OPTIMAL NEGLIGENCE RULE UNDER LIMITED LIABILITY *

Juan José Ganuza‡and Fernando Gomez‡
Universitat Pompeu Fabra, Barcelona
Ramon Trias Fargas 25-27
08005 Barcelona, Spain

First version: MARCH, 2003. This version: MAY, 2004

ABSTRACT

We study the standard economic model of unilateral accidents, in its simplest form, assuming that the injurers have limited assets. We identify a second-best optimal rule that selects as due care the minimum of first-best care, and a level of care that takes into account the wealth of the injurer. We show that such a rule in fact maximizes the precautionary effort by a potential injurer. The idea is counterintuitive: Being softer on an injurer, in terms of the required level of care, actually improves the incentives to take care when he is potentially insolvent. We extend the basic result to an entire population of potentially insolvent injurers, and find that the optimal general standards of care do depend on wealth, and distribution of income. We also show the conditions for the result that higher income levels in a given society call for higher levels of care for accidents.

KEYWORDS: Accidents, Limited Liability and Negligence Rule,

‡We wish to thank Francesco Parisi, Giuseppe Dari Mattiacci for useful discussions and suggestions. The usual disclaimer applies.

‡Department of Economics. E-mail: juanjo.ganuza@econ.upf.es. Juan-José Ganuza acknowledges the financial support of the Spanish Ministry of Science and Technology under project BEC2000-1026.

‡Department of Law. E-mail: fernando.gomez@dret.upf.es. Fernando Gomez acknowledges the financial support of the Spanish Ministry of Science and Technology under project BJU2001-0936.
1 Introduction

Individuals and firms that may cause accidents harming others do not possess infinite assets to face the resulting tort liabilities. In fact, it is commonly the case that liable tortfeasors are unable to make the victims whole for the entire amount of harm incurred by the latter. The problem of insufficient assets on the part of defendants to pay tort awards has long ago been recognized by legal systems and commentators as a major practical problem. Potential insolvency, or judgement-proofness as is also known in the Law and Economics literature, is a standard argument in legal discourse justifying policies, rules and doctrines in the field of accident Law and regulation. Mandatory insurance in certain activities posing risks to others (driving, environmentally hazardous activities); vicarious liability when agents (kids, employees) have typically less assets than their principals; joint and several liability among tortfeasors are but just examples of the awareness of the influence of limited assets on the functioning of accident Law.

It is true, however, that from a legal perspective potential insolvency is largely perceived as a source of practical concern, as an obstacle to the smooth operation of the machinery of Tort Law, rather than a crucial theoretical issue in the understanding and design of incentives with liability rules\(^1\). It is in fact one of the merits of economically oriented approaches to accident Law to have highlighted the theoretical importance of the judgement-proof problem.

Our paper in fact focuses on how the presence of limited assets on the part of potential injurers should transform the common understanding of the functioning of negligence-based liability rules, as well as the design of such rules. It is already known in the literature since the pioneering contributions from Summers (1983), and Shavell (1986) that with potential insolvency the first best in terms of accident prevention cannot be generally attained using liability rules. Our focus is the negligence rule, which the previous literature has identified as typically superior to strict liability in the basic accident setting with limited wealth of the injurer (Shavell, 1986; Dari

---

\(^1\)For instance, if one looks for the treatment of potential insolvency in one of the most comprehensive treatises of Tort Law in Europe (von Bar 1998) one does not find any specific references to the problem, just int at the practical relevance in several areas of the Law.
Mattiacci and De Geest, 2002; Miceli and Segerson, 2003). We show that, in such a setting, the optimal negligence rules is not the ordinary negligence rule that uses first-best level of care as the legally required due level of care. The second-best optimal rule we identify selects as due care the minimum of first-best care and a level of care that takes into account the wealth of the injurer. We show that such a rule in fact maximizes the precautionary effort by a potential injurer. The idea is counterintuitive: Being softer on an injurer, in terms of the required level of care, actually improves the incentives to take care when he is potentially insolvent.

We use such a modified negligence rule to analyze how to determine due levels of care for an entire population in which there is a potential judgement-proof problem. Contrary to received wisdom in the Law and Economics literature, we show that, disregarding risk-aversion, and redistribution through liability rules when other instruments for redistribution are unavailable, the legally required levels of care do depend on wealth and distribution of income. We also show that, given certain common regularity conditions, the wealthier a given population, the higher the due level of care should be. We believe that the implications of these findings are relevant for legal policy in the field of accidents, and for public policy generally.

Our paper relates to three different strands in the literature. The first is the Law and Economics, and principal-agent literature, dealing with the judgement proof problem. The second is the Law and Economics literature analyzing the role of income in the design of liability rules. Finally, our paper is related to the economic literature of context and all pay auctions. We briefly review all three in section 2. In section 3 we present the basic model in a setting of pure probability accident technology, and characterize the optimal negligence rule for a single potential injurer. In section 4 we extend the basic model to a population of potential injurers and characterize the optimal policy. Section 6 contains a discussion of the implications and concludes. All proofs are relegated to a technical appendix. In an annex we provide the proofs of the robustness of our basic result to alternative specifications of the accident technology.
As we mentioned in the introduction, our paper adds to the already long literature on judgement-proofness. The earliest Law and Economics literature on limited assets and accident Law arises from Summers (1983), and Shavell (1986). They both use a framework characterized by a safety technology in which the choice of care by the potential injurer affects external accident costs only through the probability of the accident, and by a care variable that does not affect the level of assets that can be seized by the Courts in case of the defendant being held liable. They both show that limited assets tend to reduce the incentive to take care both under strict liability and negligence. In the case of strict liability, when the asset constraint is lower than the level of harm, the injurer takes inefficiently low care. In the case of negligence, there is a critical threshold of assets, strictly lower than the level of harm, above which the potential injurer takes efficient care, and below which the latter opts for sub-optimal care. Thus, in some regions of the level of assets they both equally underperform compared to the social optimum, and in other negligence leads to the socially efficient level of care when strict liability does not. From here they conclude that negligence should be preferred over strict liability when, ceteris paribus, limited assets on the part of the potential injurer are an issue. Shavell extended the analysis to the choice of activity levels and liability insurance coverage. They both discuss some legal policy instruments related to the problem, such as awarding punitive damages and mandating or prohibiting the purchase of liability insurance to engage in potentially harmful activities.

This early contributions have been expanded upon along several lines, both in terms of the accident setting explored and the alternative legal rules and policies subject to analysis. Beard (1990) considered monetary care expenditures, that is, investments in precautionary measures that reduce the assets of the potential injurer available to pay a damage award in case the accident materializes. In this setting, under-investment in precaution need not necessarily result from limited wealth and a strict liability rule (the only rule considered by Beard) of the potential injurer, because he may have an incentive to increase the expenditures on care in order to reduce
the assets that the victim will be able to appropriate to obtain compensation for harm incurred.

Boyd and Ingberman (1994) extend the analysis of judgement-proof problems to alternative precaution and accident technologies. In addition to the standard pure probability technology, they consider safety measures that reduce the size of the accidental loss (pure magnitude technology), and safety measures that affect both the probability and the size of the loss (joint probability-magnitude technology). In the first and third scenarios (pure probability and joint probability-magnitude) they suggest supracompensatory -punitive- damages as a potential remedy for the inefficiently low incentives to adopt precaution, whereas in the second (pure magnitude) infracompensatory -capped- damages is proposed as a legal alternative conducive to more efficient care choices by potential injurers.

Dari Mattiacci and De Geest (2002) consider a fourth accident technology, which they label the separate-probability-magnitude model. In this setting, the injurer can take two different precautions, one solely affecting the probability of the accident, the other solely reducing the size of the loss. Strict liability induces for this technology either optimal precaution above a high threshold of assets, zero magnitude-reducing precaution and over or optimal probability-reducing precaution above an intermediate threshold, and zero magnitude-reducing precaution and sub-optimal probability-reducing precaution otherwise. Moreover, they also revisit the use of negligence rules, punitive damages, and under-compensation, and show the superiority of average (as distinct from actual) damages over punitive damages in the pure probability technology.

De Geest and Dari Mattiacci (2002), in turn, address the combined use of regulatory standards and liability rules when the potential injurer has limited assets, and show that under certain conditions the Government, through the use of mandated standards of care below the optimal ones, plus tort liability, is able to overcome the judgement-proof problem. This results holds, however, solely for the pure magnitude and the joint-probability-magnitude technologies, and require the potential injurer to be solvent at the optimal level of care.

Innes (1999) adds stochastic harms and asymmetric information to the setting, and analyzes a harm-contingent strict liability rule. Miceli and Segerson (2003) include litigation costs in the
analysis. They show, both for monetary and non-monetary precautions, how strict liability and negligence perform when the injurer has limited assets when compensating the victim implies costly litigation for both parties.

Others have extended the initial analysis by Shavell (1986) on legal policies regarding liability insurance when judgement-proof is a potential problem. Jost (1996), Polborn (1998), and Shavell (2000), determine optimal conditions for the requirement to purchase liability insurance prior to engaging in a risky activity. Shavell (2002) analyzes a parallel option open for the legal policy-maker, namely the legal requirement to possess a minimum amount of assets to undertake a given activity. Finally, Shavell (2004) determines the optimal conditions for the combined use of both instruments for a given activity, and makes an efficiency comparison between the joint use of compulsory liability insurance and minimum asset requirement, and the latter alone.

Pitchford (1995), along the lines of the pioneering work by Kornhauser (1982), opens a somewhat different strand of the literature dealing with judgement-proofness. He considers the extension of liability to parties other than the injurer, typically a lender who contributes capital to the activity resulting in external harm. The problem of extended liability is particularly relevant in the field of environmental externalities, in which the Law in several jurisdictions (the CERCLA legislation in the US, as paramount example) makes a definite extension of liability for cleanup costs to persons different from the material injurer (later possessors of contaminated soils, lenders, parent companies). The conclusion is that extending liability to lenders of capital to an environmentally risky undertaking, through increased lending rates, makes the no-accident state less attractive to potential injurers, thus increasing the probability of environmental accidents.

Boyd and Ingberman (1997) also conclude that extending liability to third parties transacting with the potential injurer may create inefficiencies in the capital and output choices of those third parties, as well as distortions in the choice of transacting parties (bigger instead of smaller or more specialized firms). Boyer and Laffont (1997) show that under complete information, the extension of liability to lenders induces the adequate internalization of external harms. Relationships between lenders and borrowers engaged in risky activities are subject to typical agency problems,
and when moral hazard and adverse selection are considered, neither full extended liability nor
denial of extended liability are able to implement the second best. Partial liability may be superior
but fails to achieve the second best under all conditions.

Lewis and Sappington (1999, 2001) show that when the accident technology is not binary (i.e., either a deterministic harm or zero harm materialize) the use of decoupled (a damage award
different from actual harm) and assets from the lender, depending on the kind of realization of
actual harm, increase the incentives of the potential injurer to adopt precautionary measures.
Balkenborg (2001) also argues in favor of the extension of liability, based on the key role played
by the bargaining power of the lender. When, in the lender-borrower contract, the lender has
bargaining power above some threshold, full (even punitive) lender liability is effective to induce
the potential injurer to exert optimal care to avoid external harm.

Feess (1999) incorporates the analysis of monitoring levels of potential injurers by lenders, and
compares the efficiency consequences of three regimes of lender liability: Pure strict liability, strict
liability with infracompensatory damages, and vague negligence, showing the superiority of the
second regime both for precautionary incentives and for monitoring levels. Fees and Hege (2003)
argue that mandatory liability insurance or full financial liability is superior to lender liability
and to lower forms of liability, when the insurance company can combine stochastic monitoring
with payment transfers to the potential injurer.

Hiriart and Martimort (2003) revisit the issue of third-party or lender liability in a principal-
agent setting, and argue that the extension of liability towards deep-pocket related third parties
may play a beneficial role. Moral hazard and bargaining power of the principal, on the one
side, and adverse selection in which distortions have to be corrected with monetary transfers
that might exceed the agent’s wealth, make extended liability a valuable instrument of the social
policy-maker. Finally, Boyer and Porrini (2004) compare extended liability and regulation in a
very similar setting.

The second strand of the literature related to our paper refers to the role of injurers’ wealth
in the definition of optimal standards of care. Arlen (1992) argues that the irrelevance of care
for setting optimal standards of care is true only for risk neutral individuals. When potential
injurers and victims are risk-averse, even if fully insured, the optimal level of care is increasing on
the injurer’s wealth. But this is true only for reasons of distribution when other instruments for
redistribution are unavailable: Subsidies, or even damage payments at the optimal level of care, are
zero. Miceli and Segerson (1995) reexamine the issue and conclude that liability rules abstracting
of the levels of wealth of the individuals involved can induce efficient care levels and adequate
redistribution of income under many conditions, and that the results by Arlen (1992) are highly
dependent on particular income distribution objectives favoring victims, and the unavailability of
other instruments for redistribution.

Shavell (1981), and Kaplow and Shavell (1994) also deal with the relationship between liability
rules and distribution of income, arguing that the use of liability rules for distributional purposes
is inferior to the use of taxes and subsidies to the less well-off. The core of the argument is as
follows: The use of taxes and transfers as redistributitional mechanisms just creates a distorsion,
namely in the work-leisure trade-off. Liability legal rules generate a double distorsion. One, the
same we have just described for taxes, the other, the inefficiency generated by a legal rule chosen
not on its efficiency merits, but on its redistributional effectiveness. Jolls (2000), and Sanchirico
(2000), have criticized the double distorsion argument, based on the absence of the first distorsion
with liability rules, on the one side, and on the dependence of the result on an implicit assumption
of homogeneity of individuals with respect to care efforts, on the other.

Finally, our core result, the fact that the optimal negligence rule requires lower level of care
to injurers with lower assets, has a similar flavor to some known results in the economic literature
of context and all pay auctions. Laffont and Robert (1996) state that an all-pay auction with a
reserve price is an optimal mechanism for selling a good to bidders that face a common budget
constraint. In the same vein to the present paper, they show that the optimal reserve price for
financially constrained bidders is lower than the one without constraints. Che and Gale (1998) and
Gavious, Moldovanu and Sela (2000) study context and all pay auctions , and provide conditions
under which, the sponsor can improve his outcome by introducing a price cap. This result is a
rationale for explaining why introducing salary caps in NBA (constraints on the sums of money teams are allowed to spend on salaries) can, in fact, enhance competition.

3 The basic model

We study the standard unilateral accident setting in which the behavior of an injurer\(^2\) affects the likelihood of an accident. Let \(C(x)\) be the injurer cost of the precaution effort \(x\). We assume \(C(0) = 0, \frac{\partial C(x)}{\partial x} > 0,\) and \(\frac{\partial^2 C(x)}{\partial x^2} > 0\). While the harm resulting from the accident is, \(D\), the injurer wealth is lower than this harm \(l < D\). The probability of accident depends on the injurer precaution effort \(x\), \(p(x)\). We assume \(\frac{\partial p(x)}{\partial x} < 0, \frac{\partial^2 p(x)}{\partial x^2} > 0\). Finally, we also assume that the legal system regulates behavior through the use of negligence rules implemented by Courts\(^3\).

3.1 First best solution

We start by characterizing the first best solution. Let \(x^*\) be the first best solution of the injurer precautionary effort. \(x^*\) is the solution of the following problem.

\[
x^* \in \arg \max -pD(x) - C(x)
\]

The next step is implementing this first best solution using the negligence rule. We are going to show, following Shavell (1986) and related literature, that if the injurer is protected by limited liability because his assets \(l\) are lower than \(D\), a negligence rule may not implement the first best solution. Typically, the negligence rule sets a single required level of precaution effort \(\hat{x}\), which coincides with the first best solution \(x^*\). Then, the negligence rule determines that the injurer has to pay damages equal to \(D\) if an accident materializes and the precautionary effort of the injurer is lower than \(\hat{x}\).

\(^2\)One could also think of a population of homogeneous injurers, or a population of injurers heterogeneous in terms of assets, but the level of assets being perfectly verifiable ex-post by the Court determining the level of legally required care.

\(^3\)One could also think of public regulatory standards in which enforcement comes through ex post monetary sanctions related to social harm, and not through ex ante injunctions or other preventive measures.
Lemma 1 The injurer exerts a precautionary effort of $x^*$ if $C(x^*) < p(x)l + C(x)$, where $x \in \arg\max -p(x)l - C(x)$. Otherwise, the injurer exerts precautionary effort of $\hat{x}$ which is lower than $x^*$.

Let $x^{**}$ be a level of precautionary effort such that $C(x^{**}) = p(\hat{x})l + C(\hat{x})$. Using $x^{**}$ we can rewrite the result of Lemma 1 as following: the injurer exerts a precautionary effort of $x^*$ if and only if $x^* < x^{**}$, and $\hat{x}$ otherwise. Then, the next lemma serves to characterize the negligence rule that maximizes the precautionary effort of a potential injurer with limited assets.

Lemma 2 If the injurer faces a negligence rule in which, $\hat{x} = x^{**}$, he will find optimal to exerts the level of precautionary effort $x^{**}$. This negligence rule maximizes the effort exerted by the injurer.

Notice an important implication of the previous lemma. Contrary to intuition, if $x^* > x^{**}$, by reducing the requirement of precautionary effort from $x^*$ to $x^{**}$, we in fact increase the precautionary effort exerted by the injurer from $\hat{x}$ to $x^{**}$. Then the next proposition characterizes the optimal negligence rule.

Proposition 1 The optimal negligence rule must set a precautionary effort of $\tilde{x}$, where $\tilde{x} = \min\{x^*, x^{**}\}$. There is a level of injurer’s assets $l^* < D$ such that, if and only if $l > l^*$ the first best solution can be implemented.

To illustrate the proposition, consider the following example.

Example 1 We use the following functions: $p(x) = 1 - x$ and $C(x) = -\ln(1-x)$. Then, the first best solution of the problem is $x^* = 1 - \frac{D}{D^*}$, $\hat{x} = \frac{D}{l^*}$, $x^{**} = \frac{l^*}{l}$ and $l^* = \frac{D}{e}$. Then, if $l > \frac{D}{e}$ the first best solution is feasible, and the optimal negligence rule is to set $\tilde{x} = x^* = 1 - \frac{D}{D^*}$, while that if $l < \frac{D}{e}$ the first best solution is not feasible, and the optimal negligence rule is to set $\tilde{x} = x^{**} = \frac{l^*}{l}$.

The optimal negligence rule in the presence of potential insolvency of the injurer is a different rule that the one commonly considered by the literature, that is, the one that sets due care at the first-best optimal level of care, $x^*$. The modified negligence rule that we identify sets due care
at the minimum of first-best care, on the one hand, and the maximum level of care that can be implemented with a judgement-proof injurer, on the other. This modified negligence rule, in a setting of potential insolvency, always outperforms the standard negligence rule. The result that we have shown in this section for a pure probability accident technology is robust for the rest of single-dimension accident technologies previously analyzed in the literature. The appendix contains the proofs that Proposition 1 also holds for the pure magnitude, joint probability-magnitude, and monetary care versions of accident technology.

Our modified negligence rule depends on the level of wealth of the potential injurer. In fact, we have shown that optimal required level of care is non decreasing in the level of wealth of the injurer. But note that, contrary to previous literature [Arlen (1992), Miceli and Segerson (1995)], the underlying rationale has nothing to do with concave utility functions, risk aversion and redistribution, but is a general result based on a pure incentive effect.

4 Setting a general standard of care for populations with limited assets

The result of the basic model refers to a single injurer, a homogeneous population of injurers, or a heterogeneous population in which due care can be optimally set by Courts ex post accident for each injurer based on perfect verification of the level of assets. In this section we extend the result and our modified negligence rule to an entire population of potentially insolvent injurers, given that the level of legally required care cannot be optimally set ex post accident. We can think of a rule requiring a fixed level of care enacted in legislative Statutes or in public agencies regulations and standards. In fact, many public regulations of activities (from environmental to employment hazards or motor driving) use negligence type of rules, and enforce them mainly through monetary sanctions and/or damage payments. In this section, thus, we are going to consider the same accident setting and we will characterize the optimal standard for this negligence-type of rules and regulations when injurers within given populations may be protected by their limited assets.

Consider that the wealth of the populations are distributed according to the distribution function $F_{\theta_i}(l)$ over $[l_{\text{min}}, l_{\text{max}}]$. Where $\theta_i$ is a measure of the total wealth of the population.
If \( \theta_i > \theta_j \), the population \( \theta_i \) is wealthier than population \( \theta_j \) in the first order stochastic sense, i.e \( F_{\theta_i}(l) \leq F_{\theta_j}(l) \) for all \( l \in [l_{\text{min}}, l_{\text{max}}] \). Notice that this implies that the average wealth is larger or equal in population \( \theta_i, l_{\text{mean}}(\theta_i) \geq l_{\text{mean}}(\theta_j) \).

Let \( x(\theta_i) \) be the standard of care set for population \( \theta_i \). The injurer of wealth \( l \) behaves as we have describe in the previous section, so he exerts an level of care equal to

\[
x(l, x(\theta_i)) = \begin{cases} x(\theta_i) & \text{if } x(\theta_i) < x^{**}(l) \\ \hat{x}(l) & \text{otherwise} \end{cases}
\]

The expected cost of an injurer with wealth \( l \), when the standard is \( x(\theta_i) \) is equal to

\[
U(x(\theta_i), l) = \begin{cases} -p(x(\theta_i))D - C(x(\theta_i)) & \text{if } x(\theta_i) < x^{**}(l) \\ -p(\hat{x}(l))D - C(\hat{x}(l)) & \text{otherwise} \end{cases}
\]

Let \( V(\theta_i, x(\theta_i)) \) be the expected welfare of population \( \theta_i \) with the standard \( x(\theta_i) \), it is equal to

\[
V(\theta_i, x(\theta_i)) = \int_{l_{\text{min}}}^{l_{\text{max}}} U(x(\theta_i), l) dF_{\theta_i}(l)
\]

Let \( x(\theta_i)^* \) is the optimal standard for population \( \theta_i \) and it is characterized as

\[
x(\theta_i)^* \in \text{arg max } V(\theta_i, x(\theta_i))
\]

The next proposition characterized the relationship between the expected welfare achieved by a populations and its wealth.

**Proposition 2** If \( \theta_i > \theta_j \) then \( V(\theta_i, x(\theta_i)^*) \geq V(\theta_j, x(\theta_j)^*) \).

Thus, when a population is wealthier than another, in the sense that we can order their wealth distribution according to the first order stochastic sense, then the wealthier population achieves larger welfare than the other.

A very important policy question is the relationship between the level of wealth of a population and the optimal standard of care. Building upon the result of the previous section, we could expect that if an optimally required level of care is non decreasing in the level of wealth of the injurer, we can extend this result to populations, and conclude that wealthier populations should set higher
standards of care, i.e. if \( \theta_i > \theta_j \) then \( x(\theta_i)^* \geq x(\theta_j)^* \). However, the next proposition states that, contrary to the intuition first order stochastic dominance of wealth distribution, is not a sufficient condition for setting lower standards.

**Proposition 3** \( \theta_i > \theta_j \not\Rightarrow x(\theta_i)^* \geq x(\theta_j)^* \)

Although in the appendix we provide some technical reasons for this negative result, we will here prove it by means of a counter example, a situation in which \( \theta_i > \theta_j \) and \( x(\theta_i)^* < x(\theta_j)^* \).

**Example 2** We use the same functions of the example 1: \( p(x) = 1 - x \) and \( C(x) = -\ln(1 - x) \). Consider that there are three types of injurers, \( l_i \in \{l_1, l_2, l_3\} \) with \( l_1 < l_2 < l_3 \), and two populations, \( \theta_1 \) and \( \theta_2 \), with a distribution functions over the type space characterized by the following densities, \( dF_{\theta_1} = \{0.05, 0.7, 0.25\} \) and \( dF_{\theta_2} = \{0.74, 0.01, 0.25\} \). Notice that \( F_{\theta_2}(l) \leq F_{\theta_2}(l) \) for all \( l_i \). We take the following value for the parameters, \( \{l_1, l_2, l_3\} = \{0.4, 2.4\} \), \( D = 10 \). Remember, for every type if the required level of care is larger than \( x^*(l) = \frac{l-1}{l} \), he exerts \( \hat{x}(l) = \frac{l-1}{l} \). Otherwise, the injurer exerts the care required. Given this simple rule, it is clear que the optimal standard must be \( x(\theta_j)^* \in \{x^*(l_1), x^*(l_2), x^*(l_3)\} \). For the population \( \theta_1 \), \( (l_{\text{mean}}(\theta_1) = 2.42) \), the optimal standard is \( x(\theta_1)^* = x^*(l_2) = 0.8161 \).

\[
V(\theta_1, x(l_2)^*) = -3.8559 > V(\theta_1, x(l_3)^*) = -5.3117 > V(\theta_1, x(l_1)^*) = -9.2807
\]

For the population \( \theta_2 \), \( (l_{\text{mean}}(\theta_2) = 1.316) \), the optimal standard is \( x(\theta_3)^* = x^*(l_3) = 1.3160 \).

\[
V(\theta_2, x(l_3)^*) = -8.2826 > V(\theta_2, x(l_2)^*) = -8.3185 > V(\theta_2, x(l_1)^*) = -9.2807
\]

You can notice, that \( V(\theta_1, \hat{x}) \geq V(\theta_j, \hat{x}) \) however the standard of the wealthier populations is lower that the optimal one of the poorer populations, \( x(\theta_1)^* = x^*(l_2) = 0.8161 < x(\theta_2)^* = x^*(l_3) = 0.9080 \).

\(^4\)Evidently, a reason for a wealthier society to have higher standards of care than a poorer one could be simply that social harm is higher in a richer society, given that potential victims incur higher losses when they suffer an accident. In our model, the level of social harm, \( D \), is common to both populations. Sunstein (2004) considers this issue of the value of a statistical life depending on the level of wealth.
The intuition behind this counter-example is the following. In the first population, there are many injurers of the intermediate type, and this leads to set the standard that maximizes the effort exerted by this type. On the other hand, in the poorer population, there are almost no potential injurers of the intermediate type, and the choice is basically between the lowest or the highest standard, and setting the lowest standard seems to be very costly.

Thus, first order stochastic dominance is not enough information about populations for the purposes of setting larger or lower general standards of care. In others words, we can have for wealthier populations lower optimal legal requirements of care than for poorer populations. The level of income is important, but the shape of the distribution matters. Note that we have shown that both levels of wealth and distribution of income in a population matter for the efficient choice of legal rules on care. And the source of the influence of wealth and income distribution on liability rules is wholly independent of the grounds identified in the previous literature, namely risk-aversion, and redistribution when other redistributive instruments are lacking.

We have identified, however, a sufficient condition that guarantees, that optimal standards of care should be higher in a wealthier population than in a poorer one.

**Definition 1** The population $\theta_i$ is wealthier than population $\theta_j$ in term of the likelihood ratio, $\theta_i \succ LR \theta_j$, if $f_{\theta_i}(x) \leq f_{\theta_j}(y) \frac{f_{\theta_i}(y)}{f_{\theta_j}(y)}$ for all $x < y \in [l_{\min}, l_{\max}]$.

This condition implies first order stochastic dominance, and also implies some regularity, that the density functions $f_{\theta_i}(x)$ and $f_{\theta_j}(x)$ “cross” only once. In our model, the interpretation of this condition is that, when we compare two conditional income distributions (we condition on the fact that the assets are larger than a threshold), the order asserts that we expect higher expected income from the wealthier distribution among the pair.

**Theorem 1** Assume that population $\theta_i$ is wealthier than population $\theta_j$ in term of the likelihood ratio, then the optimal standard for population $\theta_i$ is larger than the optimal standard for population $\theta_j$.

$$\theta_i \succ LR \theta_j \implies x(\theta_i)^* \geq x(\theta_j)^*$$
We have thus shown that, under some technical conditions, a wealthier population of potential injurers with limited assets should be subject to a higher general standard of due care than a poorer population. Our result implies that mandated levels of care, when the sources of harm are potentially insolvent, should be sensitive to the level of wealth and the distribution of income in populations. This result seems intuitively confirmed by the observation that wealthier societies require more stringent levels of precaution for a given kind of activity than poorer societies.

Example 3 We use the same functions of the example 1: \( p(x) = 1 - x \) and \( C(x) = -\ln(1 - x) \). Consider the following values of the parameters, \( D = 40 \), and the wealth of the population is distributed according to the distribution function \( F_a(l) = (l/10)^a \) over \([0, 10]\). Where \( a \in \{1, 2, ..., 10\} \) is a parameter that order the wealth of the populations according with the likelihood ratio, if \( a_1 > a_2 \) then \( \frac{dF_{a_1}}{dF_{a_2}} = \frac{a_1((l/10)^{a_1} - 1)}{a_2((l/10)^{a_2} - 1)} = \frac{a_1}{a_2} (l/10)^{a_1-a_2} \) is a increasing function of \( l \). The next figure show us how the optimal standard depends positively on the parameter \( a \) (level of wealth of the population).

[Figur 1 around here.]

5 Discussion and conclusions

It is a fact of life that many individuals and firms engaging in activities that may cause harm to others do not have enough assets to face all the resulting liabilities. This reality does not only leave some injured victims uncompensated, but reduces the incentives for safety efforts created by Tort Law and public regulation enforced through monetary sanctions. The Law and economics literature on judgement-proofness has been exploring the effects, extent and potential remedies to this unwelcome disturbance in the liability system. The solution we have identified and analyzed in this paper is as simple as counterintuitive. When the level of assets on the part of the injurer is exogenous, and a negligence rule is chosen by the legal system to regulate conduct in a certain area of potentially harmful behavior, the ordinary negligence rule setting due care at the first-best optimum is not the optimal negligence rule. A modified rule that takes into account the actual
level of assets of the potential injurer, although apparently "softer" on judgement-proof injurers, in fact increases the level of care that these are induced to take. In fact, we show that the rule we identify maximizes the precautionary effort of potentially insolvent liable parties. The intuition behind our result is straightforward. The negligence rule entails an implicit subsidy to the injurer over some range of the care function, given that at the optimal level of care the injurer solely faces the costs of care, but harm is borne by the victim. When limited assets reduce the incentive to take care, expanding the range along the care function in which the potential injurer receives the subsidy improves the incentive to take care under the ordinary, unmodified negligence rule. We have also shown that the modified negligence rule we identify improves the incentives for care of judgement-proof injurers over all precaution technologies, monetary and non-monetary, in which the probability of the accident, the size of the harm, or both, depend on a single care variable, thus covering most accident settings previously analyzed in the literature.

Our basic analysis explores a setting of a single injurer, a homogeneous population of injurers, or a heterogeneous population in which due care can be optimally set by Courts ex post accident for each injurer, if his level of assets is perfectly verifiable by Courts. Many, if not most, standards of care in the real world are not determined piecemeal for an individual injurer, but are determined as general rules by legislatures, or regulatory agencies, for an entire population of potential injurers. We carry over our analysis to this general setting, and observe that the use of our modified negligence rule also has important implications for the task of general rule setting in the field of care. First, that levels of wealth, but also the distribution of income in a population, have a direct impact on the optimal choice of the standard of care for the population. Second, that subject to certain regularity conditions, the standards of care that should be optimally required are increasing in the level of wealth of a population.

Our analysis does not only involve theoretical points. We think it might shed some light on existing rules or future developments in public policy, including legal policy, towards accidents. We do not dispute that our modified negligence rule does not work when the level of assets can be altered or manipulated by the potential injurer. Our rule, in such a setting, would then give
incentives to organize the risky activity with less assets. It is true that firms, and to a lesser extent, individuals, can sometimes increase or reduce the assets involved in a certain activity or undertaking: Outsourcing of risky activities, under-capitalization of subsidiaries, instrumental limited liability entities, shadow owners, and so on. And they can do this even in the presence of measures such as minimum asset requirements, or mandatory liability insurance. But we believe that our analysis can be of some relevance in several areas of accident Law and in for several policy tasks.

First, the vast majority of harms caused by individuals in everyday activities (walking, jogging, shopping, gardening, cycling, etc.) are governed by the negligence rule, which is the one we take as our starting point. Moreover, these activities pose very little risk of strategic limitation of assets by the potential injurers. The use of our modified negligence rule may affect positively the levels of care that potentially insolvent injurers currently adopt in those or similar activities. We believe, thus, that this is an ideal setting for the implementation of the modified negligence rule we have identified and described in the paper.

Second, our analysis gives additional support to the general attitude of the Law in most jurisdictions, reducing the levels of care required from some categories of vulnerable persons, such as children, and the mentally handicapped. Given that these groups typically have much lower assets than adults or non-handicapped citizens, the reduced levels of care make additional sense, that is, they may not be based solely in the higher costs of precautionary effort faced by these specially vulnerable persons. Moreover, it explains why some legal systems (Germany, Italy) contain rules making minors and handicapped persons liable for the harm done precisely when their level of assets is sufficiently high, and when an otherwise identical injurer with lower or no assets would not be held liable.

Third, the part of our analysis that refers to the determination of standards for an entire population would imply a general reconsideration of the levels of care required by Courts under

---

5 Although legal systems are often able to detect and sanction such manipulative behavior as such, at least in the extreme cases.

6 See on this rationale for the reduced levels of care, Ganuza and Gomez (2002).
certain circumstances, such as an exogenous shock to an economy, that reduces the level of assets of almost any agent in that economy. Think, for example, of the 2002 economic downturn in Argentina, that produced an overall sink in the levels of wealth of individuals and firms. In order to increase the incentives to take care, Courts should adjust the levels of care downwards so as to take into account the reduced levels of wealth of all potential injurers.

More generally, the implications of our analysis of optimal standards of care for populations with problems of limited assets suggest that the design of general rules on care and precaution should be more sensitive to wealth and distribution issues than is generally recognized. The reason for this lying not in redistributive goals, but in pure incentive motivations to increase precautionary efforts by the potential injurers. For instance, when adopting safety standards, local conditions of wealth and distribution matter, and more adaptive levels of required care may induce higher levels of precaution, and lower accident rates, than more stringent standards.

A Appendix

Proof of Lemma 1: We denote by $x$ the precaution effort of the injurer. There are two cases:

1. First, we consider that $x \geq x^*$. In this case, the injurer is not liable and consequently he has not to compensate the victim for any harm. Therefore, the injurer will never choose a care effort larger than $x^*$. Therefore, in this case the injurer exerts a precautionary effort of $x^*$ and incurs in a cost $C(x^*)$.

2. Assume now that the injurer chooses $x < x^*$. In this case, the injurer is liable for the amount $l$ of assets, and consequently he would choose a precaution effort that maximizes $-p(x)l - C(x)$. Let $\hat{x}$ be the solution of this maximization problem. It is clear that $\hat{x}$ is always lower of $x^*$. Formally, $x^*$ satisfies $-\frac{C'(x^*)}{p'(x^*)} = D$, while $\hat{x}$ satisfies $\frac{-C'(\hat{x})}{p'(\hat{x})} = l$. Given that $\frac{C''(x)}{p'(x)}$ is increasing on $x$ ($-\frac{C''(x)p'(x) + C'(x)p''(x)}{p'(x)^2} > 0$) and $D > l$, $x^*$ is larger than $\hat{x}$. Therefore, in this case the injurer exerts a precautionary effort of $\hat{x}$ and his utility is $-p(\hat{x})l - C(\hat{x})$. 

17
Finally, the injurer prefer the case 1 to the case 2 if and only if $C(x^*)$ is lower than $p(x)l + C(\hat{x})$.

**Proof of Lemma 2:** Immediate from Lemma 1.

**Proof of Proposition 1:** (i) If $\min\{x^*, x^{**}\} = x^*$, then by lemma 1, the first best solution is implemented. (ii) If $\min\{x^*, x^{**}\} = x^{**}$, $x^{**}$ is the maximum level of precautionary effort that can be implemented, as the social welfare function is concave, $-p(x)D - C(x)$, and the first best solution is not feasible, the maximum of the feasible levels of effort must be the optimal constrained optimum. Finally, notice that by definition $x^{**}$ is increasing in $l$, and that on one hand when $l \geq D$, $x^{**} > x^*$ and that if $l$ is 0, necessarily $x^{**} = 0$ and is lower than $x^*$.

**Proof of Proposition 2:** For a given standard equal or below the first best solution, $U(x, \min\{x, x^{**}\})$ is a non decreasing function of $l$. Then, given that $\theta_i > \theta_j$, implies first order stochastic dominance, i.e $F_{\theta_i}(l) \leq F_{\theta_j}(l)$ for all $l \in [l_{\min}, l_{\max}]$. Then, $V(\theta_i, \tilde{x}) = \int_{l_{\min}}^{l_{\max}} U(\tilde{x}, l) dF_{\theta_i}(l) \geq \int_{l_{\min}}^{l_{\max}} U(\tilde{x}, l) dF_{\theta_j}(l) = V(\theta_j, \tilde{x})$. This is because $V(\theta_i, \tilde{x}) - V(\theta_j, \tilde{x}) = \int_{l_{\min}}^{l_{\max}} U(\tilde{x}, l)(dF_{\theta_i}(l) - dF_{\theta_j}(l))dl$, then integrating by parts we obtain, $\int_{l_{\min}}^{l_{\max}} U'(\tilde{x}, l)(F_{\theta_i}(l) - F_{\theta_j}(l))dl \geq 0$, because $F_{\theta_i}(l) \leq F_{\theta_j}(l)$ and $U'(\tilde{x}, l) \geq 0$. We conclude with the inequalities

$$V(\theta_j, x(\theta_j)^*) \leq V(\theta_i, x(\theta_i)^*) \leq V(\theta_i, x(\theta_i)^*)$$

the first inequality follows from $V(\theta_i, \tilde{x}) \geq V(\theta_j, \tilde{x})$, and the second is due to the fact that $x(\theta_i)^*$ is the optimal standard for population $\theta_i$.

**Proof of Proposition 3:** Although, the counter example given in the main text is a good proof of proposition, here we want to give a mathematical feature of our problem which is behind of this negative result. Following to Milgrom and Shanon (1994), in problems in which $x(\theta) \in \arg\max \int H(x, s)f(s, \theta)ds$, where $\theta$ order distributions according to the first order stochastic sense, if $H(x, s)$ is supermodular, then $x(\theta)$ is non decreasing. However, in our case $U(\tilde{x}, l)$ is not supermodular.

**Lemma 3** $U(\tilde{x}, l)$ is not supermodular in $\{\tilde{x}, l\}$, in other words, if $l_1 < l_2$, $U(x, l_2) - U(x, l_1)$ is increasing on $x$. 
Proof: To simplify the notation, let \( k(x) \) be equal to \(-p(x)D - C(x)\). For definiton, \( k(x) \) is a increasing function over \([0, x^*]\).

\[
U(x, l_2) - U(x, l_1) = \begin{cases} 
0 & \text{if } x \leq x^{**}(l_1) \\
k(x) - k(x^{**}(l_1)) & \text{if } x^{**}(l_1) < x \leq x^{**}(l_2) \\
(k^{**}(l_2)) - k(x^{**}(l_1)) & \text{if } x > x^{**}(l_2)
\end{cases}
\]

\( U(x, l_2) - U(x, l_1) \) is decreasing on \( x = x^{**}(l_2) \), since \( k(x^{**}(l_2)) - k(x(l_1)) > k(x(l_2)) - k(x(l_1)) \). \( \square \)

Proof of Theorem 1: Assume, contrary to the statement, that \( \theta_i >_{LR} \theta_j \) and that \( x(\theta_i)^* < x(\theta_j)^* \). For definition of \( x(\theta_i)^* \) and \( x(\theta_j)^* \)

\[
V(\theta_j, x(\theta_j)^*) \geq V(\theta_j, x(\theta_i)^*)
\]

\[
\int_{l_{\min}}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))dF_{\theta_i}(l) \, dl \geq 0
\]

and

\[
V(\theta_i, x(\theta_i)^*) \geq V(\theta_i, x(\theta_j)^*)
\]

\[
\int_{l_{\min}}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))dF_{\theta_i}(l) \, dl \leq 0
\]

Now, we analyze the difference, \( U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l) \), when \( x^* \geq x(\theta_j)^* > x(\theta_i)^* \). To simplify the notation, let \( k(x) \) be equal to \(-p(x)D - C(x)\). For definiton, \( k(x) \) is a increasing function over \([0, x^*]\). Let \( l_i^* \) be, such that \( x(\theta_i)^* = x^*(l_i^*) \), and \( x(\theta_j)^* = x^*(l_j^*) \). Given that we are assuming that \( x(\theta_i)^* < x(\theta_j)^* \), and \( x^*(l) \) is increasing, \( l_i^* < l_j^* \).

\[
U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l) = \begin{cases} 
0 & \text{if } l < l_i^* \\
k(x(l)) - k(x^*(l_i^*)) & \text{if } l_i^* \leq l < l_j^* \\
k(x^*(l_j^*)) - k(x^*(l_i^*)) & \text{if } l \geq l_j^*
\end{cases}
\]

Notice if \( l_i^* < l \leq l_j^* \), the function is negative at \( l_i^* \), increasing for \( l_i^* \leq l < l_j^* \), in the limit of the interval, we do not know the sign \( k(x(l_i^*)) - k(x^*(l_i^*)) \geq 0 \). Finally, if \( l \geq l_j^* \), the function is positive \( k(x^*(l_j^*)) - k(x^*(l_i^*)) > 0 \). Then, \( U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l) \) is increasing for \( l \geq l_i^* \) and 0 for \( l < l_i^* \).
Given that \( \theta_i \succ_{LR} \theta_j \), \( \frac{dF_{\theta_i}(l)}{dF_{\theta_j}(l)} \) is increasing. Let \( h(l) \) be equal to \( \frac{dF_{\theta_i}(l)}{dF_{\theta_j}(l)} \), then \( h(l) \) is increasing and \( dF_{\theta_i}(l) = h(l)dF_{\theta_j}(l) \). Then,

\[
\int_{l_{\min}}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))h(l)dF_{\theta_j}(l)dl = \int_{l_{\min}}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))h(l)dF_{\theta_j}(l)dl =
\]

\[
\int_{l_{\min}}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))h(l)dF_{\theta_j}(l)dl = \int_{l_{\min}}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))h(l)dF_{\theta_j}(l)dl =
\]

Now, let \( l^R \) be the minimum level of assets such that \( (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l)) \geq 0 \). Given the shape of \( (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l)) \), for \( l < l^R \) the function is negative and for \( l > l^R \) is positive.

Then,

\[
\int_{l_i}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))h(l)dF_{\theta_j}(l)dl = \int_{l_i}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))h(l)dF_{\theta_j}(l)dl + \int_{l_i}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))h(l)dF_{\theta_j}(l)dl
\]

Now, given that \( h(l) \) is increasing, and the first term is negative, then

\[
\int_{l_i}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))h(l)dF_{\theta_j}(l)dl \geq h(l^R) \int_{l_i}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))dF_{\theta_j}(l)dl
\]

Similarly, we obtain that

\[
\int_{l_i}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))h(l)dF_{\theta_j}(l)dl \geq h(l^R) \int_{l_i}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))dF_{\theta_j}(l)dl
\]

Combining both expressions, we obtain that given that \( U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l) \) is increasing for \( l \geq l_i^* \) and 0 for \( l < l_i^* \), and that \( \theta_i \succ_{LR} \theta_j \), this leads to

\[
\int_{l_{\min}}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))dF_{\theta_i}(l)dl \geq h(l^R) \int_{l_{\min}}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))dF_{\theta_j}(l)dl
\]

Where \( h(l^R) \) is a positive constant. Then, it is not possible that \( \int_{l_{\min}}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))dF_{\theta_j}(l)dl \) is positive and \( \int_{l_{\min}}^{l_{\max}} (U(x(\theta_j)^*, l) - U(x(\theta_i)^*, l))dF_{\theta_i}(l)dl \) is negative, then we have reached a contradiction, and this concludes the proof. 

\[\Box\]
The goal of this annex is to show that our results are robust when considering different models of accident technology. In particular, we are going to consider the technologies more commonly discussed in the Law and Economics literature: (i) Care affects the level of harm, (ii) Care affects both probability of accident and level of harm, and (iii) pecuniary care, situation in which the adoption of care reduces available assets to face liability.

B.1 Model (i). Care affects the level of harm

In this alternative specification of the model we consider an accident technology in which care only affects the level of harm. Then, we consider the probability of accident constant \( p(x) = p \), while the harm resulting from the accident is decreasing and convex on the level of care, \( D(x) \),

\[
\frac{\partial D(x)}{\partial x} < 0, \quad \frac{\partial^2 D(x)}{\partial x^2} > 0.
\]

Then the first best solution \( x_{M1}^* \) is the solution of the following problem.

\[
x_{M1}^* \in \arg \max -pD(x) - C(x)
\]

The next step is to analyze whether or not this efficient level of care can be implemented in the case in which the injurer is protected by limited liability because his assets \( l \) are lower than \( D(x) \). We consider a the negligence rule that sets a single required level of precaution effort \( \tilde{x} \), which coincides with the first best solution \( x_{M1}^* \).

**Lemma 4** The injurer exerts a precautionary effort of \( x_{M1}^* \) if \( C(x_{M1}) < pl \), otherwise the injurer does not exert precautionary effort.


1. First, we consider that \( x \geq x_{M1}^* \). In this case, the injurer is not liable and consequently he has not to compensate the victim for any harm. Therefore, the injurer will never choose a precautionary effort larger than \( x_{M1}^* \). Therefore, in this case (if the injurer is constrained to
choose a effort level \( x \geq x^*_{M1} \), the injurer exerts a precautionary effort of \( x^*_{M1} \) and incurs a cost \( C(x^*_{M1}) \).

2. Consider now that the injurer chooses \( x \leq x^*_{M1} \). In this case, the injurer is liable for an amount \( l \), and consequently he would choose a care effort that maximizes \(-pl - C(x)\), which is 0.

Finally, the injurer prefer the case 1 to the case 2 if and only if \( C(x^*_{M1}) \) is lower than \( pl \).

Let \( x^*_{M1} \) be a level of precautionary effort such that \( C(x^*_{M1}) = pl \). Using \( x^*_{M1} \) we can rewrite the result of Lemma 1 as following: The injurer exerts a precautionary effort of \( x^*_{M1} \) if and only if \( x^*_{M1} < x^*_{M1} \), and 0 otherwise. Then, the next lemma characterizes the negligence rule that maximizes the precautionary effort.

**Proposition 4** The optimal negligence rule must set a precautionary effort of \( \tilde{x} \), where \( \tilde{x} = \min\{x^*_{M1}, x^*_{M1}\} \). There is a level of injurer’s assets \( l^*_{M1} < D \) such that, if and only if \( l > l^*_{M1} \) the first best solution can be implemented.

**Proof:** (i) If \( \min\{x^*_{M1}, x^*_{M1}\} = x^*_{M1} \), then by lemma 1, the first best solution is implemented.

(ii) If \( \min\{x^*_{M1}, x^*_{M1}\} = x^*_{M1} \), \( x^*_{M1} \) is the maximum level of precautionary effort that can be implemented, as the social welfare function is concave, \(-p(x)D - C(x)\), and the first best solution is not feasible, the maximum of the feasible levels of effort must be the optimal constrained optimum.

Finally, notice that by definition \( x^*_{M1} \) is increasing in \( l \), and that when \( l \geq D \), \( x^*_{M1} > x^*_{M1} \). Therefore, by continuity, there exists a level \( l^*_{M1} < D \) such that if \( l > l^*_{M1} \) the first best solution can be implemented.

**B.2 Model (ii). Care affects both probability of accident and level of harm**

We consider the following alternative specification of the model. Let \( C(x) \) be the injurer cost of the precautionary effort \( x \). We assume \( C(0) = 0 \), \( \frac{\partial C(x)}{\partial x} > 0 \), and \( \frac{\partial^2 C(x)}{\partial x^2} > 0 \). While the harm resulting from the accident and the probability of accidents are both decreasing and convex on
the level of care, \( D(x), \frac{\partial D(x)}{\partial x} < 0, \frac{\partial^2 D(x)}{\partial x^2} > 0 \) and \( p(x) \), where \( \frac{\partial p(x)}{\partial x} < 0 \) and \( \frac{\partial^2 p(x)}{\partial x^2} > 0 \). Finally, the injurer wealth is \( l \), with \( l < D \). Now, we are going to reproduce the results of the paper in this scenario.

The first best solution \( x_{M2}^* \) is the solution of the following problem.

\[
x_{M2}^* \in \arg \max -pD(x) - C(x)
\]

Notice that given the conditions of convexity we have imposed over the functions \( D(x), p(x) \) and \( C(x) \) this problem is concave, and therefore \( x_{M2}^* \) is unique. The next step is to analyze whether or not this efficient level of care can be implemented using a the negligence rule that sets a single required level of precautionary effort \( \hat{x} \), which coincides with the first best solution \( x_{M2}^* \).

\[ \text{Lemma 5} \]

The injurer exerts a precautionary effort of \( x_{M2}^* \) if \( C(x_{M2}^*) < p(\hat{x}_{M2})l + C(\hat{x}_{M2}) \), where \( \hat{x}_{M2} \in \arg \max -p(x)l - C(x) \). Otherwise, the injurer exerts a precautionary effort of \( \hat{x}_{M2} \), which is lower than \( x_{M2}^* \).

\[ \text{Proof:} \]

We denote by \( x \) the care effort of the injurer. There are two cases:

1. First, we consider that \( x \geq x_{M2}^* \). In this case, the injurer is not liable, and consequently he has not to compensate the victim for any harm. Therefore, the injurer will never choose a care effort larger than \( x_{M2}^* \). Therefore, in this case (if the injurer is constrained to choose a effort level \( x \geq x_{M2}^* \)). Then, the injurer exerts a precautionary effort of \( x_{M2}^* \) and incurs a cost \( C(x_{M2}^*) \).

2. Consider now that the injurer chooses \( x \leq x_{M2}^* \). In this case, the injurer is liable up to an amount \( l \), and consequently he would choose a precautionary effort that maximizes \(-p(x)l - C(x)\). Assume for the moment that \( l < D(x_{M2}^*) \). In this case, it is clear that \( \hat{x}_{M2} \) is always lower than \( x_{M2}^* \). Formally, \( x_{M2}^* \) satisfies \(-C'(x_{M2}^*) = p'(x_{M2}^*)D(x_{M2}^*) + p(x_{M2}^*)D'(x_{M2}^*)\), while \( \hat{x}_{M2} \) satisfies \(-C'(\hat{x}_{M2}) = p'(\hat{x}_{M2})l \). Given the convexity of \( C(x), p(x) \) and \( D(x) \), \( x_{M2}^* \)
is larger than $\hat{x}_{M2}$. Therefore, in this case the injurer exerts a precautionary effort of $\hat{x}$ and his utility is $-p(\hat{x})l - C(\hat{x})$.

Finally, the injurer prefers case 1 to case 2 if and only if $C(x^*_{M2})$ is lower than $p(\hat{x}_{M2})l + C(\hat{x}_{M2})$. Finally notice that we do not have to consider $l > D(x^*_{M2})$, because in this case, the relevant case is the case 1. In other words, if $l > D(x^*_{M2})$ then $C(x^*_{M2}) < p(\hat{x}_{M2})l + C(\hat{x}_{M2})$. This is because, $x^*_{M2}$ also minimizes $-p(x)l^+ - C(x)$, where $l^+$ is constant and equal to $D(x^*_{M2})$. By the envelope theorem, the outcome of the minimization of $-p(x)l^+ - C(x)$ has to be lower than the solution of the minimization of $-p(x)l - C(x)$. \[\]

Let $x^*_{M2}$ be a level of precautionary effort such that $C(x^*_{M2}) = p(\hat{x})l + C(\hat{x})$. Using $x^*_{M2}$, we can rewrite the result of Lemma 1 as following: The injurer exerts a precautionary effort of $x^*_{M2}$ if and only if $x^*_{M2} < x^*_{M2}$, and $\hat{x}_{M2}$ otherwise. Then, the next lemma characterizes the negligence rule that maximizes the precautionary effort.

**Proposition 5** The optimal negligence rule must set a precautionary effort of $\hat{x}$, where $\hat{x} = \min\{x^*_{M2}, x^*_{M2}\}$. There is a level of injurer’s assets $l^* < D$ such that, if and only if $l > l^*$ the first best solution can be implemented.

**Proof:** (i) If $\min\{x^*_{M2}, x^*_{M2}\} = x^*_{M2}$, then by lemma 1, the first best solution is implemented. (ii) If $\min\{x^*_{M2}, x^*_{M2}\} = x^*_{M2}$, $x^*_{M2}$ is the maximum level of precautionary effort that can be implemented, as the social welfare function is concave, $-p(x)D(x) - C(x)$, and the first best solution is not feasible, the maximum of the feasible levels of effort must be the optimal constrained optimum. Finally, notice that by definition $x^*_{M2}$ is increasing in $l$, and that when $l \geq D(x^*_{M2})$, then $x^*_{M2} > x^*_{M2}$.

**B.3 Model (iii). Care reduces the assets available to face liability (pecuniary care).**

In this alternative specification of the model we consider the basic accident technology of the paper, but we add the complication that care reduces the assets available to pay damages, in
particular we assume that the the level of liability of the potential injurer is \(l - \beta x\), where \(x\) is the level of care and \(\beta \in [0,1]\). This does not affect to the characterization of the first best solution \(x^*_M3\), and this is the same to the solution of the model, i.e \(x^*_M3 = x^*\).

\[ x^*_M3 \in \arg\max -pD(x) - C(x) \]

We consider a negligence rule that sets a single required level of precaution effort \(\hat{x}\), which coincides with the first best solution \(x^*_M3\).

\textbf{Lemma 6} The injurer exerts a precautionary effort of \(x^*_M3\) if \(C(x^*_M3) < p(\hat{x}_{M3})(l - \beta \hat{x}_{M3}) + C(\hat{x}_{M3})\), where \(\hat{x}_{M3} \in \arg\max -p(x)(l - \beta x) - C(x)\). Otherwise, the injurer exerts a precautionary effort of \(\hat{x}_{M3}\) which is lower than \(x^*_M3\).

\textbf{Proof :} We denote by \(x\) the precautionary effort of the injurer. There are two cases:

1. First, we consider that \(x \geq x^*_M3\). In this case, the injurer is not liable and consequently he has not to compensate the victim for any harm. Therefore, the injurer will never choose a precautionary effort larger than \(x^*_M3\). Therefore, in this case (if the injurer is constrained to choose an effort level \(x \geq x^*_M3\)). Then, the injurer exerts a care effort of \(x^*_M3\) and incurs a cost \(C(x^*_M3)\).

2. Consider now that, the injurer chooses \(x \leq x^*_M3\). In this case, the injurer is liable up to an amount of \(l - \beta x\), and consequently he would choose a precautionary effort that maximizes \(-p(x)(l - \beta x) - C(x)\). Therefore, in this case the injurer exerts a care effort of \(\hat{x}\), and his expected utility is \(-p(\hat{x}_{M3})(l - \beta \hat{x}_{M3}) - C(\hat{x}_{M3})\).

Finally, the injurer prefers case 1 to case 2 if and only if \(C(x^*_M3)\) is lower than \(p(\hat{x}_{M3})(l - \beta \hat{x}_{M3}) + C(\hat{x}_{M3})\). Finally notice that we are considering that \(\hat{x}_{M3}^* < x^*_M3\), assume the contrary \(\hat{x}_{M3}^* > x^*_M3\) (which could be possible given the first order condition), it is clear that then \(C(x^*_M3) < p(\hat{x}_{M3})l + C(\hat{x}_{M3})\) and the injurer exerts \(x^*_M3\).
Let $x_{M3}^{**}$ be a level of precautionary effort such that $C(x_{M3}^{**}) = p(x_{M3})(l - \beta x_{M3}) + C(x_{M3})$. Using $x_{M3}^{**}$ we can rewrite the result of Lemma 1 as following: the injurer exerts a precautionary effort of $x_{M3}^{*}$ if and only if $x_{M3}^{*} < x_{M3}^{**}$, and $x_{M3}$ otherwise. Then, the next lemma characterizes the negligence rule that maximizes precautionary effort.

**Proposition 6** The optimal negligence rule must set a precautionary effort of $\bar{x}$, where $\bar{x} = \min\{x_{M3}^{*}, x_{M3}^{**}\}$. There is a level of injurer's assets $l^* < D$ such that, if and only if $l > l^*$ the first best solution can be implemented.

**Proof:** (i) If $\min\{x_{M3}^{*}, x_{M3}^{**}\} = x_{M3}^{*}$, then by lemma 1, the first best solution is implemented. (ii) If $\min\{x_{M3}^{*}, x_{M3}^{**}\} = x_{M3}^{**}$, $x_{M3}^{**}$ is the maximum level of precautionary effort that can be implemented since if the required level is higher, the injurer would prefer $x_{M3}$, as the social welfare function is concave, $-p(x)D - C(x)$, and the first best solution is not feasible, the maximum of the feasible levels of effort must be the optimal constrained optimum. Finally, notice that by definition $x_{M3}^{**}$ is increasing in $l$, and that when $l + x_{M3}^{*} \geq D$, then $x_{M3}^{**} > x_{M3}^{*}$.  ■


Figure 1

Optimal Standard as a function of the degree of wealth of the population

Parameter a

Optimal Standard