focus on the following allocation: the best separating equilibrium allocation for the principal that leaves the agent with at least his reservation value both in state $\theta_L$ and in state $\theta_H$ (henceforth, the best separating allocation).

Formally, a menu of contracts $\hat{m} = \{c^L, c^H\}$ constitutes a best separating allocation if and only if, for all
where $\beta_L = 1$ and $\beta_H = 0$. That is, each type of principal maximizes (in independent maximizations) her own utility within the set of allocations that are incentive compatible, and regardless of the principal’s type, yields the agent a nonnegative payoff. Note two things. First, incentive compatibility depends on the agent’s investment decisions following the principal’s choice of contract $c$ in $m = \{c_L, c_H\}$, which in turn depends on the agent’s beliefs. In the definition of the best separating allocation we implicitly assume that the agent’s beliefs are: $b_L = 1$ after observing contract choice $c_L$ and $b_L = 0$ after observing contract choice $c_H$ (henceforth, separating beliefs). Second, a best separating allocation is IC given these separating beliefs.\footnote{We state and prove this result in Appendix B.} These two facts have two implications. First, although obtaining the best separating allocation involves performing two independent maximizations (one for the principal of type $\theta_L$ and another one for the principal of type $\theta_H$), the best separating allocation $\tilde{m} = \{\tilde{c}_L, \tilde{c}_H\}$ solves problem 5.9 for both types. Second, following the proposal of a best separating allocation $\tilde{m} = \{\tilde{c}_L, \tilde{c}_H\}$ by the principal, there is always a continuation equilibrium in which the agent accepts the proposal and the principal of type $\theta_j$ chooses contract $\tilde{c}_j$ from $\tilde{m}$, $j = L, H$. These two properties will be used below to obtain the best separating allocation and to characterize equilibrium contracting and investment.

In the rest of this section, we impose the following condition:

**Condition 1** The quantity that induces the agent to choose the first-best investment in state $\theta_L$, $q^0(\theta_L)$, is such that

$$q^0(\theta_L) \leq \pi(q^0(\theta_L), 1),$$

where $\pi(q, b_L)$ is the quantity level for which the expected payoffs of the agent in both states $\theta_L$ and $\theta_H$, given investment $i^*_A(q, b_L)$, are the same, i.e.,

$$\pi(q, b_L) : U^A(0, \pi(q, b_L); i^*_A(q, b_L) \mid \theta_L) = U^A(0, \pi(q, b_L); i^*_A(q, b_L) \mid \theta_H).$$

Imposing Condition 1 allows us to concentrate the attention on the payoffs in state $\theta_H$. This condition implies that when the contract specifies quantity $q^0(\theta_L)$, the payoff of the agent is decreasing in the belief $b_L$ that the principal is of type $\theta_L$. As we will see below, this has two implications. First, the payoff
of the principal of type $\theta_L$ associated with the best separating allocation is the first-best total surplus $\mathcal{T}(\bar{i}_A^L(\theta_L), \theta_L)$. Second, in state $\theta_L$ the principal can ensure herself at least $\mathcal{T}(i_A^L(\theta_L), \theta_L)$, regardless of the agent’s beliefs.

Let us derive the best separating allocation.

**Deriving the Best Separating Allocation**

We start by analyzing the payoff of the principal of type $\theta_L$ and the contract $\bar{c}^L$ associated with the best separating allocation in state $\theta_L$.

**Lemma 3** The payoff of the principal of type $\theta_L$ associated with the best separating allocation is $\mathcal{T}(i_A^L(\theta_L), \theta_L)$. Moreover, contract $\bar{c}^L = (\bar{T}_u^L, \bar{q}^L)$ is given by $\bar{q}^L = q^0(\theta_L)$ and $\bar{T}_u^L = U^A(0, q^0(\theta_L); i_A^L(q^0(\theta_L), 1) \mid \theta_L)$.

**Proof.** See Appendix B. □

Under Condition 1, given the separating beliefs, it is always possible to construct an incentive compatible menu of contracts satisfying the agent’s individual rationality constraint in each of the states $\theta_L$ and $\theta_H$, that leaves the principal of type $\theta_L$ with the first-best total surplus $\mathcal{T}(i_A^L(\theta_L), \theta_L)$. Moreover, the only contract compatible with the principal’s payoff $\mathcal{T}(i_A^L(\theta_L), \theta_L)$ and nonnegativity of the agent’s expected utility given state $\theta_L$, is given by quantity $q^L = q^0(\theta_L)$ and transfer $T_u^L$ such that the principal extracts all the surplus from the agent in state $\theta_L$ (i.e., $\text{IR}(\theta_L)$ binds).25 Thus, in the best separating allocation, this is the contract associated with the principal of type $\theta_L$.

Next, we analyze the payoff of the principal of type $\theta_H$ and the contract $\bar{c}^H$ associated with the best separating allocation. Given Lemma 3 and the observation that the best separating allocation solves (5.9) for both types of principal, finding the payoff of the principal of type $\theta_H$ associated with the best separating allocation consists solely of finding the contract $c^H = (T_u^H, q^H)$ that solves the following problem,

**Problem 1**

$$\max_{(T_u^H, q^H)} U^P(0, q^H; i_A^H(q^H, 0) \mid \theta_H) + T_u^H$$

s.t. (i) $\mathcal{T}(i_A^L(\theta_L), \theta_L) \geq T_u^H + U^P(0, q^H; i_A^H(q^H, 0) \mid \theta_L)$ (IC($\theta_L$))

(ii) $T_u^H + U^P(0, q^H; i_A^H(q^H, 0) \mid \theta_H) \geq \bar{T}_u^L + U^P(0, q^0(\theta_L); i_A^L(q^0(\theta_L), 1) \mid \theta_H)$ (IC($\theta_H$))

(iii) $U^A(0, q^H; i_A^H(q^H, 0) \mid \theta_H) - T_u^H \geq 0$ (IR($\theta_H$))

25Observe that, given state $\theta_L$, if the principal’s payoff equals the first-best total surplus then the agent’s payoff is nonnegative only if investment is efficient, i.e., $i_A^L(\theta_L)$. Moreover, when the agent’s beliefs are $b_L = 1$, only a contract specifying quantity $q^L = q^0(\theta_L)$ induces the agent to choose the first-best investment $i_A^L(\theta_L)$.
Problem 1 consists of finding the contract that maximizes the payoff of the principal in state $\theta_H$, assuming that the agent knows the true state of the world (due to separation, the agent correctly infers $\theta$ from the principal’s contract choice). The first two constraints, the usual IC conditions, impose that each type of principal prefers not to deviate and mimic the other type; otherwise separation would not occur. Constraint (iii), the IR($\theta_H$) constraint, imposes that the payoff of the agent who knows that the state is $\theta_H$ cannot be lower than his reservation value. For each quantity level $q^H$, constraints IC($\theta_L$) and IR($\theta_H$) impose an upper bound on the value of the up-front transfer $T_u^H$.

Deriving the solution to Problem 1 will offer important insights regarding the crucial effects leading to the existence of inefficient equilibria. Thus, we do it here.

First, in any solution to Problem 1, at least one of the constraints IC($\theta_L$) or IR($\theta_H$) is binding. Suppose that this was not the case; then it would be possible to increase $T_u^H$ by an arbitrarily small amount $\epsilon > 0$ and still have all the constraints in the problem satisfied (including IC($\theta_H$)) while increasing the objective function, which would be a contradiction. Second, we can be more specific about which constraint is binding.

There are two possible cases, which we consider separately.

**Case 1:** At $q^H = q^0(\theta_H)$ the binding constraint is IR($\theta_H$). This case is depicted in Figure 3.

When IR($\theta_H$) is the binding constraint at $q^H = q^0(\theta_H)$, first-best investment and full surplus extraction in both states $\theta_L$ and $\theta_H$ is incentive compatible. Therefore, the solution to Problem 1 involves $q^H = q^0(\theta_H)$ and $T_u^H$ such that IR($\theta_H$) binds. This implies that the payoff of the principal of type $\theta_H$ associated with the best separating allocation is the first-best total surplus in state $\theta_H$, $TS(i^0_A(\theta_H), \theta_H)$ and $c^H = (\hat{T}_u^H, q^0(\theta_H))$ where $\hat{T}_u^H = U^A(0, q^0(\theta_H); i^*_A(q^0(\theta_H), 0) | \theta_H)$.

\[26\] Usually in this type of problem constraint IC($\theta_H$) is not binding. Therefore it is omitted during the solution determination of the problem and checked to be satisfied \textit{ex-post}. We do this in Appendix B.
In this case, there is no conflict between surplus extraction and efficiency: the best separating allocation implies full surplus extraction and full efficiency in terms of the agent’s investment decision.

**Case 2:** At \( q^H = q^0(\theta_H) \) the binding constraint is \( IC(\theta_L) \). This case is depicted in Figure 4.

![Figure 4.](image)

To study the solution to Problem 1 in this case, we first analyze which constraint is binding for \( q^H \in [q^0(\theta_H), 1] \).

**Lemma 4** Constraints \( IC(\theta_L) \) and \( IR(\theta_H) \) intersect only once in the interval \([q^0(\theta_H), 1]\). Denote the intersection quantity by \( \overline{q} \). Then, \( IC(\theta_L) \) is the binding constraint for \( q^H \in \[q^0(\theta_H), \overline{q}\] \) and \( IR(\theta_H) \) is the binding constraint for \( q^H \in \[\overline{q}, 1\] \).

**Proof.** See Appendix B. ■

Obtaining the solution to Problem 1 involves determining how the expected payoff of the principal of type \( \theta_H \) (the objective function) evolves with \( q^H \) along the relevant binding constraint.

For \( q^H > \overline{q} \), the binding constraint is \( IR(\theta_H) \), so there is full surplus extraction and the payoff of the principal is given by \( TS(i^*_A(q^H, 0), \theta_H) \). Observe that, the expected total surplus \( TS(i^*_A(q^0(\theta_H), 0), \theta_H) \) reaches its maximum value at \( q^H = q^0(\theta_H) \), since \( i^*_A(q^0(\theta_H), 0) = i^*_A(\theta_H) \) which is the first-best investment in state \( \theta_H \). Therefore, from concavity of \( TS(i_A, \theta_H) \) in investment \( i_A \) and the fact that the agent’s investment decision \( i^*_A(q^H, 0) \) is increasing in \( q^H \), it follows that \( TS(i^*_A(q^H, 0), \theta_H) \) decreases in \( q^H \) for \( q^H \geq \overline{q} \) (recall that \( \overline{q} \geq q^0(\theta_H) \)). Thus, the payoff of principal type \( \theta_H \) decreases with \( q^H \) in the interval \([\overline{q}, 1]\) and therefore a solution to Problem 1 must satisfy \( q^H \leq \overline{q} \).

For \( q^H \in \[q^0(\theta_H), \overline{q}\] \), the binding constraint is \( IC(\theta_L) \). Thus, we now study how the expected payoff of the principal of type \( \theta_H \) evolves with \( q^H \) along the constraint \( IC(\theta_L) \). Let \( m(q^H) \) denote the function
obtained by substituting $T_u^{H}$ in the expected payoff of the principal of type $\theta_H$ by its value in the constraint IC($\theta_L$), i.e.,

$$m(q^H) = U^P(0, q^H; i_A^*(q^H, 0) | \theta_H) - U^P(0, q^H; i_A^*(q^H, 0) | \theta_L) + TS(i_A^0(\theta_L), \theta_L).$$

Differentiating with respect to $q^H$, we obtain,

$$\frac{\partial m(q^H)}{\partial q^H} = \frac{\partial}{\partial q^H}[U^P(0, q^H; i_A^*(q^H, 0) | \theta_H) - U^P(0, q^H; i_A^*(q^H, 0) | \theta_L)]$$

$$+ \frac{\partial}{\partial i_A} [U^P(0, q^H; i_A^*(q^H, 0) | \theta_H) - U^P(0, q^H; i_A^*(q^H, 0) | \theta_L)] \times \frac{\partial i_A^*(q^H, 0)}{\partial q^H}.$$

Changing $q^H$ has a direct effect, which is given by the first term, and an indirect effect through investment that corresponds to the second term. The first term, which is equal to

$$\frac{1}{2} [\mathbb{E}[v_p | \theta_H] - \mathbb{E}[v_p | \theta_L]] \geq 0$$

in the case of private information about $v_p$ and to

$$\frac{1}{2} [\mathbb{E}[v | \theta_L] - \mathbb{E}[v | \theta_H]] \geq 0$$

in the case of private information about $v$, represents the “direct surplus extraction effect” : by increasing $q^H$, the principal of type $\theta_H$ can also increase the up-front transfer $T_u^{H}$ in a way that IC($\theta_L$) continues to be respected and her total payoff increases. This is the effect obtained in the version of the model without investment developed in Section 4.2. The second term, which represents the “investment effect”, is equal to

$$v'_u(i_A^*(q^H, 0)) [P_s(i_A^*(q^H, 0), \theta_H) - P_s(i_A^*(q^H, 0), \theta_L)] \times \frac{\partial i_A^*(q^H, 0)}{\partial q^H}$$

and is also positive since $\frac{\partial i_A^*(q^H, 0)}{\partial q^H} \geq 0$ and $P_s(i_A, \theta_H) \geq P_s(i_A, \theta_L)$ for all $i_A \in I_A$. Intuitively, the “investment effect” is positive because the payoff of the principal of type $\theta_H$ is more sensitive to relationship specific investment than the payoff of the principal of type $\theta_L$. Therefore, it is the principal of type $\theta_H$ who gains more by increasing quantity and inducing the agent to invest more.

Since the “direct surplus extraction effect” and the “investment effect” are both positive, by increasing contracted quantity along IC($\theta_L$) the principal of type $\theta_H$ increases her payoff. Together with the fact obtained above that the objective function decreases along IR($\theta_H$) for $q^H > H$, this result implies that the solution to Problem 1 is given by $q^H = H$. Hence, in this case, $c^H = (T_u^{H}, H)$ where $T_u^{H}$ is such that IR($\theta_H$) binds and the payoff of the principal associated with it is $TS(i_A^0(\theta_L), \theta_H) < TS(i_A^0(\theta_H), \theta_H)$.

\footnote{Since the expected payoff of the principal of type $\theta_H$ increases when moving along the IC($\theta_L$) constraint by increasing quantity, the solution to Problem 1 cannot involve a contract specifying a quantity level $q < q^H(\theta_H)$.}
In contrast to Case 1, in this case the outcome associated with the best separating allocation is inefficient in terms of investment: in order to extract surplus, the principal sets an excessively high quantity ($\pi > q^0(\theta_H)$) leading the agent to overinvest in the relationship.

This completes the derivation of the best separating allocation.

As argued above, following the proposal of a best separating allocation $\hat{m} = \{\hat{c}^L, \hat{c}^H\}$ by the principal, there is a continuation equilibrium in which the agent accepts the principal’s proposal and then the principal chooses contract $\hat{c}^L$ if she is of type $\theta_L$ and contract $\hat{c}^H$ if she is of type $\theta_H$. Hence, the remaining question is whether both types of principal proposing $\hat{m} = \{\hat{c}^L, \hat{c}^H\}$ followed by this separating continuation equilibrium constitutes an equilibrium of the overall game, i.e., whether there exist beliefs and continuation equilibria off-the-equilibrium path such that no type of principal gains by deviating and proposing a menu $m' \neq \hat{m}$. The next two lemmas clarify this question. Let $M$ denote the set of finite menus of contracts.

**Lemma 5** If both types of principal propose a menu $m \in M$, followed by a continuation equilibrium (after menu proposal) in which the principal’s payoffs $\bar{U}^P(\theta_L)$ and $\bar{U}^P(\theta_H)$ are such that

\[
\bar{U}^P(\theta_L) \geq TS(i^0_A(\theta_L), \theta_L)
\]

and

\[
\bar{U}^P(\theta_H) \geq TS(i^0_A(\theta_H), \theta_H),
\]

then there are off-the-equilibrium path beliefs such that this proposal and continuation equilibrium constitutes an equilibrium outcome of the overall game.

**Proof.** See Appendix B. ■

**Lemma 6** Suppose that at $q = q^0(\theta_H)$ the binding constraint in Problem 1 is IC($\theta_L$) (Case 2). If both types of principal propose a menu $m \in M$, followed by a continuation equilibrium (after menu proposal) in which the principal’s payoffs $\bar{U}^P(\theta_L)$ and $\bar{U}^P(\theta_H)$ are such that

\[
\bar{U}^P(\theta_L) \geq TS(i^0_A(\theta_L), \theta_L)
\]

and

\[
\bar{U}^P(\theta_H) \geq TS(i^0_A(\theta_H), \theta_H),
\]

then there are off-the-equilibrium path beliefs such that this proposal and continuation equilibrium constitutes an equilibrium outcome of the overall game.

**Proof.** See Appendix B. ■
Lemmas 5 and 6 provide sufficient conditions for an equilibrium, in terms of the principal’s payoffs. Specifically, they ensure that both types of principal proposing a menu of contracts \( m \), followed by a continuation equilibrium (after the menu proposal) in which the principal’s payoffs Pareto dominate the payoffs associated with the best separating allocation constitutes an equilibrium.

An implication of lemmas 5 and 6 is that both types of principal proposing the best separating allocation, followed by the respective separating continuation equilibrium, always constitutes an equilibrium of the game. In particular, Lemma 6 establishes that inefficient equilibria exist. Among these are the ones in which the principal of type \( \theta_H \) distorts the contracted quantity (relative to \( q^0(\theta_H) \), which is the quantity that induces the agent to invest efficiently in state \( \theta_H \)) in order to extract more surplus from the agent.

This result is important not only because of the specific efficiency implications that it has, but also because it emphasizes that surplus extraction and efficiency of investment can in fact conflict with one another when parties contract under asymmetric information. In the next section, we allow exclusivity to be contractible and investigate its role (if any) in mitigating this conflict.

5.3 Quantity and Exclusivity Contracts

Proposition 1 (Irrelevance Result) establishes that exclusivity has no effect on the set of equilibrium outcomes when the source of private information is internal. Therefore, in this section we consider only the case of a principal with private information about the external valuation \( V \).

Throughout the section, we continue to impose Condition 1 and we consider that both quantity and exclusivity are contractible. Hence, a contract is triple \( c = (T_u, q, e) \).

In order to characterize the equilibrium allocations and payoffs, we present two propositions whose results are closely related but substantially different. While in Proposition 5 we present lower bounds for the principal’s equilibrium payoffs, in Proposition 6 we present the equilibrium payoffs themselves and characterize equilibrium investments.

**Proposition 5** Suppose that quantity and exclusivity are contractible \((Q = E = [0, 1])\). Then, in any equilibrium, the payoff of the principal of type \( \theta \) is at least the first-best expected total surplus \( TS(i^0_A(\theta), \theta) \), for all \( \theta \in \{\theta_L, \theta_H\} \).

**Proof.** See Appendix B. \[ \blacksquare \]

Contractibility of exclusivity does not play any role in ensuring to the principal of type \( \theta_L \) the first-best total surplus \( TS(i^0_A(\theta_L), \theta_L) \). Instead, the result follows from Condition 1.

However, in the case of the principal of type \( \theta_H \), it is the fact that exclusivity is contractible that allows the principal to construct a contract that guarantees her the first-best total surplus \( TS(i^0_A(\theta_H), \theta_H) \).
Note that, we have just shown in the previous section that when exclusivity is not contractible, there exist equilibria in which the payoff of the principal of type \( \theta_H \) is \( \mathcal{T}(i_A^*(\theta_H), \theta_H) < \mathcal{T}(\hat{i}_A^*(\theta_H), \theta_H) \).

To illustrate the role of exclusivity when the principal is of type \( \theta_H \), consider the expected payoff of the agent given contract \( c = (T_u, 0, q, e) \), state \( \theta \) and investment \( i_A \). In the case of private external information, this payoff can be written as:

\[
U^A(0, q, e; i_A | \theta) = \frac{1}{2} \mathbb{E}[\max\{V(i_A), \nabla\} | \theta] - \frac{1}{2} [q(v_p - v_u) + (1 - q)(1 - e)] \mathbb{E}[\nabla | \theta] - \psi_A(i_A),
\]

where \( V(i_A) = v_u(i_A) + v_p \).

When the contract prescribes full exclusivity, i.e., \( e = 1 \), the agent’s expected payoff is affected by the state \( \theta \) only through the term \( \frac{1}{2} \mathbb{E}[\max\{V(i_A), \nabla\} | \theta] \). Therefore, from the fact that the distribution of the external value \( \nabla \) in state \( \theta_L \) first order stochastically dominates the one in state \( \theta_H \), it follows that

\[
U^A(0, q, e = 1; i_A | \theta_L) \geq U^A(0, q, e = 1; i_A | \theta_H) \text{ for all } i_A \in I_A. \tag{5.10}
\]

Intuitively, when the principal promises full exclusivity, the agent is better off when the principal has a high outside option (state \( \theta_L \)), since he can appropriate part of it at the renegotiation stage by threatening to enforce the contract and prevent the principal from trading with third parties.

From (5.10), it follows that the agent’s expected payoff \( U^A(0, q, e = 1; i_A^*(q, b_L) | b_L) \) is increasing in his belief \( b_L \), for any given quantity \( q \). In particular, it holds for \( q = q^0(\theta_H) \). This implies that, regardless of his beliefs, the agent always accepts a contract \( (T_u, q = q^0(\theta_H), e = 1) \) in which the transfer is \( T_u = U^A(0, q^0(\theta_H), 1; i_A^*(q^0(\theta_H), 0) | \theta_H) \), i.e., his expected payoff when his beliefs are \( b_L = 0 \) (his worst possible payoff across beliefs).

Hence, exclusivity allows the principal to construct a contract in which the agent is better off in state \( \theta_L \) than in state \( \theta_H \). This is also possible when exclusivity is not contractible if the principal sets a sufficiently high quantity in the contract. But, the problem in doing so, is that she distorts the agent’s investment decision.

Now we turn to the question of equilibrium payoffs and investments.

**Proposition 6** Suppose that quantity and exclusivity are contractible \( (Q = E = [0, 1]) \). Then, in any equilibrium, investment levels are efficient (first-best in both states) and the principal always appropriates the first-best total surplus (extracts all the surplus), i.e., the principal’s equilibrium payoffs are \( \hat{U}^P(\theta) = \mathcal{T}(i_A^0(\theta), \theta) \) for all \( \theta \in \{\theta_L, \theta_H\} \).

**Proof.** From proposition 5 we obtain that in any equilibrium \( \hat{U}^P(\theta) \geq \mathcal{T}(i_A^0(\theta), \theta), \forall \theta \in \{\theta_L, \theta_H\} \).

From individual rationality of the agent it follows that, in any equilibrium, if \( \hat{U}^P(\theta_j) > \mathcal{T}(i_A^0(\theta_j), \theta_j) \) for some \( j \in \{L, H\} \), then \( \hat{U}^P(\theta_{-j}) < \mathcal{T}(i_A^0(\theta_{-j}), \theta_{-j}) \), where \( -j = \{L, H\} \setminus \{j\} \). These two results clearly
imply that $\tilde{U}'(\theta) = TS(\tilde{v}_d(\theta), \theta), \forall \theta \in \{\theta_L, \theta_H\}$ in any equilibrium. This fact together with IR of the agent, implies that investment must be at the first-best level in both states $\theta_L$ and $\theta_H$. ■

Proposition 6 establishes the important result that, in any equilibrium of the game when the principal can contractually use both quantity and exclusivity, the investment levels are efficient in both states $\theta_L$ and $\theta_H$. To illustrate why efficiency is always obtained when both quantity and exclusivity are contractible (as opposed to the case when only quantity is contractible), consider the derivation of the best separating allocation in Case 2 presented in the previous section (see Figure 4). Recall that, in that case, investment is inefficient due to the fact that the principal sets an excessively high quantity ($\tau > q^0(\theta_H)$) instead of $q^0(\theta_H)$ in order to extract more surplus from the agent. When exclusivity is contractible, instead of increasing quantity above $q^0(\theta_H)$ and induce the agent to overinvest in the relationship, the principal can set quantity $q^0(\theta_H)$ and use (increase) exclusivity to move along the IC($\theta_L$) constraint, signal her type and extract surplus from the agent. Surplus extraction can be done in this way because, as we showed in the version of the model without investment, the direct surplus extraction effect associated with exclusivity is positive when the source of private information is external (see Section 4.3). Moreover, since exclusivity does not affect directly the agent’s investment decision, surplus extraction using exclusivity does not conflict with provision of investment incentives.

When comparing the results in the version of the model without investment (Section 4) with the ones in this section, we shall emphasize a particularly important aspect. In contrast with the no investment case, when the agent invests, contractibility of exclusivity has important implications even when quantity is contractible. In particular, the ability to use exclusive provisions contractually eliminates inefficient equilibria that may otherwise exist. Hence, we have identified a situation in which the ability to commit to a relationship through an exclusive agreement enhances efficiency.

6 Conclusion

This paper develops a theory of commitment to relationships based on asymmetries of information. In a context of relationship formation, it shows that the level of contractual commitment can be used by a better informed party to signal information about the probability of success of the relationship.

We consider two different devices for committing to a relationship: a promise to trade in the future (contracted quantity) and a promise not to deal with anyone else (contracted exclusivity). We find that the effectiveness of each of these two devices depends on the source of asymmetry of information. Specifically, exclusivity matters only if the better informed party has private information about external values to the relationship. This result may have implications for antitrust policy, since it implies that in relationships
where private information is internal only, the use of exclusive provisions serves other purposes than signalling information.

When focusing on surplus extraction only, we obtain that if the source of asymmetry of information is external, then the better informed party can use either contracted quantity or contracted exclusivity (or both) to achieve full surplus extraction. In this case, parties commit more to the relationship when its probability of success is higher. When quantity is not contractible, this means that a better informed principal binds herself by proposing a higher level of exclusivity in the contract. This result can be related to the property rights literature (e.g., Grossman and Hart (1986), Hart and Moore (1990), Chiu (1998), De Meza (1998) and Maskin and Tirole (1999)). Interpreting an exclusive contract as not owning the asset (note that not owning the asset implies not being able to deal with outsiders) allows us to draw some inferences on how asset ownership or control is allocated in relationships born under asymmetric information. Namely, a better informed party can give up control over an asset that is essential to deal with outsiders, just to signal to the other party that the value of the outside options is low and that the relationship will be successful.

In the second part of the paper, a noncontractible investment decision by the agent is considered. This allows us to study efficiency implications. In this case, if only quantity is contractible, there are inefficient equilibria in terms of investment. Hence, the surplus extraction and the efficiency roles of contracting can conflict. This is a result similar in nature to that in Aghion and Hermalin (1992): in order to signal information through the contract, the informed party may create distortions that are undesirable in terms of efficiency. However, we show that when the source of information is external, if the principal can use both quantity and exclusivity as commitment devices, all equilibria are efficient. Therefore, allowing exclusive provisions contractually has important efficiency implications, as it resolves the conflict between full surplus extraction (for which signalling is required) and investment efficiency. From a policy design perspective, this result implies that in the presence of noncontractible investments, banning the use of exclusive provisions in environments with asymmetric information may have undesirable efficiency implications.
Appendix A: Proofs (Section 4)

**Proof.** (Proposition 2) When the principal’s private information is about $v_p$, taking expectations of (2.1) we obtain

\[ U^A(T_u, q | \theta) = \frac{1}{2} T_S(\theta) - \frac{1}{2} \{ q(\mathbb{E}[v_p | \theta] - \mathbb{E}[v_a]) + (1 - q)\mathbb{E}[\nabla] \} - T_u, \]

where $T_S(\theta) = \mathbb{E}[\max\{v_p + v_a, \nabla\} | \theta]$.

Let $h(q) \equiv U^A(0, q | \theta_H) - U^A(0, q | \theta_L)$. Since $\max\{v_p + v_a, \nabla\}$ is an increasing function of $v_p$, the fact that $F_{v_p}[, | \theta_H]$ strictly first order stochastically dominates $F_{v_p}[, | \theta_L]$ implies that $h(0) > 0$. In a similar way, since $\max\{v_p + v_a, \nabla\} - v_p$ is a decreasing function of $v_p$, the fact that $F_{v_p}[, | \theta_H]$ strictly first order stochastically dominates $F_{v_p}[, | \theta_L]$ implies that $h(1) < 0$. Moreover, differentiating $h(q)$ we obtain $h'(q) = \frac{1}{2} \{ \mathbb{E}[v_p | \theta_L] - \mathbb{E}[v_p | \theta_H] \} \leq 0$. Hence, from the fact that $h(q)$ is decreasing and $h(1) \leq h(0)$, it follows that exists $\hat{q} \in [0, 1]$ such that $h(q) \geq 0$ (i.e., $U^A(0, q | \theta_H) \geq U^A(0, q | \theta_L)$) if and only if $q \leq \hat{q}$.

With this result at hand, we show the results in the statement of the proposition.

We start by showing that in any equilibrium $\bar{U}^P(\theta) = T_S(\theta)$ for all $\theta \in \{\theta_L, \theta_H\}$. Consider a menu $m = \{c^L, c^H\}$ in which contracts $c^L = (T_u^L, q^L)$ and $c^H = (T_u^H, q^H)$ satisfy the following conditions: $0 \leq q^L \leq \hat{q}$ and $T_u^L = U^A(0, q^L | \theta_L)$, and $\hat{q} \leq q^H \leq 1$ and $T_u^H = U^A(0, q^H | \theta_H)$. By construction, contracts $c^L$ and $c^H$ imply full surplus extraction by the principal and a non-negative payoff for the agent in states $\theta_L$ and $\theta_H$, respectively, i.e., $U^P(c^j | \theta_j) = T_S(\theta_j)$ and $U^A(c^j | \theta_j) = 0$ for $j = L, H$. Moreover, menu $m$ is incentive compatible (IC) for the principal:

\[ U^P(c^L | \theta_L) = U^P(0, q^L | \theta_L) + U^A(0, q^L | \theta_H) \leq U^P(0, q^H | \theta_L) + U^A(0, q^H | \theta_L) = T_S(\theta_L) \]

and

\[ U^P(c^H | \theta_H) = U^P(0, q^H | \theta_H) + U^A(0, q^L | \theta_L) \leq U^P(0, q^L | \theta_H) + U^A(0, q^L | \theta_H) = T_S(\theta_H), \]

where the inequality in (6.1) follows from the fact that $U^A(0, q | \theta_H) \leq U^A(0, q | \theta_L)$ when $q \geq \hat{q}$, and the inequality in (6.2) from the fact that $U^A(0, q | \theta_H) \geq U^A(0, q | \theta_L)$ when $q \leq \hat{q}$. Now, since menu $m$ is IC, the agent knows that in state $\theta_L$ the principal chooses contract $c^L$ from $m$, while in state $\theta_H$ she chooses contract $c^H$. In both cases, the agent is left with his reservation value (zero). Therefore, the agent accepts a proposal of menu $m$, regardless of his beliefs about the state $\theta$. This establishes that by proposing menu $m$ the principal of type $\theta$ can always ensure herself $T_S(\theta)$ and therefore, in any equilibrium $\bar{U}^P(\theta) \geq T_S(\theta)$ for all $\theta \in \{\theta_L, \theta_H\}$. Clearly, it cannot happen in equilibrium that $\bar{U}^P(\theta) \geq T_S(\theta)$ for all $\theta \in \{\theta_L, \theta_H\}$ and $\bar{U}^P(\theta) > T_S(\theta)$ for some $\theta$. To see this, note that by (2.4) (efficient ex-post renegotiation) this would imply that $\bar{U}^A(\theta) \leq 0$, $\forall \theta \in \{\theta_L, \theta_H\}$ with at least one of the inequalities being strict, which in turn would imply contract (menu) rejection by the agent. Thus, in equilibrium $\bar{U}^P(\theta) = T_S(\theta)$ for all $\theta \in \{\theta_L, \theta_H\}$.
Next, we establish existence of equilibrium. Consider a menu $m$ as constructed above. Recall that $m$ is IC and the agent always accepts it. Thus, to show that both types of principal proposing $m$, followed by the agent accepting it, and by the principal of type $\theta$, choosing contract $c^i$ for $i = L, H$ is an equilibrium, it suffices to show that there are off-the-equilibrium path beliefs such that no type of principal gains by deviating and proposing another menu $m'$ instead of $m$. Only deviations to menus $m' \neq m$ such that

$$\max_{c \in m'} U^P(c \mid \theta) > \overline{TS}(\theta) \text{ for some } \theta \in \{\theta_L, \theta_H\}$$

may eventually be profitable for the principal. However, by setting the agent’s (off-the-equilibrium path) beliefs after observing such menus $m'$ as $b_L(m') = 1$ if $\max_{c \in m'} U^P(c \mid \theta_L) > \overline{TS}(\theta_L)$ and $b_L(m') = 0$ otherwise, we ensure that these menus are rejected by the agent.

Finally, we show that $q^L \leq \tilde{q} \leq q^H$ in any equilibrium. Consider an arbitrary equilibrium. In this equilibrium, let $c^L = (T^L_u, q^L)$ and $c^H = (T^H_u, q^H)$ denote contracts chosen (with positive probability) by the principals of types $\theta_L$ and $\theta_H$, respectively. The fact that in equilibrium there is full surplus extraction ($U^P(\theta) = \overline{TS}(\theta)$ for all $\theta \in \{\theta_L, \theta_H\}$) implies that $T^L_u = U^A(0, q^j \mid \theta_j)$ for $j = L, H$. Therefore, the (necessary) equilibrium conditions that the principal of type $\theta_L$ does not gain by deviating and choosing contract $c^H$ and vice-versa are given by (6.1) and (6.2), respectively. From $U^A(0, q \mid \theta_H) \geq U^A(0, q \mid \theta_L)$ if and only if $q \leq \tilde{q}$, it follows that a necessary condition for (6.1) to hold is $q^H \geq \tilde{q}$. In a similar way, a necessary condition for (6.2) to hold is $q^L \leq \tilde{q}$. This completes the proof. 

**Proof.** (Proposition 3) When the principal’s private information is about $\nabla$, taking expectations of (2.1) we obtain

$$U^A(T_u, q, e \mid \theta) = \frac{1}{2}\nabla S(\theta) - \frac{1}{2}(q)(\mathbb{E}[v_p] - \mathbb{E}[v_a]) + (1 - q)(1 - e)\mathbb{E}[\nabla \mid \theta] - T_u,$$

where $\nabla S(\theta) = \mathbb{E}[^{\theta}v_p + v_a, \nabla] \mid \theta$.

Let $h(q, e) \equiv U^A(0, q, e \mid \theta_H) - U^A(0, q, e \mid \theta_L)$. Function $h(q, e)$ can be rewritten as

$$h(q, e) = \frac{1}{2} \left( \nabla S(\theta_H) - \nabla S(\theta_L) \right) - \frac{1}{2}(1 - q)(1 - e)\left( \mathbb{E}[\nabla \mid \theta_H] - \mathbb{E}[\nabla \mid \theta_L] \right) \equiv \tilde{h}(1 - q)(1 - e)).$$

Since $\max\{v_p + v_a, \nabla\}$ is an increasing function of $\nabla$, the fact that $\nabla S(\theta_L \mid \theta_H]$ strictly first order stochastically dominates $\nabla S(\theta_H \mid \theta_L]$ implies that $\tilde{h}(0) < 0$. In a similar way, since $\max\{v_p + v_a, \nabla\} - \nabla$ is a decreasing function of $\nabla$, the fact that $\nabla S(\theta_H \mid \theta_L]$ strictly first order stochastically dominates $\nabla S(\theta_L \mid \theta_H]$ implies that $\tilde{h}(1) > 0$. Moreover, since $\mathbb{E}[\nabla \mid \theta_H] < \mathbb{E}[\nabla \mid \theta_L]$, $\tilde{h}(\cdot)$ is an increasing function. Hence, from the fact that $\tilde{h}(\cdot)$ is increasing and $\tilde{h}(0) \leq 0 \leq \tilde{h}(1)$, it follows that exists $\tilde{x} \in [0, 1]$ such that $h(q, e) \geq 0$ (i.e., $U^A(0, q \mid \theta_H) \geq U^A(0, q \mid \theta_L)$) if and only if $(1 - q)(1 - e) \geq \tilde{x}$. Using this result, we now show the results in the statement of the proposition.
As in the proof of proposition 2, we start by showing that in any equilibrium \( \tilde{U}^P(\theta) = TS(\theta) \) for all \( \theta \in \{\theta_L, \theta_H\} \). Consider a menu \( m = \{c^L, c^H\} \) in which contracts \( c^L = (T_u^L, q^L, c^L) \) and \( c^H = (T_u^H, q^H, c^L) \) satisfy the following conditions: \( (1 - q^L)(1 - e^L) \geq \bar{x} \) and \( T_u^L = U^A(0, q^L, e^L | \theta_L) \), and \( (1 - q^H)(1 - e^H) \leq \bar{x} \) and \( T_u^H = U^A(0, q^H, e^H | \theta_H) \). By construction, contracts \( c^L \) and \( c^H \) imply full surplus extraction by the principal and a non-negative payoff for the agent in states \( \theta_L \) and \( \theta_H \), respectively, i.e., \( U^P(c^j | \theta_j) = TS(\theta_j) \) and \( U^A(c^j | \theta_j) = 0 \) for \( j = L, H \). Moreover, menu \( m \) is incentive compatible (IC) for the principal:

\[
U^P(c^H | \theta_L) = U^P(0, q^H, e^H | \theta_L) + U^A(0, q^H, e^H | \theta_H) \leq U^P(0, q^H, e^H | \theta_L) + U^A(0, q^H, e^H | \theta_H) = TS(\theta_L) \tag{6.3}
\]

and

\[
U^P(c^L | \theta_H) = U^P(0, q^L, e^L | \theta_H) + U^A(0, q^L, e^L | \theta_L) \leq U^P(0, q^L, e^L | \theta_H) + U^A(0, q^L, e^L | \theta_H) = TS(\theta_H), \tag{6.4}
\]

where the inequality in (6.3) follows from the fact that \( U^A(0, q, e | \theta_H) \leq U^A(0, q, e | \theta_L) \) when \( (1 - q)(1 - e) \leq \bar{x} \), and the inequality in (6.4) from the fact that \( U^A(0, q, e | \theta_H) \geq U^A(0, q, e | \theta_L) \) when \( (1 - q)(1 - e) \geq \bar{x} \). Now, since menu \( m \) is IC, the agent knows that in state \( \theta_L \) the principal chooses contract \( c^L \) from \( m \), while in state \( \theta_H \) she chooses contract \( c^H \). In both cases, the agent is left with his reservation value (zero). Therefore, the agent accepts a proposal of menu \( m \), regardless of his beliefs about the state \( \theta \). This establishes that by proposing menu \( m \) the principal of type \( \theta \) can always ensure herself \( TS(\theta) \) and therefore, in any equilibrium \( \tilde{U}^P(\theta) \geq TS(\theta) \) for all \( \theta \in \{\theta_L, \theta_H\} \). Clearly, it cannot happen in equilibrium that \( \tilde{U}^P(\theta) \geq TS(\theta) \) for all \( \theta \in \{\theta_L, \theta_H\} \) and \( \tilde{U}^P(\theta) > TS(\theta) \) for some \( \theta \). To see this, note that by (2.4) (efficient ex-post renegotiation) this would imply that \( \tilde{U}^A(\theta) \leq 0, \forall \theta \in \{\theta_L, \theta_H\} \) with at least one of the inequalities being strict, which in turn would imply contract (menu) rejection by the agent. Thus, in equilibrium \( \tilde{U}^P(\theta) = TS(\theta) \) for all \( \theta \in \{\theta_L, \theta_H\} \). The proof of existence of equilibrium is identical to the one in proposition 2.

Finally, we show that \( (1 - q^H)(1 - e^H) \leq \bar{x} \leq (1 - q^L)(1 - e^L) \) in any equilibrium. Consider an arbitrary equilibrium. In this equilibrium, let \( c^L = (T_u^L, q^L, c^L) \) and \( c^H = (T_u^H, q^H, c^L) \) denote contracts chosen (with positive probability) by the principals of types \( \theta_L \) and \( \theta_H \), respectively. The fact that in equilibrium there is full surplus extraction \( \tilde{U}^P(\theta) = TS(\theta) \) for all \( \theta \in \{\theta_L, \theta_H\} \) implies that \( T_u^j = U^A(0, q^j, e^j | \theta_j) \) for \( j = L,H \). Therefore, the (necessary) equilibrium conditions that the principal of type \( \theta_L \) does not gain by deviating and choosing contract \( c^H \) and vice-versa are given by (6.3) and (6.4), respectively. From \( U^A(0, q, e | \theta_H) \geq U^A(0, q, e | \theta_L) \) if and only if \( (1 - q)(1 - e) \geq \bar{x} \), it follows that a necessary condition for (6.3) to hold is \( (1 - q^H)(1 - e^H) \leq \bar{x} \). In a similar way, a necessary condition for (6.4) to hold is \( (1 - q^L)(1 - e^L) \geq \bar{x} \). This completes the proof. □

Appendix B: Proofs (Section 5)
**Proof.** (Proposition 4) We start by establishing the following property of the expected payoff function of the agent.

**Lemma 7** The expected payoff function of the agent \( U^A(T_u; i_A^*(b_L) \mid b_L) \) is decreasing in \( b_L \).

Proof of the Lemma: From (5.1), by setting \( q = e = 0 \) we obtain that the expected payoff of the agent given transfer \( T_u \), investment \( i_A \) and state \( \theta \) can be written as:

\[
U^A(T_u; i_A \mid \theta) = \frac{1}{2} \mathbb{E}[\pi(v_u(i_A), v_p, \overline{\psi}) \mid \theta] - \psi(i_A) - T_u,
\]

where

\[
\pi(v_u(i_A), v_p, \overline{\psi}) = \max\{v_u(i_A) + v_p, \overline{\psi}\} - \overline{\psi}.
\]

Consider first the case of private information about \( v_p \). Since \( \pi(v_u(i_A), v_p, \overline{\psi}) \) is an increasing function of \( v_p \) and \( F_{v_p}[\theta_H] \) strictly first order stochastically dominates \( F_{v_p}[\theta_L] \), then \( \mathbb{E}[\pi(v_u(i_A), v_p, \overline{\psi}) \mid \theta_H] > \mathbb{E}[\pi(v_u(i_A), v_p, \overline{\psi}) \mid \theta_L] \) for all \( i_A \in I_A \). Consider now the case of private information about \( \overline{\psi} \). Since \( \pi(v_u(i_A), v_p, \overline{\psi}) \) is a decreasing function of \( \overline{\psi} \) and \( F_{\overline{\psi}}[\theta_L] \) strictly first order stochastically dominates \( F_{\overline{\psi}}[\theta_H] \), then \( \mathbb{E}[\pi(v_u(i_A), v_p, \overline{\psi}) \mid \theta_H] > \mathbb{E}[\pi(v_u(i_A), v_p, \overline{\psi}) \mid \theta_L] \) for all \( i_A \in I_A \). Hence, in both cases we obtain that \( U^A(T_u; i_A \mid \theta_H) > U^A(T_u; i_A \mid \theta_L) \) for all \( i_A \in I_A \). Since \( U^A(T_u; i_A \mid b_L) = b_L U^A(T_u; i_A \mid \theta_L) + (1 - b_L) U^A(T_u; i_A \mid \theta_H) \), this implies that, for any \( b_1, b_2 \) such that \( b_1 < b_2 \),

\[
U^A(T_u; i_A \mid b_1) > U^A(T_u; i_A \mid b_2) \text{ for all } i_A \in I_A. \tag{6.5}
\]

Finally, since this inequality holds at every investment level \( i_A \), then it must necessarily hold when evaluating each of the expected payoffs in (6.5) at their maximizing investment levels \( i_A^*(b_1) \) and \( i_A^*(b_2) \). This completes the proof of the Lemma. ■

With this result at hand, we now show the results in the proposition.

Consider first the pooling equilibria. In a pooling equilibrium, the transfer \( T_u \) cannot exceed the expected payoff of the agent (when the agent’s belief coincides with the prior \( p(\theta_L) \)), otherwise the agent rejects the contract. This establishes an upper bound of \( U^A(0; i_A^*(p(\theta_L)) \mid p(\theta_L)) \) to the set of pooling equilibrium transfers. By Lemma 7, the agent accepts a contract in which he pays \( T_u = U^A(0; i_A^*(1) \mid \theta_L) \), independently of his beliefs. Together with the facts that \( U^P(T_u; i_A \mid \theta_L) \) is increasing in \( i_A \) (recall that when \( q = 0 \) \( U^P(\zeta; i_A \mid \theta) \) is increasing in \( i_A \)) and that \( i_A^*(b_L) \) is decreasing in \( b_L \) (Lemma 1), this implies that the principal can always ensure herself \( \overline{T_S}(i_A^*(1), \theta_L) \) in state \( \theta_L \). Therefore, in a pooling equilibrium, the transfer \( T_u \) must be such that the principal of type \( \theta_L \) gets at least \( \overline{T_S}(i_A^*(1), \theta_L) \), i.e.,

\[
T_u \geq \overline{T_S}(i_A^*(1), \theta_L) - U^P(0; i_A^*(p(\theta_L)) \mid \theta_L). \tag{6.6}
\]

This establishes a lower bound on the set of pooling equilibrium transfers. Finally, off-the-equilibrium path beliefs \( b_L(t) = 1 \) for \( t \neq T_u \) sustain any of the transfers specified in the statement of the proposition.
as a pooling equilibrium. To see this, note that no type of principal gains by deviating to a transfer \( t > U^A(0;i_\lambda^*(1) \mid \theta_L) \) because it is rejected by the agent, and no type of principal gains by deviating to a transfer \( t < U^A(0;i_\lambda^*(1) \mid \theta_L) \) because, by construction of the equilibrium transfers (condition (6.6)), the potential gain for the principal associated with a bigger transfer is offset by the loss associated with a lower level of investment by the agent \((i_\lambda^*(1) < i_\lambda^*(p(\theta_L)))\).

Now, consider the separating equilibria. In any separating equilibrium

\[
T_u^H \leq T_u^L - \{U^P(0;i_\lambda^*(0) \mid \theta_L) - U^P(0;i_\lambda^*(1) \mid \theta_L)\},
\]

otherwise the principal of type \( \theta_L \) would prefer to mimic the principal of type \( \theta_H \) by proposing \( T_u^H \) and to benefit from a higher level of investment. On the other hand,

\[
T_u^H \geq T_u^L - \{U^P(0;i_\lambda^*(0) \mid \theta_H) - U^P(0;i_\lambda^*(1) \mid \theta_H)\},
\]

otherwise the principal of type \( \theta_H \) would be better off by deviating and proposing \( T_u^L \) instead of \( T_u^H \).

Moreover, in a separating equilibrium the principal of type \( \theta_L \) always obtains exactly \( TS(i_\lambda^*(1),\theta_L) \): on the one hand, she can ensure herself at least \( TS(i_\lambda^*(1),\theta_L) \) (as argued above); on the other hand, her payoff cannot exceed \( TS(i_\lambda^*(1),\theta_L) \), otherwise the agent’s expected payoff after observing contract proposal \( T_u^L \) would be negative. This implies that the principal of type \( \theta_L \) always proposes \( T_u^L = T_u^L = U^A(0;i_\lambda^*(1) \mid \theta_L) \) in a separating equilibrium. Hence, only the transfers stated in the proposition satisfy the necessary conditions for an equilibrium. Finally, beliefs \( b_L(t) = 1 \) for \( t \neq T_u^H \) sustain any of these transfers as a separating equilibrium (observe that \( T_u^L \) is the largest transfer that the agent will accept if \( b_L = 1 \)).

We next show the results in the second part of the proposition. Investment inefficiency in equilibrium follows from the fact that there is always underinvestment in state \( \theta_H \). Note that the maximum equilibrium investment by the agent when \( q = 0 \) is \( i_\lambda^*(b_L = 0) \equiv i_\lambda^*(q = 0,b_L = 0) \). Since by Lemma 1 the agent’s investment \( i_\lambda^*(\ldots) \) is increasing in \( q \), this is smaller than \( i_\lambda^*(q^0(\theta_H),0) = i_\lambda^0(\theta_H) \), which is the first-best investment in state \( \theta_H \).

To obtain that the principal of type \( \theta_H \) cannot extract all the surplus, consider an arbitrary equilibrium. In such equilibrium, any transfer \( T_u \) proposed with positive probability by either type of principal must not exceed the agent’s expected payoff given his beliefs \( b_L(T_u) \), i.e.,

\[
T_u \leq U^A(0,i_\lambda^*(b_L(T_u)) \mid b_L(T_u)); \tag{6.7}
\]

otherwise the agent would reject it. In this equilibrium, full surplus extraction in state \( \theta_H \) would imply that any transfer \( T_u' \) proposed with strictly positive probability by the principal of type \( \theta_H \) has to satisfy

\[
T_u' = U^A(0,i_\lambda^*(b_L(T_u')) \mid 0) \equiv U^A(0,i_\lambda^*(b_L(T_u')) \mid \theta_H). \tag{6.8}
\]
This is not compatible with (6.7). First, suppose that \( b_L(T_u^*) = 0 \), i.e., only the principal of type \( \theta_H \) proposes transfer \( T_u^* \). From Lemma 7 and (6.7) it follows that the principal of type \( \theta_L \) would gain by deviating and proposing transfer \( T_u^* \). Now suppose that \( b_L(T_u) > 0 \). Since \( U^A(0, i_A | b_L) \) is strictly decreasing in \( b_L \) for all \( i_A \in I_A \) (see proof of Lemma 7), a transfer satisfying (6.8) necessarily violates (6.7).

**Proposition 7** A best separating allocation \( \hat{\pi} = \{\hat{c}^L, \hat{c}^H\} \) is incentive compatible given the separating beliefs.

**Proof.** We first show that the principal of type \( \theta_H \) prefers to choose \( \hat{c}^H \) instead of \( \hat{c}^L \). Let \( \{\hat{c}^L, \hat{c}^H\} \) be a solution to the problem presented in (5.9) for the principal of type \( \theta_L \), which exists because \( \hat{c}^L \in \hat{\pi} \). The IC constraints in the problem imply that \( U^P(c^H; i_A^*(\hat{c}^H, 0) | \theta_H) \geq U^P(c^L; i_A^*(\hat{c}^L, 1) | \theta_H) \). Moreover, the set of constraints of the problem presented in (5.9) is the same whether we are solving it for principal type \( \theta_L \) or for principal type \( \theta_H \). Thus, \( U^P(c^H; i_A^*(\hat{c}^H, 0) | \theta_H) \geq U^P(c^L; i_A^*(\hat{c}^L, 0) | \theta_H) \), which implies that \( U^P(c^H; i_A^*(\hat{c}^H, 0) | \theta_H) \geq U^P(c^L; i_A^*(\hat{c}^L, 1) | \theta_H) \). The proof that the principal of type \( \theta_L \) prefers to choose \( \hat{c}^L \) instead of \( \hat{c}^H \) is perfectly analogous.

**Proof.** (Lemma 3) We start by showing that exists an allocation \( \{c^L, c^H\} \), satisfying the constraints in problem (5.9), such that \( U^P(c^L; i_A^*(c^L, 1) | \theta_L) = TS(i_A^*(\theta_L), \theta_L) \). Consider the contract \( \hat{c}^L = (\hat{T}_u^L, q^0(\theta_L)) \) in which \( \hat{T}_u^L = U^A(0, q^0(\theta_L); i_A^*(q^0(\theta_L), 1) | \theta_L) \). By construction, contract \( \hat{c}^L \) is such that \( U^P(\hat{c}^L; i_A^*(\hat{c}^L, 1) | \theta_L) = TS(i_A^*(\theta_L), \theta_L) \). For contract \( c^H \), consider the following candidate: \( c^H = (\hat{T}_u^H, q^0(\theta_L)) \), where

\[
\hat{T}_u^H = \hat{T}_u^L - \{U^P(0, q^0(\theta_L); i_A^*(q^0(\theta_L), 0) | \theta_H) - U^P(0, q^0(\theta_L); i_A^*(q^0(\theta_L), 1) | \theta_H)\},
\]

i.e., \( T_u^H \) is the transfer that leaves the principal of type \( \theta_H \) indifferent between choosing contract \( c^H \) (which is associated with the higher level of investment: \( i_A^*(q^0(\theta_L), 0) \)) and choosing contract \( c^L \) (which is associated with the lower level of investment \( i_A^*(q^0(\theta_L), 1) \)). By construction, menu \( \{\hat{c}^L, c^H\} \) satisfies constraints IR(\( \theta_L \)) and IC(\( \theta_H \)). Hence, we just have to show both constraints IC(\( \theta_L \)) and IR(\( \theta_H \)) are also satisfied.

To see that the IC(\( \theta_L \)) constraint is satisfied, note that by deviating and choosing contract \( c^H \) instead of contract \( c^L \), the principal of type \( \theta_L \) obtains:

\[
U^P(0, q^0(\theta_L); i_A^*(q^0(\theta_L), 0) | \theta_L) + \hat{T}_u^H
= TS(i_A^*(\theta_L), \theta_L) + \{U^P(0, q^0(\theta_L); i_A^*(q^0(\theta_L), 0) | \theta_L) - U^P(0, q^0(\theta_L); i_A^*(q^0(\theta_L), 1) | \theta_L)\}
- \{U^P(0, q^0(\theta_L); i_A^*(q^0(\theta_L), 0) | \theta_H) - U^P(0, q^0(\theta_L); i_A^*(q^0(\theta_L), 1) | \theta_H)\}
\leq TS(i_A^*(\theta_L), \theta_L),
\]

where the equality follows from substituting \( \hat{T}_u^H \) as given in (6.9) and the fact that \( \hat{T}_u^L + U^P(0, q^0(\theta_L); i_A^*(q^0(\theta_L), 1) | \theta_L) = TS(i_A^*(\theta_L), \theta_L) \), and the inequality follows from the fact that the expected payoff of the principal of
type $\theta_H$ responds more positively to the agent’s investment than the expected payoff of the principal of type $\theta_L$ (Lemma 2).

We now show that the IR($\theta_H$) constraint is also satisfied. If contract $c^H$ is chosen by the principal, the expected payoff of the agent (after inferring that the principal is of type $\theta_H$) is

$$U^A(0, q^0(\theta_L); i^*_A(q^0(\theta_L), 0) \mid \theta_H) - T_u^H$$

(6.10)

$$= \{U^A(0, q^0(\theta_L); i^*_A(q^0(\theta_L), 0) \mid \theta_H) - U^A(0, q^0(\theta_L); i^*_A(q^0(\theta_L), 1) \mid \theta_L)\}$$

$$+ [U^P(0, q^0(\theta_L); i^*_A(q^0(\theta_L), 0) \mid \theta_H) - U^P(0, q^0(\theta_L); i^*_A(q^0(\theta_L), 1) \mid \theta_H)],$$

where the equality follows from substituting $T_u^H$ as given in (6.9) and noting that $T_u^L = U^A(0, q^0(\theta_L); i^*_A(q^0(\theta_L), 1) \mid \theta_L)$. The term in curly brackets in (6.10) is always positive. To see this, note that Condition I implies that $U^A(0, q^0(\theta_L); i^*_A(q^0(\theta_L), b_L) \mid \theta_L) \leq U^A(0, q^0(\theta_L); i^*_A(q^0(\theta_L), b_L) \mid \theta_H)$ for all $b_L \in [0, 1]$, which in turn implies that $U^A(0, q^0(\theta_L); i^*_A(q^0(\theta_L), b_L) \mid \theta_L)$ is decreasing in $b_L$. We next show that the term in square brackets is also positive. From (5.4), we know that the derivative of the expected payoff of the principal of type $\theta_H$ with respect to investment (evaluated at $i^*_A(q^0(\theta_L), b_L)$ and $q = q^0(\theta_L)$) is given by

$$\frac{1}{2} v_u'(i_A)[P_s(i^*_A(q^0(\theta_L), b_L), \theta_H) - q^0(\theta_L)].$$

Given the facts that the agent’s investment decision $i^*_A(., b_L)$ is decreasing in $b_L$, the probability of success $P_s(i_A, .)$ is increasing in investment $i_A$ and $q^0(\theta_L) \equiv P_s(i^*_A(q^0(\theta_L), 1), \theta_L)$, we obtain that $P_s(i^*_A(q^0(\theta_L), b_L), \theta_L) - q^0(\theta_L) \geq 0$ for all $b_L \in [0, 1]$. Finally, from $P_s(i_A, \theta_L) \leq P_s(i_A, \theta_H)$ for all $i_A \in I_A$, it follows that $P_s(i^*_A(q^0(\theta_L), b_L), \theta_H) - q^0(\theta_L) \geq 0$ for all $b_L \in [0, 1]$. Hence, since $v_u'(i_A) > 0$, the payoff of the principal of type $\theta_H$ is increasing in the agent’s investment $i_A$ for $i_A \in [i^*_A(q^0(\theta_L), 1), i^*_A(q^0(\theta_L), 0)]$ when $q = q^0(\theta_L)$.

This establishes that

$$[U^P(0, q^0(\theta_L); i^*_A(q^0(\theta_L), 0) \mid \theta_H) - U^P(0, q^0(\theta_L); i^*_A(q^0(\theta_L), 1) \mid \theta_H)] \geq 0,$$

and completes the proof of existence of an allocation $\{c^L, c^H\}$, satisfying the constraints in problem (5.9), such that $U^P(c^L; i_A^*(c^L), 1 \mid \theta_L) = TS(i_A^0(\theta_L), \theta_L)$.

To see that the allocation proposed in this proof is in fact a solution to problem (5.9) for $j = L$, note that in any solution to problem (5.9) the payoff of the principal of type $\theta_L$ cannot exceed the first-best total surplus $TS(i_A^0(\theta_L), \theta_L)$, otherwise constraint IR($\theta_L$) would be violated. Finally, any menu $\{c^L, c^H\}$ solving problem (5.9) for $j = L$ must specify contract $c^L = (q^0(\theta_L), T_u^L)$, since the total surplus $TS(i_A^*(q, 1), \theta_L)$ reaches its maximum value $TS(i_A^0(\theta_L), \theta_L)$ at $q = q^0(\theta_L)$. This completes the proof of the Lemma.

Lemma 8 Both when private information is about $v_p$ and when private information is about $\bar{v}$, $U^A(T_u, q = 1; i_A \mid \theta_L) \geq U^A(T_u, q = 1; i_A \mid \theta_H)$ for all $T_u$ and all $i_A \in I_A$. 

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Proof. By taking expectations on (2.1), we obtain that the agent’s expected payoff function given contract \( c = (T_u, q) \), investment \( i_A \) and state \( \theta \) is given by

\[
U^A(c; i_A \mid \theta) = \frac{1}{2} \mathbb{E} \left[ \max \{ V(i_A), \overline{V} \} \mid \theta \right] - \frac{1}{2} \mathbb{E} \left[ q(v_p - v_a(i_A)) + (1 - q)\overline{V} \mid \theta \right] - \psi(i_A) - T_u.
\]

Hence, given that private information is either about \( v_p \) or about \( \overline{V} \) we can write

\[
U^A(T_u, 1; i_A \mid \theta_L) - U^A(T_u, 1; i_A \mid \theta_H) = \frac{1}{2} \mathbb{E} \left[ \max \{ V(i_A), \overline{V} \} - v_p \mid \theta_L \right] - \frac{1}{2} \mathbb{E} \left[ \max \{ V(i_A), \overline{V} \} - v_p \mid \theta_H \right].
\]

Suppose first that private information is about \( v_p \). From the facts that \( \max \{ V(i_A), \overline{V} \} - v_p \) is a decreasing function of \( v_p \) and \( F_{v_p}[. \mid \theta_L] \) strictly first order stochastically dominates \( F_{v_p}[. \mid \theta_L] \) we obtain that \( U^A(T_u, 1; i_A \mid \theta_L) - U^A(T_u, 1; i_A \mid \theta_H) \geq 0 \). Suppose now that private information is about \( \overline{V} \). From the facts that \( \max \{ V(i_A), \overline{V} \} - v_p \) is an increasing function of \( \overline{V} \) and \( F_{\overline{V}}[. \mid \theta_L] \) strictly first order stochastically dominates \( F_{\overline{V}}[. \mid \theta_H] \) we obtain that \( U^A(T_u, 1; i_A \mid \theta_L) - U^A(T_u, 1; i_A \mid \theta_H) \geq 0 \). This establishes the result in the Lemma. \( \blacksquare \)

Proof. (Lemma 4) Constraints IC(\( \theta_L \)) and IR(\( \theta_H \)) are given by

\[
TS(i_A^0(\theta_L), \theta_L) \geq U^P(0, q; i_A^*(q), 0 \mid \theta_L) + T_u^H
\]

and

\[
U^A(0, q; i_A^*(q), 0 \mid \theta_H) - T_u^H \geq 0,
\]

respectively. When these constraints hold with equality, they define continuous functions \( T_u^H \) of \( q \). Specifically,

\[
T_u^H(q)|_{IC(\theta_L)} = TS(i_A^0(\theta_L), \theta_L) - U^P(0, q; i_A^*(q), 0 \mid \theta_L)
\]

and

\[
T_u^H(q)|_{IR(\theta_H)} = U^A(0, q; i_A^*(q), 0 \mid \theta_H).
\]

In the remainder of this proof we focus on the difference:

\[
T_u^H(q)|_{IC(\theta_L)} - T_u^H(q)|_{IR(\theta_H)} = TS(i_A^0(\theta_L), \theta_L) - U^P(0, q; i_A^*(q), 0 \mid \theta_L) - U^A(0, q; i_A^*(q), 0 \mid \theta_H).
\]

We first show that \( T_u^H(1)|_{IC(\theta_L)} - T_u^H(1)|_{IR(\theta_H)} \geq 0 \). From Lemma 8 we obtain that

\[
U^A(0, 1; i_A^*(1, 0 \mid \theta_L) \geq U^A(0, 1; i_A^*(1, 0 \mid \theta_H),
\]

which implies that

\[
T_u^H(1)|_{IC(\theta_L)} - T_u^H(1)|_{IR(\theta_H)} \geq TS(i_A^0(\theta_L), \theta_L) - U^P(0, 1; i_A^*(1, 0 \mid \theta_L) - U^A(0, 1; i_A^*(1, 0 \mid \theta_L).\]
From (2.4) (efficient ex-post renegotiation) and the fact that $i_A^0(\theta_L)$ maximizes $TS(i_A, \theta_L)$, it follows that

$$T_u^H(1)|_{IC(\theta_L)} - T_u^H(1)|_{IR(\theta_H)} \geq TS(i_A^0(\theta_L), \theta_L) - TS(i_A^0(1,0), \theta_L) \geq 0.$$  

Finally, by continuity of $T_u^H(q)|_{IC(\theta_L)} - T_u^H(q)|_{IR(\theta_H)}$ and the Intermediate Value Theorem, we obtain that $IC(\theta_L)$ and $IR(\theta_H)$ intersect at least once in the interval $[q^0(\theta_H), 1]$.

We next show that $T_u^H(q)|_{IC(\theta_L)} - T_u^H(q)|_{IR(\theta_H)}$ is an increasing function of $q$ when $q \in [q^0(\theta_H), 1]$. Substituting $U^F(0, q; i_A^*(q, 0) | \theta_L)$ and $U^A(0, q; i_A^*(q, 0) | \theta_H)$ by their expressions obtained from taking expectations of (2.1) and (2.2), we can write:

$$T_u^H(q)|_{IC(\theta_L)} - T_u^H(q)|_{IR(\theta_H)} = TS(i_A^0(\theta_L), \theta_L) - \frac{1}{2}\{E[\max\{V(i_A^*(q,0), V)\} | \theta_L] + E[\max\{V(i_A^*(q,0), V\} | \theta_H]\}$$

$$+ \frac{1}{2}\{E[h(q, v_p, V) | \theta_H] - \frac{1}{2}E[h(q, v_p, V) | \theta_L]\}$$

$$+ \psi(i_A^*(q,0)),$$  (6.11)

where $h(q, v_p, V) = q[v_p - v_a(i_A^*(q,0))] + (1 - q)V$. Using the fact that

$$TS(i_A^*(q,0), \theta) = E[\max\{V(i_A^*(q,0), V)\} | \theta] - \psi(i_A^*(q,0))$$

for all $\theta \in \{\theta_L, \theta_H\}$,

we can rewrite (6.11) in the following way

$$T_u^H(q)|_{IC(\theta_L)} - T_u^H(q)|_{IR(\theta_H)} = TS(i_A^0(\theta_L), \theta_L) - \frac{1}{2}\{TS(i_A^*(q,0), \theta_L) + TS(i_A^*(q,0), \theta_H)\}$$

$$+ \frac{1}{2}\{E[h(q, v_p, V) | \theta_H] - E[h(q, v_p, V) | \theta_L]\}.$$  (6.12)

Only the second and third terms in (6.12) depend on $q$. Consider first the second term. From the facts that $i_A^*(q,0)$ is an increasing function of $q$ (Lemma (1)), $i_A^*(q,0) \geq i_A^0(\theta)$ for all $\theta$ when $q \geq q^0(\theta_H)$ and concavity of $TS(i_A, \theta)$ in investment, it follows that both $TS(i_A^*(q,0), \theta_L)$ and $TS(i_A^*(q,0), \theta_H)$ are decreasing functions of $q$ when $q \geq q^0(\theta_H)$. Consider now the third term in (6.12). When private information is about $v_p$ this term reduces to

$$q(E[v_p | \theta_H] - E[v_p | \theta_L]).$$

This is an increasing function of $q$, since the fact that $F_{v_p} | \theta_H$ strictly first order stochastically dominates $F_{v_p} | \theta_L$ implies that $E[v_p | \theta_H] > E[v_p | \theta_L]$. When private information is about $V$, the third term in (6.12) becomes

$$(1 - q)(E[V | \theta_H] - E[V | \theta_L]).$$

This is also an increasing function of $q$, since the fact that $F_{V} | \theta_H$ strictly first order stochastically dominates $F_{V} | \theta_L$ implies that $E[V | \theta_H] < E[V | \theta_L]$. Therefore, in both cases of private information about
\(v_p\) and private information about \(\nabla, T_u^H(q)|_{IC(\theta_L)} - T_u^H(q)|_{IR(\theta_H)}\) is increasing in \(q\) when \(q \in [q^0(\theta_H), 1]\).
Monotonicity of \(T_u^H(q)|_{IC(\theta_L)} - T_u^H(q)|_{IR(\theta_H)}\) in \(q\) implies that IC(\(\theta_L\)) and IR(\(\theta_H\)) intersect only once in the interval \([q^0(\theta_H), 1]\).

**Lemma 9** The proposed solution to Problem 1 (ignoring constraint IC(\(\theta_H\))), i.e., contract \(\tilde{c}^H = (\tilde{T}_u^H = U^A(0, q^0(\theta_H); i^*_A(q^0(\theta_H), 0) | \theta_H), q^0(\theta_H))\) if Case 1 holds and contract \(\tilde{c}^H = (\tilde{T}_u^H = U^A(0, \bar{T}; i^*_A(\bar{T}, 0) | \theta_H), \bar{T})\) if Case 2 holds, do satisfy constraint IC(\(\theta_H\)) of Problem 1.

**Proof.** We first show the result when Case 1 holds (i.e., when IC(\(\theta_L\)) is not a binding constraint in Problem 1 when \(q = q^0(\theta_H)\)). By deviating and proposing contract \(\tilde{c}^L = (\tilde{T}_u^L, q^0(\theta_L))\) instead of contract \(\tilde{c}^H = (\tilde{T}_u^H, q^0(\theta_H))\), the principal of type \(\theta_H\) obtains

\[
\begin{align*}
U^P(0, q^0(\theta_L); i^*_A(q^0(\theta_L), 1) | \theta_H) + U^A(0, q^0(\theta_L); i^*_A(q^0(\theta_L), 1) | \theta_L) & \\
\leq & \ U^P(0, q^0(\theta_L); i^*_A(q^0(\theta_L), 1) | \theta_H) + U^A(0, q^0(\theta_L); i^*_A(q^0(\theta_L), 1) | \theta_H) \\
= & \ T\mathcal{S}(i^*_A(q^0(\theta_L), 1), \theta_H) \leq T\mathcal{S}(i^*_A(\theta_H), \theta_H),
\end{align*}
\]

where the first inequality follows from Condition 1, the equality from efficient ex-post renegotiation, and the second inequality from the fact that \(i^*_A(\theta_H)\) is that investment level that maximizes expected total surplus in state \(\theta_H\).

We next show that the result also holds for Case 2 (i.e., when IC(\(\theta_L\)) is the binding constraint in Problem 1 when \(q = q^0(\theta_H)\)). Let \(\tilde{T}_u\) be the transfer such that contract \(\tilde{c} = (\tilde{T}_u, q^0(\theta_L))\) lies on the IC(\(\theta_L\)) constraint, i.e.,

\[
\tilde{T}_u = T\mathcal{S}(i^*_A(\theta_L), \theta_L) - U^P(0, q^0(\theta_L); i^*_A(q^0(\theta_L), 0) | \theta_L)
\]

\[
= \tilde{T}_u^L - \{U^P(0, q^0(\theta_L); i^*_A(q^0(\theta_L), 0) | \theta_L) - U^P(0, q^0(\theta_L); i^*_A(q^0(\theta_L), 1) | \theta_L)\}.
\]

The difference \((\tilde{T}_u^L - \tilde{T}_u) > 0\) represents the value for the principal of type \(\theta_L\) for a higher level of investment, since contracts \(\tilde{c}^L\) and \(\tilde{c}\) are associated with investment levels \(i^*_A(q^0(\theta_L), 1)\) and \(i^*_A(q^0(\theta_L), 0)\), respectively. From the fact that the expected payoff of the principal of type \(\theta_H\) responds more positively to the agent’s investment than the one of the principal of type \(\theta_L\) (Lemma 2), it follows that

\[
U^P(\tilde{T}_u, q^0(\theta_L); i^*_A(q^0(\theta_L), 0) | \theta_H) \geq U^P(\tilde{T}_u^L, q^0(\theta_L); i^*_A(q^0(\theta_L), 1) | \theta_H), \quad (6.13)
\]

where the right hand side of (6.13) corresponds exactly to the payoff of the principal of type \(\theta_H\) if she proposes contract \(\tilde{c}^L\). Finally, using the fact that the expected payoff of the principal of type \(\theta_H\) increases when moving along the IC(\(\theta_L\)) constraint by increasing quantity \(q\) (see derivation of the solution to Problem 1 in Section 5.3), we obtain that

\[
U^P(\tilde{T}_u^H, \bar{T}; i^*_A(\bar{T}, 0) | \theta_H) \geq U^P(\tilde{T}_u, q^0(\theta_L); i^*_A(q^0(\theta_L), 0) | \theta_H).
\]

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This inequality together with (6.13) imply that a deviation from \( \tilde{c}^H = (\tilde{T}^H_u, \Phi) \) to \( \tilde{c}^L = (\tilde{T}^L_u, q^0(\theta_L)) \) is not profitable for the principal of type \( \theta_H \), which completes the proof. \( \blacksquare \)

**Proof.** (Lemma 5) Let \( b_L(m') \) denote the agent’s beliefs after observing menu proposal \( m' \) by the principal. In a similar way, let \( b_L(c; m') \) denote the agents beliefs after observing that the principal chose contract \( c \) within the contracts in menu \( m' \).

The proof shows that for any menu \( m' \neq m \) there are beliefs \( b_L(m') \) and a continuation equilibrium after \( m' \) has been proposed in which either \( m' \) is rejected or equilibrium payoffs, after \( m' \) has been accepted, are \( \tilde{U}^P(m', \theta_L) \leq TS(i^0_A(\theta_L), \theta_L) \) for the principal of type \( \theta_L \) and \( \tilde{U}^P(m', \theta_H) \leq TS(i^0_A(\theta_H), \theta_H) \) for the principal of type \( \theta_H \).

The proof follows by contradiction. Suppose that exists a menu \( m' \in M \) such that, for all beliefs \( b_L(m') \) and all continuation equilibria, the agent accepts the menu proposal \( m' \) and \( \tilde{U}^P(m', \theta_L) > TS(i^0_A(\theta_L), \theta_L) \) or \( \tilde{U}^P(m', \theta_H) > TS(i^0_A(\theta_H), \theta_H) \).

Consider first beliefs \( b_L(m') = 1 \) and the continuation equilibrium in which \( b_L(c; m') = 1 \) for all \( c \in m' \) and each type of principal chooses the contract in \( m' \) that maximizes her expected payoff. In such an equilibrium, it must be the case that \( U^P(c; i^*_A(c, 1) | \theta_L) \leq TS(i^0_A(\theta_L), \theta_L) \) for all \( c \in m' \), otherwise \( \max_{c \in m'} U^P(c; i^*_A(c, 1) | \theta_L) > TS(i^0_A(\theta_L), \theta_L) \), which by (2.4) (efficient ex-post renegotiation) would imply that the expected payoff of the agent is negative after observing \( m' \) and consequently the menu would be rejected. Hence, in this specific equilibrium following the proposal of \( m' \), the deviation from \( m \) cannot be profitable for the principal of type \( \theta_L \) and therefore it must be profitable for the principal of type \( \theta_H \). Thus, \( \max_{c \in m'} U^P(c; i^*_A(c, 1) | \theta_H) > TS(i^0_A(\theta_H), \theta_H) \). Consider the set \( C_{\theta_H} = \{ c \in m' : U^P(c; i^*_A(c, 1) | \theta_H) > TS(i^0_A(\theta_H), \theta_H) \} \).

We now show that \( U^P(c; i^*_A(c, 0) | \theta_H) \leq TS(i^0_A(\theta_H), \theta_H) \) for all \( c \in C_{\theta_H} \). To see this, note that, if for some \( c \in C_{\theta_H} \) this was not true, then for beliefs \( b_L(m') = 0 \) and the subsequent equilibrium in which \( b_L(c; m') = 0 \) if \( c \in C_{\theta_H} \), \( b_L(c; m') = 1 \) if \( c \notin C_{\theta_H} \), and the principal of type \( \theta_H \) has payoff \( \max_{c \in C_{\theta_H}} U^P(c; i^*_A(c, 0) | \theta_H) > TS(i^0_A(\theta_H), \theta_H) \), the expected payoff of the agent would be negative due to (2.4) (efficient ex-post renegotiation) and therefore \( m' \) would be rejected by the agent, contradicting the initial assumption that for all beliefs \( b_L(m') \) and all continuation equilibrium, \( m' \) is accepted.

We next show that given the properties described above for \( m' \), it is always possible to find beliefs \( b_L(m') \), \( b_L(c; m') \) and a corresponding continuation equilibrium for which no type of principal wants to deviate from \( m \). Consider the following beliefs: for \( c \in \{ m' \setminus C_{\theta_H} \} \) let \( b_L(c; m') = 1 \) while for \( c \in C_{\theta_H} \) let \( b_L(c; m') \) be such that \( U^P(c; i^*_A(c, b_L(c, m'))) | \theta_H) = TS(i^0_A(\theta_H), \theta_H) \).
Denote these beliefs by \( \hat{b}(c) \). Note that these beliefs always exist. Recall that as was shown above, for all \( c \in C_{\theta_H} \), \( U^P(c; i^*_A(c,0) \mid \theta_H) \leq TS(i^*_A(\theta_H), \theta_H) \), by definition \( U^P(c; i^*_A(c,1) \mid \theta_H) > TS(i^*_A(\theta_H), \theta_H) \) and therefore from continuity of \( U^P(c; i^*_A(c,b_L) \mid \theta_H) \) in \( b_L \) and the Intermediate Value Theorem, existence of \( \hat{b}(c) \) is guaranteed. With this system of beliefs, denote by \( \tilde{c}^L \) the optimal contract in \( m' \) for the principal of type \( \theta_L \), i.e., \( \tilde{c}^L = \arg \max_{c \in m'} U^P(c; i^*_A(c, \hat{b}(c)) \mid \theta_L) \). Now, there are two possible cases.

First, suppose that \( \tilde{c}^L \notin C_{\theta_H} \). It is easy to see that beliefs \( b_L(m') = 1 \), \( b_L(c, m') = \hat{b}(c) \) for all \( c \in m' \), together with the principal of type \( \theta_L \) choosing contract \( \tilde{c}^L \) and the principal of type \( \theta_H \) choosing any contract \( c \in C_{\theta_H} \), constitutes a continuation equilibrium following the proposal of menu \( m' \). Consistency of beliefs follows from the fact that we are free to choose beliefs \( b_L(c, m') \) for all \( c \neq \tilde{c}^L \) in \( m' \), since \( b_L(m') = 1 \). Sequential rationality follows from the fact that given beliefs \( b_L(c, m') = \hat{b}(c) \), contract \( \tilde{c}^L \) is the best contract in \( m' \) for the principal of type \( \theta_L \). Regarding the principal of type \( \theta_H \), contract choice, note that \( U^P(c; i^*_A(c, \hat{b}(c)) \mid \theta_H) = TS(i^*_A(\theta_H), \theta_H) \) for all \( c \in C_{\theta_H} \), while by definition of \( C_{\theta_H} \), her payoff satisfies \( U^P(c; i^*_A(c,1) \mid \theta_H) \leq TS(i^*_A(\theta_H), \theta_H) \) for all \( c \in m' \setminus C_{\theta_H} \). Finally, with these beliefs and continuation equilibrium, the agent accepts the menu proposal \( m' \) and the equilibrium payoff of the principal of type \( \theta_j \) is lower than \( TS(i^*_A(\theta_j), \theta_j) \) for \( j = L, H \). Therefore, a deviation to menu \( m' \) is not profitable for the principal, which contradicts the initial assumption about \( m' \).

Second, suppose that \( \tilde{c}^L \in C_{\theta_H} \). In this case, we can construct the following equilibrium: \( b_L(m') = \hat{b}(\tilde{c}^L) \) (just recall that we are free to choose \( b_L(m') \)), \( b_L(c, m') \) are the same as above, i.e., \( b_L(c, m') = \hat{b}(c) \) for \( c \in m' \) and both types of principal choose contract \( \tilde{c}^L \). Note again that in the continuation game after \( m' \) has been accepted no type of principal wants to deviate to a different contract and that the system of beliefs is consistent. Moreover, in this equilibrium both types of principal have lower payoffs than the ones in menu \( m \) (for the principal of type \( \theta_H \) this is obvious, while for the principal of type \( \theta_L \) the result follows from the fact that \( U^P(\tilde{c}^L; i^*_A(\tilde{c}^L,1) \mid \theta_L) \leq TS(i^*_A(\theta_L), \theta_L) \), the fact that the payoff of the principal of type \( \theta_H \) decreases when moving from \( i^*_A(c,1) \) to the higher level of investent \( i^*_A(c, \hat{b}(c)) \) for all \( c \in C_{\theta_H} \), and from Lemma 2). Hence, a deviation from menu \( m \) to menu \( m' \) is not profitable for neither type of principal. This contradicts the initial assumption that for all beliefs \( b_L(m') \) and continuation equilibrium at least one of the types of principal wants to deviate to menu \( m' \). This completes the proof. ■

Proof. (Lemma 6) This proof follows basically the same steps as the proof of Proposition 5. However, some particular aspects are more involved and therefore a complete version of it is presented. As before, \( b_L(m') \) denotes the agent’s beliefs after observing menu \( m' \) by the principal and \( b_L(c; m') \) the agent’s beliefs after observing the principal’s choice of contract \( c \) within the contracts in \( m' \).

The proof shows that for any menu \( m' \neq m \), there are beliefs \( b_L(m') \) and a continuation equilibrium after \( m' \) has been proposed in which either \( m' \) is rejected or equilibrium payoffs, after \( m' \) has been accepted,
are \( \tilde{U}^P(m', \theta_L) \leq \mathcal{TS}(\theta_0(\theta_L), \theta_L) \) for the principal of type \( \theta_L \) and \( \tilde{U}^P(m', \theta_H) \leq \mathcal{TS}(\theta_0(\theta_H), \theta_L) \) for the principal of type \( \theta_H \).

We follow by contradiction. Suppose that exists a menu \( m' \in M \) such that, for all beliefs \( b_L(m') \) and all continuation equilibria, the agent accepts the menu proposal \( m' \) and \( \tilde{U}^P(m', \theta_L) > \mathcal{TS}(\theta_0(\theta_L), \theta_L) \) or \( \tilde{U}^P(m', \theta_H) > \mathcal{TS}(\theta_0(\theta_Q), \theta_H) \).

The remaining of the proof builds on the following Lemma.

**Lemma 10** If a contract \( c = (T_u, q) \) satisfies simultaneously \( U^P(c; i_A^*(c, 1) \mid \theta_L) \leq \mathcal{TS}(\theta_0(\theta_L), \theta_L) \) and \( U^P(c; i_A^*(c, 1) \mid \theta_H) > \mathcal{TS}(\theta_0(\theta_Q), \theta_H) \) then \( q > \bar{q} \).

Proof: Conditions \( U^P(c; i_A^*(c, 1) \mid \theta_L) \leq \mathcal{TS}(\theta_0(\theta_L), \theta_L) \) and \( U^P(c; i_A^*(c, 1) \mid \theta_H) > \mathcal{TS}(\theta_0(\theta_Q), \theta_H) \) are equivalent to

\[
T_u \leq \mathcal{TS}(\theta_0(\theta_L), \theta_L) - U^P(0, q; i_A^*(q, 1) \mid \theta_L)
\]

and

\[
T_u > \mathcal{TS}(\theta_0(\theta_Q), \theta_H) - U^P(0, q; i_A^*(q, 1) \mid \theta_H).
\]

Therefore, a necessary condition for these 2 conditions to hold simultaneously is that

\[
\mathcal{TS}(\theta_0(\theta_Q), \theta_H) - \mathcal{TS}(\theta_0(\theta_L), \theta_L) < U^P(0, q; i_A^*(q, 1) \mid \theta_H) - U^P(0, q; i_A^*(q, 1) \mid \theta_L).
\]

(6.14)

Since the right hand side of (6.14) is an increasing function of \( q \), then it suffices to show that for \( q = \bar{q} \) condition (6.14) is not satisfied. To obtain this, note that,

\[
\mathcal{TS}(\theta_0(\theta_Q), \theta_H) - \mathcal{TS}(\theta_0(\theta_L), \theta_L) = U^P(0, \bar{q}; i_A^*(\bar{q}, 0) \mid \theta_H) - U^P(0, \bar{q}; i_A^*(\bar{q}, 0) \mid \theta_L)
\]

\[
\geq U^P(0, \bar{q}; i_A^*(\bar{q}, 1) \mid \theta_H) - U^P(0, \bar{q}; i_A^*(\bar{q}, 1) \mid \theta_L),
\]

where the first equality is obtained by using the fact that \( \mathcal{TS}(\theta_0(\theta_Q), \theta_H) = U^P(0, \bar{q}; i_A^*(\bar{q}, 0) \mid \theta_H) + U^A(0, \bar{q}; i_A^*(\bar{q}, 0) \mid \theta_H) \) and the fact the incentive compatibility of type \( \theta_L \) is binding in the best separating allocation, i.e., \( \mathcal{TS}(\theta_0(\theta_L), \theta_L) = U^P(0, \bar{q}; i_A^*(\bar{q}, 0) \mid \theta_L) + U^A(0, \bar{q}; i_A^*(\bar{q}, 0) \mid \theta_L) \). The inequality follows from the fact that \( U^P(0, \bar{q}; i_A^*(\bar{q}, b_L) \mid \theta_H) - U^P(0, \bar{q}; i_A^*(\bar{q}, b_L) \mid \theta_L) \) is a decreasing function of \( b_L \), which is an implication of Lemma 2. This completes the proof of the Lemma.

Consider first beliefs \( b_L(m') = 1 \) and the subsequent continuation equilibrium in which \( b_L(c; m') = 1 \) for all \( c \in m' \) and each type of principal chooses the contract in \( m' \) that maximizes her expected payoff. In such an equilibrium, it must be the case that \( U^P(c; i_A^*(c, 1) \mid \theta_L) \leq \mathcal{TS}(\theta_0(\theta_L), \theta_L) \) for all \( c \in m' \), otherwise \( \max_{c \in m'} U^P(c; i_A^*(c, 1) \mid \theta_L) > \mathcal{TS}(\theta_0(\theta_L), \theta_L) \), which by (2.4) (efficient ex-post renegotiation) would imply that the expected payoff of the agent is negative after observing \( m' \) and therefore the menu would be rejected. Hence, in this specific equilibrium following proposal of \( m' \), the deviation from \( m \) cannot be
profitable for the principal of type $\theta_L$ and therefore it must be profitable for the principal of type $\theta_H$. Thus
\[
\max_{c \in m'} U^P(c; i_A^*(c, 1) \mid \theta_H) > T\overline{S}(i_A^*(\overline{q}, 0), \theta_H).
\]
Consider the set
\[
C_{\theta_H} = \{c \in m' : U^P(c; i_A^*(c, 1) \mid \theta_H) > T\overline{S}(i_A^*(\overline{q}, 0), \theta_H)\}.
\]
Applying Lemma (10) we obtain that for all $c \in C_{\theta_H}$, $q > \overline{q}$.

We now show that $U^P(c; i_A^*(c, 0) \mid \theta_H) \leq T\overline{S}(i_A^*(\overline{q}, 0), \theta_H)$ for all $c \in C_{\theta_H}$. To see this, note that, if for some $c \in C_{\theta_H}$ this was not true, then for beliefs $b_L(m') = 0$ and the subsequent equilibrium in which $b_L(c; m') = 0$ if $c \in C_{\theta_H}$ and $b_L(c; m') = 1$ if $c \in \{m' \setminus C_{\theta_H}\}$, the principal of type $\theta_H$ would have payoff $\max_{c \in C_{\theta_H}} U^P(c; i_A^*(c, 0) \mid \theta_H) > T\overline{S}(i_A^*(\overline{q}, 0), \theta_H)$. Together with the fact that $T\overline{S}(i_A^*(q, 0), \theta_H) < T\overline{S}(i_A^*(\overline{q}, 0), \theta_H)$ for $q > \overline{q}$, this implies that the expected payoff of the agent would be negative due to (2.4) (efficient ex-post renegotiation) and therefore $m'$ would be rejected by the agent contradicting the initial assumption that for all beliefs $b_L(m')$ and all continuation equilibrium, $m'$ is accepted.

We next show that given the properties described above for $m'$, it is always possible to find beliefs $b_L(m')$ and a continuation equilibrium for which no type of principal wants to deviate from $m$.

Consider the following beliefs: for $c \in \{m' \setminus C_{\theta_H}\}$ let $b_L(c; m') = 1$ while for $c \in C_{\theta_H}$ let $b_L(c; m')$ be such that
\[
U^P(c; i_A^*(c, b_L(c, m'))) \mid \theta_H = T\overline{S}(i_A^*(\overline{q}, 0), \theta_H).
\]
Denote these beliefs by $\widehat{b}(c)$. Note that these beliefs always exist. Recall that as was shown above, for all $c \in C_{\theta_H}$, $U^P(c; i_A^*(c, 0) \mid \theta_H) \leq T\overline{S}(i_A^*(\overline{q}, 0), \theta_H)$, by definition of $C_{\theta_H}$ the principal’s payoff satisfy $U^P(c; i_A^*(c, 1) \mid \theta_H) > T\overline{S}(i_A^*(\overline{q}, 0), \theta_H)$ and therefore from continuity of $U^P(c; i_A^*(c, b_L) \mid \theta_H)$ in $b_L$ and the Intermediate Value Theorem, existence of $\widehat{b}(c)$ is guaranteed. With this system of beliefs, denote by $\widehat{c}^L$ the optimal contract in $m'$ for the principal of type $\theta_L$, i.e., $\widehat{c}^L = \arg\max_{c \in m'} U^P(c; i_A^*(c, \widehat{b}(c)) \mid \theta_L)$. Now, there are two possible cases.

First, suppose that $\widehat{c}^L \notin C_{\theta_H}$. It is easy to see that beliefs $b_L(m') = 1$, $b_L(c, m') = \widehat{b}(c)$ for all $c \in m'$, together with the principal of type $\theta_L$ choosing contract $\widehat{c}^L$ and the principal of type $\theta_H$ choosing any contract $c \in C_{\theta_H}$ constitutes a continuation equilibrium following the proposal of menu $m'$. Consistency of beliefs follows from the fact that we are free to choose beliefs $b_L(c, m')$ for all $c \neq \widehat{c}^L$ in $m'$, since $b_L(m') = 1$. Sequential rationality follows from the fact that given beliefs $b_L(c, m') = \widehat{b}(c)$, contract $\widehat{c}^L$ is the best contract in $m'$ for the principal of type $\theta_L$. Regarding the principal of type $\theta_H$ contract choice, note that $U^P(c; i_A^*(c, \widehat{b}(c)) \mid \theta_H) = T\overline{S}(i_A^*(\overline{q}, 0), \theta_H)$ for all $c \in C_{\theta_H}$ while by definition of $C_{\theta_H}$ her payoff satisfies $U^P(c; i_A^*(c, 1) \mid \theta_H) \leq T\overline{S}(i_A^*(\overline{q}, 0), \theta_H)$ for all $c \in \{m' \setminus C_{\theta_H}\}$. Finally, with these beliefs and continuation equilibrium, the agent accepts the menu proposal $m'$ and the equilibrium payoffs of the principal of type $\theta_L$ is no larger than $T\overline{S}(i_A^*(\theta_L), \theta_L)$ and the principal of type $\theta_H$ no larger than $T\overline{S}(i_A^*(\overline{q}, 0), \theta_H)$. Therefore, a deviation to menu $m'$ is not profitable for the principal, which contradicts the initial assumption about $m'$.
Second, suppose that \( \bar{c}^L \in C_{\theta_H} \). In this case, we can construct the following equilibrium: \( b_L(m') = \hat{b}(\bar{c}^L) \) (just recall that we are free to choose \( b_L(m') \), \( b_L(c, m') \) are the same as above, i.e., \( b_L(c, m') = \hat{b}(c) \) for \( c \in m' \) and both types of principal choose contract \( \bar{c}^L \). Note again that in the continuation game after \( m' \) has been accepted no type of principal wants to deviate to a different contract and that the system of beliefs is consistent. Moreover, in this equilibrium both types of principal have lower payoffs than the ones in menu \( m \) (for the principal of type \( \theta_H \) this is obvious, while for the principal of type \( \theta_L \) the result follows from the fact that \( U^L(\bar{c}^L; i_A^s(\bar{c}^L, 1) \mid \theta_L) \leq \sigma(\hat{i}_A^0(\theta_L), \theta_L) \), the fact that the payoff of the principal of type \( \theta_H \) decreases when moving from \( i_A^s(c, 1) \) to the higher level of investment \( i_A^s(c, \hat{b}(c)) \) for all \( c \in C_{\theta_H} \), and from Lemma 2). Hence, a deviation from menu \( m \) to menu \( m' \) is not profitable for neither type of principal. This contradicts the initial assumption that for all beliefs \( b_L(m') \) and continuation equilibrium at least one of the types of principal wants to deviate to menu \( m' \). This completes the proof. \( \blacksquare \)

Proof. (Proposition 5) The payoff that the principal obtains when she proposes a contract \( c \) to the agent depends first on the agent’s decision to accept the contract and second on his investment decision. Both decisions depend on the contract \( c \) itself and on the agent’s beliefs after observing that the principal proposed \( c \). Hence, to obtain the highest payoff that the principal can ensure herself, we have to find the contract that gives her the highest payoff given the “worst” beliefs of the agent. Thus, the highest payoff a principal of type \( \theta \) can ensure herself is given by

\[
\max_{\bar{c} \in C} \bar{g}(\bar{c}, \theta),
\]

where

\[
\bar{g}(\bar{c}, \theta) = \min_{b_L} \{ U^P(\bar{c}; i_A^s(\bar{c}, b_L) \mid \theta) \} + \min_{b_L} \{ U^A(\bar{c}; i_A^s(\bar{c}, b_L) \mid b_L) \},
\]

\( \bar{c} \in C \) denotes a contract with zero transfer, i.e., \( \bar{c} = (0, q, e) \), and

\[
U^A(c; i_A \mid \theta) = \frac{1}{2} E[\max\{V(i_A), V\} \mid \theta] - \frac{1}{2} q(v_p - v_u(i_A)) + (1 - q)(1 - e)E[\psi] - \psi(i_A) - T_u.
\]

The first term in \( \bar{g}(\bar{c}, \theta) \) captures the effect of the agent’s beliefs on the principal’s expected payoff through the investment decision, while the second term captures the effect of the agent’s beliefs on the principal’s expected payoff through the acceptance decision (which limits the transfer that the principal can demand in the contract).

We first analyze how the expected payoffs \( U^P(\bar{c}; i_A^s(\bar{c}, b_L) \mid \theta) \) and \( U^A(\bar{c}; i_A^s(\bar{c}, b_L) \mid b_L) \) evolve with beliefs \( b_L \), given a contract \( \bar{c} = (0, q, e) \).

Differentiating \( U^P(\bar{c}; i_A^s(\bar{c}, b_L) \mid \theta) \) with respect to \( b_L \), we obtain

\[
\frac{1}{2} v'_u(i_A^s(q, b_L)) \times [P_s(i_A^s(q, b_L), \theta) - q] \times \frac{\partial i_A^s(q, b_L)}{\partial b_L}.
\]

(6.15)
By assumption, we have \( v_A'(i_A(q, b_L)) \geq 0 \) and by lemma 1, \( \frac{\partial v_A(q, b_L)}{\partial b_L} \leq 0 \). Thus, determining how the belief \( b_L \) affects the expected payoff of the principal \( U^P(\bar{c}; i_A^*(\bar{c}, b_L) \mid \theta) \) reduces to determining the sign of \( [P_s(i_A^*(q, b_L), \theta) - q] \).

Differentiating \( U^A(\bar{c}; i_A^*(\bar{c}, b_L) \mid b_L) \) with respect to \( b_L \) and noting that by the Envelope Theorem, the effect of beliefs on \( U^A(\bar{c}; i_A^*(\bar{c}, b_L) \mid b_L) \) through its effect on investment is zero, we obtain

\[
U^A(0, q, c; i_A^*(q, b_L) \mid \theta_L) - U^A(0, q, c; i_A^*(q, b_L) \mid \theta_H).
\]

(6.16)

We then proceed by showing that there exist contracts \( \bar{c}^L \) and \( \bar{c}^H \) such that \( \bar{g}(\bar{c}^L, \theta_L) = TS(i_A^0(\theta_L), \theta_L) \) and \( \bar{g}(\bar{c}^H, \theta_H) = TS(i_A^0(\theta_H), \theta_H) \), which is sufficient to establish the result stated in the proposition.

We begin with the principal of type \( \theta_L \). Let \( \bar{c}^L = (0, q^0(\theta_L), 0) \). In that case, \( P_s(i_A^*(q^0(\theta_L), b_L), \theta_L) \geq q^0(\theta_L) \) for all \( b_L \in [0, 1] \), so \( U^P(\bar{c}^L; i_A^*(\bar{c}^L, b_L) \mid \theta_L) \) is decreasing in \( b_L \). Thus,

\[
\min_{b_L} \{U^P(\bar{c}^L; i_A^*(\bar{c}^L, b_L) \mid \theta_L)\} = U^P(\bar{c}^L; i_A^*(\bar{c}^L, 1) \mid \theta_L).
\]

(6.17)

We also need to analyze how the agent’s payoff evolves with beliefs when the proposed contract is \( \bar{c}^L \). From the facts that \( U^A(0, x, 0; i_A^0(q^0(\theta_L), b_L) \mid \theta_L) - U^A(0, x, 0; i_A^0(q^0(\theta_L), b_L) \mid \theta_H) \) is an increasing function of \( x \) and \( b_L \), it follows by Condition 1 that (6.16) is negative for all \( b_L \in [0, 1] \) when evaluated at \( \bar{c}^L = (0, q^0(\theta_L), 0) \), which implies that \( U^A(\bar{c}^L; i_A^*(\bar{c}^L, b_L) \mid b_L) \) is decreasing in \( b_L \). Therefore,

\[
\min_{b_L} \{U^A(\bar{c}^L; i_A^*(\bar{c}^L, b_L) \mid b_L)\} = U^A(\bar{c}^L; i_A^*(\bar{c}^L, 1) \mid \theta_L).
\]

(6.18)

Hence, from (6.17) and (6.18), we obtain that

\[
\bar{g}(\bar{c}^L, \theta_L) = U^P(0, q^0(\theta_L), 0; i_A^0(q^0(\theta_L), 1) \mid \theta_L) + U^A(0, q^0(\theta_L), 0; i_A^0(q^0(\theta_L), 1) \mid \theta_L)
\]

\[
= TS(i_A^0(\theta_L), \theta_L),
\]

where the equality follows from (2.4) (efficient ex-post renegotiation) and the fact that \( i_A^0(q^0(\theta_L), 1) = i_A^0(\theta_L) \).

Consider now the case of the principal of type \( \theta_H \). Let \( \bar{c}^H = (0, q^0(\theta_H), 1) \). Now, we have \( P_s(i_A^*(q^0(\theta_H), b_L), \theta_H) \leq q^0(\theta_H) \) for all \( b_L \in [0, 1] \) and therefore \( U^P(\bar{c}^H; i_A^*(\bar{c}^H, b_L) \mid \theta_H) \) is increasing in \( b_L \). Thus,

\[
\min_{b_L} \{U^P(\bar{c}^H; i_A^*(\bar{c}^H, b_L) \mid \theta_H)\} = U^P(\bar{c}^H; i_A^*(\bar{c}^H, 0) \mid \theta_H).
\]

(6.19)

We now analyze how the agent’s payoff evolves with beliefs when the proposed contract is \( \bar{c}^H \). Since \( U^A(0, q, e = 1; i_A \mid \theta_H) \geq U^A(0, q, e = 1; i_A) \) for all \( i_A \in I_A \) and \( q \in Q \), we obtain that (6.16) is positive for all \( b_L \in [0, 1] \) when evaluated at \( \bar{c}^H = (0, q^0(\theta_H), 1) \), which implies that \( U^A(\bar{c}^H; i_A^*(\bar{c}^H, b_L) \mid b_L) \) is increasing in \( b_L \). Thus,

\[
\min_{b_L} \{U^A(\bar{c}^H; i_A^*(\bar{c}^H, b_L) \mid b_L)\} = U^A(\bar{c}^H; i_A^*(\bar{c}^H, 0) \mid \theta_H).
\]

(6.20)
Finally, from (6.19) and (6.20) we obtain that,

$$\tilde{g}(c_H, \theta_H) = U^P(0, q^0(\theta_H), 0; i_A^*(q^0(\theta_H), 0) | \theta_H) + U^A(0, q^0(\theta_H), 0; i_A^*(q^0(\theta_H), 0) | \theta_H)$$

$$= TS(i_A^*(q^0(\theta_H), 0), \theta_H),$$

where the equality follows from (2.4) (efficient ex-post renegotiation) and the fact that $i_A^*(q^0(\theta_H), 0) = i_A^0(\theta_H)$. □

References


