A Rationale for Searching (Imprecise) Health Information

Francesca Barigozzi* and Rosella Levaggi†
February 2005
Preliminary draft

Abstract

We analyse a model of patient decision-making where anxiety about the future characterizes the patient’s utility function. Anxiety corresponds to fear of bad news and results in the patient being averse to information. First, the patient chooses the accuracy of a signal which discloses information. Then he updates his beliefs according to Bayes’s rule and chooses an action. We show that the choice of imprecise information can be optimal because it allows the patient to trade off the damage deriving from complete ignorance with the anxiety raised by the news about his health level.

JEL Classification: C7; D82; I19.
Key Words: Psychological expected utility, Bayesian updating, health information.

1 Introduction

The attitude of patients towards health care and the search for health information has dramatically changed in the past few years. This phenomenon is the result of several factors, perhaps the most important being the fact that patients have now easy access to information. A spectacular example is the Internet where a huge amount of health information is available at negligible cost.¹ More generally, the media are increasingly aware of health related matters and specific magazines and programmes are devoted to explain how to preserve our "health stock" and how the most popular diseases can be cured.²

*Department of Economics, University of Bologna, Strada Maggiore 45, 40125 Bologna (Italy). E-mail: barigozz@spbo.unibo.it.
†Department of Economics, University of Brescia, Via S. Faustino, 74b, 25122 Brescia (Italy). E-mail: levaggi@eco.unibs.it;
¹Interesting issues that a decision-maker faces when searching information on the Internet are that of "information overload" and "information processing". These issues are not addressed in this paper.
²In all that cases the relevant cost for the decision-maker is in term of opportunity cost of time devoted to in searching for and processing information.
While the dissemination of general health information can be obviously useful in the case of preventive behaviors\(^3\), we should be concerned about the way not-targeted messages about diseases affect individuals. In particular, since individuals are generally ignorant on health matters, the dissemination of specific and/or general information does not necessarily allow the patient to form a precise diagnosis or to give a correct interpretation to his symptoms. A physician, on the contrary, is more efficient in this process so that his diagnosis is always more accurate. Moreover, the physician’s provision of treatment is necessarily anticipated by a medical advise and the diagnosis is generally transmitted by the physician to the patient such that, in principle, ill consumers should be completely indifferent to the dissemination of additional and general information.

In this environment, a set of interesting questions naturally arise. What is the rationale for a patient to search for information from sources other than the physician? In all the situations where the patient cannot substitute for the physician’s advise, as in the case of serious disease, is additional health information useful? Does it really increase patient’s utility?

The topic of information acquisition by a decision maker is certainly not a new one and a vast literature exists (among the others, see Moscarini and Smith, 2001). The most common issue in this case refers to the optimal stopping rule in information acquisition by comparing the cost of searching for information and processing it, against the benefits of information in terms of more accurate decisions. With this respect, the dissemination of general information, or the reduction of the costs in procuring it, is clearly a desirable process. However, when health matters are considered, fear about future health becomes relevant and information can lead to anxiety. With this respect a new issue then arises: as medical evidence indicates, fear deeply influences patients’ behavior when searching for information.

Caplin and Leahy (2001) extend expected utility theory to situations in which agents experience feelings of anticipation prior to the resolution of uncertainty. They show how these anticipatory feelings may influence decision makers. In particular they provide an example from portfolio theory to illustrate the potential impact of anticipation on portfolio decisions and asset prices. Recently, their framework has been used by Kozsegì (2003) to study the problem of anxiety related to future health and health behavior. According to such a framework, the standard model of choice under uncertainty must be enriched by adding beliefs to the description of consequences, in order to capture anticipatory feelings such as anxiety or hopefulness. In particular, decision makers have expected-utility preferences over a space of outcomes, whose description consists of the decision maker’s action, the state of nature but also the decision maker’s belief.

The aim of our analysis is to investigate how anxiety influences patients’ attitude towards information search. Using the extended expected-utility model

---

3Some empirical studies show that the dissemination of health information about appropriate life-style is welfare improving. See, for example, Wagner, Hu and Hibbard (2001) and references inside.
of Caplin and Leahy (2001) we consider a Bayesian consumer searching for health information and show that anxiety costs might induce the patient to choose information sources less precise than a physician advise or a medical test. In fact, by deliberately choosing an imprecise signal (i.e. an imprecise source of information), the patient is able to decrease the level of anxiety induced by information.4

Patients can be averse or not to information. When they are information averse they suffer from anxiety if information is disclosed. In other words information averse patients dislike bad news more than they like good news. Modelling anxiety as utility derived from the patient’s expectations about her future physical condition, the patients’ utility function is not defined over physical outcomes as in standard analysis, but instead only over beliefs about physical outcomes.

In our model, if the patient stays ignorant he pays the cost of selecting an inefficient action. In fact, the level of consumer’s health depends on nature and on a specific health maintenance action that, in order to be most effective, has to be state-contingent. In addition, since the patient derives utility directly from his beliefs, he must also consider how the information he gathers will affect those beliefs. Furthermore, we assume that decision-makers are able to indirectly influence their beliefs through the choice of signals. In our model signals can be interpreted as health information sources characterized by different content precision. The patient can decide to stay ignorant or he can decide to make a medical test or to go to the physician such that information is fully disclosed. Importantly, a third alternative is possible: the patient can search for health information from sources other than the physician. For example, by searching for information disseminated on the Internet the patient is able to choose different accuracy of information.

With this setup, we can address the analysis of the trade-off between the physical benefits and emotional costs of information for patients who are information averse. We show that the choice of imprecise information can be optimal because it allows the patient to trade off the damage deriving from complete ignorance with the anxiety raised by the news about his health level. The peculiarity of our model is in the patient choosing the precision of a signal. The signal provides information on the patient’s health status. In particular, by observing the realization of the signal, the patient updates his subjective beliefs on his health status and chooses the preferred action.

Our work is closely related to and borrows from two recent papers by Koszegi (2003) and Eliaz and Spiegler (2004) (respectively K and E-S, henceforth), which our analysis complements in several directions.

Similarly to K, in our model ignorance is costly in terms of inefficient actions. However, deriving utility directly from his belief, if the patient is information

4In a different setup, Caplin and Eliaz (2003) incidentally reach a similar conclusion as for the test for AIDS: “while there are strong health-based incentives to test for AIDS, fear may override these incentives. Our resolution of the problem is to decrease the informativeness of a bad test result, mitigating the fear of bad news, and thereby allowing the health-based incentives to reassert their primacy.”
averse, he might refuse useful information that is very cheap because of fear of bad news.

As in E-S, we allow the patient to update beliefs on health status by acquiring information. Closely related to our analysis and in a richer setup with respect to the one we study, E-S investigate whether "an enriched expected utility model, in which a Bayesian decision-maker’s belief is an argument in his vNM utility function, explain anomalous choices of information sources that the usual expected utility model cannot explain".

In the previous discussion we have analyzed the information acquisition process by an "anxious" patient. However, it should be noticed that, as already pointed out by several authors (see for example Caplin and Leahy, 2001), a similar problem may be relevant in other contexts with uncertainty where an unskilled decision maker may decide to rely on personally acquired information or, alternatively, on experts’ advice, such as, for example, in the case of the decision process concerning households’ financial plans.

The paper is organized as follows. In Section 1 we present the model setup introducing and formally stating the concept of anxiety. In Section 2 we discuss the agent’s or patient’s decision process and discuss the main results. Section 3 provides some concluding remarks.

2 The model

Health status can assume two values: $w = \{w_1, w_2\}$, with $w_1 > w_2$. Since $w_1$ is associated to greater health, it corresponds to the preferred state of nature. The (subjective) prior probabilities of the states $w_1$ and $w_2$ respectively are $p$ and $1 - p$. As in E-S, whom the notation we borrows from, a signal is a random variable, which can take two values, $s_1$ and $s_2$. A signal is characterized by a pair of conditional probabilities $(q_1, q_2)$, where $q_i \in \left[\frac{1}{2}, 1\right]$ is the probability of observing the realization $s_i$ conditional on the state being $w_i$: $q_i = \text{prob}(s_i | w_i)$. Thus, when $q_1 = q_2 = 1$ the signal is fully informative, while when $q_1 = q_2 = \frac{1}{2}$ the signal is fully uninformative. The patient’s choice of signals at every prior $p$ is rational. Therefore, the patient’s attitude to information can be summarized by a profile of preference relations $(\succsim_p)_{p \in (0, 1)}$ over the set of signals $\left[\frac{1}{2}, 1\right] \times \left[\frac{1}{2}, 1\right]$. When the patient chooses a signal, he anticipates that he will update his beliefs according to Bayes’ rule upon observing the signal’s realization. In the following we assume that $q_1 = q_2 = q$, that is a signal is simply characterized by one conditional probability $q$ (see figure 1).

Insert figure 1 about here

A signal $q$ and a prior $p$ generate the following probability distribution
\( \sigma(p, q) \) over the posterior probability of \( w_1 \):

\[
\begin{array}{ccc}
s & \text{prob}(s_i) & \text{prob}(w_1|s_i) \\
s_1 & pq + (1 - p)(1 - q) & \frac{pq}{pq + (1 - p)(1 - q)} \\
s_2 & p(1 - q) + (1 - p)q & \frac{pq}{p(1 - q) + (1 - p)q}
\end{array}
\]  \quad (T1)

As E-S observe, when a decision-maker chooses a signal, he effectively chooses a lottery over his posterior beliefs. As a consequence one can interpret choices of signals at a given prior as a revealed preference over such lotteries. In particular, given two signals \( q \) and \( r \),

\( q \succeq r \iff U(p, q) \geq U(p, r) \)

where for every \( t \in \{q, r\} \):

\[
U(p, t) = \pi u\left(\frac{pt}{\pi}\right) + (1 - \pi) u\left(\frac{p(1 - t)}{1 - \pi}\right)
\]

and \( \pi = pt + (1 - p)(1 - t) \)

where \( u : [0, 1] \to \mathbb{R} \) is a continuous and increasing vNM utility function over posterior beliefs.\(^5\) As E-S show, the assumption that only posterior beliefs enter the patient’s vNM utility function simplifies the notation and entails no loss of generality. In fact, standard expected utility (over physical outcome) can be rationalized by expected utility over beliefs. Let us consider a decision-maker who takes an action \( t \) upon observing a signal’s realization and suppose that the action \( t \) affects physical utility according to the function \( v(w_i, t) \). For every prior \( p \) and signal \( q \), \( U(p, q) \) is equal to the decision-maker’s indirect expected utility from physical outcomes when he observes \( q \). In fact expected utility is maximized w.r.t. the action \( t \) and indirect expected utility \( E[v(w_i, t')] \) finally depends on posteriors. Thus, a decision-maker whose objective is to maximize expected utility (where the expectation is taken w.r.t \( q \)) will choose signals so as to maximize the expectation of \( u \) (see E-S page 14).

Many empirical studies show that information aversion seems to well describe patients’ attitude towards information on their health status.\(^6\) Thus, we assume that the patient is information averse. Since there exists no trade-off between the physical benefits and emotional costs of information for patients who are not information averse, this also represents the most interesting case. In our simple model with two possible states of nature, \( w_1 \) and \( w_2 \), this results in \( U(p, \frac{1}{2}) > U(p, 1) \) or \( \frac{1}{2}u(p) + \frac{1}{2}u(0) = u(p) > pu(1) + (1 - pu(0)) \). In other words, starting from the prior \( p \), the gain in utility obtained from good news (\( \text{prob}(w_1|s_1) = 1 \))

\(^5\)In E-S, the unique restriction imposed on the function \( u(\cdot) \) is continuity. Since we are interested in interpreting states of nature as health outcomes and \( w_1 > w_2 \), we also assume that \( u(\cdot) \) is increasing such that \( u(1) > u(0) \).

\(^6\)See for example Lerman et al. (1998) and the references cited in K.
is lower than the utility loss derived from bad news (\(\text{prob}(w_1|s_2) = 0\)). This holds when the function \(u(\cdot)\) is concave.\(^7\) See figure 2 below.

As in K, in period 1, after observing the realization of the signal\(^8\), the patient has to take some action \(t\). Such an action has no effects on period 1, it only affects utility in period 2. We do not consider utility from physical outcome but only utility from the anticipation of health in period 2.\(^9\)

Specifically, the patient’s first period utility function takes the form:

\[
z(E[v(w_i, t)]|\text{patient’ information})
\]

where the function \(z(\cdot)\) is concave.\(^10,11\) In other words, our patient cares only about his emotions in period 1. As K, page 1076, observes: "a more plausible starting point is that the patient cares about both anticipatory utility in period 1 and actual health in period 2. But such a model would generate identical behavior to one of the above types, because the patient’s concern for future health can be included in his anticipatory utility". Each state of nature is associated with some optimal action; failure to take it entails a loss. The patient chooses in period 0 the amount of costless information to search for (i.e. the precision of the signal: \(q\)) by anticipating that, in period 1, either the optimal action (if the signal is fully informative: \(q = 1\)) or a non-optimal one (if the signal is not fully informative: \(q < 1\)) will be taken. In period 2 health outcomes are realized. The timing of the model is described in figure 3.

In period 2, the patient’s state of health depends on his health status in period 1, \(w_i\), and the action \(t > 0\) (eventually treatment consumption) taken in period 1. The patient’s level of health in period 2 is \(v(w_i, t) = w_i - l(w_i, t)\), where \(l(w_i, t) \geq 0\) is the loss-function and \(\min_t l(w_i, t) = 0\). The interpretation of the function \(w_i - l(w_i, t)\) is as follows: depending on the patient’s health status \(w_i\), there is a maximum level of health he can achieve if the appropriate action is taken. If the choice of \(t\) is not appropriate, the patient’s health worse-off. However,

\(^7\) As E-S observe, in a dynamic model, information aversion translates in preference for late resolution of uncertainty.

\(^8\) In K, the choice is simply between a signal which is fully informative (\(q = 1\)) and a signal which is fully uninformative (\(q = \frac{1}{2}\)). The author also proves that "the decision-maker will (almost) never avoid the doctor if the visit is useful and she expects to learn little from it, but may do so if very bad news are possible" (page 1074).

\(^9\) This is the same assumption K uses in his model. It will be shown later that such assumption is crucial for our result. In fact, if the patient directly derives utility from the action \(t\) taken in period 1, then choosing the fully informative signal (\(q = 1\)) is always optimal under the assumption that prior is \(p = \frac{1}{2}\).

\(^10\) This is a simple case of Caplin and Leahy’s (2001) psychological expected utility.

\(^11\) In K, as in our model, anomalous attitude towards information are explained at a given prior concerning his health status. On the contrary, the analysis by E-S is developed for all possible prior beliefs.
the highest obtainable level of health is increasing in \( w_t \). We assume that the
loss function is quadratic: \( l(w_t, t) = (w_t - t)^2 \). As a consequence \( v(w_t, t) = w_t - (w_t - t)^2 \).
Notice that the function \( v(w_t, t) \) is concave both in \( w_t \) and \( t \).
To make the analysis simple we assume that \( p = \frac{1}{2} \). Since a signal is simply
characterized by one conditional probability \( q \) and \( p = \frac{1}{2} \), \( q \) also corresponds
to the updated probability of \( w_1 \). In other words, in our model, by choosing
the precision of the signal \( q \), the patient also chooses the posterior beliefs of
the preferred health status. Thus, table T1 becomes:

\[
\begin{array}{ccc}
  s & \text{prob}(s_i) & \text{prob}(w_t|s_i) \\
  s_1 & 1/2 & q \\
  s_2 & 1/2 & 1 - q \\
\end{array}
\]

and \( U(\frac{1}{2}, q) = \frac{1}{2} u(q) + \frac{1}{2} u(1 - q) \) (2)

When signal \( s_1 \) is observed, the patient’s posterior on \( w_1 \) is \( q \) and his utility is
\( u(q) \). On the contrary, when signal \( s_2 \) is observed, the patient’s posterior on
\( w_1 \) is \( 1 - q \) and his utility is \( u(1 - q) \). Both utilities are respectively multiplied
for the probability that signal \( s_1 \) and \( s_2 \) is observed. The patient prefers a fully
uninformative signal to a fully informative one if \( U(\frac{1}{2}, 1) \leq U(\frac{1}{2}, \frac{1}{2}) \) or \( \frac{1}{2} u(1) + \frac{1}{2} u(0) \geq u(\frac{1}{2}) = u(p) \). The opposite inequality holds if the patient prefers
a fully informative signal to a fully uninformative one. Taking into account the
optimal action \( t^* \), which inequality is verified depends on the concavity of the function \( E[v(w_t, t^*)] \). If no action \( t \) has to be taken, an information-averse
patient (with a concave \( u(\cdot) \)) will always choose no information disclosure, that
is a fully uninformative signal \( q = \frac{1}{2} \). When the action \( t \) is introduced, the
choice of \( q \) is discrete, and utility only depends on the anticipation of health
in period 2, the patient must decide whether he prefers a fully informative or
a fully uninformative signal. This is precisely the situation K investigates. He
shows in observation 1 (page 1078) that, if the patient is sufficiently information
averse (the function \( z(\cdot) \) is sufficiently concave) and a suboptimal \( t \) does not yield
an extremely low expected utility, the patient will prefer to stay ignorant. In
other word, it always exists a sufficiently concave function \( z(\cdot) \) such that the
fully uninformative signal is preferred. In our model we allow the choice of \( q \) to
be continuous and we show that an intermediate level of information disclosure
\( (\frac{1}{2} < q < 1) \) can be optimal.

2.1 The choice of action \( t \) in period 1

By solving for backward induction, we look for the optimal action \( t \) given that
the realization of a particular signal has been observed by the patient in period
1. When the patient observes signal \( s_1 \), posterior beliefs for the state of health
\( w_1 \) and \( w_2 \) respectively are \( q \) and \( 1 - q \) (see table T2). Thus, the patient will
choose action \( t \) such that his expected health is maximized. Given signal \( s_1 \),
expected health is:

\[
E_q[w_1(w_t, t)] = q \left[ w_1 - (w_1 - t)^2 \right] + (1 - q) \left[ w_2 - (w_2 - t)^2 \right]
\]
Let us call \( t_1^*(q, w_i) \) the optimal value of the action given that the signal \( s_1 \) has been observed:

\[
t_1^*(q, w_i) = \arg \max_t E_q [v_1 (w_i, t)]
\]

It is easy to verify that:

\[
t_1^*(q, w_i) = qw_1 + (1 - q)w_2 = E_q (w_i)
\]  

(3)

when signal \( s_1 \) is observed, the optimal action corresponds to the mean of the two health status calculated w.r.t. the posterior belief \( q \).

In the same way, when the patient observes signal \( s_2 \), posteriors beliefs for the state of health \( w_1 \) and \( w_2 \) respectively are \( 1 - q \) and \( q \). Thus, given signal \( s_2 \), expected health is:

\[
E_{1-q} [v_2 (w_i, t)] = (1 - q) [w_1 - (w_1 - t)]^2 + q [w_2 - (w_2 - t^*)^2]
\]

and the optimal value of the action given that the signal \( s_2 \) has been observed is:

\[
t_2^*(1 - q, w_i) = \arg \max_t E_{1-q} [v_2 (w_i, t)]
\]

with:

\[
t_2^*(1 - q, w_i) = (1 - q)w_1 + qw_2 = E_{1-q} (w_i)
\]  

(4)

Since \( q \geq 1 - q \) and \( w_1 > w_2 \), \( w_1 \geq t_1^*(q, w_i) \geq t_2^*(q, w_i) \geq w_2 \). Signal \( s_1 \) represents "good news" for the patient because the preferred state of health \( w_1 \) is more likely. When \( s_1 \) is observed, the optimal action \( t_1^*(q, w_i) \) is near to \( w_1 \). On the contrary, signal \( s_1 \) represents "bad news" for the patient because the preferred state of health \( w_1 \) is more unlikely. When \( s_2 \) is observed, the optimal action \( t_2^*(q, w_i) \) is near to \( w_2 \). Notice that, when the signal is fully informative \( t_1^*(q, w_i) = w_1 \) and \( t_2^*(q, w_i) = w_1 \). Whereas, when the signal is fully uninformative, \( t_1^*(q, w_i) = t_2^*(q, w_i) = \frac{w_1 + w_2}{2} \). In general, as the signal becomes more informative, the action \( t \) becomes more accurate and the patient’s expected loss decreases.

We can now indicate indirect expected utility when signal \( s_1 \) is observed as:

\[
E_q [V_1 (q, w_i)] = q \left[ w_1 - (w_1 - t_1^*(q, w_i))^2 \right] + (1 - q) \left[ w_2 - (w_2 - t_1^*(q, w_i))^2 \right]
\]  

(5)

or, by substituting the value of \( t_1^*(q, w_i) \) expressed in (3) and rearranging:

\[
E_q [V_1 (q, w_i)] = qw_1 + (1 - q)w_2 - q(1 - q)(w_1 - w_2)^2
\]  

(6)

In the same way, indirect expected utility when signal \( s_2 \) is observed is:

\[
E_{1-q} [V_2 (1 - q, w_i)] = (1 - q) \left[ w_1 - (w_1 - t_2^*(q, w_i))^2 \right] + q \left[ w_2 - (w_2 - t_2^*(q, w_i))^2 \right]
\]  

(7)

by substituting the value of \( t_2^*(1 - q, w_i) \) expressed in (4) and rearranging:

\[
E_{1-q} [V_2 (1 - q, w_i)] = (1 - q)w_1 + qw_2 - q(1 - q)(w_1 - w_2)^2
\]  

(8)
Suppose that the patient derives utility from physical outcome in period 2 instead of his emotions in period 1. Using expression (2) with $u(q) = E_q[V_1(q, w_i)]$ and $u(1-q) = E_{1-q}[V_2(1-q, w_i)]$ one find:

$$U^P(\frac{1}{2}, q) = \frac{1}{2} [qw_1 + (1-q)w_2 - q(1-q)(w_1 - w_2)^2] + \frac{1}{2} [(1-q)w_1 + qw_2 - q(1-q)(w_1 - w_2)^2]$$

or:

$$U^P(\frac{1}{2}, q) = \frac{w_1 + w_2}{2} - q(1-q)(w_1 - w_2)^2$$

Such that the patient’s utility is maximized when $q = 1$, that is when the signal is fully informative. In fact $E_q[V_1(q, w_i)]$ and $E_{1-q}[V_2(1-q, w_i)]$ are both convex w.r.t. the accuracy of information. In other word when (extended) expected utility is calculated upon physical outcomes in period 2 and priors are $p = \frac{1}{2}$, the patient always prefers full disclosure of information. Notice that, in this case, $u(1) = E_1[V_1(1, w_i)] = w_1$ and $u(0) = E_0[V_2(0, w_i)] = w_2$. As a consequence $U^P(\frac{1}{2}, 1) = \frac{1}{2}(w_1 + w_2)$ whereas $U^P(\frac{1}{2}, \frac{1}{2}) = \frac{1}{2}(w_1 - w_2)^2$.

**Remark 1** When the patient derives utility from physical outcome in period 2 and prior is $p = \frac{1}{2}$, the patient always chooses a fully informative signal ($q = 1$).

It is interesting to verify which is the optimal value of $q$ when the prior $p$ is different from $\frac{1}{2}$. This means to maximize the function $U^P(p, q)$ w.r.t. $q$. It is easy to see that the optimal value of the action $t$ given the signal $s_1$ is:

$$t^*_1(p, q, w_i) = \frac{pqw_1 + (1-p)(1-q)w_2}{pq + (1-p)(1-q)}$$

whereas the optimal value of the action $t$ given the signal $s_2$ is:

$$t^*_1(p, q, w_i) = \frac{p(1-q)w_1 + (1-p)qw_2}{p(1-q) + (1-p)q}$$

Proceeding as before we can derive the function $U^P(p, q)$:

$$U^P(p, q) = \text{prob}(s_1)E_{s_1}[V_1(p, q, w_i)] + \text{prob}(s_2)E_{s_2}[V_2(p, q, w_i)]$$

The optimal value of the information accuracy $q$ will depend on the concavity of the functions $E_{s_i}[V_i(p, q, w_i)]$. Interestingly, here some comparative statics on the prior $p$ can be performed.

To be completed.
2.2 The function \( U \left( \frac{1}{2}, q \right) \)

Let us consider the more interesting case where the patient’s utility only depends on emotions in period 1. That is, first-period utility derives from the patient’s anticipation of health in the second period. This anticipation depends on the prior \( p = \frac{1}{2} \) and on the posterior probabilities \( q \). Using the results in the previous section:

\[
U(\frac{1}{2}, q) = \frac{1}{2}z \left[ E_q[V_1(q, w_i)] \right] + \frac{1}{2}z \left[ E_{1-q}[V_2(1 - q, w_i)] \right]
\]

and \( z(\cdot) \) is a concave function such that anxiety about the future state of health is taken into account (see expression 1).

Substituting expression (6) and (8) in (11) yields:

\[
U(\frac{1}{2}, q) = \frac{1}{2}z \left[ E_q(w_i) - q(1-q)(w_1 - w_2)^2 \right] + \frac{1}{2}z \left[ E_{1-q}(w_i) - q(1-q)(w_1 - w_2)^2 \right]
\]

(12)

Our patient anticipates that the action \( t \) is more effective the higher is information disclosure, but he fears bad news: he knows that, if the signal \( s_2 \) is observed, anxiety will make utility decrease. The function \( F(q, w_i) = q(1-q)(w_1 - w_2)^2 \) represents the utility loss due to an inaccurate action \( t \). In fact the lower is \( q \) and the higher is the loss. The loss reaches its maximum for the fully uninformative signal: \( F(\frac{1}{2}, w_i) = \frac{1}{4}(w_1 - w_2)^2 \). On the other hand, the more precise is the signal, the higher is information disclosure, and the worst are bad news. When the signal is fully informative, by observing the signal \( s_2 \) the patient infers that the bad state of nature \( w_2 \) is the true state and anxiety reaches its maximum.

Let us consider in details the function \( U(\frac{1}{2}, q) \) for the two extreme cases: the fully informative and the fully uninformative signal. When \( q = 1 \), the expected utility over beliefs is:

\[
U(\frac{1}{2}, 1) = \frac{1}{2}z(w_1) + \frac{1}{2}z(w_2)
\]

Here there is no utility loss because the action \( t \) is accurate, expected utility \( U(\frac{1}{2}, 1) \) is lower the more concave is the function \( z(\cdot) \) and the lower is \( w_2 \). In fact both concavity of \( z(\cdot) \) and a low \( w_2 \) imply that anxiety associated to bad news is high. Whereas, when the signal is fully uninformative:

\[
U(\frac{1}{2}, \frac{1}{2}) = z \left( \frac{w_1 + w_2}{2} - \frac{1}{4}(w_1 - w_2)^2 \right)
\]

Here there is no information disclosure and, thus, no anxiety arises. Anyway the utility loss reaches its maximum. Obviously the lower is the difference between the two states of nature \( w_1 \) and \( w_2 \) and the higher is expected utility \( U(\frac{1}{2}, \frac{1}{2}) \).

As a result, as in K’s observation 1, we find that:

**Remark 2** The more concave is the function \( z(\cdot) \) and the lower is the distance between \( w_1 \) and \( w_2 \), and more likely the patient prefers the fully uninformative signal \( q = \frac{1}{2} \) to the fully informative one \( q = 1 \).
Corollary 1 It always exists a sufficiently concave function \( z(\cdot) \) and a sufficiently low value of \( w_2 \) such that the patient prefers the fully uninformative signal \( (q = \frac{1}{2}) \) to the fully informative one \( (q = 1) \).

According to intuition, when the fear of bad news is sufficiently high and/or when bad news are sufficiently "bad", the patient prefers no information disclosure to full information disclosure.

As an example let us consider the simple case where \( w_1 = 1 \) and \( w_2 = 0 \).

Thus:

\[
U(\frac{1}{2}, q) = \frac{1}{2}z(q^2) + \frac{1}{2}z((1 - q)^2)
\]

Here the patient prefers the fully uninformative signal to the fully informative one if

\[
U(\frac{1}{2}, \frac{1}{2}) = z\left(\frac{1}{2}\right) > U(\frac{1}{2}, 1) = \frac{1}{2}z(1) + \frac{1}{2}z(0)
\]

With the log function the previous inequality is verified. Whereas with the quadratic function the patient is indifferent between full information disclosure and no disclosure.

3 The patient’s problem

Suppose that the patient prefers no information disclosure to full information disclosure. It is interesting to verify whether (and when) an intermediate level of \( q \) can increase patient’s expected utility such that partial information disclosure is preferred to no information disclosure. Then the patient’s problem is:

\[
\begin{cases}
\max_q U(\frac{1}{2}, q) \\
\text{s.t. : } q \in \left[\frac{1}{2}, 1\right]
\end{cases}
\]

(P1)

where \( U(\frac{1}{2}, q) \) is given by (12).

To solve problem P1 we must consider first- and second-order conditions. Let us call \( E_q[V_1(q, w_i)] = V_q \) and \( E_{1-q}[V_2(q, w_i)] = V_{1-q} \) the two arguments of the function \( z(\cdot) \).

- From first-order condition w.r.t. \( q \) of (12):

\[
z_q'[1 + (w_1 - w_2)(2q - 1)] - z_{1-q}'[1 - (w_1 - w_2)(2q - 1)] = 0
\]

where \( z_q' \) and \( z_{1-q}' \) are the derivative of respectively \( z[E_q(w_i) - F(q, w_i)] \) and \( z[E_{1-q}(w_i) - F(q, w_i)] \) w.r.t. their arguments, that is the derivative of the function \( z(\cdot) \) when the signal is \( s_1 \) and when it is \( s_2 \). From (14):

\[
\frac{z_q'}{z_{1-q}'} = \frac{1 - (w_1 - w_2)(2q - 1)}{1 + (w_1 - w_2)(2q - 1)}
\]

Notice that \( q = \frac{1}{2} \) always verifies (15). Moreover, since \( E_q[V_1(q, w_i)] > E_{1-q}[V_2(q, w_i)] \) and, as a consequence, \( 0 < \frac{z_q'}{z_{1-q}'} < 1 \), f.o.c. (15) admits a solution if and only if

\[
0 < \frac{1 - (w_1 - w_2)(2q - 1)}{1 + (w_1 - w_2)(2q - 1)} < 1 \quad \text{or:}
\]

Condition 1: \( q < \frac{1 + (w_1 - w_2)}{2(w_1 - w_2)} \)}
It easy to see that 
\[ \frac{1}{2} < \frac{1 + (w_1 - w_2)}{2(w_1 - w_2)} \] for every \( w_1 \) and \( w_2 \). Condition 1 represents a necessary condition for the (15) to admit a solution. The higher is the difference \((w_1 - w_2)\) and the more binding is Condition 1. At the limit, for \( (w_1 - w_2) \to \infty \), \( \frac{1}{2} \frac{1 + (w_1 - w_2)}{w_1 - w_2} \to \frac{1}{2} \) and the solution \( q = \frac{1}{2} \) is unique. Notice that, when \( w_1 = w_2 = 1 \), Condition 1 results in \( q \leq \frac{1}{2} \). For \( w_1 - w_2 < 1 \) (??) is always slacking and an internal solution is more likely. The previous considerations confirm Corollary 1: when the undesirable state of nature \( w_2 \) is sufficiently bad (with respect to \( w_1 \)), the patient always chooses the fully uninformative signal.

Later on we consider the following functions for \( z(\cdot) \): the CARA function \( z(t) = -e^{-at} \), with \( a \) that can be re-interpreted as the parameter of constant absolute aversion to information. The power function \( z(t) = t^{1-\gamma} \), with \( \gamma \) to be re-interpreted as the parameter of constant relative aversion to information. The quadratic function \( z(t) = b t - \frac{1}{2} t^2 \), with \( t \leq b \), which exhibits increasing absolute aversion to information.

### 3.0.1 A particular case: \( w_1 = 1 \) and \( w_2 = 0 \)

When \( w_1 = 1 \) and \( w_2 = 0 \), the function \( U(\frac{1}{2}, q) \) is as in 13 and Condition 1 is always slacking. Moreover this case presents the advantage to be particularly treatable. It is interesting to consider condition (15) because it offers a nice economic interpretation:

\[
\frac{z'_{q}}{z'_{1-q}} = \frac{1 - q}{q} \tag{16}
\]

Here, the marginal rate of substitution between the emotions the patient feels when the signal observed is \( s_1 \) and when it is \( s_2 \) equals the ratio between the two up-dated probabilities. (16) can be written as:

\[
\frac{z'_{q}}{z'_{1-q}} = \frac{\text{Prob}(w_1/s_2)}{\text{Prob}(w_1/s_1)} \tag{17}
\]

On the one hand, by increasing the information accuracy \( q \), the anticipatory utility when the signal \( s_1 \) is observed increases as well. On the other hand, the anticipatory utility when the signal \( s_2 \) is observed decreases. The optimal level of information accuracy is reached when the marginal benefit and the marginal cost of increasing \( q \) are equalized as in (17).

Let us consider second-order conditions for (13) and let us verify if and when \( q = \frac{1}{2} \) represents a maximum of the function \( U(\frac{1}{2}, q) \).

\[
\left. \frac{\partial^2 U(\frac{1}{2}, q)}{\partial q^2} \right|_{q=\frac{1}{2}} = 2 z' \left( \frac{1}{4} \right) + z'' \left( \frac{1}{4} \right)
\]

thus:

\[
\left. \frac{\partial^2 U(\frac{1}{2}, q)}{\partial q^2} \right|_{q=\frac{1}{2}} \geq 0 \iff - \frac{z'' \left( \frac{1}{4} \right)}{z' \left( \frac{1}{4} \right)} \leq 2
\]
Remark 3 When $w_1 = 1$ and $w_2 = 0$, the fully uninformative signal is a local maximum of the function $U(\frac{1}{2}, q)$ if the patient’s "absolute information aversion" is higher than 2.

As it was said before, it is interesting to verify whether the patient would prefer the fully uninformative signal when the choice is only between $q = \frac{1}{2}$ and $q = 1$ (according to the crucial result in K’s paper), but would choose an intermediate level of $q$ when the accuracy of the signal can be chosen in the interval $[\frac{1}{2}, 1]$. For this to be true, the following two conditions must hold:

\begin{align*}
\text{Condition 2} & \quad z(\frac{1}{2}) > \frac{1}{2}z(1) + \frac{1}{2}z(0) \\
\text{Condition 3} & \quad -\frac{z'(\frac{1}{2})}{z(\frac{1}{2})} < 2
\end{align*}

Condition 2 implies that the patient prefers the fully uninformative to the fully informative signal. Condition 3 implies that the fully uninformative signal is a local minimum.\(^{12}\) When both conditions are verified, since the function $U(\frac{1}{2}, q)$ is continuous, there exists an intermediate value of $q$ such that the patient prefers an imprecise signal to the fully uninformative one. Notice that condition 2 implies a high level of information aversion, whereas condition 3 implies the opposite. As a consequence, the difficult task is now to find out which functions $z(\cdot)$ verify both conditions simultaneously.

Remark 4 When $z(\cdot)$ is CARA, Conditions 2 and 3 are never simultaneously verified.

In fact all the value of $a$ such that the absolute aversion to information is less than 2 implies that the fully informative signal is preferred to the fully uninformative one. In particular, for $a < 2$, the function $U(\frac{1}{2}, q)$ is monotonically increasing in the interval $[\frac{1}{2}, 1]$ such that the optimal signal is always the fully informative one. For $a > 2$, $q = \frac{1}{2}$ is a local maximum and either $U(\frac{1}{2}, 1) > U(\frac{1}{2}, \frac{1}{2})$ or the opposite hold. Every intermediate level of $q$ is suboptimal.

Remark 5 When $z(\cdot)$ is the power function and $\gamma$ is the parameter of constant relative aversion to information; for $\gamma = \frac{1}{2}$, $U(\frac{1}{2}, q) = U(\frac{1}{2}, \frac{q}{2})$. For $\gamma > \frac{1}{2}$, $q = \frac{1}{2}$ is a global maximum in $[\frac{1}{2}, 1]$. For $\gamma < \frac{1}{2}$, $q = \frac{1}{2}$ is a global minimum in $[\frac{1}{2}, 1]$.

Because of the monotonicity property of the power function, it turns out that Conditions 2 and 3 are never simultaneously verified. When the aversion to information is higher than the threshold value given in Condition 3, the fully uninformative signal is preferred to any other signal. On the contrary, when

\[^{12}\text{The same condition 3 can be seen in terms of "relative information aversion": } -\frac{1}{2} \frac{z''(\frac{1}{2})}{z(\frac{1}{2})} < \frac{1}{7}.\]
aversion to information is lower than the threshold value, the fully uninforma-
tive signal is a global minimum and the preferred signal is always the fully
informative one.

**Remark 6** When \( z(\cdot) \) is quadratic and concave, \( q = \frac{1}{2} \) is a global maximum
in \([\frac{1}{2}, 1]\) and the function \( U(\frac{1}{2}, q) \) reaches its maximum in \( q = 1 \).

With a quadratic function \( z(\cdot) \), the function \( U(\frac{1}{2}, q) \) has good monotonicity
properties and the fully uninformative signal is always a global minimum. The
preferred signal is the fully informative one.

All the previous remarks show that when \( w_1 = 1 \) and \( w_2 = 0 \), the patient
never prefers an intermediate level of \( q \). When the difference between good and
bad news is low, there is no place for trading off anxiety and efficacy of the
action \( t \).

**Result 1** When \( w_1 = 1 \) and \( w_2 = 0 \), depending on the patient’s aversion to
information it can be optimal either to choose the fully informative or the fully
uninformative signal, but no intermediate level of information accuracy will be
chosen.

In the next section we show that, when the difference between \( w_1 \) and \( w_2 \) is
high enough, it can be optimal to search for imprecise information.

### 3.1 The case \( w_1 = w \) and \( w_2 = 0 \)

This case can be analyzed without generality loss since the value of \( w \) also
represents the difference between good and bad news. The function \( U(\frac{1}{2}, q) \)
becomes:

\[
U(\frac{1}{2}, q) = \frac{1}{2} z \left[ qw - q(1 - q)w^2 \right] + \frac{1}{2} z \left[ (1 - q)w - q(1 - q)w^2 \right]
\]  

(18)

and from the first-order condition w.r.t. \( q \):

\[
\frac{z'_{v_1}}{z'_{v_1-q}} = \frac{1 - w(2q - 1)}{1 + w(2q - 1)}
\]

(19)

such that Condition 1 is:

\[
\text{Condition 1bis: } q < \frac{1 + w}{2w}
\]

The second-order condition of program P1 is:

\[
z'_{v_1} + z'_{v_1-q} + \frac{1}{2} z''_{v_1-q} (1 - w + 2qw)^2 + \frac{1}{2} z''_{v_1-q} (-1 - w + 2qw)^2 \leq 0
\]

(20)

Let us consider the solution \( q = \frac{1}{2} \). It is easy to verify that:

\[
\left. \frac{\partial^2 U(\frac{1}{2}, q)}{\partial q^2} \right|_{q = \frac{1}{2}} = 2z' \left( \frac{1}{2} w - \frac{1}{4} w^2 \right) + z'' \left( \frac{1}{2} w - \frac{1}{4} w^2 \right)
\]
thus:
\[
\frac{\partial^2 U(q, q)}{\partial q^2} \bigg|_{q = \frac{1}{2}} \geq 0 \Leftrightarrow -\frac{z'' \left( \frac{1}{2} w - \frac{1}{4} w^2 \right)}{z' \left( \frac{1}{2} w - \frac{1}{4} w^2 \right)} \leq 2
\]

**Remark 7** The fully uninformative signal is a local maximum if the patient’s aversion to information is higher than 2 and is a local minimum if the opposite holds.

It can be easily verified that when \( z(\cdot) \) is the power function, \( q = \frac{1}{2} \) is a local minimum for \( 0 < w \leq 2 \) and \( w - \frac{1}{2} w^2 > \gamma > 0 \). While when \( z(\cdot) \) is CARA, \( q = \frac{1}{2} \) is a local minimum for \( a \leq 2 \).

The following figures show cases where \( z(\cdot) \) is the power function and \( q = \frac{1}{2} \) is a local minimum. It can be either \( U(\frac{1}{2}, \frac{1}{2}) > U(\frac{1}{2}, 1) \) or the opposite. Interestingly, an internal solution for \( q \) is preferred in both situations.

Insert figure 4 and 5 about here

On the contrary no internal solution for \( q \) can be found with the CARA utility function. In this case the function \( U(q, q) \) is monotonically increasing in the interval \( \frac{1}{2}, 1 \) : the fully informative signal is always the optimal choice.

Finally, with the quadratic function an internal solution is possible, as for the power function it can be either \( U(\frac{1}{2}, \frac{1}{2}) > U(\frac{1}{2}, 1) \) or the opposite and imprecise information can lead to higher expected utility in both cases.

Insert figure 6 and 7 about here

**Proposition 1** When the fully uninformative signal is a local minimum, with both a power and a quadratic function \( z(\cdot) \) an information accuracy \( q \), with \( \frac{1}{2} < q < 1 \), can be optimal. Concerning the extreme values of \( q \), it can be either that the fully informative signal is preferred to the fully uninformative one or the opposite.

As Remark 7 indicates, the fully uninformative signal is a local minimum when the aversion to information is not too high. Thus, according to intuition, an intermediate level of information accuracy can be optimal only when anticipatory feelings are non characterized by too much anxiety. In this case, the patient can increase his (extended) expected utility by choosing to be partially informed. With an intermediate level of information accuracy the action \( t \) results to be more appropriate than in the case of ignorance and anxiety does not increase too much.

4 **Conclusion**

In this paper we have addressed the issue of information acquisition on health status by a patient who fears about future health. Following the strand of economic and psychology literature on anticipatory feelings, we have investigated a model of choice under uncertainty enriched by adding beliefs to the description
of uncertain consequences, so as to capture anticipatory feelings such as anxiety or hopefulness. Our Bayesian decision makers have expected-utility preferences over outcomes that are described with the decision maker’s action, the state of nature and also the decision maker’s belief.

In this framework, we have shown that anxiety costs might induce the patient to choose information sources less precise than a physician advise or a medical test. By deliberately choosing an imprecise source of information, the patient is able to decrease the level of anxiety induced by information. The peculiarity of our model is that a third alternative with respect to staying ignorant and going to the physician (who provides full information) is considered. In fact, we discuss the possibility to search for other signals that can be interpreted as health information sources characterized by different content precision, for example, by searching for information disseminated on the Internet the patient is able to choose different accuracy of information.

Information acquisition by anxious patients is not the only relevant context for our analysis. In fact, a similar issue arises in other contexts with uncertainty where an unskilled decision maker may decide to rely on personally acquired information or, alternatively, on experts’ advice, such as, for example, in the case of the decision on financial plans.

References


