Collusion in Audit∗

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Abstract

In an infinitely repeated situation, a monopoly auditor faces each period the choice between being honest and keep her reputation intact or collude and lose her business. We show that the auditor is honest in all periods only if she is patient enough. Otherwise, the probability of collusion is always positive, although never equal to 1, and increasing with the degree of impatience. Competition among auditors and mandatory audit both increase the probability of collusion. We extend our model to look at incentive problems inside the audit company. We find that both the audit price and direct monitoring are instruments to fight collusion at the lower levels. The higher the monitoring cost, the higher the optimal audit price.

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1 Introduction

In recent years, a growing literature in economics has been devoted to the analysis of certification intermediaries and their incentives to disclose information to uninformed parties. The fact that firms (resp. managers) cannot credibly transmit information to the market (resp. owners of the firm) provides a rationale to such third parties to emerge. The research in this area has however by and large ignored the possibility of collusion since concerns for reputation were implicitly viewed as an immunization against it. This is typically the case of the audit market where the credibility of auditors has long relied on the belief that they would not survive if they were not trustworthy. Among others, the Enron, Parmalat, CUC International affairs have raised the concern that no longer can reputation be a sufficient guarantee against collusion.

In this paper we provide, in a repeated game framework, a model where an auditor who cares about future profits faces, in each period, a decision between colluding with her client at the cost of going out of business or being honest and survive. The quality offered by the firm at each period is his private information and the auditor owns a technology that allows her to discover the true quality. Depending on the report made by the auditor, the market takes a decision regarding the price to pay for the good. Our model is then closely related to the literature on cheap talk initiated by Crawford and Sobel (1982). A major difference is that the auditor, by setting a particular audit price, affects the set of potential auditees and thereby, the space of messages that can be sent in equilibrium.

We show that there is an equilibrium in which the auditor colludes with positive probability if the discount factor if high, but such probability of collusion is strictly lower than 1. In particular, the auditor only colludes with the lowest types among those asking for audit. As a direct implication, an auditor who is known to be strategic may survive for a long period of time before colluding. Even though this result seems somewhat close to those of Sobel (1985), Bénabou and Laroque (1992) or Morris (2001), the underlying approach is different. Their results rely on the assumption that there exists a positive probability that the sender of information is always honest. Truthful revelation of information enhances the assessment of the market regarding the credibility of the sender of information. The higher the probability that the auditor is honest (credibility), the larger is the willingness to pay of the receiver and, therefore, the larger become the gains from collusion. We provide a complementary explanation by extending the space of messages that can be sent by the auditor. For all true quality of the
auditee, the auditor has then the possibility to offer a collusion contract. Whenever her client is of a high enough quality, the auditor may prefer to forgo the potential bribe and, instead, continue in business. This occurs despite the fact that the auditor is known to be strategic with probability one.

The possibility of collusive agreements and its impact on the design of contracts in an agency relationship has received much focus starting with Tirole (1986), Kofman and Lawarrée (1993) and Laffont and Martimort (1997), (1998) and (2000). By applying a result close to the revelation principle (the collusion-proof principle) they show that collusion does not emerge as an equilibrium phenomenon. Moreover, much of the literature on audit in agency relationships (Kofman and Lawarrée (1993), (1996), Laffont and Martimort (1999)) endow the principal with some commitment power to a pre-specified audit policy. Again, a reputation argument is generally invoked to defend that assumption. Some notable exceptions are Baiman, Evans, and Nagarajan (1991), Straužs (1997) and Khalil and Lawarrée (2003) which, by preventing the principal to commit generate collusive behavior at equilibrium.

The philosophy of our approach is different since we consider that it is the auditor who acts as the principal.1 By constraining the auditor to fully disclose some information to the market, we leave the auditor with no instruments to ex ante screen the firm’s type. Thus, collusion becomes an equilibrium phenomenon which occurs despite the fact that rational agents in the market expect it to happen sooner or later. However, the possibility of collusion forces the market to assign different levels of reliability to different reports made by the auditor. We show that the market perfectly believes the report sent by the auditor if and only if it is low enough. That is, whenever the market receives a quality report from the auditor, it takes into account that it may be the outcome of a collusive agreement and rationally update its belief regarding the true quality offered by the manager. While this process does not prevent collusion from happening, it sharply reduces its probability of occurrence.

We also highlight a new conflict of interest that differs from the standard price/volume dilemma in the literature on monopoly behavior. Indeed, assuming that her valuation of future profit is not too large, the monopolistic auditor is torn between decreasing her audit price in order to generate volume but also attract lower types with whom stakes for collusion are large and increasing this price to increase her margin but only attract higher quality firms not willing to offer large bribes. We show that, the lower the value of future profits (the higher

1We believe that this assumption is more realistic when applied to certification markets.
the discount factor), the lower the price of audit. That is, an auditor with a high discount factor is tempted to reduce the audit price and decrease the average quality of the firm that is being audited in order to increase the chances of a very profitable collusive agreement. A mirror image of this is that an auditor with a low discount factor “ties her hands” by setting a high price and increasing the average quality of the auditees. The mark-up of an auditor with a well established reputation (low discount rate) then stems from their willingness to refrain from attracting firms with whom the stakes for collusion are large.

We then successively consider the impact of two institutional constraints on the probability of collusion: competition in the audit market and mandatory audit with a regulated audit price below the monopoly price. In both cases, the probability of collusion is higher than with a monopoly auditor who can freely set her own price. The reason is simple. Both frameworks restrict the ability of the auditor to affect the space of types who ask for audit although through different channels. Competition, by driving the audit price down, strategically modifies the decision of the firm to remain or not out of the audit market. Under mandatory audit, any type of firm is forced to be audited. We highlight two pervasive effects of such institutional changes on collusion. First, by lowering the price below the monopoly price the auditor’s future profits decrease and, hence, the potential loss from collusion is lower. Moreover, both mechanisms increase the probability that the auditor will face a low quality firm with whom the stakes for collusion are high. This latter effect is nevertheless mitigated by the fact that the market discounts even more high announcements made by the auditor.

Finally, we analyze the agency problem within the audit company. A board of directors immune to collusion has to provide the right incentives to an audit department who does not value reputation. We show that the former has two costly ways to limit the stakes for collusion between the audit department and the manager. She can increase the audit price to make collusion less attractive or she can increase her monitoring effort. We show that as monitoring becomes more costly, the auditor modifies her strategy by increasing the audit price and lowering her monitoring effort.

The paper is organized as follows. In Section 2, we describe the structure of the model. In Section 3, we show that if the value of reputation is high enough, there is an informative equilibrium in which collusion never occurs. In Section 4 we show that with an impatient auditor, collusion occurs in equilibrium with positive probability and that an auditor with a high discount factor can build up a reputation of honesty for many periods before deciding to
collude. We then show that price competition and mandatory audit increase the probability of collusion, compared to the monopoly case. In Section 5 we show how the audit contract is modified when there are conflicts inside the audit company. Finally, we conclude in Section 6.

2 The model

We present a dynamic model of audit composed of three risk neutral players: an auditor (she), a manager or firm (he), and a continuum of buyers (the market). There is an infinity of periods indexed by $t$. The auditor is the only infinitely lived agent and all other agents live only one period. At each period $t$, the manager owns a good of quality $\theta_t$, which is his private information. It is common knowledge that $\Theta_t \sim U[0,1]$ and is iid across periods. We assume that the manager has no alternative way to signal his quality to buyers other than asking for audit.

The auditor owns a costless technology that allows her to test and discover the true value of $\theta_t$. Namely, the auditor receives, at no cost, a signal perfectly correlated with the manager’s quality. This signal is soft information. Once the auditor has received her signal she reports it to the buyers but could, in cooperation with the manager and in exchange for a bribe, decide to report $r(\theta) \neq \theta$. We simplify the bargaining problem between the manager and the auditor by considering that the latter has all the bargaining power in the collusion agreement. Finally, following Kofman and Lawarrée (1993), we assume that the auditor cannot make a false announcement without the approval of the manager and that there exists some mechanism which makes side-contracts enforceable.

We restrict the set of contracts by assuming that the auditor cannot offer contracts contingent on the true quality of the firm. That is, at each period $t$, the auditor offers an upfront audit price $p_t$. If the manager decides to pay the price, the auditor learns her signal and then chooses a report to send to the buyers. Throughout the paper, we also restrict the auditor’s strategy to precise announcements about the manager’s type. We believe that these assump-

\[\text{We call } \Theta \text{ the random variable and } \theta \text{ a particular value of the random variable.}\]

\[\text{In a environment where all agents are risk neutral, \cite{maskin1992} shows that if the seller has sufficient credibility to behave as an informed principal à la Maskin and Tirole (1992) by directly offering a contract, then intermediaries cannot be active on the market. Otherwise, they will be.}\]

\[\text{Since collusion happens under symmetric information, the allocation of the bargaining power at the collusion stage is irrelevant.}\]

\[\text{The auditor could in principle make imprecise announcements, such as “} \theta \text{ is low”}. \text{We do not allow for}\]
tions are relevant, given the types of contracts that are empirically observed. However, we discuss at the end of the analysis how our results would change if the auditor were able to set contingent pricing schemes. Once the audit contract is signed and the signal obtained, the auditor can directly offer a collusion contract to the manager.

Buyers form expectations based on the information they have. They attach a value $\theta$ to a good of quality $\theta$. We assume that they do not observe the price proposed by the auditor but only observe whether the manager accepts or rejects the audit contract. Whenever collusion occurs, they detect any false announcement, once they have experienced the good, with probability one.

**Timing**

At each period $t$, the timing of the game is the following:

1. The manager learns $\theta$.
2. The auditor fixes an audit price $p$, which is not observable by the market.
3. The manager accepts or rejects the offer. If he rejects the offer, there is no audit in period $t$ and the game proceeds to stage 7. If he accepts the offer, the auditor learns her signal and the game proceeds to stage 4.
4. The auditor may offer an enforceable collusion contract, $(r(\theta), b(\theta))$ to the manager, where $r(\theta)$ is the announcement made to the buyers and $b(\theta)$ is the bribe.
5. The manager accepts or refuses the auditor’s collusion offer.
6. The auditor discloses information to the buyers depending on the collusion agreement, if any.
7. Given the manager’s action and the information disclosed, buyers update their beliefs and compete for the good.

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6 Changing this assumption would modify the equilibrium but all our results would qualitatively hold.
7 Changing such assumption would only reinforce our result since it is the least favorable environment to sustain collusion.
8 We omit the subscript $t$ when it does not lead to confusion.
9 The assumption of a continuum of buyers implies that, at each period $t$, the good will be sold at a price equal to the expected quality given the information available.
3 Benchmark: Equilibrium without collusion

We first analyze, as a benchmark case, a setting where the auditor always truthfully reveals the signal she obtains regarding the quality of the manager. Buyers, rationally anticipating that the report is correct pay \( \hat{r} \) for any report \( \hat{r} \). Even though collusion does not happen in equilibrium we need to specify the (out-of-equilibrium) continuation game if collusion were ever observed. Because this is a cheap talk game it is straightforward to show that an equilibrium with no auditor at every period always exists.

**Proposition 1** There is an uninformative sequential equilibrium of the game. The auditor charges any non-negative price, no type of manager accepts to be audited and the auditor’s stage profit is \( \pi_t = 0 \) \( \forall t \). This equilibrium is sustained by out-of-equilibrium beliefs \( \Pr(\Theta_t = 0 | \text{Acceptance, } r) = 1 \forall t, r \).

**Proof.**
The proof is straightforward. Given that at each period \( t \) and whatever the auditor’s report \( E(\Theta_t | \text{Acceptance, } r) = 0 \forall r \) and \( E(\Theta_t | \text{Rejection}) = \frac{1}{2} \), no type of manager is willing to pay a positive price for audit. Thus, it is an equilibrium to reject the audit contract. Acceptance is then an out-of-equilibrium event and any beliefs at this point are possible. 

The aim of our paper is to investigate the existence of more informative equilibria when considering collusion between the auditor and the manager. To do so, we make the following assumption:

**Assumption 1** If \( r(\theta) \neq \theta \) in period \( t \), then buyers coordinate on the zero profit equilibrium from date \( t + 1 \) on.

This equilibrium selection once collusion is observed is certainly not neutral but aims at showing that even by selecting the least favorable equilibrium for sustaining collusion, such behavior has major consequences on the degree of informativeness of the equilibrium.\(^{10}\)

\(^{10}\)There always exists another uninformative equilibrium where all types of manager ask for audit, the price of audit and the auditor’s spot profit are \( \frac{1}{2} \) and any announcement of the auditor is not taken into account by buyers who are then ready to pay \( \frac{1}{2} \) whatever the announcement of the auditor. If players coordinated on such uninformative equilibrium, collusion would happen from the very beginning, and there would be no possibility of an informative equilibrium.
It is well known from the existing literature in dynamic games that the less the firm discounts the future, the less shall we witness collusion. This is certainly the case in our model and this is summarized in the following proposition.

**Proposition 2** Suppose that the auditor does not collude with the manager. Then,

i. the most informative equilibrium of the stage game is such that the auditor offers an audit price \( p = \frac{1}{3} \) and the manager accepts the offer if and only if \( \theta \geq \tilde{\theta} = \frac{2}{3} \). If the manager accepts the offer, the auditor makes an announcement \( r(\theta) = \theta \) and consumers rationally believe it and pay \( \beta(r) = r \). Consumers pay a price \( \beta = \frac{1}{3} \) if the manager did not accept the contract.

ii. there is no other equilibrium with full disclosure of information.

iii. such “no-collusion equilibrium” exists if and only if \( \delta \geq \frac{3}{4} \).

**Proof.**

Suppose there is an equilibrium in which the auditor offers a contract \( p \) and the manager accepts the contract if and only if \( \theta \geq \tilde{\theta} \). We solve the game backwards.

A manager of type \( \theta \) who accepts the offer knows that his type will be fully revealed because there is no collusion, so \( \beta(r(\theta)) = r(\theta) = \theta \) is the price buyers will pay for the good. The manager then gets \( u^m(\theta) = \theta - p \). Instead, if he rejects the offer, buyers will be willing to pay \( E[\Theta|\text{Rejection}] \). Therefore, a manager of type \( \theta \) accepts the offer whenever

\[
\theta - p \geq E[\Theta|\text{Rejection}] . \tag{1}
\]

Given that the left hand side of equation (1) is increasing in \( \theta \), there exists a threshold \( \tilde{\theta} \) such that only managers with type \( \theta \geq \tilde{\theta} \) ask for certification. The manager \( \tilde{\theta} \) who is indifferent is defined by

\[
\tilde{\theta} = E[\Theta|\text{Rejection}] + p \tag{2}
\]

and, at equilibrium, \( E[\Theta|\text{Rejection}] = \frac{\tilde{\theta}}{2} \).

That is, the manager accepts the contract whenever \( \theta \geq p + \frac{\tilde{\theta}}{2} \). So only types above \( \tilde{\theta} \) accept the contract if and only if \( \tilde{\theta} = p + \frac{\tilde{\theta}}{2} \). This implies that in equilibrium

\[
p = \frac{\tilde{\theta}}{2} \tag{3}
\]

Finally, the auditor chooses \( p \) to maximize her profit given the continuation strategies. The auditor’s spot profit is

\[
\pi = (1 - \tilde{\theta})p, \tag{4}
\]
where $\hat{\theta}$ is the type who is indifferent between accepting or rejecting the auditor’s offer given the consumers’ beliefs and the price $p$. So, $\hat{\theta} = p + \tilde{\theta}$.

Plugging it in (4) and maximizing with respect to $p$ we obtain that

$$p = \frac{1}{2} - \frac{\tilde{\theta}}{4}.$$  

(5)

Combining (3) and (5), we get that the unique equilibrium is with $\hat{\theta} = \frac{2}{3}$, which implies $p = \frac{1}{4}$.

The spot profit of the auditor at equilibrium is $\pi^* = \frac{1}{9}$. Given that the auditor does not collude with the manager, this is the profit she will get in all future periods. Therefore, her life-time total profit at an equilibrium without collusion is $\Pi^* = \frac{\pi^*}{1 - \delta} = \frac{1}{9(1 - \delta)}$.

Once the manager has accepted the audit contract and the auditor has discovered the true quality of the manager, she may want to offer a collusion contract. Given that the auditor loses her reputation in case of any untruthful announcement, any collusion contract with any type $\theta$ will propose $r(\theta) = 1$ and a bribe $b(\theta) = 1 - \theta$. The auditor also takes into account the fact that she loses all future profits whatever the deviation from the truth. Since a lower quality manager is ready to pay more for a high announcement than a higher quality manager, the auditor never colludes if and only if she has no incentives to collude with the lowest possible type $\tilde{\theta}$ who asks for audit. If she colludes with the lowest type in period $t$ she gets the spot profit in (4) plus the bribe $b(\tilde{\theta}) = 1 - \tilde{\theta}$.

$$\pi^*_t + b(\tilde{\theta}) = \frac{1}{9} + 1 - \tilde{\theta} = \frac{4}{9}.$$  

Thus, the auditor tells the truth if and only if $\Pi^* \geq \pi^* + b(\tilde{\theta})$, or $\frac{1}{9(1 - \delta)} \geq \frac{4}{9}$. This condition is satisfied if $\delta \geq \frac{3}{4}$.

For $\delta$ high enough, the value of telling the truth is so high that even the largest possible bribe cannot offset it. This is no longer true if $\delta$ is low. In particular, if $\delta < \frac{3}{4}$, the strategies and beliefs described in Proposition 2 cannot form an equilibrium, since the auditor has incentives to deviate from truth-telling when the type of the manager is low enough. It is the purpose of next section to analyze this case in details.

4 Equilibrium with an impatient auditor

Let us now assume that $\delta < \frac{3}{4}$ such that collusion becomes an issue. To remain as general as possible, we allow the audit company to play according to mixed strategies. That is, once she has learned the manager’s type, she offers a collusion contract in which she sends an announcement “$r$” according to a cdf function $G(r|\theta)$, where $G(r|\theta)$ is the probability that the announcement is lower than $r$ when the true type is equal to $\theta$. When observing an announcement equal to $r$, buyers try to infer the true type given the equilibrium strategy of
the auditor. As in the previous section, we look for a threshold equilibrium in which all types above $\tilde{\theta}$ accept the audit contract and all types below $\tilde{\theta}$ reject it.

4.1 Collusion with a monopolist auditor

Suppose that the auditor is a monopolist and freely sets her price in order to maximize profits. The goal of this section is to stress how collusion affects the standard price/volume dilemma of the monopolist. Before doing so, let us analyze the equilibrium behavior of both the manager and the buyers. We obtain the following result:

**Lemma 1** *In any equilibrium neither the manager nor the buyers play mixed strategies, i.e.:

i. buyers buy the good with probability 1 at a price equal to the expected quality given the information available (decision of the manager and announcement by the auditor),

ii. the manager accepts with probability 1 any collusion offer made by the auditor that gives him at least the same utility he would obtain if the auditor were truthful. Any other collusion offer is rejected with probability 1.*

**Proof.**
We prove the lemma by contradiction. Buyers are indifferent between buying or not the good, when they pay the expected valuation. Suppose that a buyer wants to buy the good at its expected quality, but with probability $\alpha < 1$. Then, any other buyer can get the good by offering a price slightly lower but buying with probability 1. There is always a price sufficiently close to the expected quality such that the manager will switch to this new buyer. Therefore, buying with probability lower than 1 cannot be an equilibrium.

The same reasoning applies for the second part of the lemma. Since collusion occurs under symmetric information, whenever there are gains from colluding the auditor makes an offer. If the manager rejects this offer with positive probability, he must be indifferent between accepting or rejecting. If the manager rejects the collusion offer, the auditor tells the truth (otherwise, she will lose her reputation without getting any bribe). But then, the auditor can slightly reduce the bribe she asks from the manager and the latter will accept with probability 1. Therefore, rejecting collusion with positive probability cannot be part of an equilibrium.

We can now turn to the equilibrium of the whole game, summarized in the following result.
Proposition 3  In the most informative equilibrium of the game, the auditor offers a price \( p(\delta) \) and the manager accepts the offer if and only if \( \theta \geq \bar{\theta}(\delta) = 2p(\delta) \). There are functions \( \theta^*(\delta) \) and \( \bar{\theta}(\delta) = \theta^*(\delta) - \delta \Pi(\delta) \) such that if the manager accepts the offer

i. there is no collusion with types above \( \bar{\theta}(\delta) \). That is, \( r(\theta) = \theta \) and \( b(\theta) = 0 \) if \( \theta \geq \bar{\theta}(\delta) \),

ii. there is always collusion with types below \( \bar{\theta}(\delta) \). Stated differently, if \( \theta < \bar{\theta}(\delta) \), \( r(\theta) \in (\theta^*(\delta), 1] \) and is defined according to a mixed strategy such that \( \mathbb{E}[\Theta| r > \theta^*(\delta)] = \theta^*(\delta) \) and \( b(\theta) = \theta^*(\delta) - \theta \),

iii. any announcement \( r < \bar{\theta}(\delta) \) is out of equilibrium. The strategies defined above form an equilibrium under the reasonable out of equilibrium beliefs that \( \mathbb{E}[\Theta| r < \bar{\theta}(\delta)] = r \). That is, buyers believe low reports that are not supposed to be made in equilibrium.

iv. the functions \( \bar{\theta}(\cdot) \), \( \theta^*(\cdot) \) and \( p(\cdot) \) are increasing in \( \delta \), while \( \theta(\cdot) \) is a decreasing function of \( \delta \).

The proof is provided in the appendix.

Figure 1 summarizes the main results of Proposition 3 and represents the functions \( \theta^*(\delta) \), \( \bar{\theta}(\delta) \) and \( \bar{\theta}(\delta) \). The intuition for the result is as follows. When \( \delta < \frac{3}{4} \), a high announcement cannot be fully trusted since, with some probability, it is the outcome of a collusive contract with a low quality manager. Buyers then discount the price they are ready to pay whenever a high announcement is made. For all \( \delta < \frac{3}{4} \), there exists a range \( A \equiv [\theta^*(\delta), 1] \) for which \( \mathbb{E}[\Theta| r \in A] = \theta^*(\delta) \ \forall r \in A \).

An auditor then never colludes when the true quality is in \( A \) since she goes out of business without extracting any bribe. Similarly, the auditor is not ready to collude with a manager of
type $\theta^*(\delta) - \epsilon$, for $\epsilon$ small since, at most, she can obtain a bribe equal to $\epsilon$ but will lose $\delta \Pi(\delta)$ on future profit. Collusion can then only occur with some types $\theta$ below $\bar{\theta}(\delta) = \theta^*(\delta) - \delta \Pi(\delta)$.

The more an auditor discounts the future, the more she is prone to offer collusive agreements to low quality managers and, at the same time, the more she is willing to attract even lower quality managers to increase the probability of extracting a bribe. That is, the price offered by the auditor, and consequently the quality $\bar{\theta}$ of the manager who is indifferent between being audited or not decreases with the degree of impatience of the auditor. This is anticipated by rational buyers who increase their discount on high announcements. This is illustrated in Figure 1 where $\theta^*$ is an increasing function of $\delta$. The range $A$ over which the auditor mixes when she colludes is then larger the higher the degree of impatience of the auditor. As a result we have that

**Corollary 1** The probability of collusion decreases with $\delta$ and is bounded away from 1.

**Proof.**
The probability of collusion is equal to the probability that the manager asks for audit and his type is below $\bar{\theta}$. That is $\Pr(\text{Collusion}) = \bar{\theta} - \tilde{\theta}$. Since $\bar{\theta}$ decreases with $\delta$ and $\tilde{\theta}$ increases with $\delta$, we get the result. The highest probability of collusion is achieved when $\delta = 0$, in which case $\bar{\theta} = \frac{3}{4}$ and $\tilde{\theta} = \frac{1}{2}$. 

It is worth noting that, contrary to the intuition one can have, when $\delta$ converges to 0, the monopoly price does not converge to 0 but to $\frac{1}{4}$. Thus, even when the auditor does not attach any value to the future, she does not want to attract all types of manager. This result illustrates well the trade-off we want to highlight. By lowering her price and attracting the lowest tail of the distribution, the auditor can play a volume strategy and, at the same time, increase the proportion of firms from which extracting a bribe is profit enhancing. At the same time, when $\delta$ is low, the market rationally discounts any payment for reports larger than $\theta^*(\delta)$, lowering the margin the auditor gains on each firm, whether she decides to collude or not. When $\delta$ converges to zero, by leaving some types outside the audit market the auditor substantially increases the bribe she can extract from those who accept to be audited.

Another interesting feature of the result in the proposition above is that an audit firm with high discount factor may survive a certain number of periods just because she has faced high quality managers with whom the stakes for collusion were small. However, such an impatient auditor will lose her reputation sooner or later.

In the monopoly approach, the price and, indirectly, the proportion of firms asking for audit are chosen by the auditor in order to maximize profits. We now investigate how the
equilibrium collusion changes when we depart from the monopoly framework, by looking at two different cases: Bertrand competition among auditors and mandatory audit.

### 4.2 Collusion in a competitive audit market

The benefits of introducing competition in the audit market come from the belief that in a more competitive environment, more firms will be audited because the audit price is small, and therefore, a more informative equilibrium will prevail. This is in line with the decision of the FTC who, “concerned with a potential oligopoly behavior by the large audit firms, required the profession to change its standards in order to allow audit firms to advertise and compete aggressively with each other for clients” (Healy and Palepu (2003)). However, tough competition among auditors may give them incentives to recover some lost profits through collusion since the auditor is indeed a monopoly at the collusion stage. In this section we aim at analyzing the impact of competition on the probability of collusion. To do so, we assume that at least two auditors compete in prices to attract the manager for audit.

Bertrand competition among auditors together with no costs of audit implies that the equilibrium audit price is equal to 0 and, consequently, all types of firm is audited in equilibrium. The only profit that the auditor can expect is then derived from collusion. However, it turns out that collusion is never a probability 1 event.

**Proposition 4** Suppose Bertrand competition in the audit market. In the most informative equilibrium, the probability of collusion is higher than in the monopoly case, but it is bounded away from 1.

**Proof.**

The proof is the analogous to that of Proposition 3 and 1, except that we fix $p = 0$ and, therefore $\tilde{\theta} = 0$ in equilibrium.

Figure 2 shows how the probability of collusion changes with the value of $\delta$ in the cases of monopoly and Bertrand competition. The intuition for the result can be sketched as follows. Since buyers discount high announcements, there always exists a subset of types, $[\theta^*, 1]$ from whom the auditor is unable to extract any positive bribe.\(^{11}\) It is therefore never an optimal strategy to collude with those types. Only when a type lower than $\theta^*$ is being audited, the bribe that can be extracted is strictly positive.

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\(^{11}\)When $\delta = 0$ the auditor is indifferent between colluding or not with types above $\theta^*$. We assume that the
Finally, since competition drives the audit price down to zero, the auditor has to rely on complementary contracts to generate positive profits, such as consulting services. In our framework, the complementary source of profit stems from collusion and, contrary to the monopoly case, for any degree of impatience $\delta \in [0, 1)$, there exists a subset of types with whom the auditor is prone to collude.

4.3 Collusion and mandatory audit

In many cases, audit is not a choice for firms, but rather a legal obligation. As with competition, the underlying objective of such a law is to increase the disclosure of information. Hence, the question is how making audit mandatory may affect the probability of collusion in the audit market. In order to provide an answer to this question suppose now that audit becomes mandatory. Mandatory audit implies that all types of firm ask for audit at any given price that we assume below the monopoly price. We have the following result.

**Proposition 5** If audit is mandatory and the price of audit is fixed at $p$, below the monopoly price, then

(i) the probability of collusion decreases with the audit price.

(ii) the probability of collusion is higher under mandatory audit than under voluntary audit.

**Proof.**

See appendix.

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auditor does not collude in that case, which is the limit of the equilibrium when $\delta$ goes to 0.
There are two countervailing forces at hand when the price of audit increases. On the one hand, $\theta^* = E[\Theta|r \in A]$ increases, that is the market reduces the discount it applies on high reports. This generates higher willingness to collude since a higher bribe can be extracted. On the other hand, collusion entails forgoing profit flows in subsequent periods which are increasing with the audit price. This lowers the incentives of the auditor to collude. The proposition above shows that this second effect dominates.

This has some interesting policy implications. Indeed, mandatory audit generates a match between the auditor and some low quality managers who would have decided to remain out of the certification market otherwise. Assume for an instant that buyers are risk averse. An interesting trade-off then emerges. On the one hand, forcing the firm to be audited decreases the degree of asymmetric information in the economy and increases social welfare by improving risk sharing as long as the auditor honestly reports her signal (for $\delta$ large enough). On the other hand, mandatory audit increases the likelihood of collusion since it matches two parties for which it can be Pareto-optimal to misreport the auditor’s signal. If the latter effect dominates, it might be welfare improving to leave some types out of the certification market.

We have conducted our analysis assuming that the auditor was banned from offering ex post prices contingent on the true quality of the firm. This deserves some discussion. Suppose that the auditor could charge contingent prices. An honest (high $\delta$) auditor’s best strategy
would be to charge no upfront fee and an ex post payment \( p(\theta) = \theta \). With this pricing policy all types of firm get audited, all the information is revealed and the auditor captures the entire surplus. This is indeed an equilibrium as long as the market expects a firm who does not get audited to be type 0. Note that contingent prices imply that the auditor gets a higher fee, the higher the announcement. This strict monotonicity also generates incentives to misreport the quality, and when \( \delta \) is low, these incentives are greater than any reputation concern. Thus, our qualitative results on the shape of collusion are robust to such consideration. The only important difference is that the value of \( \delta \) would have no effect on the mass of firms asking for audit.

Contingent prices may bring about some other incentive problems that are not captured by our simple model. In particular, in a more general framework where the firm has to invest ex ante in quality, if the auditor captures ex post the entire surplus, the firm’s incentives to invest are very low. This is just a simple example of the hold-up problem. Finally, as the market for audit is characterized both by a high degree of concentration (market power on the side of the auditor) and non-contingent prices, our modelling choice then appears to be the most empirically relevant.

5 Incentives within the audit company

So far, we have modelled the audit company as a unified entity. However, taking into account that the different units of the audit company may have different reputation concerns, there is no doubt that some conflicts may arise inside the audit company. As Healy and Palepu (2003) argue “incentives inside the audit firms need to encourage audit professionals to exercise judgment and walk away from clients that don’t deserve their certification - even when they are big and important...”. We introduce such conflicting interest inside the audit firm by assuming that the cost of losing reputation is higher for the company itself than for the particular unit that is actually in charge of the audit process. To simplify the analysis, we consider in what follows that the reputational cost for the unit that carries out the auditing is nil.

The model is almost identical to the one developed in Section 3. The difference is that the audit firm is composed of a board of directors (the principal \( P \)) who is a long-lived agent never prone to collusion (\( \delta = 1 \)) and an audit department (the agent \( A \)) who lives only one period and, therefore, is ready to collude with the manager.
Whenever the manager decides to be audited, A receives a signal \( s \), perfectly correlated with the manager’s private information. Given that the principal cares about reputation in future periods, she has to offer an incentive contract to the audit department in order to obtain truthful revelation of the agent’s signal. We assume that, if the manager and the auditor collude, by making an effort equal to \( \alpha \in [0,1] \) at a cost \( c\alpha^2 \), the principal finds verifiable evidence of the bribe \( b \) the auditor has received with probability \( \alpha \). She can, therefore, size this bribe.\(^{12}\) The incentive contract associates to all reports \( r \) a financial investigation of the auditor with precision \( \alpha(r) \) and a transfer to the auditor \( w(r) \).

The utility of an auditor with signal \( s \) who makes a report equal to \( r \) is then equal to:

\[
u_A(s, r) = b(r, s) + w(r) - \alpha(s)b(r, s) \quad (6)
\]

Given that the auditor has perfect information about the manager’s quality at the collusion stage he is able to extract all the rent. That is, if the signal is \( s \) and he commits to report \( r \), the bribe he receives is equal to \( r - s \). Therefore, (6) becomes \( u_A(s, r) = w(r) + (1 - \alpha(r))(r - s) \), and the utility when the auditor reports the true signal is \( u_A(s) = w(s) \). We then get the following result,

**Lemma 2** When the lowest type who accepts the audit contract is \( \hat{\theta} \), the optimal incentive contract is given by:

\[
u_A(s) = \int_s^1 (1 - \alpha(x))dx = w(s) \quad \forall s \in [\hat{\theta}, 1],
\]

\[
\alpha(s) = \begin{cases} 
\frac{s - \hat{\theta}}{c} & \forall s \text{ such that } \hat{\theta} \leq s \leq c + \hat{\theta} \\
1 & \forall s \text{ such that } c + \hat{\theta} < s \leq 1
\end{cases}
\]

**Proof.**

The incentive compatibility constraint is

\[
s = \arg \max_r u_A(r, s) \Rightarrow \frac{du_A}{ds}(s, s) = -(1 - \alpha(s)), \forall s \in [\hat{\theta}, 1].
\]

The participation constraints, \( u_A(s) \geq 0, \forall s \), are reduced to \( u_A(1) \geq 0 \). This implies that \( u_A(s) = \int_s^1 (1 - \alpha(x))dx \). The second order incentive compatibility condition is \( \alpha(s) \) non-decreasing in \( s \).

The principal maximizes her spot profit, which is going to be the same all over the periods subject to the incentive and participation constraints. The principal’s spot profit given that all types of manager above \( \hat{\theta} \) accept the auditing contract is

\[
\pi = \int_\hat{\theta}^1 \left( p - w(s) - c\frac{\alpha(s)^2}{2} \right) ds.
\]

\(^{12}\)To simplify the analysis, we assume that only the bribe can be sized if collusion is found.
Replacing \( w(s) \) by \( \int_{s}^{1} (1 - \alpha(x)) dx \) and maximizing with respect to \( \alpha(s) \in [0, 1] \) we get the specified solution, which satisfies the second order incentive compatibility condition.

The principal offers a contract in which the effort in finding evidence, \( \alpha \), is an increasing function of the report made by the agent about the firm’s type. Indeed, the agent has no incentives to lie by sending a report \( r = \hat{\theta} \), and therefore, if such a report is made, it must be true. The higher the report, the larger the probability that the agent is colluding and, thus, the higher the effort in finding evidence.\(^{13}\) Moreover, the pricing strategy is also influenced by the need to provide the agent with the right incentives to transmit information. This is the essence of the following proposition.

**Proposition 6** For any \( c > 0 \), there is a value \( \bar{\delta}(c) > \frac{3}{4} \) such that if \( \delta \geq \bar{\delta}(c) \) and the agent is offered an incentive contract according to Lemma 2, then,

i. the most informative equilibrium is characterized by: \( p^*(c) > \frac{1}{3} \forall c > 0 \)

ii. \( p^*(c) \) is continuous and increasing in \( c \).

**Proof.**

See Appendix.\(^{18}\)

The Principal has two alternative ways to limit the stakes for collusion. First, she can directly increase her monitoring effort \( \alpha \), and look more aggressively for evidence of collusion. Second, she can increase the price of audit in order to limit the amount of the bribe that the auditor can extract from side-contracting. This is an indirect way of preventing collusion. Both mechanisms are costly for the principal and she always trades them off when defining the optimal audit price and incentive contract. To avoid having to spend too much in monitoring the auditor, the principal decreases her monitoring effort and offsets the increase in the stakes for collusion by increasing the price of the audit process, which, in turn, increases the lowest type of firm who asks for audit.

When there are incentive issues inside the audit company, the audit process is no longer a costless economic activity. Although the actual cost of acquiring information about the firm’s type is still equal to 0, there is a cost in terms of monitoring the agent to provide the right incentives and prevent collusion. The higher this cost, the lower the audit company’s

\(^{13}\)This incentive contract suffers from the problem that, ex post it is not optimal to exert any effort, once the principal knows that the agent is telling the truth. However, the repetition of the game gives incentives to the principal to exert effort in order to convince future auditors that she will do it in the future too.
incentives to audit low quality firms. This implies, in particular, that this equilibrium is less informative than the equilibrium of Proposition 2, since less types are audited. Furthermore, if we assume that $\delta$ is randomly drawn from some distribution on $[0, 1]$, the probability that this no collusive equilibrium exists is lower when there are conflicts inside the audit company.

This model can be extended to the case in which $\delta < \delta(c)$ and the results are just a combination of this section and Section 4.1. To do so, it is enough to see that, even if the principal wants to collude with some types of firm, she still needs to induce the agent to truthfully reveal the firm’s type, because she wants to be able to get the bribe for herself.

6 Concluding Remarks

We have analyzed how collusion affects the contract offered by an auditor in a simplified three-agents model. We have shown how impatient auditors will propose lower audit prices in order to attract low quality managers with whom the stakes for collusion are large. The market anticipates such behavior and discounts high reports made by the auditor. This discount is however not enough to prevent collusion from occurring at equilibrium. Collusion is however not a probability one event. Indeed, we show that even an impatient auditor can remain truthful for a large number of periods as long as she is asked to audit high quality managers.

We have then extended the analysis in mainly two different directions. We have first considered the impact of competition and mandatory audit on the probability of collusion. Both aspects provide, through different channels, incentives to low quality managers to ask for audit and, in this respect, increase the stakes for collusion. We have then introduced an agency relationship within the audit company by considering that a long lived board of director who does not discount future payoffs has to provide incentives to a short lived impatient audit department to refrain him from colluding. We have identified two instruments to prevent collusion: a monitoring effort, which increases the probability of gathering evidence of collusion, and a high audit price, which reduces the bribe the audit department can extract. We show that the optimal audit price increases with the monitoring cost.

To conclude, let us discuss an interesting extensions which could have a positive impact on collusion. To do so, suppose that the probability of detecting collusion is bounded away from one. This clearly increases the tendency of the auditor to initiate collusive agreements. To be more specific, assume that the probability of detection increases with the size of the lie. Then,
a “small” lie would remain almost undetected whereas “large” misreportings would always be
detected. A new trade-off would then appear. The further from the truth the larger might be
the bribe but the higher is the probability of going out of business.

A Proof of Proposition 3

Call $\mu(\theta)$ the probability that the auditor colludes with type $\theta$, so $\mu(\theta) = \int_0^1 dG(r|\theta) \leq 1$. Playing a
mixed strategy at the collusion stage is optimal if and only if the payoff of all the strategies played with
positive density is equal.

Let us call $g(r/\theta)$ the equilibrium probability or density corresponding to the equilibrium cdf. The
auditor can then expect a bribe $b(r, \theta) = E[\Theta/r] - E[\Theta/\theta]$ from any misreport $r \neq \theta$.

The payoff of announcing the truth is the discounted expected profit that the auditor will get in
all future periods. Call that profit $\Pi$ which is constant in each period since the same game is repeated
successively. Then we have that

$$E[\Theta/r] - E[\Theta/\theta] \geq \delta \Pi, \forall r \text{ such that } g(r/\theta) > 0,$$

with equality if $\mu(\theta) < 1$.

Given this equilibrium strategy, the payoff of type $\theta$ manager is $E[\Theta/\theta] - p$, if he accepts the audit
contract and $\frac{\hat{\theta}}{2}$ if he does not accept.\footnote{This is true both with or without collusion since the manager only accepts to collude if he gets at least the
same as without collusion.} Therefore, the manager’s strategy is an equilibrium if

$$E[\Theta/\theta] - p \geq \frac{\hat{\theta}}{2} \forall \theta \geq \hat{\theta},$$

and

$$E[\Theta/\theta] - p < \frac{\hat{\theta}}{2} \forall \theta < \hat{\theta}.$$

Suppose first that there is an equilibrium without collusion. If the auditor is always truthful, buyers
should believe any announcement and the auditor will deviate by, at least, announcing 1 when the
manager is of type $\hat{\theta}$ since, by assumption, $\delta < \frac{3}{4}$.

Suppose now that there is an equilibrium with collusion. It has to satisfy the following properties:

$\forall \theta \geq \hat{\theta}$

$$E[\Theta/r] \geq E[\Theta/\theta] + \delta \Pi \forall r : g(r/\theta) > 0,$$

$$E[\Theta/r'] \leq E[\Theta/r] \forall (r, r') : g(r/\theta) > 0, \quad g(r'/\theta) = 0.$$

The first condition tells that the payoff when announcing a type $r$ different from the truth (the
bribe) has to be at least as high as the payoff of announcing the truth, making the auditor willing
to collude. The second condition implies that the auditor does not want to deviate to some other announcements. This also implies that, \( \forall \theta \geq \bar{\theta}, \) if \( g(r/\theta) > 0 \) then \( r \in \arg \max_a E[\Theta/a] \).

In what follows, define \( A \subseteq [\bar{\theta}, 1] = \{ r : r \in \arg \max_a E[\Theta/a] \}, \) so \( \forall (r, r') \in A^2, E[\Theta/r] = E[\Theta/r'] \) and \( \forall r \in A, \forall r'' \notin A, E[\Theta/r] > E[\Theta/r''] \) and let us consider a pair \((\theta, r) \in A^2\). The true type is \( \theta \). If the auditor announces \( \theta \) she earns no bribe but gets the discounted future profit \( \delta \Pi \). If she announces \( r \) instead she gets a bribe equal to

\[
E[\Theta/r] - E[\Theta/\theta] = 0 < \delta \Pi
\]

because \((\theta, r) \in A^2\). Therefore, the auditor strictly prefers to announce the truth for any type in \( A \). So \( \mu(\theta) = 0 \ \forall \ \theta \in A \).

Notice that \( E[\Theta/r] = r \ \forall r \notin A \), because the auditor announces types not in \( A \) only when she does not collude. Define by \( \bar{A} \) the complement of \( A \), we then have:

\[
\int_\bar{\theta}^1 \theta \mu(\theta) g(r|\theta) dF(\theta) = \int_\theta^A \theta \mu(\theta) g(r|\theta) dF(\theta)
\]

Then, it is easy to show that \( 1 \in A \). Suppose this is not the case. Then, \( E[\Theta/1] = 1 \). But by definition of \( A \), we have that \( E[\Theta/1] < E[\Theta/r'] \) for any \( r' \in A \), implying that \( E[\Theta/r'] > 1 \), which is a contradiction. Therefore we have \( 1 \in A \).

Moreover, \( E[\Theta/1] < 1 \) since some lower types are revealed as type 1 sometimes. Let us now define \( \theta^* = E[\Theta/1] = E[\Theta/r \in A] < 1 \). All \( \theta \geq \theta^* \) must then be in \( A \). Suppose it is not the case. Then if \( \theta \geq \theta^* \notin A \), we have \( E[\Theta/\theta] = \theta \geq \theta^* \). This contradicts the definition of \( A \). This means that the auditor colludes only with some types below \( \theta^* \), because there is no collusion with types in \( A \) and that, \( \forall r \in A, E[\Theta/r] = \theta^* \).

For any possible announcement \( r \), we have

\[
E[\Theta/r] = \frac{(1 - \mu(r)) f(r) r + \int_\bar{\theta}^1 \theta \mu(\theta) g(r|\theta) dF(\theta)}{(1 - \mu(r)) f(r) + \int_\bar{\theta}^1 \theta \mu(\theta) g(r|\theta) dF(\theta)}
\]

Now, we know that \( \theta^* \in A \) and, therefore, \( E[\Theta/\theta^*] = \theta^* \) and \( \mu(\theta^*) = 0 \). That is, we have:

\[
\frac{\theta^* + \int_{\theta \in A} \theta \mu(\theta) g(\theta^*|\theta) d\theta}{1 + \int_{\theta \in A} \mu(\theta) g(\theta^*|\theta) d\theta} = \theta^*.
\]

Rearranging terms, we get:

\[
\int_{\theta \in A} (\theta^* - \theta) \mu(\theta) g(\theta^*|\theta) d\theta = 0.
\]

Given that all \( \theta \in \bar{A} \) are such that \( \theta < \theta^* \), then such equality can only be verified if, \( \forall \theta \in \bar{A} \), either \( \mu(\theta) = 0 \) or \( g(\theta^*|\theta) = 0 \). The former condition cannot be satisfied since we have shown that there is no equilibrium without collusion. Therefore, we have \( g(\theta^*|\theta) = 0 \), meaning that the auditor never sends message \( \theta^* \) when she decides to collude.

Consider now another announcement \( r \neq \theta^* \) in \( A \). As above, we have:

\[
\frac{r + \int_{\theta \in A} \theta \mu(\theta) g(r|\theta) d\theta}{1 + \int_{\theta \in A} \mu(\theta) g(r|\theta) d\theta} = \theta^*.
\]
rearranging terms again, we get:

$$\int_{\theta \in \hat{A}} (\theta^* - \theta) \mu(\theta) g(r|\theta) d\theta = r - \theta^* \quad (10)$$

which can only be satisfied for $r > \theta^*$.

Let us now analyze the behavior of the auditor with managers whose quality is in $\hat{A}$. The maximum bribe that the auditor can extract with a type $\theta \in \hat{A}$ is $\theta^* - \theta$. The auditor then offers a collusion contract if and only if

$$\theta^* - \theta > \delta \Pi$$

Two cases must then be distinguished.

i. $\theta^* - \delta \Pi < \tilde{\theta}$. Then the auditor will never propose a collusion contract. This is the case when $\delta > \frac{3}{4}$, in which case $\theta^* = 1$ and $\tilde{\theta} = \frac{2}{3}$. However, for $\delta < \frac{3}{4}$, this can never happen.

ii. $\theta^* - \delta \Pi > \tilde{\theta}$. Then define, $\hat{\theta} \equiv \theta^* - \delta \Pi$. The auditor will collude with types in $[\tilde{\theta}, \hat{\theta})$. That is

$$\mu(\theta) = 1 \text{ when } \theta \in [\tilde{\theta}, \hat{\theta})$$

We can then rewrite condition (10) as follows for all $r \in (\theta^*, 1]$:

$$\int_{\tilde{\theta}}^{\hat{\theta}} (\theta^* - \theta) g(r|\theta) d\theta = r - \theta^*. \quad (11)$$

Moreover, by definition of a density, we also have, $\forall \theta \in [\tilde{\theta}, \hat{\theta})$

$$\int_{r \in \hat{A}} g(r|\theta) dr = 1 \quad (12)$$

We need to show that such a density exists for any $\theta \in [\tilde{\theta}, \hat{\theta})$. Since condition (11) must be true for any $r \in A$, a necessary condition for existence is that

$$\int_{x \in \hat{A}} \left( \int_{\tilde{\theta}}^{\hat{\theta}} (\theta^* - \theta) g(x|\theta) d\theta \right) dx = \int_{\tilde{\theta}}^{\hat{\theta}} (\theta^* - \theta) d\theta = \int_{r \in \hat{A}} (r - \theta^*) dr = \frac{1}{2} (1 - \theta^*)^2. \quad (13)$$

Then, we need to show that there is a probability measure over $A$ such that, for any $C \subseteq A$ we have

$$\int_{x \in C} \left( \int_{\tilde{\theta}}^{\hat{\theta}} (\theta^* - \theta) g(x|\theta) d\theta \right) dx = \int_{r \in C} (r - \theta^*) dr.$$

Using Fubini’s theorem, this can be written as

$$\int_{\tilde{\theta}}^{\hat{\theta}} (\theta^* - \theta) \left( \int_{x \in C} g(x|\theta) dx \right) d\theta = \int_{r \in C} (r - \theta^*) dr.$$

We define

$$\gamma(C|\theta) = \int_{x \in C} g(x|\theta) dx$$

as the probability that the report is in $C$ given that the true value is $\theta$. Then, it is easy to show that a function $\gamma(C|\theta)$ exists.
Using the fact that $\bar{\theta} = \theta^* - \delta\Pi$, we can rewrite condition (13) as

$$
\theta^*(1 - \bar{\theta}) + \frac{1}{2}(\bar{\theta}^2 - (\delta\Pi)^2) = \frac{1}{2} \quad (14)
$$

Call $\hat{\theta}(p)$ the type who is indifferent between accepting the contract or not, i.e., $\hat{\theta}(p) = \frac{\bar{\theta}}{2} + p$. In equilibrium, $\bar{\theta} = \hat{\theta}$ or $\bar{\theta} = 2p$. We can now define the expected profit of the auditor. It is given by

$$
\Pi = p(1 - \hat{\theta}(p)) + (\bar{\theta} - \hat{\theta}(p)) \left( \theta^* - \frac{\hat{\theta} + \hat{\theta}(p)}{2} \right) + (1 - \bar{\theta} + \hat{\theta}(p)) \delta\Pi \quad (15)
$$

which can be rewritten as follows:

$$
\Pi = \frac{p(1 - \hat{\theta}(p)) + (\bar{\theta} - \hat{\theta}(p)) \left( \theta^* - \frac{\hat{\theta} + \hat{\theta}(p)}{2} \right)}{1 - \delta(1 - \theta + \hat{\theta}(p))}. \quad (16)
$$

Thus, the auditor chooses the price to maximize $\Pi$. This gives an optimal price, $p(\delta, \bar{\theta}, \theta^*)$. Combining this with the equilibrium conditions $2p = \bar{\theta}$ and equation (14), we obtain $\hat{\theta}(\delta)$, $p(\delta)$, and $\theta^*(\delta)$. The numerical solution is provided in Figure 1.

**Proof of proposition 5**

We start by showing that $\theta^*$ is an increasing function of the price under mandatory audit. Under the constraint that $\bar{\theta} = 0$, life-time expected profit can be written as follows:

$$
\Pi = \frac{p + \bar{\theta} \left( \theta^* - \frac{\bar{\theta}}{2} \right)}{1 - \delta(1 - \theta)} = \frac{1 - \sqrt{1 + \delta^2(1 - 2p - 2\theta^*) - 2\delta(1 - \theta^*) - \delta(1 - \theta^*)}}{\delta^2},
$$

since $\bar{\theta} = \theta^* - \delta\Pi$. Similarly, rewriting the equilibrium condition, we get

$$
\theta^* - \frac{1}{2} (\delta\Pi)^2 = \frac{1}{2}. \quad (17)
$$

Using the implicit function theorem, we obtain that

$$
\frac{\partial \theta^*}{\partial p} \left( 1 - \delta^2 \Pi \frac{\partial \Pi}{\partial \theta^*} \right) = \delta^2 \Pi \frac{\partial \Pi}{\partial p}.
$$

Moreover, it is easy to show that

$$
1 - \delta^2 \Pi \frac{\partial \Pi}{\partial \theta^*} > 0.
$$

To do this, we just need to use condition (17) to show that

$$
\frac{\partial \Pi}{\partial \theta^*} = \frac{\theta^* - \sqrt{2\theta^* - 1}}{1 - \delta(1 - \theta^*) - \delta\sqrt{2\theta^* - 1}}.
$$

and hence

$$
1 - \delta^2 \Pi \frac{\partial \Pi}{\partial \theta^*} = \frac{1 + \delta(3\theta^* - 2) + \delta(1 + \theta^*)\sqrt{2\theta^* - 1}}{1 - \delta(1 - \theta^*) - \delta\sqrt{2\theta^* - 1}}.
$$
which is positive because \( \theta^* \geq \frac{1}{2} \) in equilibrium.

Thus, the sign of \( \frac{\partial \theta^*}{\partial p} \) is the same as the one of \( \frac{\partial \Pi}{\partial p} \), which is positive. \( \theta^* \) then increases with the audit price \( p \).

The probability of collusion is defined by \( \bar{\theta} = \theta^* - \tilde{\theta} \). By definition, \( \bar{\theta} = \theta^* - \delta \Pi \) and, therefore

\[
\frac{\partial \bar{\theta}}{\partial p} = \frac{\theta^* - \delta \Pi}{1 - \delta^2 \Pi \frac{\partial \Pi}{\partial \theta^*}} \frac{\partial \Pi}{\partial p}.
\]

(18)

We know from the implicit function theorem used above that \( \frac{\partial \theta^*}{\partial p} = \frac{\delta \Pi}{1 - \delta^2 \Pi \frac{\partial \Pi}{\partial \theta^*}} \frac{\partial \Pi}{\partial p} \). Plugging this back in (18), we get that

\[
\frac{\partial \bar{\theta}}{\partial p} = \frac{\delta (\delta \Pi - 1)}{1 - \delta^2 \Pi \frac{\partial \Pi}{\partial \theta^*}} \frac{\partial \Pi}{\partial p}.
\]

(19)

Given that \( \frac{\partial \Pi}{\partial p} > 0 \), the sign of the derivative depends on the sign of \( (\delta \Pi - 1) \). From the equilibrium condition we know that \( \theta^* - \frac{1}{2}(\delta \Pi)^2 = \frac{1}{2} \) which is equivalent to \( \delta \Pi = \sqrt{2\theta^* - 1} < 1 \), since \( \theta^* \leq 1 \). Therefore, \( \frac{\partial \Pr(\text{collusion})}{\partial p} < 0 \).

To show point (ii), we compute the price that would be needed under mandatory audit to obtain the same probability of collusion as in the case of voluntary audit. This is shown in Figure 3. Since this price is far above the monopoly price, and the probability of collusion increases when the price decreases, if the regulated price is below the monopoly price, the probability of collusion must be higher than in the case of a monopoly with voluntary audit.

**Proof of Proposition 6**

As long as \( c + p + \frac{\tilde{\theta}}{2} < 1 \), the spot profit of the principal is:

\[
\pi = \left( 1 - p - \frac{\tilde{\theta}}{2} \right) p - \int_{p + \frac{\tilde{\theta}}{2}}^{c + p + \frac{\tilde{\theta}}{2}} \left( s - p - \frac{\tilde{\theta}}{2} \right) \left( 1 - \frac{s - p - \tilde{\theta}}{c} \right) ds
- \int_{p + \frac{\tilde{\theta}}{2}}^{c + p + \frac{\tilde{\theta}}{2}} \frac{c}{2} \left( s - p - \frac{\tilde{\theta}}{2} \right)^2 ds - \int_{c + p + \frac{\tilde{\theta}}{2}}^{1} \frac{c}{2} ds
\]

(20)

We maximize with respect to \( p \) and we know that at any equilibrium \( p = \frac{\tilde{\theta}}{2} \). This gives

\[
p^*(c) = \frac{2 + c}{6}.
\]

But this is true as long as \( c + 2p^*(c) \leq 1 \), which implies that \( c \leq \frac{1}{4} \).

Now, if \( c \leq \frac{1}{4} \), then given (20) the profit of the principal if there is no collusion is

\[
\frac{2 - 4c + 5c^2}{18(1 - \delta)}
\]

(21)

If the principal colludes with the lowest type, \( \tilde{\theta}^*(c) \), she gets the spot profit plus the bribe \( b = 1 - \tilde{\theta}^*(c) \). So her profit is

\[
\frac{8 - 10c + 5c^2}{18}
\]

(22)
Comparing (21) and (22), the principal does not want to collude even with the lowest type if
\[ \delta \geq \delta_1(c) = \frac{6(1 - c)}{8 - 10c + 5c^2} \]

For \( c > \frac{1}{4} \), the spot profit is given by:
\[
\pi = \left( 1 - p - \tilde{\theta} \right) p - \int_{p + \frac{\tilde{\theta}}{2}}^{1} \left( s - p - \tilde{\theta} \right) \left( 1 - s - p - \tilde{\theta} \right) ds \\
- \int_{p + \frac{\tilde{\theta}}{2}}^{1} \frac{c}{2} \left( s - p - \tilde{\theta} \right)^2 ds
\]
and therefore the equilibrium price is
\[ p^*(c) = \frac{2 - 5c + \sqrt{25c^2 - 4c}}{4} \]

In this case, the profit of the principal if there is no collusion is
\[ \frac{12c - 25c^2 + (5c - 2)\sqrt{25c^2 - 4c}}{12(1 - \delta)} \]  
(23)

If the principal colludes with the lowest type, \( \tilde{\theta}^*(c) \), she gets the spot profit plus the bribe \( b = 1 - \tilde{\theta}^*(c) \). So her profit is
\[ \frac{42c - 25c^2 - (8 - 5c)\sqrt{25c^2 - 4c}}{12} \]  
(24)

Comparing (23) and (24), the principal does not want to collude even with the lowest type if
\[ \delta \geq \delta_2(c) = \frac{6 \left( 5c - \sqrt{25c^2 - 4c} \right)}{42c - 25c^2 - (8 - 5c)\sqrt{25c^2 - 4c}} \]

Note that \( \delta_1 \left( \frac{1}{4} \right) = \delta_2 \left( \frac{1}{4} \right) \) and \( \delta_1 ' \left( \frac{1}{4} \right) = \delta_2 ' \left( \frac{1}{4} \right) \).

Moreover, \( \delta_1 \) reaches a maximum at \( c < \frac{1}{4} \) and \( \delta_2 \) is decreasing for \( c > \frac{1}{4} \).

References


