Heterogeneous Information and the Benefits of Transparency*

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Abstract

Should central banks or other government agencies always commit to provide timely and accurate information about economic fundamentals? The answer to this question depends on the extent to which markets make efficient use of the available information. In this paper, I study the welfare effects of public and private information provision, when economic agents have access to heterogeneous sources of information. I first consider a monopolistic price-setting model with incomplete, heterogeneous information, in which the use of private information gives rise to a social tradeoff between output volatility and price dispersion. In equilibrium, pricing decisions undervalue the social costs of price dispersion, implying that prices rely too heavily on private information; contrary to results reported in Morris and Shin (2002), it follows that the provision of public information is always beneficial, but better private information may reduce welfare. Finally, I consider a general class of linear-quadratic interaction models, in which the use of private information gives rise to a tradeoff between the aggregate volatility and cross-sectional dispersion of individual decisions, and I show how the welfare implications of public and private information provision, and the efficiency of information use depend on different types of externalities in this tradeoff. Which externalities are present in a given environment has to be determined from underlying microfoundations.

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1 Introduction

Should central banks or other government agencies always commit to provide timely and accurate information about economic fundamentals? In principle, the provision of better information should allow for a better inference of sector-specific and aggregate shocks and enable markets to allocate resources more efficiently, both across sectors and over time. This argument, however, relies on the answer to a second, closely related question: *To what extent do markets make efficient use of the available information?* In response to these questions, Morris and Shin (2002 - henceforth MS) have recently argued that public information may carry too much weight in individual decisions and thereby lead to inefficiently high volatility; consequently, the releases of such public information may be welfare-reducing. They formalize this idea in a 'beauty contest' game in which a large number of agents with access to heterogeneous sources of information, and hence different beliefs about economic conditions, all seek to coordinate their decisions. Although stylized, this game captures important features of many macro-economic models with decision complementarities, such as business cycle models of incomplete nominal adjustment, or investment models with technological spill-overs.

In this paper, I provide a detailed analysis of the welfare effects of public and private information provision and the efficiency of information use when economic agents have access to heterogeneous sources of information, first in the context of a monopolistic price-setting model, then for a general class of linear-quadratic interaction models with decision complementarities. To assess how efficiently information is used in equilibrium, I introduce the *Decentralized Information Optimum* as a first-best benchmark of comparison. The decentralized information optimum is established as the decision rule which makes socially optimal use of the information locally available to each firm; formally it is obtained as the solution to a planner's problem, where the planner can dictate to each firm, how it should set its price, conditional on its public and private information.[1] The monopolistic price setting model has all the characteristics of Morris and Shin's beauty contest game, but gives rise to the exact opposite conclusions: equilibrium strategies rely too heavily on private information; consequently the provision of public information is always welfare-improving, but improved access to private sources of information may reduce welfare. The contrast between

[1] If instead the planner's problem were formulated as a direct revelation mechanism, the planner could completely aggregate all private information. The decentralized information optimum rules out this possibility, but it also abstracts from incentive compatibility issues.

these results and MS is resolved in the general linear-quadratic model, where I show how the efficiency of information use and the welfare effects of information provision depend on underlying payoffs, in particular the existence of different types of externalities. However, which externalities are applicable in a given environment has to be established from the underlying microfoundations.

More specifically, the first half of the paper considers a fully micro-founded, stochastic dynamic general equilibrium model of monopolistic price-setting, in which, in the spirit of Phelps (1970) and Lucas (1972), incomplete nominal adjustment emerges endogenously because firms are imperfectly informed about economic fundamentals and may have access to heterogeneous sources of information. As in Woodford (2002), heterogeneity of information gives rise to persistent delays of price adjustment, even when firms have precise, but private information about underlying fundamental changes; consequently, nominal shocks can have important real effects. The provision of public information improves price adjustment and reduces the real effects of nominal shocks, but the complementarity in pricing decisions amplifies the impact of public signals, which may therefore increase, rather than decrease output volatility. These results, which are summarized in theorem 1, establish the main insights of the literature following Woodford (2002) within the context of a fully micro-founded general equilibrium model, and they illustrate the main argument of MS.[2] The logic behind these results can be described rather simply: When firms have access to heterogeneous sources of information, they face a non-trivial problem of forecasting the other firms' prices. For this, public information is disproportionately more informative than private information, hence, in equilibrium, it receives a larger weight in individual firms' pricing decisions, while private information is discounted, relative to their respective information content about fundamentals. In the absence of public information the discount on private signals reduces the response of prices to nominal shocks and increases their real effects; with public information, the excess weight on public signals amplifies the impact of public signal noise on output volatility.

I then examine the welfare effects of information provision within the context of this price-setting model. Theorem 2 shows that despite possibly increasing output volatility, the provision of better public information is always welfare-improving. More precise private information, on the other hand, may reduce welfare. Next, I derive the decentralized information optimum to examine how efficiently the available information is used in equilibrium. Theorem 3 establishes that the equilibrium is inefficient: Although equilibrium strategies discount private signals relative to their

[2] See Hellwig (2002) and Amato and Shin (2003) for discussions of public information in Woodford's model, and Ui (2003) for related results in the original Lucas-island model.

information content about fundamentals, they still rely too heavily on private information and too little on public information, compared to the decentralized information optimum.

To understand these welfare results, it is useful to decompose the costs of incomplete, heterogeneous information into a component due to output volatility, and a component due to price dispersion. The conditioning of prices on public information only affects output volatility, trading off incomplete nominal adjustment against public signal noise. The use of private information, on the other hand, leads to a tradeoff between output volatility and price dispersion: More conditioning on private information improves nominal adjustment and reduces output volatility, but increases price dispersion. Theorems 2 and 3 follow from a distortion in this tradeoff: Private decisions attach too much weight to output volatility, and too little weight to price dispersion, relative to the social planner. The source of this distortion is an externality in information processing. In equilibrium, the firms' conditioning on private information affects the overall level of uncertainty about prices, and hence uncertainty the other firms have about the demand for their product. When firms decide how much to rely on private information, they do not internalize this *strategic uncertainty.* Private decisions thus don't take into account the full social cost of price dispersion.and attribute too much weight to private information, relative to the social planner.[3]

While these results are in striking contrast with MS, they are consistent with recent contributions by Angeletos and Pavan (2004 a,b - henceforth AP), who study an investment model with complementarities due to technological spill-overs. AP also come to the conclusion that public information is beneficial, because it allows for a better coordination of investment decisions. Better private information, on the other hand, may be socially costly, because it leads to an inefficient amount of dispersion in investment activity.

In the last part of this paper, I develop a linear-quadratic framework, in which I compare and reconcile the different results of MS, AP and the present model. Within this framework, the use of private information gives rise to a social tradeoff between *aggregate volatility*, i.e. the aggregate response to fundamental shocks, and the *dispersion* of actions across the population. The extent to which individual decisions internalize this tradeoff determines how efficiently the available information is used. I link the welfare effects of public and private information provision to two types of externalities, namely, wedges in the tradeoff between volatility and dispersion, and common

[3] The idea that information externalities may lead to suboptimal use of information also appears in other contexts, for example asset pricing models, in which heterogeneously informed traders do not internalize the information value of market prices - see Kyle (1989) and Stein (1987) for related discussions.

knowledge externalities that would lead to inefficient responses to shocks even if the latter were perfectly observable. In general, the welfare effects of public and private information provision and the efficiency of equilibrium information use depend on what type of externality, if any, is present.

Absent any distortions, the equilibrium attains the decentralized information optimum. Distortions in the volatility-dispersion tradeoff imply that decisions rely too heavily on one type of information, at the expense of the other: if individual agents attach too much weight to the cost of action dispersion, relative to the social planner, their decisions rely too heavily on public information, and too little on private information, and improving public information may be welfare reducing, and vice versa. Common knowledge externalities on the other hand affect both types of signals symmetrically: if actions respond too little to shocks under common knowledge, the fact that better information improves the overall response to shocks magnifies the benefits of better public or private information and mitigates any negative welfare effect that might result from distortions in the volatility-dispersion tradeoff. However, if there is overreaction to shocks under common knowledge, any positive welfare effects of better public or private information provision are mitigated, while negative welfare effects are amplified.

The different welfare results in MS, AP and here are therefore the result of different externalities in the volatility-dispersion tradeoff. For any specific application, the nature of these externalities has to be determined from micro-foundations; however the different models in MS, AP and here already give some useful hints as to what externalities are likely to be present. Here, the externality is caused by strategic uncertainty, as a result of decreasing returns to scale in production and complementarities in pricing decisions. In AP, technological spill-overs imply that agents don't fully internalize the social value of their investment, and they put too much weight on the cost of volatility, and not enough weight on dispersion.[4] In contrast, in MS, the complementarity results from a zero-sum game, due to which agents attribute too little weight to volatility.

Section 2 presents the model, defines the equilibrium, and derives a series of preliminary results. In section 3, I discuss the effects of information heterogeneity for nominal adjustment and output volatility. In section 4, I present the main welfare results. In section 5, I provide the general linear-quadratic analysis, and the comparison with MS and AP. All proofs are collected in an appendix.

[4] In various extensions, Angeletos and Pavan discuss other channels which mihgt affect the welfare effects of information provision, such as congestion externalities which offset the positive technological spill-overs, or differences in inequality aversion between the planner and agents.

2 The Model

Apart from the information structure, I consider a standard model of incomplete nominal adjustment with monopolistic firms, along the lines of Blanchard and Kiyotaki (1987), with nominal prices being preset, conditional on available information, before markets open. Time is discrete and infinite. There is a measure 1 continuum of different intermediate goods, indexed by $i \in [0, 1]$, each produced by one monopolistic firm using labor as the unique input into production. There is a final consumption good, which is produced by a perfectly competitive final goods sector using the continuum of intermediates according to a Dixit-Stiglitz CES technology with constant returns to scale. On the consumption side, there is an infinitely-lived representative household, with preferences defined over the final consumption good and labor supply in each period. The household faces a Cash-in-Advance constraint, and has to finance consumption out of the current period's nominal balances. Each period is separated into two stages: at the beginning of the period, a nominal shock is realized in the form of a stochastic lump sum transfer to the representative household. Each intermediate goods producer receives a noisy private signal about this shock, in addition there is a noisy public signal which is commonly available to everyone. On the basis of these signals, each intermediate producer then sets the nominal price for his intermediate good. In the second stage, markets open. Intermediates are traded at the posted prices, and intermediate producers hire labor to satisfy the demand for their products at the posted prices. The wage rate and the final goods price adjust to clear the labor, goods and money markets.

Household Preferences: The representative household's preferences over final good consumption and labor supply $\{C_{t+\tau}, n_{t+\tau}\}_{\tau=0}^{\infty}$ are given by

$$U_t = \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} \left(\log C_{t+\tau} - n_{t+\tau} \right) \right] \tag{1}$$

where $\beta < 1$ denotes the discount rate, and $\mathbb{E}_t(\cdot)$ denotes the household's expectations as of date t. The household's objective is to maximize (1) subject to its sequence of flow budget constraints, for $\tau = 0, 1, ...$

$$P_{t+\tau} C_{t+\tau} + M_{t+\tau}^d = W_{t+\tau} n_{t+\tau} + M_{t+\tau-1}^d + T_{t+\tau} + \Pi_{t+\tau} \tag{2}$$

where $M_{t+\tau}^d$ denotes the household's demand for nominal balances, $P_{t+\tau}$ the price of the final consumption good, $W_{t+\tau}$ the nominal wage rate, $T_{t+\tau}$ a stochastic monetary transfer the household receives at the beginning of each period, and $\Pi_{t+\tau}$ the aggregate profits of the corporate sector, which are rebated to the household. Wage payments and corporate profits are transferred to the household at the end of each period. In addition, the household has to satisfy a Cash-in-Advance constraint and finance its purchases of the consumption good out of its nominal balances after receiving the monetary transfer; i.e. for $\tau = 0, 1, ...$

$$P_{t+\tau} C_{t+\tau} \leq M^d_{t+\tau-1} + T_{t+\tau} \tag{3}$$

The nominal money supply is stochastic, with the government making a lump sum transfer $T_{t+\tau} = M^s_{t+\tau} - M^s_{t+\tau-1}$ to the representative household at the beginning of each period. Specifically, I assume that $m_t \equiv \log M^s_t$ follows a random walk,

$$m_t = m_{t-1} + \mu_t.$$

$\mu_t \sim \mathcal{N}\left(0, \tau_\mu^{-1}\right)$ is i.i.d. over time, and τ_μ is a scaling parameter representing the inverse of the shock's variance. In each period, the household chooses final good consumption C_t, labor supply n_t, and money demand M^d_t to maximize (1), subject to the constraints (2) and (3). Finally, I assume that $\gamma^{-1} \equiv \beta e^{\frac{1}{2\tau_\mu}} < 1$. As I will show below, this assumption guarantees that the Cash-in-Advance constraint is binding in every state and date.

Final Good Producers: A large number of final goods producers uses the intermediate goods to produce the final output according to a constant returns to scale technology, which is given by the CES aggregator

$$C_t = \left[\int_0^1 \left(c^i_t \right)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \tag{4}$$

Final goods producers maximize profits, taking as given the market prices of intermediate and final goods. For a total demand C_t of the final good by the household, a final goods price P_t, and input prices p^i_t, the demand for intermediate good i by the final good sector is given by

$$c^i_t = c\left(p^i_t\right) = C_t \left(\frac{p^i_t}{P_t} \right)^{-\theta}. \tag{5}$$

The final goods price P_t is given by the Dixit-Stiglitz aggregator

$$P_t = \left[\int_0^1 \left(p^i_t \right)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{6}$$

Intermediate Good Producers: Each intermediate good is produced by a single monopolistic firm using labor as the only input into production, according to a technology with decreasing returns to scale. In order to produce y units of good i, firm i needs to hire $n(y)$ units of labor, where $n(y)$ is given by

$$n(y) = \frac{1}{\delta} y^{\delta}, \tag{7}$$

with $\delta > 1$.[5] In the first stage of each period, intermediate producers receive noisy signals about m_t, when they must set their prices. Specifically, firms receive a private signal about m_t, denoted x_t^i:

$$x_t^i = m_t + \xi_t^i,$$

where $\xi_t^i \sim \mathcal{N}\left(0, \tau_\xi^{-1}\right)$ is i.i.d. over time and across the population, and is independent of μ_t. τ_ξ represents the precision of the private signal. In addition, all firms observe a public signal z_t,

$$z_t = m_t + v_t,$$

where $v_t \sim \mathcal{N}\left(0, \tau_v^{-1}\right)$ is i.i.d. over time, and independent of μ_t and all ξ_t^i, and τ_v represents the precision of the public signal. Finally, I assume that m_{t-1} is commonly known at the beginning of date t, which follows immediately from the public observation of the market wage rate W_{t-1} at the end of the previous period. Firm i's information set $\Im_t^i$ is then given by $\Im_t^i = \left\{x_{t-s}^i, z_{t-s}, m_{t-s-1}\right\}_{s=0}^{\infty}$. i's nominal profits π_t^i, as a function of its price p_t^i, are given by:

$$\pi_t^i = p_t^i c\left(p_t^i\right) - W_t n\left(c\left(p_t^i\right)\right), \tag{8}$$

where $c\left(p_t^i\right)$ denotes the stochastic demand firm i faces for its product, given by (5).

If information were homogeneous and asset markets complete, the firm's objective would be determined simply by evaluating profits according to state-prices. Here, such an approach leads to the added complication that, if available to the firms, these asset prices would fully and commonly reveal the underlying state; on the other hand, when markets are incomplete, the firm's objective need not be unambiguously specified. To get around this issue, I assume that Arrow-Debreu prices are not available to firms, but instead each firm sets its price p_t^i to maximize *expected shareholder value*.[6] Let $E_t^i(\cdot) \equiv E\left(\cdot \mid \Im_t^i\right)$ denote the expectations operator, conditional on $\Im_t^i$. Then firm i's

[5] Alternatively, the production function $y(n)$ is given by $y(n) = [\delta n]^{1/\delta}$

[6] The idea behind this objective is that each firm is instructed to set prices to maximize the representative household's welfare, taking as given the other firms' equilibrium pricing behavior. Under complete information, this approach is equivalent to evaluating profits according to state-prices.

expected shareholder value is defined as

$$E_t^i\left(\beta E_t\left(\frac{1}{C_{t+1}P_{t+1}}\right)\pi_t^i\right)=E_t^i\left(\beta E_t\left(\frac{1}{C_{t+1}P_{t+1}}\right)\left[p_t^i c\left(p_t^i\right)-W_t n\left(c\left(p_t^i\right)\right)\right]\right). \tag{9}$$

To aggregate prices and profits, I assume that the realized distribution of private signals across firms (conditional on m_t) is given by the conditional distribution of x_t^i, almost surely, which implies that the population average of the private signal, $\int x_t^i di$ equals m_t, almost surely.[7] Since prices p_t^i are measurable with respect to private signals x_t^i, the CES price index P_t is given by $P_t = \int \left(p_t^i\right)^{\frac{\theta-1}{\theta}} d\Phi\left(x_t^i \mid m_t\right)$ and aggregate profits are given by $\Pi_t \equiv \int \pi_t^i di = \int \pi_t^i d\Phi\left(x_t^i \mid m_t\right)$, almost surely, where I let $\Phi\left(\cdot \mid m_t\right)$ denote the normal cdf of the private signal distribution, conditional on a realization m_t.

Equilibrium Definition: I focus on stationary equilibria, in which (i) intermediate good prices p_t^i are functions of the firms' contemporaneous information sets $\mathcal{I}_t^i \equiv \left\{z_t, x_t^i, m_{t-1}\right\}$, and (ii) the representative household's equilibrium demand for the final good and nominal balances and its supply of labor, as well as the final good price and the nominal wage rate, are all functions only of $\{z_t, m_t, m_{t-1}\}$. This leads to the following equilibrium definition:

Definition 1 *A symmetric, stationary equilibrium is defined as a set of functions* $C(\cdot)$, $M^d(\cdot)$, $n(\cdot)$, $P(\cdot)$, $W(\cdot)$, *and* $p(\cdot)$, *such that:*

(i) $\left\{C(\cdot), M^d(\cdot), n(\cdot)\right\}$ *maximize (1) subject to (2) and (3).*

(ii) *zero profits for final good producers:* $P(\cdot)$ *is given by (6), where* $p_t^i = p\left(\mathcal{I}_t^i\right)$.

(iii) $p(\cdot)$ *maximizes (9), where* $c(p) = C(\cdot)\left[P(\cdot)\right]^{\theta} p^{-\theta}$.

(iv) *All markets clear.*

The equilibrium definition imposes symmetry across intermediate good producers, i.e. all firms use an identical pricing rule $p(\cdot)$. Furthermore, by Walras Law, it is sufficient for market clearing that the money market clears, or $\log M_t^d = m_t$.

Preliminary Results: To characterize the equilibrium, I first characterize optimal household behavior and ex post market-clearing. I then use these results to characterize optimal price-setting by the intermediate firms as the solution to a fixed point problem. Lemma 1 characterizes the household's optimal behavior.

[7] see Judd 1985 for the measure-theoretic issues involved in applying the Law of Large Numbers to a continuum of random variables.

Lemma 1 *The Cash-in-Advance constraint is always binding, and the household's optimal consumption in equilibrium is given by*

$$C_t = \frac{M_t^s}{P_t} \tag{10}$$

The equilibrium wage rate satisfies

$$W_t = \gamma M_t^s. \tag{11}$$

As a consequence, it follows immediately that $\beta E_t\left(\frac{1}{C_{t+1}P_{t+1}}\right) = \frac{1}{W_t}$, and hence the expected shareholder value of firms is defined as $E_t^i\left(\beta E_t\left(\frac{1}{C_{t+1}P_{t+1}}\right)\pi_t^i\right) = E_t^i\left[\frac{\pi_t^i}{W_t}\right]$. After substituting (5), (10) and (11) into (9), the intermediate firms' maximization problem is given by:

$$\max_{p_t^i} E_t^i\left[\left(p_t^i\right)^{1-\theta} P_t^{\theta-1} - \frac{\gamma}{\delta}\left(p_t^i\right)^{-\theta\delta}\left(M_t^s\right)^{\delta} P_t^{\delta(\theta-1)}\right] \tag{12}$$

The corresponding first-order condition for p_t^i is

$$\left(p_t^i\right)^{1+\theta\delta-\theta} = \frac{\gamma\theta}{\theta-1}\frac{E_t^i\left[\left(M_t^s\right)^{\delta} P_t^{\delta(\theta-1)}\right]}{E_t^i\left[P_t^{\theta-1}\right]}. \tag{13}$$

Conjecture that $\log P_t$ and m_t, conditional on $\mathcal{I}_t^i$, will be jointly normally distributed in equilibrium. (13) can then be rewritten as:

$$\log p_t^i = (1-r)\left[\frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{1}{2\delta}V\right] + (1-r)\,E_t^i\left(m_t\right) + rE_t^i\left(\log P_t\right) \tag{14}$$

where

$$r \equiv \frac{(\theta-1)(\delta-1)}{1+\theta\delta-\theta} \text{ and } V \equiv \delta^2 V_t^i\left[\log\left(M_t^s P_t^{\theta-1}\right)\right] - V_t^i\left[\log\left(P_t^{\theta-1}\right)\right].$$

$r \in (0,1)$ denotes the degree of strategic complementarities in price-setting. (14) captures the strategic interaction that results from monopolistic price competition: a firm's optimal pricing decision is an increasing function of its expectation about the average price in the market. The novel aspect of the model presented here lies in the assumption that firms are *heterogeneous* in their information. Hence, firms need to form expectations not only about nominal spending, but also about the pricing decisions of other firms.

Public Information Benchmark: As a useful benchmark, I first suppose that information is homogeneous, i.e. $\tau_\xi = 0$. All firms then set the same price, and (14) can immediately be solved for the equilibrium pricing rule and the resulting output level:

$$\begin{aligned}
\log p_t^i &= \log P_t = \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{\delta/2}{\tau_v+\tau_\mu} + m_{t-1} + \frac{\tau_v}{\tau_v+\tau_\mu}\left(z_t - m_{t-1}\right) \qquad (15)\\
\log C_t &= \frac{1}{\delta}\log\left(\frac{\theta-1}{\gamma\theta}\right) - \frac{\delta/2}{\tau_v+\tau_\mu} + \frac{\tau_\mu}{\tau_v+\tau_\mu}\mu_t - \frac{\tau_v}{\tau_v+\tau_\mu}v_t
\end{aligned}$$

This equilibrium characterization has the following properties:

Proposition 1 *In the absence of informational heterogeneity,*

1. *Equilibrium prices respond to the public signal exactly according to its Bayesian weight* $\frac{\tau_v}{\tau_v+\tau_\mu}$.
2. *Firms face no strategic uncertainty: Equilibrium prices can be perfectly forecast, and the risk premium* $\frac{\delta/2}{\tau_v+\tau_\mu}$ *only takes into account the exogenous uncertainty about fundamentals.*
3. *The volatility of output is given by* $\frac{1}{\tau_v+\tau_\mu}$*, and is strictly decreasing in the precision of public information.*

With homogeneous information, equilibrium prices make efficient use of the available information, i.e. the weight that pricing decisions attribute to the public signal minimizes output volatility. The only inefficiency in the market arises from the firms' market power and the inflation tax, and is measured by the mark-up $\frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right)$.

3 Equilibrium characterization

Equation (14) implicitly defines the equilibrium pricing rule as the solution to a fixed point problem. The resulting equilibrium characterization is provided in proposition 2.

Proposition 2 *In the unique equilibrium in linear strategies, firms set prices according to*

$$\log p_t^i = \Gamma_0 + m_{t-1} + \frac{\tau_\xi\left(1-r\right)}{\tau_\mu+\tau_v+\left(1-r\right)\tau_\xi}\left(x_t^i - m_{t-1}\right) + \frac{\tau_v}{\tau_\mu+\tau_v+\left(1-r\right)\tau_\xi}\left(z_t - m_{t-1}\right) \qquad (16)$$

and P_t and C_t satisfy:

$$\begin{aligned}
\log P_t &= \Gamma + m_{t-1} + \frac{\tau_v+\tau_\xi\left(1-r\right)}{\tau_\mu+\tau_v+\left(1-r\right)\tau_\xi}\mu_t + \frac{\tau_v}{\tau_\mu+\tau_v+\left(1-r\right)\tau_\xi}v_t \qquad (17)\\
\log C_t &= -\Gamma + \frac{\tau_\mu}{\tau_\mu+\tau_v+\left(1-r\right)\tau_\xi}\mu_t - \frac{\tau_v}{\tau_\mu+\tau_v+\left(1-r\right)\tau_\xi}v_t \qquad (18)
\end{aligned}$$

where Γ_0 and Γ are given by:

$$\Gamma = \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right)+\frac{\delta}{2}\frac{\tau_\mu+\tau_v+\theta(1-r)\tau_\xi}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2} \tag{19}$$

$$\Gamma_0 = \Gamma+\frac{\theta-1}{2}\frac{\tau_\xi(1-r)^2}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2}$$

Proposition 2 characterizes the response of prices and consumption to monetary and informational shocks, and the effect of incomplete, heterogeneous information on the expected level of prices and output, as measured by Γ. The first term in Γ measures the effect of monopolistic competition and the inflation tax on the price level and would arise identically under common knowledge. The second term measures the effect of information heterogeneity. This term can be decomposed into a component due to output volatility, denoted σ_C^2, and a component due to price dispersion, denoted Σ_p^2 and defined as the cross-sectional variance of prices across firms. Σ_p^2 and σ_C^2 are given by:

$$\Sigma_p^2=\frac{\tau_\xi(1-r)^2}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2} \text{ and } \sigma_C^2=\frac{\tau_\mu+\tau_v}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2}.$$

and hence $\Gamma=\frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right)+\frac{\delta}{2}\left(\sigma_C^2+\frac{\theta}{1-r}\Sigma_p^2\right)$.

Theorem 1 discusses in detail how the dynamic adjustment of prices and output depends on the informational parameters τ_ξ, τ_v, and τ_μ. It further discusses how σ_C^2, Σ_p^2, and Γ vary with the informational parameters.

Theorem 1 *Informational heterogeneity has the following implications for prices and consumption:*

1. ***Informational heterogeneity reduces the response of prices to monetary shocks:*** *Holding the overall amount of information constant, a shift in signal precision from public to private information reduces the adjustment of prices in response to monetary shocks.*

2. ***The impact of public information:*** *The higher is r, i.e. the more complementary pricing decisions are, the larger is the impact of public information.*

3. ***Output volatility:*** *σ_C^2 is decreasing in τ_ξ, but non-monotonic in τ_v and τ_μ. Whenever $\tau_v+\tau_\mu>\tau_\xi(1-r)$, σ_C^2 is decreasing in τ_v and τ_μ, otherwise it is increasing.*

4. ***Price dispersion:*** *Σ_p^2 is decreasing in τ_v and τ_μ, but non-monotonic in τ_ξ. Whenever $\tau_\xi(1-r)>\tau_v+\tau_\mu$, Σ_p^2 is decreasing in τ_ξ, otherwise it is increasing.*

5. ***Deadweight output loss:*** *Γ is decreasing in τ_v and τ_μ, but if $\theta > 2$, Γ is non-monotonic in τ_ξ. Whenever $\tau_\xi (1-r) > \frac{\theta-2}{\theta} (\tau_v + \tau_\mu)$, Γ is decreasing in τ_ξ, otherwise it is increasing.*

Theorem 1 outlines the implications of information heterogeneity for the dynamics of output and prices, and establishes the tradeoff between output volatility σ_C^2 and the price dispersion Σ_p^2 that was discussed in the introduction.

As can be observed from the equilibrium pricing rule, the impact of private signals on prices is reduced, while the weight on public signals and the prior is amplified, relative to their information content; moreover, these shifts become larger the higher is r. As a consequence of the shift towards the prior, prices respond less to monetary shocks than they would if all information was common so that monetary shocks have more important real effects. As a consequence of the shift towards the public signal, prices and consumption also respond more to noise in the public signal: Holding the overall precision of information constant, the impact of public signal noise is amplified, if decisions become more complementary. The first two points of Theorem 1 thus summarize the main insights of Woodford (2002), Hellwig (2002), and Amato and Shin (2003) in a simple model of incomplete nominal adjustment.

The third, fourth and fifth points lay out the implications for output volatility, price dispersion and the deadweight output loss. Due to the disproportionate weight on public signals, more precise public information may increase output volatility, but output volatility is unambiguously decreasing in the precision of private information. Price dispersion is unambiguously decreasing in the precision of public information, but may be increasing in the precision of private signals: When private signals are infinitely noisy, they do not affect prices, and hence there is no price dispersion. On the other hand, when private signals are infinitely precise, firms almost exclusively condition on private signals, but the signal dispersion across the population is small. Price dispersion is therefore largest when the precision of private signals is neither too high, nor too low. Finally, the deadweight loss of output Γ depends both on output volatility and on price dispersion, but its comparative statics with respect to τ_ξ, τ_v and τ_μ are driven by the latter, and an increase in the precision of private information may increase the expected price level and reduce the expected level of output.

4 Welfare Results

I now turn to the welfare implications of public and private information in the present model of monopolistic price-setting. The main results are stated in two theorems: In Theorem 2, I establish the comparative statics of the representative household's expected utility with respect to the informational parameters. Theorem 3 compares the equilibrium to the socially optimal use of the available information.

For the results, I focus on log-linear pricing rules of the form

$$\log p_t^i = m_{t-1} + \Lambda_0 + \Lambda_1 \left(x_t^i - m_{t-1}\right) + \Lambda_2 \left(z_t - m_{t-1}\right) \tag{20}$$

that are stationary over time, and fully incorporate the past money supply into the price.[8] Any rule of the form of (20) implies that the resulting consumption realizations $\log C_t$ are i.i.d. normally distributed over time, and furthermore, prices are lognormally distributed across firms. Moreover, $\mathbb{U} \equiv \mathbb{E}_{t-1}(\log C_t) - \mathbb{E}_{t-1}(n_t)$ is constant over time, and $\mathbb{U}/(1-\beta)$ equals the representative household's expected life-time utility prior to knowing the current period's monetary shock. As a function of the parameters $\{\Lambda_0, \Lambda_1, \Lambda_2\}$ of the linear pricing rule (20), $\mathbb{U}$ is given by:

$$\mathbb{U} = -\Lambda - \frac{1}{\delta} \exp\left\{-\delta\Lambda + \frac{\delta^2}{2}\left[(1-\Lambda_1-\Lambda_2)^2 \frac{1}{\tau_\mu} + \Lambda_2^2 \frac{1}{\tau_v} + \Lambda_1^2 \frac{\theta}{1-r}\frac{1}{\tau_\xi}\right]\right\}, \tag{21}$$

where $\Lambda = \Lambda_0 - (\Lambda_1)^2 \frac{\theta-1}{2}\frac{1}{\tau_\xi}$.

4.1 Equilibrium welfare

Using (21) and the equilibrium pricing coefficients, the equilibrium welfare level is given by:

$$\mathbb{U}^{eq} = \frac{1}{\delta}\left[\log\left(\frac{\theta-1}{\gamma\theta}\right) - \frac{\theta-1}{\gamma\theta}\right] - \frac{\delta}{2}\frac{\tau_\mu + \tau_v + \theta(1-r)\tau_\xi}{\left[\tau_\mu + \tau_v + (1-r)\tau_\xi\right]^2} \tag{22}$$

This characterization of equilibrium welfare separates the social cost of the inflation tax and the market power from the social cost of incomplete and heterogeneous information. The first component of the RHS of (22) measures the inflation tax and the market power, this term is maximized when $\frac{\theta-1}{\gamma\theta} = 1$, i.e. in the limiting case when $\theta \to \infty$ and $\gamma \to 1$ (eliminating the mark-ups and the inflation tax). The second component of the RHS of (22) measures the social cost of heterogeneous information, and the resulting deadweight loss of output.

Theorem 2 *(i)* $\mathbb{U}^{eq}$ *is strictly increasing in* τ_v.

[8] Restricting attention to linear decision rules is obviously not without loss of generality. Nevertheless, it provides a very simple first step towards understanding the difference between private and social costs and benefits of information use.

(ii) $\mathbb{U}^{eq}$ *is monotonically increasing in* τ_ξ*, iff* $\theta \leq 2$*. If* $\theta > 2$*,* $\mathbb{U}^{eq}$ *non-monotonic in* τ_ξ*, and reaches a minimum when* $\tau_\xi(1-r) = \frac{\theta-2}{\theta}(\tau_v + \tau_\mu)$*.* $\mathbb{U}^{eq}$ *is decreasing in* τ_ξ*, when* $\tau_\xi(1-r) < \frac{\theta-2}{\theta}(\tau_v + \tau_\mu)$*, and increasing, when* $\tau_\xi(1-r) > \frac{\theta-2}{\theta}(\tau_v + \tau_\mu)$*.*

Theorem 2 states that equilibrium welfare is strictly increasing in the precision of public information, and non-monotonic in the precision of private information: initially decreasing, but increasing if τ_ξ is sufficiently large. The welfare considerations are therefore dominated by the cost of price heterogeneity, which is decreasing in the precision of public, and non-monotonic in the precision of private information.[9]

The conclusion of Theorem 2 is diametrically opposite to the results obtained by MS. While the qualitative effects of information provision on volatility is identical in the two papers, the different results are a consequence of different preferences. In contrast to MS, the micro-foundations in the present model reveal that the cost of price dispersion is the dominant component in determining the welfare effects of better public and private information.

It is possible to replicate the results of MS in a reduced-form version of the present model that evaluates social welfare according to a quadratic loss function in inflation and output, along the lines of Barro and Gordon (1983). While the Barro-Gordon model implies that it is always in the social interest to improve public information to reduce aggregate volatility, when information is homogeneous, it does not take into account the tradeoff between output volatility and price dispersion that arises with heterogeneous information. In the last section of this paper, I propose an alternative model, which takes this trade-off into account, and illustrates how the welfare effects of public and private information provision depend on the underlying payoff structure.

4.2 Decentralized Information Optimum

How efficiently is the available information used in equilibrium? To answer this question, I next derive the *Decentralized Information Optimum*, i.e. the linear decision rule that makes socially optimal use of the locally available information. Maximizing (21) with respect to the coefficients

[9] The welfare effects of heterogeneous information also depend on the degree of monopolistic competition, which is parametrized by the substitution elasticity θ. Increasing θ not only reduces mark-ups, but in addition increases the complementarity in pricing decisions, r. It follows from (22) that the cost of informational heterogeneity is increasing in θ; furthermore, when $\tau_\xi/(\tau_\mu + \tau_v)$ is large, the welfare cost of heterogeneous information can become arbitrarily large, as $\theta \to \infty$. Thus, in an economy with heterogeneous information, enhanced competition may by costly, if the benefits of reduced mark-ups are more than outweighed by the increased cost of information heterogeneity.

Λ, Λ_1 and Λ_2, the coefficients characterizing the decentralized information optimum are given by:

$$\Lambda^* = \frac{\delta/2}{\tau_\mu + \tau_v + \frac{1-r}{\theta}\tau_\xi}; \qquad \Lambda_1^* = \frac{\frac{1-r}{\theta}\tau_\xi}{\tau_\mu + \tau_v + \frac{1-r}{\theta}\tau_\xi}; \qquad \Lambda_2^* = \frac{\tau_\mu}{\tau_\mu + \tau_v + \frac{1-r}{\theta}\tau_\xi} \tag{23}$$

The decentralized information optimum differs from the equilibrium pricing rule in two ways: first, it eliminates the positive mark-up that was due to monopoly power and the inflation tax. Second, the decentralized information optimum shifts the weight in pricing decisions from private towards public information. We immediately have the second theorem:

Theorem 3 *Whenever $\tau_\xi > 0$, the equilibrium pricing rule puts too little weight on public information, and too much weight on private information, relative to a socially optimal use of the available information, i.e. $\Lambda_1^{eq} > \Lambda_1^*$ and $\Lambda_2^{eq} < \Lambda_2^*$.*

Even though the equilibrium weight on private information is smaller than its Bayesian weight in forecasting first-order expectations, the market equilibrium puts too much weight on private information, relative to the decentralized information optimum. By comparing the first-order conditions of the social planner's problem to the coefficients of the linear equilibrium rule, we can trace this inefficiency to a negative externality in the processing of private information. The decentralized information optimum satisfies the social planner's first-order conditions for Λ_1 and Λ_2:

$$\begin{aligned}
(1 - \Lambda_1^* - \Lambda_2^*)\frac{1}{\tau_\mu} &= \Lambda_2^* \frac{1}{\tau_v} && \text{for } \Lambda_2 \\
(1 - \Lambda_1^* - \Lambda_2^*)\frac{1}{\tau_\mu} &= \Lambda_1^* \frac{\theta}{1-r}\frac{1}{\tau_\xi} && \text{for } \Lambda_1.
\end{aligned}$$

The equilibrium coefficients Λ_1^{eq} and Λ_2^{eq}, on the other hand, satisfy:

$$\begin{aligned}
(1 - \Lambda_1^{eq} - \Lambda_2^{eq})\frac{1}{\tau_\mu} &= \Lambda_2^{eq} \frac{1}{\tau_v} \\
(1 - \Lambda_1^{eq} - \Lambda_2^{eq})\frac{1}{\tau_\mu} &= \Lambda_1^{eq} \frac{1}{1-r}\frac{1}{\tau_\xi} < \Lambda_1^{eq}\frac{\theta}{1-r}\frac{1}{\tau_\xi}.
\end{aligned}$$

Therefore, while the equilibrium makes optimal use of public information to minimize volatility, it does not fully internalize the trade-off between aggregate volatility and cross-sectional price heterogeneity that results from the conditioning of prices on private information. This distortion in the conditioning of prices on private information can be traced to a wedge between the private and the social cost of price dispersion. The more prices are conditioned on private signals, the larger is the aggregate degree of strategic uncertainty about equilibrium prices, i.e. the harder it becomes for any individual firm to forecast the average price. Uncertainty about equilibrium prices is reflected in the firm's pricing equation in the constant term Λ, which incorporates a risk premium due to the strategic uncertainty that firms face. In conditioning their prices on private information, firms do not take into account their own contribution to the aggregate strategic risk. This creates a wedge between the private and the social costs of price heterogeneity and implies that the private benefit of conditioning prices on private signals exceeds the social benefit, which results in the excess reliance on private information.

5 A general linear-quadratic model

The results presented in the previous section are in stark contrast with those reported in MS. This raises several issues: First, which of the modeling features drive the differences in results between MS and the present analysis? Second, what underlying principles determine whether it is socially beneficial to provide public or private information? In this section, I explore these questions and show how the different results presented here, as well as in MS and AP can be reconciled within one simple quadratic model.

5.1 Set-up

Consider the following static normal-form game: There is a continuum of agents, indexed by $i \in [0,1]$. Each agent simultaneously chooses an action $a_i \in \mathbb{R}$. Let $\mathbf{a}(\cdot) : [0,1] \to \mathbb{R}$ denote an action profile. Each agent's preferences are given by u, which is a function of the player's own action a_i, the action profile $\mathbf{a}(\cdot)$, and a stochastic state variable μ:

$$\begin{aligned} u\left(a_i, \mathbf{a}(\cdot), \mu\right) &= -(1-r)\left(a_i-\mu\right)^2 - r\left(a_i-a\right)^2 \\ &\quad + k_1 \int \left(a_j-a\right)^2 dj + (1-r)\, k_2 \left(a-\mu\right)^2 + 2\left(1-r\right) k_3 \mu \left(a-\mu\right), \end{aligned} \tag{24}$$

where $a = \int a_j dj$.

μ is normally distributed with mean 0 and precision τ_μ. $r < 1$ denotes the degree of strategic complementarities. Since the last three terms do not depend on a_i, they do not affect equilibrium strategies, and enter purely as externalities. If μ is common knowledge, the best-response profile is given by $a_i = (1-r)\mu + ra$, and $a_i = a = \mu$ constitutes the unique equilibrium of this game.

To define the social planner's problem, I consider a utilitarian welfare criterion, $W(\mathbf{a}(\cdot), \mu) = \int u(a_j, \mathbf{a}(\cdot), \mu)\, dj$. $W(\mathbf{a}(\cdot), \mu)$ can be written as:

$$\begin{aligned} W(\mathbf{a}(\cdot), \mu) &= -(1-k_1)\int (a_j - a)^2\, dj \\ &\quad -(1-r)(1-k_2)(a-\mu)^2 + 2(1-r)k_3\mu(a-\mu). \end{aligned} \tag{25}$$

The first term corresponds to the social cost of the *dispersion* of actions. The second term measures the social cost of *aggregate volatility*, i.e. the cost associated with having individual decisions out of line with the aggregate state. The coefficients k_1 and k_2 measure the extent to which individual decisions internalize the social costs of action dispersion and volatility. I assume that $k_1 < 1$ and $k_2 < 1$, so that action dispersion and volatility are both socially costly. For further analysis, I define $\chi \equiv \frac{1-k_2}{1-k_1}$, which measures the importance of the externality on aggregate volatility, relative to the externality in action dispersion in agent preferences. The last term in $W(\mathbf{a}(\cdot), \mu)$ measures the social cost that arises from under-reaction or over-reaction to shocks: under common knowledge, social welfare is maximized by $a_i = a = (1+\rho)\mu$, where $\rho = \frac{k_3}{1-k_2}$. ρ thus measures the over- or under-reaction of the common knowledge equilibrium, relative to the social planner solution. If $\rho = 0$, the equilibrium attains the social planner solution. If $\rho > 0$, the equilibrium responds too little, if $\rho < 0$, the equilibrium responds too much to a change in μ. Here, I assume that $\rho > \min\left\{-\frac{1}{2}, -\frac{1}{2\chi}\right\}$, which imposes an upper bound on any negative aggregate externality.[10]

As before, I assume that agents have access to a private signal x_i, $x_i \sim \mathcal{N}\left(\mu, \tau_\xi^{-1}\right)$, and a public signal z, $z \sim \mathcal{N}\left(\mu, \tau_v^{-1}\right)$. Private signals are iid across the population, and all signals are conditionally independent of each other. Agents maximize $\mathbb{E}(u(a_i, \mathbf{a}(\cdot), \mu) \mid x_i, z)$, implying that the best-response profile is $a_i = (1-r)\mathbb{E}(\mu \mid x_i, z) + r\mathbb{E}(a \mid x_i, z)$. The unique equilibrium of this game is

$$a_i = \frac{(1-r)\tau_\xi}{\tau_\mu + \tau_v + (1-r)\tau_\xi} x_i + \frac{\tau_v}{\tau_\mu + \tau_v + (1-r)\tau_\xi} z. \tag{26}$$

To solve the social planner's problem, I focus on the *Decentralized Information Optimum*, in which the planner dictates what decision rule has to be used by all agents. For simplicity, I again restrict attention to linear decision rules of the form $a_i = \lambda_1 x_i + \lambda_2 z$. After substituting into the

[10] The social planner's objective is equivalent to

$$\widetilde{W}(\mathbf{a}(\cdot), \mu) = -(1-k_1)\int (a_j - a)^2\, dj - (1-r)(1-k_2)(a - (1+\rho)\mu)^2,$$

which measures aggregate volatility as the squared deviation of the average action from the socially optimal level.

social planner's problem, the expected social welfare, as a function of λ_1 and λ_2, is:

$$EW(\lambda_1, \lambda_2) = -(1-r)(1-k_2)\left[\frac{\lambda_1^2}{\chi(1-r)\tau_\xi} + \frac{(1-\lambda_1-\lambda_2)^2}{\tau_\mu} + \frac{\lambda_2^2}{\tau_v} + 2\rho\frac{1-\lambda_1-\lambda_2}{\tau_\mu}\right] \quad (27)$$

I conclude the discussion of the model set-up with the following useful benchmark result:

Proposition 3 *When $\chi = 1$ and $\rho = 0$, the equilibrium pricing rule coincides with the decentralized information optimum.*

This proposition suggests a natural interpretation for the class of objective functions described by (24): When $\chi = 1$, the equilibrium fully internalizes the social costs and benefits of action dispersion and aggregate volatility; $\rho = 0$ implies that the common knowledge equilibrium attains the social optimum. On the other hand, when $\chi \neq 1$, there is a wedge between the private and social costs of action dispersion ($k_1 \neq 0$) and/or aggregate volatility ($k_2 \neq 0$), so that agents do not weigh them relative to each other in the same way as the social planner. And when $\rho \neq 0$, the common knowledge equilibrium is inefficient.

5.2 Main Results

I now establish the general equivalents of the previous theorems 2 and 3. Computing equilibrium welfare from (26), one finds:

$$W^{eq} = -(1-r)(1-k_2)\left[\frac{\tau_\mu + \tau_v + \frac{1}{\chi}(1-r)\tau_\xi}{[\tau_\mu + \tau_v + (1-r)\tau_\xi]^2} + 2\rho\frac{1}{\tau_\mu + \tau_v + (1-r)\tau_\xi}\right]. \quad (28)$$

The first term inside the square bracket measures the welfare cost of action dispersion and aggregate volatility. The second term measures the common knowledge inefficiency. The general counterpart to theorem 2 follows immediately:

Theorem 4 *Define $\widehat{\chi} = \chi\frac{1+2\rho}{1+2\chi\rho}$.*

(i) If $\widehat{\chi} < \frac{1}{2}$, W^{eq} is monotonically increasing in τ_μ and τ_v, but non-monotonic in τ_ξ, reaching a local minimum for given τ_μ and τ_v, when $(1-r)\tau_\xi = (1-2\widehat{\chi})(\tau_\mu + \tau_v)$. For lower τ_ξ, W^{eq} is decreasing, for higher τ_ξ, W^{eq} is increasing in τ_ξ.

(ii) If $\widehat{\chi} \in \left[\frac{1}{2}, 2\right]$, W^{eq} is monotonically increasing in τ_μ, τ_v, and τ_ξ.

(iii) If $\widehat{\chi} > 2$, W^{eq} is monotonically increasing in τ_ξ, but non-monotonic in τ_μ and τ_v, reaching a local minimum for given τ_ξ, when $\tau_\mu + \tau_v = \frac{\widehat{\chi}-2}{\widehat{\chi}}\tau_\xi(1-r)$. For lower $\tau_\mu + \tau_v$, W^{eq} is decreasing, for higher $\tau_\mu + \tau_v$, W^{eq} is increasing in $\tau_\mu + \tau_v$.

Taking first-order conditions of (27) with respect to λ_1 and λ_2, I compute the decentralized information optimum, characterized by λ_1^* and λ_2^*:

$$\lambda_1^* = (1+\rho)\,\frac{\chi\,(1-r)\,\tau_\xi}{\tau_\mu + \tau_v + \chi\,(1-r)\,\tau_\xi}; \qquad \lambda_2^* = (1+\rho)\,\frac{\tau_v}{\tau_\mu + \tau_v + \chi\,(1-r)\,\tau_\xi} \tag{29}$$

The decentralized information optimum scales up the coefficients of the decision rule by a factor $1+\rho$ to correct for the common knowledge inefficiency, and adjusts the relative weights of the two signals to optimally trade off between volatility and dispersion. The next theorem establishes the general counterpart to theorem 3:

Theorem 5 *(i) If $\chi = 1$, in equilibrium, the relative weight on private information corresponds to the optimal relative weight, i.e. $\frac{\lambda_1^*}{\lambda_2^*} = \frac{\lambda_1^{eq}}{\lambda_2^{eq}}$.*

(ii) If $\chi < 1$, in equilibrium, the relative weight on private information is too large compared to the optimum, i.e. $\frac{\lambda_1^*}{\lambda_2^*} < \frac{\lambda_1^{eq}}{\lambda_2^{eq}}$.

(iii) If $\chi > 1$, in equilibrium, the relative weight on private information is too large compared to the optimum, i.e. $\frac{\lambda_1^*}{\lambda_2^*} > \frac{\lambda_1^{eq}}{\lambda_2^{eq}}$.

When $\rho = 0$, these comparisons also hold in absolute terms, i.e. if $\chi = 1$, $\lambda_1^{eq} = \lambda_1^*$ and $\lambda_2^{eq} = \lambda_2^*$; if $\chi < 1$, $\lambda_1^{eq} > \lambda_1^*$ and $\lambda_2^{eq} < \lambda_2^*$; and if $\chi > 1$, $\lambda_1^{eq} < \lambda_1^*$ and $\lambda_2^{eq} > \lambda_2^*$.

The two theorems discuss how the welfare implications of improving public or private information depend on χ and ρ. For understanding the results, it is useful to begin with the case where the common knowledge equilibrium is efficient, i.e. when $\rho = 0$, so that $\widehat{\chi} = \chi$. The parameter χ allows for a simple comparison between the private and the social tradeoff between aggregate volatility and action dispersion. When $\chi = 1$, individual decisions fully internalize the social tradeoff. When $\chi < 1$, agents put too much weight on aggregate volatility, and too little weight on action dispersion, relative to the social planner, and the opposite is true, when $\chi > 1$. The implications of these wedges for equilibrium decisions become clear from comparing the equilibrium to the social planner's first-order conditions for λ_1^* and λ_2^*, are given by

$$\frac{1-\lambda_1^*-\lambda_2^*}{\tau_\mu} = \frac{\lambda_2^*}{\tau_v} \quad \text{for } \lambda_2, \quad \text{and} \quad \frac{1-\lambda_1^*-\lambda_2^*}{\tau_\mu} = \frac{1}{\chi\,(1-r)}\frac{\lambda_1^*}{\tau_\xi} \qquad \text{for } \lambda_1.$$

The equilibrium coefficients λ_1^{eq} and λ_2^{eq} satisfy:

$$\frac{1-\lambda_1^{eq}-\lambda_2^{eq}}{\tau_\mu} = \lambda_2^{eq}\frac{1}{\tau_v} \qquad \text{and} \qquad \frac{1-\lambda_1^{eq}-\lambda_2^{eq}}{\tau_\mu} = \frac{1}{1-r}\frac{\lambda_1^{eq}}{\tau_\xi}.$$

λ_1^{eq} and λ_2^{eq} solve the first-order condition for λ_2, but violate the condition for λ_1, if $\chi \neq 1$. Once λ_1 is given, λ_2 only affects aggregate volatility. Since private and social incentives are aligned in minimizing aggregate volatility, λ_2 fully internalizes the social benefit of reducing aggregate volatility in equilibrium. The conditioning of actions on private information on the other hand generates a tradeoff between action dispersion, which increases with λ_1, and aggregate volatility, which decreases with λ_1. Private and social incentives with regard to this tradeoff, and hence the use of private information, are aligned, if and only if $\chi = 1$, in which case the equilibrium reaches the decentralized information optimum, and the provision of any type of information is welfare-improving. If $\chi > 1$, individual incentives put too much weight on action dispersion and the equilibrium rule puts too much weight on public signals. If this distortion is sufficiently large (if $\chi > 2$), better public information may be welfare-reducing. If $\chi < 1$, individual incentives put too little weight on action dispersion and the equilibrium rule puts too much weight on private signals. If $\chi < \frac{1}{2}$, this distortion is so large that better private information may be welfare-reducing.

When $\rho \neq 0$, $\widehat{\chi}$ modifies χ to account for the interaction between the common knowledge inefficiency ρ and the informational distortion χ. If $\chi = 1$, $\widehat{\chi} = \chi = 1$ and welfare is increasing in all signal precisions, and the decentralized information optimum merely scales up the equilibrium decision rule by ρ to correct for the common knowledge inefficiency. If $\chi \neq 1$, the decentralized information optimum in addition adjusts the relative weights on public and private information to correct for the wedge between the private and the social costs of action dispersion and aggregate volatility; since these two effects may offset each other, the absolute comparison between the equilibrium and the decentralized information optimum remains ambiguous. Different scenarios arise for the welfare effects of public and private information provision, depending on whether there is over- or under-reaction in the common knowledge equilibrium: If $\rho > 0$, i.e. if there is a positive aggregate externality, the negative effect of more precise informaton that may result from a distortion in the volatility-dispersion tradeoff is mitigated by the fact that better information improves the overall response to shocks (to see this, note that either $1 > \widehat{\chi} > \chi$, if $1 > \chi$, or $1 < \widehat{\chi} < \chi$, if $1 < \chi$). If $\rho < 0$, i.e. if there is a negative externality, then more precise information overall increases the cost of this aggregate externality, which further increases any negative welfare effect that may be due to a distortion χ in the volatility-dispersion tradeoff (to see this note that, if $\rho < 0$, then either $1 > \chi > \widehat{\chi}$, if $1 > \chi$, or $1 < \chi < \widehat{\chi}$ if $1 < \chi$.[11]

[11] Note that the present analysis did not rely on the sign or the magnitude of the complementarity coefficient r, as long as $r < 1$. The present results therefore also apply to environments in which $r < 0$, i.e. individual actions are

5.3 Applications

To conclude, I discuss how the results presented in section 2-4 of this paper, as well as in MS and AP map into this quadratic model.

This paper: In the model of incomplete nominal adjustment developed in sections 2-4, the social objective was given by (21) as

$$\begin{aligned} \mathbb{U} &= -\Lambda - \frac{1}{\delta}\exp\left\{-\delta\Lambda + \frac{\delta^2}{2}W(\lambda_1,\lambda_2)\right\}, \\ \text{where } W(\lambda_1,\lambda_2) &= \frac{(1-\lambda_1-\lambda_2)^2}{\tau_\mu} + \frac{(\lambda_2)^2}{\tau_v} + \frac{\theta}{1-r}\frac{(\lambda_1)^2}{\tau_\xi}. \end{aligned}$$

The first two terms of $W(\lambda_1,\lambda_2)$ account for the cost of aggregate volatility, while the third accounts for the cost of action dispersion. This objective maps into the general model for parameters $k_1 = 1-\theta < 0$ and $k_2 = k_3 = 0$, which implies that $\chi = \theta^{-1}$ and $\rho = 0$. There is a negative externality associated with action dispersion; individual decisions do not fully internalize the social cost of action dispersion, but they fully internalize the cost of aggregate volatility. Equilibrium prices rely too much on private information, relative to the social optimum. When θ is sufficiently large, providing better private information may reduce welfare. The economic reason for this externality is that firms do not internalize the cost of strategic risk that is imposed on others by conditioning prices on private information.

Morris and Shin (2002): In MS, individual objectives are given by:

$$\begin{aligned} u(a_i,\mathbf{a}(\cdot),\mu) &= -(1-r)(a_i-\mu)^2 - r\left(L_i - \overline{L}\right) \\ \text{where } L_i &= \int_0^1 (a_i-a_j)^2\,dj \text{ and } \overline{L} = \int_0^1 L_j dj, \end{aligned}$$

while the social welfare function is

$$W(\mathbf{a}(\cdot),\mu) = \int_0^1 u(a_i,\mathbf{a}(\cdot),\mu)\,di = -(1-r)\int_0^1 (a_i-\mu)^2\,di$$

The stark modelling assumption in MS is that the coordination motive enters individual preferences through a zero-sum component that washes out in the social welfare function. Rewriting

strategic substitutes.

L_i, $\overline{L}$ and $u(a_i, \mathbf{a}(\cdot), \mu)$ yields

$$\begin{aligned} L_i &= (a_i - a)^2 + \int_0^1 (a_j - a)^2 \, dj \text{ and } \overline{L} = 2\int_0^1 (a_j - a)^2 \, dj \\ u(a_i, \mathbf{a}(\cdot), \mu) &= -(1-r)(a_i - \theta)^2 - r(a_i - a)^2 + r\int_0^1 (a_j - a)^2 \, dj \end{aligned}$$

The objective function in MS maps into the present framework as the case where $k_1 = r$ and $k_2 = k_3 = 0$, which implies that $\chi = (1-r)^{-1} > 1$. Relative to the social tradeoff, agents place too little weight on the costs of aggregate volatility, and too much weight on dispersion. The equilibrium hence relies too heavily on public information, and too little on private information. Furthermore, when $r > 1/2$, better public information may be welfare-reducing.[12]

Angeletos and Pavan (2004 a,b): AP consider a static investment model with technological spillovers. In their set-up, agents take investment decisions a_i to maximize profits, which are given by

$$u(a_i, \mathbf{a}(\cdot), \mu) = 2Aa_i - a_i^2$$

where the productivity parameter A is given by $A = ra + (1-r)\mu$, and $r < 1/2$. μ is interpreted as an aggregate technology shock. This objective can be rewritten as:

$$u(a_i, \mathbf{a}(\cdot), \mu) = -(1-r)(a_i - \mu)^2 - r(a_i - a)^2 + r(a - \mu)^2 + 2r\mu(a - \mu) + \mu^2. \tag{30}$$

AP's model translates into the present framework as $k_1 = 0$, $k_2 = \frac{r}{1-r}$ and $k_3 = \frac{r}{1-r}$, implying $\chi = \frac{1-2r}{1-r} < 1$, $\rho = \frac{r}{1-2r}$, and $\widehat{\chi} = \frac{1}{1+r}$. The model with investment complementarities generates under-investment under common knowledge, and an informational distortion under heterogeneous information, which leads strategies to attribute too much weight to the costs of aggregate volatility, relative to the social planner. The decentralized information optimum corrects for the two inefficiencies as discussed above; however, $\lambda_1^{eq} < \lambda_1^*$ and $\lambda_2^{eq} < \lambda_2^*$, i.e. the equilibrium responds too little to both types of signals, and equilibrium welfare is increasing in both signal precisions. In contrast to MS and this paper, the investment complementarity in AP not only distorts the private weights on action dispersion and aggregate volatility, but also implies that the common knowledge equilibrium is inefficient. While the first distortion shifts strategies towards more reliance on private information, the common knowledge inefficiency dominates the informational distortion in the

[12] While Morris and Shin motivate their choice of an objective function by an intuitive appeal to Keynes' analogy of portfolio investment decisions with a beauty contest, they do not model such an investment game explicitly.

welfare discussion and implies that the equilibrium reacts too little to either signal. Consequently, improving either type of information is welfare-enhancing, because it increases the response to a shock in μ, which reduces underinvestment.[13]

6 Concluding Remarks

Following Woodford (2002), this paper has developed a fully-microfounded model of nominal adjustment, in which heterogeneous information among monopolistically competitive firms increases persistence of nominal shocks and amplifies the impact of public signals on prices. The paper then discusses the welfare effects of public and private information provision in terms of a tradeoff between output volatility and price dispersion. Overall, the welfare effects of informational dispersion are determined by the latter: Improving private information may be welfare-reducing, but better public information always increases welfare. This result follows from an information externality that prevents firms from internalizing the full social cost of price dispersion for the allocation of resources, and thereby creates a wedge between the private and the social benefits of conditioning prices on private information.

Finally, to relate the present analysis to contrasting results by Morris and Shin (2002) and similar results by Angeletos and Pavan (2004 a, b), the last part of this paper explores the tradeoff between volatility and dispersion in abstract terms. Within this general framework, the welfare effects of improved public or private information are traced to distortions in the volatility-dispersion tradeoff and to externalities that generate inefficiencies, even when shocks are common knowledge.

While the model does not address policy issues, it raises new questions in that direction. Traditionally, the importance of transparent information provision for monitoring purposes is emphasized within a principal-agent framework with time-inconsistency in the absence of policy commitment. Recently this literature has received renewed attention in contributions that focus on the interplay between a privately informed policy maker, and a homogeneously informed private sector (see, for example, Athey, Atkeson and Kehoe 2003, and Moscarini 2003). When private sector agents are heterogeneously informed, there are additional channels through which a privately informed policy

[13] In an alternative version, AP consider the case where there are investment complementarities, but no aggregate under-investment. In that case, the equilibrium conditions too heavily on private information and too little on public information, and welfare is non-monotonic in the precision of private information, when $r \in \left(\frac{1}{3}, \frac{1}{2}\right)$. This case maps into the present model, when $k_2 = \frac{r}{1-r}$ and $k_1 = k_3 = 0$, so that $\chi = \frac{1-2r}{1-r} < 1$ and $\rho = 0$.

maker may influence market outcomes: First, the disclosure of such private information through public announcements or policy actions provides public information which may improve the coordination of private decisions among heterogeneously informed market participants.[14] Second, even without conveying information, policies interventions which alter the distortions in the tradeoff between volatility and dispersion or affect the costs of information acquisition and processing have novel implications for an economy with heterogeneously informed agents. I leave an analysis of these questions to future work.

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7 Appendix: Proofs

Proof of lemma 1. Substituting the budget constraint, the household's optimization problem can be rewritten as:

$$U_t = \max_{\left\{C_{t+\tau}, M^d_{t+\tau}\right\}_{\tau=0}^{\infty}} \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \left(\log C_{t+\tau} - \frac{P_{t+\tau} C_{t+\tau}}{W_{t+\tau}} - \frac{M^d_{t+\tau} - M^d_{t+\tau-1} - T_{t+\tau}}{W_{t+\tau}} + \frac{\Pi_{t+\tau}}{W_{t+\tau}} \right) \right]$$

$$\text{s.t. } P_{t+\tau} C_{t+\tau} \leq M^d_{t+\tau-1} + T_{t+\tau}$$

Let $\theta_{t+\tau}$ denote the Lagrange multiplier on the period $t+\tau$ Cash-in-Advance Constraint. The resulting first-order conditions are:

$$\begin{aligned} \frac{1}{P_{t+\tau} C_{t+\tau}} &= \frac{1}{W_{t+\tau}} + \theta_{t+\tau} \\ \frac{1}{W_{t+\tau}} &= \beta \mathbb{E}_t \left(\frac{1}{W_{t+\tau+1}} + \theta_{t+\tau+1} \right) \end{aligned}$$

while the transversality condition is

$$\lim_{\tau \to \infty} \beta^\tau \mathbb{E}_t \left(\frac{M^d_{t+\tau}}{W_{t+\tau}} \right) = 0.$$

We check that the proposed solution $C_{t+\tau} = \frac{M^s_{t+\tau}}{P_{t+\tau}}$, $W_{t+\tau} = \gamma M^s_{t+\tau}$ and $M^d_{t+\tau} = M^s_{t+\tau}$ satisfies the first-order conditions and transversality condition. For this solution, we find that $\theta_{t+\tau} = \frac{1}{P_{t+\tau} C_{t+\tau}} - \frac{1}{W_{t+\tau}} = \frac{1}{M^s_{t+\tau}} \left(1 - \frac{1}{\gamma}\right) > 0$, so that the Cash-in-Advance constraint is binding in every period and in every state (justifying that $C_{t+\tau} = \frac{M^s_{t+\tau}}{P_{t+\tau}}$). Next, we check that the second first-order condition is satisfied. To see that this is the case, note that

$$\begin{aligned} \frac{1}{W_{t+\tau}} - \beta \mathbb{E}_t \left(\frac{1}{W_{t+\tau+1}} + \theta_{t+\tau+1} \right) &= \frac{1}{\gamma M^s_{t+\tau}} - \beta \gamma \mathbb{E}_t \left(\frac{1}{\gamma M^s_{t+\tau+1}} \right) \\ &= \frac{1}{\gamma M^s_{t+\tau}} \left[1 - \beta \gamma \mathbb{E}_t \left(\frac{M^s_{t+\tau}}{M^s_{t+\tau+1}} \right) \right] \\ &= \frac{1}{\gamma M^s_{t+\tau}} \left[1 - \beta \gamma e^{\frac{1}{2\tau_\mu}} \right] = 0 \end{aligned}$$

Thus, the two first-order conditions are satisfied. It is also immediate that the transversality condition holds, and that the money market clears. ■

Proof of proposition 1. With homogeneous information, all firms set identical prices,

$$\log p^i_t = \log P_t = \frac{1}{\delta} \log \left(\frac{\gamma \theta}{\theta - 1} \right) + E\left(m_t \mid m_{t-1}, z_t\right) + \frac{\delta}{2} V\left(m_t \mid m_{t-1}, z_t\right),$$

where $E(m_t \mid m_{t-1}, z_t) = m_{t-1} + \frac{\tau_v}{\tau_\mu + \tau_v}(z_t - m_{t-1})$ and $V(m_t \mid m_{t-1}, z_t) = \frac{1}{\tau_\mu + \tau_v}$. ■

Equilibrium characterization (proposition 2). A standard approach for characterizing the equilibrium pricing rule is to conjecture and verify a linear pricing rule for $\log p_t^i$. Here, I provide an alternative which highlights the role of higher-order expectations about fundamentals and provides some insights into the effects of private and public information for dynamic price adjustment, along lines similar to Woodford (2002), MS, and Hellwig (2002). To begin, it is useful to rewrite (14). Using the definition of P_t, we have

$$(1-\theta)\log P_t = \log \int (1-\theta)\log p_t^i di = (1-\theta)\overline{\log p_t} + \frac{(1-\theta)^2}{2}\Sigma_p^2$$

and therefore

$$\begin{aligned} \log p_t^i &= (1-r)\Gamma_0 + (1-r)E_t^i(m_t) + rE_t^i\left(\overline{\log p_t}\right), \\ \text{where } \Gamma_0 &= \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{1}{2\delta}V - r\frac{\theta-1}{2(1-r)}\Sigma_p^2. \end{aligned}$$

Σ_p^2 denotes the conjectured cross-sectional variance of prices, and $\overline{\log p_t} \equiv \int \log p_t^i di = \log P_t + \frac{\theta-1}{2}\Sigma_p^2$, almost surely. Taking averages and substituting forward, $\log p_t^i$ is given by

$$\log p_t^i = \Gamma_0 + (1-r)\sum_{s=0}^{\infty} r^s E_t^i\left[\overline{E}_t^{(s)}(m_t)\right] + \lim_{k\to\infty}\left[r^{k-1}\overline{E}_t^{(k)}(m_t) + r^k\overline{E}_t^{(k)}\left(\overline{\log p_t}\right)\right]$$

where $\overline{E}_t^{(s)}(m_t)$ is recursively defined by: $\overline{E}_t^{(0)}(m_t) = m_t$; $\overline{E}_t(m_t) = \int E_t^i(m_t)\, d\Phi\left(x_t^i \mid m_t\right)$; and $\overline{E}_t^{(s)}(m_t) = \overline{E}_t\left[\overline{E}_t^{(s-1)}(m_t)\right]$. It follows from Theorem 1 in Samet (1998) that $\lim_{k\to\infty}\overline{E}_t^{(k)}\left(\overline{\log p_t}\right) = E_t\left[\log p_t \mid z_t, m_{t-1}\right]$ and $\lim_{k\to\infty}\overline{E}_t^{(k)}(m_t) = E_t\left[m_t \mid z_t, m_{t-1}\right]$, so that $\lim_{k\to\infty} r^k\overline{E}_t^{(k)}\left(\overline{\log p_t}\right) = 0$ and $\lim_{k\to\infty} r^{k-1}\overline{E}_t^{(k)}(m_t) = 0$, hence $\log p_t^i$ satisfies

$$\log p_t^i = \Gamma_0 + (1-r)\sum_{s=0}^{\infty} r^s E_t^i\left[\overline{E}_t^{(s)}(m_t)\right].$$

We have thus transformed the fixed point problem of forecasting equilibrium prices into one of determining a weighted average of higher-order expectations; i.e. i's expectation of the average expectation of the average expectation ... (repeat s times) ... of m_t. The solution to this problem is uniquely determined from the information structure. When all information is common, the law of iterated expectations implies that the higher-order expectations all collapse to the common first-order expectation. However, the law of iterated expectations does not apply to average expectations in the presence of information heterogeneity. Nevertheless, we can characterize all higher-order expectations individually. The first-order expectation of m_t is given by:

$$E_t^i\left(m_t\right)=m_{t-1}+\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\left(x_t^i-m_{t-1}\right)+\frac{\tau_v}{\tau_\mu+\tau_v+\tau_\xi}\left(z_t-m_{t-1}\right)$$

Averaging over i, we find the first-order average expectation:

$$\overline{E}_t\left(m_t\right)=m_{t-1}+\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\left(m_t-m_{t-1}\right)+\frac{\tau_v}{\tau_\mu+\tau_v+\tau_\xi}\left(z_t-m_{t-1}\right)$$

and the second-order expectation:

$$\begin{aligned}
E_t^i\left[\overline{E}_t\left(m_t\right)\right] &= m_{t-1}+\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\left[E_t^i\left(m_t\right)-m_{t-1}\right]+\frac{\tau_v}{\tau_\mu+\tau_v+\tau_\xi}\left(z_t-m_{t-1}\right)\\
&= m_{t-1}+\left(\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)^2\left(x_t^i-m_{t-1}\right)+\left[1+\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right]\frac{\tau_v}{\tau_\mu+\tau_v+\tau_\xi}\left(z_t-m_{t-1}\right)\\
&= m_{t-1}+\left(\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)^2\left(x_t^i-m_{t-1}\right)+\frac{\tau_v}{\tau_\mu+\tau_v}\left(1-\left(\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)^2\right)\left(z_t-m_{t-1}\right)
\end{aligned}$$

Successively averaging and substituting forward, we find:

$$E_t^i\left[\overline{E}_t^{(s)}\left(m_t\right)\right]=m_{t-1}+\left(\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)^{s+1}\left(x_t^i-m_{t-1}\right)+\frac{\tau_v}{\tau_\mu+\tau_v}\left(1-\left(\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)^{s+1}\right)\left(z_t-m_{t-1}\right)$$

And summing over s:

$$\begin{aligned}
&(1-r)\sum_{s=0}^{\infty}r^sE_t^i\left[\overline{E}_t^{(s)}\left(m_t\right)\right]\\
=\ & m_{t-1}+(1-r)\sum_{s=0}^{\infty}r^s\left(\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)^{s+1}\left(x_t^i-m_{t-1}\right)\\
&+(1-r)\sum_{s=0}^{\infty}r^s\left[1-\left(\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)^{s+1}\right]\frac{\tau_v}{\tau_\mu+\tau_v}\left(z_t-m_{t-1}\right)\\
=\ & m_{t-1}+(1-r)\frac{\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}}{1-r\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}}\left(x_t^i-m_{t-1}\right)+\left[1-(1-r)\frac{\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}}{1-r\frac{\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}}\right]\frac{\tau_v}{\tau_\mu+\tau_v}\left(z_t-m_{t-1}\right)\\
=\ & m_{t-1}+\frac{(1-r)\,\tau_\xi}{\tau_\mu+\tau_v+(1-r)\,\tau_\xi}\left(x_t^i-m_{t-1}\right)+\frac{\tau_v}{\tau_\mu+\tau_v+(1-r)\,\tau_\xi}\left(z_t-m_{t-1}\right)
\end{aligned}$$

This characterization illustrates the shift in weights away from private signals towards the prior and the public signal in the higher-order expectations. The reason for this shift can be seen by considering the inference problem underlying higher-order expectations: The impact of the public information on the other firms' expectations is perfectly forecastable, because public information is shared by all firms. The private signal, only enters into the forecast of the other firms' private signals, and hence its weight is declining in higher orders of expectations.

To complete the characterization, I solve for the remaining undetermined coefficients. From the coefficients of the pricing rule, it immediately follows that $\Sigma_p^2 = \frac{(1-r)^2\tau_\xi}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2}$. Using the fact that $(\theta-1)(1-r) = \frac{r\delta}{\delta-1}$ and $1+\frac{r}{\delta-1} = \theta(1-r)$, we have

$$\begin{aligned}
V &= \delta^2 V_t^i\left[\log\left(M_t^s P_t^{\theta-1}\right)\right] - V_t^i\left[\log\left(P_t^{\theta-1}\right)\right] \\
&= \frac{\delta^2}{\tau_\mu+\tau_v+\tau_\xi}\left[1+\frac{(\theta-1)(1-r)\tau_\xi}{\tau_\mu+\tau_v+(1-r)\tau_\xi}\right]^2 - \frac{1}{\tau_\mu+\tau_v+\tau_\xi}\left[\frac{(\theta-1)(1-r)\tau_\xi}{\tau_\mu+\tau_v+(1-r)\tau_\xi}\right]^2 \\
&= \frac{\delta^2}{\tau_\mu+\tau_v+\tau_\xi}\left[\frac{\tau_\mu+\tau_v+\tau_\xi+\frac{r}{\delta-1}\tau_\xi}{\tau_\mu+\tau_v+(1-r)\tau_\xi}\right]^2 - \frac{\delta^2}{\tau_\mu+\tau_v+\tau_\xi}\left[\frac{\frac{r}{\delta-1}\tau_\xi}{\tau_\mu+\tau_v+(1-r)\tau_\xi}\right]^2 \\
&= \delta^2\frac{\tau_\mu+\tau_v+\tau_\xi}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2}\left[\left(1+\frac{\frac{r}{\delta-1}\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)^2 - \left(\frac{\frac{r}{\delta-1}\tau_\xi}{\tau_\mu+\tau_v+\tau_\xi}\right)^2\right] \\
&= \delta^2\frac{\tau_\mu+\tau_v+\tau_\xi+2\frac{r}{\delta-1}\tau_\xi}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2}
\end{aligned}$$

Now, for Γ and Γ_0,

$$\begin{aligned}
\Gamma &= \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{1}{2\delta}V - \frac{1}{1-r}\frac{\theta-1}{2}\Sigma_p^2 \\
&= \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{\delta}{2}\frac{\tau_\mu+\tau_v+\tau_\xi+2\frac{r}{\delta-1}\tau_\xi}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2} - \frac{\theta-1}{2}\frac{(1-r)\tau_\xi}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2} \\
&= \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{\delta}{2}\frac{\tau_\mu+\tau_v+\tau_\xi+\frac{r}{\delta-1}\tau_\xi}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2} \\
&= \frac{1}{\delta}\log\left(\frac{\gamma\theta}{\theta-1}\right) + \frac{\delta}{2}\frac{\tau_\mu+\tau_v+\theta(1-r)\tau_\xi}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2}.
\end{aligned}$$

■

Proof of Theorem 1. Apart from the non-monotonicity results, the theorem follows immediately from proposition 2. For the non-monotonicity results, note that for generic $x_1, x_2 > 0$,

$$\frac{\partial}{\partial x_1}\left[\ln x_1 - 2\log(x_1+x_2)\right] = \frac{1}{x_1} - \frac{2}{x_1+x_2} = \frac{x_2-x_1}{x_1(x_1+x_2)}$$

which is positive if $x_1 < x_2$ and negative otherwise. For part 5,

$$\begin{aligned} & \frac{\partial}{\partial \tau_\xi}\left[\ln\left(\tau_\mu+\tau_v+\theta\left(1-r\right)\tau_\xi\right)-2\log\left(\tau_\mu+\tau_v+\left(1-r\right)\tau_\xi\right)\right] \\ = & \frac{\theta\left(1-r\right)}{\tau_\mu+\tau_v+\theta\left(1-r\right)\tau_\xi}-\frac{2\left(1-r\right)}{\tau_\mu+\tau_v+\left(1-r\right)\tau_\xi} \\ = & \frac{\left(1-r\right)\left[\theta\left(\tau_\mu+\tau_v+\left(1-r\right)\tau_\xi\right)-2\left(\tau_\mu+\tau_v+\theta\left(1-r\right)\tau_\xi\right)\right]}{\left(\tau_\mu+\tau_v+\theta\left(1-r\right)\tau_\xi\right)\left(\tau_\mu+\tau_v+\left(1-r\right)\tau_\xi\right)} \end{aligned}$$

which is positive, if and only if $(\theta-2)(\tau_\mu+\tau_v) > \theta(1-r)\tau_\xi$, and negative otherwise. ■

Derivation of equation (21). For any linear pricing rule given by (20), prices are normally distributed in the cross section, and consumption is i.i.d. normally distributed over time. Let $\widetilde{\log C}$ denote the mean of log-consumption, σ_C^2 the variance of consumption, Σ_p^2 the cross-sectional dispersion of prices, and $\overline{\log p_t}$ the mean of log-prices. In terms of the parameters of the linear pricing rule, the first three are given by:

$$\widetilde{\log C} = -\Lambda_0 + \frac{\theta-1}{2}\Sigma_p^2 = \Lambda;\ \sigma_C^2 = \left(1-\Lambda_1-\Lambda_2\right)^2\frac{1}{\tau_\mu} + \Lambda_2^2\frac{1}{\tau_v},\ \text{and}\ \Sigma_p^2 = \Lambda_1^2\frac{1}{\tau_\xi}$$

We can express the per period expected utility in terms of $\widetilde{\log C}$, σ_C^2 and Σ_p^2 as:

$$\begin{aligned} \mathbb{U} &= \widetilde{\log C} - \mathbb{E}_{t-1}\left(n_t\right) = \widetilde{\log C} - \mathbb{E}_{t-1}\left(\frac{1}{\delta}\int_0^1\left[c\left(p_t^i\right)\right]^\delta di\right) \\ &= \widetilde{\log C} - \mathbb{E}_{t-1}\left(\frac{1}{\delta}C_t^\delta P_t^{\delta\theta}\int_0^1\left(p_t^i\right)^{-\theta\delta}di\right) \\ &= \widetilde{\log C} - \mathbb{E}_{t-1}\left(\frac{1}{\delta}C_t^\delta P_t^{\delta\theta}\exp\left\{-\theta\delta\overline{\log p_t}+\frac{\left(\theta\delta\right)^2}{2}\Sigma_p^2\right\}\right) \\ &= \widetilde{\log C} - \mathbb{E}_{t-1}\left(\frac{1}{\delta}C_t^\delta\exp\left\{\frac{\left(\theta\delta\right)}{2}\left[1-\theta+\theta\delta\right]\Sigma_p^2\right\}\right) \\ &= \widetilde{\log C} - \frac{1}{\delta}\exp\left\{\delta\widetilde{\log C}+\frac{\delta^2}{2}\sigma_C^2+\frac{\theta\delta^2}{2\left(1-r\right)}\Sigma_p^2\right\} \end{aligned}$$

The proof is completed by substituting the above expressions for $\widetilde{\log C}$, σ_C^2 and Σ_p^2 in terms of Λ, Λ_1 and Λ_2. ■

Proof of theorem 2. The theorem follows directly from (22). To solve for (22), note that

$$\frac{\delta}{2}\left[\frac{\left(1-\Lambda_1^{eq}-\Lambda_2^{eq}\right)^2}{\tau_\mu}+\frac{\left(\Lambda_2^{eq}\right)^2}{\tau_v}+\frac{\theta}{1-r}\frac{\left(\Lambda_1^{eq}\right)^2}{\tau_\xi}\right]-\Lambda^{eq} = \frac{1}{\delta}\log\left(\frac{\theta-1}{\gamma\theta}\right)$$

from which it follows that $\mathbb{U}^{eq} = \frac{1}{\delta}\left[\log\left(\frac{\theta-1}{\gamma\theta}\right) - \frac{\theta-1}{\gamma\theta}\right] - \frac{\delta}{2}\left[\frac{\tau_\mu+\tau_v+\theta(1-r)\tau_\xi}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2}\right]$. ■

Derivation of equation (23). Taking first-order conditions of (21) with respect to Λ, Λ_1, and Λ_2, one finds:

$$\begin{aligned} 1 &= \exp\left\{-\delta\Lambda + \frac{\delta^2}{2}\left[(1-\Lambda_1-\Lambda_2)^2\frac{1}{\tau_\mu} + \Lambda_2^2\frac{1}{\tau_v} + \Lambda_1^2\frac{\theta}{1-r}\frac{1}{\tau_\xi}\right]\right\} \\ (1-\Lambda_1-\Lambda_2)\frac{1}{\tau_\mu} &= \Lambda_2\frac{1}{\tau_v} \\ (1-\Lambda_1-\Lambda_2)\frac{1}{\tau_\mu} &= \Lambda_1\frac{\theta}{1-r}\frac{1}{\tau_\xi} \end{aligned}$$

Solving these conditions with respect to Λ, Λ_1, and Λ_2 yields Λ^*, Λ_1^*, and Λ_2^*, and $\Lambda_0^* = \Lambda^* + \frac{\theta-1}{2}\frac{(\Lambda_1^*)^2}{\tau_\xi}$. ■

Proof of theorem 3. Follows immediately from Propositions 2 and equation (23). ■

Proof of proposition 3. When $\chi = 1$ and $\rho = 0$, the first-order conditions of (27) w.r.t. λ_1 and λ_2 are $\frac{1}{1-r}\frac{\lambda_1}{\tau_\xi} = \frac{1-\lambda_1-\lambda_2}{\tau_\mu} = \frac{\lambda_2}{\tau_v}$, from which the proposition follows immediately. ■

Proof of theorem 3. Rewriting (28), we have

$$\begin{aligned} W^{eq} &= -(1-r)(1-k_2)\left[\frac{(1+2\rho)(\tau_\mu+\tau_v) + \left(\frac{1}{\chi}+2\rho\right)(1-r)\tau_\xi}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2}\right] \\ &= -(1-r)(1-k_2)(1+2\rho)\left[\frac{\tau_\mu+\tau_v+\frac{1}{\widehat{\chi}}(1-r)\tau_\xi}{\left[\tau_\mu+\tau_v+(1-r)\tau_\xi\right]^2}\right] \end{aligned}$$

where $\widehat{\chi} = \chi\frac{1+2\rho}{1+2\rho\chi}$. Now, taking derivatives of $\ln(-W^{eq})$ w.r.t. τ_v and τ_ξ, we have

$$\begin{aligned} \frac{\partial\ln(-W^{eq})}{\partial\tau_v} &= \frac{1}{\tau_\mu+\tau_v+\frac{1}{\widehat{\chi}}(1-r)\tau_\xi} - \frac{2}{\tau_\mu+\tau_v+(1-r)\tau_\xi} \\ &= \frac{\tau_\mu+\tau_v+(1-r)\tau_\xi - 2\left(\tau_\mu+\tau_v+\frac{1}{\widehat{\chi}}(1-r)\tau_\xi\right)}{\left(\tau_\mu+\tau_v+\frac{1}{\widehat{\chi}}(1-r)\tau_\xi\right)(\tau_\mu+\tau_v+(1-r)\tau_\xi)} \\ &= \frac{-(\tau_\mu+\tau_v) + \left(1-\frac{2}{\widehat{\chi}}\right)(1-r)\tau_\xi}{\left(\tau_\mu+\tau_v+\frac{1}{\widehat{\chi}}(1-r)\tau_\xi\right)(\tau_\mu+\tau_v+(1-r)\tau_\xi)} \end{aligned}$$

$$\begin{aligned}
\frac{\partial \ln\left(-W^{eq}\right)}{\partial\left(1-r\right)\tau_\xi} &= \frac{1}{\widehat{\chi}\left(\tau_\mu+\tau_v+\frac{1}{\widehat{\chi}}\left(1-r\right)\tau_\xi\right)}-\frac{2}{\tau_\mu+\tau_v+\left(1-r\right)\tau_\xi} \\
&= \frac{\left(\tau_\mu+\tau_v+\left(1-r\right)\tau_\xi\right)-2\widehat{\chi}\left(\tau_\mu+\tau_v\right)-2\left(1-r\right)\tau_\xi}{\widehat{\chi}\left(\tau_\mu+\tau_v+\frac{1}{\widehat{\chi}}\left(1-r\right)\tau_\xi\right)\left(\tau_\mu+\tau_v+\left(1-r\right)\tau_\xi\right)} \\
&= \frac{\left(\tau_\mu+\tau_v\right)\left(1-2\widehat{\chi}\right)-\left(1-r\right)\tau_\xi}{\left(\tau_\mu+\tau_v+\frac{1}{\widehat{\chi}}\left(1-r\right)\tau_\xi\right)\left(\tau_\mu+\tau_v+\left(1-r\right)\tau_\xi\right)}
\end{aligned}$$

Therefore, W^{eq} is increasing in τ_v, if and only if $\tau_\mu+\tau_v>\left(1-\frac{2}{\widehat{\chi}}\right)\left(1-r\right)\tau_\xi$, and increasing in τ_ξ, if and only if $\left(\tau_\mu+\tau_v\right)\left(1-2\widehat{\chi}\right)>\left(1-r\right)\tau_\xi$. ■

Derivation of equation (29) and theorem 5. Taking first-order conditions of (27) with respect to λ_1 and λ_2, one finds

$$\frac{1}{\chi\left(1-r\right)}\frac{\lambda_1}{\tau_\xi}=\frac{1-\lambda_1-\lambda_2}{\tau_\mu}+\frac{\rho}{\tau_\mu}=\frac{\lambda_2}{\tau_v}$$

These can be rearranged to find

$$\begin{aligned}
\lambda_1+\lambda_2 &= \left(1+\rho\right)\frac{\tau_v+\chi\left(1-r\right)\tau_\xi}{\tau_\mu+\tau_v+\chi\left(1-r\right)\tau_\xi} \\
\frac{1}{\chi\left(1-r\right)}\frac{\lambda_1}{\tau_\xi} &= \frac{\lambda_2}{\tau_v}
\end{aligned}$$

from which the result follows immediately. ■