Domestic and Cross-Border Mergers in Unionized Oligopoly

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Abstract

We re-examine the model of Lommerud et al. (2006) which shows that cross-border mergers should be expected in the presence of powerful unions. In contrast, we obtain a domestic mergers outcome whenever firms are sufficiently heterogeneous (both in terms of productive efficiency and product differentiation). Cost asymmetries tend to dampen labor unions’ wage demands and allow the merged firms to partially reallocate production from the high cost to the low cost plant. When cost asymmetries become smaller and products more substitutable, then cross-border mergers are the unique equilibrium. However, cross-border mergers may be either between symmetric or asymmetric firms. Finally, we show that a domestic merger outcome is less desirable from a social welfare perspective when compared with cross-border mergers.


Keywords: Unionization, International Oligopoly, Endogenous Mergers.

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1 Introduction

Mergers and acquisitions between firms may affect labor markets and wages, the direction of the effect is however not quite clear (see, e.g., Gokhale et al., 1995; McGuckin et al., 2001; Lehto and Boeckermann, 2008). With increasing integration of international product markets, especially the impact of cross-border mergers and the option of outsourcing production to low-wage countries has received considerable attention in the literature (e.g., Zhao, 1995, 1998; Bughin and Vannini, 1994 for the impact of FDI on unionized labor).

Cross-border mergers have become the predominant form of FDI. A salient feature of international mergers and acquisitions is the threat effect associated with cross-border transactions vis-à-vis (national or local) input suppliers (Freeman, 1995). Intuitively, by creating an “outside option” abroad, a firm can threaten to move production abroad which creates downward pressure on input prices. Clougherty et al. (2011) find that a higher degree of unionization in a country increases the likelihood of a wage decrease after an international merger, which may hint at the presence of threatening to move production abroad by firms.

Accordingly, the presence of labor unions adds a strategic motive to cross-border mergers, as firms can increase their bargaining power vis-à-vis monopoly unions by optimally scaling up production abroad. Despite an increasing trend towards more cross-border mergers, however, the vast majority of mergers and acquisitions still occurs at national levels. In addition, mergers typically involve asymmetric firms (Breinlich, 2008). Gugler et al. (2003) report that target firms are on average only about 16 percent of the size of their acquirers.

Our paper builds on a growing literature which analyzes mergers in a vertical structure where upstream firms (or, unions in the case of labor) have market power vis-à-vis downstream oligopolists. Making the vertical structure explicit this literature has uncovered new incentives for downstream mergers resulting from improved purchasing conditions on input markets.

We depart from those works by analyzing an international setting and we apply the ap-

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1 For example, in 1999 cross-border mergers and acquisitions already accounted for approximately 80 percent of global foreign direct investments (UNCTAD, 2000).

2 See Fabbri et al. (2003) for another empirical study which shows that labor demand of UK and US firms for low skilled workers between 1958 and 1991 (UK data are available until 1986) has become more elastic. They argue that increased activity of multinationals is (partially) responsible for this trend. Choi (2006) provides empirical evidence for the presence of the threat effect associated with cross-border M&A. Barba-Navaretti et al. (2003) provide a cross country firm-level study of European countries where they find that multinationals adjust their labor demand more rapidly than domestic firms in response to shocks. However, they report a more inelastic demand curve with respect to wages for multinationals which they contribute to differences in skill structure.

3 Gugler et al. (2003) utilize a dataset on 44,600 mergers and acquisitions between 1981 and 1998. Cross-border mergers accounted only for 22% of all deals. By now, the share of cross-border mergers has increased to approximately 33% of total M&A.

4 Works which assume linear wholesale prices (or, the right-to-manage approach in the case of labor) include Horn and Wolinsky (1988a), Dobson and Waterson (1997), von Ungern-Sternberg (1997), Zhao (2001), and Symeonidis (2010). Another approach is to assume “efficient contracts” in the input market (see, for an early contribution, Horn and Wolinsky, 1988b).
Our analysis is closely related to Lommerud et al. (2006) which we extend by considering asymmetric firms. Lommerud et al. (2006) analyze a two-country model with four symmetric firms (two in each country) each producing an imperfect substitute. In each country a monopoly union sets wages at the firm level. Within such a symmetric setting, Lommerud et al. obtain their main result that the endogenous merger equilibrium only exhibits cross-country mergers. Under the resulting market structure wages reach their minimum as both merged firms can most effectively threaten to scale up production abroad if a union raises its wage.

Considering asymmetric firms in each country we can qualify that finding as follows: i) When firms’ products are differentiated, a domestic merger equilibrium follows whenever cost asymmetries between national firms are large enough. ii) When products become more substitutable, then the cross-country merger equilibrium emerges; however, both a symmetric and an asymmetric cross-border merger outcome are possible.

Our results show that Lommerud et al.’s result remains largely valid if products are close substitutes. In those instances a cross-border merger induces intense competition between the unions to the benefit of the international firm. If, however, products are relatively differentiated then this effect of “internal” union competition becomes less effective. Considering cost asymmetries gives then rise to our main result that a domestic merger equilibrium emerges. From the perspective of the low-cost firm, a national merger with the high cost firm becomes attractive as this constrains the wage demand of the domestic union. It is, therefore, the “uniformity” effect of a domestic merger that prevents the labor union from extracting rents from a low-cost plant in order to maintain employment at a high-cost plant. The merged entity can partially shift production domestically from a less towards a more efficient plant, rendering a domestic merger even more profitable. As a result, we find that either domestic or cross-border mergers may result in equilibrium, depending on cost asymmetries among firms.

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5Horn and Persson (2001b) analyze how international merger incentives depend on input market price setting and, in particular, on trade costs. They show how trade costs affect cross-country merger incentives and the type of mergers (unionized or non-unionized firms).

6Related are also Lommerud et al. (2005) and Straume (2003). Straume (2003) considers international mergers in a three-firm, three-country model where labor is unionized only in some firms. Lommerud et al. (2005) examine how different union structures affect downstream merger incentives in a three-firm Cournot oligopoly.

7Specifically, we assume that total costs are the sum of labor and non-labor costs. With regard to non-labor costs we suppose a high-cost and a low-cost firm in each country. Firms do not differ with regard to labor productivity.

8We assume that a union cannot discriminate workers in a single firm which appears to be standard current practice. In Germany, for instance, the tariff unity principle (“Tarifeinheit”) stipulates that only one collective bargaining agreement can govern workers’ labor conditions in a single firm. This principle implies that a firm must “unify” labor contracts after a merger. A recent example is the merger between RWTÜV and TÜVNORD in 2011. Both firms had different collective agreements before the merger. After the merger, the merged entity settled for a new collective wage agreement with the labor union (see Verdi, 2011) which now defines a uniform wage profile for all workers of the company.
and the degree of product differentiation.\footnote{Interestingly, a merger between asymmetric firms remains an equilibrium of the merger formation process even when firms become highly asymmetric; a result which stands in contrast to previous works on mergers between asymmetric firms (Barros, 1998).}

The remainder of the paper is organized as follows: Section 2 presents the model and the merger formation process. We analyze firms’ merger incentives in Section 3. In Section 4 we determine the equilibrium ownership structure of the industry. Welfare implications of our model are discussed in Section 5. Finally, Section 6 concludes.

\section{The Model}

We consider an oligopolistic industry with initially four independent firm owners. Each owner operates a single plant to produce a differentiated product. There are two countries $A$ and $B$: owners 1 and 2 are located in country $A$, while owners 3 and 4 reside in country $B$. The firms compete in quantities in the internationally integrated product market (i.e., we consider Cournot competition).

The price for brand $i$ is given by the linear inverse demand function $p_i = 1 - q_i - \beta \sum_k q_k$ for $i = 1, 2, 3, 4$ and $i \neq k$, where $q_i$ denotes the quantity supplied by plant $i$, and $\beta \in (0, 1)$ measures the degree of product differentiation. As $\beta$ approaches 1, brands become perfect substitutes, while for $\beta$ close to 0 products can be considered to be independent.

In order to produce their output, firms use labor and non-labor inputs in fixed proportions. We consider a simple constant-returns-to-scale production technology, such that one unit of output of brand $i$ requires one unit of labor, i.e., $q_i = l_i$, at wage $w_i$ and a variable amount of non-labor inputs at price $c_i$.

Firms differ in their non-labor production costs. We assume that firms 1 and 3 are the low-cost firms with $c_1 = c_3 = 0$, while firms 2 and 4 are the high-cost producers with $c_j = c > 0$ for $j = 2, 4$.\footnote{To ensure that each plant produces a non-negative output in every industry structure to be analyzed, we assume that $0 \leq c \leq \bar{c}$. We derive this upper bound $\bar{c}$, in the Appendix.} For $c = 0$, all firms are ex ante identical and we are back in the model analyzed by Lommerud et al. (2006). We can express firm $i$’s cost function as

$$C_i(q_i) = (w_i + D_i c)q_i,$$

where $D_i \in \{0, 1\}$ such that $D_i = 1$ for the high-cost firms $i = 2, 4$ and $D = 0$ for the low-cost firms $i = 1, 3$.\footnote{We abstract from the option that mergers induce efficiency gains with respect to marginal costs. We calculated another version of this model where mergers induced marginal cost savings for the high cost plants by $\mu c$, where $\mu \in (0, 1)$ measures the degree of efficiency gains. Our results are not affected by the introduction of merger synergies, only the scope for domestic mergers is reduced the larger the cost savings through mergers becomes. The results are available from the authors upon request.}

The profit of a producer supplying brand $i$ is thus given by

$$\pi_i = (p_i - w_i - D_i c)q_i, \text{ for } i = 1, 2, 3, 4.$$
Workers are organized in centralized labor unions in their respective countries.\footnote{A crucial assumption is that workers are unable to organize in unions across borders. This assumption is sensible in the light of empirical evidence. Although there have be tendencies towards more cooperation between labor unions on a European level, in general, labor market regimes are bound locally at the national level (Traxler and Mermet, 2003).} We consider a monopoly union model and adopt the right-to-manage approach, which stipulates that labor unions set wages for the firms residing in their respective countries, whereas the responsibility to determine employment remains with the firms. Unions make take-it or leave-it wage offers to firms, which resembles a situation in which the labor unions possess all bargaining power. The wage-setting regimes of labor unions are exogenously given and adjust to the industry structure which the plant owners determine cooperatively. Therefore, we consider two different cases. First, unions set plant-specific wages for the firms located in its country when there has been no or cross-border mergers. Labor unions in country \(A\) and country \(B\) maximize their wage bills \(U_A\) and \(U_B\), respectively, which are given by

\[
U_A = (w_1 - \bar{w}_A)l_1 + (w_2 - \bar{w}_A)l_2 \quad \text{and} \quad U_B = (w_3 - \bar{w}_B)l_3 + (w_4 - \bar{w}_B)l_4,
\]

where \(w_i\) denotes the wage paid by owner \(i\) and \(l_i\) is the derived labor demand of firm \(i\), with \(i = 1, 2, 3, 4\). By \(\bar{w}_A\) and \(\bar{w}_B\) we denote the outside option (wage) of workers in country \(A\) and country \(B\).\footnote{This can either be a wage level which can be achieved outside the considered industry or unemployment benefits.} We do not examine the effect of country specific reservation wages and, therefore, normalize the outside option wages to zero.

Second, when a domestic merger has occured, the responsible labor union is required to set a uniform wage for the merged firm, because it cannot discriminate between the workers. For example, consider a domestic merger between owners 1 and 2 in country \(A\) (the case of a merger in country \(B\) is obviously perfectly analogue). In this case the wage bill of labor union in country \(A\) is given by

\[
U_A = (w_{12} - \bar{w})(l_1 + l_2),
\]

where \(w_{12}\) is the (uniform) wage charged from firms 1 and 2.

We analyze a three-stage game. In the first stage, owners of plants cooperatively determine the equilibrium ownership structure of the industry and form firms according to the cooperative merger formation process proposed by Horn and Persson (2001a). In the second stage, after having observed the outcome of the firm formation process, labor unions simultaneously and non-cooperatively set wages. Finally, in the third stage of the game, firms compete in quantities in the final product market.

For any possible ownership structure we solve our model by backward induction to obtain the subgame perfect equilibrium. The solution concept for the merger formation process in the first stage of the game is the core.
Merger formation process. Before presenting the equilibrium analysis, we describe the merger formation process.\textsuperscript{14} We apply the method developed by Horn and Persson (2001a, 2001b) by modelling the merger formation process as a cooperative game of coalition formation. We let an ownership structure $M^r$ be a partition of the set $N = \{1, 2, 3, 4\}$ of firms into voluntary coalitions. As in Lommerud et al. (2006), we consider only two-firm mergers, so that the highest possible industry concentration is given by a duopoly.\textsuperscript{15} Considering the asymmetry between firms, we obtain eight possible industry structures which are possible outcomes of the merger formation process:

1. No merger: $M^0 = \{1, 2, 3, 4\}$
2. One domestic merger: $M^{D1} = \{12, 3, 4\}, M^r_{N1} = \{1, 2, 34\}$
3. Two domestic mergers: $M^{D2} = \{12, 34\}$
4. One symmetric cross-border merger between the efficient plants: $M^{C1se} = \{13, 2, 4\}$
5. One symmetric cross-border merger between the inefficient plants: $M^{C1si} = \{1, 3, 24\}$
6. Two symmetric cross-border mergers: $M^{C2s} = \{13, 24\}$
7. One asymmetric cross-border merger: $M^{C1a} = \{14, 2, 3\}, M^{C1a'} = \{1, 4, 23\}$
8. Two asymmetric cross-border mergers: $M^{C2a} = \{14, 23\}$

As firms are not identical, cross-border mergers can take place in different constellations. First, firms with the same non-labor production costs can merge, which we denote by symmetric cross-border mergers in structures 4) to 6). When there is only one international symmetric merger, it can either be the two efficient ($M^{C1se}$) or the two inefficient ($M^{C1si}$) firms that merge. The ownership structure with two mergers between the symmetric (low cost and high cost) firms is represented by structure $M^{C2s}$. Thus, in structure $M^{C2s}$ there is one efficient firm producing brands 1 and 3 at low cost, and one inefficient firm producing brands 2 and 4 at high cost.

Second, there can be a cross-border mergers between two firms of different cost types, which is denoted by asymmetric cross-border mergers in structures 7) and 8). If there is only one asymmetric cross-border merger, the outcome is obviously identical for structures $M^{C1a}$ and $M^{C1a'}$. Ownership structure $M^{C2a}$ indicates that there have been two cross-border mergers between one low cost and one high cost plant each. As a result each merged firm produces one brand at low cost and the other brand at high cost.

\textsuperscript{14}For a detailed description of the approach, see Horn and Persson (2001a, 2001b).
\textsuperscript{15}We are interested in highlighting the incentives for domestic versus cross-border mergers and the role asymmetries between firms play in this formation process. If firms have the opportunity to monopolize the market, an all-encompassing merger is the obvious outcome, regardless of firm asymmetries. In addition, three- or four-firm mergers are more likely to be blocked. Finally, cost of administering a merger may grow overproportionally making mergers of three or four plants unprofitable.
To determine the outcome of the cooperative merger formation process, the main concept is the determination of dominance relations between the structures. If an ownership structure is dominated by another structure, it cannot be the equilibrium outcome of the cooperative merger formation game. The approach involves a comparison of each structure $M^r$ against all other structures $M^{-r}$ separately. $M^r$ dominates a structure $M^l$ if the combined profits of the decisive group of owners in structure $M^r$ exceeds those in structure $M^l$.

Decisive owners can influence which coalition of plants is built. Thus, a group of owners which belongs to identical coalitions in ownership structures $M^r$ and $M^l$ is not decisive as we exclude the possibility of transfer payments between coalitions. Within a coalition, owners are free to distribute the joint profit among each other. Thus, an ownership structure $M^r$ dominates another structure $M^l$ if all decisive owners prefer $M^r$ to $M^l$. Naturally, a decisive group of owners will only prefer $M^r$ over $M^l$ if the combined profit of this group is larger in $M^r$ than in $M^l$.

According to the bilateral dominance relationship, it is possible to rank different ownership structures. We are interested in the equilibrium industry structure (EIS), i.e. a structure which is undominated. As we observe heterogeneous firms, it is possible that an ownership structure is only undominated for certain parameter constellations. Consequently, there may be more than one EIS depending on the parameter values of $c$ and $\beta$. We apply the core as our solution concept. That is, those structures that are undominated constitute the core which defines the equilibrium industry structure (EIS).

### 3 Merger Incentives

We solve our model for all possible ownership structures in the Appendix. Our main focus of interest are the driving forces behind domestic and cross-border mergers between asymmetric firms in the presence of labor unions. Before we derive the equilibrium industry structure(s) it is instructive to analyze the impact of different types of mergers on wages and employment. As the empirical literature on merger effects on labor has shown that the impact can be rather diverse, it is worth investigating whether different merger types affect labor differently. Consequently, different wage and employment effects may provide a better understanding why plant owners may choose one type of merger over another in equilibrium.

**Lemma 1.** The no-merger ($M^0$) and all one-merger structures ($M^{D1}, M^{C1se}, M^{C1si}, M^{C1a}$) are dominated by at least one two-merger structure ($M^{D2}, M^{C2a}, M^{C2s}$).

**Proof.** See Appendix.

A comparison of profit levels reveals that industry structures involving two mergers ($M^{D2}, M^{C2a}, M^{C2s}$) unambiguously provide higher total profits for the pivotal firms than industry structures in which more than two firms prevail in the market. The equilibrium outcome of the merger formation process will therefore always result in a duopoly, the most concentrated industry structure which is allowed. This confirms the result by Horn and Persson (2001a).
that the industry tends towards concentration when (1) a merger to monopoly is profitable and (2) the formation of either a monopoly or a duopoly is allowed.

As a consequence, when analyzing possible candidates for equilibrium industry structures, only structures with two merged firms should be considered. Therefore, we restrict our attention in the following to three possible industry structures: \( M^{D2} \), \( M^{C2a} \) and \( M^{C2s} \), i.e. we focus on the incentives for either two domestic or two cross-border mergers, where we distinguish between coalitions of symmetric plants (two efficient and two inefficient plants merge) and coalitions between asymmetric plants (one efficient producer merges with one inefficient producer each).

### 3.1 Wage and Employment Effects of Domestic and Cross-Border Mergers

As wage rates are determined endogenously in our model, unions may react to each market structure by adjusting their wage responses accordingly. Therefore, we begin with an analysis of the effects of different kinds of mergers on wage rates. As we restrict our attention to two-merger industry structures, wage rates in countries \( A \) and \( B \) will be symmetric, although there will be differences in the wage rates paid by efficient and inefficient plants if labor unions set plant-specific wages (in structures \( M^{C2s} \) and \( M^{C2a} \)). For expositional purposes, therefore, denote by subscript \( I \) wages paid by inefficient plants (plants 2 and 4) and by subscript \( E \) those paid by efficient plants (1 and 3). As there is only one uniform wage for \( M^{D2} \), there is no subscript.

When we compare the wage rates set by the labor unions in countries \( A \) and \( B \) for structures \( M^{D2} \), \( M^{C2s} \) and \( M^{C2a} \), we find that the plant-specific wages in ownership structures involving international mergers can be ranked unambiguously, while such a ranking is not distinctly possible when including the uniform wage set for national merger participants. The relation between the wage rates in the different industry structures then depends on the relation between product differentiation (\( \beta \)) and cost asymmetry between firms (\( c \)).

**Proposition 1.**

(a) The ranking of wage rates set by labor unions in ownership structures \( M^{C2s} \) and \( M^{C2a} \) is unambiguously given by \( w^C_{E2s} > w^C_{E2a} > w^C_{I2s} > w^C_{I2a} \) \( \forall \beta \in (0,1) \) and \( c \in (0,\bar{c}] \).

(b) Comparing the uniform wage rate set by the labor unions in structure \( M^{D2} \), the ranking depends on the degree of product differentiation and the cost asymmetry between firms.

**Proof.** See Appendix.

Efficient plants pay unambiguously higher plant-specific wage rates than inefficient plants. Obviously, labor unions are able to extract a higher surplus from efficient plants. However, post-merger wages depend on which type of plants have formed a coalition. Recall that in structure \( M^{C2a} \) each merged firm operates one efficient and one inefficient plant.
To save on non-labor cost of production, each merged firm will partially reallocate production from the high to the low-cost plant. The magnitude of this reallocation depends on the degree of substitutability between brands. Consequently, the efficient plants increase their market shares in $M^{C2a}$ giving labor unions the opportunity to raise wages $w^{C2a}_E$ while balancing wage demands and respective effects on employment.

In contrast, instructure $M^{C2s}$ firms of the same cost type merge, thereby creating no option to reallocate production among each other to save on non-labor cost. Unions adjust their wage demands to these different constellations of ownership. The respective production shifting opportunities in the two structures yield higher wages for efficient plants in $M^{C2a}$ than $M^{C2s}$. For inefficient plants, obviously the reverse holds true.

Comparing the uniform wage $w^{D2}$ to the plant specific wage rates is less obvious, because there does not exist a unique ranking. Rather the position of $w^{D2}$ in relation to the plant specific wages depends on the degree of product differentiation and the asymmetry between firms. Figure 1 illustrates the different rankings.

The three areas are defined as follows:

\begin{align*}
A & : w^{C2a}_E > w^{C2a}_I > w^{D2} > w^{C2s}_I > w^{C2s}_E; \\
B & : w^{C2a}_E > w^{D2} > w^{C2s}_I > w^{C2s}_E > w^{C2a}_I; \\
C & : w^{D2} > w^{C2a}_E > w^{C2s}_I > w^{C2s}_E > w^{C2a}_I.
\end{align*}

As is the case for structure $M^{C2a}$, as a result of domestic mergers firms have an incentive to reallocate production nationally towards the more efficient plant. This reallocation becomes more pronounced the more asymmetric firms are or the less differentiated products are. If the labor unions were allowed to discriminate between workers in a merged firm, they would optimally increase the wage rates at the efficient plant in anticipation of increased production. However, as the wage-setting regime prohibits the unions to discriminate, they become
constrained in their ability to raise the wage.

More specifically, each union has to balance an increase in the wage to obtain a higher rent from the efficient plant against an associated loss in employment at the inefficient plant. Thus, the more asymmetric firms become with respect to non-labor cost, the more constrained the unions become in their wage choices. Thus, as can be seen from Figure 1, for higher values of \( c \), \( w_{D^2} \) is driven below the levels of \( w_{E}^{2a} \) and \( w_{E}^{2s} \).

The reason for this result is that the non-labor cost of the inefficient firms affects wage rates differently. Note that

\[
\frac{\partial w_{D^2}}{\partial c} = -\frac{1}{4 + 2\beta} < 0
\]

for \( \beta \in (0, 1) \). When the non-labor cost of production of the inefficient plants marginally increases, the wage rate paid by the merged firm falls. As uniformity of wages restricts the labor union in exploiting the production efficiency of the low cost producer, it limits its wage demand when firms become more asymmetric in order to maintain employment at the high cost plant. In contrast, in cross-border merger structures, low cost plants’ wages rise if non-labor costs of high cost plants increase, as

\[
\frac{\partial w_{E}^{2a}}{\partial c} = \frac{\beta}{4 - \beta} > 0,
\]

and

\[
\frac{\partial w_{E}^{2s}}{\partial c} = \frac{2(1 - \beta)\beta}{(4 - \beta)(4 - 3\beta)} > 0.
\]

Next to the impact of firm asymmetry and uniformity of wages, a merger further affects the choice of wage rates through changes in the elasticities of labor demand at the merged firms. Different merger types may result in different changes in labor demand elasticities due to the relation between national labor unions and international firms. While for a domestic merger plants with relation to the same labor union merge, cross-border mergers induce rivalry between nationally organized labor unions due to the threat effect.

To analyze the changes in labor demand elasticities, first consider structure \( M_{D^2} \) in relation to the no-merger case. From the first order conditions of the unions’ maximization problems in Stage II, we can derive the slopes of the labor demand curves

\[
\frac{\partial \tilde{l}_E^0}{\partial w_E^0} = \frac{\partial \tilde{l}_I^0}{\partial w_I^0} = \frac{2 + 2\beta^2}{(\beta - 2)(2 + 3\beta)}, \quad \text{and}
\]

\[
\frac{\partial \tilde{l}_E^{D^2}}{\partial w_E^{D^2}} = \frac{\partial \tilde{l}_I^{D^2}}{\partial w_I^{D^2}} = \frac{2 - 2\beta^2}{4(\beta - 1)(1 + 2\beta)}
\]

for the pre- and post-merger cases, where \( \tilde{l}_E^k \) and \( \tilde{l}_I^k \), \( k \in \{0, D^2\} \) are the derived labor demand of efficient and inefficient plants, respectively. Comparison of the two expression reveals that

\[
\left| \frac{\partial \tilde{l}_E^{D^2}}{\partial w_E^{D^2}} - \frac{\partial \tilde{l}_E^0}{\partial w_E^0} \right| = \frac{\beta(1 + \beta)(4 + 3\beta)}{2(-4 - 12\beta - 5\beta^2 + 6\beta^3)} < 0.
\]
Reduced product market competition after the two domestic mergers reduces the responsiveness of firms’ labor demand. Ceteris paribus, labor demand becomes less elastic in a domestic merger case and the labor unions have an incentive to raise wages. However, the previously described effect of uniformity countervails this incentive, because the union would raise wages for all workers in both plants.

On the other hand, a cross-border merger induces union rivalry through the threat effect. The slope of labor demand in both cross-border merger structures $M^{C2s}$ and $M^{C2a}$ is given

$$\frac{\partial I_{E}^{C2s}}{\partial w_{E}^{C2s}} = \frac{\partial I_{I}^{C2s}}{\partial w_{I}^{C2s}} = \frac{\partial I_{E}^{C2a}}{\partial w_{E}^{C2a}} = \frac{\partial I_{I}^{C2a}}{\partial w_{I}^{C2a}} = \frac{2 + 2\beta + \beta^2}{4(\beta - 1)(1 + 2\beta)}.$$  \hspace{1cm} (11)

Comparison with the labor demand slope in the no-merger case reveals that

$$\left| \frac{\partial I_{E}^{C2s}}{\partial w_{E}^{C2s}} \right| - \left| \frac{\partial I_{0}^{0}}{\partial w_{E}^{0}} \right| = \frac{3\beta^2(2 + 2\beta + \beta^2)}{4(4 - 8\beta - 7\beta^2 - 11\beta^3 + 6\beta^4)} > 0.$$  \hspace{1cm} (12)

Ceteris paribus, cross-border mergers increase the responsiveness of labor demand of the firms, which would lead to a decrease in wage demands by unions. The difference in labor demand responsiveness for different merger types is in line with the results by Lommerud et al. (2006), however, a countervailing effect may arise in our model increasing firms’ incentives to merge domestically: the constraining effect of a uniform wage on a labor union’s ability to extract surplus from efficient firms.

To understand which types of mergers will be chosen in equilibrium, it is also instructive to look at employment effects of different merger types. The following Lemma summarizes the impact of different merger types on total employment.

**Lemma 2.** (a) Total employment is always lower in the domestic merger structure than in the international merger structures: $Q^{C2s} = Q^{C2a} > Q^{D2}$. (b) Comparing employment in the two-merger structures to no merger, we find that

(i) $Q^{0} > Q^{C2s} = Q^{C2a} > Q^{D2}$ if $\beta \in (0, 0.5)$, and

(ii) $Q^{C2s} = Q^{C2a} > Q^{0} > Q^{D2}$ if $\beta \in (0.5, 1)$.

**Proof.** The proof involves a mere comparison of total employment in the four relevant industry structures.

Three interesting observations can be made from Lemma 3. First of all, we find that total employment is always lowest in the domestic merger structure compared to cross-border merger structures and the no-merger benchmark.\footnote{Note that uniformity of wages in the domestic merger structure $M^{D2}$ does not influence this result. Essentially, uniformity has no effect on total employment compared to plant-specific (discriminatory) wages when market demand is linear (Schmalensee, 1981; Yoshida, 2000). Assuming symmetric firms, total employment is the same as in the model analyzed by Lommerud et al. (2006). Differences in total employment are therefore only a result of firm asymmetries.} Inspection of the plant-specific employment rates
(see Appendix) reveals that this mainly hinges upon the low employment of inefficient plants in the domestic merger structure. The increase in market concentration leads to a contraction of total employment.

Second, total employment in the two cross-border merger structures is identical, although different types of mergers are formed in the two structures. The reason for this result becomes obvious from the ranking of wage rates above. In the two cross-border merger structures, labor unions set wages as to balance total costs for the firms in the two structures. Note that, however, this does not mean that the distribution of output across plants is identical for the merger structures. This is not the case, as firms shift production towards more efficient plants in structure $M^{C2a}$ while this not possible for structure $M^{C2s}$, where plants with identical technologies merge.

Third, for a lower degrees of product differentiation, total employment is higher with cross-border mergers than in the no merger case. If products are closer substitutes ($\beta$ close to 1) the opportunity for firms to shift production, for either labor or non-labor cost savings, becomes larger.

## 4 Equilibrium Industry Structure

We now turn to the industry structures which will result in equilibrium as the outcomes of the merger formation process. A comparison of the relevant profit functions according to the method proposed by Horn and Persson (2001a, 2001b) yields that the equilibrium ownership structure is two domestic mergers ($M^{D2}$), if firms differ according to their non-labor production costs and asymmetries between firms are sufficiently large.

**Proposition 2.** Depending on the degree of product differentiation and the cost asymmetry between firms, either two-merger structure can result in equilibrium:

(a) If $\beta \in (0, 0.396)$ and $c \in (\Psi, \bar{c}]$, then the equilibrium industry structure is given by $M^{D2}$.

(b) If $\beta \in (0, 0.396)$ and $c \in (0, \Psi) \cup \beta \in (0.396, 0.913)$ and $c \in (0, \bar{c}]$ then the equilibrium industry structure is given by $M^{C2s}$.

(c) If $\beta \in (0.913, 1)$ and $c \in (0, \bar{c}]$ then the equilibrium industry structure is given by $M^{C2a}$.

**Proof.** See Appendix.

In contrast to previous work with homogeneous firms and purely plant-specific wages, the equilibrium industry structure in our model can consist of either domestic or cross-border mergers. Two domestic mergers will be the unique equilibrium if products are sufficiently differentiated and sufficient firm asymmetries. Figure 3 illustrates these results. The result that either domestic or cross-border mergers can occur is in contrast to the findings by Lommerud et al (2006), where domestic mergers never occur in equilibrium.
The incentives for firms to merge domestically when plants are sufficiently asymmetric stem from the two effects described above. A domestic merger induces the labor unions to limit their wage demands from the efficient plant in order to maintain employment at the inefficient plant. The mergers decrease competition in the product market and induce a reduction in overall employment. Concerning the distribution of employment among the plants, merged firms domestically shift production from an inefficient to the efficient plant.

When these two effects dominate the gains from cross-border mergers - namely the reduction of market power of labor unions through the threat of reallocation- two domestic mergers will emerge as an equilibrium industry structure. More specifically, merging domestically becomes more attractive the more asymmetric firms become. Thus, we should expect that the threat effect will conversely dominate the uniformity effect if firms are rather symmetric or products are closer substitutes.

Interestingly, for $c$ sufficiently large, two domestic mergers will occur in equilibrium, which results in two asymmetric firms merging. This is in contrast to previous work on mergers between heterogeneous firms, which shows that when asymmetries are large, mergers will occur (if any) between more symmetric firms (see e.g. Barros, 1998).

For a wide range of parameter values, we observe that cross-border mergers between symmetric firms ($M^{C2s} = \{13, 24\}$) will occur in equilibrium. In this region, the threat effect of cross-border mergers dominates the benefits of uniformity for the firms. Note that equilibrium cross-border mergers will not necessarily lead to higher employment compared to a no-merger case. Only for the region $\beta \in (0.5, 1)$ cross-border mergers will increase total employment.

Finally, note that cross-border mergers between asymmetric firms will only be the equilibrium outcome of the merger formation process when products are close substitutes. In this case the opportunity to shift production is largest, because products are only slightly differentiated.
and at the same time, firms will be rather homogeneous due to the upper bound on non-labor marginal cost, $\bar{c}$.

5 Welfare

Finally, we inspect the welfare implications of our results. At first glance, a domestic merger might have welfare improving effects because of the redistribution of production from less efficient to more efficient firms (Barros, 1998). However, the employment effect of domestic mergers gives rise to the following result.

**Proposition 3.** The ownership structure involving two domestic mergers $M^{D2}$ is never socially optimal. The optimal industry structure from a welfare perspective can be either no merger ($M^0$), one domestic merger ($M^{D1}$), one cross-border merger between the inefficient plants ($M^{C1si}$), two symmetric cross-border mergers ($M^{C2s}$) or two asymmetric cross-border mergers ($M^{C2a}$).

**Proof.** See Appendix.

Calculating the global welfare as the sum of firms’ profits, labor union wage bills and consumer surplus, we see that industry structure $M^{D2}$ is never welfare optimal. For all parameter constellations of $\beta$ and $c$, it is welfare dominated by other structures. Although a domestic merger results in a partial reallocation of production from less towards more efficient plants, the reduction in overall quantity in the market causes this structure to be never optimal from a welfare perspective.

Establishing which industry structure is welfare optimal (from a global welfare point of view) is, however, not easy in practice. Since the production asymmetry may cause a reallocation of production from inefficient to efficient plants in some structures, total quantity sold in the market does not necessarily indicate when a structure is also most desirable from a welfare perspective. Figure 3 summarizes the industry structures, which can be welfare optimal in given parameter regions. Interestingly, there can be also welfare optimal industry structures which will never be the equilibrium outcome of the merger formation process between firms ($M^0$, $M^{D1}$, and $M^{C1si}$). Most notably, while two domestic mergers are never optimal from a welfare perspective, an industry structure with one domestic merger can be when firms are rather asymmetric and product differentiation is rather strong. This parameter constellation roughly coincides with the area where two domestic mergers are the equilibrium industry structure (see Figure 2). From a welfare perspective, two many domestic mergers occur for these parameter constellations.

We do find, following Lommerud et al. (2006), that two cross-border mergers ($M^{C2s}$ or $M^{C2a}$) always welfare dominate the two domestic mergers structures. In contrast, we cannot establish a pattern leading to the conclusion that cross-border mergers are the welfare optimal industry structure for a wide range of parameters. Interestingly, for intermediate degrees of product differentiation, the welfare optimal industry structure is given by $M^{C2a}$. 

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To illustrate the above results, Figure 4 shows the global welfare functions of the five possible structures for $\beta = 0.25$ and $\beta = 0.75$. In the upper panel, $M^{D1}$ is displayed by the red curve. It becomes obvious, that structure $M^{D1}$ produces lower welfare than all other structures when firms are rather symmetric. However, when asymmetry increases, global welfare in this structure increases as well and finally dominates all other structures from a welfare perspective. When firms are more symmetric, the no merger structure $M^0$ produces the highest global welfare with strong product differentiation.

When products are less differentiated ($\beta = 0.75$) this picture changes. One domestic merger is clearly dominated from a welfare perspective by the other candidate structures for welfare maximization. When $c$ increases, two mergers between asymmetric firms produce the highest global welfare ($M^{C2a}$). In these instances, production rescaling from less efficient to more efficient plants becomes more effective, thereby increasing global welfare.
A comparison with the results of the equilibrium outcomes of the merger formation shows that firms will not choose the optimal equilibrium industry structure from a welfare point of view for the given parameter range in Proposition 4. The result does support, however, empirical findings, namely an increasing trend in cross-border mergers where target and acquiring firms may strongly differ in size (Gugler et al., 2003).
6 Concluding Remarks

We have presented an extension of the model analyzed by Lommerud et al. (2006) to uncover the role of cost asymmetries among firms in a unionized oligopoly. Our results suggest that domestic mergers may result as an equilibrium outcome of the cooperative coalition formation process when firms are asymmetric in their non-labor costs of production. This result holds even when asymmetries become very large, which is in contrast to previous work on horizontal mergers between asymmetric firms without endogenous input costs (see e.g. Barros, 1998).

The incentives for domestic mergers critically depend labor unions inability to discriminate among workers belonging to the same employer. Thereby, firms face a trade-off between domestic and cross-border mergers in the coalition formation game: cross-border mergers give rise to the threat effect -the opportunity to reallocate production from one country to another-which puts downward pressure on wages. Domestic mergers constrain the labor unions in their freedom to extract surplus from efficient plants. This uniformity effect provides incentives for firms to merge domestically. On the one hand a domestic merger may lower the wage paid by the efficient plant, on the other hand production may be reshuffled within one country from the less to the more efficient producer.

We obtain, therefore, both domestic and cross-border mergers in equilibrium, depending on the degree of product substitutability and the asymmetry between firms. If cross-border mergers occur, mergers between symmetric plants will be the prevailing industry structure for the widest range of parameter constellations. However, asymmetric international merger outcomes are also possible whenever products are sufficiently homogenous.

A comparison with the optimal industry structures from a global welfare perspective reveals that firms do not choose the welfare optimal industry structures. While two domestic mergers are never welfare optimal, no unambiguous pattern in the industry structures according to welfare effects can be established. The welfare optimal structure can involve no mergers at all, one, or two mergers. It can be said, however, that for a wide range of parameter constellations, the global welfare maximizing industry structures involves two mergers between asymmetric firms, i.e. between an efficient and an inefficient plant each. Obviously, this result is enforced by the positive welfare effect of these mergers because of the reallocation of production from less to more efficient firms. A comparison to the equilibrium industry structure chosen by firms, reveals that such an industry structure is however rarely chosen by firms.
Appendix

In the following, we explicitly solve our model for all possible industry structures. We report firms’ profits, employment (quantities) and wages for each merger structure.

No merger \((M^0)\):

\[
\begin{align*}
\Pi_1^0 &= \Pi_3^0 = \frac{(8+\beta^2(-2+c)+6\beta c)^2}{64(4+4(-3+\beta)\beta)^2}, \\
\Pi_2^0 &= \Pi_4^0 = \frac{(8+\beta^2(-2+c)-8c-6\beta c)^2}{64(4+4(-3+\beta)\beta)^2}, \\
q_1 &= q_3 = \frac{8+\beta^2(-2+c)+6\beta c}{8(4+4(-3+\beta)\beta)}, \\
q_2 &= q_4 = \frac{8+\beta^2(-2+c)-8c-6\beta c}{8(4+4(-3+\beta)\beta)}, \\
w_1 &= w_3 = \frac{(4+\beta(-2+c)-4c)}{8}, \\
w_2 &= w_4 = \frac{(4+\beta(-2+c)-4c)}{8}.
\end{align*}
\]

One domestic merger \((M^{D1})\): Assume, firms 1 and 2 in country A merge

\[
\begin{align*}
\Pi_{12}^{D1} &= \frac{-2(-1+\beta^2)(-4-6+\beta)^2(4+\beta(6+\beta))e+\Gamma e^2}{4(1-\beta)(2+(-3+\beta)^2)(4+\beta(6+\beta))^2}, \\
q_1 &= q_2 = \frac{2(-2+\beta)(2+\beta(5+\beta(-1+c)-9c-6(2+c))}{2(-1+\beta)(-2+(-3+\beta)^2)(4+\beta(6+\beta))^2}, \\
q_3 &= q_4 = \frac{2(-2+\beta)(2+\beta(5+\beta(-1+c)-9c-6(2+c))}{2(-1+\beta)(-2+(-3+\beta)^2)(4+\beta(6+\beta))^2}, \\
w_1 &= w_2 = \frac{(4+\beta(-2+\beta(-1+c))}{2(4+\beta(6+\beta))}, \\
w_3 &= w_4 = \frac{(4+\beta(-2+\beta(-1+c))}{2(4+\beta(6+\beta))}.
\end{align*}
\]

Two national mergers \((M^{D2})\): When both national firms merge in each country, we get:

\[
\begin{align*}
\Pi_{12}^{D2} &= \Pi_{34}^{D2} = \frac{-4(-1+\beta)(1+\beta)^3+4(-1+\beta)(1+\beta)^3+5+\beta(22+3\beta(11+\beta(6+\beta)))}{8(1-\beta)(2+\beta)^2(1+2\beta)^2}, \\
q_1 &= q_2 = \frac{2+\beta(5+\beta^2(-2+3c)}{8+12\beta-12\beta^2-8\beta^3}, \\
q_3 &= q_4 = \frac{2+\beta(5+\beta^2(-2+3c)}{8+12\beta-12\beta^2-8\beta^3}, \\
w_1 &= w_2 = \frac{4(4+\beta(-2+\beta(-1+c))}{2(4+\beta(6+\beta))}, \\
w_3 &= w_4 = \frac{4(4+\beta(-2+\beta(-1+c))}{2(4+\beta(6+\beta))}.
\end{align*}
\]

One efficient symmetric international merger \((M^{C1se})\): Firms 1 and 3 merge

\[
\begin{align*}
\Pi_{13}^{C1se} &= \Pi_{31}^{C1se} = \frac{(-2+\beta)^2(1+\beta)(-8+\beta^2-63c)^2}{2[16+(-1+\beta)(2+\beta^2(-2+(-3+\beta)^2))]^2}, \\
q_1^{C1se} &= q_3^{C1se} = \frac{2[16+(-1+\beta)(2+\beta^2(-2+(-3+\beta)^2))]^2}{(8(-1+\beta)) + \beta(4+\beta(-1+c)-7c+4c)^2}, \\
q_2^{C1se} &= q_4^{C1se} = \frac{2[16+(-1+\beta)(2+\beta^2(-2+(-3+\beta)^2))]^2}{2(2-\beta)(8+\beta^2-63c)}, \\
w_1^{C1se} &= w_3^{C1se} = \frac{2[16+(-1+\beta)(2+\beta^2(-2+(-3+\beta)^2))]^2}{2(2-\beta)(8+\beta^2-63c)}, \\
w_2^{C1se} &= w_4^{C1se} = \frac{2[16+(-1+\beta)(2+\beta^2(-2+(-3+\beta)^2))]^2}{2(2-\beta)(8+\beta^2-63c)}.
\end{align*}
\]
One inefficient symmetric international merger ($MC_{1si}$): Firms 2 and 4 merge

\[ \Pi_{C1si}^{\text{24}} = \frac{(-2+\beta)^2(1+\beta)(8+\beta^2(1-c)-8c-6\delta c)^2}{2(16-12\beta+\beta^2)^2(1-2-3\beta+\beta^2)^2} \]

\[ \Pi_{C1si}^{\text{14}} = \Pi_{C3si}^{\text{14}} = \frac{(8+\beta^2-\beta^2(4+3c)+\beta(-2+6c))}{(16-12\beta+\beta^2)^2(1-2-3\beta+\beta^2)^2} \]

\[ q_{C1si}^{\text{14}} = q_{C3si}^{\text{14}} = \frac{-8+\beta(2-6c+\beta(4-\beta+3c))}{(16+(-12+\beta)\beta(-2+3\beta+\beta(4-\beta+3c)))} \]

\[ w_{C1si}^{\text{14}} = w_{C3si}^{\text{14}} = \frac{2(-2+\beta)(4\beta(-3+c))}{16+(-12+\beta)\beta} \]

Two symmetric international mergers ($MC_{2s}$):

\[ \Pi_{C2s}^{\text{13}} = \frac{(1+\beta)(8+\beta(2-3\beta+6(-4+\beta)\beta)c)^2}{2(4-\beta)(4-\beta)^2(1+\beta)^2(1+\beta)^2} \]

\[ \Pi_{C2s}^{\text{24}} = \frac{(1+\beta)(8+\beta(2-3\beta+6(-4+\beta)\beta)c)^2}{2(4-\beta)(1-3\beta)(4-\beta)^3} \]

\[ q_{C2s}^{\text{13}} = q_{C2s}^{\text{24}} = \frac{8+\beta(-2-3\beta+6(-4+\beta)\beta)c}{8-8c-8\beta(2-3\beta+4(1-7+\beta)\beta)c} \]

\[ q_{C2s}^{\text{13}} = q_{C2s}^{\text{24}} = \frac{2(-2+\beta)(4\beta(-3+c))}{16+(-12+\beta)\beta} \]

\[ u_{C2s}^{\text{13}} = u_{C2s}^{\text{24}} = \frac{(4-\beta)(4-\beta+3+c)}{(4-\beta)(4-\beta)} \]

\[ u_{C2s}^{\text{13}} = u_{C2s}^{\text{24}} = \frac{(4-\beta)(4-\beta+3+c)}{(4-\beta)(4-\beta)} \]

One asymmetric international merger ($MC_{1A}$): Assume firms 1 and 4 merge

\[ \Pi_{C1A}^{\text{13}} = \frac{2(-2+\beta)^2(8+\beta^2)^2(-1+\beta^2)(-16+\beta(-12+7\beta))^2-2(-2+\beta)^2(8+\beta^2)^2(-1+\beta^2)(-16+\beta(-12+7\beta))^2c+6c^2}{4(-1+\beta)^2(16+(-12+7\beta))^2(-2+(-3+\beta)\beta)^2(-16+\beta(-12+7\beta))^2} \]

where \( \Phi = (-65536 + \beta(-98304 + \beta(159744 + \beta(219136 + \beta(-218880 + \beta(-160640 + \beta(188192 + \beta(\Psi))))))) \)

and \( \Psi = (7568 + \beta(-64620 + \beta(27968 + 3\beta(-1551 + \beta(71 + 4\beta)))) \)

\[ \Pi_{C2A}^{\text{13}} = \frac{2(-2+\beta)(-2+\beta)(-16+\beta(-12+7\beta))+(512+\beta(384+\beta(-736+\beta(-168+\beta(380+\beta(-120+11\beta)))))))c^2}{(4-2+\beta)^2(16+(-12+7\beta))^2(-16+\beta(-12+7\beta))^2} \]

\[ \Pi_{C2A}^{\text{13}} = \frac{(4-2+\beta)^2(16+(-12+7\beta))^2(-16+\beta(-12+7\beta))^2}{(4-2+\beta)^2(16+(-12+7\beta))^2(-16+\beta(-12+7\beta))^2} \]

\[ q_{C2A}^{\text{13}} = q_{C2A}^{\text{13}} = \frac{-256+\beta(384+\beta(-736+\beta(-168+\beta(380+\beta(-120+11\beta))))))c^2}{-2(-1+\beta)(16+(-12+7\beta)+2(-3+\beta)\beta)(-16+\beta(-12+7\beta))} \]

\[ q_{C2A}^{\text{13}} = q_{C2A}^{\text{24}} = \frac{-12+4+\beta(2-1+\beta)(2-4+\beta)}{(16+(-12+7\beta)+2(-3+\beta)\beta)\beta(-16+\beta(-12+7\beta))} \]

\[ q_{C2A}^{\text{13}} = q_{C2A}^{\text{24}} = \frac{2(-2+\beta)(-16+12+7\beta)\beta(-2+(-3+\beta)\beta)(-16+12+7\beta)\beta}{2(-2+\beta)(-16+12+7\beta)\beta(-2+(-3+\beta)\beta)(-16+12+7\beta)\beta} \]

\[ q_{C2A}^{\text{13}} = q_{C2A}^{\text{24}} = \frac{2(-2+\beta)(-16+12+7\beta)\beta(-2+(-3+\beta)\beta)(-16+12+7\beta)\beta}{2(-2+\beta)(-16+12+7\beta)\beta(-2+(-3+\beta)\beta)(-16+12+7\beta)\beta} \]

\[ u_{C2A}^{\text{13}} = u_{C2A}^{\text{24}} = \frac{-(2+\beta)-(-1+\beta)(-16+\beta(-12+7\beta))-(64+\beta(8+\beta(-50+13\beta))))c}{-256+\beta^2(240+\beta(-967+\beta))} \]

\[ u_{C2A}^{\text{13}} = u_{C2A}^{\text{24}} = \frac{128(-1+\beta)+(-32+2\beta)(128(-120+\beta(-106+3(7-4c)+74c)))}{-256+\beta^2(240+\beta(-967+\beta))} \]

\[ u_{C2A}^{\text{13}} = u_{C2A}^{\text{24}} = \frac{-128+\beta(-32+2\beta)+\beta(-8+(-16+\beta(106+\beta(21-9c)+32c)))}{-256+\beta^2(240+\beta(-967+\beta))} \]

\[ u_{C2A}^{\text{13}} = u_{C2A}^{\text{24}} = \frac{-(2+\beta)-(1+\beta)(-16+\beta(-12+7\beta))+(128-\beta(32+\beta(128+\beta(-82+15\beta))))c}{-256+\beta^2(240+\beta(-967+\beta))} \]

Two asymmetric international mergers ($MC_{2A}$):
\[ \Pi_{14}^{C2a} = \Pi_{23}^{C2a} = \frac{4(2+\beta)^2(-1+\beta^2)-4(2+\beta)^2(-1+\beta^2)(-8-\beta(-16+\beta)(2(16-3\beta)))c^2}{8(1+\beta)(4+(7-2\beta)\beta)^2} \]
\[ q_1^{C2a} = q_3^{C2a} = \frac{4-\beta^3(-2+\beta(-2+4c)}{4(4+\beta-9\beta^2+2\beta^3)} \]
\[ q_2^{C2a} = q_4^{C2a} = \frac{4-\beta^3(-2+\beta(-2+4c)}{4(2+\beta(-2+4c)} \]
\[ w_1^{C2a} = w_3^{C2a} = \frac{2+\beta(-2+4c)}{4-\beta} \]
\[ w_2^{C2a} = w_4^{C2a} = \frac{2-2\beta-2\beta^2+\beta^3}{4-\beta} \]

\[ c = \frac{-4-2\beta+6\beta^2+\beta^3-\beta^4}{-6-13\beta-5\beta^2+2\beta^3} \]

In the following we provide proofs of Lemmas 1 to 3 and Propositions 1 to 4.

**Proof for Restriction on Non-Labor Costs** We need to restrict the non-labor production cost \( c \) in that way, that each plant produces a non-negative output in each of the eight possible industry structures. To this aim, consider for each structure when \( q_k^i \geq 0, i = \{1, 2, 3, 4\} \) and \( k = \{0, D1, D2, C1se, C1si, C2s, C1a, C2a\} \). From all the restrictions on the parameter \( c \) we obtain, we choose the most stringent one in order to ensure that plants produce non-negative output in every instance. The following solutions can be easily verified:

\[ q_2^0 \geq 0 \Leftrightarrow c \leq \frac{8-2\beta^2}{8+6\beta-\beta^2} \tag{13} \]
\[ q_2^D1 \geq 0 \Leftrightarrow c \leq \frac{4+2\beta-6\beta^2-\beta^3+\beta^4}{6+13\beta+5\beta^2-2\beta^3} \tag{14} \]
\[ q_4^D1 \geq 0 \Leftrightarrow c \leq \frac{8+12\beta-2\beta^2-5\beta^3+\beta^4}{8+18\beta+7\beta^2-4\beta^3} \tag{15} \]
\[ q_2^D2 \geq 0 \Leftrightarrow c \leq \frac{2-2\beta^2}{3+5\beta+\beta^2} \tag{16} \]
\[ q_2^{C1se} \geq 0 \Leftrightarrow c \leq \frac{8-2\beta^2-4\beta^3+\beta^4}{8+4\beta-7\beta^2+\beta^3} \tag{17} \]
\[ q_2^{C1si} \geq 0 \Leftrightarrow c \leq \frac{8-\beta^2}{8+6\beta-\beta^2} \tag{18} \]
\[ q_2^{C2s} \geq 0 \Leftrightarrow c \leq \frac{8-2\beta-3\beta^2}{8+4\beta-7\beta^2+\beta^3} \tag{19} \]
\[ q_2^{C1a} \geq 0 \Leftrightarrow c \leq \frac{512-704\beta^2+216\beta^3+196\beta^4-108\beta^5+14\beta^6}{512+384\beta-736\beta^2-168\beta^3+380\beta^4-120\beta^5+11\beta^6} \tag{20} \]
\[ q_2^{C1a} \geq 0 \Leftrightarrow c \leq \frac{256-192\beta-304\beta^2+288\beta^3-22\beta^4-33\beta^5+7\beta^6}{256+64\beta-384\beta^2+44\beta^3+140\beta^4-56\beta^5+6\beta^6} \tag{21} \]
\[ q_2^{C1a} \geq 0 \Leftrightarrow c \leq \frac{4-2\beta-2\beta^2}{4+2\beta-3\beta^2} \tag{22} \]

As can be seen, the restrictions on \( c \) are only relevant for the output produced by the high cost firms. By means of comparison, we can establish that the most restrictive threshold is given by the inequality in (14). Thus, for plant level output to be non-negative in all possible
industry structures, it must hold that
\[ c \leq \frac{4 + 2\beta - 6\beta^2 - \beta^3 + \beta^4}{6 + 13\beta + 5\beta^2 - 2\beta^3} \equiv \tau. \quad (23) \]

**Proof of Lemma 1.** The Lemma states that industry structures involving no merger or one merger cannot be equilibrium industry structures, because they are always dominated by at least one of the industry structures involving two mergers. The proof is based on the establishment of bilateral dominance relationships between the two-merger structures \( M^{D2}, M^{C2a}, M^{C2s} \) and the remaining no- and one-merger industry structures.

**No merger \((M^0)\):** Suppose \( M^{D2} \) dom \( M^0 \), then it must hold that \((\Pi_2^{D2} + \Pi_0^{D2}) - (\Pi_1^0 + \Pi_0^0 + \Pi_2^0) > 0 \). Substituting the expressions derived in this appendix and simplifying, we obtain the following quadratic function:
\[
\frac{1}{16(1 - \beta) (-6\beta^4 - 7\beta^3 + 22\beta^2 + 28\beta + 8)^2} \phi_0(\beta, c),
\]
where \( \phi_0(\beta, c) = r_0c^2 + s_0c + t_0 \) and
\[
\begin{align*}
 r_0 &= (4\beta^9 + 124\beta^8 + 485\beta^7 - 9\beta^6 - 1324\beta^5 + 276\beta^4 + 3536\beta^3 + 3472\beta^2 + 1344\beta + 192), \\
 s_0 &= (80\beta^8 - 16\beta^9 - 20\beta^7 - 332\beta^6 + 192\beta^5 + 480\beta^4 - 192\beta^3 - 192\beta^2), \quad \text{and} \\
 t_0 &= (16\beta^9 - 80\beta^8 + 20\beta^7 + 332\beta^6 - 192\beta^5 - 480\beta^4 + 192\beta^3 + 192\beta^2). \\
\end{align*}
\]
Note that
\[
\frac{1}{16(1 - \beta) (-6\beta^4 - 7\beta^3 + 22\beta^2 + 28\beta + 8)^2} > 0
\]
for all \( \beta \in (0, 1) \) and \( t_0 > 0 \) for all \( \beta \in (0, 1) \) such that \( \phi_0(\beta, c) > 0 \) when \( c \to 0 \). Inspection of \( s_0 \) and \( r_0 \) reveals that \( s_0 < 0 \) and \( r_0 > 0 \) for all \( \beta \in (0, 1) \). We therefore know that \( \phi_0(\beta, c) \) is a convex function since \( \partial^2 \phi_0 / \partial^2 c = 2t_0 > 0 \). Deriving numerical results for the quadratic equation
\[
\left( \frac{1}{16(1 - \beta) (-6\beta^4 - 7\beta^3 + 22\beta^2 + 28\beta + 8)^2} \right) \phi_0(\beta, c) = 0 \quad (26)
\]
we find that there does not exist a real and feasible root. Since \( t_0 > 0 \) and \( \phi_0(\beta, c) \) is convex, we can conclude that \( M^{D2} \) dom \( M^0 \) for all \( \beta \in (0, 1) \).
One domestic merger \((M^{D1})\)  Suppose that \(M^{D2} \text{ dom } M^{D1}\), i.e. \((\Pi_{12}^{D2} + \Pi_{34}^{D2}) - (\Pi_{12}^{D1} + \Pi_{34}^{D1} + \Pi_{41}^{D1}) > 0\). Substituting and simplifying we obtain

\[
\left( \frac{1}{4 (1 - \beta) (2\beta^7 + 7\beta^6 - 37\beta^5 - 92\beta^4 + 76\beta^3 + 248\beta^2 + 160\beta + 32)^2} \right) \phi_{D1}(\beta, c) > 0 \quad (27)
\]

where \(\phi_{D1}(\beta, c) = r_{D1}c^2 + s_{D1}c + t_{D1}\) and

\[
\begin{align*}
t_{D1} &= \left( \begin{array}{c} 4\beta_{12}^{14} - 58\beta_{12}^{11} + 100\beta_{10}^{10} - 346\beta_{9}^{9} + 1266\beta_{8}^{8} + 912\beta_{7}^{7} \\ -2616\beta_{6}^{6} - 2208\beta_{5}^{5} + 1120\beta_{4}^{4} + 1536\beta_{3}^{3} + 384\beta_{2}^{2} \\
\end{array} \right), \\
s_{D1} &= \left( \begin{array}{c} 58\beta_{12}^{12} - 4\beta_{12}^{11} + 100\beta_{10}^{10} + 346\beta_{9}^{9} - 1266\beta_{8}^{8} - 912\beta_{7}^{7} \\ +2616\beta_{6}^{6} + 2208\beta_{5}^{5} - 1120\beta_{4}^{4} - 1536\beta_{3}^{3} - 384\beta_{2}^{2} \\
\end{array} \right), \quad \text{and} \\
r_{D1} &= \left( \begin{array}{c} 3\beta_{12}^{14} + 44\beta_{12}^{12} - 408\beta_{11}^{11} + 3189\beta_{10}^{10} + 5164\beta_{9}^{9} - 5600\beta_{8}^{8} \\ -13600\beta_{6}^{6} + 5840\beta_{5}^{5} + 31896\beta_{4}^{4} + 32320\beta_{3}^{3} + 15680\beta_{2}^{2} + 3840\beta + 384 \end{array} \right).
\]

As before, we see that \(t_{D1} > 0\) for \(\beta \in (0, 1)\) such that \(\phi_{D1}(\beta, 0) > 0\). Additionally, inspection of \(s_{D1}\) and \(r_{D1}\) reveals that \(s_{D1} < 0\) and \(r_{D1} > 0\) for all \(\beta \in (0, 1)\). Again, we know that \(\phi_{D1}(\beta, c)\) is a convex function since \(\partial^2 \phi_{D1}/\partial^2 c = 2r_{D1} > 0\).

Note that

\[
\frac{1}{4 (1 - \beta) (2\beta^7 + 7\beta^6 - 37\beta^5 - 92\beta^4 + 76\beta^3 + 248\beta^2 + 160\beta + 32)^2} > 0 \quad (28)
\]

for \(\beta \in (0, 1)\). As for the previous case, we can derive a numerical solution for

\[
\left( \frac{1}{4 (1 - \beta) (2\beta^7 + 7\beta^6 - 37\beta^5 - 92\beta^4 + 76\beta^3 + 248\beta^2 + 160\beta + 32)^2} \right) \phi_{1}(\beta, c) = 0 \quad (29)
\]

and obtain the result that there does not exist a real and feasible solution. Therefore, we can conclude that \(M^{D2} \text{ dom } M^{D1}\) for all \(\beta \in (0, 1)\).

One symmetric cross-border merger between the efficient plants \((M^{C1sc})\)  Suppose \(M^{C2s} \text{ dom } M^{C1se}\). Then, it must hold that \((\Pi_{13}^{C2s} + \Pi_{24}^{C2s}) - (\Pi_{13}^{C1sc} + \phi_{3}(\beta, c)\Pi_{21}^{C1si} + \Pi_{4}^{C1si}) > 0\). We can substitute the expressions derived in this appendix, which yields the following quadratic function

\[
\frac{1}{2 \left(16 - 12\beta + \beta^2\right)^2 \left(-2 - 3\beta + \beta^2\right)^2 \left(16 + 16\beta - 29\beta^2 + 6\beta^3\right)^2} \phi_{C1se}(\beta, c) > 0 \quad (30)
\]

where \(\phi_{C1se}(\beta, c) = r_{C1se}c^2 + s_{C1se}c + t_{C1se,}\) and

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We immediately see that

\[ \frac{1}{2 \left( 16 - 12\beta + \beta^2 \right)^2 \left( -2 - 3\beta + \beta^2 \right)^2 \left( 16 + 16\beta - 29\beta^2 + 6\beta^3 \right)^2} \] (31)

is positive. Further, we can establish that \( r_{C1se} > 0 \), \( s_{C1se} < 0 \) and \( t_{C1se} > 0 \) for \( \beta \in (0,1) \) such that \( \phi_2(\beta, c) > 0 \) when \( c \to 0 \). Additionally, we know that \( \phi_2(\beta, c) \) is a convex function since \( \partial^2 \phi_2 / \partial^2 c = 2r_{C1se} > 0 \). Solving the quadratic equation

\[ \frac{1}{2 \left( 16 - 12\beta + \beta^2 \right)^2 \left( -2 - 3\beta + \beta^2 \right)^2 \left( 16 + 16\beta - 29\beta^2 + 6\beta^3 \right)^2} \phi_{C1se}(\beta, c) = 0, \] (32)

we obtain two solutions, namely

\[ c'_{C1se} = \frac{\zeta - 2\sqrt{\eta}}{2\tau} \] (33)

\[ c''_{C1se} = \frac{\zeta + 2\sqrt{\eta}}{2\tau} \] (34)

where

\[ \zeta = 65536 + 278528\beta - 167936\beta^2 - 844288\beta^3 + 610048\beta^4 + 643104\beta^5 - 765856\beta^6 + 111488\beta^7 + 152412\beta^8 - 81442\beta^9 + 15950\beta^{10} - 1218\beta^{11} + 18\beta^{12}, \]

\[ \eta = 1224736768\beta^2 + 4370464768\beta^3 - 632350464\beta^4 - 28918153216\beta^5 + 23871619072\beta^6 + 84199810696\beta^7 - 81075236864\beta^8 - 118825334784\beta^9 + 174103576832\beta^{10} + 39944605696\beta^{11} - 181449165312\beta^{12} + 83007743808\beta^{13} + 50792655952\beta^{14} - 68880716816\beta^{15} + 25543059624\beta^{16} + 2600329872\beta^{17} - 5973909972\beta^{18} + 2729342180\beta^{19} - 693762431\beta^{20} + 109851188\beta^{21} - 10637738\beta^{22} + 535476\beta^{23} - 2303\beta^{24} - 1068\beta^{25} + 36\beta^{26}, \] and
\[
\tau = 32768 + 135168\beta - 49152\beta^2 - 409344\beta^3 + 231552\beta^4 + 432448\beta^5 - 477888\beta^6 - 1040\beta^7 + 250056\beta^8 - 175467\beta^9 + 59801\beta^{10} - 11633\beta^{11} + 1313\beta^{12} - 80\beta^{13} + 2\beta^{14}.
\]

Of these two roots only the first one is feasible. It is sufficient to show that \(\Delta c_{C_{1se}} = c'_{C_{1se}} - \bar{c} > 0\) in order to establish that \(M^{C_{2s}} \text{ dom } M^{1Ce}\) for \(c \in (0, \bar{c})\). It can easily be checked that \(\lim_{\beta \to 0} \Delta c_{C_{1se}} = 1/3\) and \(\lim_{\beta \to 1} \Delta c_{C_{1se}} = 197/514\). Solving \(\Delta c_{C_{1se}} = 0\) and looking for a numerical solution, we find that there exists no real and feasible root. From the previous calculations, we can therefore conclude that \(\Delta c_{C_{1se}} > 0\). Consequently, \(M^{C_{2s}} \text{ dom } M^{1Ce}\) for all \(c \in (0, \bar{c})\).

One asymmetric cross-border merger between the inefficient plants \((M^{C_{1s}})\) Suppose that \(M^{C_{2s}} \text{ dom } M^{C_{1s}}\). Then it must hold that \((\Pi_{13}^{C_{2s}} + \Pi_{24}^{C_{2s}}) - (\Pi_{13}^{C_{1s}} + \Pi_{24}^{C_{1s}} + \Pi_{3}^{C_{1s}}) > 0\). Substituting and simplifying the LHS yields
\[
\frac{1}{2} \left( \frac{\beta}{(6\beta^7 - 119\beta^6 + 751\beta^5 - 1818\beta^4 + 1064\beta^3 + 1344\beta^2 - 896\beta - 512)^2} \right) \phi_{C_{1s}}(\beta, c) > 0 \quad (35)
\]
where \(\phi_{C_{1s}}(\beta, c) = r_{C_{1s}}c^2 + s_{C_{1s}}c + t_{C_{1s}}\) and
\[
\begin{align*}
r_{C_{1s}} &= \left( 2\beta^{14} - 80\beta^{13} + 1277\beta^{12} - 10601\beta^{11} + 49420\beta^{10} - 127072\beta^9 + 152140\beta^8 - 11812\beta^7 - 95232\beta^6 - 56016\beta^5 + 130560\beta^4 + 9984\beta^3 - 36864\beta^2 \right), \\
s_{C_{1s}} &= \left( 54\beta^{12} - 846\beta^{11} + 4812\beta^{10} - 15348\beta^9 + 43420\beta^8 - 89944\beta^7 + 5444\beta^6 + 33824\beta^5 - 408064\beta^4 + 5632\beta^3 + 143360\beta^2 - 8192\beta \right), \quad \text{and} \\
t_{C_{1s}} &= \left( 5569\beta^{10} - 186\beta^{11} - 18\beta^{12} - 33047\beta^9 + 54496\beta^8 + 100716\beta^7 - 383200\beta^6 + 154640\beta^5 + 509056\beta^4 - 424960\beta^3 - 155648\beta^2 + 143360\beta + 32768 \right).
\end{align*}
\]
We see that \(t_{C_{1s}} > 0\) for \(\beta \in (0, 1)\) and that
\[
\frac{1}{2} \left( \frac{\beta}{(6\beta^7 - 119\beta^6 + 751\beta^5 - 1818\beta^4 + 1064\beta^3 + 1344\beta^2 - 896\beta - 512)^2} \right) > 0 \quad (36)
\]
for all \(\beta \in (0, 1)\).

In contrast to the previous cases, the sign of \(s_3\) and \(r_3\) cannot be determined unambiguously. Deriving a numerical solution for the quadratic equation
\[
\frac{1}{2} \left( \frac{\beta}{(6\beta^7 - 119\beta^6 + 751\beta^5 - 1818\beta^4 + 1064\beta^3 + 1344\beta^2 - 896\beta - 512)^2} \right) \phi_{C_{1s}}(\beta, c) = 0 \quad (37)
\]
we obtain no real and feasible solution. Since \(t_{C_{1s}}\) is positive, we can conclude that \(M^{C_{2s}} \text{ dom } M^{C_{1s}}\).

One asymmetric cross-border merger \((M^{C_{1a}})\) Suppose \(M^{C_{2s}} \text{ dom } M^{C_{1a}}\). Then it must hold that \((\Pi_{13}^{C_{2s}} + \Pi_{24}^{C_{2s}}) - (\Pi_{14}^{C_{1a}} + \Pi_{2}^{C_{1a}} + \Pi_{3}^{C_{1a}}) > 0\). Substituting the expressions derived in this Appendix and simplifying we obtain the following expression:

\[
\tau = 32768 + 135168\beta - 49152\beta^2 - 409344\beta^3 + 231552\beta^4 + 432448\beta^5 - 477888\beta^6 - 1040\beta^7 + 250056\beta^8 - 175467\beta^9 + 59801\beta^{10} - 11633\beta^{11} + 1313\beta^{12} - 80\beta^{13} + 2\beta^{14}.
\]
where $C_1a(c) = rC_1ac^2 + sC_1ac + tC_1a$ and

$$\lambda = \frac{\beta}{(\beta - 1)\Phi^2}$$

where

$$\Phi = -42\beta^{10} + 989\beta^9 - 8399\beta^8 + 33012\beta^7 - 56916\beta^6 +$$
$$8768\beta^5 + 90880\beta^4 - 64512\beta^3 - 49152\beta^2 + 32768\beta + 16384,$$

and

$$rC_1a = \begin{pmatrix}
196\beta^{21} - 9492\beta^{20} + 198930\beta^{19} - 249424\beta^{18} + 18666864\beta^{17} - 97241491\beta^{16} + \\
340342617\beta^{15} - 756937592\beta^{14} + 832319840\beta^{13} + 447909760\beta^{12} - 2781569024\beta^{11} + \\
2892334016\beta^{10} + 1397113664\beta^9 - 4997983232\beta^8 + 1711283200\beta^7 + 3569582080\beta^6 - \\
2469265408\beta^5 - 1391460352\beta^4 + 1237581824\beta^3 + 356515840\beta^2 - 239075328\beta - 67108864
\end{pmatrix}$$

$$sC_1a = \begin{pmatrix}
1764\beta^{19} + 3360\beta^{18} - 657926\beta^{17} + 8239928\beta^{16} - 45808390\beta^{15} + 119173072\beta^{14} - \\
35799520\beta^{13} - 612564896\beta^{12} + 1424402432\beta^{11} - 329771264\beta^{10} - 3074486144\beta^9 + \\
3738042368\beta^8 + 1438300160\beta^7 - 5019041792\beta^6 + 1223688192\beta^5 + 2652635136\beta^4 - \\
1209532416\beta^3 - 603979776\beta^2 + 260046848\beta + 67108864
\end{pmatrix}$$

$$tC_1a = \begin{pmatrix}
657926\beta^{17} - 3360\beta^{18} - 1764\beta^{19} - 8239928\beta^{16} + 45808390\beta^{15} - 119173072\beta^{14} + \\
35799520\beta^{13} + 612564896\beta^{12} - 1424402432\beta^{11} + 329771264\beta^{10} + 3074486144\beta^9 - \\
3738042368\beta^8 - 1438300160\beta^7 + 5019041792\beta^6 - 1223688192\beta^5 - \\
2652635136\beta^4 + 1209532416\beta^3 + 603979776\beta^2 - 260046848\beta - 67108864
\end{pmatrix}$$

It can be verified that $\lambda < 0$ for $\beta \in (0, 1)$. Further, we have that $tC_1a < 0$, such that $\lambda \phi_{C1a}(\beta, c) > 0$ when $c \to 0$, and $sC_1a > 0$. The sign of $rC_1a$ cannot be determined unambiguously, but derivation of a numerical solution reveals that $rC_1a < 0$ if $\beta \in (0, 0.953785)$. Solving

$$\frac{1}{4}\lambda \phi_4(\beta, c) = 0,$$

we obtain two roots

$$c'_{C1a} = \frac{\alpha^2\omega - \sqrt{\rho - \varphi}}{\mu}$$

$$c''_{C1a} = \frac{\alpha^2\omega + \sqrt{\rho + \varphi}}{\mu}$$

where

$$\alpha = (-2(4 - 3\beta)^2(-2 + \beta)^2(-1 + \beta)(-16 + \beta(-12 + 7\beta))$$
\[ \omega = (-2048 + \beta(-12032 + \beta(-7168 + \beta(22576 + \beta(6080 \\
+ \beta(-13244 + \beta(664 + \beta(2151 + \beta(-553 + 2\beta(13 + \beta)))))))))) \\
+ \beta(-13244 + \beta(664 + \beta(2151 + \beta(-553 + 2\beta(13 + \beta)))))))))))) \\
\]

\[ \rho = 4(4 - 3\beta)^4(-2 + \beta)^4(-1 + \beta)^2(-16 + \beta(-12 + 7\beta))^4 \\
(-2048 + \beta(-12032 + \beta(-7168 + \beta(22576 + \beta(6080 \\
+ \beta(-13244 + \beta(664 + \beta(2151 + \beta(-553 + 2\beta(13 + \beta)))))))))) \\
\]

\[ \varphi = 8(4 - 3\beta)^2(-2 + \beta)^2(-1 + \beta)(-16 + \beta(-12 + 7\beta))^2 \\
(-2048 + \beta(-12032 + \beta(-7168 + \beta(22576 + \beta(6080 + \\
\beta(-13244 + \beta(664 + \beta(2151 + \beta(-553 + 2\beta(13 + \beta)))))))))) \\
(-67108864 + \beta(-239075328 + \beta(356515840 + \beta(1237581824 + \\
\beta(-1391460352 + \beta(-2469265408 + \beta(3569582080 + \beta(1711283200 + \\
\beta(-4997983232 + \beta(1397113664 + \beta(2892334016 + \beta(-2781569024 + \\
\beta(447909760 + \beta(832319840 + \beta(-756937592 + \beta(340342617 + \\
\beta(-97241491 + 2\beta(9333432 + \beta(-1202124 + \\
\beta(99465 + 14\beta(-339 + 7\beta))))))))))))))))))))))) \\
\]

\[ \mu = (2(-67108864 + \beta(-239075328 + \beta(356515840 + \beta(1237581824 + \\
\beta(-1391460352 + \beta(-2469265408 + \beta(3569582080 + \beta(1711283200 + \\
\beta(-4997983232 + \beta(1397113664 + \beta(2892334016 + \beta(-2781569024 + \\
\beta(447909760 + \beta(832319840 + \beta(-756937592 + \beta(340342617 + \\
\beta(-97241491 + 2\beta(9333432 + \beta(-1202124 + \beta(99465 + \\
14\beta(-339 + 7\beta)))))))))))))))))))) \\
\]

of which only \( c''_{C1a} \) is feasible and only defined on the interval \((0.953785, 1)\). Since \( \frac{1}{4} \lambda \phi_{C1a}(\beta, c) > 0 \) when \( c \to 0 \), it must be that \( \frac{1}{4} \lambda \phi_{C1a}(\beta, c) \) is positive for all \( c < c''_{C1a} \). To establish that \( M^{C2s} \text{ dom } M^{C1a} \) for \( \beta \in (0, 1) \) and \( c \in (0, \bar{c}] \), it is therefore sufficient to show that \( \Delta c_{C1a} = c''_{C1a} - \bar{c} > 0 \) for \( \beta \in (0.953785, 1) \). Inspection of \( \Delta c_{C1a} \) reveals that \( \lim_{\beta \to 0.953785} \Delta c_{C1a} = 0.980751 \) and \( \lim_{\beta \to 1} \Delta c_{C1a} = 0 \). Looking for a numerical solution to \( \Delta c_{C1a} = 0 \), we find no real and feasible solution. From the previous considerations, we can conclude that \( \Delta c_{C1a} > 0 \), which shows that \( M^{C2s} \text{ dom } M^{C1a} \).

Within the range of valid parameter values, i.e. \( \beta \in (0, 1) \) and \( c \in (0, \bar{c}] \) each of the industry structures \( M^{D1}, M^0, M^{C1se}, M^{C1si} \) and \( M^{C1a} \) is dominated by at least one of the two-merger structures. Thus, neither of the five structures is a candidate for an equilibrium industry structure, as only undominated structures are in the core.
Proof of Proposition 1. Let us first consider Part (a) of the Proposition. The ranking presented is unique. It is sufficient to show that \( w_{E}^{C2a} - w_{E}^{C2s} = w_{I}^{C2s} - w_{I}^{C2a} = \frac{(2-\beta)e}{(4-\beta)(4-3\beta)} \), and \( w_{E}^{C2s} - w_{I}^{C2s} = \frac{2(1-\beta)e}{4-3\beta} \) are positive. Obviously, this is the case for \( \beta \in (0,1) \) and \( e \in (0,\bar{e}] \).

Now, we turn to Part (b) of the Proposition.

**Area C:** \( w^{D2} > w_{E}^{C2a} > w_{E}^{C2s} > w_{I}^{C2s} > w_{I}^{C2a} \) We begin with an analysis of Area C and consider when \( w^{D2} > w_{E}^{C2a} \). Substituting the wage expressions and simplifying the inequality reduces to

\[
\frac{\left( -2\beta^2 - 3\beta - 4 \right) c + (4\beta^2 + 2\beta)}{2(4 - \beta)(2 + \beta)} > 0.
\]

As the denominator of the previous expression is clearly positive for \( \beta \in (0,1) \), we only need to consider the sign of the numerator which is linear in \( e \). We see immediately that \( -2\beta^2 - 3\beta - 4 \) is negative for \( \beta \in (0,1) \). Therefore, there must exist a root \( e' \) such that

\[
\left( -2\beta^2 - 3\beta - 4 \right) e' + (4\beta^2 + 2\beta) = 0.
\]

Solving the previous equation, the unique solution is given by

\[
e' = \frac{4\beta^2 + 2\beta}{2\beta^2 + 3\beta + 4}.
\]

Next, we need to consider when \( e' < \bar{e} \). Using the expression for \( \bar{e} \), this inequality reduces to

\[
\frac{(2\beta^6 + 9\beta^5 - 27\beta^4 - 80\beta^3 - 60\beta^2 + 8\beta + 16)}{\left( -4\beta^5 + 4\beta^4 + 33\beta^2 + 71\beta^2 + 70\beta + 24 \right)} > 0
\]

We find that the denominator is again positive for \( \beta \in (0,1) \). The sign of the numerator cannot be determined unambiguously, however a numerical solution reveals that \( (2\beta^6 + 9\beta^5 - 27\beta^4 - 80\beta^3 - 60\beta^2 + 8\beta + 16) \) is positive for \( \beta \in (0,0.442426) \). Thus, for any \( \beta \in (0.442426,1) \) it must hold that \( c < \bar{e} \) for \( w^{D2} > w_{E}^{C2a} \). Hence, for any \( c < \min \{e', \bar{e}\} \) and \( \beta \in (0,1) \), \( w^{D2} > w_{E}^{C2a} \).

**Area B:** \( w_{E}^{C2a} > w^{D2} > w_{E}^{C2s} > w_{I}^{C2s} > w_{I}^{C2a} \) From the previous analysis we can conclude that \( w_{E}^{C2a} > w^{D2} \) whenever \( e' < c \leq \bar{e} \). Therefore, we turn to the case when \( w^{D2} > w_{E}^{C2s} \) holds. Substituting the expressions for wages and simplifying we obtain

\[
\frac{8\beta + 10\beta^2 - 12\beta^3 - 16c + 8\beta c + \beta^2 c + 4\beta^3 c}{2(2 + \beta)(16 - 16\beta + 3\beta^2)} > 0
\]

As the denominator is clearly positive, we need to show when the sign of the numerator is positive. It can be rewritten as

\[
(4\beta^3 + \beta^2 - 8) c + (10\beta^2 - 12\beta^3 + 8\beta)
\]
which is a linear function in $c$. Note that $(10eta^2 - 12\beta^3 + 8\beta) > 0$ and $(4\beta^3 + \beta^2 - 8) < 0$ for all $\beta \in (0, 1)$, therefore there must exist a root $e''$ such that

$$(4\beta^3 + \beta^2 - 8) e'' + (10\beta^2 - 12\beta^3 + 8\beta) = 0. \quad (48)$$

The solution is given by

$$e'' = -\frac{(10\beta^2 - 12\beta^3 + 8\beta)}{(4\beta^3 + \beta^2 - 8)}. \quad (49)$$

To demonstrate that this root is feasible, we need to show that $e'' < \bar{c}$. It must then hold that

$$\frac{4\beta^3 + 12\beta^2 - 81 - 32\beta - 32}{8\beta^3 + 18\beta^2 + 57\beta^3 + 53\beta^2 - 34\beta^2 - 104\beta - 48} > 0 \quad (50)$$

Note that the denominator is strictly negative for $\beta \in (0, 1)$. The sign of the numerator cannot be determined unambiguously. A numerical solution provides that $4\beta^7 + 21\beta^6 - 105\beta^5 - 128\beta^4 + 124\beta^3 + 216\beta^2 + 32\beta - 32 < 0$ for $\beta \in (0, 0.304974)$. Thus, $e'' < \bar{c}$ for $\beta \in (0, 0.304974)$. Hence, for $c < c''$ and $\beta \in (0, 0.304974)$ or $c \leq \bar{c}$ and $\beta \in (0.304974, 1)$, which is equivalent to $c < \min\{c'', \bar{c}\}$ and $\beta \in (0, 1)$ it holds that $w_E^{C_{2a}} > w_I^{D_2}$.

**Area A:** $w_E^{C_{2a}} > w_E^{C_{2s}} > w_I^{D_2} > w_I^{C_{2s}} > w_I^{C_{2a}}$ Finally, the relevant wage comparisons are $w_E^{C_{2s}} - w_I^{D_2} > 0$ and $w_I^{D_2} - w_I^{C_{2s}} > 0$. The first threshold has been determined above. Consider therefore $w_I^{D_2} - w_I^{C_{2s}} > 0$. The expression reduces to

$$\frac{(8\beta + 16c - 11\beta^2 c + 8\beta^3 c - 16\beta c + 10\beta^2 - 12\beta^3)}{2(\beta - 4)(\beta + 2)(3\beta - 4)} > 0. \quad (51)$$

Since the denominator is clearly positive for $\beta \in (0, 1)$ we need to establish when $8\beta + 16c - 11\beta^2 c + 8\beta^3 c - 16\beta c + 10\beta^2 - 12\beta^3 > 0$. Rewriting the LHS expression yields $(8\beta^3 - 11\beta^2 - 16\beta + 16)c + (10\beta^2 - 12\beta^3 + 8\beta)$. We can immediately see that $(10\beta^2 - 12\beta^3 + 8\beta) > 0$ for $\beta \in (0, 1)$. The sign of $(8\beta^3 - 11\beta^2 - 16\beta + 16)$ cannot be determined unambiguously, but a numerical solution reveals that $(8\beta^3 - 11\beta^2 - 16\beta + 16) > 0$ for $\beta \in (0, 0.814126)$. Thus, we have that $w_I^{D_2} - w_I^{C_{2s}} > 0$ for sure if $\beta \in (0, 0.814126)$. It remains to check whether $(8\beta^3 - 11\beta^2 - 16\beta + 16)c + (10\beta^2 - 12\beta^3 + 8\beta)$ has a root $e'''$ for $\beta \in (0.814126, 1)$ such that $(8\beta^3 - 11\beta^2 - 16\beta + 16)e''' + (10\beta^2 - 12\beta^3 + 8\beta) = 0$. The solution is given by

$$e''' = -\frac{(10\beta^2 - 12\beta^3 + 8\beta)}{(8\beta^3 - 11\beta^2 - 16\beta + 16)}. \quad (52)$$

It is now sufficient to show that $e''' > \bar{c}$ for $\beta \in (0.814126, 1)$. Using $\bar{c}$ and simplifying the expression, we obtain

$$\frac{(8\beta^7 + 5\beta^6 - 133\beta^5 - 8\beta^4 + 18\beta^3 - 8\beta^2 + 16\beta + 64)}{16\beta^6 - 62\beta^5 - 81\beta^4 + 207\beta^3 + 194\beta^2 - 112\beta - 96} > 0. \quad (53)$$
Note that the denominator is strictly positive for \( \beta \in (0.814126, 1) \), and
\[
(8\beta^7 + 5\beta^6 - 133\beta^5 - 8\beta^4 + 188\beta^3 - 8\beta^2 + 16\beta + 64) > 0
\]
for \( \beta \in (0, 1) \). We can therefore conclude that \( \sigma'' \beta^2 > \bar{\sigma} \) for \( \beta \in (0.814126, 1) \). Consequently, \( w'^D - w'^{C*} > 0 \) for all \( \beta \in (0, 1) \) and \( c \in (0, \bar{c}] \).

**Proof of Lemma 2.** The proof of Lemma 2 is straightforward. For Part (a), the inequality \( Q^{C*} = Q^D > 0 \) reduces to \( \frac{\beta(c-2)}{(-4+3\beta(2+3\beta)} > 0 \), which holds -given the restrictions on parameters- for all \( \beta \in (0, 1) \) and \( c \in (0, \bar{c}] \). Concerning Part (b), we need to inspect \( Q^D - Q^{C*} \) which is given by \( \frac{-\beta(2+\beta)(-1+2\beta)(-2+c)}{2(4+\beta)(1+2\beta)(2+3\beta)} \). Note that the denominator of the previous expression is always negative. Thus, the sign of the above expression hinges upon the term \( (-1 + 2\beta) \). Obviously, the whole expression changes sign at \( \beta = 1/2 \). More specifically, the difference \( Q^D - Q^{C*} \) is positive for \( \beta < 1/2 \) and negative for \( \beta > 1/2 \).

**Proof of Proposition 2.** From Lemma 1, we know that the only candidates for equilibrium industry structures are those structures involving two mergers. In order to determine the equilibrium industry structures, we need to compare bilaterally the profits of the decisive owners in each of the two-merger industry structures against those of the other two structures. Only when a structure dominates both other structures, it will be the equilibrium outcome of the merger formation process. We say that in these instances, an industry structure is in the core. We consider the comparison of all merger structures sequentially.

**Equilibrium M^D: For two domestic mergers (M^D) to be an equilibrium, it needs to hold that M^D^2 dom M^C^2 and M^D^2 dom M^C^3, i.e. \( \Pi^D_{12} + \Pi^D_{34} - (\Pi^C_{12} + \Pi^C_{34}) > 0 \) and \( \Pi^D_{12} + \Pi^D_{34} - (\Pi^C_{12} + \Pi^C_{34}) > 0 \). Consider first M^D^2 dom M^C^2: \( \Pi^D_{12} + \Pi^D_{34} - (\Pi^C_{12} + \Pi^C_{34}) > 0 \). Substituting the results for profits derived in the Appendix yields an expression which is quadratic in \( c \):
\[
\frac{1}{4 (-2\beta^2 + \beta + 1)} (3\beta^3 - 10\beta^2 - 16\beta + 32)^2 \delta_1(c, \beta) > 0,
\]
where \( \delta_1(c, \beta) = r_1c^2 + s_1c + t_1 \) and
\[
\begin{align*}
t_1 &= (252\beta^6 - 384\beta^5 - 572\beta^4 + 896\beta^3 + 320\beta^2 - 512\beta), \\
s_1 &= (384\beta^5 - 252\beta^6 + 572\beta^4 - 896\beta^3 - 320\beta^2 + 512\beta), \text{ and} \\
r_1 &= (2\beta^9 - 15\beta^8 + 24\beta^7 + 103\beta^6 - 316\beta^5 + 381\beta^4 + 512\beta^3 - 1728\beta^2 + 512\beta + 768).
\end{align*}
\]
We see that
\[
\frac{1}{4 (-2\beta^2 + \beta + 1)} (3\beta^3 - 10\beta^2 - 16\beta + 32)^2 > 0
\]
for all $\beta \in (0, 1)$. Additionally, we can unambiguously determine the signs of the rest of the terms. We have that $t_1 < 0$ for all $\beta \in (0, 1)$ such that $\delta_1(\beta, c) < 0$ when $c \to 0$. Further, $s_1 > 0$ and $r_1 > 0$ for $\beta \in (0, 1)$.

The existence of a unique critical value $c^* > 0$ such that $\delta_1(c^*, \beta) = 0$ while $\delta_1(c, \beta) > 0$ for all $c > c^*$ follows from noting that $\partial^2 \delta_1 / \partial^2 c = 2r_1 > 0$. Solving the quadratic equation

$$\frac{1}{4} \left( -2\beta^2 + \beta + 1 \right) (3\beta^3 - 10\beta^2 - 16\beta + 32)^2 \delta_1(c, \beta) = 0$$

yields two real roots, namely

$$c'(\beta) = 2 (3\beta - 4) \left( \frac{32\beta^2 - 2\beta \sqrt{96\beta + 116\beta^2 - 292\beta^3 - 152\beta^4 + 280\beta^5 + 3\beta^6 - 70\beta^7 + 33\beta^8 - 14\beta^9}}{512\beta - 1728\beta^2 + 512\beta^3 + 381\beta^4 - 316\beta^5 + 103\beta^6 + 24\beta^7 - 15\beta^8 + 2\beta^9 + 768} \right)$$

$$c''(\beta) = 2 (3\beta - 4) \left( \frac{32\beta^2 + 2\beta \sqrt{96\beta + 116\beta^2 - 292\beta^3 - 152\beta^4 + 280\beta^5 + 3\beta^6 - 70\beta^7 + 33\beta^8 - 14\beta^9}}{512\beta - 1728\beta^2 + 512\beta^3 + 381\beta^4 - 316\beta^5 + 103\beta^6 + 24\beta^7 - 15\beta^8 + 2\beta^9 + 768} \right)$$

Of these two roots, only one is feasible such that the unique solution is $c^* = c''$. We need to show that $\Delta_{c_1} = \tau - c^* > 0$. We can easily check that $\lim_{\beta \to 0} \Delta_{c_1} = 2/3$ and $\lim_{\beta \to 1} \Delta_{c_1} = 0$. Since $\partial^2 \Delta_{c_1} / \partial^2 \beta > 0$ for $\beta \in (0, 1)$, we know that $\Delta_{c_1}$ is a convex function and that there must exist some critical value of $\beta$ such that $\Delta_{c_1} = 0$. Looking for a numerical solution, we find that $\Delta_{c_1} = 0$ for $\beta = 0.39597$. Thus, for all $\beta < 0.39597$, $c^* < \tau$. Hence, we know that there exists a feasible critical value $c^*$ such that $M_{D2} \text{ dom } M_{C2a}$ whenever $c^* < c \leq \tau$.

Next we need to check $M_{D2} \text{ dom } M_{C2a}$, i.e., $\Pi_{12}^{D2} + \Pi_{34}^{D2} - (\Pi_{14}^{C2a} + \Pi_{23}^{C2a}) > 0$. We can use the expressions derived in the Appendix, so that we obtain for the LHS of the inequality

$$\frac{1}{4} \left( -\beta^2 + 2\beta + 8 \right)^2 \left( -2\beta^2 + \beta + 1 \right) \delta_2(c, \beta) > 0,$$

where $\delta_2(c, \beta) = r_2 c^2 + s_2 c + t_2$ and

$$r_2 = (53\beta^2 - 4\beta^3 - \beta^4 + 120\beta + 48),$$

$$s_2 = (28\beta^2 - 32\beta^3 - 28\beta^4 + 32\beta),$$

$$t_2 = (28\beta^2 - 32\beta^3 - 28\beta^4 + 32\beta).$$

We can easily see that

$$\frac{1}{4} \left( -\beta^2 + 2\beta + 8 \right)^2 \left( -2\beta^2 + \beta + 1 \right) > 0,$$

(61)
for $\beta \in (0, 1)$. By the same reasoning as above, we can establish that $t_2 < 0$ for all $\beta \in (0, 1)$ such that $\delta_2(\beta, c) < 0$ when $c \to 0$, while $s_2 > 0$ and $r_2 > 0$ for $\beta \in (0, 1)$. Thus it must be that $\delta_2(c, \beta)$ is a convex function since $\partial^2 \delta_2(c, \beta)/\partial^2 c = 2r_2 > 0$ and there must exist a critical value $\tilde{c}$ such that $\delta_2(\tilde{c}, \beta) = 0$ and $\delta_2(\tilde{c}, \beta) > 0$ for $c > \tilde{c}$. Solving $\delta_2(c, \beta) = 0$ we obtain two real solutions

$$c'' = \frac{-16\beta + 8\sqrt{\beta} (2\beta + 1) (7\beta + 8) (\beta - 1) (\beta + 1) (\beta - 3)}{120\beta + 53\beta^2 - 4\beta^3 - \beta^4 + 48}$$

and

$$c''' = \frac{16\beta + 8\sqrt{\beta} (2\beta + 1) (7\beta + 8) (\beta - 1) (\beta + 1) (\beta - 3)}{120\beta + 53\beta^2 - 4\beta^3 - \beta^4 + 48}$$

of which only the second one is feasible such that $\tilde{c} = c'''$. It remains show now that $\tilde{c} < \tau$ for at least some values of $\beta$. Thus, substituting and simplifying yields the expression $\Delta c_2 = \tau - \tilde{c}$, which is convex in $\beta$. We can easily check that $\lim_{\beta \to 0} \Delta c_2 = 2/3$ and $\lim_{\beta \to 1} \Delta c_2 = 0$, so that there must exist some critical value of $\beta$ such that $\Delta c_2 > 0$. Deriving a numerical solution for $\Delta c_2 = 0$, we find the only real and feasible solution to be $\beta = 0.413423$, such that $\tilde{c} < \tau$ whenever $\beta < 0.413423$. Hence, we know that there exists a feasible critical value $\tilde{c}$ such that $M^{D2} dom M^{C2a}$ whenever $\tilde{c} < c \leq \tau$.

Finally, we need to show that $M^{D2} dom M^{C2s}$ and $M^{D2} dom M^{C2a}$ at the same time. To this aim, we compare $c^*$ to $\tilde{c}$ in the range $\beta \in (0, 0.395977)$. To this aim, we inspect the expression

$$\Delta c_3 = c^* - \tilde{c}.$$  

Mathematical manipulation reveals that $\Delta c_3(\beta)$ is always positive for $\beta \in (0, 0.395977)$. Consequently, for $c > c^*$, $M^{D2} dom M^{C2s}$ and $M^{D2} dom M^{C2a}$.

**Equilibrium $M^{C2s}$**: For two symmetric cross-border mergers to be the equilibrium result of the merger formation process, we need to determine when $\Pi_{13}^{C2s} + \Pi_{24}^{C2s} - (\Pi_{12}^{D2} + \Pi_{34}^{D2}) > 0$ and $\Pi_{13}^{C2s} + \Pi_{24}^{C2s} - (\Pi_{14}^{C2a} + \Pi_{23}^{C2a}) > 0$.

$M^{C2s} dom M^{D2}$: From the previous case we can deduce that for $c < c^*(\beta)$, it holds that $M^{C2s} dom M^{D2}$.

$M^{C2s} dom M^{C2a}$: To establish when two symmetric cross-border mergers dominate the two asymmetric cross border mergers, it must hold that $\Pi_{13}^{C2s} + \Pi_{24}^{C2s} - (\Pi_{14}^{C2a} + \Pi_{23}^{C2a}) > 0$. Substitution of the expressions derived in the Appendix and simplification yields

$$r_4c^2 > 0,$$
where
\[ r_4 = \left( -\frac{1}{4} \frac{\beta}{\beta - 1} \frac{(\beta - 2)^2}{3\beta^2 - 16\beta + 16} \right)^2 \left( 8\beta^2 - 3\beta - 24\beta + 16 \right). \]

Inspection of this term yields that \( \lim_{\beta \to 0} r_4 = 0 \). The sign of \( r_4 \) cannot be determined unambiguously. A numerical solution yields that \( r_4 > 0 \) for \( \beta \in (0, 0.91262) \) and \( r_4 < 0 \) otherwise. Therefore, we can immediately conclude that \( M^{C2a} \) dom \( M^{C2a} \) if \( \beta \in (0, 0.91262) \). As there is no restriction on \( c \), it must only hold that \( c \leq \overline{c} \).

**Equilibrium \( M^{C2a} \):** Finally, we can establish when two asymmetric cross-border mergers will result in equilibrium. This is the case whenever \( \Pi^{C2a}_{14} + \Pi^{C2a}_{23} - (\Pi^{D2}_{12} + \Pi^{D2}_{34}) > 0 \) and \( \Pi^{C2a}_{14} + \Pi^{C2a}_{23} - (\Pi^{C2a}_{13} + \Pi^{C2a}_{24}) > 0 \).

The proof for \( M^{C2a} \) dom \( M^{D2} \) and \( M^{C2a} \) dom \( M^{C2s} \) follows from the solutions above. Most simply, we know from the prior case that \( M^{C2a} \) dom \( M^{C2s} \) whenever \( \beta > 0.91262 \) (otherwise, \( M^{C2s} \) dom \( M^{C2a} \)). Further, we know from above considerations that there exists a threshold \( c < \overline{c}(\beta) < \overline{c}(\beta) \) such that \( M^{C2a} \) dom \( M^{D2} \).

However, we know that \( \tilde{c} < \overline{c} \) only for \( \beta \in (0, 0.413423) \). Thus, for \( \beta \in (0.91262, 1) \) it must be that \( c < \overline{c} \) in order that \( M^{C2a} \) dom \( M^{D2} \).

**Proof of Proposition 3.** To prove Proposition 4, we need to calculate the global welfare resulting in each of the industry structures analyze. We define global welfare as the sum of consumer surplus, firm profits and union wage bills. More specifically, denote global welfare in structure \( M^I \) to be
\[
W^I = V^I - \sum_{i=1}^{4} (p_i^I q_i^I) + \sum_{i=1}^{4} \Pi_i^I + U_\beta^I + U_\alpha^I,
\]
where \( V^I \) denotes the utility function of a representative consumer in industry structure \( M^I \) and is given by
\[
\frac{4}{\sum_{i=1}^{4} q_i^I - \frac{1}{2}} \left( \sum_{i=1}^{4} q_i^2 + \beta \sum_{i=1}^{4} \sum_{j=1}^{4} q_i q_j \right) + z,
\]
where \( z \) denotes the outside numeraire good and \( i, j = 1, 2, 3, 4; i \neq j \). The expressions for global welfare in all merger structures are given by
\[
W^0 = -\frac{14\beta^2(-2 + c)^2 + \beta^3(-2 + c)^2 + 112(2 + (-2 + c)c + 16\beta (-1 + c + 5c^2)}{64(-2 + \beta)(2 + 3\beta)} + z
\]
\[
W^{D1} = -\frac{1}{8(-2 + \beta)(2 + \beta - 4\beta^2 + \beta^3)} \left( 4 + \beta(6 + \beta) \right)^2 \xi_D(\beta, c) + z
\]
where $\xi_{D1} = d_{D1}c^2 + f_{D1}c + g_{D1}$, and

$$d_{D1} = (528 + \beta(1776 + \beta(380 + \beta(-3648 + \beta(-2202 + 
\beta(2176 + \beta(1072 + \beta(-567 + \beta(-13 + \beta(13 + \beta))))))))),$$

$$f_{D1} = (-2 + \beta)(-1 + \beta)^2(448 + \beta(2112 + \beta(3320 + \beta(1620 + 
\beta(-434 + \beta(-353 + 2\beta(19 + 5\beta)))))), \text{ and}$$

$$g_{D1} = (-2 + \beta)(-1 + \beta)^2(448 + \beta(2112 + \beta(3320 + 
\beta(1620 + \beta(-434 + \beta(-353 + 2\beta(19 + 5\beta)))))).$$

$$W_{D2} = \frac{1}{16(1 + 2\beta)^2 (-2 + \beta + \beta^2)^2} \xi_{D2}(\beta, c) + z \tag{69}$$

where $\xi_{D2} = d_{D2}c^2 + f_{D2}c + g_{D2}$, and

$$d_{D2} = (6 + \beta(-15 + \beta(2 + \beta)(-41 + 17\beta^2)), \text{ and}$$

$$f_{D2} = -4(-1 + \beta)^2(1 + \beta)(14 + \beta(35 + 13\beta)), \text{ and}$$

$$g_{D2} = 4(-1 + \beta)^2(1 + \beta)(14 + \beta(35 + 13\beta)).$$

$$W_{C1se} = \frac{1}{8(16 + (-12 + \beta)\beta)(-2 + (-3 + \beta)\beta)^2} \xi_{C1se}(\beta, c) + z \tag{70}$$

where $\xi_{C1se} = d_{C1se}c^2 + f_{C1se}c + g_{C1se}$, and

$$d_{C1se} = (3584 - 4\beta(\beta(\beta(\beta(\beta(\beta(5\beta - 88) + 566) - 1432) + 652) + 1864) - 1088)), \text{ and}$$

$$f_{C1se} = (4\beta(\beta - 6)(\beta(\beta(\beta(\beta(\beta(12\beta - 101) + 216) + 136) - 672) + 128) - 7168), \text{ and}$$

$$g_{C1se} = (7168 - \beta(\beta - 6)(\beta(\beta(\beta(\beta(\beta(37\beta - 292) + 500) + 864) - 2672) - 2304)).$$

$$W_{C1si} = \frac{1}{8(16 - 12\beta + \beta^2)^2 (-2 - 3\beta + \beta^2)^2} \xi_{C1si}(\beta, c) + z \tag{71}$$

where $\xi_{C1si} = d_{C1si}c^2 + f_{C1si}c + g_{C1si}$, and

$$d_{C1si} = (174\beta^6 - 9\beta^5 - 1228\beta^6 + 3224\beta^4 + 704\beta^3 - 6848\beta^2 + 3584\beta + 3584), \text{ and}$$

$$f_{C1si} = (26\beta^7 - 336\beta^6 + 1216\beta^5 + 368\beta^4 - 9760\beta^3 + 15424\beta^2 - 1536\beta - 7168), \text{ and}$$

$$g_{C1si} = (514\beta^6 - 37\beta^5 - 2252\beta^5 + 2136\beta^4 + 7856\beta^3 - 16032\beta^2 + 2304\beta + 7168).$$

$$W_{C2s} = \frac{1}{8(4 - 3\beta)^2 (-4 + \beta)^2(1 + 2\beta)^2} \xi_{C2s}(\beta, c) + z \tag{72}$$

where $\xi_{C2s} = d_{C2s}c^2 + f_{C2s}c + g_{C2s}$, and

$$d_{C2s} = (896 + \beta(1344 + \beta(-1992 + \beta(-1108 + \beta(1662 + \beta(-457 + 2\beta(11 + \beta)))))), \text{ and}$$

$$f_{C2s} = 2(4 - 3\beta)^2(2 + \beta)(-28 + \beta(-48 + 19\beta)), \text{ and}$$

$$g_{C2s} = -2(4 - 3\beta)^2(2 + \beta)(-28 + \beta(-48 + 19\beta)).$$

$$W_{C1a} = \frac{1}{8(-2 + \beta)(-1 + \beta)^2(16 + (-12 + \beta)\beta)^2(-2 + (-3 + \beta)\beta)^2(16 + \beta(-12 + 7\beta))^2} \xi_{C1a}(\beta, c) + z \tag{33}$$
where $\xi_{C_1a} = d_{C_1a}c^2 + f_{C_1a}c + g_{C_1a}$, and

\[
d_{C_1a} = (-1835008 + \beta(-393216 + \beta(9183232 + \beta(-2584576 + \\
\beta(-16067072 + \beta(11342592 + \beta(9244896 + \beta(-12799536 + \\
\beta(2339576 + \beta(3528244 + \beta(-2703070 + \beta(894811 + \\
\beta(-160671 + (15311 - 613\beta)\beta)))))))))))
\]

\[
f_{C_1a} = (-2 + \beta)(-1 + \beta)^2(-16 + \beta(-12 + 7\beta))^2 \\
\quad \times (-7168 + \beta(-2304 + (-6 + \beta)\beta(-267 + \\
\quad 2 + \beta(864 + \beta(500 + \beta(-292 + 37\beta)\beta))))),
\]

\[
g_{C_1a} = -(-2 + \beta)(-1 + \beta)^2(-16 + \beta(-12 + 7\beta))^2 \\
\quad \times (-7168 + \beta(-2304 + (-6 + \beta)\beta(-2672 + \\
\quad \beta(864 + \beta(500 + \beta(-292 + 37\beta)\beta))))).
\]

Finally,

\[
W^{C_2a} = \frac{1}{16(1 + 2\beta)^2 (4 - 5\beta + \beta^2)^2} \xi_{C_2a}(\beta, c) + z
\]

where $\xi_{C_2a} = d_{C_2a}c^2 + f_{C_2a}c + g_{C_2a}$, and

\[
d_{C_2a} = (112 + \beta(136 - \beta(308 + \beta(122 + \beta(-220 + 47\beta)\beta)))))
\]

\[
f_{C_2a} = 4(-1 + \beta)^2(2 + \beta)(-28 + \beta(-48 + 19\beta)),
\]

\[
g_{C_2a} = -4(-1 + \beta)^2(2 + \beta)(-28 + \beta(-48 + 19\beta)).
\]

Comparing (69) to (70) to (74), it can be easily confined that $W^{D2}$ is lower than the global welfare in all other industry structures for all $\beta \in (0, 1)$ and $c \in (0, \pi]$.

For the second part of the proposition, we have plotted the regions for each welfare dominant industry structure in Figure 3. We have calculated these boundaries using Mathematica.
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