Information Exchange and Competition in Communications Networks

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Abstract

We develop a model of information exchange between calling parties. We characterize the equilibrium when networks compete for such users that can be charged both for outgoing and incoming calls. We show that when calls originated and received are complements in the information exchange, then networks have reduced incentives to use off-net price discrimination to induce a connectivity breakdown. This breakdown totally disappears if operators are allowed to negotiate reciprocal access charges. We also show that a “bill-and-keep” system over access charges can approximate an efficient regime and we discuss when this system emerges from private negotiations.

JEL Classification Codes: L41, L96

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1. Introduction

People use electronic communications in order to exchange information. This exchange may take very different forms. Take, as an example, a situation with two individuals, A and B. A and B are academics trying to work on a joint paper but live in different places, thus they rely on telephone conversations and e-mail exchanges to conduct their work. One day, A and B are struggling to solve an equation. A finds the solution and calls B: B is obviously happy, but then does not need to call A back since the problems has been solved. Another day, A has an embryo of an idea and calls B. B needs some time to ponder over this rudimental idea and then calls A back to tell what he thinks about it: in this second case B is still happy of having received a call from A since it may lead to an improvement of their paper, but now the original call has initiated a chain reaction that has led B to return the call.

These simple examples illustrate a more general feature, namely that communications services are not consumed in isolation and involve interdependencies between callers. They also exemplify a phenomenon which seems particularly relevant for business users, where an exchange of information may create the need for further exchanges of information, leading to a stimulation of calls. From a theoretical standpoint, two things are worth noticing. Firstly, receivers of an electronic communications enjoy a positive benefit (a call externality). Secondly, each individual has a demand for calls that depends not only on the price she has to pay, but also on the amount of calls received from the other party. Thus the demand functions depend on the prices charged to both parties. This paper studies a model of network competition when demand functions depend on the way information is exchanged between users.

Existing models of network competition typically assume that consumers derive utility only from making calls. This literature,\(^1\) initiated by the seminal works of Armstrong (1998) and Laffont et al. (1998), asks if free negotiations over inter-carrier (wholesale) access charges can lead operators to choose (retail) fees that are detrimental to social welfare, for instance because they may be used to sustain collusive linear prices in the retail markets, or because operators of differing size may have divergent interests over the level of reciprocal interconnection fees (Carter and Wright, 2003), or because interconnection fees provide a commitment mechanism to limit competition over investments in infrastructure (Valletti and Cambini, 2005). However, these works ignore the role played by call externalities which, as we argued above, are pervasive in electronic communications.

\(^1\) See the surveys of Armstrong (2002) and Vogelsang (2003).
Call externalities are a relevant missing ingredient from the literature given its normative intent since they have a strong impact on the (efficiency properties of the) competitive process. Recent literature has started to look into the problem of how to set retail charges in an environment with call externalities (Kim and Lim, 2001; DeGraba, 2003; Jeon et al., 2004; Berger, 2004). These works, however, ignore the interdependencies between users and assume that the sets of calls/messages sent and received are independent of each other. An exception is represented by Hermalin and Katz (2004) who analyze two-way communications. Their focus is on the possible strategic game between the communicating parties as to who will be the sender and who the receiver. The concern of Hermalin and Katz (2004) is on pricing under uncertainty about the parties’ values of an exchange while they do not analyze multiple networks and the problem of interconnection.

Our work is theoretical but is also motivated by findings from an empirical literature on telecommunications demand, started by the work of Larson et al. (1990) on point-to-point demand. This framework disaggregates traffic into two distinct elements: autonomous traffic – independent of the traffic level between the networks – and induced traffic, which is influenced by the traffic volume. Induced traffic strictly depends on how information between two customers is exchanged. Taylor (2004) reviews the empirical literature from the early 90’s that applied point-to-point models to communications between city pairs in Canada (inter-provincial traffic) and in the US (inter-LATA traffic), and between countries (Canada/US among others). This empirical literature rejects separability between outgoing and incoming traffic, and finds strong and statistically relevant reverse-calling phenomena. Calls in one direction tend to generate calls in return that are independent of price and income. These ideas found applications in studies of long distance telephony in monopolistic markets but have not been applied to a competitive environment.

In this paper we take at face value these empirical findings that have been neglected by the theoretical literature that has either ignored call externalities, or modeled them in a simple fashion by assuming perfect separability between incoming and outgoing calls. We start by studying the process of information exchange which is the primitive that leads parties to enjoy utility from calls. Utility is not generated by calls per se, but rather calls are inputs to the production of information. Calls made and received could be substitutes (if the information received does not require the receiver to call back) or complements (if information be-

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2 As summarized by Taylor (2004, p. 129): “a call in one direction stimulates something like one-half to two-thirds of a call in return”.
comes “productive” only by responding to queries initiated by one the parties). Thus we do not take call externalities as given, but rather consider the way a caller and a receiver generate information. We then embed the fundamental process of information exchange between two parties calling each other into the IO framework of network competition.

Jeon et al. (2004; JLT hereafter) is the paper closest to ours. They determine competitive retail and reception charges when the utility function is perfectly separable between outgoing and incoming calls (i.e., there is no induced traffic). They show that, in the presence of a reception charge, the call price goes down as it reflects a pecuniary externality: when a network lowers its price, the subscribers to the rival network will have to pay more as they receive more calls. Hence an increase in the rival’s reception charge makes it more desirable for a network to expand its output. JLT also show that, in the presence of network-based discrimination both on final prices and on receiving charges, “connectivity breakdowns” arise. In an attempt to discourage subscribers from connecting to the rival, a network has an incentive to charge extremely high off-net call prices and off-net receiver prices. By doing so, the subscribers to the rival network could not benefit from making or receiving calls off-net. In particular, connectivity breakdown can occur according to different mechanisms:

- When the benefit from receiving calls is small compared to originating calls, then it is the recipient network that wants to reduce off-net calls as they would benefit mostly the originating rival; the instrument to achieve this purpose is a very high off-net receiver charge.
- When the benefit from receiving calls is high compared to originating calls, then it is the originating network that wants to reduce off-net calls as they would benefit mostly the recipient rival; the instrument to achieve this is a very high off-net originating charge.

These results are worrying from a policy perspective and call for some sort of intervention (e.g., regulation of reception charges). We assess their relevance under a more general formulation of the information exchange process. We also consider if mild forms of intervention might allow the industry to self-regulate. Firstly, we show that when access charges are taken as given, the risk of a “connectivity breakdown” is much diluted compared to JLT when induced traffic is positive (i.e., calls made and received are complements in the information production function). Secondly, we show that this risk is completely eliminated if operators can set reciprocal access charges (JLT do not endogenize access charges). Intui-
tively, although reciprocal deals give firms an instrument to tacitly collude, there is no need to destroy off-net traffic. On the contrary, operators prefer to choose very low access charges so that off-net prices are also very low: in this way the intensity of competition for the market is reduced as customers find a “big” network less appealing than a “small” one. While concerns for a breakdown are weakened, we still show how there exists some (limited) room for intervention to improve the outcome of private negotiations, for instance by using suitable reception charges. Finally, we consider the interaction between termination charges and retail prices, both for outgoing and incoming calls. We show that the level of reception charges depends quite crucially on access charges.3

From the positive point of view our paper is about competitive pricing in the presence of externalities. Our analysis is also relevant from the normative point of view to assess the impact of recent regulatory interventions over inter-network charges. The debate on the regulation of fixed-to-mobile termination charges in Europe, where a Calling Party Pays (CPP) system is in place, includes the adoption of charges based on Long Run Incremental Costs (LRIC), and the introduction of RPP (Crandall and Sidak, 2004). The determination of mobile-to-mobile termination is typically less intrusive and it includes reciprocal arrangements, and in some instances “bill-and-keep” deals. “Bill-and-keep” has been advocated as a way of sharing efficiently the value created by a call when both callers and receivers benefit from it (DeGraba, 2002). We give conditions under which this system is indeed efficient and we study if it can ever emerge from private negotiations. We also show how RPP and access charges are related to each other.

The remainder of the paper is organized as follows. Section 2 introduces a model of information exchange between calling parties. Section 3 solves a model of network competition with uniform pricing. Section 4 analyzes the case with network-based price discrimination. Since price discrimination typically does not exhibit the result of profit neutrality with respect to access charges, in section 4 we also ask what level would be chosen by a negotiating pair of operators as opposed to a benevolent social planner. Section 5 considers the case of market-determined reception charges and section 6 concludes.

3 The operators’ practice of charging customers both for originating and receiving calls is usually referred to as the Receiver Pays Principle (RPP) which is applied in the mobile markets of countries such as Canada, China, Hong Kong, and the United States. Clearly, positive reception charges make sense only if the receiver cares about being called, otherwise no one would ever answer a call.
2. The information exchange process and call externalities

Imagine there are two users. One user subscribes to network $i$ and the other user to network $j$ and they communicate with each other. For the moment assume the networks are different, hence $i$ can also denote the first user and $j$ the second user. Each user is assumed to derive utility from the amount of information created from originating and receiving calls in a typical point-to-point connection. Information for user $i$, denoted $I^{ij}$, is produced according to the following production function: $I^{ij} = f^{ij}(Q^{ij}, Q^{ji})$, where $Q^{ij}$ is the quantity of outgoing calls user $i$ originates to contact user $j$ and $Q^{ji}$ is the quantity of incoming calls that user $i$ receives from user $j$. The production function $f(\cdot, \cdot)$ is one of the important primitives in our model and it can describe different cases when calls are substitutes or complements in the information creation process. $\partial I^{ij} / \partial Q^{ij} = f^{ij}_1$ and $\partial I^{ij} / \partial Q^{ji} = f^{ij}_2$ denote respectively how information changes at the margin with calls sent and received by consumer $i$ when communicating with $j$, and they are both assumed to be positive. We use superscripts $ij$ to denote the exchange of information between $i$ (the originator) and $j$ (the receiver). This notation can also accommodate the case when both users are connected to the same network, in which case $j = i$ and $I^{ii} = f^{ii}(Q^{ii}, Q^{ii})$.

Denote with $p_i$ (respectively $p_j$) the price charged by network $i$ (j) for calls originated by user $i$ (j). Users have identical preferences over consumption of information and $U_i(I^{ij})$ denotes the utility function of user $i$ when communicating with user $j$. We assume for the moment that users cannot be charged for receiving calls. Users $i$ and $j$ determine simultaneously the quantity of calls to consume by maximizing the following problems:

**User $i$:**

$$\begin{align*}
\max_{Q^{ij}} U_i(I^{ij} - p_i Q^{ij}) \\
\text{s.t.} \quad I^{ij} = f^{ij}(Q^{ij}, Q^{ji})
\end{align*}$$

**User $j$:**

$$\begin{align*}
\max_{Q^{ji}} U_j(I^{ji} - p_j Q^{ji}) \\
\text{s.t.} \quad I^{ji} = f^{ji}(Q^{ji}, Q^{ij})
\end{align*}$$

The first-order conditions are respectively $U'_i f^{ij}_1 = p_i$ and $U'_j f^{ji}_1 = p_j$, which determine the equilibrium level of calls originated by user $i$ (directed to $j$) and user $j$ (directed to $i$): $Q^{ij} = Q^{ij}(p_i, p_j)$ and $Q^{ji} = Q^{ji}(p_j, p_i)$. In general, the quantity originated by a user is affected both by the price (s)he pays and by the price other people pay. Let $Q^{ij}_1 = \partial Q^{ij}(p_i, p_j) / \partial p_i$ denote the “direct” price effect on quantities and $Q^{ij}_2 = \partial Q^{ij}(p_i, p_j) / \partial p_j$ the “induced” price effect. Their sign is found by totally differentiating the first-order conditions. Denoting with
\[ A^{ii} = U''(f_i^1)^2 + U'_i f_{1i}^i < 0 \] the second order derivative with respect to own prices and \\
\[ B^{ii} = U''_i f_{1i}^i f_{2i} + U'_i f_{12} \] \\
the cross derivative with respect to own and rival prices, it results:

\[
\begin{bmatrix}
A^{ii} & B^{ii} \\
B^{ii} & A^{ii}
\end{bmatrix}
\begin{bmatrix}
dQ^{ii} \\
dQ^{ii}
\end{bmatrix}
= 
\begin{bmatrix}
dp_i \\
dp_j
\end{bmatrix}
\]

In a symmetric equilibrium with \( p_i = p_j \), then \( A^{ii} = A^{ii} = A, B^{ii} = B^{ii} = B \) and:

\[
\begin{cases}
Q_1^{ii} = A/(A^2 - B^2) \\
Q_2^{ii} = -B/(A^2 - B^2)
\end{cases}
\]

If both users are connected to the same network, i.e. calls are “on-net”, then \( p_i = p_j \).

For “on-net” calls the total effect of a price change is:

\[
dQ^{ii} / dp_j = Q_1^{ii} + Q_2^{ii} = 1/(A + B).
\]

In summary, a point-to-point exchange of information between two parties leads them to call each other, generating demand functions, \( Q^{ii}(p_i, p_j) \) and \( Q^{ii}(p_j, p_i) \). These demand functions are determined as a Nash equilibrium of the game played by the two users, where each user controls only one input of the information exchange. Demand functions thus react to the prices charged to both parties. The direct effect has a standard sign: \( \text{sign} \left[ \partial Q^{ii} / \partial p_i \right] = \text{sign}[A] = \text{sign} \left[ U''(f_i^1)^2 + U'_i f_{1i}^i \right] < 0 \). There is also an indirect effect that induces additional traffic: \( \text{sign} \left[ \partial Q^{ii} / \partial p_j \right] = \text{sign}[-B] = \text{sign}[-(U'_i f_{1i}^i + U'_i f_{12})] \). The sign of \( B \) is ambiguous and it depends on the properties of the information production function. It turns out to be zero only when the utility function is perfectly separable between the benefit derived from outgoing and incoming calls, as it is assumed by JLT. As anticipated by the Introduction, separability is a strong assumption and does not seem to hold in the data. According to the empirical literature, calls tend to generate additional calls in return, thus \( \text{sign}[B] > 0 \). This implies that incoming and outgoing calls should be complements in the information production function (\( f_{12} > 0 \)).

It is easy to accommodate the framework of information exchange also in the presence of reception charges. If network \( i \) (\( j \)) charges its customers for receiving calls at a price \( r_i \) (\( r_j \)
per call, then the previous Nash equilibrium that generates \( Q^i(p_i, p_j) \) and \( Q^j(p_j, p_i) \) is unchanged as long as customers accept every call, which indeed would happen if \( U'_j f^j_2 \geq r_j \) and \( U'_i f^i_2 \geq r_i \). We call this situation where reception charges do not “bind”, either because they are zero or “low” enough, a situation of “sender sovereignty”.

When reception charges do bind, the same analysis can be applied. In particular, if both reception charges are “high” enough compared to sending charges, then each customer effectively controls the amount of calls received. The first-order conditions in this case are respectively \( U'_i f^i_1 = r_i \) and \( U'_j f^j_1 = r_j \), which determine the volume of calls at the Nash equilibrium: \( Q^i(r_j, r_i) \) and \( Q^j(r_i, r_j) \). This situation of “receiver sovereignty” arises when sending charges do not bind: \( U'_i f^i_1 \geq p_i \) and \( U'_j f^j_1 \geq p_j \).

Finally, a last possibility arises when the binding prices (both for sending and receiving calls) are set by one network alone. This case is denoted as a situation of “network sovereignty”. For instance, if network \( i \) is sovereign, then user \( i \) chooses both inputs in the information exchange. The first-order conditions in this case are \( U'_i f^i_1 = p_i \) and \( U'_i f^i_2 = r_i \), which determine the volume of calls: \( Q^i(p_i, r_i) \) and \( Q^j(r_i, p_i) \). Notice that this is a simple optimization problem conducted by user \( i \), since user \( j \) is passive. This situation of “network \( i \) sovereignty” arises when user \( j \) accepts all calls received (\( U'_j f^j_2 \geq r_j \)) and cannot send additional calls since they are truncated by \( i \) (\( U'_j f^j_1 \geq p_j \)).

The four situations of sender/receiver/network \( i \)/network \( j \) sovereignty exhaust all possible cases for the determination of the volume of calls. Most of our analysis in the remainder concentrates on sender sovereignty since it is the more relevant case from an empirical standpoint. We do so by considering reception charges that are regulated and “low” enough, so that they do not bind. However, we do examine also the other situations when we analyze market-determined reception charges in Section 5.

### 3. The model of network competition

Demand for calls between a pair of users is generated according to the model described in the previous Section and summarized by the price effects of eq. (1). The model of competition follows Armstrong (1998), Laffont et al. (1998) and its extension by JLT. There are two net-
works, differentiated à la Hotelling, competing in a communications market. A unit mass of consumers is uniformly located on the segment $[0,1]$ while the network operators are located at the two extremities. We denote by $1$ (respectively $2$) the firm located at the origin (respectively at the end) of the line. Customers pay a three-part tariff, $T_i(q) = F_i + p_i q_o + r_i q_r$, $i = 1, 2$, where the fixed fee $F_i$ can be interpreted as a subscriber line charge, $p_i$ as the marginal price for originating $q_o$ calls, $r_i$ as the unit price that network $i$’s consumer pays for receiving $q_r$ calls.\(^4\)

When a consumer located at $l$ buys from firm $i$ located at $l_i$, her utility is given by $y + v_0 - |l - l_i|/(2\sigma) + w_i$, where $y$ is the income, $v_0$ is a fixed surplus component from subscribing (sufficiently large such that all customers always choose to subscribe to a network) and $w_i$ is the indirect utility derived from making and receiving calls that we describe below. The parameter $\sigma$ represents an index of substitutability between the networks.

The consumer indifferent between the two networks determines the market share of the two firms. In particular firm $i$’s share is $\alpha_i$ where:

\[
(2) \quad \alpha_i = \alpha(w_i, w_2) = 1/2 + \sigma(w_i - w_j)
\]

where:

\[
(3) \quad w_i = \alpha_i v(p_i, p_i) + \alpha_j v(p_i, p_j) - r_i \left( \alpha_i Q^u_i(p_i, p_i) + \alpha_j Q^u_j(p_j, p_i) \right) - F_i
\]

\[
(4) \quad v(p_i, p_j) = U_i(f^i(Q^u_i(p_i, p_j), Q^u_j(p_j, p_i)) - p_i Q^u_i(p_i, p_i)
\]

\[
(5) \quad v(p_j, p_j) = U_i(f^i(Q^u_i(p_i, p_j), Q^u_j(p_j, p_j)) - p_i Q^u_i(p_i, p_j)
\]

This system of equations describes a situation where a customer has many point-to-point information exchanges. A customer belonging to network $i$ has $\alpha_i$ exchanges “on net” and $\alpha_j$ exchanges “off net”. Each on-net exchange generates a quantity of calls $Q^u_i$ originated by the user connected to network $i$ and destined to a user connected to the same network. This quantity depends only on $p_i$, and $v(p_i, p_j)$ is the corresponding (indirect) utility derived from the single point-to-point exchange on net. Similarly, $v(p_j, p_j)$ is the (indirect) utility derived

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\(^4\) The case with network-based price discrimination is considered in Section 4.
from each off-net exchange that involves $Q^{ij}$ calls originated by the user connected to network $i$ and destined to a user connected to network $j$ (this depends not only on $p_i$ but also on $p_j$ since the reverse traffic is originated by the rival network).

Both networks have full coverage. Serving a customer involves a fixed cost $f$ of connection and billing. The marginal cost is $c$ per call at the originating end and $t$ at the terminating end. The total marginal cost for a call is $c + t$. Networks pay each other a reciprocal two-way access charge, denoted by $a$, for terminating each others calls. Thus, network $i$’s profit is given by:

$$
\pi_i = \alpha_i \left\{ \alpha_i (p_i - c - t + r_i) Q^{ii} (p_i, p_i) + \alpha_j (p_i - c - a) Q^{ii} (p_i, p_j) + F_i - f \right\} + \alpha_i \alpha_j (a - t + r_i) Q^{ii} (p_j, p_i)
$$

Network $i$’s profit in (6) is given by the sum of the following components: the profit deriving from originating calls terminated on the same network, including the reception charge ($\alpha_i^2 (p_i - c - t + r_i) Q^{ii}$); the profit from originating calls destined to and terminated on the rival network ($\alpha_i \alpha_j (p_i - c - a) Q^{ii}$); the profit from termination and reception of incoming calls originated by customers of the rival network ($\alpha_i \alpha_j (a - t + r_i) Q^{ii}$).

We consider the following timing of the game. First interconnection ($a$) and reception ($r$) terms are set in stage I. Then operators compete in two-part prices ($F$ and $p$) in stage II. In particular, we assume that reception charges are always set “low” enough. In this way, we can concentrate on the case of “sender sovereignty” only. This analysis obviously includes the case where reception charges do not exist ($r = 0$) as is the case in many countries that adopt CPP. It also deals with the case of reception charges regulated in stage I by a benevolent social planner that makes sure that they do not “bind” in stage II.

3.1. Equilibrium with uniform pricing under “sender sovereignty”

In the Appendix we obtain the following expression for calling prices at equilibrium:

$$
p_i^* = c + t + \alpha_j (a - t) (\Phi - \Gamma) - \alpha_i r_j \Phi + (\alpha_i U'_i f'_i \Phi - U'_i f'_i \Theta)
$$
where:

- $\Phi = \frac{\partial Q^o_i}{\partial p_i} = \frac{Q^o_i}{Q^o_i + \alpha_i(Q^o_i + Q^r_i)}$, where $Q^o_i = \alpha_i Q^u_i + \alpha_j Q^u_j$ denotes the total quantity of calls originated by network $i$ and destined both on-net and off-net. $\Phi$ represents the ratio between the marginal change in outgoing off-net calls and the average marginal change in all outgoing calls (both off- and on-net, including the return effect on the latter), caused by an increase of price $p_i$; $\Phi$ is a summary of the “outgoing return phenomenon”.

- $\Gamma = \frac{\partial Q^i_j}{\partial p_i}$ is the ratio between the marginal change in incoming off-net calls and the average marginal change in all outgoing calls, caused by an increase of price $p_i$. $\Gamma$ is a summary of the “incoming return phenomenon”.

- $\Theta = \frac{\partial Q^i_j}{\partial p_i}$, with $Q^i_j = \alpha_i Q^u_i + \alpha_j Q^u_j$ denoting the total quantity of calls received by users connected to network $i$. $\Theta$ represents the ratio between the average marginal change in all incoming calls and the average marginal change in all outgoing calls, caused by an increase of price $p_i$. $\Theta$ is a summary of “incoming-to-outgoing” calls.

Eq. (7) can be interpreted as follows:

- $c + t + \alpha_j(a - t)(\Phi - \Gamma)$ represents the network’s direct perceived marginal cost. The possible termination mark-up for off-net calls is multiplied by the difference between the off-net outgoing and incoming ratios, reflecting the two-way effect of call propagation;\(^5\)

- $-\alpha_j r_j \Phi$ is the pecuniary externality due to the presence of the reception charge $r_j$. This is multiplied only by the factor $\Phi$, the outgoing return ratio from $i$ to $j$, since this is the only direction that matters for the pecuniary externality imposed on $j$’s customers.

- $\alpha_j f'_2 \Phi - \alpha_i f'_2 \Theta$ represents the difference between the marginal utility of user $j$ from receiving information through calls originated by users $i$ (this is why the marginal utility per point-to-point exchange is multiplied by the off-net outgoing ratio $\Phi$ and by $i$’s share) and the marginal utility of receiving calls by user $i$ (user $i$ receives information both on-\(^5\) One would expect $\Phi \geq \Gamma$. Also notice that the sign of the termination mark-up does not matter if $\Phi = \Gamma$.}
net and off-net, thus the marginal utility per exchange is multiplied by the incoming-to-
outgoing calls ratio \( \Theta \).

Eq. (7) generalizes previous results obtained in the literature. When the utility func-
tion is perfectly separable between outgoing and incoming calls, as it is in JLT, it results
\( Q_1^i = Q_1^j \) and \( Q_2^i = Q_2^j = 0 \), so that the incoming ratio is \( \Gamma = 0 \) and the outgoing ratio is \( \Phi = 1 \), while the incoming-to-outgoing ratio simplifies to \( \Theta = \alpha_i \). Then eq. (7) simplifies to
\[
p^*_i = c + t + \alpha_j (a - t) - \alpha_i \tilde{r}_j + \alpha_i (U'_j f'_2 - U'_i f'_2),
\]
that corresponds to (and partly amends) the result of JLT.\(^6\) If users do not receive utility from being called \( (f_2 = 0) \) and are not charged for
reception \( (r = 0) \), then (7) becomes simply \( p^*_i = c + t + \alpha_j (a - t) \), as in Laffont et al. (1998).

Under our more general specification we obtain the following:\(^7\)

**Proposition 1.** Under “sender sovereignty”, the symmetric equilibrium with uniform multi-
part pricing is characterized by the following conditions:

1. **a)** the call price is given by:

\[
p^* = c + t + \frac{2}{2 + x} \left[ \frac{a - t}{2} (1 - x) - \frac{r}{2} - xy \right]
\]

where \( x \) is a “propagation” factor given by: \( x = \frac{\Gamma}{\Phi} = \frac{\partial Q^i}{\partial p_i} \);

2. **b)** the fixed charge is given by: \( F^* = f + 1/2 \sigma - (p^* + r - c - t)Q(p^*) \);

3. **c)** each network’s profit is equal to \( 1/4 \sigma \).

The result of profit neutrality (part c)) is common in symmetric models of network com-
petition with multi-part uniform pricing. The more interesting feature of Proposition 1 lies in
the expression for the call price given by eq. (8) where we have introduced a new element,
the propagation factor \( x \). This denotes the ratio between the change of off-net calls when the
rival price changes and the direct change of on-net calls when the own price changes. In other
words the propagation factor measures the relative impact of the reverse traffic phenomenon.

\(^6\) Proposition 1 of JLT (p. 95) misses the last term in brackets, which is zero only in a symmetric equilibrium. This term represents the difference in users’ marginal utility for receiving calls. It shows that if the marginal utility from receiving calls is higher for a rival user than for a own user, then, ceteris paribus, a network would increase its own price in order to reduce its outgoing calls and reduce the benefits of users of the rival network.

\(^7\) JLT show that a symmetric equilibrium exists if \( \sigma \) is small enough or \( |a - t - r| \) is small enough.
Using eq. (1) and the notation from Section 2, the expression for the propagation factor in a symmetric equilibrium is:

\[
x = \frac{-B}{A} = \frac{-\left( U'f_1f_2 + U'f_{12} \right)}{U'(f_1)^2 + U'f_{11}}.
\]

Since \(A\) is negative, the sign of \(x\) is the same as the sign of \(B\). As we argued in the Introduction, the empirical evidence available to us suggests that incoming and outgoing calls are complements in the information production function, hence \(x\) should take positive values, although there is nothing in theory to prevent substitution effects, leading to negative values of \(x\). It is also realistic to imagine that \(x < 1\), while \(x = 1\) corresponds to the case of perfect complements. This factor reduces to zero when the separability condition on utility holds, as in JLT, in which case eq. (8) reduces to \(p^* = c + t + (a - t - r)/2\).

The impact of the propagation factor on the operators’ pricing is two-fold. On the one hand, a higher \(x\) tends to decrease the price in order to induce the “call generates calls” phenomenon which is enjoyed by customers: this is the last term of the square bracket in eq. (8). On the other hand, call propagation also impacts on call prices via access imbalances and reception charges. The “perceived” off-net cost \((a - t)/2\) is pushed down the higher is the propagation factor \(x\) as every call off-net (for which \(a - t\) is paid) induces a fraction of return calls (for which \(a - t\) is received).

### 3.2 Welfare analysis

We now compare the equilibrium call charges with the social optimum. This is found by maximising total welfare, i.e. the utility of the calling and the receiving parties minus the total cost, leading to \(U'(f_1 + f_2) = c + t\). The optimal outcome reflects the externality and, under sender sovereignty (i.e., \(U'f_2 \geq r\)), could be implemented by setting the following call charge:

\[
p = c + t - U'f_2.
\]
The optimal call charge should be set below the marginal cost, accounting for the positive externality from receiving calls. By equating eq. (8) to the optimal charge, one derives the following condition that would ensure efficiency:

\[(9) \quad (a-t)(1-x)-r = -(2-x)U'f_2.\]

Since there are two prices to be set \((a\) and \(r\)), several cases can arise:

- Imagine there is no reception charge \((r = 0)\), as is the case under a CPP system, then efficiency can be ensured only with below-cost access charges. In this case in fact from eq. (9) one gets \(a = t - (2-x)U'f_2/(1-x) < t\). To understand this result, recall that efficiency dictates that the call price is set below cost: without a reception charge the only way to decrease the retail price is to cut the “perceived” marginal cost of a network, thus \(a < t\). If the benefit from receiving calls is high, this might imply a “bill-and-keep” \((a = 0)\).

- Imagine termination is set at the LRIC level, \(a = t\), then efficiency can be reached only with a positive reception charge; however this is not compatible with “sender sovereignty” if \(x < 1\). In this case in fact from eq. (9) one gets \(r = (2-x)U'f_2\). The level of the reception charge decreases with the level of the propagation factor. However, the reception charge should be set so high that it violates the constraint \(U'f_2 \geq r\). Only the case of perfect complements \((x = 1)\) makes LRIC regulation compatible with efficiency and sender sovereignty. In fact the last result is more general: if \(x = 1\) efficiency can always be ensured for any access charge if \(U'f_2 = r\), as can be seen directly from eq. (9).

- A corollary of the previous case is that efficiency cannot be ensured under sender sovereignty if \(a > t\) since from eq. (9) \(r = (2-x)U'f_2 + (a-t)(1-x) > U'f_2\); thus no reception charge can induce to “call more” since the receivers would hang up.

- Imagine the reception charge is set at the network termination cost, \(r = t - a\), then efficiency can be reached only if access is priced below marginal cost. In this case in fact from eq. (9) one gets \(a - t = -U'f_2 < 0\). Notice that these conditions imply \(U'f_2 = r\) and \(a = t - r\), thus they just satisfy the constraint of “sender sovereignty”. When the externality is high, this might imply a zero termination charge and a reception charge equal to the marginal cost of termination.
4. Network-based discrimination

Under uniform pricing operators are indifferent with respect to the level of the reciprocal access charge. This makes the model not very interesting to understand if the industry could “self-regulate” with a minimal degree of intervention, e.g., by simply imposing a reciprocity clause. To address this question we now consider the case of network-based discrimination, both on final prices and on receiving charges. In other words, customers are offered five-part tariffs of the form: $T_i(q) = F_i + p_i q_o + \hat{p}_i \hat{q}_o + r_i q_r + \hat{r}_i \hat{q}_r$, $i = 1, 2$, where $\hat{r}_i$ is the unit price for receiving calls originated on network $j \neq i$ and $\hat{p}_i$ is the unit price for off-net calls.

Firm $i$’s share is still given by eq. (2) where now:

$$w_i = \alpha_i v(p_i, p_i) + \alpha_j v(\hat{p}_i, \hat{p}_j) - r_i \alpha_i Q^\alpha(p_i, p_i) - \hat{r}_i \alpha_j Q^\alpha(\hat{p}_j, \hat{p}_i) - F_i$$

is the net surplus for customers connected to network $i$. Notice that with the notation above we have again implicitly assumed that reception charges are “low” enough so that they do not bind in determining call volumes that are determined instead by callers alone. Network $i$’s profit becomes:

$$\pi_i = \alpha_i \{\alpha_i (p_i - c - t)Q^\alpha(p_i, p_i) + \alpha_j (\hat{p}_i - c - t)Q^\alpha(\hat{p}_i, \hat{p}_j) + F_i - f\} +$$

$$+ \alpha_i \{\alpha_j r_i Q^\alpha(p_i, p_i) + \alpha_j \hat{r}_i Q^\alpha(\hat{p}_j, \hat{p}_i)\} + \alpha_i \alpha_j (a - t)\left[Q^\alpha(\hat{p}_j, \hat{p}_i) - Q^\alpha(\hat{p}_i, \hat{p}_j)\right]$$

(10)

The timing of the game is as before. First interconnection ($a$) and reception ($r$) terms are set in stage I. Then operators compete in multi-part prices ($F$, $p$ and $\hat{p}$) in stage II.

4.1. Equilibrium in stage II: price competition

 Operators can internalize everything on-net, resulting in an efficient amount of on-net calls. On the contrary, there are uninternalized externalities off-net and thus inefficiency can arise with off-net communications. In the Appendix we prove the following:
Proposition 2. When a symmetric equilibrium with sender sovereignty and termination-based discrimination exists, this equilibrium is characterized by:

\begin{enumerate}
\item the on-net unit price is at the socially optimal value: \( p^* = c + t - Uf_2 \);
\item the off-net unit price is given by the following expression:
\begin{equation}
\hat{p}^* = c + a - (a - t)x - \hat{r} + Uf_2(1 - x);
\end{equation}
\item the fixed charge is given by the following expression:
\begin{equation}
F' = f - (p^* - c - t)Q(p^*, p^*) + 1/2\sigma - \hat{r}Q(\hat{p}^*, \hat{p}^*) - (v(p^*, p^*) - v(\hat{p}^*, \hat{p}^*));
\end{equation}
\item a “connectivity breakdown” is less (more) likely to arise when the propagation factor \( x \) is positive (negative).
\end{enumerate}

JLT find that when the utility function is separable and the utility from incoming calls is a fraction \( \beta \) of the utility from outgoing calls, there is always a “connectivity breakdown” if \( \beta \geq 1 \) via infinitely high off-net prices. To compare our results with JLT, denote with \( \beta = f_2 / f_1 \) so that \( Uf_2 = \beta \hat{p} \). From eq. (11) the equilibrium off-net price is re-written as:

\begin{equation}
\hat{p}^* = \frac{c + a - (a - t)x - \hat{r}}{1 - \beta(1 - x)},
\end{equation}

which takes finite values for \( 0 \leq \beta < \frac{1}{1 - x} \). Then, even if \( \beta \) tends to 1 we do not have connectivity breakdown if the propagation factor is positive. In our framework, the breakdown appears (and is even more pronounced) when calls are substitutes \( (x < 0) \) in the production of information since it could occur also for values of \( \beta < 1 \), while the phenomenon of network breakdown is less likely when calls are complements \( (x > 0) \). Notice that when \( x = 1 \), i.e. when a change of price induces the same change in outgoing and incoming volumes, then the off-net price simplifies to \( c + t - \hat{r} \). The access charge \( a \) does not matter at all since for any outgoing call (for which \( a \) has to be paid) there will also be an incoming return call (for which \( a \) will be received).

A positive propagation factor helps ensure that the off-net exchange of information is not cut via a breakdown induced by high retail charges when senders determine volumes.\(^8\) A mirror result arises in the case of “receiver sovereignty” when calling prices do not bind. In

\(^8\) Since we have assumed that the volume of traffic is determined by callers, it must be \( \hat{p} \hat{r} \geq \hat{r} \), which from eq. (11bis) can be re-written as \( \hat{r} \leq \beta(a(1 - x) + c + tx) / (1 + \beta x) \).
JLT a connectivity breakdown would occur if $\beta > 1$ via infinitely high off-net reception prices. In our model, even if $\beta$ tends to 1 we do not have connectivity breakdown if the propagation factor is positive since reception charges take finite values for $\beta > 1 - x$.\(^9\) All in all, the magnitude of the connectivity problem is reduced compared to JLT if calls originated and received are complements in the information production function. To understand this, recall that the breakdown typically happens because one network (say the receiving) has the opposite interest to the other network (say the originating). However, once calls are complements to generate information, then the receiving network will need to allow termination of calls if it wants also its own originating calls to become “productive”.

### 4.2 Stage I and the role of access charges

We now ask what level of access charges would be chosen in the first stage by negotiating firms as opposed to a social planner, to see if and how private and social interests diverge. In case of a discrepancy, in stage I a benevolent regulator may also set a reception charge.

The stage-II profit in a symmetric equilibrium is given by:

$$
\pi^* = \frac{1}{4} \left( p^* + r - c - t \right) Q(p^*, p^*) + \frac{1}{4} \left( \hat{p}^* + \hat{r} - c - t \right) Q(\hat{p}^*, \hat{p}^*) + \frac{1}{2} \left( F^* - f \right),
$$

where the expressions for the equilibrium off-net prices and fixed fees are given respectively by eq. (11bis) and (12): these are the only prices affected by access charges and by off-net reception charges. We imagine that in the first stage network operators are free to negotiate reciprocal access prices. We also suppose that a low-enough $\hat{r}$ is set exogenously and may be controlled only by the regulator. Given the symmetry, negotiations over access charges are equivalent to maximizing each operator’s profit. Thus the reciprocal access charge is set such that

$$
\frac{\partial \pi^*}{\partial a} = \frac{1}{4} \left[ \frac{\partial Q(p^*, \hat{p}^*)}{\partial a} \left( \hat{p}^* - c - t + \hat{r} \right) + Q(p^*, \hat{p}^*) \frac{\partial \hat{p}^*}{\partial a} \right] + \frac{1}{2} \frac{\partial F^*}{\partial a} \leq 0.
$$

The effect of $a$ on the volume of off-net calls is:

\(^9\) A formal proof is available from the authors.
\[
\frac{\partial Q(\hat{p}^*, \hat{p}^*)}{\partial a} = \left[ Q_1(\hat{p}^*, \hat{p}^*) + Q_2(\hat{p}^*, \hat{p}^*) \right] \frac{\partial \hat{p}^*}{\partial a} = (1 + x) Q_1(\hat{p}^*, \hat{p}^*) \frac{\partial \hat{p}^*}{\partial a}.
\]

The effect of \(a\) on the off-net price \(\hat{p}^* = \frac{c + a - (a-t)x - \hat{r}}{1 - \beta(1-x)}\) is quite intricate in general. We cannot suppose that the ratio \(\beta\) of marginal utilities does not change with \(a\), since this would be true only by taking the special separable functional form of JLT. To simplify the problem, we assume from now that the propagation effect does not substantially change with a change in prices. In other words, if calls “propagate” at certain vector of prices – for instance every call sent induces 0.5 calls in return – a change of prices would not change the information exchange and calls would still “propagate” in the same manner with \(x = 50\%\). This assumption is restrictive but is arguably realistic for small price changes. Under this simplifying assumption, we obtain the following:

**Proposition 3.** If an interior solution exists, network operators jointly agree on the following reciprocal access charge:

\[
\hat{a}^* = t + \frac{1}{(1-x)(1+2\beta)} \left[ -\beta(3-x)(c + t) + \hat{r}(2 + \beta + \beta x) + \frac{Q(\hat{p}^*, \hat{p}^*)}{Q_1(\hat{p}^*, \hat{p}^*)} \frac{1 - \beta(1-x)}{1 + x} \right].
\]

In order to understand eq. (15), consider the standard case with no reception charge \((\hat{r} = 0)\) and no benefit from receiving calls \((\beta = 0)\). Then eq. (15) simplifies to \(\hat{a}^* = t + Q / Q_1 < t\). This is the finding of Gans and King (2001): firms “collude” by setting below-cost access charges, in the limit by adopting a “bill-and-keep” system. This is because the off-net calling charge becomes cheaper than the on-net charge, and customers then prefer to belong to the “smaller” network so that they make many cheap off-net calls. Operators are less aggressive, since it pays to be small. In equilibrium, this results in higher profits. This mechanism carries forward to our more general framework. The first term in the square bracket of eq. (15) is negative when customers care about receiving calls: part of this benefit is internalized by the operators, and they tend to push off-net calling prices down via lower access charges. The second term in the bracket reflects the impact of the reception charge. As operators try to avoid competition by pushing off-net prices down, and as the reception
charge already achieves this via the pecuniary externality, then there is no need to push \( a \) too low when this effect is ensured by the reception charge. The last term is the effect of Gans and King, corrected by the call externality and by the propagation factor.

Eq. (15) is only a candidate equilibrium charge since, as we discuss in the Appendix, restrictions are needed to ensure that profit is concave in \( a \). Also, this solution holds so long as the reception charge does not bind, i.e. \( \hat{r} \leq \hat{\beta}p \). When it exists and is positive, an interesting implication is that connectivity breakdown would not happen even when \( \beta \) is very high: when \( a \) is endogenized and takes the value of eq. (15), then \( \hat{p}^* \) becomes:

\[
\hat{p}^* = \frac{c + t + \hat{r}}{1 + 2\hat{\beta}} + \frac{Q}{Q_1(1 + 2\hat{\beta})(1 + x)},
\]

that does not go to infinity for any value of the propagation factor and of the utility from receiving calls. This contrasts with Proposition 2, where we found that, under some parameter configuration, a connectivity breakdown would happen. The difference is that now operators can adjust the access charge as well: as the utility from receiving calls increases or as the propagation factor goes up, the access charge is pushed down so that the off-net price always takes finite values. In fact, it is easy to see that if the reception charge is zero, then from eq. (16) the off-net price is strictly lower than the on-net price: \( \hat{p}^* < p^* = \frac{c + t}{1 + \beta} \).

This result has two immediate implications. Firstly, connectivity breakdown concerns are totally eliminated if operators are allowed to negotiate over access charges. Secondly, in the absence of reception charges, negotiations would not necessarily deliver efficient outcomes, since they would result in off-net prices that are “too” low. The latter point leads to our main policy question. How does the privately chosen access charge compare to the socially optimal one? The access charge that maximizes social welfare is the one that makes the privately optimal off-net price in eq. (11bis) equal to the socially efficient price:

\[
\hat{p}^* = p^* \Rightarrow \frac{c + a - (a - t)x - \hat{r}}{1 - \beta(1 - x)} = \frac{c + t}{1 + \beta}.
\]

Thus, the socially optimal access charge is:
\[ a^w = t - \frac{(c + t)\beta(2 - x)}{(1 - x)(1 + \beta)} + \frac{\hat{r}}{1 - x}. \]

Note that \( a^w = t \), the so-called LRIC rule, is typically not socially optimal in the absence of reception charges, as we already discussed in Section 3.2 under uniform pricing. If \( \hat{r} = 0 \), then \( a^w < t \) is socially efficient and so is a “bill-and-keep” system when \( \beta \) is sufficiently high. The same arguments apply here. Evaluating the difference between eq. (15) and eq. (17), we get:

\[ a^* - a^w = \frac{1 - \beta(1 - x)}{(1 + \beta)(1 + 2\beta)(1 - x)} \left[ -\beta(c + t) + \hat{r}(1 + \beta) + \frac{Q_1 1 + \beta}{Q_1 1 + x} \right]. \]

Eq. (18) is informative about the difference between private and social optimal access charges. Notice that in the special case when \( \beta = 1/(1 - x) \) there is no wedge between the two charges: the industry would self-regulate with no need of intervention. However, this result does not hold in general. In the absence of reception charges, negotiations over reciprocal access charges would set them “too” low from a social point of view, as both the first and the third term of the RHS of (18) are negative. Still, this may not be a problem if the socially-optimal charge from eq. (17) is already equal to zero: operators could not go below this level, thus they would agree on an efficient “bill-and-keep” system without having to mandate it.

If, on the other hand, both \( a^* \) and \( a^w \) take positive values, then it possible for the regulator to use a regulated reception charge to achieve efficiency. This level of the reception charge is determined by letting the LHS of (18) equal to zero, finding:

\[ \hat{r}^w = -\frac{Q(p^*, p^*)}{Q(p^*, p^*)} \frac{1}{1 + x} + \frac{\beta}{1 + \beta}(c + t). \]

Our results on welfare are summarized in the following proposition:

**Proposition 4.** In stage I of the game:

a) unregulated negotiations over reciprocal access can eliminate the “connectivity breakdown” problem for off-net calling charges;

b) an efficient “bill-and-keep” system can be chosen by negotiating operators;
c) when negotiations lead to a positive access charge, the regulator can use positive reception charges to get closer to efficient choices, although he never achieves the first-best.

In summary, we have found two encouraging results, and a mildly encouraging one. From a policy perspective, we have shown that free negotiations over the reciprocal access charge would ensure that connectivity breakdown problems disappear. This is the first encouraging result. The intuition can be obtained once again by recalling that operators are better off by trying not to compete too hard against each other: only by allowing cheap off-net calls they can ensure that this is achieved. Their mutual interest goes opposite to a breakdown that is equivalent to infinitely high off-net charges. We have also seen that under some circumstances both a regulator and private firms would choose a “bill-and-keep” system, in which case efficient outcomes are reached with minimal intervention (only a reciprocity clause is needed). This is the second encouraging result.

When negotiations are not efficient, it is still possible for a regulator to improve efficiency with an appropriate reception charge, although a lot of information on costs and demand is required on the regulator’s side to achieve this outcome: this is our “mildly” encouraging result. In fact, it may be impossible even for a perfectly informed regulator to achieve the first best via the regulation of reception charges: they should be set so high that they would violate sender sovereignty. The best a regulator could do is to set reception charges at the highest possible level such that all calls received are also accepted.

5. Market-determined reception charges
In the previous section we discussed the role that can be played by reception charges when set by a benevolent regulator. If they were instead negotiated by operators in stage I, it is clear that operators would have an interest divergent from the regulator’s. The intuition that we gave for negotiated access charges would carry forward to negotiated reception charges as well. Operators would try to obtain cheap off-net calls to reduce the intensity of competition. When off-net charges are already the lowest possible (i.e., “bill-and-keep” or \( a = 0 \)), then the off-net price from (11bis) is \( \hat{p}^\ast = \frac{c + tx - \hat{r}}{1 - \beta(1 - x)} \): operators would then start introducing positive reception charges as this allows them to push off-net retail prices even lower, via the pecuniary externality. This typically results in reception charges that are “too high” from the social point of view. Conversely, if the freely negotiated access charge is positive
(but “too low” from a social point of view), then eq. (16) holds and the off-net prices increases with the reception charge. The regulator and the negotiating operators have conflicting interests: the former would like to introduce a positive reception charge as the overall result is to push up the price, but the latter have exactly the opposite interest and they would set the reception charge to zero, the lowest possible level.

It is doubtful however that operators would be allowed to negotiate over reciprocal reception charges as these are retail prices, contrary to access charges that are wholesale prices. An antitrust authority would almost surely forbid this kind of retail contracts given their collusive nature. In this last section we ask what type of retail reception charges would be chosen individually by competing operators.

In order to answer this question we must amend our model since, as explained in Section 2, the amount of incoming and outgoing traffic is determined by two “binding” prices only. For instance, under “sender sovereignty” only outgoing call charges matter as long as reception charges are low enough, but the exact level of reception charges is indeterminate. Similarly, under “receiver sovereignty”, only reception charges matter as long as outgoing charges are low enough.

In order to determine simultaneously the four call prices (outgoing and reception charges for both operators) we follow JLT by introducing some noise when receiving calls. This captures a random element, for instance the recipient of a call may not be in the ideal situation for answering the call (e.g., because driving or engaged in other activities). Its technical role is that both the sender and the receiver have positive probability of cutting a communication. We suppose that the utility from an exchange of information is as in section 2 plus an additive term with noise from receiving calls. For consumer $i$ the gross utility is $U_i(I^i) - \varepsilon_i Q^i$ where $\varepsilon_i$ has a distribution function $G(\cdot)$, with wide enough support $[\varepsilon, \bar{\varepsilon}]$, zero mean, and strictly positive density $g(\cdot)$ over the support. $\varepsilon_i$ is i.i.d. for each pair of customers. Consider an information exchange between $i$ and $j$:

- If $\varepsilon_i \geq \hat{\varepsilon}_i = r_i - U_i^i f_i^i$ and $\varepsilon_j \geq \hat{\varepsilon}_j = r_j - U_j^j f_j^j$, then there is a “sender sovereignty” regime that generates calls $Q^i = Q_i^i(p_i, p_j)$ and $Q^j = Q_j^j(p_j, p_i)$;
- If $\varepsilon_i < \hat{\varepsilon}_i$ and $\varepsilon_j \geq \hat{\varepsilon}_j$, then there is a “network $i$ sovereignty” regime that generates calls $Q^i = Q_i^i(p_i, r_i - \varepsilon_i)$ and $Q^j = Q_j^j(r_i - \varepsilon_i, p_i)$;
• If $\epsilon_i \geq \hat{\epsilon}_i$ and $\epsilon_j < \hat{\epsilon}_j$, then there is a “network $j$ sovereignty” regime that generates calls $Q_{ij}^j = Q_{ij}(r_j - \epsilon_j, p_j)$ and $Q_{ji}^j = Q_{ji}(p_j, r_j - \epsilon_j);

• If $\epsilon_i < \hat{\epsilon}_i$ and $\epsilon_j < \hat{\epsilon}_j$, then there is a “receiver sovereignty” regime that generates calls $Q_{ij}^j = Q_{ij}(r_j - \epsilon_j, r_i - \epsilon_i)$ and $Q_{ji}^j = Q_{ji}(r_i - \epsilon_i, r_j - \epsilon_j).

We now illustrate the equilibrium in stage II, when operators compete in five-part prices. As before, on-net prices are efficient as everything is internalized and their expressions are not reported. The following proposition characterizes off-net prices:

**Proposition 5.** As the noise vanishes,

- the only symmetric candidate equilibrium without connectivity breakdown when senders are sovereign most of the time is given by:

\[
\begin{align*}
\hat{p}^* &= \frac{2a + c - t}{1 - \beta} \quad \text{iff } t - \beta c < a \leq t - \frac{c \beta (c + t)}{1 - \beta + x(\beta + 1)} \text{ and } \beta < 1; \\
\hat{r}^* &= t - a - \frac{a - t + \beta(c + a)}{1 - \beta} \\
\end{align*}
\]

- The only symmetric candidate equilibrium without connectivity breakdown when senders are sovereign most of the time is given by:

\[
\begin{align*}
\hat{p}^* &= \frac{c + a - (a - t)x}{1 - \beta(1 - x)} \quad \text{iff } a \geq t - \frac{c \beta (c + t)}{1 - \beta + x(\beta + 1)} \text{ and } \beta < 1/(1 - x). \\
\hat{r}^* &= 0
\end{align*}
\]

- The only symmetric candidate equilibrium without connectivity breakdown when senders are sovereign most of the time is given by:

\[
\begin{align*}
\hat{p}^* &= a + c + x \frac{a - t + \beta(c + a)}{\beta - 1} \quad \text{iff } -c + \frac{x(c + t)}{\beta - 1 + x(\beta + 1)} \leq a < \frac{t - \beta c}{1 + \beta} \text{ and } \beta > 1; \\
\hat{r}^* &= \frac{\beta(-2a - c + t)}{\beta - 1} \\
\end{align*}
\]

\[
\begin{align*}
\hat{p}^* &= 0 \\
\hat{r}^* &= \frac{\beta(-a(1 - x) + cx)}{\beta - 1 + x} \quad \text{iff } a \leq -c + \frac{x(c + t)}{\beta - 1 + x(\beta + 1)} \text{ and } \beta > 1 - x.
\end{align*}
\]

We briefly discuss eq. (20) and (21) as they relate to the case of sender sovereignty most of the time, which is the case people are familiar with in practical terms, as the number and length of calls is predominantly set by the caller and not the receiver for price reasons. The range of validity of the two equations is also plotted in Figure 1, where we have put the
marginal utility from receiving calls on the horizontal axis and the level of the access charge on the vertical axis. Connectivity breakdown occurs in the bottom left corner (the lowest curve describes the limiting condition for having sender sovereignty most of the time) and to the right of the diagram (the vertical line is $1/(1 - x)$). In the other regions there is a candidate for a symmetric equilibrium with senders determining volumes most of the time.

Eq. (21) says that when termination charges are set at or above cost (LRIC), then competing operators should set negative reception fees. By paying consumers to receive calls, operators directly promote increased termination revenue. However, negative reception charges raise several problems. For instance, some time ago in Italy mobile operators did set negative reception fees for a while only to find that people were calling their mobile phones from office lines so as to obtain these rebates. The scheme had to be withdrawn. Thus in Proposition 5 and Figure 1 we also impose a non-negativity constraint on reception charges. Our results then explain why consumers are not charged for receiving calls in countries with high termination charges, even if there is nothing in principle to prevent operators from introducing such charges. This is in line, for instance, with the European experience of fixed-to-mobile charges. On the contrary, if termination charges are set below cost then operators may start introducing reception charges if they have the ability to do so. This is in line with the North-American experience of mobile telephony, as we discuss in the concluding section.

![Diagram](image)

**Figure 1 – Market-determined reception charges under “sender sovereignty”**

[Parameter values: $x = 0.2$, $t = 0.1$, $c = 0.15$]
6. Conclusions

This paper has developed a model of information exchange between calling parties and used it to analyze competition between network operators. We have found that a relevant role is played by the “propagation factor”, which measures how return traffic is induced by sending calls. When inter-network (access) charges are taken as given, we have shown that the “connectivity breakdown” problem highlighted by Jeon et al. (2004) is enhanced when calls made and received are substitutes in the information production function, and reduced when they are complements. When access charges are endogenized and chosen by negotiating operators, we have also shown that the breakdown problem can be totally eliminated: operators are better off by allowing cheap off-net traffic.

The empirical literature has found that “calls generate calls”, i.e. the propagation factor is positive. As mentioned by Taylor (2004), the earlier literature has used aggregate data and might have captured both the propagation factor and network externalities. It is an interesting question for further empirical research to study the information exchange process and correctly identify the propagation factor using micro-data. Until better analysis is available, however, the evidence available to us so far points to a positive propagation factor, thus our results suggest that connectivity breakdowns should not be seen as a major source of concern. In this respect, the industry is quite likely to be able to self-regulate. However, we have also discussed how the industry would almost never achieve the first-best when operators can negotiate over access charges. To the extent that regulatory authorities are typically worried that off-net prices may be set too high, for instance via high termination charges, we have not found support for this concern under a mild form of regulation (reciprocity). If anything, negotiating operators would agree on deals that tend to push off-net prices too low from a social point of view.

Our results also suggest that the debate of the merits of CPP versus RPP must be assessed against the level set for the access charge. When the benefit from receiving calls is sufficiently high, below-cost termination charges (in the limit a “bill-and-keep” system) have good properties and may be selected by operators themselves. We have also shown that, in the presence of below-cost termination charges, operators may also introduce reception charges. Given that optimal regulation is very difficult and costly to achieve, our policy message is to allow operators to negotiate over reciprocal access charges. Only extremely high
off-net charges may be subject to further scrutiny, especially if they are very different from on-net charges.

We conclude by reporting some evidence available from the U.S. that seems to support our positive findings. Marcus (2004) discusses how the call termination system in the U.S. has a strong tendency toward symmetry in the rates charged for reciprocal compensation, and toward identical access charge rates for wired carriers in the same geographic area. In particular he shows that:

- ILEC-CLEC and ILEC-mobile reciprocal compensation rates are generally symmetric, and set at a rate that reflects the marginal cost of the ILEC.\(^\text{10}\)
- ILEC-ILEC, CLEC-CLEC, CLEC-mobile, and mobile-mobile reciprocal compensation rates are determined through voluntary negotiations, and in many cases are set to zero ("bill-and-keep"), in particular for ILEC-ILEC and mobile-mobile interconnection.

The mobile sector is a particularly good candidate to test our positive findings since it is indeed a case of network-based competition where roughly symmetric mobile firms have to terminate calls on each other’s network. The rate for reciprocal compensation is established through unregulated commercial negotiations. These agreements are generally on a "bill-and-keep" basis.\(^\text{11}\) Mobile operators also charge their customers for receiving calls (RPP): this is expected from our analysis only in the presence of below-cost termination rates.

Our findings can be applied when operators are symmetric and calling patterns balanced. If traffic were significantly imbalanced, voluntary negotiations of symmetric rates would be more difficult to achieve. We believe that further work is needed in this area to see if “light” regulation (e.g., reciprocity deals) could work also in asymmetric settings.

---

\(^{10}\) ILECs are the incumbent fixed-line operators, while CLECs are their competitors in the fixed-line market. This is in stark contrast with most European countries where large asymmetries in termination rates exist between fixed-line operators (who are typically subject to termination rate regulation) and mobile operators (who historically have not been subject to termination rate regulation).

\(^{11}\) FCC, In the Matter of developing a Unified Intercarrier Compensation Regime, CC Docket 01-92, §95.
References


Appendix

Derivation of eq. (7).
Following Jeon et al. (2004), we study the optimization program of network $i$ assuming that $\alpha_i$ is given. Let $\tilde{F}_i = F_i + r_i [\alpha_i Q^u(p_i, p_i) + \alpha_j Q^u(p_j, p_j)]$ be a generalized fixed fee incorporating also the cost for receiving calls. Given eq. (2) we can write $w_i - w_j = (\alpha_i - 1/2) / \sigma$ and, using eq. (3), we obtain:

$$\tilde{F}_i = \tilde{F}_j + (1/2 - \alpha_i) / \sigma + \alpha_i \nu(p_i, p_i) + \alpha_j \nu(p_i, p_j) - \alpha_j \nu(p_j, p_j) - \alpha_i \nu(p_i, p_i).$$

Network $i$'s profit in eq. (6) can be rearranged in the following way:

$$\pi_i = \alpha_i \left\{ (p_i - c - t) \left[ \alpha_i Q^u(p_i, p_i) + \alpha_j Q^u(p_i, p_j) \right] + \tilde{F}_i - f_i \right\} +$$

$$+ \alpha_i \alpha_j (a - t) \left[ Q^u(p_j, p_i) - Q^u(p_i, p_j) \right].$$

The FOC with respect to $p_i$ is given by:

$$\frac{\partial \pi_i}{\partial p_i} = \alpha_i \left[ \alpha_i Q^u(p_i, p_i) + \alpha_j Q^u(p_i, p_j) + (p_i - c - t) \left( \alpha_i \frac{\partial Q^u}{\partial p_i} + \alpha_j \frac{\partial Q^u}{\partial p_i} \right) \right] +$$

$$+ \alpha_i \frac{\partial \tilde{F}_i}{\partial p_i} + \alpha_i \alpha_j (a - t) \left( \frac{\partial Q^u}{\partial p_i} - \frac{\partial Q^u}{\partial p_i} \right),$$

where

$$\frac{\partial \tilde{F}_i}{\partial p_i} = \frac{\partial \tilde{F}_j}{\partial p_i} + \alpha_i \frac{\partial \nu(p_i, p_i)}{\partial p_i} + \alpha_j \frac{\partial \nu(p_i, p_j)}{\partial p_i} - \alpha_i \frac{\partial \nu(p_j, p_i)}{\partial p_i},$$

$$\frac{\partial \tilde{F}_j}{\partial p_i} = \frac{r_i \alpha_i \partial Q^u}{p_i}.$$ 

For calls originated by network $i$ and terminated on network $j$, the indirect utility is given by eq. (5). Thus it results:

$$\frac{\partial \nu(p_i, p_j)}{\partial p_i} = -Q^u(p_i, p_i) + U_j f_j \frac{\partial Q^u}{\partial p_i}.$$

The effect of a price change on the indirect utility of customers connected to network $i$ is given by the sum of two factors: a “standard” loss in utility on each originated call ($-Q^u$), and a term that reflects the change in utility due to way the recipient reacts according to the “information exchange” function, thus leading him/her to change the number of calls sent to customer $i$ ($U_j f_j \partial Q^u / \partial p_i$).
For calls originated by network \( j \) and terminated on network \( i \), where \( v(p_j, p_i) = U_j(f'(Q^\mu(p_j, p_i), Q^\nu(p_i, p_j)) - p_j Q^\mu(p_j, p_i)) \), it results:

\[
\frac{\partial v(p_j, p_i)}{\partial p_i} = U_j f'_2 \frac{\partial Q^\nu}{\partial p_i}.
\]

In this case, the marginal change in indirect utility depends only on the impact that network \( i \)'s price has on the calls received by users \( j \).

For calls originated and terminated on the same network, where the indirect utility is given by eq. (4), it results:

\[
\frac{\partial v(p_j, p_i)}{\partial p_i} = -Q^\mu(p_j, p_i) + U_j f'_2 \frac{\partial Q^\nu}{\partial p_i}.
\]

Note that in the last term the effect of \( p_i \) influences not only call origination but also call reception, \( \frac{\partial Q^\nu}{\partial p_i} = Q^\mu + Q^\nu_2 \).

After substituting the previous expressions in eq. (A1), one gets eq. (7). QED

**Proof of Proposition 1.**

In a symmetric equilibrium \( p_j^* = p_i^* = p^* \), \( r_i = r_j = r \), \( U_i = U_j = U \), \( \alpha = \frac{1}{2} \). We also have \( Q^\nu_i = Q^\mu_i \) and \( Q^\nu_2 = Q^\mu_2 \). Thus it results \( \Phi|_{sym} = \frac{2}{2 + x} \), \( \Gamma|_{sym} = \frac{2x}{2 + x} \), \( \Theta|_{sym} = \frac{2x + 1}{2 + x} \), where \( x \) is a propagation factor defined as \( x = \Gamma/\Phi = Q^\mu_2/Q^\mu_i \). Substitution into eq. (7) gives eq. (8).

The level of the fixed fee is determined by studying the derivative w.r.t. \( \alpha_i \):

\[
\pi_i = \alpha_i \left\{ \left[p_i - c - t \left[ \alpha_i, Q^\mu + \alpha_j, Q^\nu \right] \right] + \tilde{F}_j - f + (1/2 - \alpha_i)/\sigma \right\} +
\alpha_i \left[ \alpha_i v(p_i, p_i) + \alpha_j v(p_i, p_j) - \alpha_i v(p_j, p_j) - \alpha_j v(p_j, p_i) \right] + \alpha_i \alpha_j (a - t)\left[ Q^\mu - Q^\nu \right]
\]

Since in \( p_i = p_i^* \) it results \( \frac{d\pi_i}{d\alpha_i} = \frac{\partial \pi_i}{\partial \alpha_i} + \frac{\partial \pi_i}{\partial p_i^*} \frac{\partial p_i^*}{\partial \alpha_i} = \frac{\partial \pi_i}{\partial \alpha_i} \), we have:

\[
\frac{\partial \pi_i}{\partial \alpha_i} = \left\{ \left[p_i - c - t \left[ \alpha_i, Q^\mu + \alpha_j, Q^\nu \right] \right] + \tilde{F}_j - f + (1/2 - \alpha_i)/\sigma \right\} +
\alpha_i \left[ \alpha_i v(p_i, p_i) + \alpha_j v(p_i, p_j) - \alpha_i v(p_j, p_j) - \alpha_j v(p_j, p_i) \right] + (1 - 2\alpha_i)(a - t)\left[ Q^\mu - Q^\nu \right] +
\alpha_i \left[ \left[p_i - c - t \left[ Q^\mu - Q^\nu \right] \right] - 1/\sigma + v(p_i, p_i) - v(p_i, p_j) + v(p_j, p_j) - v(p_j, p_i) \right] = 0.
\]

In a symmetric equilibrium the above expression reduces to:

\[
\left. \frac{\partial \pi_i}{\partial \alpha_i} \right|_{sym} = \frac{1}{2} \left( p_i^* - c - t \right) Q(p_i^*) + \tilde{F}_i^* - f - \frac{1}{2\sigma} = 0
\]
where $F^* = F^* + rQ(p^*)$ and $Q(p^*) = Q^u(p^*, p^*) + Q^u(p^*, p^*)$, giving part b). Finally, substituting $F^*$ and $p^*$ in eq. (6) gives part c): $\pi^* = 1/4\sigma$. QED

Proof of Proposition 2.

We follow the same procedure as in Section 3 with $\alpha_i$ given. Result a) is standard, as for on-net prices a network maximizes joint surplus with all customers, hence reaching the socially optimal value. To get result b) considering only the profit for off-net calls (both outgoing and incoming). After manipulations of eq. (10) we get:

\begin{equation}
\hat{\pi}_i(\hat{p}_i; \alpha_i, \hat{r}_j) = \alpha_i \left\{ \alpha_j \left[ v(\hat{p}_i, \hat{p}_j) + (\hat{p}_i - c - a)Q^u(\hat{p}_i, \hat{p}_j) \right] + \alpha_i \hat{r}_j \right\} + \alpha_i \left\{ \alpha_j (a - t)Q^u(\hat{p}_j, \hat{p}_i) - \alpha_i v(\hat{p}_j, \hat{p}_i) \right\}
\end{equation}

Contrary to JLT, it is not possible to split $\hat{\pi}_i$ in outgoing profit (related only to retail prices) and incoming profit (related only to reception charges). The FOC is:

$$\frac{\partial \hat{\pi}_i}{\partial \hat{p}_i} = \alpha_i \left\{ \alpha_j \left[ -Q^u(\hat{p}_i, \hat{p}_j) + U'(f_1 \hat{p}_i)Q^u + Q^u(\hat{p}_i, \hat{p}_j) + (\hat{p}_i - c - a)Q^u \right] + \alpha_i \hat{r}_j \right\} +$$
$$+ \alpha_i \left\{ \alpha_j (a - t)Q^u - \alpha_i U'(f_1 \hat{p}_i) \right\}$$

that can be rewritten as:

$$\frac{\partial \hat{\pi}_i}{\partial \hat{p}_i} = \alpha_i Q^u \left\{ \alpha_j \left[ \hat{p}_i - (c + a - xU'(f_1 \hat{p}_i) - x(a - t)) \right] + \alpha_i \hat{r}_j \right\}$$

where $x = Q^u / Q^u$ is the propagation effect. Solving $\frac{\partial \hat{\pi}_i}{\partial \hat{p}_i} = 0$ we get:

$$\hat{p}_i^* = c + a - (a - t)x - U'(f_1 \hat{p}_i)x - \frac{\alpha_i}{\alpha_i} \left( \hat{r}_j - U'(f_1 \hat{p}_i) \right)$$

that, if a symmetric equilibrium exists, reduces to eq. (11). In order to show under what conditions this price is indeed a maximum, we have to determine the SOC. Denote with $\beta^i = f_1^i / f_1^i$ such that $U'(f_1^i) = \beta^i \hat{p}_i$. In the same way, define $\beta^j = f_1^j / f_1^j$. Since $\hat{\varphi}(U'(f_1^i)) / \partial \hat{p}_i = \beta^i + \hat{p}_i \hat{\varphi}(\beta^i / \partial \hat{p}_i)$ and $\hat{\varphi}(U'(f_1^j)) / \partial \hat{p}_j = \hat{p}_j \hat{\varphi}(\beta^j / \partial \hat{p}_j)$, it results:

$$\frac{\partial^2 \hat{\pi}_i}{\partial \hat{p}_i^2} = \alpha_i Q^u \left\{ \alpha_j \left[ \hat{p}_i - (c + a - x\beta^i \hat{p}_i - x(a - t)) \right] + \alpha_i \hat{r}_j - \beta^i \hat{p}_j \right\} +$$

$$+ \alpha_i \left\{ \alpha_j \left[ 1 + x \left( \beta^i + \hat{p}_i \frac{\partial \beta^i}{\partial \hat{p}_i} + x' \left( \beta^i \hat{p}_i + a - t \right) \right) \right] - \alpha_i \hat{r}_j \frac{\partial \beta^i}{\partial \hat{p}_i} \right\}$$

that reduces in a symmetric equilibrium to:
\[
\frac{\partial^2 \hat{\pi}_i}{\partial \hat{p}_i^2} \bigg|_{\text{sym}} = \frac{1}{4} Q_i^{\|} \left\{ 1 + x \left( \beta + \hat{p} \frac{\partial \beta}{\partial \hat{p}_i} + x'(\beta \hat{p} + a - t) \right) - \hat{p} \frac{\partial \beta}{\partial \hat{p}_i} \right\}.
\]

Consider first the case of JLT with independent outgoing and incoming call, i.e. \( x = 0 \).

The SOC becomes
\[
\frac{\partial^2 \hat{\pi}_i}{\partial \hat{p}_i^2} \bigg|_{\text{sym}} = \frac{1}{4} Q_i^{\|} < 0.
\]

More in general, recalling the definition of the propagation factor, we can write:
\[
\frac{\partial \beta}{\partial \hat{p}_i} = \frac{1}{(f_i^i)^2} (f_1^i f_{12}^i - \beta^i f_{11}^i + x(f_{22}^i - \beta^i f_{12}^i)) f_i^i.
\]

In a symmetric equilibrium the SOC becomes:
\[
\frac{\partial^2 \hat{\pi}_i}{\partial \hat{p}_i^2} \bigg|_{\text{sym}} = \frac{1}{4} Q_i^{\|} \left\{ 1 + x[\beta + x'(\beta \hat{p} + a - t)] - \hat{p} Q_i^{\|} (f_{22}^i - \beta f_{12}^i)(1-x^2) \right\}.
\]

Suppose that calls are perfect complements, i.e. \( x = 1 \). A sufficient condition to have
\[
\frac{\partial^2 \hat{\pi}_i}{\partial \hat{p}_i^2} \bigg|_{\text{sym}} < 0
\]

is that \( x' \approx 0 \), i.e. the propagation factor is not affected by a change in prices.

Since we have assumed sender sovereignty, it must be that the reception charge is low enough, i.e. \( U f_i^f \geq \hat{r} \) in a symmetric equilibrium. Finally, to determine the fixed fee of the multi-part tariff, we maximize network \( i \)'s total profit given by eq. (10) w.r.t. to \( F_i 
\]

\[
\frac{\partial \pi_i}{\partial F_i} = \frac{\partial \alpha_i}{\partial F_i} \left\{ \left( p_i - c - t \right) Q_i^{\|} + \alpha_j \left( \hat{p}_j - c - t \right) Q_j^{\|} + F_i - f + \alpha_i r Q_i^{\|} + \alpha_j \hat{r} Q_j^{\|} \right\} + \alpha_j \left( \frac{\partial \alpha_j}{\partial F_i} \left( p_i - c - t \right) Q_j^{\|} - \frac{\partial \alpha_j}{\partial F_i} \left( \hat{p}_j - c - t \right) Q_j^{\|} + 1 + \frac{\partial \alpha_j}{\partial F_i} r Q_j^{\|} - \frac{\partial \alpha_j}{\partial F_i} \hat{r} Q_j^{\|} \right) + (1 - 2 \alpha_i) \frac{\partial \alpha_i}{\partial F_i} (a - t) (Q_i^{\|} - Q_i^{\|}) = 0.
\]

with
\[
\frac{\partial \alpha_i}{\partial F_i} = -\frac{\sigma}{1 - \sigma \left[ v(p_i, p_i) + v(p_j, p_j) - v(p_i, p_j) - v(p_j, p_i) - r Q_i^{\|} - r Q_j^{\|} + \hat{r} Q_i^{\|} + \hat{r} Q_j^{\|} \right].
\]

In a symmetric equilibrium (A3) is
\[
\frac{\partial \alpha_i}{\partial F_i} \bigg|_{\text{sym}} \left\{ (p^* - c - t) Q(p^*, p^*) + F^* - f + r Q(p^*, p^*) \right\} + \frac{1}{2} = 0.
\]

After manipulations we obtain eq. (12). QED

**Proof of Proposition 3.**

The off-net price is given by eq. (11bis), and under the assumption \( x' \approx 0 \), we have:
\[ \frac{\partial p^*}{\partial a} = \frac{(1-x)[1-\beta(1-x)]+(1-x) \frac{\partial \beta}{\partial a} [c + a - (a-t)x - \hat{r}]}{(1 - \beta(1-x))^2} = \frac{1-x}{1 - \beta(1-x)} \left( 1 + \hat{p}^* \frac{\partial \beta}{\partial a} \right), \]

where \( \frac{\partial \beta}{\partial a} = \frac{\partial \beta}{\partial \beta} \). Notice how \( \frac{\partial p^*}{\partial a} \) is positive under rather general conditions (it suffices that \( \frac{\partial \beta}{\partial a} \) is positive or not “too” negative). From eq. (12) the effect on \( F^* \) is

\[ \frac{\partial F^*}{\partial a} = \frac{\partial}{\partial a} [Q(\hat{p}^*, \hat{p}^*) - r Q(\hat{p}^*, \hat{p}^*)], \]

where \( \frac{\partial}{\partial a} [Q(\hat{p}^*, \hat{p}^*)] = U f_1 \frac{\partial Q}{\partial a} + U f_2 \frac{\partial Q}{\partial a} - \hat{p}^* \frac{\partial Q}{\partial a} = \]

\[ [-Q + \beta \hat{p}^* Q(1+x)] \frac{\partial \hat{p}^*}{\partial a}. \]

Therefore, it results:

\[ \frac{\partial \pi^*}{\partial a} = \frac{\partial}{\partial a} \left\{ Q(\hat{p}^*, \hat{p}^*) + Q(\hat{p}^*, \hat{p}^*) (1+x) \hat{p}^*(1+2 \beta - c - t - \hat{r}) \right\} \leq 0 \]

which gives eq. (15) at an interior solution when \( x \neq 1 \). In order to determine if the optimal access charge is a local maximum, we consider the sign of the SOC. Denoting by \( \Psi = -Q + Q(1+x) [\hat{p}^*(1+2 \beta) - c - t - \hat{r}] \), the SOC evaluated in \( a = a^* \) reduces to

\[ \frac{\partial^2 \pi^*}{\partial a^2} \bigg|_{a=a^*} = \frac{1}{2} \frac{\partial \pi^*}{\partial a} \frac{\partial \Psi}{\partial a}. \]

Since \( \frac{\partial p^*}{\partial a} \) is positive, we need to study the sign of \( \frac{\partial \Psi}{\partial a} \) :

\[ \frac{\partial \Psi}{\partial a} = (1+x) \left\{ \frac{\partial}{\partial a} \left[ Q_{11} + Q_{12} \right] \hat{p}^*(1+2 \beta) + Q_1 \left[ 2 \beta \frac{\partial \hat{p}^*}{\partial a} + \hat{p}^*(1+2 \frac{\partial \beta}{\partial a}) \right] \right\}, \]

\[ \left. \frac{\partial \Psi}{\partial a} \right|_{a=a^*} = (1+x) \left\{ \frac{\partial}{\partial a} \left[ Q_{11} + Q_{12} \right] Q_{11} + Q_{12} \left[ 2 \beta \frac{\partial \hat{p}^*}{\partial a} + \hat{p}^*(1+2 \frac{\partial \beta}{\partial a}) \right] \right\}. \]

The sign of the last expression is not obvious a priori. If demand functions are linear, and \( \frac{\partial \beta}{\partial a} \) is positive or not “too” negative, then it results \( \frac{\partial \Psi}{\partial a} \bigg|_{a=a^*} < 0 \) and the access charge in (15) is a local maximum. However, this is not a general property. QED

**Proof of Proposition 4.**

Parts a) and b) are already discussed in the main text. To complete part b) imagine a “bill-and-keep” system is in place, \( a = 0 \). From (17) the regulator can induce efficiency by choosing a reception charge equal to:

\[ \hat{r}^* = \frac{\beta(c+t)(2-x)}{1+\beta} - t(1-x), \]
which satisfies the constraint \( \hat{r} \leq \frac{\beta}{1+\beta}(c+t) \) if \((\beta c-t) (1-x)/(1+\beta) \leq 0 \). In other words the first-best can be achieved if either the propagation factor is 1, or if \( x < 1 \) and \( \beta c < t \). In particular, if \( \beta \) is sufficiently low, reception charges may not be needed at all \((\beta \leq \frac{t}{(1-t)} \Rightarrow \hat{r} = 0) \). Finally, part c) points to a potential problem that the regulator faces when trying to introduce a regulated termination charge in order to drive \( a^* \) towards efficiency. Such charge, if it exists, is given by eq. (19) when both \( a^* \) and \( a^\mu \) are positive. This charge has also to satisfy “sender sovereignty”, which is written as \( \hat{r} \leq \frac{\beta}{1+\beta}(c+t) \) when the efficient retail price is induced. It is straightforward to see that this inequality is always violated by (19), unless the propagation factor is infinitely high. Thus the regulator sets the highest possible reception charge that does not violate the constraint. QED

Proof of Proposition 5 (sketch).
A formal proof (available from the authors) is very long and involves direct maximization of the profit as the noise vanishes while keeping a large support to avoid price indeterminacy. Instead of conducting the full analysis we sketch here a more heuristic proof. We first consider each one of the four regimes of “sender”, “receiver”, “network 1” and “network 2” sovereignty independently. As the noise vanishes, we then consider the equilibria in which the volume is determined by senders (respectively the receivers) most of the time.

1. “Sender sovereignty”.
The analysis with respect to outgoing charges when senders are sovereign most of the time parallels the analysis conducted in Proposition 2 and we report here the expression of the symmetric equilibrium for convenience:

\[
\hat{p}^* = \frac{c + a - (a-t)x - \hat{r}}{1-\beta(1-x)}.
\]

2. “Receiver sovereignty”.
This case is a mirror of “sender sovereignty” and the same analysis conducted in Section 2 applies here, with the appropriate change in notation. The indirect utilities from the information exchange between \( i \) and \( j \) are:

\[
v(\hat{r}, \hat{r}) = U_i(f^i(Q^\mu(\hat{r}, \hat{r}), Q^\mu(\hat{r}, \hat{r})) - \hat{p}, Q^\mu(\hat{r}, \hat{r}))
\]

\[
v(\hat{r}, \hat{r}) = U_j(f^j(Q^\mu(\hat{r}, \hat{r}), Q^\mu(\hat{r}, \hat{r})) - \hat{p}, Q^\mu(\hat{r}, \hat{r})).
\]

Since \( \hat{r} = U'_i f^i_2 \) when the receivers are sovereign it results:

\[
\frac{\partial v(\hat{r}, \hat{r})}{\partial \hat{r}_i} = \frac{\partial Q^\mu}{\partial \hat{r}_i} (U'_i f^i_2 - \hat{p}) + \hat{r}_i \frac{\partial Q^\mu}{\partial \hat{r}_i}.
\]

\[12 \text{To get the results we impose a regularity condition. Given a function } F(\cdot), \text{ a regular sequence of distributions } G_n(\varepsilon) \text{ of the random variable } \varepsilon \text{ with zero mean satisfies: } \lim_{n\to\infty} G_n(F(\varepsilon) \mid \varepsilon \geq \varepsilon_n) = F(\varepsilon = \varepsilon_n) \text{ for } \varepsilon_n > 0, \] \[\lim_{n\to\infty} G_n(F(\varepsilon) \mid \varepsilon \leq \varepsilon_n) = F(\varepsilon = 0) \text{ for } \varepsilon_n > 0.\]
After the appropriate change of notation, from (A2) the FOC for firm \(i\) is:

\[
\frac{\partial v(\hat{r}_i, \hat{p}_i)}{\partial \hat{r}_i} = \frac{\partial Q^h_i}{\partial \hat{r}_i} (U'_j f'_i - \hat{p}_j) + \hat{r}_i \frac{\partial Q^h_i}{\partial \hat{r}_i}.
\]

where \(\frac{\partial Q^h_i}{\partial \hat{r}_i} = Q^h_2, \frac{\partial Q^h_i}{\partial \hat{r}_i} = Q^h_i\), and \(Q^h_2 / Q^h_i\) is the propagation effect when the receiver is sovereign. Solving \(\frac{\partial v(\hat{r}_i, \hat{p}_i)}{\partial \hat{r}_i} = 0\) we get:

\[
\hat{r}_i^* = -(a-t) - \frac{x}{\alpha_i} [U'_j f'_i - (c + a)] + \frac{\alpha_i U'_j f'_i - \hat{p}_j}{\alpha_i}
\]

that, if a symmetric equilibrium exists, reduces to:

(A9) \(\hat{r}_i^* = -(a-t) - \hat{p}_i^* + x_c (c + a) + U'_j f'_i (1-x_c)\);

Denoting \(\beta = f_2 / f_1\), eq. (A9) can be re-written as:

(A9bis) \(\hat{r}_i^* = \frac{\beta [t - \hat{p}_i - a (1-x) + cx]}{\beta - 1 + x}\).

3. “Network \(j\)” sovereignty.

This regime arises when network \(j\) increases its reception and outgoing charges to sufficiently high levels such that it determines the volume of all calls. The analysis of Section 2 has to be modified since traffic is only determined by \(j\). The first-order conditions are \(U'_j f'_j = \hat{p}_j\) and \(U'_j f'_j = \hat{r}_j\), which determine the optimal level of calls \(Q^h(\hat{p}_j, \hat{r}_j)\) and \(Q^u(\hat{r}_j, \hat{p}_j)\). The expressions for the change in off-net quantities caused by a change in price are:

\[
\begin{bmatrix}
A^h & B^h \\
B^h & C^h
\end{bmatrix}
\begin{bmatrix}
dQ^h \\
dQ^u
\end{bmatrix}
= \begin{bmatrix}
d\hat{p}_j \\
d\hat{r}_j
\end{bmatrix}
\]

where \(A^h = U'_j (f'_j)^2 + U'_j f'_j < 0\), \(C^h = U'_j (f'_j)^2 + U'_j f'_j < 0\) and \(B^h = U'_j f'_j f'_j + U'_j f'_j < 0\). The indirect utilities derived from the information exchange between \(i\) and \(j\) are:

\[\frac{\partial^2 \hat{r}_i}{\partial \hat{r}_i^2} < 0\] are that either \(x\) is small, or that it is close to 1 and \(x' \approx 0\).
\[ v(\hat{r}_j, \hat{p}_j) = U_j(f^\prime(Q^\prime(\hat{r}_j, \hat{p}_j), Q^\prime(\hat{p}_j, \hat{r}_j)) - \hat{p}_j Q^\prime(\hat{r}_j, \hat{p}_j) \]

\[ v(\hat{p}_j, \hat{r}_j) = U_j(f^\prime(Q^\prime(\hat{p}_j, \hat{r}_j), Q^\prime(\hat{r}_j, \hat{p}_j)) - \hat{p}_j Q^\prime(\hat{p}_j, \hat{r}_j) \]

\[ \frac{\partial v(\hat{r}_j, \hat{p}_j)}{\partial \hat{r}_j} = \frac{\partial Q^\mu}{\partial \hat{r}_j} (U'_j f'_i - \hat{p}_i) + U'_j f'_i \frac{\partial Q^\mu}{\partial \hat{r}_j} \]

\[ \frac{\partial v(\hat{p}_j, \hat{r}_j)}{\partial \hat{p}_j} = \frac{\partial Q^\mu}{\partial \hat{p}_j} (U'_j f'_i - \hat{p}_i) + U'_j f'_i \frac{\partial Q^\mu}{\partial \hat{p}_j} \]

\[ \frac{\partial v(\hat{p}_j, \hat{r}_j)}{\partial \hat{r}_j} = U'_j f'_i \frac{\partial Q^\mu}{\partial \hat{r}_j} \]

After the appropriate change of notation, from (A2) the FOCs w.r.t. \( \hat{r}_j \) and \( \hat{p}_j \) for firm \( j \) are the following:

\[ \frac{\partial \hat{r}_j}{\partial \hat{r}_j} = \alpha_j \left\{ \alpha_i \left[ \hat{r}_j \frac{\partial Q^\mu}{\partial \hat{r}_j} + (\hat{p}_j - c - a + \alpha_i \hat{r}_i) \frac{\partial Q^\mu}{\partial \hat{r}_j} + (a - t) \frac{\partial Q^\mu}{\partial \hat{r}_j} \right] - \alpha_j \left[ \frac{\partial Q^\mu}{\partial \hat{r}_j} (U'_j f'_i - \hat{p}_i) + U'_j f'_i \frac{\partial Q^\mu}{\partial \hat{r}_j} \right] \right\} \]

\[ \frac{\partial \hat{p}_j}{\partial \hat{p}_j} = \alpha_j \left\{ \alpha_i \left[ -Q^\mu + U'_j f'_i \frac{\partial Q^\mu}{\partial \hat{p}_j} + Q^\mu + (\hat{p}_j - c - a) \frac{\partial Q^\mu}{\partial \hat{p}_j} \right] + \alpha_i \hat{r}_i \frac{\partial Q^\mu}{\partial \hat{p}_j} + \alpha_i (a - t) \frac{\partial Q^\mu}{\partial \hat{p}_j} \right\} - \alpha_j \left[ \frac{\partial Q^\mu}{\partial \hat{p}_j} (U'_j f'_i - \hat{p}_i) + U'_j f'_i \frac{\partial Q^\mu}{\partial \hat{p}_j} \right] \right\} \]

Denote with \( x^\mu = \frac{\partial Q^\mu}{\partial \hat{r}_j} = \frac{-B^\mu}{A^\mu} \) the incoming propagation effect related to the calls received by network \( j \) who sovereigns the information exchange, and let \( x^{out} = \frac{\partial Q^\mu}{\partial \hat{p}_j} = \frac{-B^\mu}{C^\mu} \) be the outgoing propagation effect related to the calls originated by network \( j \). Recalling that \( U'_j f'_j = \hat{r}_j \), the above FOCs can be rewritten as:

\[ (A10) \frac{\partial \hat{r}_j}{\partial \hat{r}_j} = \alpha_j \left\{ \alpha_i \left[ \hat{r}_j + a - t + (\hat{p}_j - c - a) x^\mu \right] - \alpha_j \left[ U'_j f'_j - \hat{p}_i \right] + x^\mu (U'_j f'_j - \hat{r}_j) \right\} \]

\[ (A11) \frac{\partial \hat{p}_j}{\partial \hat{p}_j} = \alpha_j \left\{ \alpha_i \left[ \hat{p}_j - (c + a - x^{out} \hat{r}_j - x^{out} (a - t)) \right] - \alpha_j \left[ U'_j f'_j - \hat{p}_i \right] + x^{out} (U'_j f'_j - \hat{r}_j) \right\} \]

4. Equilibrium analysis when the noise vanishes.

We consider candidates for symmetric equilibria. Consider first the case where \( \beta \hat{p} \geq \hat{r} \) and senders determine most volume in equilibrium. Then the relevant conditions are (11bis) and
(A10), where the latter simplifies to \( \frac{\partial \hat{r}_j}{\partial \hat{r}_j} = \alpha_j \frac{\partial Q_j^\#}{\partial \hat{r}_j} [\hat{r}_j + a - t + (\hat{p}_j - c - a)x] \). In a symmetric equilibrium \( \hat{r}^* = t - a - (\hat{p}^* - c - a)x \) and \( \hat{p}^* = \frac{c + a - (a - t)x - \hat{r}^*}{1 - \beta(1 - x)} \). This system gives eq. (20) as the interior solution. This solution is valid if \( \beta < 1 \) and compatible with the regime of sender sovereignty if \( \frac{\beta \hat{p}}{\hat{r}} > \hat{r}^* \Rightarrow a > (t - \beta c)/(1 + \beta) \). The reception charge \( \hat{r}^* \) must take positive values \( \hat{r}^* \geq 0 \Rightarrow a \leq t - x \frac{\beta(c + t)}{1 - \beta + x(\beta + 1)} \). If \( a \) is above this threshold, then the reception charge \( \hat{r}^* \) is zero and \( \hat{p}^* \) is obtained directly from (11bis). In this case the range of validity is \( \beta < 1/(1 - x) \). The case where \( \beta \hat{p} < \hat{r} \) and receivers determine most volume in equilibrium can be analyzed in a similar manner, giving eq. (22) and (23). QED