Asset Diversion, Input Allocation and Capital Structure\(^1\)

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Abstract

The paper investigates the determinants of bank and trade credit demands and their interactions with the input combination optimally chosen by the entrepreneur. Within an incomplete contract setting with uncertainty, two-input technology and collateralised credit contracts, the paper provides a theoretical foundation for several stylized facts for which the extant literature lacks a clear understanding: (1) Why do financially unconstrained firms take trade credit when they can access cheaper credit? (2) Why in situations of bankruptcy the suppliers are given priority on debt claims through the liquidation of the inputs supplied to the firm? (3) Why do suppliers lend inputs but only seldom lend cash? The paper also investigates the presence of regularities between the documented cross-country differences in financing decisions (bank versus trade credit) and the ones in technological choices (tangible versus intangible assets).

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1 Introduction

Trade credit is known to be a very important source of external finance to many firms in developed and developing countries (see Petersen and Rajan (1997); Rajan and Zingales (1995); Fisman and Love (2003)), and to be even more important than bank credit for some countries (Fabbri and Padula 2004).\footnote{Petersen and Rajan (1997) provide evidence that account payables average 4.4% of sales for a sample of small US firms and 11.6% for large firms. Firms in most of the other industrialized nations are comparably reliant on trade credit. Rajan and Zingales (1995) report that in the G-7 nations accounts payables of a sample of large firms range from 17% of assets in France to 11.5% in Germany, compared with 15% in US, 15.4% in Japan and 14.7% in Italy. Fisman and Love (2003) provide evidence that in developing countries trade credit may be even more important. Fabbri and Padula (2004) document that on average trade credit is larger than bank debt for Italian firms in all industries. In particular, in the wholesale and retail sectors, trade credit is twice as big as bank credit, while in the manufacturing sector is 1.5 times larger.}

Compared to the vast literature on banking, however, trade credit has received modest attention from researchers. In particular, there are several stylized facts for which the existing theories lack a clear understanding. First, why do wealthy unconstrained firms take trade credit when they can access cheaper credit? Second, why in situations of bankruptcy the suppliers are given priority on debt claims through the liquidation of the inputs supplied to the firm? Third, why do suppliers lend inputs but only seldom lend cash? Last, is there any link between the documented cross-country differences in financing decisions and the ones in technological choices?

There are several empirical findings suggesting that non-rationed firms take trade credit. The World Business Environment Survey (WBES) conducted by the World Bank in 1999 provides unique information on the presence of financing obstacles to firm’s activity. In particular, using self-reported answers, the survey allows to classify firms in four groups depending on whether they are more or less likely to be credit-constrained.\footnote{WBES asks firms to "judge on a four point scale how problematic is the financing factor for the operation and growth of its business", choosing one of the following categories: no obstacle, minor obstacle, moderate obstacle and major obstacle.} For each firm in each of these groups, WBES reports balance-sheet information including the use of supplier credit. In table 1, we report this information for a subsample of countries. Table 1 shows that in several countries the percentage of supplier credit over total firm’s financing is positive for non-rationed firms (see the first two columns). Interestingly, in Italy, Sweden and Canada that percentage is even higher than the one for constrained firms. Similarly, Marotta (2001) records that in 1994 in the Italian manufacturing sector, trade credit financed on average 38.1% of the input purchase of non-rationed firms and 37.5% of rationed ones.\footnote{Marotta (2001) uses data from the Italian survey on manufacturing sector conducted by Mediocredito Centrale in 1994. Credit constrained firms are identified by two questions: "In 1994, has the firm applied for, but not obtained, more bank loans?" and "In 1994, would the firm have accepted tighter terms (higher interest rates or higher collateral requirements) to obtain more bank loan?". Notice that the measure of trade credit reported in Marotta (2001) is not comparable with the data reported in Table 1. Marotta (2001) uses the percentage of inputs financed by trade credit, while WBES provides the share of trade credit over total financing.} These findings raise a puzzle.
Why do financially unconstrained firms in some countries make massive use of trade credit when they can access cheaper credit?

Existing theories have tried to rationalise the evidence on the use of trade credit by financially constrained firms but they have been silent on the factors driving trade credit demand for unconstrained firms. These theories rely in different ways on the sellers having private information that other creditors do not have (Freixas (1993), Biais and Gollier (1997), Burkart and Ellingsen (2004)). However, as noted by Burkart and Ellingsen (2004, p. 2), if one accepts the supplier’s information advantage theory, it remains unexplained “why the supplier’s lending is so closely tied to the value of the input transaction”, i.e. why suppliers extend credit by allowing delayed payment for their products but seldom lend cash.

As earlier contributions in the field, the present paper also identifies the supplier’s information advantage as a possible determinant of trade credit demand. However, it argues that there are other comparative advantages in supplier’s lending, namely a higher ability in liquidating the assets sold to the firm. This liquidation advantage explains the reliance on trade credit by non-rationed firms, in spite of the higher cost of this source of financing. Furthemore, this same advantage is at the heart of the priority rule followed in situations of bankruptcy by many countries, among which Italy and Sweden, where the suppliers are given priority on debt claims through the liquidation of the inputs supplied to the firm. An implication of this analysis is that the structure of financial contracts between the entrepreneur and the financiers (say, bank and supplier) depends on the motives underlying the demand for trade credit. When the firm asks for trade credit to exploit the supplier’s liquidation advantage, the supplier gets a contract with flat repayments across states, whilst the bank gets a
contract with repayments increasing in cash flows. Conversely, when the firm asks for trade credit to exploit the supplier’s information advantage, both the bank and the supplier get a contract with repayments increasing in cash flows.

A further contribution of this paper lies in its ability to reconcile two different pieces of evidence related to corporate governance structures. Recent empirical literature has highlighted cross-country differences in the use of trade credit, which have been ascribed to the quality of corporate governance (Rajan and Zingales (1995); La Porta et al. (1998); Frank and Maksimovic (1998)). At the same time, firms in developing countries have higher proportions of fixed assets to total assets and fewer intangible assets than firms in developed countries (e.g., Demirguc-Kunt and Maksimovic (2001)). This evidence has been explained by cross-country differences in the degree of property rights protection by Claessens and Laeven (2003). While each of these two aspects - financing and technological decisions - has been investigated separately in the literature, this paper argues that they are closely related and thus only a study of the interplay between them can reconcile theory with evidence.

Our idea is that, in determining the investment level, an entrepreneur chooses among different combinations of assets (e.g., tangible and intangible). This decision depends not only on the absolute level of the available sources of funding, but also on the nature (type) of these sources, namely own wealth, banking finance and trade credit. On the one hand, a change in one of the available resources is likely to change both the total investment size and the optimal combination of inputs, since, due to their different characteristics, these types of resources are not perfect substitutes. On the other hand, by altering the optimal combination of inputs, the entrepreneur can change the availability and the combination of financing resources, since each input has different features that can be used to leverage the credit constraints. It follows that the channels through which corporate governance standards may affect firm’s behaviour go through the interplay of financing and technological decisions, since potentially relevant feedback effects going from the one to the other may arise. Based on this idea, we expect regularities between technological choices and capital structures to arise among countries with different corporate governance institutions.

To investigate these aspects, we introduce a multi-input technology, where tangible (e.g., physical capital) and intangible (e.g., human capital) assets have different properties in terms of liquidity degree,

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4The use of trade credit relative to banking finance is higher in countries providing weaker protection of creditor’s rights, as documented by In US trade credit is the single largest source of short-term finance and, as shown by Lee and Stowe (1993), in 1985 far exceeded the business lending of the entire banking system.
5In particular, in countries where the value of intangible assets is weakly secured by property rights legislation, firm’s investment is biased towards tangible assets.
6For example, intangible assets cannot be financed in general by trade credit.
7Here, we interpret corporate governance standards with a very broad meaning as the legal rules disciplining the rights of all the contracting parties, i.e. entrepreneurs, workers and investors, such as banks and suppliers.
observability and sources of financing. While we allow the supplier to finance the purchase of physical capital, we assume that any unit of intangible inputs is fully paid for in cash. Moreover, we assume tangible assets have higher liquidation value than the intangible ones. Finally, while the bank lends cash but cannot observe whether this is used to purchase the inputs (tangible and intangible assets), the supplier cannot observe the purchase of intangible assets, but she can automatically observe the purchase of tangible ones. Given that, imperfect creditor rights protection creates the potential for the entrepreneur’s opportunistic behaviour, since the funding (either in cash or in kind) raised from the creditors might be used for purposes other than the investment in the venture. However, differential input observability implies that this incentive problem is less severe when trade credit is used, at least up to a certain extent.

Under these assumptions, we find that the use of trade credit is beneficial, as it allows the entrepreneur to increase the investment size for any level of wealth (and not only for poor entrepreneurs as in previous papers). This effect goes together with a bias of the input combination towards tangible assets, which is stronger for lower levels of wealth. If we consider two entrepreneurs with different wealth, we find that the poor one not only invests a lower amount of inputs, as expected, but also uses a higher share of trade credit over total funds (trade credit intensity henceforth) and prefers a technology more intensive in tangible assets.

For sufficiently poor entrepreneurs, the optimal degree of asset tangibility is likely to be so high that the production activity would benefit from having more intangible assets, since the marginal productivity of these inputs is very high when they are underinvested. Notwithstanding, we derive a rule of no overfunding: it is never optimal for the supplier to lend cash to finance the purchase of other inputs. This allows us to explain why, in spite of the supplier’s information advantage, they do not lend cash. Our rationale for this is that the information advantage is limited to the transaction between the supplier and the entrepreneur and does not extend to the transactions between the entrepreneur and the providers of other inputs (e.g., intangible assets). If we allow the supplier to at least partially observe these transactions, cash lending arises endogenously. As a side result, we find that in this case the share of intangible inputs financed by the supplier equals the difference in the degree of information asymmetry between lenders. This can help explaining why there is evidence available on cash lending only for Japan (Uesugi and Yamashiro (2004)), where giant trading companies provide many different services, including trade credit and cash lending, to units belonging to the same vertically integrated structure. The organizational structure of these firms guarantees a continuous flow of information among the units. In line with our intuition, this evidence seems to suggest that the informational advantage is crucial to have cash lending.
Finally, we derive testable predictions on how the optimal firm’s financing policy (i.e., the mix of banking finance and trade credit) and the optimal input combination (i.e., the mix of tangible and intangible assets) adjust to changes in corporate governance standards. Interestingly, we find that the effect of creditor rights vulnerability on firms’ choices is likely to be firm-specific: a weaker protection of creditor rights increases both the degree of asset tangibility and the trade credit intensity for sufficiently poor firms, while it has no effect for richer ones.

The paper closest to ours is Burkart and Ellingsen (2004). Like earlier contributions, they use an information advantage argument to explain firm’s use of trade credit. However, and this is the line we follow in this paper, the information advantage lies in the input transaction rather than in the supplier-lender relationship. The direct observability of the input transaction and the lower liquidity of inputs relative to cash helps mitigating the entrepreneur’s potential for opportunistic behaviour and eventually justifies the use of trade credit by financially constrained firms. However, their focus on deterministic production (no uncertainty) allows them to capture only the financing motive that is behind the trade credit demand and cannot account for its use by non-rationed firms. Moreover, they do not provide any prediction on the optimal priority rule for debt claims in bankruptcy. Last, focusing on a single input technology, their setup cannot capture the interrelations between financing and technological choices and does not allow for cash lending.

Another related paper is Frank and Maksimovic (2004). Within an adverse selection framework, the paper explains firm’s trade credit reliance with the supplier’s advantage in liquidating the collateral. However, as earlier theories, the paper predicts once again a higher reliance on trade credit by financially distressed firms rather than by more creditworthy ones. Using a similar collateral liquidation advantage argument, our theory can account for this case, as well as for the case in which creditworthy firms use trade credit.

The paper is organised as follows. In Section 2, we describe the model. In Section 3, we analyse the determinants of trade credit and bank demand and the characteristics of the financing contracts. In Section 4, we derive the optimal input combination and we investigate how input choice and financing policy adjust to changes in the entrepreneur’s wealth and in the degree of creditor rights vulnerability. In Section 5, we isolate the necessary conditions to obtain cash lending from the supplier. In Section 6, we endogenise the cost of funds of the supplier. Section 7 concludes.
2 Setup and model assumptions

A risk-neutral entrepreneur needs funding to carry out an investment project which requires the purchase of two inputs, called tangible (e.g., physical capital) and intangible (e.g., human capital) and denoted by $q_k$ and $q_L$, respectively. The amount of purchased inputs that is invested, denoted by $I_k, I_L$ and observable only to the entrepreneur, is transformed into verifiable output according to the increasing and strictly concave production function $f_\sigma(I_k, I_L)$, where $\sigma \in \{G, B\}$ is the state of nature and $p$ is the probability of the good state, namely $p = Pr(\sigma = G)$. Uncertainty affects production through demand (i.e., production is demand-driven): in times of high demand, all inputs are used in production; in times of low demand, no output is produced ($f_B(I_k, I_L) = 0$) and the firm is worth only the scrap value of unused inputs, which can therefore be pledged as collateral to the financiers.

We assume that inputs are substitutes in production, but each one is essential. The following conditions summarise the main assumptions on the production function:

$$f_G(I_k, 0) = f_B(I_k, 0) = f_G(0, I_L) = f_B(0, I_L) = 0$$

$$f_G(I_k, I_L) > f_B(I_k, I_L) = 0$$

$$\frac{\partial f_G(I_k, I_L)}{\partial I_k} > \frac{\partial f_B(I_k, I_L)}{\partial I_k} = 0$$

$$\frac{\partial f_G(I_k, I_L)}{\partial I_L} > \frac{\partial f_B(I_k, I_L)}{\partial I_L} = 0$$

$$\frac{\partial^2 f_\sigma(I_k, I_L)}{\partial I_k^2} < 0; \frac{\partial^2 f_\sigma(I_k, I_L)}{\partial I_L^2} < 0$$

$$\frac{\partial^2 f_\sigma(I_k, I_L)}{\partial I_k \partial I_L} > 0$$

The entrepreneur is a price taker in three markets: the output price is normalised to 1, the capital input price is $\rho$ and the competitive wage is $w$.

The input purchase $q_k$ and $q_L$ can be financed with the entrepreneur’s own wealth ($A$), which is observable to all parties, and with the credit ($L$) raised from competitive banks ($L_B$) and suppliers ($L_S$).

Banks and suppliers play different roles. Banks only lend cash. Suppliers sell inputs, but they can also act as financiers, lending inputs and cash. Of the two inputs provided by the suppliers, we assume that only the supplier of the tangible inputs (physical capital) can finance its purchase. Each unit of intangible inputs instead is fully paid for in cash.\footnote{If we interpret intangible assets as high-skilled employees working in research and development units inside the firm, the assumption implies that we do not allow workers to finance the hiring of labor units, i.e., there is no labour credit. Similarly, if we interpret intangible assets as expenditure in research and development services provided by external firms, our assumption implies that these costs have to be paid for entirely in cash.}
Banks and suppliers also differ in the type of information they have. Suppliers of each input can costlessly observe the amount of that input which is purchased by the entrepreneur, but not the amount that is purchased from the other supplier. Thus, the supplier of tangible inputs \( q_k \) (resp. intangible inputs \( q_L \)) observes which is the amount \( q_k \) (resp. \( q_L \)) purchased by the entrepreneur. The cost of observing the purchase of inputs by parties other than their direct suppliers is too high to make observation worthwhile. It follows that the bank, which supplies no input but only lends cash, cannot observe whether any input transaction takes place, i.e. whether the loan made is used for input purchase.\(^9\)

Of the inputs \( q_k \) and \( q_L \) purchased, the entrepreneur can choose to invest an amount \( I_k \leq q_k \), \( I_L \leq q_L \). Neither the bank, nor the supplier can observe the actual amount invested in the venture.\(^10\)

Under the assumption of imperfect creditor rights protection, investment unobservability introduces a problem of moral hazard on the entrepreneur, since the funding (either in cash or in kind) raised from the creditors might not be invested in the venture but diverted to other uses. Diverted resources can be enjoyed in full and are only repaid to the extent that project returns are available (except for \( q_L \) that is paid for in cash when it is purchased). If all resources are diverted, nothing is repaid.

The profitability of the diversion activity depends on the vulnerability of creditor rights (\( \phi \)) and on the characteristics of the inputs, particularly their degree of liquidity. The more vulnerable creditor rights are (the higher \( \phi \)), the higher the return from diversion. Thus, one unit of diverted cash gives the entrepreneur \( \phi < 1 \) units of private benefit. Inputs are less liquid than cash and thus less easily divertable. In particular, one unit of input \( q_k \) costs \( \rho \) and gives \( \rho \phi \beta_k \) units of private benefit, where \( \beta_k \) is the degree of capital input liquidity;\(^11\) similarly, one unit of input \( q_L \) costs \( \omega \) and gives \( \omega \phi \beta_L \) units of private benefit, where \( \beta_L \) is the degree of labour input liquidity, with \( \beta_k \rho \geq \beta_L \omega = 0 \).

Thus, not only inputs are less liquid than cash, but input \( q_L \) is less liquid than input \( q_k \). Diversion of cash or inputs implies therefore a loss on the entrepreneur which is higher the better the

\(^9\)We assume that the bank cannot condition the contract on \( q_k \) or a share of that. Assuming full unobservability of the tangible input from the bank is not necessary to get our results. We could also assume that the bank can partially observe \( q_k \). What matters is that the degree of observability is different between the two financiers and higher for the supplier. Even assuming that the cost to get information on the balance sheet is sufficiently low to be paid, it is easy to show that the bank will never pay such cost. Provided they are risk neutral, banks only care about getting the loan \( L_B \) back. Moreover, free entry implies that they compete for clients. Thus, they have no incentive in making lending conditions tighter (by charging higher interests rate that include the cost of getting information), since these would reduce the entrepreneurs’ profits and banks would lose their clients.

\(^10\)If they could, they would write a contract which is conditional on the realised investment.

\(^11\)Buyers may want to buy capital inputs directly from the dealer, as the supply is often accompanied by other services which cannot be provided by the entrepreneur in second-hand sale.

\(^12\)Interpreting this input as labour, a limited divertability would be possible in spite of the many labour protection laws, for example by assigning workers to tasks different from those they were employed for.
degree of creditor’s legal protection. (an example? Peter trova questa parte poco chiara). It follows that differential input observability and differences in input characteristics imply that the entrepreneur’s moral hazard problem is less severe for the input that is financed through trade credit and with lower input liquidity.

Another difference between the two financiers concerns the ability to liquidate the inputs, should these be repossessed from a defaulting firm. We assume that the supplier has a better knowledge of the resale market, which gives her a more efficient liquidation technology relative to the bank. This implies that in case of failure, tangible assets are worth more if pledged as collateral to the supplier than to the bank, $\beta_S C \geq \beta_B C$, where $C$ is the collateral value ($C = \beta_k p_k$) and $\beta_S \geq \beta_B$. Notice that intangible inputs have zero collateral value, since their degree of liquidity is equal to zero.

The last difference concerns the price of the two sources of fundings. Consistent with existing evidence (see among others Ng, Smith and Smith (1999) and Wilner (2000)), we assume that bank credit is cheaper than trade credit, $r_B < r_S$. We argue that this is without loss of generality for two reasons. First, the supplier is likely to be credit constrained and have higher cost of funds than banks. Alternatively, we can assume that, even if the two sources of funding cost the same, the supplier enjoys some market power (and there is free entry in the banking sector) due to her informational advantage over input transactions and her better liquidation technology. This increases the price of credit above $r_B$.

Contracts

The entrepreneur-bank contract specifies \{ $L_B, R_B^B (f_\sigma (\cdot, \cdot), L_B)$ \}, where $L_B$ is the loan from the bank to the entrepreneur and $R_B^B$ is the state-contingent repayment obligation, depending on the size of the loan and on the output realised.

The contract between the entrepreneur and the supplier of the tangible input specifies \{ $q_k, L_S, R_S^S (f_\sigma (I_k, I_L), q_k, L_S)$ \}, where $L_S$ is the credit given by the supplier to the entrepreneur, $R_S^S$ is the state contingent repayment obligation, depending on the size of the loan, the output realised and the input purchase $q_k$. Unlike the bank, the supplier can condition the contract on the input purchase.

Because the intangible input is fully repaid when it is hired, the relevant contract between the entrepreneur and the supplier of the intangible input specifies \{ $q_L, w$ \}, i.e. the amount of the input

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13 First, the supplier has an advantage over the bank as she can observe the amount of inputs purchased by the entrepreneur and can therefore mitigate the entrepreneur’s moral hazard problem. Second, she has a more efficient collateral liquidation technology, which allows the entrepreneur to reduce the repayment due to the supplier and hence increase the profits from the venture. Because of this, by taking trade credit the entrepreneur extracts a higher surplus from the contractual relationship than if he contracted with the bank only. Knowing that, the supplier asks for a share of this surplus, under the threat to withhold her participation from the venture. This increases the price of credit above $r_B$. In Section 6, we show that the analysis developed under the assumption $r_B < r_S$ can be seen as a reduced form of a model in which the supplier has bargaining power.
purchased \( q_L \) and its remuneration.

Each party is protected by limited liability.

**Time-line**

1. Competitive banks and suppliers make contract offers given \( A \);
2. the entrepreneur accepts or rejects;
3. if he accepts he takes the investment/diversion decision (choose \( q_k^*, q_L^* \); and \( I_k^*, I_L^* \)).
4. the uncertainty resolves;
5. the payoff realises and repayments are made.

We can now determine how uncertainty and the multi-input technology shape the properties of the entrepreneur-bank-supplier contract.

### 3 Determinants of bank and trade credit

In our model there are firms, banks and suppliers of the tangible and of the intangible input. Firms carry out production, with the funding provided by banks -in cash- or by suppliers of the tangible input -in cash or in kind. The cost of bank credit is assumed to be lower than the cost of supplier credit, which makes the firm prefer the former means of financing to the latter. However, supplier’s credit has two advantages: one is that, since inputs are less liquid than cash, input transactions reduce the scope for diversion; the second has to do with the better opportunities of the supplier relative to other creditors in liquidating the inputs in case of repossession from a defaulting firm. In this section we show how these three effects interact.

Firms maximise profits, which can be split into two components: the return from production \((EP)\) and the return from diversion \((D)\). The expected return from production is given by:

\[
EP = p \left[ f_G (I_k, I_L) - R^B_G - R^S_G \right]
\]

where \( R^B_G, R^S_G \geq 0 \) are the good state repayments to bank and supplier respectively. Because low state output is zero \((f_B (I_k, I_L) = 0)\) and because of bank and supplier limited liability, the repayments they get in the bad state \( R^B_G, R^S_G \) are both zero.

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14 A remark is warranted here regarding the terminology. We will henceforth use the term trade credit to identify the credit, either in cash or in kind, provided by the supplier. To be rigorous, however, the term trade credit should be used only to define in-kind finance and not include any cash lending. We will show that under our assumptions cash lending by the supplier never arises in equilibrium and the supplier only lends inputs, which reconciles with our terminology. We will come back to this point in Section 5.
The return from diversion is

\[ D = \phi \{ \beta_k \rho (q_k - I_k) + \beta_L w (q_L - I_L) + [A + L_B + L_S - w q_L - \rho q_k] \} \]

where the first two terms in round brackets denote the return from input diversion and the term in square brackets denotes the return from cash diversion (the difference being the amount of cash not spent in input purchase).

We first show that the entrepreneur never purchases intangibles for diversion purposes, i.e. \( q_L = I_L = 0 \). This is because: a) the transaction in intangible inputs is a cash transaction, which implies that it cannot be used to raise credit from its suppliers; b) it is unobservable to any creditor, which implies that no contract can be conditioned on the amount of intangibles that is actually purchased. These two considerations, along with the lower degree of liquidity of intangibles relative to cash, imply that the entrepreneur prefers to divert cash rather than buy intangibles and then divert them.

A similar argument can be used to show that the entrepreneur does purchase tangibles for diversion purposes, even though they are less liquid than cash. This is because the supplier is willing to extend credit to the firm and the contract can be conditioned on the input transaction taking place. Thus, a potentially diverting entrepreneur will purchase the desired \( q_k \) from the supplier and keep the residual resources in cash. This gives a return from diversion:

\[ D = \phi \{ \beta_k \rho q_k + [A + L_B + (L_S - \rho q_k)] \} \]

However, the inefficient diversion technology and the uncertain investment return at the time of the investment/diversion decision imply that partial diversion is never optimal. Thus, either all funds (and inputs) are used for investment \((D = 0)\), or they are diverted, which implies that neither share of the purchased inputs is invested: \( I_k = I_L = 0 \). The return from diversion reduces to:

\[ D = \phi \{ \beta_k \rho q_k + [A + L_B + (L_S - \rho q_k)] \} \]

15 This gives \( \beta_L w (q_L - I_L) = 0 \) independently of the fact that \( \beta_L = 0 \).

16 We assume that the supplier of either input cannot credibly communicate to other parties whether an input transaction has taken place.

17 Two cases are conceivable here: (i) invest in the venture an amount sufficient to repay the loan in full (it has sufficient revenues to repay \( L_B r_B \)). To see it better, suppose the entrepreneur has to decide whether to invest all resources or to invest almost all and divert the marginal unit: say he invests 10 units of \( I_k \) and 10 of \( I_L \). By profit maximisation \( E f_k = \rho r_B, E f'_L = w r_B \). Is it worthwhile to divert, say, the 10th unit of \( I_k ? \) Invested in production this gives \( \rho r_B \), but if diverted it gives \( \rho o \), i.e. the value of the input scaled by \( \phi \). Thus, the entrepreneur always prefer to invest this unit, and even more so for the last but one units (the \( n - 1 \) unit gives a return even higher than \( E f'_k \) and so forth the \( n - i \) unit). Thus once he decides to invest an amount sufficient to repay the loan, he better invests all the resources available.

(ii) invest in the venture an amount not sufficient to repay the loan in full \((\pi f_G (I_k^u, I_L^u) + (1 - \pi) (1 - \phi) \beta_B \rho I_k^u < L_B r_B \), where \( I^u \) is a level of investment lower than the minimum required to repay the loan in full). Because output is observable, any positive return will be claimed by the bank and the entrepreneur will get a return only on the return from residual cash not invested and diverted. This implies that there is no point in investing such amount, and in general any positive amount not sufficient to repay the loan in full. In this case it is better to divert all.

Thus, diversion is a zero-one decision.
The entrepreneur’s problem is therefore given by program \( P^G \) below:

\[
\begin{align*}
\underset{L_B, L_S, I_k, I_L, R^B_G, R^S_G, \gamma}{\text{max}} \quad & EP + \phi [\beta_k q_k + A + L_B - (\rho q_k - L_S)] \quad (1) \\
\text{st} \quad & EP \geq \phi \{\beta_k q_k + (A + L_B - (\rho q_k - L_S))\} \quad (2) \\
& EP \geq \phi (A + L_B) \quad (3) \\
& pR^B_G + (1 - p) \gamma \beta_B C \geq L_B r_B \quad (4) \\
& pR^S_G + (1 - p) (1 - \gamma) \beta_S C \geq L_S r_S \quad (5) \\
& A + L_B + L_S \geq wI_L + \rho I_k \quad (6) \\
& R^S_G \geq (1 - \gamma) \beta_S C \quad (7)
\end{align*}
\]

where \( EP = p \left[ f_G (I_k, I_L) - R^B_G - R^S_G \right] \). \( L_S \) is the amount of trade credit raised from the supplier costing \( r_S \). The parameters \( \beta_S \) and \( \beta_B \) are the values of the unused inputs to the supplier and to the bank respectively, with \( \beta_S > \beta_B \). \( \gamma \) is the share of collateral accruing to the bank and \( (1 - \gamma) \) the one accruing to the supplier.

Constraint (2) is the incentive constraint towards the supplier, responding to the temptation of the entrepreneur of diverting the input \( q_k \) purchased from the supplier (with return \( \beta_k \rho q_k \)) plus any spare cash left after the inputs have been acquired with bank credit and own wealth financing \( (A + L_B - (\rho q_k - L_S)) \); (3) is the incentive constraint towards the bank, which prevents the entrepreneur from diverting his own cash and that raised from the bank; constraints (4) and (5) are the bank and the supplier’s participation constraints respectively; (6) is the resource constraint.

Constraint (7) ensures that repayments to the supplier are non-decreasing in revenues. Without this constraint, the entrepreneur would use the supplier, at least for a sufficiently high level of wealth, only to liquidate the unused inputs in the event of default and never to finance input purchase.\(^{18}\) In exchange for the proceeds from liquidation in bad states, the supplier would transfer an equal amount to the entrepreneur in the good state, thereby acting as a pure liquidator. Being interested in the supplier’s dual role of liquidator and financier, we do not allow for such contracts in this analysis and require repayments to be non-negative and non-decreasing in revenues.

When the entrepreneur finds it convenient to demand trade credit for liquidation motives, constraint (7) becomes binding. Using this in the supplier’s participation constraint (5) gives trade credit demand as \( L_S = \frac{1}{r_S} (1 - \gamma) \beta_S C \).\(^{18}\)

\(^{18}\)Because \( r_B < r_S \), wealthy entrepreneurs always prefer to finance production using bank credit rather than trade credit.
With perfect protection of creditor rights ($\phi = 0$), there are no diversion opportunities and even a zero-wealth firm can carry out the first best investment $I^F_k, I^F_L$. To rule this out, we assume that $\phi > \phi_1(\beta_k, \beta_S)$ where

$$\phi_1 = \frac{pf_G(I^F_k, I^F_L) - r_B(wI^{FB}_k + \beta_S \rho I^{FB}_k - A) - r_S(1-\beta_L)\rho I^{FB}_k + (1-p)(\gamma \beta_B + (1-\gamma)\beta_S)\beta_k \rho I^F_k}{\beta_k \rho I^F_k + wI^{FB}_k}$$

where $I^F_k, I^F_L$ are those solving the first order conditions for $I_k$ and $I_L$ of the unconstrained optimisation problem.

We introduce the following assumption on the parameter space.

**Assumption 1** $p\beta_S \leq \frac{\beta_B}{r_B} \beta_S - (1-p)\beta_B$.

This condition identifies the parameter space in which the entrepreneur prefers to use the supplier rather than the bank to liquidate the collateral. By doing that, first, the firm is substituting bank credit with trade credit. Second, it is taking away the unit of collateral from the bank to pledge it to the supplier. The cost of this reallocation is the positive return that the entrepreneur now has to pay back to the supplier in the good state, namely $p\beta_S$. The benefit is the lower repayment due to the bank in the good state of nature. Since the firm receives $\beta_S/r_S$ units of credit, which would cost $r_B$ if granted by the bank, and each unit of collateral is worth $\beta_B$ for the bank, the firm saves $\left(\frac{\beta_S}{r_S}r_B - (1-p)\beta_B \right)$ units of cash. When the cost is lower than the benefit, the firm has clear incentives to take trade credit for liquidation purposes. Two elements are important in this inequality: first, how cheap is bank credit with respect to trade credit, namely $(r_S/r_B)$ and second, how efficient is the supplier in liquidating the collateral with respect to the bank, namely $(\beta_S - \beta_B)$. We can interpret this inequality as a condition on the maximum interest rate charged by the supplier, namely $r_S \leq r_B \left\{ \frac{\beta_S}{p\beta_S + \beta_B (1-p)} \right\}$. Notice that there is no loss of generality in restricting to this parameter space. This condition can be seen as a reduced form of a model in which, because of her more efficient liquidation technology, the supplier has a market power which she can use to increase the price of credit. In Section 7 we show that, assuming the same cost of credit for the two financiers and letting the supplier exploit her market power, the effective interest rate charged by the supplier is higher than the one charged by the bank but never so high to induce the entrepreneur to give up supplier’s credit altogether. This implies that the interest rate chosen in equilibrium always satisfies Assumption 1.

Under this assumption, we derive the following propositions:

**Proposition 1** If $(1-\beta_k) > \beta_k \beta_S/r_S$, there exist three critical levels of wealth, $\hat{A}_1 < \hat{A}_2 < \hat{A}_3$ such that: (i) if $A \geq \hat{A}_3$, entrepreneurs are not credit constrained, take trade credit for liquidation motive ($L_S = \beta_S \beta_k \rho I^*_k/r_S$) and bank credit, and invest $I^*_k$ solving program $PG$, with (2) and (3)
slack; (ii) if \( \hat{A}_2 \leq A < \hat{A}_3 \), entrepreneurs fully exploit the bank credit line, take trade credit still for liquidation motive \( (L_S = \beta_S \beta_k \rho \hat{I}^*_k / r_S) \) but do not fully exploit such credit line and invest \( \hat{I}^*_k < \hat{I}^*_k \) solving program \( P_G \), with (2) slack; (iii) if \( \hat{A}_1 \leq A < \hat{A}_2 \), entrepreneurs take trade credit for financing motive \( (\beta_S \beta_k \rho \hat{I}^*_k / r_S < L_S \leq (1 - \beta_k) \rho \hat{I}^*_k / r_S) \) and invest \( \hat{I}^*_k < \hat{I}^*_k \) solving program \( P_G \), with (2) and (7) slack; (iv) if \( A < \hat{A}_1 \), entrepreneurs are constrained on both credit lines, invest \( \hat{I}^*_k \), with \( L_S = (1 - \beta_k) \hat{I}^*_k \) solving program \( P_G \), with (7) slack.

**Proof.** In the appendix ■

**Proposition 2** If \( (1 - \beta_k) < \beta_S / r_S \), there exists one critical level of wealth, \( \hat{A}_1 \), such that: (i) if \( A \geq \hat{A}_1 \), entrepreneurs are unconstrained on either credit line, take trade credit for liquidation motive \( (L_S = \beta_S \beta_k \rho \hat{I}^*_k / r_S) \), and invest \( \hat{I}^*_k \) solving program \( P_G \), with (2) and (3) slack; (ii) if \( A < \hat{A}_1 \), entrepreneurs exhaust the entire trade credit line, take bank credit without fully exploiting the credit line offered by the bank and invest \( \hat{I}^*_k < \hat{I}^*_k \) solving program \( P^G \), with (3) slack.

**Proof.** In the appendix ■

The content of Propositions 1 and 2 is illustrated in Figures 1 and 2, respectively. Being in the first or in the second scenario depends on whether the maximum share of tangible inputs that the supplier is willing to finance still preserving the entrepreneur’s incentive constraint, namely \( (1 - \beta_k) \), is larger (as in Figure 1) or lower (as in Figure 2) than the maximum trade credit line demanded for liquidation motives. This condition depends on three different parameters: the degree of liquidity of the tangible input \( \beta_k \), its degree of collateralisability for the supplier \( \beta_S \) and the supplier cost of funds \( r_S \).

For firms with sufficiently low degrees of liquidity and collateralisability of capital inputs \( \beta_k, \beta_S \) low) and high trade credit prices, the evolution of trade and bank credit demands for different levels of wealth is represented in Figure 1. Rich entrepreneurs who are unconstrained on bank credit \( (A > \hat{A}_3) \) take trade credit to exploit the better liquidation technology of the supplier. In this wealth area, the non-decreasing repayment condition (see constraint 7) is binding, which implies that the share of tangible inputs financed through trade credit is equal to \( \beta_k \beta_S / r_S \). When wealth decreases and entrepreneurs become constrained on bank credit \( (\hat{A}_2 < A < \hat{A}_3) \), they keep using trade credit for liquidation motives as in the previous case. Only when wealth is lower than \( \hat{A}_2 \), they start demanding trade credit also for financing motives spiegare meglio intuitivamente il financing motive. The share of tangible inputs financed by trade credit increases until it reaches its maximum \( (1 - \beta_k) \) where also the supplier incentive constraint binds \( (\hat{A} = A_1) \). For \( A < \hat{A}_1 \), the entrepreneur is constrained on both credit lines. The total amount of trade credit decreases, but the share of tangible inputs financed by trade credit stays constant and equal to \( (1 - \beta_k) \).
For firms with sufficiently high degrees of liquidity and collateralisability of capital inputs and low trade credit prices, the evolution of trade and bank credit demands for different levels of wealth is represented in Figure 2. Again, wealthy firms (those with $A > \tilde{A}_1$) take trade credit to exploit the liquidation technology of the supplier. However, they do not increase the share of inputs financed with trade credit when wealth decreases below $\tilde{A}_1$, as they have already fully exploited the maximum credit line conceded by the supplier.

The comparison between the two figures is useful to derive predictions on the relation between trade credit line, input or sector characteristics and entrepreneur’s wealth. In sectors where the capital inputs supplied are sufficiently illiquid (or less collateralisable), the share of inputs financed by trade credit is higher for financially constrained firms than for the unconstrained ones. Conversely, in sectors

<table>
<thead>
<tr>
<th>wealth</th>
<th>A_1</th>
<th>A_2</th>
<th>A_3</th>
</tr>
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<tbody>
<tr>
<td>γ=0 unconstrained on BC; constrained on TC</td>
<td>γ=0 unconstrained: TC (liquidation), BC (financing)</td>
<td>γ=0: flat contract</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 1: Liquidation and financing motives** ($(1 - \beta_k) > \beta_k\beta_S/r_S$)

**Fig. 2: Liquidation motive and unused bank credit line** ($(1 - \beta_k) < \beta_k\beta_S/r_S$)
where inputs are very liquid, the share of tangible inputs financed by trade credit is equal for both constrained and unconstrained firms. Interestingly, this last prediction is consistent with the evidence provided by Marotta (2001) for the Italian manufacturing sector.

Proposition 2 allows us to derive the following corollary:

**Corollary 1** If \((1 - \beta_k) < \beta_k \beta_S / r_S\), entrepreneurs exhaust the trade credit line for any \(A \leq \bar{A}_1\), but they never fully exploit the bank credit line, which remains slack for any level of wealth.\(^{19}\)

The intuition is the following. Taking the extra credit from the bank is shown to be suboptimal or to violate the incentive constraint of the entrepreneur, unless he gives up part of the trade credit and provides guarantees to the bank by pledging her part or all of the collateral. It turns out that also this choice is suboptimal. Let’s proceed using a proof by contradiction. Suppose the entrepreneur takes the extra amount of cash the bank is willing to provide. He has two options. First, he can keep the extra credit in cash in order to divert it. However, this is never optimal as it costs \(r_B\) and generates a benefit \(\phi < r_B\). Second, he can divert all the credit raised in inputs and cash.\(^{20}\) In this case, the maximum return he gets is given by \(\phi [\beta_k \rho I_k^* + wI_L^* + (EP - D_B)]\), where \((EP - D_B)\) is the extra credit (which corresponds to the slack on the bank incentive constraint, that is, the difference between the left hand side and the right side of the incentive constraint with the bank (equation 3)). It is easy to show that the return from diversion is higher than the return he would get from production without taking any extra credit from the bank, namely \(\phi (\beta_k \rho I_k^* + wI_L^*)\). The bank anticipates the entrepreneur’s incentive to take the entire credit line for diversion and does not provide the extra credit demanded, unless the entrepreneur grants her a share of the collateral \(\gamma = 1 - \frac{1 - \beta_k}{\beta_k \beta_S} r_S\) in case of default. This lowers the credit line provided by the supplier to \((1 - \beta_k) \rho I_k^*\) and restores the incentives for production. However, this is suboptimal since any increase in \(\gamma\) decreases the entrepreneur’s profits (by Assumption 1, the objective function is decreasing in \(\gamma\)). Thus the entrepreneur prefers not to take the extra credit line provided by the bank and give full priority to the supplier in case of default (\(\gamma = 0\)).

**Corollary 2** The share of tangible input expenditure financed by trade credit \(\frac{L_S}{\rho I_k}\) is never higher than

\[
\max \left\{ (1 - \beta_k) , \frac{\beta_k \beta_S}{r_S} \right\}.
\]

\(^{19}\)Notice that in the intermediate case, where \((1 - \beta_k) = (\beta_k \beta_S / r_S)\), there exists a threshold level of wealth \(\bar{A}_1\) such that entrepreneurs with \(A < \bar{A}_1\) are unconstrained both on bank credit and trade credit, while firms with \(A \geq \bar{A}_1\), do not exhaust either credit line. In either case, trade credit is used for liquidation motives.\(^{20}\)Since we do not allow for decreasing repayment contracts, \(r_B\) is the lowest repayment due to the bank in the good state for any extra unit of cash lent.
The supplier has no incentive to increase the share above the limit, because, if she did, the entrepreneur would have a clear incentive in diverting the inputs rather than investing them in production. The reason is the following. When the firm is financially constrained (3 binding), the maximum amount of trade credit offered, using the two incentive constraints, is

\[ L_{S_{\text{max}}}^S = (1 - \beta_k) \rho I_k. \]

This requires that the income from diverting inputs plus cash, \( \phi [\beta_k \rho I_k + A + L_B - (\rho I_k - L_S)] \), be no higher than the income from diverting all cash, \( \phi (A + L_B) \). It follows that the incentive constraint is preserved if and only if the cash that remains after buying all the inputs, \( (A + L_B) - (\rho I_k - L_S) \), is sufficiently low, or the value of the inputs paid for in cash, \( (\rho I_k - L_S) \), is sufficiently high. A share of tangible input expenditure higher than \( (1 - \beta_k) \) will never be financed by the seller, as this gives the credit rationed firm an incentive to diversion. An amount of trade credit higher than that can nevertheless be obtained by giving up part of the bank credit, so as to reduce sufficiently the cash that remains after buying all the inputs, \( (A + L_B) - (\rho I_k - L_S) \). In this way, the liquidity provided by the extra amount of trade credit \( (\beta_k \rho I_k) \) is compensated by the lower amount of bank credit \( (L_B) \). Thus, the entrepreneur can substitute bank credit with trade credit so as to keep the incentive constraints preserved.

The issue is whether and when the entrepreneur finds it convenient to substitute bank credit with trade credit.

Because \( r_S > r_B \), it is never the case that the entrepreneur substitutes bank credit with trade credit for financing reasons, neither when he is not credit rationed \( (r_S > r_B) \) nor when he is (he is indifferent between the two sources of financing in this case). Instead he might want to substitute the two forms of financing to exploit the seller’s liquidation technology. Because of the restriction on repayments to be non decreasing, from the seller’s participation (5) we deduce that the maximum amount of trade credit demanded to exploit the seller’s liquidation technology is given by

\[ L_{S_{\text{max}}}^D = \frac{\beta_S \beta_k}{r_S} \rho I_k. \]

Thus, if \( L_{S_{\text{max}}}^D \leq L_{S_{\text{max}}}^S \), the entrepreneur will first take a share of trade credit equal to \( \frac{\beta_S \beta_k}{r_S} \rho I_k \) for the liquidation motive and will increase this share up to \( 1 - \beta_k \) when financial constraints kick in. At this point he is constrained on both credit lines, the two forms of financing have the same (shadow) cost and he is indifferent between the two, which implies that he has no incentive to substitute one with the other.

If \( L_{S_{\text{max}}}^D > L_{S_{\text{max}}}^S \), the entrepreneur demands a share of trade credit equal to \( \frac{\beta_S \beta_k}{r_S} \rho I_k \) for the liquidation motive higher than the maximum share provided to a financially constrained entrepreneur,
The only way to be granted such a share, is to reduce the bank credit exposition, which implies having some unused bank credit line that makes sure that there are not too many liquid resources to divert.

From Corollary 2 we derive an interesting prediction on the relation between trade credit line and input liquidity. As shown in Figure 3, this is not monotonic, but it has a \( V \) shape. Among sectors where the liquidity of capital inputs is low (\( \beta_k < \frac{r_S}{r_S + \beta_S} \)), we should observe financially constrained firms using inputs with higher liquidity to have a lower share of these inputs financed by trade credit. Conversely, among sectors where the liquidity of capital inputs is high (\( \beta_k \geq \frac{r_S}{r_S + \beta_S} \)), financially constrained firms using inputs with higher liquidity should have a higher share of these inputs financed by trade credit. The size of the two areas depends on supplier’s characteristics, \( r_S \) and \( \beta_S \). If the capital input has zero collateral value, the switching threshold is equal to 1, and the right hand side area disappears. Thus, we have that the trade credit line is a decreasing function of input liquidity. Conversely, if the input collateral value is positive, the threshold is lower than 1, and the right hand side area is bigger the lower the supplier cost of funds and the higher the collateral value. The two regimes capture the two motives underlying the demand for trade credit in our model. When the supplier’s information advantage is the main determinant of trade credit demand, the area on the left is big. Conversely, when the liquidation advantage is the driver, the area on the right becomes big.

In Burkart and Ellingsen (2004), the assumption of certainty implies that the choice between bank credit and trade credit is fully driven by the relative price of the two sources of financing (\( r_B \) vs. \( r_S \)). Since bank credit is assumed to be cheaper than trade credit, trade credit only arise to slacken the firm’s financial constraints. In our setting, the uncertainty introduces an extra factor affecting the demand for the two sources of funding: the liquidation technology available (\( \beta_S \) vs. \( \beta_B \)). Assuming \( \beta_S > \beta_B \) implies the existence of a trade-off: bank credit is cheaper than trade credit (\( r_B < r_S \)), but
the supplier is more efficient in the liquidation of the collateral. Thus, even financially unconstrained firms ask for trade credit to exploit the better liquidation technology of the supplier.

\textit{difference with Frank and Maksimovic}

\textbf{Corollary 3} The properties of the entrepreneur-bank-supplier financial contract depend on the motives underlying the demand for trade credit.

- When the firm asks for trade credit for the liquidation motive, the supplier gets a contract with flat repayments across states, whilst the bank gets a contract with repayments increasing in cash flows, with full priority in the liquidation of the collateral to the supplier.

- When the firm asks for trade credit for the financing motive both the bank and the supplier get a contract with repayments increasing in cash flows, with full priority in the liquidation of the collateral to the supplier.

- Whatever the motivation underlying the demand for trade credit, the bank always gets a contract with repayments increasing in cash flows and never has priority in liquidation of the collateral.

\textbf{Proof.} In the appendix

One important message delivered in proposition 3 is that the right to repossess and liquidate the collateral in bankruptcy should be given to the supplier as long as she has access to a better liquidation technology. This is true even when the price of trade credit is higher than bank credit.

A second result is that the form of the financing contract between the entrepreneur and the supplier changes depending on the reasons behind the demand for trade credit. This appears by comparing Figure 1 with Figure 2. Being in the first or in the second scenario depends on whether the input illiquidity \((1 - \beta_k)\) is larger or smaller than \(\beta_k \beta_S / r_S\). The term \((1 - \beta_k)\) represents the maximum share of tangible inputs that the supplier can finance by preserving the entrepreneur's incentive constraint. The second term is the trade credit demand for liquidation motives.

When inputs are illiquid and the liquidation value is relatively low (see Figure 1), we can distinguish between two cases according to the entrepreneur's wealth. When \(A \geq A_2\), the entrepreneur takes both bank credit and trade credit, but he demands less trade credit than the supplier is willing to extend. In particular, he takes an amount is just sufficient to exploit the supplier's better liquidation technology: \(L_S = \frac{\beta_S}{r_S} C = \frac{\beta_k \beta_S}{r_S} \rho I_k\). The supplier gets a flat contract across states with full priority in the liquidation of the collateral, while the bank gets an increasing repayment contract with no rights in defaulting states.
For \( A < A_2 \), the entrepreneur is bank credit constrained and needs to raise the residual trade credit line offered by the supplier to finance production. Because the collateral has been already fully pledged to the supplier, any extra unit of TC above \( L_S = \frac{\beta_S}{r_S} C \) but no higher than \( (1 - \beta_k) \rho I_k \) can be provided only if a higher repayment is promised in good states. She gets therefore an increasing repayment contract.

When inputs are liquid (see Figure 2) and the liquidation value for the supplier is sufficiently high, the maximum credit line offered by the supplier that preserves both incentive constraints is lower than the amount demanded for liquidation purposes \( ((1 - \beta_k) \rho I_k < \frac{\beta_S}{r_S} \rho I_k) \). Again, we can distinguish between two cases according to the entrepreneur’s wealth. Rich entrepreneurs \( (A \geq A_1) \) face no financial constraint and can therefore take all the TC necessary to exploit the supplier’s better liquidation technology. Thus \( L_S = \frac{\beta_k \beta_S}{r_S} \rho I_k \) and the supplier gets a flat contract across states. As wealth decreases \( (A < A_1) \), the entrepreneur faces a trade-off: he can keep the same share of trade credit over capital inputs as when he is unconstrained, \( \frac{\beta_k \beta_S}{r_S} \), but this implies that the bank credit constraint stays slack and there is some unused bank credit line. Or else he can fully use the bank credit line, but in order to do that he has to grant the bank a share of the collateral, which implies reducing the trade credit demand to \( 1 - \beta_k \). As argued in the discussion of Proposition 2, the entrepreneur will find convenient to keep the higher share of trade credit and leave some unused bank credit because of the supplier’s more efficient liquidation technology. In either case, the financial contract with the supplier has constant repayments across states for any level of wealth, while the contract with the bank has increasing repayments with no priority in the liquidation of the collateral.

4 Input choice, financing decisions and the role of creditor protection

**Proposition 3** If the production function is homogeneous of degree \( j \leq 1 \), both the degree of asset tangibility \( (I_k/I_L) \) and the trade credit intensity \( (\frac{L_S}{L_S + L_B + A}) \) are decreasing in wealth.

**Proof.** In the Appendix. ■

**Proposition 4** If the production function is homogeneous of degree \( j \leq 1 \), an increment in creditor rights vulnerability increases both the degree of asset tangibility \( (I_k/I_L) \) and the trade credit intensity \( (\frac{L_S}{L_S + L_B + A}) \) for any \( A \leq A_1 \) and \( A_2 < A < A_3 \), but has no effect either on \( (I_k/I_L) \) or on \( (\frac{L_S}{L_S + L_B + A}) \) for any \( A_1 < A < A_2 \) and \( A > A_3 \).

**Proof.** In the Appendix. ■
Propositions 3 and 4 share a common intuition which is related to the determinants of the relative price ratio of the two inputs, namely $P_k/P_L$. Under the assumption of homogeneity of the production function, the optimal input combination only depends on the relative factors’ price ratio. Each input price reflects the financing cost and the cost of moral hazard. The size of these two components not only depends on the entrepreneur’s wealth, but also differs between the two inputs due to their different degree of liquidity (in case of diversion and default), investment observability and financing sources. The combination of wealth and inputs characteristics explains the pattern of the optimal input combination (Figure 4) and the trade credit intensity (Figure 5) for different levels of wealth and their reaction to changes in the legal variable (Figures 6 and 7, respectively).

Let us consider first how the input ratio ($I_k/I_L$) and the trade credit intensity, ($\frac{L_s}{L_s+L_B+\text{A}}$) change for different levels of wealth (Figures 4 and 5 respectively).

![Fig. 4: Input tangibility with varying wealth](image-url)
The optimal share of tangible over intangible inputs increases as wealth decreases, although not strictly monotonically. Starting from high levels of wealth \((A > A_3)\), the optimal input combination remains constant since the firm is unconstrained on bank credit. This implies that the input price ratio \(\frac{P_k}{P_L} = \frac{\rho h_B - \beta_k \beta_s r_B}{\rho h_s} - p\) is independent of \(A\). Given that wealth enters the trade credit intensity only through the input combination, also trade credit intensity remains constant for sufficiently rich entrepreneurs.

When the incentive constraint towards the bank binds \((A = A_3)\), the entrepreneur shifts towards technology more intensive in tangible assets, and keeps doing so as wealth decreases, until \(A = A_2\). The reason is that the problem of moral hazard, that makes the bank unwilling to extend extra bank credit, affects the input price ratio, and thus the inputs demand, in different ways. Reductions in wealth increase the shadow price of bank credit by the term \(\phi \lambda_1 \frac{1}{1+\lambda_1}\), where the parameter \(\lambda_1\) represents the multiplier of the incentive constraint towards the bank. Such increment fully translates into a higher price of intangibles, given that such purchases are totally financed by bank credit. Conversely, the price of tangibles is only increased by a fraction \((1 - \frac{\beta_s \beta_k}{\rho})\), which represents the share of capital inputs financed by bank credit. It follows that the price of intangibles increases more than the price of tangibles, thereby inducing the entrepreneur to increase input tangibility. Such increase goes together with an increase in trade credit intensity for the same reason as before.

When wealth falls below \(A_2\), trade credit is used to relax the credit constraints. This implies that the entrepreneur is not anymore financially constrained. Therefore, further drops in wealth do not
force the entrepreneur to change the input combination so long as wealth is no less than $A_1$. Again the reason is that the input price, given by $\frac{P_k}{P_L} = \rho \left[ \beta_k r_B + \frac{\lambda \phi}{1 + \lambda} \right] + (1 - \beta_k) r_S - \beta_S \beta_k (1 - p) w r_B$, does not depend on wealth. In this case, wealth enters the definition of trade credit intensity also through the share of tangible inputs financed by trade credit. This share decreases when wealth increases, which implies that trade credit intensity is also decreasing in wealth.

When wealth falls short of $A_1$, the entrepreneur becomes again financially constrained. As in the previous case ($A_2 < A < A_3$), the input price ratio decreases as $A$ goes down, but for a different reason. In this case, the factor price ratio is given by $\frac{P_k}{P_L} = \rho \left[ \beta_k r_B + \frac{\lambda \phi}{1 + \lambda} \right] + (1 - \beta_k) r_S - \beta_S \beta_k (1 - p) \frac{w}{w r_B + \frac{\lambda \phi}{1 + \lambda}}$, where $\lambda$ is the multiplier of the global incentive constraint (17). The financing cost of each unit of capital is split among the supplier and the bank according to their participation ($\beta_k r_B$ for the bank and $(1 - \beta_k) r_S$ for the supplier), while it is equal to the bank interest rate ($r_B$) for labour, since labour expenditure are fully financed through bank credit. This cost is scaled down by the liquidation value of the input in the low state of nature. Given that only capital is a collateralisable asset, the liquidation value is positive for capital, (namely, $(1 - p) \beta_S \beta_k$) and zero for labour. When $A$ goes down, the moral hazard problem becomes more severe, since the entrepreneur has a higher incentive to divert resources. This is captured by an increment in the parameter, $\lambda$. Although it enters the definition of the two factor prices, its effect is scaled down by the parameter $\beta_k < 1$ in the price of capital. The intuition for this has to be found in the different verifiability of labour and capital input purchase. This in turn affects the source of funding which is made available. Since labour purchase are not verifiable to anybody, unless a monitoring cost is paid (and labour credit is not allowed), they are financed only through bank credit. Moreover, because labour has zero liquidation value in case of diversion, the potentially diverting entrepreneur always prefers to divert cash rather than hire workers and not employ them in the production process. This implies that the liquidation income of labour is the same as cash. Conversely, the capital investment, which is financed through both bank and trade credit, is costlessly observable by the supplier and has a positive liquidation value in case of diversion. Because of this, in order to ease the credit constraints, the diverting entrepreneur prefers to buy the capital inputs and then divert them, rather than divert cash. It follows that the income from diverting one unit of capital is lower than the income from diverting one unit of cash, since inputs are less liquid than cash. This makes the moral hazard problem behind the use of capital less severe than the one behind the labour investment. It follows that both asset tangibility and trade credit intensity increase when wealth goes down.

By comparing Figure 4 and 5, we can see that in general input tangibility and trade credit intensity react very similarly to changes in wealth. The only exception is when $A_1 \leq A \leq A_2$, where
entrepreneurs take trade credit to relax financial constraints. In this case, they can keep asset tangibility constant, because they have a higher share of tangible inputs to be financed by trade credit.

*questo non e’ chiaro*

IO MI ASPETTO CHE QUESTO ACCADA TRA A3 E A1, CIOE’ NEL TRATTO IN CUI SONO VINCOLATO SOLO SU BC E POI ANCHE SU TC

Consider now how input tangibility and trade credit intensity react to an increase in creditor rights vulnerability ($\phi$) (see Figures 6 and 7).

![Changes in $\phi$](image)

**Fig. 6:** Input tangibility and creditor rights protection
Notice that any increase in $\phi$ moves to the right the threshold levels of $A$. Consider first the case in which $A < A_1$. This change makes the moral hazard problem of the entrepreneur towards both the bank and the supplier more severe, since the benefits from diversion increases. This effect appears in both prices through higher $\phi$ and $\lambda$, but its magnitude is bigger for intangibles than for tangibles, since only the fraction $\beta_k$ of tangibles is divertable. It follows that an increase in creditor rights vulnerability reduces the input price ratio and therefore increases the degree of asset tangibility. Therefore, the line shifts up for any $A \leq A_1$. Similar comments hold for $A_2 < A \leq A_3$. Conversely, for any $A_1 < A \leq A_2$ and $A > A_3$, the input price ratio is not affected by changes in the degree of creditor rights vulnerability, which implies that the optimal input ratio does not depend on $\phi$.

Finally, from the proof of proposition 4, it follows that trade credit intensity and input tangibility are positively correlated. This implies that whenever creditor rights vulnerability induces the entrepreneurs to switch to tangible assets, the relevance of trade credit as source of finance increases and vice versa.

The previous discussion provides interesting testable predictions across firms with different levels of initial wealth and across countries. First, for a given degree of creditor vulnerability, we should observe poorer firms having a technological bias towards tangible assets and using relatively more trade credit than wealthy firms (same $\phi$ but different $A$). Second, we should observe poorer firms having a technological bias towards tangible assets and using relatively more trade credit if located in countries with higher creditor rights vulnerability (different $\phi$ but same $A < A_1$). Conversely, we should not observe any difference in input combinations and capital structures among firms located in
different countries as long as these firms are not credit constrained (different \( \phi \) but same \( A > A_3 \)).

5 Do suppliers ever lend cash?

Corollary 2 rules out the possibility of having cash lending by the supplier. This result is not surprising when the entrepreneur is sufficiently rich to be unconstrained on the bank credit market. In fact, there is no point in asking for cash to the supplier, since the cost of cash loan is higher if provided by the supplier. However, this result is surprising if we consider less wealthy entrepreneurs. For sufficiently low levels of wealth, the entrepreneur is constrained on bank credit and the input combination is likely to be biased towards capital inputs. In this case, the marginal productivity of labour is very high and thus lending money to increase the investment in labour units would benefit the production level. However, the supplier is not willing to provide any cash, since he anticipates that this money would be used for diversion rather than for production: any extra unit of cash provided to the entrepreneur would violate the incentive compatibility conditions towards both the bank and the supplier.

Crucial to obtain this result is the assumption that not only the bank but also the supplier is unable to verify the purchase of labour units. The complete unobservability of labour implies that the liquidation degree of each unit of labour is the same as cash.

5.1 Conditions for cash lending

In this section, we relax the assumption that the capital input purchase is observable only to the supplier and that the labour input purchase is unobservable to both the bank and the supplier. We assume instead that the bank can observe, although partially the capital input purchase and that both the creditors can partially but asymmetrically observe the labour input purchase. Denoting by \( \delta_B \) and \( \delta_S \) the degree of observability of labour input purchase by the bank and the supplier respectively, and by \( \zeta \) the degree of observability of capital input by the bank, the incentive constraint for the bank and the supplier can be respectively written as:

\[
EP \geq \phi (A + L_B - \delta_B wI_L - \zeta \rho I_k)
\]

\[
EP \geq \phi \left( \beta_k \rho I_k + A + L_B + (L_S - \rho I_k - \delta_S wI_L) \right)
\]

Using the resource constraint, namely \( (A + L_B + L_S = \rho I_k + wI_L) \), we can rewrite the two incentive compatibility condition in the following way:

\[
EP \geq \phi (\rho I_k + wI_L - L_S - \delta_B wI_L - \zeta \rho I_k)
\]

\[
EP \geq \phi (\beta_k \rho I_k + wI_L - \delta_S wI_L).
\]
Assuming that both constraints bind, we can find the maximum credit line that the supplier is willing to provide. Notice that \( \bar{L}_S \) denotes the maximum amount of total credit (trade credit plus any cash) that the supplier is willing to provide.

\[
\bar{L}_S = (1 - \beta_k - \zeta) \rho I_k + (\delta_S - \delta_B) wI_L
\]

and the maximum credit line that the bank is willing to provide as

\[
\bar{L}_B = \rho I_k (\beta_k + \zeta) + (1 - \delta_S + \delta_B) wI_L - A.
\]

Thus, if we assume that it is less costly for the supplier to observe the labour input purchase, i.e. \( \delta_S > \delta_B \), and that the degree of observability by the bank of capital input purchase (\( \zeta \)) is not too high, we have that the supplier not only provides the inputs and accepts a partial repayment equal to \( (\beta_k + \zeta) \) of their value, but also provides an amount of cash to finance the hiring of labour equal to \( (\delta_S - \delta_B) \) of its value.

The previous discussion implies that in order to have cash lending, the supplier must have an informational advantage with respect to the bank on the labour input purchase. The bigger this advantage \( (\delta_S - \delta_B) \), the larger the amount of input purchase the supplier is willing to finance by loan.

Interestingly, Uesugi and Yamashiro (2004) show that in Japan cash lending is provided by giant trading companies, with a peculiar organisational structure. They are vertically integrated firms that carry out many different businesses transactions including sometimes all the stages of the production and commercialisation of a good. Thus, trade credit take place among firms which are units of the same company. Moreover, often the commercial transactions are supported by a variety of activities such as investment, financial and management assistance or information provision. This implies that the supplier is providing many types of services to the same buyer. Consider, for example a computer company not only selling computers but also providing human capital formation, consultancy and assistance to the computers users. In this case, it would be relatively easy for the seller company to control whether the buyer really hires workers. The organizational structure of these firms guarantees a continuous flow of information that allows the seller to have control on the behavior of its customer. In line with our intuition, this evidence seems to suggest that the informational advantage is crucial to have cash lending.

6 Supplier with market power

So far, we have assumed \( r_S \) to be exogenous and higher than \( r_B \), but not so high to induce the entrepreneur to give up trade credit, even for high levels of wealth. In this section, we assume that the
price of the two sources of financing is the same and equal to \( r \). Under this assumption we show that the analysis developed so far can be seen as a reduced form of a model in which the supplier has bargaining power. Such bargaining power depends on two features of the supplier activity: input observability and liquidation technology. First, the supplier has an advantage over the bank as she can observe the amount of inputs purchased by the entrepreneur and can therefore mitigate the entrepreneur’s moral hazard problem. Second, she has a more efficient collateral liquidation technology, which allows the entrepreneur to reduce the repayment due to the supplier and hence increase the profits from the venture.

Because of this, by taking trade credit the entrepreneur extracts a higher surplus from the contractual relationship than if he contracted with the bank only. Let us define \( \Pi_S \) as the profit obtainable when the entrepreneur can access trade credit and \( \Pi_B \) as the profit obtainable when the entrepreneur cannot access trade credit. The spread in profits is the surplus generated by the presence of the supplier in the credit relationship. Knowing this, the supplier will ask for a share \( \alpha \) of this surplus, under the threat to withhold her participation from the venture. The parameter \( \alpha \) is a measure of the bargaining power enjoyed by the supplier.

Notice that the supplier derives market power exclusively from her role as financier, rather than as supplier of goods. If the goods are paid for in cash, the supplier can no longer act as a liquidator and her presence cannot be used to mitigate the entrepreneur’s moral hazard problem.

The entrepreneur problem becomes:

\[
\begin{align*}
\max_{\ell_k, \ell_L, L_B, L_S, \gamma, R_G^S, R_G^B} \Pi & = p \left( f_G(I_k, I_L) - R_G^S - R_G^B \right) \\
\text{st} \: \Pi & \geq \phi \{ \beta_k \rho I_k + A + L_B + L_S - \rho I_k \} \\
\Pi & \geq \phi (A + L_B) \\
pR_G^S + (1 - p)(1 - \gamma) \beta_S C & \geq L_S \left[ r + \alpha \frac{\Pi_S - \Pi_B}{\Pi_B} \right] \\
pR_G^B + (1 - p)\gamma \beta_B C & \geq L_B r \\
A + L_B + L_S & = wI_L + \rho I_k \\
\Pi & \geq \Pi_B
\end{align*}
\]

Notice that the market power enters the problem through its impact on the supplier’s reservation utility. The supplier ask for positive profits proportional to the amount of trade credit provided \( (L_S) \) and to the increment in the entrepreneur’s surplus generated by its presence \( \left[ \alpha - \frac{\Pi_S - \Pi_B}{\Pi_B} \right] \).

Moreover, relative to the original problem formulation, we have an extra constraint on the minimal profits required by the entrepreneur to find it convenient to ask for trade credit \( (\Pi \geq \Pi_B) \). Using this
constraint we can derive the maximum share of surplus $\alpha^*$ that can be appropriated by the supplier without triggering the incentive of the entrepreneur to ask for trade credit. For any $\alpha \leq \alpha^*$, we can derive the effective price of charged by the supplier for each unit of trade credit, $r_S = r + \frac{\alpha(\Pi_S - \Pi_B)}{\Pi_S} > r_B$. Such price is bigger than the bank credit interest rate, but never so high to induce the entrepreneur to take only bank credit. This implies that $r_S$ always satisfies the condition $\pi S \beta S \leq \frac{r_S}{r_S} \beta S - (1 - p) \beta_B$.

One difference between this formulation and the previous one is that $r_S$ is now endogenous and therefore not anymore constant. In particular, changes in the initial entrepreneur wealth change the surplus and therefore the interest rate asked by the supplier.

7 Conclusions

The paper uses an incomplete contract approach with uncertainty, two-input technology and collateralised credit contracts, to investigate the determinants of the firm’s optimal financing policy, in particular trade credit demand, and its interactions with the optimal input combination.

Our analysis provides a theoretical foundation for some commonly observed stylized facts, for which the extant literature lacks a clear understanding. First, it explains why even firms without financial constraints may want to take trade credit, even if they have access to cheaper bank credit. Besides the traditional financing constraints motivation, we show that trade credit is also taken to exploit the seller’s advantage in the liquidation of the inputs in case these are repossessed from a defaulting firm. Second, it rationalizes the determinants of different priority rules and in particular why, if in default, priority in liquidation of collateral is given to the supplier. Third, it investigates the reasons why traditionally suppliers do not lend cash and derives conditions for cash lending.

Finally, the paper shows that the channels through which corporate governance standards may affect firm’s behaviour go through the interplay of financing and technological decisions. It follows that interesting testable regularities between input choice, capital structure and corporate governance institutions arise in the analysis.
A Appendix

Proof of Proposition 1. The proof proceeds as follows. We set up the Lagrangean of the maximisation problem given by $\mathcal{P}_G$ under the constraints (2)-(7) and derive the FOC’s. Then, we derive the properties of the optimal contracts for decreasing levels of wealth. In particular, we isolate four relevant wealth areas and within each of these areas we derive the properties of the contracts.

When the entrepreneur asks for trade credit for liquidation motives, constraint (7) is binding and trade credit is given by $L_S = \frac{1}{r_S} (1 - \gamma) \beta_S C$. Substituting out in the objective function and the constraints, and solving the resource constraint for $L_B$, we can rewrite the general program $\mathcal{P}_G$ under the binding constraint (7) in the following way:

$$\max_{I_L, I_k, \gamma} EP$$

$$EP \geq \phi \left( wI_L + \rho I_k - \frac{1 - \gamma}{r_S} \beta_S C \right)$$

$$EP \geq \phi \left( wI_L + \beta_k \rho I_k \right)$$

where $EP = p [f_G (I_k, I_L) - (1 - \gamma) \beta_S C] - \left( wI_L + \rho I_k - A - \frac{1 - \gamma}{r_S} \beta_S C \right) r_B + (1 - p) \gamma \beta_B C$. The first constraint is the incentive compatibility condition of the entrepreneur towards the bank, while the second is the incentive constraint towards the bank.

Notice that $\phi \left( wI_L + \rho I_k - \frac{1 - \gamma}{r_S} \beta_S C \right) \geq \phi (\beta_k \rho I_k + wI_L)$. Setting up the Lagrangean:

$$\Lambda = EP + \lambda_1 \left[ EP - \phi \left( wI_L + \rho I_k - \frac{1 - \gamma}{r_S} \beta_S C \right) \right] + \lambda_2 \left[ EP - \phi (\beta_k \rho I_k + wI_L) \right]$$

and working out the FOC’s:

$$\frac{\partial \Lambda}{\partial I_k} = \left\{ \frac{p \partial f_G}{\rho \partial I_k} - r_B - \beta_k \left( p (1 - \gamma) \beta_S - \frac{(1 - \gamma) r_B}{r_S} \beta_S - (1 - p) \gamma \beta_B \right) \right\} (1 + \lambda_1 + \lambda_2) = \lambda_1 \phi \left( 1 - \frac{1 - \gamma}{r_S} \beta_S \beta_k \right) + \lambda_2 \phi \beta_k$$

$$\frac{\partial \Lambda}{\partial I_L} = \left( \frac{p \partial f_G}{w \partial I_L} - r_B \right) (1 + \lambda_1 + \lambda_2) = \phi (\lambda_1 + \lambda_2)$$

$$\frac{\partial \Lambda}{\partial \gamma} = \left( p (\beta_S - \beta_B) - \frac{1}{r_S} (r_B \beta_S - r_S \beta_B) \right) (1 + \lambda_1 + \lambda_2) - \lambda_1 \frac{1}{r_S} \phi \beta_S \beta_k \leq 0$$

$$\frac{\partial \Lambda}{\partial \lambda_1} = EP - \phi \left( wI_L + \rho I_k - \frac{1 - \gamma}{r_S} \beta_S C \right) \geq 0$$

$$\frac{\partial \Lambda}{\partial \lambda_2} = EP - \phi (\beta_k \rho I_k + wI_L) \geq 0$$

Notice that $\frac{\partial \Lambda}{\partial \gamma} \leq 0$ given Assumption 1. Therefore, we can without loss of generality solve the problem assuming that $\gamma = 0$ for any level of wealth.

Wealth area 1: $A \geq A_3$ In this case the entrepreneur is sufficiently rich so that neither of the incentive constraints binds and $\lambda_1 = \lambda_2 = 0$. Since $\gamma = 0$, the firm pledges all of the collateral to the
supplier in case of default. The supplier gets flat repayments across states for the funding provided and is repaid in full upon liquidation, while the bank gets an increasing repayment contract. The optimal financial contract has the following properties:

\[
R_S^B = \beta_S \beta_k \rho \hat{I}_k^*,
\]

\[
L_S = \frac{1}{r_S} \beta_S \beta_k \rho \hat{I}_k^*,
\]

\[
L_B = w \hat{I}_L^* + \frac{1}{r_S} (r_S - \beta_S \beta_k) \rho \hat{I}_k^* - A,
\]

\[
R_G^B = \frac{r_B}{p} \left( w \hat{I}_L^* + \frac{1}{r_S} (r_S - \beta_S \beta_k) \rho \hat{I}_k^* - A \right)
\]

where \( \hat{I}_k^* \) and \( \hat{I}_L^* \) solve

\[
\frac{\partial f_G}{\partial I_k} - \rho r_B - \beta_k \rho \beta_S \left( \frac{p - r_B}{r_S} \right) = 0
\]

\[
\frac{\partial f_G}{\partial I_L} - w r_B = 0
\]

Notice that the financial structure that is actually chosen is the one that minimises the cost of capital and thus maximises the tangibility of the inputs.

**Wealth area 2: \( \hat{A}_2 \leq A < \hat{A}_3 \)**

The bank incentive constraint binds, while the supplier incentive constraint is still slack. The entrepreneur takes trade credit for the liquidation motive and the optimal contract has the following properties:

\[
R_S^G = \beta_S \beta_k \rho \hat{I}_k^*,
\]

\[
L_S = \frac{1}{r_S} \beta_S \beta_k \rho \hat{I}_k^*,
\]

\[
L_B = w \hat{I}_L^* + \frac{1}{r_S} (r_S - \beta_S \beta_k) \rho \hat{I}_k^* - A,
\]

\[
R_G^G = \frac{r_B}{p} \left( w \hat{I}_L^* + \frac{1}{r_S} (r_S - \beta_S \beta_k) \rho \hat{I}_k^* - A \right)
\]

where \( \hat{I}_k^* \) and \( \hat{I}_L^* \) solve

\[
\frac{p}{\delta_f G} \frac{\partial f_G}{\partial I_k} - \rho r_B - \beta_k \rho \beta_S \left( \frac{p - r_B}{r_S} \right) = 0
\]

\[
\frac{p}{\delta_f G} \frac{\partial f_G}{\partial I_L} - w r_B = 0
\]

where \( \lambda_1 \) is the multiplier of the incentive compatibility condition of the entrepreneur towards the bank.

**Wealth areas 3 and 4: \( \hat{A}_1 \leq A < \hat{A}_2 \) and \( A < A_1 \)**

Since the trade credit taken for liquidation motive does not exhaust the maximum credit line offered by the supplier, the entrepreneur starts demanding trade credit also for the financing motive. However,
to persuade the supplier to increase the financing, the entrepreneur has to offer her a contract with repayments increasing in cash flows. This implies that the optimal contract solves programme $\mathcal{P}_G$ with the constraint 7 slack. We can therefore rewrite the programme in the following way:

$$\max_{I_k,I_L,L_B} EP \quad \text{st } EP \geq \phi (A + L_B) \quad \text{EP} \geq \phi (\beta_k \rho I_k + w I_L)$$

where the constraints have the usual meaning and

$$EP = pf_G (I_k, I_L) - L_B r_B - (\rho I_k + w I_L - A - L_B) r_S + (1 - p) \beta_S \beta_k \rho I_k.$$ 

Setting up the Lagrangean $\Lambda_I = EP + \lambda_1 (EP - \phi (A + L_B)) + \lambda_2 (EP - \phi (\beta_k \rho I_k + w I_L))$, working out the FOC’s

$$\frac{\partial \Lambda_I}{\partial I_k} = \left(p \frac{\partial f_G}{\partial I_k} - r_S + \beta_S \beta_k (1 - p) \right) (1 + \lambda_1 + \lambda_2) = \lambda_2 \beta_k \phi$$

$$\frac{\partial \Lambda_I}{\partial I_L} = \left(\frac{p}{w} \frac{\partial f_G}{\partial I_L} - r_S \right) (1 + \lambda_1 + \lambda_2) = \lambda_2 \phi$$

$$\frac{\partial \Lambda_I}{\partial L_B} = (r_S - r_B) (1 + \lambda_1 + \lambda_2) = \lambda_1 \phi$$

and solving for $\lambda_1$

$$\lambda_1 = \frac{r_S - r_B}{\phi - r_S + r_B} (1 + \lambda_2)$$

we get

$$\frac{p}{\rho} \frac{\partial f_G}{\partial I_k} - r_S + \beta_S \beta_k (1 - p) = \frac{\lambda_2}{1 + \lambda_2} \beta_k (\phi - r_S + r_B)$$

(13)

$$\left(\frac{p}{w} \frac{\partial f_G}{\partial I_L} - r_S \right) = \frac{\lambda_2}{1 + \lambda_2} (\phi - r_S + r_B)$$

(14)

When $A_1 \leq A < A_2$, the entrepreneur incentive constraint towards the supplier is slack, which implies that $\lambda_2 = 0$. Therefore the optimal contract is given by:

$$\frac{\beta_S \beta_k \rho I_k^3}{r_S} < L_S < (1 - \beta_k) \rho I_k^3,$$

$$L_B > w I_L^3 + \beta_k \rho I_k^3 - A,$$

$$R_{G}^{B} = \frac{1}{p} \left(\frac{1}{p} - (1 - \beta_k) r_S - (1 - p) \beta_S \beta_k \right) \rho I_k^3$$

$$(qui non sono sicura dei repayments. dipendono dalla quota di trade credit che viene presa sul totale degli input. forse non e' corretto.)$$
where $I_k^{4*}, I_L^{4*}$ solve the following conditions:

\[
\frac{p}{\rho} \frac{\partial f_G}{\partial I_k^{4*}} - r_S + \beta_S \beta_k (1 - p) = 0
\]  

(15)

\[
\left( \frac{p}{w} \frac{\partial f_G}{\partial I_L^{4*}} - r_S \right) = 0
\]  

(16)

When $A < A_1$, the trade credit incentive constraint is binding ($\lambda_2 > 0$) and the optimal contract is given by the following equations:

\[
L_S = (1 - \beta_k) \rho I_k^{4*},
\]

\[
L_B = wI_L^{4*} + \beta_k \rho I_k^{4*} - A,
\]

\[
R_G^S = \frac{1}{p} ((1 - \beta_k) r_S - (1 - p) \beta_S \beta_k) \rho I_k^{4*}
\]

\[
R_B^G = \frac{1}{p} (wI_L^{4*} + \beta_k \rho I_k^{4*} - A) r_B
\]

where $I_k^{4*}, I_L^{4*}$ solve the following optimality conditions:

\[
\frac{p}{\rho} \frac{\partial f_G}{\partial I_k^{4*}} - r_S + \beta_S \beta_k (1 - p) = \frac{\lambda_2}{1 + \lambda_2} \beta_k (\phi - r_S + r_B)
\]

(15)

\[
\left( \frac{p}{w} \frac{\partial f_G}{\partial I_L^{4*}} - r_S \right) = \frac{\lambda_2}{1 + \lambda_2} (\phi - r_S + r_B)
\]

(16)

The last two optimality conditions can also be expressed in terms of the multiplier $\lambda$ of the global incentive constraint

\[
EP \geq \phi (\beta_k \rho I_k + w I_L)
\]

(17)

where $EP = pf_G(I_k, I_L) - (wI_L + \beta_k \rho I_k) r_B - (1 - \beta_k) \rho I_k r_S + Ar_B + (1 - p) \beta_S \beta_k \rho I_k$. This gives the Lagrangean as follows: $\Lambda_I = EP + \lambda [EP - \phi (\beta_k \rho I_k + w I_L)]$. If we derive the previous expression with respect to the two inputs we obtain the following optimality conditions:

\[
\frac{p}{\rho} \frac{\partial f_G}{\partial I_k^{4*}} = \left[ \beta_k \left( r_B + \frac{\lambda}{1 + \lambda} \phi \right) + r_S (1 - \beta_k) - \beta_S \beta_k (1 - p) \right]
\]

\[
\frac{p}{w} \frac{\partial f_G}{\partial I_L^{4*}} = \left[ r_B + \frac{\lambda}{1 + \lambda} \phi \right]
\]

Proof of Proposition 2. This is the case in which $1 - \beta_k < \frac{\beta_k \beta_S}{r_S}$. Thus, the maximum amount of trade credit the seller is willing to extend when the entrepreneur is financially constrained (the one resulting when both incentive constraints (2) and (3) are binding) is lower than the amount demanded to exploit the seller’s liquidation technology.

The line of the proof is similar to that followed in proposition 1. We set up the Lagrangean of the maximisation problem given by $\mathcal{P}_G$ under the constraints (2)-(7) and derive the FOC’s. Then, we derive the properties of the optimal contract for decreasing levels of wealth.
Unlike the previous case, we have now only one relevant threshold of wealth $A_1$ and thus two wealth areas, according to whether $A \geq A_1$. In either case, constraint (7) is binding and we need to solve the optimisation problem (8) through (10) with $L_S = \frac{1}{r_S} (1 - \gamma) \beta_S C$.

From the incentive constraints, because $1 - \beta_k < \frac{\beta_k \beta_S}{r_S}$, we deduce that:

$$\phi \left( w I_L + \rho I_k - \frac{1 - \gamma}{r_S} \beta_S C \right) < \phi (\beta_k \rho I_k + w I_L)$$

which implies that, as wealth decreases, constraint (2) binds first.

Having restricted the parameter space to that identified by Assumption 1, we can set $\gamma = 0$.

**Wealth area 1: $A \geq A_1$**

In this case the entrepreneur is sufficiently rich to be unconstrained on both credit lines ($\lambda_1 = \lambda_2 = 0$). The optimal financial contract has the following properties:

- $R_S^S = \beta_S \beta_k \rho \tilde{I}_k^*$,
- $L_S = \frac{1}{r_S} \beta_S \beta_k \rho \tilde{I}_k^*$,
- $L_B = w \tilde{I}_L^* + \frac{1}{r_S} (r_S - \beta_S \beta_k) \rho \tilde{I}_k^* - A$,
- $R_B^S = \frac{r_B}{p} \left( w \tilde{I}_L^* + \frac{1}{r_S} (r_S - \beta_S \beta_k) \rho \tilde{I}_k^* - A \right)$

where $\tilde{I}_k^*$, $\tilde{I}_L^*$ solve the following FOC’s:

$$\frac{\partial \Lambda}{\partial I_k} : \frac{p}{\rho} \frac{\partial f_G}{\partial I_k} - r_B - \beta_k \left( p \beta_S - \frac{r_B}{r_S} \beta_S \right) = 0$$

$$\frac{\partial \Lambda}{\partial I_L} : \frac{p}{w} \frac{\partial f_G}{\partial I_L} - r_B = 0$$

**Wealth area 2: $A < A_1$**

As wealth decreases, the incentive constraint towards the supplier becomes binding (18 holds) and the one towards the bank stays slack. Hence $\lambda_1 = 0$, $\lambda_2 > 0$ and the contract has the following properties:

- $R_S^S = \beta_S \beta_k \rho \tilde{I}_k^{*2}$,
- $L_S = \frac{1}{r_S} \beta_S \beta_k \rho \tilde{I}_k^{*2}$,
- $L_B = w \tilde{I}_L^{*2} + \frac{1}{r_S} (r_S - \beta_S \beta_k) \rho \tilde{I}_k^{*2} - A$,
- $R_B^S = \frac{r_B}{p} \left( w \tilde{I}_L^{*2} + \frac{1}{r_S} (r_S - \beta_S \beta_k) \rho \tilde{I}_k^{*2} - A \right)$

where $\tilde{I}_k^{*2}$, $\tilde{I}_L^{*2}$ solve the following FOC’s:

$$\frac{p}{\rho} \frac{\partial f_G}{\partial I_k} - r_B - \beta_k \left( p \beta_S - \frac{r_B}{r_S} \beta_S \right) = \frac{\lambda_2}{1 + \lambda_2} \phi \beta_k$$

$$\frac{p}{w} \frac{\partial f_G}{\partial I_L} - r_B = \frac{\lambda_2}{1 + \lambda_2} \phi$$
and \( \lambda_2 \) is the multiplier of the incentive constraint \( EP = \phi (\beta_k p_k + wI_L) \).

**Proof of Proposition 3.** Under the assumption that the production function is homogeneous of degree \( j \leq 1 \), the input combination only depends on the input price ratio. Using the proof of proposition 1, we can write the input price ratio as a function of the parameters of the model. Thus we can investigate how the input price ratio, and therefore the input combination, depends on the those parameters. Since trade credit intensity depends on wealth only through the input ratio, we can easily derive the sensitivity of trade credit intensity to changes in wealth. Let us consider the four wealth areas separately.

When \( A > A_3 \), \( \frac{P_k}{L_k} = \frac{\mu}{w} - \frac{\beta_k}{\lambda_2} \left( \frac{\beta_k}{\lambda_2} \phi + wI_L \right) \) and \( \frac{L_S}{L_S + L_B + A} = \frac{\beta_S}{\lambda_2} \frac{1}{w} \left( \frac{\beta_k}{\lambda_2} + \frac{\phi}{w} \right) + r_S \beta_S \mu \). It follows that \( \frac{\partial \left( \frac{P_k}{L_k} \right)}{\partial \lambda} = 0 \). Thus, both input tangibility and trade credit intensity are independent of \( A \).

When \( A_2 < A < A_3 \): \( \left( \frac{P_k}{L_k} \right)_{A_2 < A < A_3} = \frac{\mu}{w} - \frac{\beta_k}{\lambda_2} \left( \frac{\beta_k}{\lambda_2} \phi + wI_L \right) = \left( \frac{P_k}{L_k} \right)_{A > A_3} \) and \( \frac{L_S}{L_S + L_B + A} = \frac{\beta_S}{\lambda_2} \frac{1}{w} \left( \frac{\beta_k}{\lambda_2} + \frac{\phi}{w} \right) + r_S \beta_S \mu \). It follows that \( \frac{\partial (P_k/L_k)_{A_2 < A < A_3}}{\partial \lambda} = \left( \frac{1 - \beta_k}{\lambda_2} \right) - \frac{\phi}{w} \frac{\partial \lambda}{\partial \lambda} > 0 \), since \( \frac{\partial \lambda}{\partial \lambda} \leq 0 \) and \( \left( \frac{1 - \beta_k}{\lambda_2} \right) - \frac{\phi}{w} \frac{\partial \lambda}{\partial \lambda} = -\beta_S \beta_k \phi \leq 0 \). Thus, both input tangibility and trade credit intensity are decreasing in wealth.

When \( A_1 < A < A_2 \): \( \left( \frac{P_k}{L_k} \right)_{A_1 < A < A_2} = \frac{\mu}{w} - \frac{\beta_k}{\lambda_2} \left( \frac{\beta_k}{\lambda_2} \phi + wI_L \right) = \left( \frac{P_k}{L_k} \right)_{A > A_3} \) and \( \frac{L_S}{L_S + L_B + A} = \frac{\beta_S}{\lambda_2} \frac{1}{w} \left( \frac{\beta_k}{\lambda_2} + \frac{\phi}{w} \right) + r_S \beta_S \mu \), where \( \frac{\beta_S}{\lambda_2} \phi \leq \mu \leq (1 - \beta_k) \). Since \( \frac{\partial (P_k/L_k)_{A_1 < A < A_2}}{\partial \lambda} = 0 \) and \( \frac{\partial \mu}{\partial \lambda} \leq 0 \), while input tangibility is independent of \( A \), trade credit intensity is decreasing in \( A \).

When \( A < A_1 \): \( \left( \frac{P_k}{L_k} \right)_{A_1 < A < A_2} = \frac{\mu}{w} - \frac{\beta_k}{\lambda_2} \left( \frac{\beta_k}{\lambda_2} \phi + wI_L \right) = \left( \frac{P_k}{L_k} \right)_{A > 1} \) and \( \frac{L_S}{L_S + L_B + A} = \frac{\beta_S}{\lambda_2} \frac{1}{w} \left( \frac{\beta_k}{\lambda_2} + \frac{\phi}{w} \right) + r_S \beta_S \mu \). It follows that \( \frac{\partial (P_k/L_k)_{A < A_1}}{\partial \lambda} = \frac{\beta_k}{\lambda_2} \left( \frac{\beta_k}{\lambda_2} \phi + wI_L \right) \frac{\partial \lambda}{\partial \lambda} > 0 \), since \( \frac{\partial \lambda}{\partial \lambda} \leq 0 \) and \( \left( \beta_S - \frac{P_k}{L_k} \right) \beta_k = \beta_S \beta_k (1 - p) - (1 - \beta_k) r_S \leq 0 \) when \( 1 - \beta_k > \frac{\beta_k}{\lambda_2} \). Therefore, both input tangibility and trade credit intensity are decreasing in wealth.

**Proof of Proposition 4.** From the proof of proposition 3, we know both the optimal input tangibility and the trade credit intensity for the four relevant wealth areas. Since the legal variable \( \phi \), affects trade credit intensity only through the input combination, and trade credit intensity is increasing in the degree of asset tangibility, it is easy to show that the two variables react in the same way to changes in the degree of legal enforcement in each of the four wealth areas.

When \( A > A_3 \), \( \frac{\partial (P_k/L_k)_{A > A_3}}{\partial \phi} = 0 \). It follows that \( \frac{L_k}{L_k} \) does not depend on \( \phi \). Thus, both asset tangibility and trade credit intensity are independent of \( \phi \).

When \( A_2 < A < A_3 \), \( \frac{\partial (P_k/L_k)_{A_2 < A < A_3}}{\partial \phi} < 0 \), which implies that \( \frac{L_k}{L_k} \) is decreasing in \( \phi \). Thus, both asset tangibility and trade credit intensity increase after an increase in \( \phi \).
When \( A_1 < A < A_2 \), \( \frac{\partial (P_k/P_L)_{A_1 < A < A_2}}{\partial \phi} = 0 \), which implies that \( \frac{L}{T_k} \) is independent of \( \phi \). Thus, asset tangibility is constant. Moreover, since \( \frac{\partial \mu}{\partial \phi} = 0 \), also trade credit intensity is independent of \( \phi \).

QUINDI NON VARIA LA TRADE CREDIT INTENSITY IN QUEST'AREA?

When \( A < A_1 \), \( \frac{\partial (P_k/P_L)_{A < A_1}}{\partial \phi} < 0 \). It follows that both asset tangibility and trade credit intensity increase after an increase in \( \phi \). ■
References


