Hold-up and Credit Rationing in Oligopoly

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Abstract
This paper develops a theory of equilibrium credit rationing in oligopoly, which features strategic interaction in R&D-product market competition, and endogenous market structure. Endogenous credit rationing arises as a rational response to a hold-up problem between entrepreneurs and external investors, due to incompleteness of contracts. This model is capable of generating both an ‘investment size rationing’, and a ‘denial of credit’ to an ‘unsatisfied fringe’. It shows an interesting interplay between the two: only by denying credits to an ‘unsatisfied fringe’, is the capital market able to relax rationing the investment size for the rest.

Key words: credit rationing, oligopoly, R&D, hold-up, corporate governance, market structure

1 Introduction
It has long been a belief of economists that corporate investments in R&D suffer from limited credit availability and under-investment (see Himmelberg and Petersen (1994), Hall (2002)). This is in tune with a broader view that corporate investments in general may be a victim of financial constraints, particularly for small start-up firms (see Fazzari, Hubbard and Petersen (1988), Hubbard (1998)). Theories of equilibrium credit rationing have been advanced substantially to rationalize credit rationing as a profit maximizing behavior (see Hodgman (1960), Freimer and Gordon (1965), Jaffee and Modigliani (1969), Jaffee and Russell (1976), Stiglitz and Weiss (1981), Allen (1983), Gale and Hellwig (1985), Williamson (1987)).

The existing theories of endogenous credit rationing mainly study investments in working capital in settings of perfectly competitive (or monopoly) product market. They have been able to rationalize two types of credit rationing. The first type, which we call the ‘Type I credit rationing’, rations

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1This is independent from the argument that underinvestment in R&D occurs because of the existence of externality.
credit size, i.e., the amount of credit extended is positive but below the level that maximizes the borrowing firm’s profit (Gale and Hellwig (1985)) \(^2\). The second type, which we refer to as the ‘Type II credit rationing’, denies credits to an ‘unsatisfied fringe’, i.e., only a fraction of the applicants can obtain a positive amount of credit (Stiglitz and Weiss (1981) \(^3\), Williamson (1987) \(^4\)). These theories have established a solid foundation for understanding the issues of credit rationing related to investments in working capital; especially, they have contributed significantly to underpinning the ‘credit availability doctrine’ in monetary economics (Roosa (1951)). However, these theories have not been particularly useful for understanding the determinants and impacts of credit rationing in R&D-intensive industries, which usually involve large amounts of fixed investments in the setting of oligopoly markets. The key missing element from the existing theories is the interplay of the strategic interaction in R&D-product markets and optimal (the second-best) financial contracting. This omission results in the existing theories’ inability to address the following important questions: First, what are the effects of credit availability on R&D investments and market structure in the high-tech industries? Second, what is the effect of competition in the R&D-product markets on financing decisions (e.g., credit rationing rules)? Third, what are the relationship, and the direction of causality thereof, between credit availability, R&D and market structure? This limitation hinders our understanding of the relationship between finance and technological progress in a market economy.

This paper sets out to develop a theory of equilibrium credit rationing in oligopoly, which features strategic interaction in R&D-product market competition, and endogenous market structure. It draws from two related literatures: one is the literature on technology and market structure (Dasgupta and Stiglitz (1980), Sutton (1991), Sutton (1998)); the other is the literature on incomplete contracts and optimal allocation of property/control rights (Grossman and Hart (1986), Hart and Moore (1988), Hart and Moore (1990), Hart and Moore (1994), Hart (1995), Hart and Moore (1998)). Whereas the former literature gives us a natural basis to build our model of R&D as a strategic investment, we need to explain why we also rely heavily on the latter. To understand credit rationing as a meaningful financial functionality, one may need, as has been exemplified by the existing credit rationing literature, to carefully examine the underlying financial structure (e.g., the form of financial contracts). It has now been well

\(^2\)In this model, credit rationing (under-investment) arises in debt contract, which is the optimal (second-best) financial contract subject to asymmetric information between borrowers and lenders and costly state verification (monitoring) by lenders.

\(^3\)In this model, credit denial arises in debt contract as a response to an adverse selection problem. One criticism of this model is that debt contract is imposed rather than optimally chosen. De Meza and Webb (1987) points out that the optimal contract in the setting of this model should use equity rather than debt financing.

\(^4\)Credit rationing arises in this model as a response to the positive expected monitoring cost to lenders (as is similar to Gale and Hellwig (1985)), and debt contract is the optimal contract. The reason why credit rationing takes the form of credit denial rather than rationing credit size is driven by the assumption that each investment project demands an indivisible one unit of capital.
understood, that when long-term investments and/or complicated projects (e.g., R&D projects) are involved in a transaction, an enforceable complete contract that governs every aspect of the transaction (ex ante) is simply not feasible or desirable (Williamson (1975), Williamson (1985), Hart and Moore (1999)); ex post negotiation (bargaining) is inevitable. This literature has had an extensive investigation on the so call ‘hold-up’ problem, and drawn two general conclusions. First, the ‘hold-up’ problem in general results in some form of under-investment. Second, certain observable organization forms (e.g., vertical integration/separation, allocation of property rights, and financial structure) can be understood as an optimal (the second-best) solution to mitigate the ‘hold-up’ problem. In this paper, we relate the under-investment result to credit rationing, particularly on R&D investments.5

The existing incomplete contract theory of the firm has not been concerned with role of wealth constraints on investment decisions (see Hart (1995)). To tackle the issue of credit rationing, we look at the environments where entrepreneurs of the firms are wealth-constrained, and they have to ensure the participation of external investors who themselves may not be good at running the business at all. The investment by the external investors is essential only because the entrepreneur of the firm does not have sufficient internal funding. As a result, the under-investment outcome caused by the hold-up problem between the entrepreneur and the external investors, resides with the latter but not the former party’s investment decision. This results in an endogenous rationing of credit size. Applying this to R&D investments in a setting of R&D-product market competition, our model is capable of characterizing the interplay of R&D financing and strategic interaction in the R&D-product market competition. This enables us to contribute to both literatures of credit rationing and R&D investment.

The technically most challenging part of the analysis lies in examining the existence of credit-rationing equilibrium. We find that if we confine the analysis to the Type I credit rationing, i.e., rationing of credit size, only, the existence of equilibrium is not guarantied. This problem is overcome when we allow the Type II credit rationing, i.e., credit denial. Not only can we guaranty the existence of credit-rationing equilibrium, our model also becomes the first model which predicts a coexistence of both types of credit rationing in equilibrium. The model shows an interesting interplay between the two: only by denying credits to an ‘unsatisfied fringe’, is the capital market able to relax rationing the investment size for the rest.

The organization of the remainder of the paper is as follows. Section 2 constructs the model. Section 3 characterizes the symmetric equilibrium outcome, particularly, the equilibrium outcome with credit size rationing; we also establish the necessary and sufficient condition for this kind of equilibrium outcome to exist. In Section 4, we characterize asymmetric equilibrium outcomes where both types of credit rationing (credit size rationing and credit denial) coexist. In

5The issue of incompleteness of contracts is particularly salient when the transactions involve R&D, which is typically associated with complexity in technicality and a high degree of uncertainty.
section 5 we discuss possible augmented settings which can provide foundations for some of the exogenous features of our model before we conclude in Section 6.

2 The Model

There are a number $S$ of identical consumers in the market of a vertically differentiable product. Hence parameter $S$ measures the size of the market. Each consumer’s preference is given by the following quasi-linear utility function:

$$U = \sum_{i=1}^{N} (u_i q_i) - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} q_i q_j + m$$  \hspace{1cm} (1)

subject to:

$$\sum_{i=1}^{N} p_i q_i + m \leq I$$

where $N$ is the number of active firms that offer a positive quantity of the vertically differentiable product; $u_i$ and $q_i$ are the quality and quantity supplied by firm $i$; $m$ is the numeraire; $I$ is the income.

The first order necessary condition of the above maximization program implies the following inverse demand function

$$p_i = u_i - \sum_{j=1}^{N} q_j.$$  \hspace{1cm} (2)

Each firm has a production technology which has constant returns to scale, and the marginal cost is $c_i$ for firm $i$. When the number of firms and their respective levels of quality and marginal costs are fixed, the firms compete in the manner a la Cournot. Hence, each firm objective is given by the following maximization program:

$$\max \Pi_i = S (p_i - c_i) q_i$$  \hspace{1cm} (3)

subject to:

$$p_i = u_i - \sum_{j=1}^{N} q_j$$

which is equivalent to

$$\max \Pi_i = S \left( u_i - \sum_{j=1}^{N} q_j - c_i \right) q_i,$$  \hspace{1cm} (4)

which has the first order condition:

$$u_i - \sum_{j=1}^{N} q_j - c_i - q_i = 0.$$  \hspace{1cm} (5)
The implied best reaction function is

$$q_i = \frac{u_i - \sum_{j \neq i} q_j - c_i}{2} \text{ for } i = 1, 2, \ldots, N.$$  \hspace{1cm} (6)

Summating the above family of equations and solving for $$\sum_{i=1}^{N} q_i$$ yield

$$\sum_{i=1}^{N} q_i = \frac{1}{(N + 1)} \left( \sum_{i=1}^{N} \kappa_i \right),$$  \hspace{1cm} (7)

where $$\kappa_i \triangleq u_i - c_i$$ is firm $$i$$’s level of ’capability’. So a firm can have a high capability by possessing a high quality or a low marginal cost. Inserting eq. (7) into eq. (6) and (2) gives us the equilibrium output and price-cost margin at individual firm level as follows

$$q_i = \kappa_i = \frac{1}{(N + 1)} \sum_{j=1}^{N} \kappa_j,$$  \hspace{1cm} (8)

$$p_i - c_i = \kappa_i = \frac{1}{(N + 1)} \sum_{j=1}^{N} \kappa_j.$$  \hspace{1cm} (9)

More precisely,

$$q_i = p_i - c_i = \max \left( 0, \kappa_i - \frac{1}{(N + 1)} \sum_{j=1}^{N} \kappa_j \right)$$  \hspace{1cm} (10)

for all $$i = 1, 2, \ldots, N$$, since $$q_i \geq 0$$ and $$p_i - c_i \geq 0$$ must be satisfied in equilibrium.

The implied equilibrium profit of each firm is given by

$$\Pi_i (\kappa_i; \kappa_{-i}) = S\pi_i (\kappa_i; \kappa_{-i}) = \frac{S}{(N + 1)^2} \left( \kappa_i + \sum_{j=1}^{N} (\kappa_i - \kappa_j) \right)^2,$$  \hspace{1cm} (11)

conditional on all firms: $$i = 1, 2, \ldots, N$$ remain active; where $$\kappa_{-i} = (\kappa_j)_{N-1}$$ for $$j \neq i$$ is the vector of all rivals’ levels of capability; $$\pi_i (\kappa_i; \kappa_{-i})$$ is the normalized profit function (for unit market size). Most of the properties of $$\Pi_i (\kappa_i; \kappa_{-i})$$ are the properties of $$\pi_i (\kappa_i; \kappa_{-i})$$, upon which our analysis will focus in what follows.

To start with, $$\pi_i (\kappa_i; \kappa_{-i})$$ is twice continuously differentiable conditional on the composition of the set of active firms remains unchanged\(^6\), non-decreasing in own capability, and non-increasing in any rival’s capability, i.e.,

$$\frac{\partial \pi_i}{\partial \kappa_i} \geq 0, \frac{\partial \pi_i}{\partial \kappa_j} \leq 0.$$

\(^6\)In case of any change in the composition of the set of active firms, additional active firms must be included in the expression, and inactive firms must be excluded from it. As a result, such a change may produce a kink (i.e., continuous but non-differentiable point) in the profit function.
We introduce the following notations:

\[ \pi (\kappa, N) \triangleq \pi_i (\kappa; (\kappa)_{N-1}) \]  

(12)

is the profit of a firm in an symmetric configuration with \( N \) firms each having capability level of \( \kappa \), denoted by \((\kappa, N)\).

\[ \zeta_\kappa (N) \triangleq \frac{\partial \pi_i (\kappa; (\kappa)_{N-1})}{\partial \kappa_i} \frac{\kappa}{\pi (\kappa, N)} \]  

(13)

is the elasticity of \( \pi_i \) with respect to \( \kappa_i \), valued at the symmetric configuration \((\kappa, N)\).

We interpret the economic meanings of the following terms:

- \( \frac{\partial \pi (\kappa, N)}{\partial \kappa} \) measures the market expansion effect of investment in capability;
- \( \frac{\partial \pi (\kappa, N)}{\partial N} \) measures the competition effect of entry;
- \( \zeta_\kappa (N) \) measures the private benefit of investment in capability;
- \( \left( \zeta_\kappa (N) - \frac{\partial \pi (\kappa, N)}{\partial \kappa} \frac{\kappa}{\pi (\kappa, N)} \right) \) measures the business stealing (competition) effect of investment in capability;
- \( \frac{\zeta_\kappa (N)}{\zeta_\kappa (N)} \) measures the competition effect of entry on private benefit of investment in capability.

It is easy to verify that the profit function (11) has the following properties:

\[ \pi (0, N) = 0 \quad \text{and} \quad \frac{\partial \pi (\kappa, N)}{\partial \kappa} > 0, \]  

(14)

i.e., a positive level of capability is necessary for this market to be active, and increasing capability levels of a symmetric configuration has a positive market expansion effect. The magnitude of this effect, however, is determined by

\[ \frac{\partial \pi (\kappa, N)}{\partial \kappa} \frac{\kappa}{\pi (\kappa, N)} = 2. \]  

(15)

There is a negative competition effect: increasing the number of firms in a symmetric configuration reduces the profit of each firm, i.e.,

\[ \frac{\partial \pi (\kappa, N)}{\partial N} < 0, \]  

(16)

and the magnitude of this effect is given by

\[ \frac{\partial \pi (\kappa, N)}{\partial N} \frac{N}{\pi (\kappa, N)} = \frac{2N}{N + 1}. \]  

(17)
It happens that the elasticity of $\pi_i$ with respect to $\kappa_i$ at a symmetric configuration, is independent of the level of capability, but increases with the number of firms, more precisely,

$$\zeta_\kappa (N) = \zeta (N) = 2N > 0.$$  \hspace{1cm} (18)

A positive business stealing effective exists in oligopoly, as shown by

$$\zeta (N) - \frac{\partial \pi (\kappa, N)}{\partial \kappa} \frac{\kappa}{\pi (\kappa, N)} = 2(N - 1) > 0 \text{ for } N > 1,$$  \hspace{1cm} (19)

which gives each individual firm some extra incentive for investing in capability.

From

$$\zeta' (N) = 2 > 0,$$  \hspace{1cm} (20)

it is clear that increasing the number of firms of a symmetric configuration increases each firm’s proportional marginal incentive for investment, regardless of the current level of capability. The elasticity of $\zeta (N)$ is given by

$$\frac{\zeta' (N) N}{\zeta (N)} = 1 > 0.$$  \hspace{1cm} (21)

Finally, note the fact that

$$\frac{\partial \pi_i (\kappa_i, \kappa_j)}{\partial \kappa_i \pi_i} = \frac{2N}{1 + \sum_{j=1}^{N} \left( 1 - \frac{\kappa_j}{\kappa_i} \right)},$$  \hspace{1cm} (22)

which implies that a firm’s proportional marginal incentive for investment is non-increasing in its level of capability in an oligopoly, i.e.,

$$\frac{\partial \left( \frac{\partial \pi_i (\kappa_i, \kappa_j)}{\partial \kappa_i \pi_i} \right)}{\partial \kappa_j} < 0 \text{ for } N \geq 2 \text{ and } \pi_i > 0,$$  \hspace{1cm} (23)

with strict inequality holding in all symmetric configurations.

So far we have comprehensively characterized the (proportional) marginal benefit of investing in capability. Next we look at the proportional marginal cost of investing in capability by considering the following simple increasing and convex fixed cost function:

$$F(\kappa) = \kappa^\beta, \hspace{0.5cm} (\beta > 2),$$  \hspace{1cm} (24)

which has a constant elasticity or proportional marginal cost, i.e.,

$$\frac{\partial F (\kappa)}{\partial \kappa} = \beta.$$  \hspace{1cm} (25)

In what follows we study how the levels of capability are determined in equilibrium when there are free entry and conflict of interest between the entrepreneurs and the external investors of the firms. The conflict of interest origins
from the lack of commitment power by the entrepreneurs (corporate insiders) to pledge the entirety of profit to external investors. It takes the form of ex post bargaining over the split of the profit. Suppose the ex post bargaining outcome is parameterized by the share of profit that accrues to the external investors, \( \theta \); then the pledgeable income\(^7\) for the external investors is \( \theta \Pi_i (\kappa_i; \kappa_{-i}) \). The simplest setting which delivers this feature is the one where none of the two parties has an (ex post) outside option and the entrepreneur’s bargaining power is \( (1 - \theta) \) (and the external investors, \( \theta \)). We treat \( \theta \) as exogenous.

Consider a game with the following three stages:

1. A (sufficiently large) number of potential entrepreneurs simultaneously decide whether to enter the market.

2. Upon entry, entrepreneurs simultaneously announce their target capability levels, and each proposes a take-it-or-leave-it offer to the external investors in the competitive capital market. So each entrepreneur’s action (project) is a vector \( (\kappa_i, \tilde{\theta}_i) \), where \( \kappa_i \) is the capability level and \( \tilde{\theta}_i \) is the share of capital that is demanded from the external investors. Each entrepreneur has to commit the insider equity: \( (1 - \tilde{\theta}_i) F (\kappa_i) \).
   
   • If the offer is accepted by sufficient external investors, then it becomes a binding contract and is implemented, namely, the funds are collected from the external investors, the investments are sunk and target capability is developed.
   
   • If the offer is not accepted by sufficient external investors, it is not binding; and the de facto investment level is \( (1 - \tilde{\theta}_i) F (\kappa_i) \), with no external investors involved.

3. After the investments are sunk, the entrepreneurs and external investors bargain over the sharing of ex post profits,\(^8\) \( \Pi_i (\kappa_i; \kappa_{-i}) \).
   
   • The bargaining outcomes are determined by the Nash bargaining solution with external investors’ bargaining power being \( \theta \).\(^9\)
   
   • The profit of a solely internally financed firm is accrued solely to the entrepreneur.

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\(^7\)The term ‘pledgeable income’ is coined by Tirole (2001).

\(^8\)This is to implicitly assume that the sharing of profit cannot be contracted ex ante, or the contracts will have to be renegotiated ex post. It is an assumption of incompleteness of contacts.

\(^9\)For tractability, we use a reduced-form Nash bargaining game with optimal allocation of control/property rights; it has a single parameter \( \theta \) to represent the equilibrium bargaining strength of the external investors. Information about the optimal (second-best) allocation of control/property rights (which gives the maximal external investor protection) and the associated set of outside options has been subsumed into \( \theta \). This modeling choice, in line with the incomplete-contract/property-right theory of the firm literature (Grossman and Hart (1986), Hart and Moore (1988), Hart and Moore (1990), Hart and Moore (1994), Hart and Moore (1998)). In a nutshell, we assume \( \theta \) summarizes all the relevant information on the second-best corporate governance terms.
We characterize the equilibrium of the game by backward induction. At stage 3, the Nash bargaining solution dictates that the profit share for the external investors is $\theta$, hence, $(1 - \theta)$ for the entrepreneur.

At stage 2, each entrepreneur’s program is to choose $(\kappa_i; \tilde{\theta}_i)$ to maximize his or her net income:

$$y_i (\kappa_i; \kappa_{-i}; \tilde{\theta}_i) = (1 - \theta) S\pi_i (\kappa_i; \kappa_{-i}) - \left(1 - \tilde{\theta}_i\right) F (\kappa_i),$$

conditional on the external investors being willing to participate and the requirement on insider equity does exceeds the entrepreneurial wealth level $\omega > 0$, i.e.,

$$\max_{(\kappa_i, \tilde{\theta}_i)} (1 - \theta) S\pi_i (\kappa_i; \kappa_{-i}) - \left(1 - \tilde{\theta}_i\right) F (\kappa_i) \quad (26)$$

subject to:

$$\theta S\pi_i (\kappa_i; \kappa_{-i}) - \tilde{\theta}_i F (\kappa_i) \geq 0, \quad (27)$$

$$\left(1 - \tilde{\theta}_i\right) F (\kappa_i) \leq \omega, \quad (28)$$

where the first constraint (27) is the external investors’ participation constraint, the second, (28), is the entrepreneurial wealth constraint.

The Lagrangian function of the program is

$$\mathcal{L} = \left( (1 - \theta) S\pi_i (\kappa_i; \kappa_{-i}) - \left(1 - \tilde{\theta}_i\right) F (\kappa_i) \right) + \lambda_i \left( \theta S\pi_i (\kappa_i; \kappa_{-i}) - \tilde{\theta}_i F (\kappa_i) \right) + \nu_i \left( \omega - \left(1 - \tilde{\theta}_i\right) F (\kappa_i) \right) \quad (29)$$

where $\lambda_i$ and $\nu_i$ are the Lagrangian multipliers, which are the ‘shadow prices’ of the slacks in the external investors’ participation constraint, and entrepreneurial wealth constraint respectively.

The first order necessary conditions for the solution of the program are:

$$\frac{\partial \mathcal{L}}{\partial \kappa_i} = (1 - \theta) S \frac{\partial \pi_i}{\partial \kappa_i} - \left(1 - \tilde{\theta}_i\right) F' (\kappa_i) + \lambda_i \left( \theta S \frac{\partial \pi_i}{\partial \kappa_i} - \tilde{\theta}_i F' (\kappa_i) \right) - \nu_i \left(1 - \tilde{\theta}_i\right) F' (\kappa_i) = 0 \quad (30)$$

$$\frac{\partial \mathcal{L}}{\partial \tilde{\theta}_i} = F (\kappa_i) - \lambda_i F (\kappa_i) + \nu_i F (\kappa_i) = 0 \quad (31)$$

$$\begin{align*}
\theta S\pi_i (\kappa_i; \kappa_{-i}) &= \tilde{\theta}_i F (\kappa_i) \quad \text{as } \lambda_i \neq 0 \\
\theta S\pi_i (\kappa_i; \kappa_{-i}) &> \tilde{\theta}_i F (\kappa_i) \quad \text{as } \lambda_i = 0 \\
\lambda_i &\geq 0
\end{align*} \quad (32)$$

$$\begin{align*}
\omega - \left(1 - \tilde{\theta}_i\right) F (\kappa_i) &= 0 \quad \text{as } \nu_i \neq 0 \\
\omega - \left(1 - \tilde{\theta}_i\right) F (\kappa_i) &> 0 \quad \text{as } \nu_i = 0 \quad \nu_i \geq 0
\end{align*} \quad (33)$$
Eq. (31) gives rise to
\[ \nu_i = -1 + \lambda_i, \] (34)
which is useful to simplify (30), hence,
\[ \lambda_i = \frac{-(1 - \theta) S \frac{\partial \pi_i}{\partial \kappa_i}}{\theta S \frac{\partial \pi_i}{\partial \kappa_i} - F' (\kappa_i)}. \] (35)
The implied value of \( \nu_i \) is
\[ \nu_i = \frac{-S \frac{\partial \pi_i}{\partial \kappa_i} + F' (\kappa_i)}{\theta S \frac{\partial \pi_i}{\partial \kappa_i} - F' (\kappa_i)}. \] (36)

When \( \theta < 1 \), \( \nu_i \geq 0 \) if and only if
\[ \theta S \frac{\partial \pi_i}{\partial \kappa_i} \leq F' (\kappa_i) \leq S \frac{\partial \pi_i}{\partial \kappa_i}. \] (37)
This rules out the possibility of \( \frac{\partial \pi_i}{\partial \kappa_i} = 0 \) for \( \kappa_i > 0 \) and according to (35), demands that \( \lambda_i \neq 0 \). As result, (32) implies:

**Lemma 1** Two necessary conditions for the solution of program (26) are that the external investors’ participation constraint (27) must be binding, i.e.,
\[ \theta S \pi_i (\kappa_i; \kappa_{-i}) = \tilde{\theta}_i F (\kappa_i), \] (38)
and
\[ \theta S \frac{\partial \pi_i}{\partial \kappa_i} < F' (\kappa_i). \] (39)

Depending on the range of parameters, one (and only one) of the following two possible cases happens in equilibrium.

**Case 1** If \( \nu_i = 0 \), then \( \lambda_i = 1 \), and
\[ S \frac{\partial \pi_i}{\partial \kappa_i} = F' (\kappa_i), \] (40)
i.e., there is neither “under-investment” nor “over-investment” from the firm’s profit-maximizing point of view, and the entrepreneurial wealth constraint is not binding.

**Case 2** If \( \nu_i > 0 \), then condition (33) implies that the entrepreneurial wealth constraint (28) must be binding, i.e.,
\[ \left(1 - \tilde{\theta}_i\right) F (\kappa_i) = \omega \] (41)
which, in combination with condition (38), implies
\[ \theta S \pi_i (\kappa_i; \kappa_{-i}) - F (\kappa_i) + \omega = 0. \] (42)
The left-hand side of the above equation is the slack in the external investors’ participation constraint. To emphasize the important intuition involved, we denote it by $Z_i (\kappa_i; \kappa_{-i})$, hence,

$$Z_i (\kappa_i; \kappa_{-i}) \triangleq \theta S \pi_i (\kappa_i; \kappa_{-i}) - F'(\kappa_i) + \omega.$$  \hspace{1cm} (43)

Then condition (42) can be reformulated as

$$Z_i (\kappa_i; \kappa_{-i}) = 0.$$ \hspace{1cm} (44)

Since the solution to this equation is not necessarily unique, the following lemma specifies which one of them should be picked up in equilibrium.

**Lemma 2** When the entrepreneurial wealth constraint is binding, the global maximum of program (26) corresponds to the largest of the solution(s) to Eq. (44).

**Proof.** Suppose equilibrium picks up a non-maximal solution to condition (44), say $\kappa_i^*$, and there exists $\tilde{\kappa}_i > \kappa_i^*$ which is the maximal solution to condition (44). Given that the entrepreneurial wealth constraint is binding, increasing $\kappa_i$ from $\kappa_i^*$ to $\tilde{\kappa}_i$ which is solely financed by external investors is a Pareto improvement, i.e., it is acceptable to the external investors, and makes the entrepreneur better off. This constitutes a feasible and profitable deviation for the the entrepreneur, which contradicts the assumption that $\kappa_i^*$ is an equilibrium value.

When $\nu_i > 0$, condition (36) implies the following:

**Lemma 3** When the entrepreneurial wealth constraint is binding, there is an under-investment in capability from a firm’s profit-maximizing point of view, i.e.,

$$S \frac{\partial \pi_i}{\partial \kappa_i} > F'(\kappa_i).$$ \hspace{1cm} (45)

Since it is an option for a firm not to invest, i.e., to choose $(\kappa_i, \tilde{\theta}_i) = (0, 0)$, which generates net payoff zero, in equilibrium no entrepreneur should make an expected net loss in the market, i.e.,

$$(1 - \theta) S \pi_i (\kappa_i; \kappa_{-i}) - (1 - \tilde{\theta}_i) F'(\kappa_i) \geq 0.$$ \hspace{1cm} (46)

From (38), it follows:

$$S \pi_i (\kappa_i; \kappa_{-i}) \geq F'(\kappa_i)$$ \hspace{1cm} (47)

and

$$\bar{\theta}_i \geq \theta.$$ \hspace{1cm} (48)

If strict equality holds for any two of above three conditions, it must hold for the third one as well.
3 Symmetric Equilibrium Outcome

To simplify the analysis at this stage, we delay the characterization of any asymmetric equilibrium outcomes to Section 4. In this section we characterize the symmetric equilibrium outcome, i.e., $\kappa_i = \kappa^*$ and $\tilde{\theta}_i = \tilde{\theta}^*$ for all $i = 1, 2, \cdots, N^*$, where $N^*$ is the number of entry. Given our interest in the effect of competition, we only consider environments which warrant $N^* \geq 2$.

In a symmetric equilibrium outcome with free entry, the following external investors’ break-even condition arises:

$$
(1 - \theta) S\pi (\kappa^*, N^*) = \left(1 - \tilde{\theta}^*\right) F (\kappa^*). \tag{49}
$$

Now the external investors’ participation constraint takes a simpler form:

$$
\theta S\pi (\kappa^*, N^*) = \tilde{\theta}^* F (\kappa^*). \tag{50}
$$

The above two conditions determine a firm’s break-even condition:

$$
S\pi (\kappa^*, N^*) = F (\kappa^*) \tag{51}
$$

and the condition

$$
\tilde{\theta}^* = \theta. \tag{52}
$$

**Proposition 4** In a symmetric equilibrium outcome with free entry, the external investors’ capital share equals to the external investors’ cash flow right share, which equals the ex post external investors’ bargaining power, i.e., $\tilde{\theta}^* = \theta$.

3.1 Without binding wealth constraint

When the entrepreneurial wealth constraint (28) is not binding, conditions (40) and (51) both hold and jointly imply that

$$
\frac{\partial \pi_i (\kappa_i)}{\partial \kappa_i} \bigg|_{(\kappa, N_n)} = \zeta (N_{nb}) = \beta, \tag{53}
$$

i.e.,

$$
N_{nb} = \zeta^{-1} (\beta) = \frac{\beta}{2}, \tag{54}
$$

where $N_{nb}$ and $\kappa_{nb}$ are the equilibrium number of firms and capability level in this case, and subscript $nb$ indicates that the wealth constraint (28) is not binding. Consequently,

$$
\kappa_{nb} = \left(\frac{S}{\left(\frac{\beta}{2} + 1\right)^2}\right)^{\frac{1}{2}}. \tag{55}
$$
The necessary and sufficient condition for the entrepreneurial wealth constraints not to be binding is

\[ \omega \geq (1 - \theta) \left( \frac{S}{(\frac{S}{\beta} + 1)^2} \right)^{\frac{\beta}{2}}, \]  

or equivalently,

\[ S \leq S(\theta, \omega) \triangleq \left( \frac{\beta}{S} + 1 \right)^2 \left( \frac{\omega}{1 - \theta} \right)^{\frac{\beta - 2}{\beta}}. \]  

**Proposition 5** The symmetric market configuration \((\kappa_{nb}, N_{nb})\) is an equilibrium if the entrepreneurial wealth level is sufficiently high, i.e., \(\omega \geq (1 - \theta) \left( \frac{S(\beta)}{(\frac{S}{\beta} + 1)} \right)^{\frac{\beta}{2}}\), or the market size is not too large, i.e., \(S \leq S(\theta, \omega)\); and \(\kappa_{nb}\) increases with market size \(S\), i.e., \(\frac{\partial \kappa_{nb}}{\partial S} > 0\); \(N_{nb}\) increases with \(\beta\), i.e., \(\frac{\partial N_{nb}}{\partial \beta} > 0\).

**Proof.** The key is to show that given the market configuration \((\kappa_{nb}, N_{nb})\), the contract \((\kappa_{nb}, \theta)\) is a global maximal point for the maximization program (26). When the wealth constraint is not binding, we can substitute the binding external investors’ participation constraint (38) into the objective function of program (26) to eliminate variable \(\tilde{\theta}_i\). Then the maximization program (26) is reduced to

\[ \max_{\kappa_i} S \pi_i (\kappa_i; \kappa_{-i}) - F (\kappa_i) \]  

i.e., to choose \(\kappa_i\) to maximize the firm’s net profit. We need to show the \(\kappa_{nb}\) is a global maximal point of

\[ S \pi_i (\kappa_i; (\kappa_{nb})_{N_{nb} - 1}) - F (\kappa_i). \]

It is known that

\[ S \pi (\kappa_{nb}, N_{nb}) = F (\kappa_{nb}) \]

and

\[ \frac{\partial \pi_i \kappa_i}{\partial \kappa_i \pi_i} \bigg|_{(\kappa_{nb}, N_{nb})} = \frac{dF (\kappa_i)}{d\kappa_i} \frac{\kappa_i}{F (\kappa_i)} \bigg|_{(\kappa_{nb})} = \beta. \]

These and property (23), i.e., \(\frac{\partial \pi_i}{\partial \kappa_i \pi_i}\) is decreasing in \(\kappa_i\) conditional on \(\pi_i > 0\), jointly imply that

\[ S \pi_i (\kappa_i; (\kappa_{nb})_{N_{nb} - 1}) < F (\kappa_i) \text{ for } \kappa_i > \kappa_{nb}, \]
\[ S \pi_i (\kappa_i; (\kappa_{nb})_{N_{nb} - 1}) < F (\kappa_i) \text{ for } \kappa_i < \kappa_{nb} \text{ and } \pi_i > 0, \]

i.e., \(\kappa_{nb}\) is the maximal point for the range whereof \(\pi_i > 0\). For the range whereof \(\pi_i = 0\), the maximal point is \(\kappa_i = 0\). Overall, \(\kappa_i = 0\) and \(\kappa_{nb}\) are the two global maximal points.
3.2 With binding wealth constraints

The entrepreneurial wealth constraint is binding if

\[ \omega < (1 - \theta) \left( \frac{S}{\left( \frac{\theta}{\beta} + 1 \right)^{2}} \right)^{\frac{\beta}{2 - \beta}}, \]  

or

\[ S > S(\theta, \omega). \]  

Then condition (41) is satisfied; together with condition (52), it implies

\[ \frac{S(\kappa_b)^2}{(N_b + 1)^2} = (\kappa_b)^{\beta} = \frac{\omega}{1 - \theta}. \]  

where \( N_b \) and \( \kappa_b \) the equilibrium number of firms and capability level in this case; and subscript \( b \) indicates that the wealth constraint (28) is binding. Consequently,

\[ \kappa_b = \left( \frac{\omega}{1 - \theta} \right)^{1/\beta} \]  

and

\[ N_b = \left( \left( \frac{1 - \theta}{\omega} \right)^{1-2/\beta} \right)^{1/2} - 1 \]  

Proposition 6 In the symmetric equilibrium outcome with binding wealth constraints, the level of each firm’s capability is increasing in the entrepreneurial wealth level, i.e., \( \frac{\partial \kappa_b}{\partial \omega} > 0 \); is increasing in the ex post external investors’ bargaining power, i.e., \( \frac{\partial \kappa_b}{\partial \theta} > 0 \); does not change with the size of the market, i.e., \( \frac{\partial \kappa_b}{\partial S} = 0 \). The number of firms, \( N_b \), is decreasing in the entrepreneurial wealth level, i.e., \( \frac{\partial N_b}{\partial \omega} < 0 \); is decreasing in the ex post external investors’ bargaining power, i.e., \( \frac{\partial N_b}{\partial \theta} < 0 \); is increasing in the size of the market, i.e., \( \frac{\partial N_b}{\partial S} > 0 \).

The symmetric equilibrium outcome without binding wealth constraints can be used as a benchmark to measure the impacts of binding wealth constraints on the equilibrium market structure and capability level.

Proposition 7 Compared with the symmetric equilibrium outcome without binding wealth constraints, the symmetric equilibrium outcome with binding wealth constraints has lower level of capability and more active firms in the market, i.e., \( \kappa_b \leq \kappa_{nb} \) and \( N_b \geq N_{nb} \).

Proof. It follows from Proposition 6 that \( \frac{\partial \kappa_b}{\partial \omega} > 0, \frac{\partial \kappa_b}{\partial \theta} < 0 \). It is also a fact that \( \kappa_{nb} = \kappa_b \) and \( N_{nb} = N_b \) when \( \omega = \frac{\theta S}{(\frac{\theta}{\beta} + 1)^2} \). As a result, \( \kappa_b \leq \kappa_{nb} \) and \( N_b \geq N_{nb} \) as \( \omega \leq \frac{\theta S}{(\frac{\theta}{\beta} + 1)^2} \).
The above comparison reveals a form of credit rationing that occurs in the symmetric equilibrium outcome, and affects the level of capability and the market structure. From the perspective of maximization of a firm’s profit, there is an under-investment in capability (and R&D). Each firm as a whole is not playing its best response, because of the conflict of interest between the entrepreneur and the external investors. We can easily see that each entrepreneur is credit rationed; more importantly, each firm is credit rationed as well. In this type of credit rationing, each firm still obtains some positive level of credit, though below the level which is optimal for the firm as a whole. This is the ‘Type I credit rationing’ defined in the introduction.

Proposition 6 is informative on the determinants and the consequences of the Type I credit rationing in the oligopoly market. The level of the entrepreneurial wealth and the external investor protection provided by the corporate governance system are the key determinants of the Type I credit rationing. More entrepreneurial wealth and more external investor protection are shown to reduce the severity of the credit rationing and under-investment in capability. The size of the market, although does not affect the level of (credit-constrained) investment in capability, does affects the relative severity of credit rationing and under-investment since it affects the level of capability which is optimal for the firm as a whole. The consequences of the Type I credit rationing are an under-investment in R&D at the firm and industry levels (see the proposition below), and a rise in the number of active firms.

Proposition 8 The industry level of R&D expenditure under the Type I credit rationing is lower than the level without credit rationing. And it is increasing in the level of entrepreneurial wealth \( \omega \) and the level of external investor protection \( \theta \).

Proof. The industry level of R&D expenditure under the Type I credit rationing is given by

\[ N_b(\kappa_b)^\beta. \]

The level without credit rationing is given by

\[ N_{nb}(\kappa_{nb})^\beta. \]

It is known both \((\kappa_b, N_b)\) and \((\kappa_{nb}, N_{nb})\) are solutions to eq. (51), more precisely, the following equation:

\[ \frac{S\kappa^2}{(N+1)^2} = (\kappa)^\beta, \]

which implies that the industry level of R&D expenditure is now given by

\[ N(\kappa)^\beta = N\left(\frac{S}{(N+1)^2}\right)^{\frac{\beta}{\beta - 2}}. \]
Differentiating the logarithm of both sides of the above equation w.r.t. $N$ reveals that

$$
\frac{d \ln \left( N (\kappa)^{\beta} \right)}{dN} = -\frac{\beta N - \beta + 2N + 2}{N (\beta - 2)(N + 1)} < 0 \text{ since } N > 1 \text{ and } \beta > 2.
$$

Given that $N_b > N_{nb}$, it follows that $N_b (\kappa_b)^{\beta} < N_{nb} (\kappa_{nb})^{\beta}$. Similarly, since $N_{nb}$ is decreasing in $\omega$ and $\theta$, it follows that $N_b (\kappa_b)^{\beta}$ should be increasing in $\omega$ and $\theta$.

The result of under-investment in R&D at the industry level under the Type I credit rationing has important implications for technological progress and economic growth.\(^{10}\) This result holds despite the fact that the under-investment at the individual firm level is associated with a larger number of active firms in the market.

One property of the symmetric equilibrium outcome with Type I credit rationing revealed in Proposition 6 is that the number of firms increases with the size of the market. This property resembles the relationship between market size and market structure in industries where the sunk (fixed) cost is exogenous. The difference between them though, is that while the lower bound of some measure of market concentration approaches zero as the market size approaches infinity in the exogenous sunk industries (Sutton (1991)), this ‘convergence’ result does not hold in the kind of markets modeled here. The reason for the difference is that as the size of market is sufficiently large, the symmetric outcome with Type I credit rationing ceases to be equilibrium. To see why this is the case, in what follows we turn to the necessary and/or sufficient conditions for such equilibrium outcomes.

A necessary condition for any equilibrium is condition (39), which in combination with eq. (51) implies a necessary condition for symmetric equilibrium outcome with Type I credit rationing as follows:

$$
\theta \zeta (N_b) < \beta. \quad (64)
$$

Define a threshold value of $S$, $\bar{S}(\theta, \omega)$, such that when $S = \bar{S}(\theta, \omega)$,

$$
\theta \zeta (N_b) \equiv \beta, \quad (65)
$$

or equivalently,

$$
\bar{S}(\theta, \omega) \equiv \left( \frac{\omega}{1 - \theta} \right)^{1 - 2/\beta} \left( \frac{\beta}{2\theta} + 1 \right)^2. \quad (66)
$$

**Lemma 9** A symmetric equilibrium outcome with the Type I credit rationing exists only if

$$
\bar{S}(\theta, \omega) < S \leq \bar{S}(\theta, \omega). \quad (67)
$$

\(^{10}\)See Tong (2005) for a formal discussion.
For the existence of symmetric equilibrium outcome with the Type I credit rationing, the necessary condition (67) may be weaker than also being sufficient. In what follows we characterize the necessary and sufficient condition for the market configuration \((\kappa_b, N_b)\) to be an equilibrium.

The analysis here is built on the following intuition: when the market size is sufficiently large, e.g., \(S = \tilde{S} > S(\theta, \omega)\), some higher capability level, \(\kappa_i > \kappa_b\), can generate a positive slack in the external investors’ participation constraint, hence a Pareto improvement for both the entrepreneur and the external investors, and a profitable deviation. Decreasing \(S\) reduces the set of profitable deviations. There exists a lower limit to the set of values of \(S\) that support profitable deviations. This critical value of \(S\) is what we look for and characterize below.

To formalize this intuition, we first establish the following relationship between the slack in external investors’ participation constraint and \(S\). Note that the slack for firm \(i\) with capability level \(\kappa_i\) competing with \((N_b - 1)\) rivals all with capability level \(\kappa_b\) is given by

\[
Z_i (\kappa_i; (\kappa_b)_{N_b-1}) = \frac{\theta \left( \kappa_i + \left( \frac{S}{1-\theta} \right)^{1-2/\beta} - \frac{2}{1-\theta} \left( \frac{\kappa_i}{\omega} \right)^{1/\beta} \right)^2}{\left( \frac{1-\theta}{\omega} \right)^{1-2/\beta}} - (\kappa_i)^3 + (68)
\]

It is easy to see that

\[
\frac{\partial Z_i (\kappa_i; (\kappa_b)_{N_b-1})}{\partial S} \geq 0 \text{ for } \kappa_i \geq \kappa_b,
\]

i.e., the slack is increasing (decreasing) in \(S\) for \(\kappa_i > \kappa_b\) (\(\kappa_i < \kappa_b\)). This relationship is illustrated in Figure 1 and 2, where the horizontal axis is the level of capability, the vertical axis is the slack in the external investors’ participation constraint, the dashed vertical lines indicate capability level \(\kappa_b\), the thick solid curves correspond to the critical values of \(S\), the dotted curves correspond to larger values of \(S\), and the thin solid curves correspond to smaller values of \(S\). The thick dashed curves correspond to the net incomes of the entrepreneur, which are negative for \(\kappa_i < \kappa_b\).

We look for a critical value of \(S\), i.e., \(S_S (\theta, \omega)\), defined such that when \(S(\theta, \omega) < S < S_S (\theta, \omega)\), \((\kappa_b, N_b)\) is an equilibrium outcome; but when \(S \geq S_S (\theta, \omega)\), \((\kappa_b, N_b)\) is not an equilibrium outcome. It is illustrated as the thick solid curves in Figures 1 and 2, for two different possible cases: for the first case, \(S_S (\theta, \omega) = \tilde{S} (\theta, \omega)\); for the second case, \(S_S (\theta, \omega) > \tilde{S} (\theta, \omega)\). The following lemma establishes a key property of \(S_S (\theta, \omega)\).

**Lemma 10** \(S_S (\theta, \omega) \leq \tilde{S} (\theta, \omega)\). When \(S = S_S (\theta, \omega)\), there exists \(\kappa_i \geq \kappa_b\) such that

\[
Z_i (\kappa_i; (\kappa_b)_{N_b-1}) = 0 \quad (69)
\]
Figure 1: Effect of $S$ on slack $Z_i$; Case: $S_S(\theta, \omega) = \bar{S}(\theta, \omega)$

Figure 2: Effect of $S$ on slack $Z_i$; Case: $S_c(\theta, \omega) < \bar{S}(\theta, \omega)$
and
\[
\frac{\partial Z_i(\kappa_i; (\kappa_b)_{N_b-1})}{\partial \kappa_i} = 0. \tag{70}
\]

**Proof.** If \( S \in (S(\theta, \omega), S_S(\theta, \omega)) \), i.e., the necessary and sufficient condition is satisfied, then \( S \in (S(\theta, \omega), S(\theta, \omega)) \) since a necessary condition must be satisfied. Consequently, \( S_S(\theta, \omega) \leq S(\theta, \omega) \).

If \( S_S(\theta, \omega) = S(\theta, \omega) \) (see Figure 1), then when \( S = S_S(\theta, \omega) \), it follows from the definition of \( S(\theta, \omega) \) that \( \kappa_i = \kappa_b \) satisfies conditions (69) and (70).

If \( S_S(\theta, \omega) < S(\theta, \omega) \) (see Figure 2), then when \( S = S_S(\theta, \omega) \), by the definition of \( S_S(\theta, \omega) \), \( \kappa_b \) is not an equilibrium capability level; then according to Lemma 2, \( \kappa_b \) is not the largest solution to eq. (69). Therefore there exists \( \kappa_i > \kappa_b \) which satisfies condition (69), i.e., \( \kappa_i \) is a larger solution to eq. (69). We claim that \( \kappa_i \) must also satisfy condition 70. To prove this claim, suppose \( \frac{\partial Z_i(\kappa_i; (\kappa_b)_{N_b-1})}{\partial \kappa_i} \neq 0 \), then there exists \( \tilde{\kappa}_i > \kappa_b \) such that \( |\tilde{\kappa}_i - \kappa_i| > 0 \) and \( Z_i(\tilde{\kappa}_i; (\kappa_b)_{N_b-1}) > 0 \); then exists \( \varepsilon > 0 \) such that \( Z_i(\tilde{\kappa}_i; (\kappa_b)_{N_b-1}) \big|_{S=S_S(\theta,\omega)-\varepsilon} > 0 \), i.e., a positive slack exists, which contradicts the assumption that when \( S(\theta, \omega) < S < S_S(\theta, \omega) \), \((\kappa_b, N_b)\) is an equilibrium outcome. \[\]

Define \( r \overset{\Delta}{=} \frac{\kappa_i}{(\tau^\omega)^{1/\beta}} \), hence
\[
\kappa_i = r \left( \frac{\omega}{1-\theta} \right)^{1/\beta}. \tag{71}
\]

Substituting (71) into eq. (69) and (70) gives rise to the following two equations in two unknowns: \( r \) and \( S \):
\[
\theta \left( r + \left( \left( S \left( \frac{1-\theta}{\omega} \right) \right)^{1-2/\beta} \right)^{1/2} - 2 \right) (r - 1) = r^\beta - (1 - \theta) \tag{72}
\]
and
\[
2\theta \left( r + \left( \left( \frac{1-\theta}{\omega} \right)^{1-2/\beta} \right)^{1/2} - 2 \right) (r - 1) \left( \left( S \left( \frac{1-\theta}{\omega} \right) \right)^{1-2/\beta} \right)^{1/2} - 1 = \beta r^{\beta-1}, \tag{73}
\]
which imply
\[
2\theta \left( \frac{r^\beta - (1 - \theta)}{\theta} \right)^{1/2} \left( \left( \frac{r^\beta - (1 - \theta)}{\theta} \right)^{1/2} - 1 \right) = \beta r^{\beta-1} (r - 1) \tag{74}
\]
and
\[
S_S(\theta, \omega) = \left( \frac{\omega}{1-\theta} \right)^{1-2/\beta} \left( \frac{\beta}{2\theta} \left( \frac{r^{\beta-1}}{1 + S_S(\theta, \omega)^{\beta-1}} + 1 \right) \right)^2. \tag{75}
\]
where $r_S$ is the largest solution to equation (74). It is straightforward to verify that $r = 1$ is a solution to (69), so $r_S \geq 1$. Figure 3 illustrates the relationship between $r_S$ and parameter $\theta$. There exists a threshold value of $\theta$, $\bar{\theta}$, below which $r_S > 1$ and above which $r_S = 1$. To solve for $\bar{\theta}$, we transform eq. (74) into

\[
2 \left( \frac{r^\beta - (1 - \theta)}{\theta} \right)^\frac{1}{2} \left( \theta \frac{r^\beta - (1 - \theta)}{(r - 1)} \right)^\frac{1}{2} - 1) = \beta r^{\beta-1}. \tag{76}
\]

Then $r_S > 1$ is a solution to the above equation; it is a function of $\theta$, and $r_S \to 1 \Leftrightarrow \theta \to \bar{\theta}$. Consequently,

\[
\bar{\theta} = \lim_{r \to 1} \theta \text{ subject to Eq. (76)}.
\]

As a first step, we note that

\[
\lim_{r \to 1} \left( \theta \frac{r^\beta - (1 - \theta)}{(r - 1)} \right)^\frac{1}{2} - 1 = 2.
\]

Then we substitute

\[
\left( \theta \frac{r^\beta - (1 - \theta)}{(r - 1)} \right)^\frac{1}{2} - 1 = 2
\]

to eq. (76) and solve for $\theta$, with the solution:

\[
\theta = \frac{r^\beta - 1}{r^{(\beta-1)} - 1}.
\]

Finally, we solve for $\bar{\theta}$ by the following:

\[
\bar{\theta} = \lim_{r \to 1} \frac{r^\beta - 1}{r^{(\beta-1)} - 1} = \frac{2}{3}.
\]

**Lemma 11** The threshold value $S_S(\theta, \omega)$ exists. And $S_S(\theta, \omega) = \bar{S}(\theta, \omega)$ if and only if $\theta \geq \frac{2}{3}$; $S_S(\theta, \omega) < \bar{S}(\theta, \omega)$ if $\theta < \frac{2}{3}$.

**Proposition 12** The market configuration $(\kappa_b, N_b)$ is an equilibrium if and only if $S(\theta, \omega) < S < S_S(\theta, \omega)$.

**4 Asymmetric Equilibrium Outcome**

Proposition 12 implies that for given $\theta$ and $\omega$, if the market size is sufficiently large, namely, $S > S_S(\theta, \omega)$, then no symmetric equilibrium outcome can exist. Then a natural question to ask is whether any equilibrium exists in the model when the market size is so large. In this section, this question will be answered.
For tractability we consider a simple asymmetric equilibrium outcome with symmetric strategies (financial contractual terms) but asymmetric credit allocation. Suppose there are $N$ firms that enter at stage one, all offer the same contractual terms that aim at a large investment project in stage two. Unfortunately, only a fraction $\phi \in [0, 1]$ of the offers are accepted in equilibrium, giving rise to $N_h = \phi N$ high-capability firms with implemented contracts $(\kappa_h, \tilde{\theta})$; the other $N_l = (1 - \phi)$ offers are rejected in equilibrium, giving rise to low-capability firms who have $\kappa_l$ with no external finance. Each firm has an equal chance of winning the selection, with probability $\phi$ when their offers are identical.\footnote{This assumption of ex ante equal chance of winning the selection can be consistent with a setting where there is ex ante uncertainty about some efficiency-related firm characteristics, and the ex post selection is based on the relative efficiency of firms. This assumption can be consistent with an equilibrium outcome that features ‘survival of the most efficient’. A more detailed discussion of this point is placed in Section 5.} If the offers differ, the external investors always choose to accept the offers which give them higher pledgeable returns for their investments. The comparison of the two realized levels of capability are as follows: $\kappa_h > \kappa_l$. We assume that as a necessary condition for the entrepreneurs to gain external finance, in stage 2 each entrepreneur has to commit his or her promised insider equity. Each entrepreneur maximizes his or her expected net income.

In equilibrium the entrepreneurial wealth constraints must be binding, i.e.,

$$\left(1 - \tilde{\theta}\right) (\kappa_h)^\beta = \omega, \quad (77)$$

and

$$\kappa_l = \omega^{1/\beta} \quad (78)$$

otherwise, an entrepreneur can always slightly increase his or her own equity commitment and secure external finance with certainty, hence increases the expected net income by a finite amount.
Since the capital market is perfectly competitive, in equilibrium no external investors should earn higher than the market rate, therefore the external investors’ participation constraints must be binding, i.e.,

$$\theta S\pi_h (\kappa_h; \kappa_{-h}) = \tilde{\theta} (\kappa_h)^\beta$$

or

$$\theta S\pi_h (\kappa_h; \kappa_{-h}) = (\kappa_h)^\beta - \omega$$

where $\kappa_{-h}$ denotes $[(\kappa_h)_{N_h - 1}, (\kappa_l)_{N_l}]$.

Now some remarks on the fact that a fraction of entrepreneurs are denied (external) credits are in order. This is what we call the ‘Type II credit rationing’. Ex ante, these entrepreneurs have a positive probability of obtaining credit, that is why they apply for credit in the first place, and even commit their own equity participation in the face of the risk of credit denial. Besides the Type II credit rationing which happens to the fraction $(1 - \phi)$ of entrants, the other fraction $\phi$ of entrepreneurial entrants still face the Type I credit rationing.

Consider free entry. So the following ex ante zero profit condition should hold in equilibrium:

$$\phi (1 - \theta) S\pi_h (\kappa_h; \kappa_{-h}) + (1 - \phi) S\pi_l (\kappa_l; \kappa_{-l}) = \omega.$$  

(81)

In equilibrium the net income of an entrepreneur who is Type I credit-rationed must be higher than that of an entrepreneur who is Type II credit-rationed, i.e.,

$$(1 - \theta) S\pi_h (\kappa_h; \kappa_{-h}) > \omega > S\pi_l (\kappa_l; \kappa_{-l}).$$

(82)

The firms that are Type I credit-rationed, i.e., the high capability firms, earn positive net profits, while the firms that are Type II credit-rationed, i.e., the low capability firms, make net losses. The reason why the high capability firms can make positive net profits when there is free entry is because the low capability firms are denied (external) credits therefore are inferior competitors. There is a coexistence of the two types of credit rationing, and their interaction has an impact on Type I credit rationing and market structure, allows a smaller number of high capability firms a higher level of credit than the level of a symmetric outcome with Type I credit rationing, as established by the following proposition.

**Proposition 13** In comparison with the symmetric outcome with Type I credit ration, the high capability firms in the asymmetric equilibrium outcome obtain more credits and the high capability level is higher, i.e.,

$$\tilde{\theta}_h > \theta,$$

(83)

and

$$\kappa_h > \kappa_h.$$

(84)

Also, the number of high capability firms is smaller than the number of firms of a symmetric outcome with Type I credit rationing, i.e.,

$$N_h < N_h.$$  

(85)
Proof. The high capability firms earn positive net profits, i.e.,

\[(1 - \theta) S\pi_h (\kappa_h; \kappa_{-h}) = \left(1 - \hat{\theta}_h\right) (\kappa_h)^3 > 0. \quad (86)\]

Using eq. (79) and (86) to eliminate \(\pi_h (\kappa_h; \kappa_{-h})\) results in

\[\hat{\theta}_h > \theta. \]

From (62) and (77), then it follows that

\[\kappa_h = \left(\frac{\omega}{1 - \hat{\theta}_h}\right)^\frac{1}{\beta} > \kappa_b = \left(\frac{\omega}{1 - \theta}\right)^\frac{1}{\beta}. \quad (87)\]

From the inequality

\[S\pi (\kappa_h, N_h) = S\frac{(\kappa_h)^2}{(N_h + 1)^2} \geq S\pi (\kappa_h; \kappa_{-h}) = \frac{\hat{\theta}_h}{\theta} (\kappa_h)^3 > (\kappa_h)^3\]

it follows that

\[N_h < S^\frac{1}{\beta} (\kappa_b)^{\frac{2}{\beta}} - 1.\]

Since \(\kappa_h > \kappa_b\) and \(\beta > 2\), it is immediate that

\[N_h < S^\frac{1}{\beta} (\kappa_b)^{\frac{2}{\beta}} - 1 = N_b. \quad (88)\]

When the Type II credit rationing exists in equilibrium, it creates winners and losers among the entrepreneurs. Ex ante, when an entrepreneur chooses his or her optimal contract, he or she has to balance two elements: one is to maximize the net income conditional on the contract being acceptable; the other is to maximize the chance of the contract being selected. As a result of the competition for the rationed contract acceptance and free entry, the equilibrium outcome must exhaust all the opportunities of positive slack in the external investors’ participation constraints. If the profit function \(\pi_i\) is continuously differentiable\(^{12}\) at the point \((\kappa_h; \kappa_{-h})\), then to rule out any profitable local deviation (i.e., positive slack in external investors’ participation constraint), the following condition should hold:

\[\theta S \frac{\partial \pi_i}{\partial \kappa_i} = F'(\kappa_i) \quad (89)\]

which is a necessary condition for equilibrium in this circumstance. Hence,

\[-\theta \frac{\partial \pi_i}{\partial \kappa_i} \frac{\kappa_i}{\pi_i} = \frac{F'(\kappa_i) \kappa_i F(\kappa_i)}{F(\kappa_i) - S\pi_i} = \beta \frac{\theta}{\theta_i}\]

\(^{12}\)The case of a “kink”, i.e., the function is not continuously differentiable, is discussed in Subsubsection 4.2.1.
It follows that
\[ \frac{\partial \pi_i}{\partial \kappa_i \pi_i} \bigg|_{\kappa_i = \kappa_h, \kappa_{-i} = \kappa_{-h}} = \beta. \]  
\hfill (90)

Using (77) and (90) to eliminate \( \tilde{\theta}_h \) yields
\[ \left( 1 - \frac{\omega}{(\kappa_h)^\beta} \right) \frac{\partial \pi_i}{\partial \kappa_i \pi_i} \bigg|_{\kappa_i = \kappa_h, \kappa_{-i} = \kappa_{-h}} = \beta. \]  
\hfill (91)

When we discuss an asymmetric outcome, we have to pay particular attention to whether it is optimal for a low capability firm to sell a positive quantity in the market, i.e., to have a positive market share. It is possible that if the low capability is relatively too low, then the firm’s optimal choice is not to produce at the third stage of the game.

Denote by \( m(\kappa_i; \kappa_{-i}) \) the equilibrium market share of firm \( i \).

\[ m(\kappa_i; \kappa_{-i}) = \max \left( 0, \frac{\kappa_i + \sum_{j=1}^{N} (\kappa_i - \kappa_j)}{\sum_{j=1}^{N} \kappa_j} \right). \]  
\hfill (92)

Define the normalized ‘limit capability’ \( \bar{\kappa}(N_h) \) such that
\[ m(\kappa_i; \kappa_{-i})|_{\kappa_i = \omega^{1/\beta}, \kappa_h = \omega^{1/\beta} \bar{\kappa}(N_h)} = 0 \]  
\hfill (93)

and
\[ m(\kappa_i; \kappa_{-i})|_{\kappa_i = \omega^{1/\beta}, \kappa_h = \omega^{1/\beta} \bar{\kappa}(N_h) - \varepsilon} > 0 \]  
\hfill (94)

for any \( \varepsilon > 0 \). So
\[ \bar{\kappa}(N_h) = \left( 1 + \frac{1}{N_h} \right). \]  
\hfill (95)

There are three types of possible asymmetric equilibrium outcomes as far as \( \kappa_h \) and \( m(\kappa_i; \kappa_{-i}) \) are concerned, corresponding to the low capability firms being:

1. Active low capability firms: \( \kappa_h < \omega^{1/\beta} \bar{\kappa}(N_h) \) and \( m(\kappa_i; \kappa_{-i}) > 0 \).

2. Inactive but effective low capability firms: \( \kappa_h = \omega^{1/\beta} \bar{\kappa}(N_h) \) and \( m(\kappa_i; \kappa_{-i}) = 0 \), and the low capability firms do not produce any positive quantity, but they still have an influence on every high capability firm’s marginal incentive for investment.

3. Ineffective low capability firms: \( \kappa_h > \omega^{1/\beta} \bar{\kappa}(N_h) \) and \( m(\kappa_i; \kappa_{-i}) = 0 \), and the low capability firms have no influence on any high capability firm’s marginal incentive for investment in stage.

These three cases are analyzed respectively in the following three subsections.
4.1 Active low capability firms

In this case, the high capability must be below the limit capability, i.e.,

\[ \kappa_h \leq \bar{\kappa} (N_h) \omega^{1/\beta} = \left( 1 + \frac{1}{N_h} \right) \omega^{1/\beta}. \] (96)

The profits for high and low capability firms are respectively

\[ \Pi_h (\kappa_h; \kappa_{-h}) = \frac{S}{\left( \frac{N_h}{\phi} + 1 \right)^2} \left( \kappa_h + (1 - \phi) \frac{N_h}{\phi} (\kappa_h - \omega^{1/\beta}) \right)^2, \] (97)

\[ \Pi_l (\kappa_l; \kappa_{-l}) = \frac{S}{\left( \frac{N_h}{\phi} + 1 \right)^2} \left( \omega^{1/\beta} + N_h \left( \omega^{1/\beta} - \kappa_h \right) \right)^2. \] (98)

The binding external investors’ participation constraint (80) now has the following implementation:

\[ \theta \frac{S \left( \kappa_h + (1 - \phi) \frac{N_h}{\phi} (\kappa_h - \omega^{1/\beta}) \right)^2}{\left( \frac{N_h}{\phi} + 1 \right)^2} = (\kappa_h)^\beta - \omega. \] (99)

The ex ante zero-profit condition (81) now takes the following form:

\[ (1 - \theta) S \left( \kappa_h + (1 - \phi) \frac{N_h}{\phi} (\kappa_h - \omega^{1/\beta}) \right)^2 + (1 - \phi) S \left( \omega^{1/\beta} + N_h \left( \omega^{1/\beta} - \kappa_h \right) \right)^2 = \omega. \] (100)

The necessary condition (91) takes the form:

\[ \frac{\left( 1 - \frac{\omega}{(\kappa_h)\beta} \right) ^2 \frac{N_h}{\phi}}{1 + (1 - \phi) \frac{N_h}{\phi} \left( 1 - \frac{\omega^{1/\beta}}{\kappa_h} \right)} = \beta, \] (101)

which features Figure 4. Where the thick solid curve is the slack in the external investors’ participation constraint. Although it is positive when \( \kappa_i < \omega^{\frac{1}{\beta}} \) (\( \omega^{\frac{1}{\beta}} \) is indicated by the dotted vertical line), it is always associated with a negative net income for the entrepreneur (which is illustrated by the thin solid curve) when \( \kappa_i < \omega^{\frac{1}{\beta}} \). The slack is maximal and continuously differentiable at \( \kappa_i = \kappa_h \) for \( \kappa_i > \omega^{\frac{1}{\beta}} \).

Condition (101) implies

\[ \phi = \phi_A (\kappa_h, N_h) = N_h \left( \frac{1 - \frac{\omega}{(\kappa_h)\beta}}{1 + \left( \frac{\omega^{1/\beta}}{\kappa_h} - 1 \right) N_h} \right). \] (102)
The characterization of this type of equilibrium eventually boils down to the following equations system with 2 equations (and an additional inequality) in 2 unknowns: $N_h$ and $\kappa_h$, as follows:

$$
\begin{cases}
\frac{\theta}{\Phi_A(\kappa_h, N_h)} \left( \left( \kappa_h + \frac{1 - \Phi_A(\kappa_h, N_h)}{\Phi_A(\kappa_h, N_h) + 1} N_h (\kappa_h - \omega^{1/\beta}) \right)^2 \right) = (\kappa_h)^\beta - \omega \\
\frac{\Phi_A(\kappa_h, N_h)(1 - \theta)}{\Phi_A(\kappa_h, N_h)} \left( \left( \kappa_h + \frac{1 - \Phi_A(\kappa_h, N_h)}{\Phi_A(\kappa_h, N_h) + 1} N_h (\kappa_h - \omega^{1/\beta}) \right)^2 \right) = \omega \left( 1 + \frac{1}{N_h} \right) \omega^{1/\beta}
\end{cases}
$$

(103)

\[ 4.2 \text{ Inactive low capability firms} \]

When the low capability firms are not active, the profit of a high capability firm is given by

$$
\Pi_h(\kappa_h; \kappa_{-h}) = \frac{S(\kappa_h)^2}{(N_h + 1)^2}. 
$$

(104)

The ex ante zero-profit condition (81) now takes the following form:

$$
\phi \left( 1 - \theta \right) \frac{S(\kappa_h)^2}{(N_h + 1)^2} = \omega. 
$$

(105)

The binding external investors’ participation constraint (80) now has the following implementation:

$$
\theta \frac{S(\kappa_h)^2}{(N_h + 1)^2} = (\kappa_h)^\beta - \omega. 
$$

(106)
which implies
\[ N_h = \left( \frac{\theta S (\kappa_h)^2}{(\kappa_h)^\beta - \omega} \right)^{\frac{1}{\beta}} - 1. \]  
(107)

Using eq. (105) and (106) to solve for \( \phi \) yields
\[ \phi = \frac{\theta}{1 - \theta} \frac{\omega}{(\kappa_h)^\beta - \omega}. \]  
(108)

### 4.2.1 Effective low capability firms

In this case the high capability level equals the limit capability, i.e.,
\[ \kappa_h = \omega^{1/\beta} \kappa (N_h) = \left( 1 + \frac{1}{N_h} \right) \omega^{1/\beta}, \]  
(109)

which implies
\[ N_h = \frac{1}{\omega^{1/\beta}} - 1. \]  
(110)

The profit function \( \Pi_i \) is not continuously differentiable, i.e., it has a “kink”, at the equilibrium point. This kink point must be global maximum of the slack in the external investors’ participation constraint, as is shown in Figure 5.

![Figure 5: Asymmetric equilibrium outcome with effective low capability firms (the kink maximum point)](image)

Inserting the above two equations into eq. (106) to eliminate \( \kappa_h \) and \( N_h \) respectively, we have
\[ \left( 1 + \frac{1}{N_h} \right)^\beta (N_h)^2 \omega^{1-2/\beta} = \theta S \]  
(111)
and

\[ \theta S \left( \kappa_h - \omega^{1/\beta} \right)^2 = (\kappa_h)^\beta - \omega. \]  
(112)

The above two equations imply the following comparative statics:

\[
\frac{\partial N_h}{\partial S} = -\frac{1}{S_N} \left( \frac{(N_h+1)^{\beta-1} - (N_h)^{\beta}}{\beta - 2} \right),
\]  
(113)

\[
\frac{\partial N_h}{\partial \theta} = -\frac{1}{N_h} \left( \frac{(N_h+1)^{\beta-1} - (N_h)^{\beta}}{\beta - 2} \right),
\]  
(114)

\[
\frac{\partial \kappa_h}{\partial S} = \frac{1}{N_h S} \left( \frac{(N_h+1)^{\beta-1} - (N_h)^{\beta}}{\beta - 2} \right),
\]  
(115)

\[
\frac{\partial \kappa_h}{\partial \theta} = \frac{1}{N_h \theta} \left( \frac{(N_h+1)^{\beta-1} - (N_h)^{\beta}}{\beta - 2} \right).
\]  
(116)

Inserting eq. (109) and the relationship

\[ N = \frac{N_h}{\phi} \]

into the necessary condition (for the kink maximum point):

\[
\frac{\beta}{1 + (1 - \phi) N \left( 1 - \frac{\omega^{1/\beta}}{\kappa_h} \right)} \geq \beta
\]  
(117)

yields

\[
\beta \frac{(N_h+1)^{\beta-1}}{(N_h+1)^{\beta} - 1 (N_h)^{\beta}} - 2 \leq -\frac{\beta \phi}{N_h} \frac{(N_h+1)^{\beta-1}}{(N_h+1)^{\beta} - 1 (N_h)^{\beta}}.
\]  
(118)

It follows that

\[
\frac{(N_h+1)^{\beta-1}}{(N_h+1)^{\beta} - (N_h)^{\beta}} \beta - 2 < 0.
\]  
(119)

This condition ensures definite signs for the comparative statics (113) - (116), as stated in the following proposition.

**Proposition 14** In the asymmetric equilibrium outcome with effective low-capability firms, the number of high-capability firms \( N_h \) increases with market size \( S \) and the external investors’ bargaining power \( \theta \), i.e., \( \frac{\partial N_h}{\partial S} > 0 \) and \( \frac{\partial N_h}{\partial \theta} > 0 \); the high capability level decreases with market size \( S \) and the external investors’ bargaining power \( \theta \), i.e., \( \frac{\partial \kappa_h}{\partial S} < 0 \) and \( \frac{\partial \kappa_h}{\partial \theta} < 0 \).
4.2.2 Ineffective low capability firms

In this case, the high capability level must exceed the limit capability level, i.e.,

\[ \kappa_h > \bar{\kappa}(N_h)\omega^{1/\beta} = \left(1 + \frac{1}{N_h}\right)\omega^{1/\beta}. \]  \hspace{1cm} (120)

This equilibrium outcome does not involve a kink maximum point (see Figure 6), hence the necessary condition (91) should apply and is implemented as follows:

\[ \left(1 - \frac{\omega}{(\kappa_h)^{\beta}}\right)2N_h = \beta, \] \hspace{1cm} (121)

or equivalently,

\[ N_h = \frac{\beta(\kappa_h)^{\beta}}{2\left((\kappa_h)^{\beta} - \omega\right)}, \] \hspace{1cm} (122)

or

\[ \kappa_h = \left(\frac{\omega N_h}{N_h - \frac{\beta}{2}}\right)^{1/\beta}. \] \hspace{1cm} (123)

It follows that

\[ N_h > \frac{\beta}{2} = N_{lb}. \] \hspace{1cm} (124)

The binding external investors’ participation constraint (80) now takes the form as

\[ \theta \frac{S(\kappa_h)^2}{(N_h + 1)^2} = (\kappa_h)^{\beta} - \omega. \] \hspace{1cm} (125)
Using Equations (123) and (125) to eliminate \( \kappa_h \) and \( N_h \) respectively results in

\[
\frac{\theta S}{(N_h + 1)^{2/\beta}} \left( \frac{\omega N_h}{N_h - \frac{\beta}{2}} \right)^{-2} = \left( \frac{\omega N_h}{N_h - \frac{\beta}{2}} - \omega \right),
\]

and

\[
\frac{S(\kappa_h)^2}{\left( \frac{\beta(\kappa_h)^{\beta}}{2(\kappa_h)^{\beta} - \omega} + 1 \right)^2} = (\kappa_h)^{\beta} - \omega,
\]

which imply the following set of comparative statics:

\[
\frac{\partial N_h}{\partial \theta} = -\frac{1}{S \frac{N_h}{N_h^2} \left( 1 + \frac{\beta - 2}{2N_h - \beta} - \frac{2N_h}{N_h + 1} \right)}
\]

\[
\frac{\partial N_h}{\partial S} = -\frac{1}{S \frac{N_h}{N_h^2} \left( 1 + \frac{\beta - 2}{2N_h - \beta} - \frac{2N_h}{N_h + 1} \right)}
\]

\[
\frac{\partial \kappa_h}{\partial S} = \frac{1}{S \frac{N_h}{N_h^2} \left( 1 + \frac{\beta - 2}{2N_h - \beta} - \frac{2N_h}{N_h + 1} \right)} \left( 2N_h - \beta \right)
\]

\[
\frac{\partial \kappa_h}{\partial \theta} = \frac{1}{S \frac{N_h}{N_h^2} \left( 1 + \frac{\beta - 2}{2N_h - \beta} - \frac{2N_h}{N_h + 1} \right)} \left( 2N_h - \beta \right)
\]

The equilibrium entails the following second order necessary condition for maximization:

\[
\theta \frac{\partial^2 \pi_i}{\partial \kappa_i^2} \bigg|_{(\kappa_h, N_h)} = -\beta (\beta - 1) (\kappa_h)^{(\beta - 2)} \leq 0,
\]

which implies

\[
\frac{\partial^2 \pi_i}{\partial \kappa_i^2} \bigg|_{(\kappa_h, N_h)} \leq \beta - 1.
\]

Given the following property of the profit function:

\[
\frac{\partial^2 \pi_i}{\partial \kappa_i^2} \bigg|_{(\kappa_h, N_h)} = N_h,
\]

it follows that the number of high capability firms is bounded from above by:

\[
N_h \leq \beta - 1,
\]

and the inequality below is satisfied.

\[
1 + \frac{\beta - 2}{2N_h - \beta} = \frac{2N_h}{N_h + 1} > \frac{2}{N_h + 1} > 0.
\]

30
As a result, we obtain definite signs for the comparative statics (128) - (131) as follows:

\[
\frac{\partial N_h}{\partial S} < 0, \quad \frac{\partial N_h}{\partial \theta} < 0, \quad \frac{\partial \kappa_h}{\partial S} > 0 \quad \text{and} \quad \frac{\partial \kappa_h}{\partial \theta} > 0. \tag{137}
\]

**Proposition 15** In the asymmetric equilibrium outcome with ineffective low-capability firms, the high capability \(\kappa_h\) increases with market size \(S\) and the external investors’ bargaining power \(\theta\), i.e., \(\frac{\partial \kappa_h}{\partial S} > 0 \) and \(\frac{\partial \kappa_h}{\partial \theta} > 0\); the number of high capability firms \(N_h\) decreases with market size \(S\) and the external investors’ bargaining power \(\theta\), i.e., \(\frac{\partial N_h}{\partial S} < 0 \) and \(\frac{\partial N_h}{\partial \theta} < 0\); the number of low capability firms \(N_l\) increases with market size \(S\) and the external investors’ bargaining power \(\theta\), i.e., \(\frac{\partial N_l}{\partial S} > 0 \) and \(\frac{\partial N_l}{\partial \theta} > 0\).

The reason for \(N_l\) to increase with \(S\) and \(\theta\) is as follows: When \(S\) (or \(\theta\)) increases, the high capability firms as a whole earn more positive net profits; then the ex ante zero-profit condition implies that in balance more net losses must be made by the low capability firms. Given each low capability firm makes a fixed amount of net loss, it follows that the larger amount of total loss must be shared by more low capability firms.

### 4.3 Existence and multiplicity of equilibrium

Now we discuss the existence, the type(s), and multiplicity of equilibrium. The discussion is divided according to the market size \(S\), for any given pair of parameters \((\theta, \omega)\).

Consider the lower bound of \(S\), denoted by \(S_I(\theta, \omega)\), for the existence of an asymmetric equilibrium outcome with ineffective low-capability firms. Figure 7 features the slack in the external investors’ participation constraint for \(S = S_I(\theta, \omega)\): there is a kink at the equilibrium level of high capability, particularly, the slope at the immediate left of the kink is flat, while it is upward at the immediate right of the kink.

When \(S = S_I(\theta, \omega)\), there exists \((\kappa_h, N_h)\) which satisfies conditions (122), (127) and (110). Define \(r_I = \frac{\kappa_h}{\omega^{1/\beta}}\). It follows that

\[
\frac{r_I^{\beta} - 1}{r_I - 1} = \frac{\beta}{2} r_I^{\beta - 1} \tag{138}
\]

and

\[
S_I(\theta, \omega) = \frac{\omega^{1-2/\beta}}{\theta} \left( \left( \frac{\theta}{\omega} + 1 \right) r_I^\beta - 1 \right)^2 \tag{139}
\]

Consider the lower bound of \(S\), denoted by \(S_E(\theta, \omega)\), for the existence of an asymmetric equilibrium outcome with effective low-capability firms. Figure 8 features the slack in the external investors’ participation constraint for \(S = S_E(\theta, \omega)\): there is a kink at the equilibrium level of high capability, particularly, the slope at the immediate left of the kink is flat, while it is downward at the immediate right of the kink.
Figure 7: Asymmetric equilibrium outcome with effective low capability firms (threshold case: almost ineffective)

Figure 8: Asymmetric equilibrium outcome with effective low capability firms (the threshold case: almost active)
When $S = S_E(\theta, \omega)$, there exists $(\kappa_h, N_h)$ which satisfies conditions (110), (108), and (112). Define $r_E \triangleq \frac{1}{\omega}$. It follows that

$$
\frac{\theta}{1 - \theta} \left( \frac{1}{r_E} \right)^2 - 1 = \left( \frac{1 - \frac{1}{(r_E)^2}}{r_E - 1} \right) \left( 1 - \frac{1}{r_E} \right) + \left( \frac{1}{r_E - 1} \right)
$$

(140)

$$
S_E(\theta, \omega) = \frac{\omega^{1-2/\beta} \beta^2}{\beta} - 1
\frac{\beta^2 - 1}{r_E - 1}
$$

(141)

Consider the lower bound of $S$, denoted by $S_A(\theta, \omega)$, for the existence of an asymmetric equilibrium outcome with active low-capability firms. When $S = S_A(\theta, \omega)$, there exists only one high capability firm (i.e., $N_h = 1$) with $\kappa_h$, that satisfies the system of equations (103). Define $r_A \triangleq \frac{1}{\omega}$. It follows that

$$
\begin{align*}
\phi &= r_A \left( 1 - \frac{1}{(r_A)^2} \right)^{2/\beta} - 1 + 1 \\
\frac{\theta}{(r_A)^2 - 1} &= \phi (1 - \theta) + \frac{\beta^2 (1 - \theta)^2 (2 - r_A)^2}{(r_A - (1 - \phi))^2} \\
S_A &= \frac{(1 + \phi)^2 ((r_A)^2 - 1)^{1-2/\beta} \omega^{1-2/\beta}}{r_A (r_A - (1 - \phi))^2}
\end{align*}
$$

(142)

Proposition 16 There exists critical values of $S$: $S_1(\theta, \omega) > S_E(\theta, \omega) > S_A(\theta, \omega)$, such that an asymmetric equilibrium outcome with ineffective low-capability firms exists if and only if $S > S_1(\theta, \omega)$; an asymmetric equilibrium outcome with ineffective low-capability firms exists if and only if $S \in [S_E(\theta, \omega), S_1(\theta, \omega)]$; an asymmetric equilibrium outcome with ineffective low-capability firms exists if and only if $S \in (S_A(\theta, \omega), S_E(\theta, \omega))$.

Proposition 17 There exists critical values of $S$: $S_A(\theta, \omega) > S_S(\theta, \omega)$ such that both symmetric and asymmetric equilibrium outcomes exist if and only if $S \in (S_A(\theta, \omega), S_S(\theta, \omega))$.

By now, we have characterized all the symmetric equilibria\(^\text{13}\) of this model. In summary: when $S$ is small, the entrepreneurial wealth constraints are not binding; increasing $S$ will eventually lead to Type I credit rationing with symmetric equilibrium outcomes; further increasing $S$ will lead to both Type I and Type II credit rationing with asymmetric equilibrium outcomes. There is a possibility of multiplicity of equilibrium. When the size of the market $S$ is sufficiently large, the Type II credit rationed low capability firms become ineffective, then $\kappa_h$ increases with $S$ but $\kappa_h < \kappa_{nh}$; $N_h$ decreases with $S$ but $N_h < N_{nh}$.

4.4 Very large market: the limiting case

In this section, we study the behavior of equilibrium when the market size $S$ approaches infinity. We are particularly interested in the asymptotic behavior of variables: $\kappa_h$ and $N_h$.

\(^{13}\) Although the outcomes of these equilibria may be asymmetric, the strategies involved are all symmetric.
Since $\kappa_h$ increases with $S$, and approaches to infinity as $S$ approaches infinity. It makes more sense if we compare $\kappa_h$ with the benchmark capability whereof there is no credit rationing, $\kappa_{nb}$, i.e., we calculate $\lim_{S \to \infty} \frac{\kappa_h}{\kappa_{nb}}$ and relate it to parameters $\theta$ and $\omega$.

From eq. (127) it follows that

$$
(k_h)^{\beta-2} - \frac{\omega}{(k_h)^2} = \frac{\theta S}{\left( \frac{\beta(k_h)^{\beta}}{2((k_h)^{\beta}-\omega)} + 1 \right)^2},
$$

which, as $k_h \to \infty$, can be approximated as

$$
(k_h)^{\beta-2} \approx \frac{\theta S}{\left( \frac{\beta}{2} + 1 \right)^2},
$$

which has the solution:

$$
\kappa_h \approx \left( \frac{\theta S}{\left( \frac{\beta}{2} + 1 \right)^2} \right)^{1/2},
$$

hence,

$$
\frac{\kappa_h}{\kappa_{nb}} \approx \left( \frac{\theta S}{\left( \frac{\beta}{2} + 1 \right)^2} \right)^{1/2} \to \theta \frac{3}{\beta} \text{ as } S \to \infty
$$

From eq. (126), it follows

$$
N_h = \frac{\beta}{2} + \frac{\omega N_h}{(N_h+1)^2} \left( \frac{N_h}{N_h-\frac{\beta}{2} + \omega} \right)^{2/\beta} \to N_{nb} = \frac{\beta}{2}.
$$

When low capability firms are inactive, industry R&D level equals

$$
N_h \frac{S(k_h)^2}{(N_h+1)^2} \to \theta \frac{3}{\beta} \left( \frac{S}{\left( \frac{\beta}{2} + 1 \right)^2} \right)^{1/2},
$$

which is a fraction $\theta \frac{3}{\beta}$ of the level whereof there is no credit rationing. It does not depend on the level of entrepreneurial wealth. In this circumstance, using eq. (108), the number of low capability firms is given by

$$
N_l = \left( \frac{1}{\phi} - 1 \right) N_h = \left( \frac{1 - \theta (k_h)^{\beta} - \omega}{\theta \omega} - 1 \right) N_h,
$$
which implies that

\[
N_t \rightarrow \frac{\beta (1 - \theta) \theta^{\frac{\omega}{\beta}}}{2 \omega \left(\frac{\theta}{2} + 1\right)^{\frac{3 \theta}{2}}} \text{ as } S \rightarrow \infty.
\]  

(150)

Hence,

\[N_t \rightarrow \infty \text{ as } S \rightarrow \infty\]

and \(N_t\) increases faster (proportionally) than \(S\). Nevertheless, since the low capability firms collective market share is zero, the entry of many low capability firms do not reduce market concentration. Here in this limiting case, the financial functionality (of allocating capital) played by the Type II credit rationing is the most clearly in display: it limits the number of major R&D firms, and allows the size of R&D investments (by the major R&D firms) to increase. The Type II credit rationing occurs in big markets, its impact on the market structure (composition of firms) generates ‘a lot of start-ups, and a very few stars’. Its is an ingenious way of the markets to mitigate (but not yet to eliminate) the problem of under-investment in R&D.

**Proposition 18** When the size of market \(S\) approaches infinity, the high capability level \(\kappa_h\) approaches infinity, but still it is only a fraction \(\theta^{\frac{\omega}{\beta}}\) of the level whereof there is no credit rationing; the number of high capability firms approaches to the numbers of firms whereof there is no credit rationing; the level of industry R&D approaches infinity, but is still only a fraction \(\theta^{\frac{\omega}{\beta}}\) of the level whereof there is no credit rationing.

A consistent moral lesson we can learn from Propositions 8 and 18 is that the industry level of R&D investment increases with the level of external investor protection provided by the corporate governance system.

5 Discussion

**Corporate governance: endogeneity and determinants** The current paper is silent on the detail of the relationship between financial structure, or allocation of property/control rights, and the value of the (external investor protection) parameter \(\theta\) of the model. This position does not suggest that we ignore the potential importance of corporate governance structure on the current topic. On the contrary, we hypothesize that the value of \(\theta\) (a specific corporate governance functionality) could be consistent with the second-best solution of the endogenous variable of corporate governance structure (including financial structure and allocation of property/control rights) of some suitably crafted model. This paper has also been silent on the determinants of the value of \(\theta\). Some main results (e.g., those stated in Propositions 6, 8 and 18) of the current study have been reported as comparative statics with respect to parameter \(\theta\), i.e., we have treated \(\theta\) as an exogenous variable. This theoretical limitation demands necessary caution in drawing conclusions from these
results; they nevertheless strongly suggest an important role of corporate governance (structure and functionality), which deserves more research energy. The fast-growing literature on corporate governance - law and finance (La Porta, de Silanes, Shleifer and Vishny (1998)), suggests that some measures of legal efficiency/quality should be among the determinants of parameter $\theta$.

Selection in the competition for credits: randomness and efficiency

In Section 4, we have assumed that each entrepreneur who proposes the (symmetric equilibrium) offer has an equal chance of winning the competition for rationed acceptance. This assumption of equal opportunity in the an ex ante random selection can be consistent with an efficiency-based deterministic selection rule and an equilibrium outcome which is characterized by ‘survival of the most efficient’. Consider the following example setting which augments our model.

Suppose among potential entrants of the market, there are two types of entrepreneurs: a good type, who can adopt the best practice of corporate governance which leads to the external investor protection parameter being of value $\theta$; a bad type, who can not adopt the best practice of corporate governance, hence can only ensure a much weaker protection to external investors. Only a good entrepreneur is able provide a signal (proof) of adopting the best practice of corporate governance within a finite time period (at stage two). The speed of a good entrepreneur producing the signal (proof) is a random draw from an identical independent distribution; a bad entrepreneur can never produce the signal. In equilibrium, entrepreneurs self-select: only good entrepreneurs enter the market. External investors only accept the offers of the first $N$ entrepreneurs who have produced the signals (proofs) of adopting of the best practice when their offers are identical. This outcome is an equilibrium because no party wants to unilaterally deviate from this outcome. This equilibrium outcome also constitutes an ‘efficiency’-based selection process, which deter bad entrepreneurs from entry and gives good entrepreneurs (who enter) equal opportunities to win the competition for credits ex ante.

6 Conclusion

In this paper we develop a theory of credit rationing in oligopoly. The theory predicts two types of equilibrium credit rationing on R&D investments: ‘credit size rationing’ and ‘credit denial’. This result suggests a novel interpretation for the stylized fact that some small high-tech start-up firms are denied credits while bigger R&D firms are treated more favorably by the capital market. According the currently theory, this kind of discrimination may have an efficiency justification: only by denying credits to an ‘unsatisfied fringe’, is the capital market able to relax the rationing of credit size for some major R&D firms in equilibrium.

This theory thereby predicts a ‘winners take it all’ type of competition for credits among R&D firms. It suggests an important role played by finance and
corporate governance in R&D-product market competition and technological progress. More future research is clearly needed in this area, particularly on, first, the selection process, second, the optimal contractual forms, and third, the optimal allocation of property/control rights, which help mitigate the problem of credit constraints on R&D.

References


