Colluding through Suppliers*

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Abstract

This article investigates downstream firms’ ability to collude in a repeated game of competition between supply chains. We show that downstream firms with buyer power can collude more easily in the output market if they also collude on their input supply contracts. More specifically, an implicit agreement on input supply contracts with above-cost wholesale prices and negative fixed fees (that is, slotting fees) facilitates collusion on downstream prices. Banning slotting fees or information exchange about wholesale prices decreases the scope for collusion. Moreover, high downstream prices are more difficult to sustain if upstream rather than downstream firms make contract offers.

Keywords. Collusion, vertical contracting, slotting fees, buyer power, information exchange

JEL Classification. D21, D43, L42.

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1 Introduction

Many cartels involve industries in which firms obtain key inputs through bilateral supply contracts. Yet, with a few recent exceptions that focus on collusion between upstream firms when selling to downstream firms, existing theories of collusion ignore supply contract negotiations. In this article, we investigate the strategic design of input supply contracts as a way to facilitate collusion between downstream firms. What features of input supply contracts facilitate collusion? How do bans on certain contentious contractual practices affect the sustainability of collusion? And how do the relative bargaining positions of upstream and downstream firms affect supply contracts and the risk of collusion?

Our analysis contributes to antitrust policy debates about two trends in retailing: first, the major shift in bargaining power from manufacturers to retailers in recent decades, which triggered widespread concerns about the competitive effects of “buyer power” (Inderst and Mazzarotto, 2008); second, the contemporaneous rise of complex contractual arrangements in the retail industry, in particular slotting fees paid by manufacturers to retailers for shelf space access (Federal Trade Commission, 2001 and 2003; Competition Commission, 2000). The existing literature reaches mixed conclusions about the competitive effects of buyer power in conjunction with slotting fees. Some theories predict that strong retailers use contracts with slotting fees to exclude weaker competitors (Marx and Shaffer, 2008), whereas others show that slotting fees can dampen downstream competition (Shaffer, 1991; Miklós-Thal et al., 2011; Rey and Whinston, 2011; Miklós-Thal, 2012). This literature, however, has taken a strictly static view of the interaction between firms. In contrast, we analyze a dynamic game of repeated interaction between upstream and downstream firms.

We consider an industry with $N$ supplier-retailer pairs. The retailers purchase an intermediate good from their respective suppliers, transform it into a homogeneous final good, and compete in prices. In each period, retailers make secret two-part tariff offers to their suppliers. Retailers observe each other’s

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1 The dairy cartel among supermarkets in the United Kingdom (Office of Fair Trading, 2007, 2011) provides a recent example.

2 A recent literature analyzes the effects of various vertical restraints on collusion between upstream firms when selling to downstream firms: Nocke and White (2007, 2010) and Norman (2009) investigate the impact of vertical mergers, Jullien and Rey (2007) show that resale price maintenance facilitates collusion, Piccolo and Reisinger (2010) analyze the effects of exclusive territories, and Schinkel et al. (2008) investigate the impact of banning indirect purchaser lawsuits. The only extant article on wholesale contracting and downstream collusion we are aware of is Doyle and Han (2012). They show that a buyer group, which centrally negotiates input contracts for all downstream firms, can facilitate collusion in the output market. A related literature analyzes the impact of firms’ capital structure on collusion (Maksimovic, 1988; Spagnolo, 2004).
wholesale prices before they set retail prices, either due to mandatory disclosure rules or as the result of voluntary information exchange between retailers. We show that collusion is sustainable for some discount factors strictly below the standard threshold \((N - 1)/N\) in symmetric Bertrand games if and only if retailers can offer tariffs with above-cost wholesale prices and negative fixed fees (that is, slotting fees). Intuitively, the retailers can limit their own incentives to undercut the retail price by agreeing to pay their suppliers high wholesale prices. For any given retail price, a higher wholesale price reduces the downstream margin and thereby decreases the short-term profit gain from undercutting the retail price. Profits along the collusive path, on the other hand, are unaffected by a wholesale price increase as long as the increase is coupled with a fixed fee reduction that fully compensates retailers. By agreeing on a wholesale price as high as the retail price, the retailers could completely eliminate their own incentives to undercut the retail price. However, another possible deviation for each retailer is to offer its supplier a wholesale price below the one agreed upon by the cartel, coupled with a fixed fee that extracts the supplier’s entire (expected) rent. Although such a “contract deviation” is detected before the retailers set final prices, it can be profitable if the deviator gains enough market share thanks to its wholesale price advantage. A higher collusive wholesale price makes contract deviations more attractive, because it weakens the other retailers’ ability to punish the deviator in the output market. The optimal collusive wholesale price that arises from this trade-off between retail price and contract deviation incentives lies between the suppliers’ marginal cost and the collusive retail price. Slotting fees are essential because they enable the retailer cartel to agree on a wholesale price above marginal cost without harming their own profits on the collusive path.

A ban on slotting fees, as in the British groceries industry, hence lowers the risk of collusion between retailers in our model.\(^3\) If bargaining power rests with the suppliers, however, then collusion on any retail price above marginal cost is sustainable if and only if the discount factor is at least \((N - 1)/N\), and a ban on slotting fees has no impact. Intuitively, upstream bargaining power makes collusion more difficult because, unless the downstream margin is zero, industry profits must be shared between upstream and downstream firms to prevent both contract deviations by suppliers and retail price deviations by retailers.

\(^3\)In the United Kingdom, The Groceries Supply Code of Practice, in force since 2010, forbids retailers to require slotting fees from suppliers unless the payment is in relation to a promotion or product introduction. In the United States, slotting fees are not regulated although their competitive effects have been the subject of intense debates (Federal Trade Commission, 2001, 2003).
The supply contract that maximizes the scope for collusion then features no fixed fee and a wholesale price equal to the retail price, which implies a critical discount factor of \((N - 1)/N\). Our theory thus predicts that the incidence of slotting fees is correlated with buyer power, which is consistent with anecdotal evidence collected by the U.S. Federal Trade Commission (2001) and the U.K. Competition Commission (2000). Moreover, if slotting fees are legal, a shift of bargaining power from suppliers to retailers, as witnessed in the grocery industry in recent years, increases the risk of collusion.

Our analysis also has implications for policy debates about disclosure standards for vertical contracts (Arya and Mittendorf, 2011) and information exchange between firms (Kühn, 2001; Swedish Competition Authority, 2006). We show that wholesale price transparency is crucial for firms’ ability to sustain an agreement in the input supply market, which, in turn, facilitates collusion in the output market. Mandatory disclosure of wholesale pricing contracts, as recently advocated in the U.S. health care industry (Hahn et al., 2008; Pauly and Burns, 2008),\(^4\) therefore raises the risk of collusion between downstream firms. Moreover, because downstream firms benefit from supply contract transparency, they have incentives to exchange information even in the absence of mandatory disclosure rules. More specifically, we show that voluntary information exchange can fully replace mandatory disclosure as long as firms are able to share hard information about wholesale prices.\(^5\) The cartel can then credibly agree to punish any participant that refuses to share information, which implies that the set of sustainable retail prices at any discount factor remains the same as with exogenous contract transparency. Although firms collude tacitly on retail prices, communication about accepted supply contracts is thus necessary to sustain collusion at low discount factors, and a ban on information sharing about input prices decreases the risk of collusion between downstream firms.\(^6\)

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\(^4\) Another example of mandatory disclosure is the infamous decision by the Danish competition authority to gather and disseminate data on supply contracts for ready-mix concrete. Wholesale prices increased by 15-20% after the rule was implemented (Albaek et al., 1997), prompting a rescission of the mandate.

\(^5\) To ensure truthful revelation of the current wholesale price, the firms could rely on an independent auditor. Faced with a similar truth-telling problem regarding sales levels, the citric acid cartel, for example, hired an international auditing firm to independently audit sales reports (Connor, 2001). Research that analyzes communication and truth-telling incentives in cartels includes Compte (1998), Kandori and Matsushima (1998), Athey and Bagwell (2001), Athey, Bagwell and Sanchirico (2004), Gerlach (2009), and Harrington and Skrzypacz (2011).

\(^6\) As far as we are aware, the argument that information exchange about input prices can facilitate collusion was first raised by Kühn and Vives (1995) in a discussion of the Wood Pulp case (OJ L85/1, 1985). Although the case itself dealt with collusion between upstream wood pulp producers, the investigation uncovered that downstream paper producers were exchanging detailed information about input prices. Kühn and Vives (1995) speculate that this information exchange “might have been used by paper producers to sustain agreements in the final goods market.” Our theory formalizes this idea.
The main part of our analysis focuses on collusion sustained by infinite “Nash reversion” strategies. Any type of deviation, whether in the retail or in the wholesale market, triggers infinite reversion to the competitive equilibrium with zero profits for all firms from the next period onwards. In addition, contract deviations, which are detected before retailers set retail prices, trigger a within-period punishment in the form of a switch to the competitive Bertrand equilibrium given the current wholesale prices. These strategies are a natural generalization of standard grim-trigger punishments (Friedman, 1971) to the repeated extensive-form game in this article. Yet, as we show, the scope for collusion can be enhanced by means of more complex punishments of contract deviations. Unlike in a repeated normal-form game, however, making the punishment more severe (that is, reducing the deviator’s post-detection continuation profit) does not necessarily make collusion easier. Instead, the retailers need to “collude” on a credible punishment scheme in which the deviator’s market share in the deviation period is so small that the deviator cannot obtain a slotting fee high enough to make the deviation profitable. We construct such punishment strategies and show that collusion can be sustained for an even wider range of discount factors than in the baseline model.

The rest of the article is organized as follows. In Sections 2 and 3, we set out and analyze the baseline model where retailers make contract offers and observe each other’s accepted supply contracts. We show that collusion is sustainable for some discount factors below the standard threshold \((N - 1)/N\), but only if slotting fees and above-cost wholesale prices are feasible. In Section 4, we show that collusion becomes more difficult to sustain if suppliers (rather than retailers) make contract offers. In Section 5, we endogenize the observability of supply contracts by adding a communication stage to the game, and we show that a ban on information exchange hinders collusion. In Section 6, we discuss alternative punishments of contract deviations. Section 7 concludes. All proofs are relegated to the Appendix.

## 2 Baseline model

Consider a vertically related industry with \(N \geq 2\) identical downstream firms (or retailers) \(R_1, R_2, \ldots, R_N\), and \(N\) identical upstream firms (or suppliers) \(S_1, S_2, \ldots, S_N\). The suppliers produce an intermediate good at a constant marginal cost, which for simplicity is normalized to zero. Supplier \(S_i\) and retailer \(R_i\)
are in an exclusive relationship with each other.\footnote{One possible interpretation of the exclusivity assumption is that the suppliers are spin-offs of the retailers and that there are (unmodeled) fixed costs of setting up those units. The results in our baseline model would remain unchanged if each retailer could choose among multiple competing suppliers. Key for our findings is that no retailer can foreclose its competitors' input supplies. Jullien and Rey (2007) and Schinkel et al. (2008) take similar approaches.} The retailers transform the intermediate good into a final good using a one-to-one technology with zero marginal cost of production, and sell it to consumers. Consumers view the final goods offered by different retailers as homogeneous.\footnote{The qualitative insights would remain unchanged if retailers were differentiated.}

The retailers make simultaneous and secret take-it-or-leave-it contract offers to their suppliers. \(R_i\)'s offer to \(S_i\) takes the form of a two-part tariff \(C_i = (F_i, w_i)\), specifying a per unit wholesale price \(w_i\) and a fixed fee \(F_i\). The fixed fee must be paid up-front at the time the offer is accepted. The wholesale price is paid after the retailer sets its downstream price and decides how much input to order. If \(F_i < 0\), we will say that \(R_i\) demands a slotting fee from \(S_i\). All suppliers have an outside option of zero, and we assume that a supplier who is indifferent between accepting and rejecting an offer will accept it.

We consider an infinitely repeated game with discrete time \(\tau = 1, 2, ...\). In each period, aggregate demand for the retailers’ output as a function of price is \(D(p)\). We make the following regularity assumptions:

\textbf{A1} There exists a finite choke price \(\bar{p} > 0\) such that \(D(p) > 0\) if \(p < \bar{p}\) and \(D(p) = 0\) if \(p \geq \bar{p}\). \(D(p)\) is continuous and strictly decreasing on \([0, \bar{p}]\), and twice continuously differentiable on \((0, \bar{p})\).

\textbf{A2} For every \(w \in [0, \bar{p}]\), \(D(p) \cdot (p - w)\) is strictly concave on \([w, \bar{p}]\) and reaches a unique maximum at \(p = p^m(w) \in (w, \bar{p})\).

A standard argument implies that \(p^m(w)\) is increasing in \(w\) for \(w \in [0, \bar{p}]\). To save on notation, the maximizer of \(\phi(p) \equiv D(p) \cdot p\) is also denoted as \(p^m \equiv p^m(0)\).

Given a vector \(p = (p_1, p_2, ..., p_N)\) of downstream prices, the entire market demand \(D(\min \{p\})\) goes to the lowest priced retailer(s). In case of a price tie at the lowest price, the main part of our analysis will use the standard market-sharing rule that total demand is symmetrically shared between the lowest-cost of the lowest-price retailers — i.e., demand is symmetrically shared between all retailers in the set \(\{j : p_j = \min \{p\}\text{ and there is no }k \text{ with } p_k = \min \{p\} \text{ and } w_k < w_j\}\).
$T = 1$ Contracting stage: Retailers make simultaneous and secret contract offers to their suppliers. Suppliers decide whether to accept their offers. If $S_i$ accepts $C_i$, then $F_i$ is paid. Accepted contracts become public knowledge.\(^9\)

$T = 2$ Price-setting stage: Retailers simultaneously set retail prices and consumers decide how much to buy. Retailers order the quantities demanded by consumers from the suppliers at the relevant wholesale prices, and revenues are realized.

The equilibrium concept is subgame-perfect Nash equilibrium. Each firm aims to maximize the discounted sum of its future profits over an infinite horizon, using the common discount factor $\delta \in (0, 1)$. We focus on equilibria that are stationary and symmetric along the equilibrium path: in each period, all retailers offer the contract $C^c = (F^c, w^c)$, set the price $p^c \in [0, p^m]$, and each retailer sells $\frac{1}{N}D(p^c).$\(^10\) Attention is restricted to contracts with $w^c \geq 0$. As supply contracts contain no quantity restrictions, it would be in each retailer’s interest to order an infinite quantity if $w^c < 0$; hence, no supplier would ever accept a supply contract with $w^c < 0$. For simplicity, the main part of the analysis will assume that retailers sustain collusion by means of infinite “Nash reversion”.\(^11\)

- If a retailer deviates to a price different from $p^c$, then all retailers play the competitive equilibrium of the stage game $\mathcal{G}$ in all subsequent periods.
- If a retailer has an accepted wholesale contract different from $C^c$ in period $\tau$,\(^12\) then the retailers play the competitive equilibrium of the price-setting game with asymmetric wholesale prices at $T = 2$ of period $\tau$, and play the competitive equilibrium of the stage game $\mathcal{G}$ in all subsequent periods.

We will say that collusion is sustainable if there is an equilibrium in which $p^c > 0$, and that full collusion is sustainable if there is an equilibrium in which $p^c = p^m$.

\(^9\)It is crucial for our analysis that retailers observe each other’s supply contracts prior to setting prices. In Section 5, we analyze information exchange between retailers as a way to endogenize this contract transparency. Whether suppliers observe the accepted supply contracts of competing vertical chains or not is irrelevant for our results.

\(^10\)Restricting attention to $p^c \leq p^m$ comes without loss of generality. For any $p' > p^m$ there exists a $p'' < p^m$ such that the industry profit $\phi(p'') = \phi(p')$, but there are fewer potentially profitable deviations to lower prices for $p''$ than for $p'$. Hence, $p''$ weakly dominates $p'$ for the retailer cartel.

\(^11\)In Section 6, we will consider alternative punishment strategies.

\(^12\)If a retailer has no accepted wholesale contract at the beginning of $T = 2$, then no punishment is started. However, it is never rational for a retailer to offer a wholesale contract that will be rejected.
3 Equilibrium analysis

In this section, we derive the set of retail prices sustainable by infinite Nash reversion strategies. We begin with the competitive equilibrium of the price-setting subgame of the stage game $G$, which will serve as a within-period punishment for deviations from the collusive wholesale contract. The price-setting subgame of $G$ is a Bertrand game between sellers that potentially have asymmetric marginal costs due to wholesale price asymmetries. The standard competitive equilibrium of this game is well known.\footnote{We call this the “standard” competitive equilibrium because, as discussed in the literature (Deneckere and Kovenock, 1996; Blume, 2003; Miklós-Thal, 2011), a Bertrand game with asymmetric costs typically has a continuum of equilibria. If $w_1 < w_2 < \ldots < w_N \leq p^m$, for instance, then any price $p \in [w_1, w_2]$ can be sustained in a static equilibrium where $p_1 = p_2 = p$ and $R_1$ serves the entire demand. However, equilibria in which $p_i < w_i$ are usually considered unappealing because $R_i$ plays a weakly-dominated strategy that cannot be obtained as a limit of undominated strategies.}

\textbf{Definition 1} Given any vector of wholesale prices $(w_1, w_2, \ldots, w_N)$, denote $w^{\min}$ as the lowest wholesale price and $w^{\sec}$ as the second-lowest wholesale price.\footnote{$w^{\sec} = w^{\min}$ if several retailers have wholesale price $w^{\min}$.} Suppose that $w^{\min} < p$. The competitive equilibrium of the price-setting game at $T = 2$ of $G$ is:

$$p_i = \begin{cases} \min \{w^{\sec}, p^m(w^{\min})\} & \text{if } w_i = w^{\min}, \\ w_i & \text{if } w_i > w^{\min}. \end{cases}$$

- $R_i$’s demand is $D_i(p) = \begin{cases} D(p) & \text{if } p_i = \min \{p\} \text{ and } w_i = w^{\min}, \\ 0 & \text{otherwise.} \end{cases}$

If $R_i$ has a wholesale cost advantage over the other retailer(s) that all pay $w^c \leq p^m$, then the competitive equilibrium of the price-setting subgame is such that all firms charge $w^c$ and $R_i$ serves the entire demand.

The stage game $G$ has multiple subgame-perfect Nash equilibria. We focus on the \textit{symmetric competitive equilibrium} in which, along the equilibrium path, all retailers offer the efficient contract $C^* = (0, 0)$, all suppliers accept this offer, and all retailers charge $p^* = 0$.\footnote{This is the unique symmetric equilibrium outcome of $G$ if we exclude implausible strategies that permit $p_i < w_i$ in the price-setting subgame.} In the price-setting subgame at $T = 2$, retailers play the competitive equilibrium from Definition 1, both on and off the equilibrium path. It is easy to see that no retailer can gain through a unilateral deviation at the contracting stage. As the wholesale price of the other retailer(s) is 0, a deviant retailer’s profit in the competitive equilibrium of
the price-setting subgame is at most 0, regardless of its own wholesale price. Hence, any deviant contract offer that yields a strictly positive profit for the deviator would be rejected by the deviator’s supplier.

Now consider the infinitely repeated game. There are two types of potentially profitable deviations from the equilibrium path for a retailer. First, a retailer could cheat by undercutting $p^c$ after offering the contract $C^c$. Second, a retailer could cheat by offering its retailer a wholesale price below $w^c$ to obtain a marginal cost advantage in the price-setting subgame of the deviation period. As retailers observe each other’s accepted supply contracts, such a “contract deviation” can be punished at the price-setting stage of the period in which it occurs. A “price deviation,” on the other hand, allows the deviator to steal the entire demand at price $p^c$ before other retailers can react.

In view of the fact that we are interested in collusion between retailers, we assume that suppliers play the statically optimal strategy of accepting any contract that promises a non-negative profit in the period in which it is offered. Retailers play the following infinite Nash reversion strategy:

(i) Offer $C^c$ and charge $p^c$ if no deviation (of either type) was detected in any past period and all accepted contracts in the current period are $C^c$;

(ii) Play the symmetric competitive equilibrium of the stage game $\mathcal{G}$ if a deviation (of either type) was detected in any past period;

(iii) Play the competitive equilibrium of the price-setting subgame of period $\tau$ if one of the accepted contracts in period $\tau$ is different from $C^c$.\footnote{A contract deviation is detected only if it is successful in the sense that the supplier accepts the deviant offer. We assume that if a retailer has no accepted contract, which could be due to a deviation by either the retailer or its supplier, then no punishment is started; instead the retailers with accepted contracts charge $p^c$ and share the resulting demand symmetrically. No retailer ever has an incentive to deviate to a contract that will be rejected by its supplier.}

This strategy maximally punishes price deviations by infinite reversion to the symmetric competitive equilibrium with zero profits for all firms. Contract deviations trigger not only reversion to the symmetric competitive equilibrium of the stage game in future periods, but also play of the competitive equilibrium of the price-setting subgame (as described in Definition 1) at $T = 2$ of the deviation period. As all punishments are composed of static equilibria, there are no profitable deviations from any punishment.\footnote{Parts (ii) and (iii) of the strategy are compatible: if all retailers offer the competitive contract $C^* = (0,0)$, then the equilibrium of the price-setting subgame of $\mathcal{G}$ is $p_i = 0$ for all $i$.}
Suppliers have no incentive to deviate from the equilibrium path by rejecting an offer if $C^c$ satisfies the following “participation constraint”:

$$\frac{1}{N} D(p^c) w^c + F^c \geq 0. \quad (1)$$

For retailers, both price and contract deviations from the equilibrium path must be unprofitable. Consider price deviations first. If $w^c < p^c$, the most attractive deviation at the price-setting stage is to marginally undercut $p^c$ to obtain the entire demand; this deviation is unprofitable if and only if the long-term profit loss exceeds the short-term profit gain from deviating:

$$\frac{\delta}{1 - \delta} \left( \frac{1}{N} D(p^c)(p^c - w^c) - F^c \right) \geq \frac{N - 1}{N} D(p^c)(p^c - w^c). \quad (2)$$

If $w^c > p^c$, the most profitable price deviation is to increase price above $p^c$ so as to earn zero instead of negative profits at the price-setting stage; this deviation is unprofitable if and only if:

$$\frac{\delta}{1 - \delta} \left( -\frac{1}{N} D(p^c)(w^c - p^c) - F^c \right) \geq \frac{1}{N} D(p^c)(w^c - p^c). \quad (3)$$

If $w^c = p^c$, no short-term gain can be made by deviating to a price either above or below $p^c$; hence, there is no profitable price deviation for any discount factor as long as retailers earn non-negative profits along the equilibrium path (that is, as long as $(1/N)D(p^c)(p^c - w^c) \geq F^c$).

Second, contract deviations must be unprofitable. Suppose a retailer offers a deviant contract $(\tilde{F}, \tilde{w}) \neq C^c$ and its supplier accepts the offer. Because retailers play the competitive equilibrium (from Definition 1) in the price-setting subgame of the deviation period, the deviator’s total profit in the deviation period is $D(\min \{w^c, p^m(\tilde{w})\})(\min \{w^c, p^m(\tilde{w})\} - \tilde{w}) - \tilde{F}$ if $\tilde{w} < w^c$, and $-\tilde{F}$ otherwise. The deviator’s profit in all future periods is 0, regardless of $\tilde{w}$. The deviant offer $(\tilde{F}, \tilde{w}) = (0, 0)$ therefore maximizes the deviator’s profit subject to the supplier’s contract acceptance constraint,\(^{18}\) and there is

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\(^{18}\) If $w^c > 0$, the deviator obviously wants to offer a wholesale price $\tilde{w} < w^c$ and a fixed fee $\tilde{F} = -D(\min \{w^c, p^m(\tilde{w})\})\tilde{w}$ that extracts the entire profit from supplier. This yields a deviation profit of $\phi(\min \{w^c, p^m(\tilde{w})\})\tilde{w}$, the maximum of which is $\phi(\min \{w^c, p^m\})$ and can always be reached by setting $\tilde{w} = 0$. For $w^c = 0$, the highest attainable deviation profit is 0.
no profitable contract deviation if and only if:

\[
\frac{1}{1 - \delta} \left( \frac{1}{N} D (p^c) (p^c - w_c) - F^c \right) \geq \phi (\min \{w^c, p^{m}\}).
\]  

(4)

The benchmark for our analysis of collusion in both the output and the input market will be the case where retailers use Nash reversion strategies to sustain collusion on the retail price \(p^c\), but continue to offer the supply contract \(C^* = (0, 0)\) from the symmetric competitive equilibrium in every period of the repeated game.\(^{19}\) The next proposition characterizes the set of sustainable retail prices in this benchmark case, and shows that retailers must agree on contract offers \(C^c\) with slotting fees and above-cost wholesale prices in order to increase the scope for collusion relative to the benchmark case.

**Proposition 1**

(i) Suppose that retailers offer the contract \(C^* = (0, 0)\) in every period. Then, collusion on any price \(p^c \in (0, p^{m}]\) is sustainable if and only if \(\delta \geq (N - 1) / N\).

(ii) Collusion is sustainable for \(\delta < (N - 1) / N\) only if the collusive contract \(C^c\) features a wholesale price above marginal cost \((w^c > 0)\) and a slotting fee \((F^c < 0)\).

To understand why slotting fees are necessary to sustain collusion for discount factors below \((N - 1) / N\), consider condition (2), which excludes retail price deviations when the downstream margin is positive.\(^{20}\) If \(F^c = 0\), (2) is the standard no-cheating constraint in a symmetric Bertrand model with \(N\) firms and marginal cost \(w_c\), which holds if and only if \(\delta \geq (N - 1) / N\). Increasing the fixed fee so that \(F^c > 0\) makes collusion even more difficult to sustain, because it decreases the retailers’ profits on the equilibrium path. A slotting fee (i.e., \(F^c < 0\)) is thus necessary to ensure that the price deviation constraint in (2) holds for discount factors below \((N - 1) / N\). It directly follows that \(w^c > 0\) is another necessary condition for collusion if \(\delta < (N - 1) / N\), as the suppliers would refuse to pay slotting fees otherwise.

In the following, we will show that collusion is indeed sustainable for some discount factors strictly below \((N - 1) / N\) if retailers can offer contracts with slotting fees and above-cost wholesale prices. As long as slotting fees are feasible, the retailers can obtain the entire industry profit along the equilibrium

\(^{19}\)One can also think about this benchmark as a situation where all \(R_i - S_i\) pairs are vertically integrated and cannot commit to internal transfer prices above marginal costs (Bonanno and Vickers, 1988).

\(^{20}\)For \(w^s \geq p^s > 0\), the contract deviation constraint in (4) directly implies that \(F^c < 0\), otherwise retailers earn non-positive profits on the equilibrium path but could make a strictly positive profit by deviating to the contract \((0, 0)\).
path even if the wholesale price exceeds upstream marginal costs. Intuitively, a higher $w^c$ then facilitates collusion in the output market because it decreases the margin that the deviator can earn on additional units if it undercuts the retail price. However, contract deviations limit the extent to which the retailers can use a high wholesale price to facilitate collusion, as we will discuss in more detail later.

The retailer cartel optimally sets $p^c \in [0, p^m]$, $w^c \geq 0$, and $F^c \in \mathbb{R}$ so as to maximize:

$$D(p^c)(p^c - w^c) - NF^c,$$

subject to the retailers’ “no-cheating” constraints and the supplier’s participation constraint. Given any $p^c$ and $w^c$, it is clearly optimal for the retailers to set the fixed fee $F^c$ such that the suppliers’ participation constraint in (1) is binding. Each retailer’s per period profit on the collusive path is thus $\phi(p^c)/N$, regardless of $w^c$. This implies that it is without loss of generality to restrict attention to equilibria in which $w^c \leq p^c$. Starting from any $w^c > p^c$, reducing the wholesale price to $p^c$ eliminates price deviation incentives and weakens contract deviation incentives (as $\phi(\min\{w^c, p^m\})$ is increasing in $w^c$). Given $F^c = - (1/N) D(p^c) w^c$ and $w^c \leq p^c \leq p^m$, the cartel problem, thereafter denoted by $\mathcal{P}$, boils down to:

$$\max_{p^c \in [0, p^m], w^c \in [0, p^c]} \phi(p^c)
\text{s.t.}
\frac{\phi(p^c)}{N(1-\delta)} \geq \psi(w^c; p^c) \equiv \max \left\{ D(p^c) \left( p^c - \frac{w^c(N-1)}{N} \right), \phi(w^c) \right\},
$$

where (5) is obtained by combining (2) and (4).

**Lemma 1** For any $p^c \in (0, p^m]$, there exists a unique $w(p^c) \in (0, p^c)$ such that:

$$D(p^c) \left( p^c - \frac{w(p^c)(N-1)}{N} \right) = \phi(w(p^c)),
$$

and:

$$\psi(w^c; p^c) = \begin{cases} D(p^c) \left( p^c - \frac{w^c(N-1)}{N} \right) & \text{if } w^c \in [0, w(p^c)], \\ \phi(w^c) & \text{if } w^c \in (w(p^c), p^c]. \end{cases}$$
The following trade-off determines the cartel’s choice of $w^c$. On the one hand, an increase in $w^c$ makes price deviations less attractive, because undercutting the collusive price yields a smaller profit gain the smaller the downstream margin. On the other hand, an increase in $w^c$ makes contract deviations more attractive, as the deviator’s profit $\phi(w^c)$ is strictly increasing in $w^c$ for $w^c < p^c \leq p^m$. Intuitively, a higher $w^c$ weakens the ability of the punishor(s) to drastically cut their own price in the deviation period, which makes the within-period punishment of a contract deviation less severe. Contract deviation incentives thus limit the extent to which $w^c$ can be increased in order to make price deviations unprofitable. Given this trade-off between price and contract deviation incentives, the wholesale price $w^c = w(p^c)$, at which the price and contract deviation constraints are simultaneously binding, maximizes the scope for collusion on any given price $p^c \in (0, p^m]$.

Building on these insights, the next proposition fully characterizes the solution $(p^c(\delta, N), w^c(\delta, N))$ of the cartel problem $\mathcal{P}$, with the understanding that the optimal fixed fee is given by:

$$
F^c(\delta, N) = -\frac{1}{N} D(p^c(\delta, N)) w^c(\delta, N).
$$

**Proposition 2** The solution $(p^c(\delta, N), w^c(\delta, N))$ of the cartel problem $\mathcal{P}$ is as follows:

(i) $p^c(\delta, N) = p^m$ if and only if $\delta \geq \tilde{\delta}_1(N)$, where:

$$
\tilde{\delta}_1(N) = 1 - \frac{\phi(p^m)}{N \phi(w(p^m))} \in (0, \frac{N-1}{N}).
$$

For $\delta \in [\tilde{\delta}_1(N), (N-1)/N)$, any $w^c \in [w_1^c(\delta, N), w_2^c(\delta, N)]$, where:

$$
w_1^c(\delta, N) = \frac{N (1 - \delta) - 1}{(N - 1)(1 - \delta)} p^m \in (0, w(p^m]], \quad \text{and}
$$

$$
w_2^c(\delta, N) = \phi^{-1} \left( \frac{\phi(p^m)}{N (1 - \delta)} \right) \in [w(p^m), p^m),
$$

is optimal for the retailers. For $\delta \geq (N-1)/N$, any $w^c \in [0, p^m]$ is optimal for the retailers.

(ii) There exists a $\tilde{\delta}_0(N) \in (0, \tilde{\delta}_1(N))$ such that $p^c(\delta, N) \in (0, p^m)$ if and only if $\delta \in (\tilde{\delta}_0(N), \tilde{\delta}_1(N))$.

For any $\delta \in (\tilde{\delta}_0(N), \tilde{\delta}_1(N))$, the cartel problem has a unique solution characterized by the follow-
ing two conditions:

\[
\begin{align*}
\phi (p^c (\delta, N)) &= (1 - \delta) N \phi (w^c (\delta, N)), \\
w^c (\delta, N) &= w (p^c (\delta, N)).
\end{align*}
\]

\(\delta^0 (N)\) is the unique solution of \(p^c (\delta, N) = 0\).

By offering contracts with above-cost wholesale prices and slotting fees to their suppliers, retailers can sustain collusion for discount factors below the standard threshold \((N - 1)/N\) in symmetric Bertrand games. As cartel profits are strictly increasing in the retail price, it is always optimal for the cartel to select the highest sustainable price in \([0, p^m]\); hence, \(p^c (\delta, N)\) is unique for all \(\delta\) and \(N\). The optimal contract, on the other hand, need not be unique: for discount factors high enough to sustain full collusion \((p^c = p^m)\) with slack in the “no-cheating” constraint (5), there is a range of optimal wholesale prices and associated fixed fees, including \(w^c = F^c = 0\) if and only if \(\delta \geq (N - 1)/N\).

**Linear example** A numerical example is helpful to illustrate these results. Consider the case \(N = 2\) and \(D (p) = \max \{0, 1 - p\}\), so that \(p^m = \frac{1}{2}\) and \(\frac{N - 1}{N} = \frac{1}{2}\). Solving (6) for \(w (p^m)\) yields:

\[
w (p^m) = \frac{1}{4}.
\]

Using (7), we obtain the critical discount factor for full collusion:

\[
\delta^1 (2) = \frac{1}{3} < \frac{1}{2}.
\]

For \(\delta = \delta^1 (2) = \frac{1}{3}\), the unique optimal wholesale price is \(w (p^m) = \frac{1}{4}\). For \(\delta \in \left(\frac{1}{3}, \frac{1}{2}\right)\), there is a range of optimal wholesale prices with lower bound:

\[
w^c_1 (\delta, 2) = \frac{1 - 2\delta}{2(1 - \delta)} \in \left(0, \frac{1}{4}\right),
\]
and upper bound:

\[ w_2^* (\delta, 2) = \frac{1}{2} - \frac{1}{2} \sqrt{\frac{1 - 2\delta}{2 (1 - \delta)}} \in \left( \frac{1}{4}, \frac{1}{2} \right). \]

Note that the range \([w_1^* (\delta, 2), w_2^* (\delta, 2)]\) is expanding in \(\delta\). For \(\delta \geq \frac{1}{2}\), the range of optimal wholesale prices is \([0, \frac{1}{2}]\).

Now consider the region of parameters where full collusion is not sustainable — i.e., \(\delta < \frac{1}{3}\). Solving the system of equations (10)-(11), we obtain:

\[ p^c (\delta, 2) = \frac{(1 - \delta) (1 - 4\delta)}{1 + 8\delta^2 - 7\delta}, w^c (\delta, 2) = \frac{(1 - 2\delta) (1 - 4\delta)}{1 + 8\delta^2 - 7\delta}. \]

\(p^c (\delta, 2) = 0\) at \(\delta = \frac{1}{4}\), hence some collusion is sustainable if and only if the discount factor (strictly) exceeds:

\[ \delta^0 (2) = \frac{1}{4}. \]

Figure 1 provides a graphical illustration.

Our results have clear-cut implications for the policy controversy around slotting fees. Whereas a ban on slotting fees has no impact on retail prices if \(\delta \geq (N - 1)/N\) or \(\delta \leq \delta^0 (N)\), collusion is sustainable for discount factors in the range \((\delta^0 (N), (N - 1)/N)\) if and only if slotting fees are allowed. In our model, a ban on slotting fees hence (weakly) increases welfare by reducing the scope for collusion on retail prices.

We considered cartel agreements that include the fixed fee \(F\) so far, but our insights would remain unchanged if the retailers colluded only on wholesale and retail prices. In the latter case, each retailer would set its fixed fee at the lowest level permitted by its supplier’s participation constraint and legal rules. In the absence of a ban on slotting fees, the suppliers’ participation constraint in (1) would therefore bind. As (1) was also binding at the solution of the cartel problem that includes the fixed fee, Proposition 2 would thus continue to hold. Similarly, in the presence of a ban on slotting fees, each retailer would set its fixed fee equal to 0, in which case the no-cheating constraints (4) and (2) imply that collusion is sustainable if and only if \(\delta \geq (N - 1)/N\). Hence, a ban on slotting fees would again reduce the scope for collusion.
4 Buyer versus seller power

So far we assumed that all bargaining power rests with retailers. Now consider the polar case where suppliers make take-it-or-leave-it contract offers to their retailers. To keep the analysis as close as possible to the baseline model in all other respects, we focus on infinite Nash reversion strategies where contract offer and pricing decisions are made to maximize discounted profits in the repeated game, but retailers accept any contract that promises a non-negative profit in the period in which it is offered.\footnote{The assumption that contract acceptance decision are myopic also allows us to exclude implicit agreements between retailers aimed solely at increasing their share of industry profits, in which case the concept of bargaining power would lose its meaning.}

More specifically, consider the following strategy profile $\sigma = (\sigma_S, \sigma_R)$. The strategy $\sigma_S$ of suppliers is:

- Offer the contract $C^c = (F^c, w^c)$ as long as there was no deviation either by a supplier to a contract different from $C^c$ or by a retailer to a price different from $p^c$ in any past period.
- Offer the contract $(0, 0)$ otherwise.

The strategy $\sigma_R$ of retailers is:

- Charge $p^c$ as long as there was no deviation by either a retailer or a supplier in any past period and all accepted contracts in the current period are $C^c$.
- Play the competitive Nash equilibrium of the price-setting game (as described in Definition 1) otherwise.
- Accept any contract that gives a non-negative (expected) profit in the period in which it is offered.

The punishments induced by this strategy profile are counterparts of the infinite Nash reversion punishments in the baseline model. As before, a price deviation triggers infinite reversion to the symmetric competitive equilibrium of the stage game with zero profits for all firms,\footnote{It is easy to see that the new stage game has a subgame-perfect equilibrium in which all suppliers offer the contract $(0, 0)$, all retailers accept this offer, and all retailers charge 0. For any set of contract offers, the standard competitive equilibrium of the price-setting subgame (at $T = 2$) is as described in Definition 1, and if retailers play this equilibrium at $T = 2$ both on and off the equilibrium path, it is optimal for each supplier to offer $(0, 0)$ at $T = 1$ given that the other supplier(s) offer $(0, 0)$.} and a contract deviation triggers play of the competitive equilibrium (as described in Definition 1) of the price-setting subgame in...
the deviation period and infinite reversion to the symmetric competitive equilibrium of the stage game with zero profits for all firms in future periods. All punishments are again composed of static Nash equilibria and therefore credible.

The condition that rules out price deviations by retailers remains the same as in the baseline model where retailers make contract offers (condition (2) or (3) depending on whether \( w^c \) is above or below \( p^c \)). Thus, there is no profitable price deviation if and only if:

\[
\frac{\delta}{1 - \delta} \left( \frac{1}{N} D(p^c) (p^c - w^c) - F^c \right) \geq \max \left\{ \frac{N - 1}{N} D(p^c) (p^c - w^c), \frac{1}{N} D(p^c) (w^c - p^c) \right\}.
\]  (12)

Condition (12) implies that, although suppliers have all the bargaining power, retailers must earn strictly positive rents on the collusive path whenever \( p^c \neq w^c \). Intuitively, suppliers must reward retailers for not undercutting the retail price if \( p^c > w^c \), and reward retailers for not deviating to a higher price if \( p^c < w^c \). If \( p^c = w^c \), there is no profitable price deviation for retailers as long as \( F^c \leq 0 \).

The most attractive contract deviation for a supplier is to offer \((0, 0)\) so as to earn \( \phi(\min \{w^c, p^m\}) \) in the deviation period. This deviation is unprofitable if and only if each supplier’s discounted profit on the collusive path exceeds the deviation profit:

\[
\frac{1}{1 - \delta} \left( \frac{1}{N} D(p^c) w^c + F^c \right) \geq \phi(\min \{w^c, p^m\}).
\]  (13)

Note that the deviation profit, \( \phi(\min \{w^c, p^m\}) \), is the same as in the contract deviation constraint of the baseline model (condition (4)).

Our main result is that collusion is sustainable if and only if the discount factor exceeds the standard threshold \((N - 1)/N\) for collusion in a symmetric Bertrand game:

**Proposition 3** If suppliers make take-it-or-leave-it contract offers to retailers, then collusion on any price \( p^c \in (0, p^m] \) is sustainable by the strategy profile \((\sigma_S, \sigma_R)\) if and only if \( \delta \geq \frac{N-1}{N} \).

The comparison between the two polar cases of downstream and upstream bargaining power suggests that buyer power facilitates collusion between vertical chains. For \( \delta \geq (N-1)/N \), full collusion is sustainable regardless of whether suppliers or retailers make contract offers, but for \( \delta \in \left(2^0(N), (N-1)/N \right), \)
collusion is sustainable by infinite Nash reversion punishments if and only if retailers make contract offers.

Intuitively, upstream bargaining power makes collusion more difficult because suppliers are unable to extract the entire industry surplus on the equilibrium path unless \( w^c = p^c \). This implies that, for any \( w^c \neq p^c \), contract deviation incentives are stronger when suppliers make offers than when retailers make offers. As shown in the proof of Proposition 3, it follows that a cartel agreement with \( w^c \neq p^c > 0 \) can only be sustained if \( \delta > (N - 1)/N \). The only cartel agreement in which suppliers extract the entire industry surplus without inducing a price deviation is \( w^c = p^c \) and \( F^c = 0 \). However, in this case the within-period punishment for a contract deviation is ineffective as the retailers of non-deviating suppliers continue to charge \( p^c \). A contract deviation hence allows the deviator to make a short-term profit gain of \( \frac{N-1}{N} \phi(p^c) \), which implies that the collusive agreement is sustainable if and only if \( \delta \geq (N - 1)/N \).

A ban on slotting fees has no impact on the sustainability of collusion if bargaining power rests upstream. For \( \delta \geq (N - 1)/N \), any price \( p^c \in (0, p^m] \) can be sustained in equilibrium by means of the supply contract \( (F^c, w^c) = (0, p^c) \), and for \( \delta < (N - 1)/N \), collusion is unsustainable regardless of whether slotting fees are feasible. In contrast, if retailers make contract offers, then slotting fees arise as part of the optimal collusive agreement for any \( \delta \in (\delta^0(N), (N - 1)/N) \). Our model thus predicts that the incidence of slotting fees is correlated with buyer power, which is in line with anecdotal evidence from the retail industry, as mentioned in the introduction.

5 Information exchange between retailers

So far we assumed that each retailer observes its competitors’ supply contracts once they are accepted. This ex post contract transparency can be endogenized rather straightforwardly by modeling communication about supply contracts as part of the collusive strategy. Consider the following stage game \( \mathcal{G}' \):

\[ T = 1 \text{ Contracting stage: Retailers make simultaneous and secret contract offers to their suppliers.} \]

Suppliers decide whether to accept their offers. If \( S_i \) accepts \( C_i \), then \( F_i \) is paid.

\[ T = 2 \text{ Information exchange stage: Retailers simultaneously decide whether to make the terms of their} \]

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supply contracts observable to all other retailers.  

\[ T = 3 \] Price-setting stage: Retailers simultaneously set retail prices and consumers decide how much to buy. Retailers order the quantities demanded by consumers from the suppliers at the relevant wholesale prices, and revenues are realized.

All other features of the model remain unchanged. The stage game \( G' \) has a symmetric competitive equilibrium in which, along the equilibrium path, all retailers offer the contract \( C^* = (0, 0) \), all suppliers accept this offer, retailers do not share information about their supply contracts, and all retailers set \( p^* = 0 \). Consistent with the equilibrium contract offers, the retailers believe that a retailer who does not share information has signed \( C^* \). If none of the retailers share information at \( T = 2 \), it is therefore indeed optimal for a retailer with contract \( C^* \) to set \( p^* = 0 \) at \( T = 3 \) given that the other retailer(s) play the same strategy. This implies that it is unprofitable to offer a contract different from \( C^* \) at \( T = 1 \) and keep the deviation secret at \( T = 2 \): the other retailer(s) will charge 0 at \( T = 3 \) in this case, which implies that there is no contract offer that will be accepted by the supplier and give a strictly positive profit to the deviator. Finally, if a retailer unilaterally deviates at \( T = 1 \) and then reveals its deviant contract at \( T = 2 \), the deviator again earns 0 in any Nash equilibrium of the price-setting subgame at \( T = 3 \), because the other retailer(s) have wholesale price 0. Thus, any deviant contract offer that increases the retailer’s profit would be rejected by the supplier.

In the infinitely repeated game, suppose that retailers play an infinite Nash reversion strategy as in the baseline model, adapted as follows to incorporate communication. Along the equilibrium path, all retailers exchange information. Refusal to reveal one’s contract triggers the following punishment: \( (i) \) at the price-setting stage of the deviation period, all punishers charge \( w^c \) and the deviator sets its best-response to the price \( w^c \) given its own wholesale price, \( (ii) \) from the next period onwards, the retailers play the symmetric competitive equilibrium of \( G' \) with zero profits for all firms.  

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23 As discussed in the introduction, we assume that retailers can exchange hard information, such as signed contracts and invoices. If retailers could lie about their supply contracts, collusion would be sustainable only if \( \delta \geq (N - 1)/N \).

24 The punishments for price deviations and contract deviation that are revealed at the information exchange stage remain as in the baseline model: retailers play the competitive equilibrium of the price-setting subgame if a retailer revealed a contract deviation in the current period, and revert to the symmetric competitive equilibrium of \( G' \) with zero profits in all future periods after either type of deviation.
If a retailer deviates from the equilibrium path by refusing to reveal its contract, then the other retailer(s) does (do) not know whether the deviator offered \((F^c, w^c)\) and deviated only at the information exchange stage or whether it already deviated at the contracting stage. A sensible restriction on beliefs in this situation is that the deviator’s wholesale price is at most \(w^c\). This restriction is based on the following forward induction argument: regardless of the price that the punisher(s) threaten to charge in the deviation period if a retailer refuses to reveal its contract, it is always optimal for a contract deviator to offer \((0, 0)\). Given this restriction on beliefs, it is optimal for the punisher(s) to charge \(w^c\) at the price-setting stage of the deviation period. Hence, the above punishment for a refusal to share information is clearly credible.\(^{25}\)

Given these strategies, the set of sustainable retail prices (given \(\delta\) and \(N\)) remains the same as in the infinitely repeated stage game \(G\) where accepted contracts are observable for exogenous reasons. First, because retailers reveal their supply contracts on the collusive path, the added information exchange stage has no impact on deviation incentives at the price-setting stage. Second, no retailer that signed the collusive contract \(C^c\) wants to deviate at the information exchange stage by refusing to make its contract public, as this would trigger zero instead of positive continuation profits. Third, given that the within-period punishment price for refusal to share information is \(w^c\), a retailer with an accepted contract \(\tilde{C} \neq C^c\) is indifferent between admitting its deviation and refusing to reveal its contract at the information sharing stage of the deviation period. In either case, the deviator’s profit is

\[
D(\min \{w^c, p^m(\tilde{w})\})(\min \{w^c, p^m(\tilde{w})\} - \tilde{w}) - \tilde{F} \quad \text{if} \; \tilde{w} < w^c, \quad \text{and} \quad -\tilde{F} \quad \text{otherwise.}
\]

Hence, it is optimal for a contract deviator to admit its deviation to the other retailers at \(T = 2\), which implies that the contract deviation constraint remains the same as in the baseline model (condition (4)). This discussion implies that voluntary information exchange between retailers can fully substitute for exogenous transparency of supply contracts:

**Proposition 4** If an outcome \((p^c, F^c, w^c)\) is sustainable in the infinitely repeated stage game \(G\) with exogenous contract transparency, then it is also sustainable in the infinitely repeated stage game \(G'\) with voluntary information exchange.

\(^{25}\)The belief restriction is necessary only if \(N = 2\). For \(N \geq 3\), it is optimal for each punishor to charge \(w^C\) given that the other punishor(s) charge \(w^C\), regardless of their beliefs about the deviator’s wholesale price.
If information exchange about input contracts is banned, retailers are unable to detect contract deviations. A possible deviation from the collusive path is therefore to offer the contract \((0,0)\) and marginally undercut the collusive price \(p^c\) in the same period. Such a deviation is unprofitable only if:

\[
\max_{w^c,F^c\leq -\frac{1}{\delta} D(p^c)w^c} \frac{1}{1-\delta} \left( \frac{1}{N} D(p^c) (p^c - w^c) - F^c \right) = \frac{\phi(p^c)}{N(1-\delta)} \geq \phi(p^c),
\]

or \(\delta \geq (N-1)/N\). In contrast, when retailers are allowed to exchange information, collusion is sustainable for any \(\delta > \delta^0(N)\) where \(\delta^0(N) < (N-1)/N\) — see Propositions 4 and 2. A ban on information exchange about input contracts thus reduces the scope for collusion.

Discussions about the role of communication in collusion often focus on communication about intended future conduct; our analysis, on the other hand, highlights the importance of communication about verifiable past conduct. Information exchange about input supply contracts facilitates collusion between downstream firms with a strong bargaining position because it allows the firms to collude on the terms they offer to their suppliers, which, in turn, makes it easier to sustain collusion on final output prices. Although collusion on retail prices is tacit in our model, information exchange about input supply contracts is necessary to sustain collusion for low discount factors.

6 Alternative punishments of contract deviations

Infinite Nash reversion is an optimal punishment for any price deviation in our game, but it is a priori unclear whether the scope for collusion could be enhanced through alternative punishments of contract deviations. In this section, we show that it is possible to design punishments of contract deviation that further facilitate collusion; however, such punishments require complex agreements where punishers accept losses in the deviation period in order to reduce the deviator’s market share.

Unlike most models of tacit collusion, ours is a repeated extensive-form (rather than normal-form) game. Punishments of contract deviations begin within the deviation period, which implies that the

\[^{26}\text{In fact, it is easy to see that for } \delta \geq \frac{N-1}{N}, \text{ collusion on any price } p^c \in (0,p^m] \text{ is sustainable in an equilibrium with } w^c = F^c = 0.\]

\[^{27}\text{See Kühn (2001) for a discussion of communication in cartels distinguishing between soft information regarding intentions and hard information regarding market conditions and past conduct.}\]

\[^{28}\text{Mailath, Nocke and White (2004) discuss the failure of simple penal codes (Abreu 1986, 1988) in repeated extensive-form games. They show through examples that it can be necessary to tailor the punishment to the nature of the deviation}\]
details of the punishment affect the slotting fee that the deviator can obtain from its supplier. Unlike in repeated normal-form games where short-term deviation gains are independent of future play, the short-term deviation gain that can be achieved by means of a contract deviation thus depends on the exact nature of the punishment. One implication is that even if a credible punishment forces the deviator down to zero continuation profits (the minmax of our game), the punishment is not necessarily optimal, that is, it does not necessarily maximize the scope for collusion. For illustration, consider two alternative punishments of deviations to contracts with \( \bar{w} < w^c \), both forcing the deviator down to zero continuation profits:

(i) Maximal “grim trigger” punishment:

In the price-setting subgame of the deviation period, retailers play the (non-standard) equilibrium in which all retailers charge \( \bar{w} \) and the deviator serves the entire demand;\(^{30}\) thereafter, the game reverts to the symmetric competitive equilibrium of \( G \) with zero profits for all firms. This punishment is credible for any \( \delta \geq 0 \).

(ii) Maximal stick-and-carrot punishment:

At the price-setting stage of the deviation period, all firms charge \( p^P(\bar{w}) \) defined by:\(^{31}\)

\[
D \left( p^P(\bar{w}) \right) \left( p^P(\bar{w}) - \bar{w} \right) + \frac{\delta}{1 - \delta} \frac{\phi \left( p^C \right)}{N} = 0, \tag{14}
\]

and the deviator serves the entire demand; in the next period, the firms revert to the equilibrium path, with per-period profit \( \phi \left( p^C \right) / N \) for each retailer. This punishment is credible whenever the equilibrium path (the carrot of the punishment) is sustainable. First, if the deviator deviates from its own punishment to a price above \( p^P(\bar{w}) \) to avoid a profit loss in the current period, this will trigger reversion to the competitive equilibrium of \( G \) with zero profits in all future periods, which is no better than the total continuation profit from complying. Second, the punishers have

\(^{29}\) Deviations to contracts with \( \bar{w} \geq w^c \) are unattractive even if no punishment ensues and the deviator still serves \( \frac{1}{N} \) of total demand in the deviation period.

\(^{30}\) See footnote 13 and the references therein for a discussion of non-standard equilibria in Bertrand games with asymmetric marginal cost.

\(^{31}\) Note that condition (14) may imply \( p^P(\bar{w}) < 0 \) for small \( \bar{w} \). However, it would be easy to rule out negative prices by assuming strictly positive downstream production costs and \( \lim_{p \to 0} D \left( 0 \right) = \infty \).
a strict incentive to comply with the punishment so as to obtain strictly positive instead of zero continuation profits. The stick-and-carrot punishment is therefore credible whenever $\delta$ is high enough such that, given $p^c$ and $C^c$, there is no profitable deviation from the collusive equilibrium path.

In a repeated normal-form game, any credible punishment that forces the deviator down to minmax continuation profits is optimal. In our repeated extensive-form game, this is no longer true. If the retailers use the maximal grim trigger punishment described in (i), the highest slotting fee the deviator can obtain is $D(\tilde{w})\tilde{w}$, which is maximal and (almost) equal to $\phi(w^c)$ if the deviator marginally undercuts $w^c$. Although the punishment yields zero continuation profits for the deviator, the incentives for a contract deviation are hence the same as with the Nash reversion punishments considered in Section 3: in both cases, the no-cheating constraint is $\frac{\phi(p^c)}{N(1-\delta)} \geq \phi(w^c)$.

With the stick-and-carrot punishment in (ii), the highest slotting fee the deviator can ask for is $D(p^P(\tilde{w}))\tilde{w}$. (14) implies that $p^P(\tilde{w}) < \tilde{w}$, which, in turn, implies that $D(p^P(\tilde{w}))\tilde{w} > D(\tilde{w})\tilde{w}$. Hence, if the deviator marginally undercuts $w^c$, it can obtain a slotting fee that strictly exceeds $\phi(w^c)$. It follows that the constraint to rule out contract deviations becomes more stringent than $\frac{\phi(p^c)}{N(1-\delta)} \geq \phi(w^c)$. Although the stick-and-carrot punishment is harsher than the standard Nash reversion punishment (it gives zero instead of strictly positive continuation profits to the deviator), it is hence less effective at deterring contract deviations.

The key to weakening contract deviation incentives is to reduce the quantity that the deviator sells in the deviation period, so as to reduce the slotting fee it can obtain.\(^{32}\) This means that the punisher(s) must be willing to sell a positive quantity even if the punishment price lies below their marginal input cost, which requires future rewards (a carrot) for the punisher(s). The proof of the following proposition shows that stick-and-carrot punishments in which the punisher(s) sell below cost can indeed expand the scope for collusion:\(^{33}\)

\(^{32}\)This requires that the market-sharing rule in case of a tie at the lowest price is no longer necessarily the standard rule that the lowest-cost of the lowest-price firms serve the entire demand. Instead, the analysis permits any market sharing rule that is consistent with the equilibrium.

\(^{33}\)Note that, for tractability, we continue to assume that suppliers play the statically optimal strategy of accepting any contract that promises a non-negative profits in the period in which it is offered. In general, the scope for collusion in an intermediate goods industry could be further increased by considering dynamic strategies with rewards to parties that
Proposition 5 *Full collusion is sustainable for some discount factors strictly below $\delta_1(N)$. 

Linear example Consider $N = 2$ and $D(p) = \max\{0, 1 - p\}$. As shown in Section 3, we then have $p^m = \frac{1}{2}$, $w(p^m) = \frac{1}{4}$, and $\delta_1(2) = \frac{1}{3}$. We now show that the collusive agreement $p^c = p^m = \frac{1}{2}$, $w^c = 0.255 > w(p^m)$, and $F^c = -\frac{D(p^m)0.255}{2}$ is sustainable for discount factors strictly below $\delta_1(2)$.

Suppose that price deviations are punished through infinite Nash reversion as before, and that any deviation to a contract with $\tilde{w} < w^c$ triggers the following punishment:

- at the price-setting stage of the deviation period, all retailers charge $0.12$ and each retailer serves half of the total demand $D(0.12)$,
- from the next period onwards, the game reverts to the equilibrium path.

Collusion is sustainable if the following four conditions are met: (i) no retailer has an incentive to deviate in price from the equilibrium path, (ii) no retailer has an incentive to deviate in contract from the equilibrium path, (iii) no punisher has an incentive to deviate from a punishment in play, (iv) a deviator has no incentive to deviate from its own punishment. Because the punishments of contract deviations involve reversion to the equilibrium path in the following period, and price deviations are punished by reversion to a Nash equilibrium of the stage game, (iii) and (iv) boil down to ruling out deviations from punishments of contract deviations at the price-setting stage of the deviation period.

In our example, no firm has an incentive to undercut the collusive price if:

$$\frac{\delta}{1 - \delta} \frac{\phi(0.5)}{2} \geq \frac{0.5(0.5 - 0.255)}{2} \iff \delta \geq \delta_P \approx 0.329.$$ 

Second, no retailer has an incentive to deviate to a contract with a lower wholesale price if:

$$\frac{1}{1 - \delta} \frac{\phi(0.5)}{2} \geq \frac{\phi(0.12)}{2} + \frac{\delta}{1 - \delta} \frac{\phi(0.5)}{2} \iff \phi(0.5) \geq \phi(0.12),$$

which holds because $0.5$ is the maximizer of $\phi$. Third, no punisher has an incentive to deviate from a reject deviation contract offers. See Nocke and White (2007) for a related discussion in a setting in which suppliers make contract offers.
contract deviation punishment at the price-setting stage of the deviation period if:

\[
\frac{1}{2} D (0.12) (0.12 - 0.255) + \frac{\delta}{1 - \delta} \phi (0.5) \geq 0 \quad \Leftrightarrow \quad \delta \geq \delta_C \simeq 0.192.
\]

Fourth, if \( \bar{w} \in [0.12, 0.255] \), then the deviator has no incentive to deviate from its own punishment at the price-setting stage of the deviation period if:

\[
\frac{1}{2} D (0.12) (0.12 - \bar{w}) + \frac{\delta}{1 - \delta} \phi (0.5) \geq 0.
\]

This condition is more difficult to satisfy the larger \( \bar{w} \), but as just shown it holds for \( \bar{w} = 0.255 \) as long as \( \delta \geq \hat{\delta} \simeq 0.192 \). Finally, if \( \bar{w} < 0.12 \), then the deviator must be kept from undercutting the price 0.12 to obtain the entire demand, which would trigger reversion to the symmetric competitive equilibrium of \( \mathcal{G} \) with zero profits. Such a deviation is unprofitable for all \( \bar{w} \geq 0 \) if:

\[
\frac{1}{2} \phi (0.12) \leq \frac{\delta}{1 - \delta} \phi (0.5) \quad \Leftrightarrow \quad \delta \geq \delta_D \simeq 0.174.
\]

Hence, the critical discount factor is \( \delta_P \simeq 0.329 < \delta_1 (2) = \frac{1}{3} \).

This example illustrates that it is possible to design punishments that decrease the critical discount factor for collusion on the monopoly price (conversely, it is possible to increase the highest sustainable price for a given discount factor). However, making the punishment more severe does not suffice to achieve this goal. Instead, the punishment must be designed so as to reduce the deviation gains that the deviator can attain prior to the start of the punishment.\(^{34}\)

The trade-off between price and contract deviation incentives discussed earlier persists. A decrease in \( w^c \) increases the short-term gain from a price deviation, but it also makes contract deviations less attractive. First, because the only potentially attractive deviation wholesale prices are those below \( w^c \), any decrease in \( w^c \) restricts the range of potentially profitable contract deviations. Second, for any given

\(^{34}\)A full analysis of optimal punishments is beyond the scope of this paper. One difficulty is that a priori the simple stick-and-carrot punishment structure used above need not be optimal. It is, for instance, conceivable that the optimal punishment assigns a market share below \( \frac{1}{N} \) to the deviator in future (carrot) periods, with the goal of reducing the market share that the deviator serves in the deviation period and, ultimately, the deviator’s slotting fee. In this case, the credibility of the carrot part of the punishment would no longer follow from the sustainability of the collusive agreement.
\( \bar{w} \), a decrease in \( w^c \) makes it easier for the other retailers to punish the deviator: the lower \( w^c \), the easier it is for the punishers to force the deviator to serve a smaller share of total demand, which, for any given (punishment) price, decreases the slotting fee that the deviator can obtain.

The result in Proposition 1(ii), that above-cost wholesale prices and slotting fees are necessary to sustain collusion for discount factors below \( (N - 1) / N \), is also robust to the use of alternative punishments of contract deviations. The price deviation constraint in (2) implies this result, which is therefore independent of the nature of punishments of contract deviations.

7 Concluding remarks

In this article, we show that retailers with buyer power can sustain collusion on retail prices more easily if they also collude on the terms of their input supply contracts. By agreeing to pay above-cost wholesale prices to their suppliers, retailers limit their own short-term profit gain from undercutting the retail price. Slotting fees enable retailers to obtain the entire industry profits in spite of above-cost wholesale prices, which implies that the long-term loss from deviating remains high. Inefficient wholesale prices and slotting fees thus emerge as part of a mechanism to facilitate collusion on downstream prices.

Our analysis yields several antitrust policy implications. First, in industries with powerful retailers, a ban on slotting fees decreases the risk of collusion. Second, in industries with powerful retailers, a ban on information exchange between retailers about their input supply terms decreases the risk of collusion. Third, a shift in bargaining power from suppliers to retailers increases the risk of collusion.

The analysis was cast in the context of supplier-retailer relationships where slotting fees and buyer power are particularly prominent topics in policy discussion. However, our insights also apply to horizontal collusion at intermediate levels of the vertical chain. Whenever firms have a strong bargaining position in their input supply negotiations, the scope for collusion on output prices is greater if the firms also reach an implicit agreement on their input supply contracts. An open question for future research is to what extent firms can use contracts other than two-part tariffs, for example, tariffs with quantity constraints, to facilitate collusion.
Appendix

Proof of Proposition 1. We prove parts (i) and (ii) in turn.

(i) : Suppose that all retailers offer \( C^* = (0,0) \) on the equilibrium path. In this case, the contract deviation constraint in (4) and the suppliers’ participation constraint in (1) hold for any \( \delta \) and any \( p^c \in (0,p^m] \). The relevant constraint to exclude price deviations is (2), which, given \( F^c = w^c = 0 \), holds for any \( p^c \in (0,p^m] \) if and only if \( \delta \geq (N-1)/N \).

(ii) : To show that \( F^c < 0 \) is a necessary condition for collusion if \( \delta < \frac{N-1}{N} \), we distinguish two cases: either \( w^c \geq p^c > 0 \) or \( p^c > w^c \). First, if \( w^c \geq p^c > 0 \), then the contract deviation constraint in (4) directly implies that \( F^c < 0 \). Second, if \( p^c > w^c \), then the relevant price deviation constraint is (2), which is violated for any \( \delta < \frac{N-1}{N} \) if \( F^c \geq 0 \). To see this, note that at \( F^c = 0 \), (2) holds if and only if \( \delta \geq \frac{N-1}{N} \), and that (2) is more difficult to satisfy the larger \( F^c \). We conclude that collusion is sustainable for \( \delta < \frac{N-1}{N} \) only if \( F^c < 0 \). This directly implies that collusion is sustainable for \( \delta < \frac{N-1}{N} \) only if \( w^c > 0 \), because otherwise suppliers would refuse the equilibrium contract offer (see condition (1)).

Proof of Lemma 1. Let \( p^c \in (0,p^m] \) and \( w^c \in [0,p^c] \). Recall that:

\[
\psi(w^c;p^c) = \max \left\{ D(p^c) \left( p^c - \frac{w^c(N-1)}{N} \right), \phi(w^c) \right\}.
\]

It is easy to see that \( D(p^c) \left( p^c - \frac{w^c(N-1)}{N} \right) > \phi(w^c) \) for \( w^c = 0 \), but \( D(p^c) \left( p^c - \frac{w^c(N-1)}{N} \right) < \phi(w^c) \) for \( w^c = p^c \). Moreover, \( D(p^c) \left( p^c - \frac{w^c(N-1)}{N} \right) \) is strictly decreasing in \( w^c \), but \( \phi(w^c) \) is strictly increasing in \( w^c \). Together, these observations imply that there exists a unique \( w(p^c) \in (0,p^c) \) such that:

\[
D(p^c) \left( p^c - \frac{w(p^c)(N-1)}{N} \right) = \phi(w(p^c)) = \psi(w(p^c);p^c),
\]

\[
\psi(w^c;p^c) = D(p^c) \left( p^c - \frac{w^c(N-1)}{N} \right) > \phi(w^c) \quad \forall w^c \in [0,w(p^c)),
\]

\[
\psi(w^c;p^c) = \phi(w^c) > D(p^c) \left( p^c - \frac{w^c(N-1)}{N} \right) \quad \forall w^c \in (w(p^c),p^c].
\]

Proof of Proposition 2. Recall the cartel problem:

\[
P : \left\{ \begin{array}{l}
\max_{p^c \in (0,p^m],w^c \in [0,p^c]} \phi(p^c) \\
\text{s.t.} \\
\frac{\phi(p^c)}{N(1-\delta)} \geq \psi(w^c;p^c) \equiv \max \left\{ D(p^c) \left( p^c - \frac{w^c(N-1)}{N} \right), \phi(\min \{w^c,p^m\}) \right\}.
\end{array} \right\}
\]
The proof proceeds by establishing two intermediate results from which the statements in the Proposition will readily follow.

**Step 1:** For any $p^c \in (0, p^m]$, there exists a unique $\delta(p^c, N) \in (0, (N - 1) / N)$ such that collusion on $p^c$ is sustainable if and only if $\delta \geq \delta(p^c, N)$.

Collusion on $p^c$ and $w^c$ is sustainable by Nash reversion trigger strategies if and only if:

$$\phi(p^c) \geq N (1 - \delta) \max \left\{ D(p^c) \left( p^c - \frac{w^c(N - 1)}{N} \right), \phi(w^c) \right\}. \quad (A1)$$

By Lemma 1, for any $p^c \in (0, p^m]$, there exists a unique $w(p^c) \in (0, p^c)$ such that:

$$D(p^c) \left( p^c - \frac{w(p^c)(N - 1)}{N} \right) = \phi(w(p^c)). \quad (A2)$$

As $D(p^c) \left( p^c - \frac{w(p^c)(N - 1)}{N} \right)$ is decreasing in $w^c$ but $\phi(w^c)$ is increasing in $w^c$, setting $w^c = w(p^c)$ minimizes the right-hand side of (A1). Hence, $w^c = w(p^c)$ maximizes the range of parameters for which (A1) holds. Solving (A1) for $\delta$ at $w^c = w(p^c)$ yields the following necessary and sufficient condition for collusion on price $p^c$:

$$\delta \geq \delta(p^c, N) \equiv 1 - \frac{p^c}{N \left( p^c - \frac{w(p^c)(N - 1)}{N} \right)} = 1 - \frac{\phi(p^c)}{N \phi(w(p^c))}. \quad (A3)$$

Because $\phi(w(p^c)) < \phi(p^c)$ for any $p^c \in (0, p^m]$, $\delta(p^c, N) < (N - 1) / N$. It is also easy to show that $\delta(p^c, N) > 0$ for any $p^c > 0$: (A2) can be rewritten as $N \phi(w(p^c)) - \phi(p^c) = (N - 1) \frac{D(p^c)(p^c - w(p^c))}{N \phi(w(p^c))}$, which, given that $w(p^c) < p^c$, implies $N \phi(w(p^c)) > \phi(p^c)$.

**Step 2:** $\frac{d\delta(p^c, N)}{dp^c} > 0$ for $p^c \in (0, p^m]$.

Applying the Implicit Function Theorem to (A2) yields:

$$\frac{dw(p^c)}{dp^c} = \frac{N \phi'(p^c) - (N - 1) D'(p^c) w}{N \phi'(w) + (N - 1) D(p^c)}. \quad (A4)$$

The definition of $\delta(p^c, N)$ in (A3) implies that $\frac{d\delta(p^c, N)}{dp^c} > 0$ if and only if:

$$\frac{d}{dp^c} \left( \frac{w(p^c)}{p^c} \right) < 0 \iff \frac{dw(p^c)}{dp^c} < \frac{w(p^c)}{p^c}. \quad (A5)$$

Using (A3), we can express $\frac{w(p^c)}{p^c}$ as follows:

$$\frac{w(p^c)}{p^c} = \frac{(1 - \delta(p^c, N)) (N - 1)}{(1 - \delta(p^c, N)) (N - 1)}. \quad (A6)$$
Straightforward calculations show that:

\[
\frac{N\phi'(p^c) - (N - 1) D'(p^c) w}{N\phi'(w) + (N - 1) D(p^c)} < \frac{(1 - \hat{\delta}(p^c, N)) \cdot N - 1}{(1 - \hat{\delta}(p^c, N)) \cdot (N - 1)}
\]

\[
\iff
\[(1 - \hat{\delta}(p^c, N)) \cdot N - 1 \left\{ - \left[ \phi'(w) - \phi'(p^c) \right] - \frac{N - 1}{N} \left[ D(p^c) + D'(p^c) w \right] \right\} < 0. \tag{A8}
\]

\(\hat{\delta}(p^c, N) < (N - 1)/N\) (from step 1) implies that \((1 - \hat{\delta}(p^c, N)) \cdot N > 1\), hence the first term on the left-hand-side of (A8) is strictly positive. The second term is strictly negative: \(w(p^c) < p^c\) and the strict concavity of \(\phi\) imply that \(\phi'(w) > \phi'(p^c)\), and \(w(p^c) < p^c\) and \(D'(p^c) < 0\) imply that \(D(p^c) + D'(p^c) w > \phi'(p^c)\), which is strictly positive for \(p^c \in (0, p^m)\). Hence, (A8) holds, which implies that \(\hat{\delta}(p^c, N)\) is strictly increasing in \(p^c\) for all \(p^c \in (0, p^m)\).

We now use the results from steps 1 and 2 to derive the statements in the proposition. Denote the solution of \(P\) by \((p^c(\hat{\delta}, N), w^c(\hat{\delta}, N))\), with the understanding that \(F^c = -(1/N) D(p^c) w^c\). Define:

\[\hat{\delta}^1(N) \equiv \hat{\delta}(p^m, N).\]

By step 1, full collusion is sustainable if and only if \(\delta \geq \hat{\delta}^1(N)\). As cartel profits reach a unique maximum at \(p^m\), this implies that \(p^c(\delta, N) = p^m\) if and only if \(\delta \geq \hat{\delta}^1(N)\).

For \(\delta = \hat{\delta}^1(N)\), the only wholesale price compatible with full collusion is \(w(p^m)\), so the optimal wholesale price is unique in this case. For \(\delta > \hat{\delta}^1(N)\), however, full collusion can be sustained by means of a continuum of wholesale prices around \(w(p^m)\). By Lemma 1, the relevant constraint for \(w_c < w(p^m)\) is the price deviation constraint:

\[\phi(p^m) \geq N(1 - \delta) D(p^m) \left( p^m - w^c(N - 1) \frac{N}{N} \right),\]

which is binding at:

\[w = w_1^c(\delta, N) \equiv \frac{N(1 - \delta) - 1}{(N - 1)(1 - \delta)} p^m.\]

It is easy to see that \(w_1^c(\delta, N) > 0\) if and only if \(\delta < (N - 1)/N\). For \(w_c > w(p^m)\), the relevant constraint is the contract deviation constraint:

\[\phi(p^m) \geq N(1 - \delta) \phi(w^c),\]

which is binding at a unique \(w_2^c(\delta, N) < p^m \iff \delta < (N - 1)/N\). Hence, for \(\delta \in (\hat{\delta}^1(N), (N - 1)/N)\), the range of optimal wholesale prices is \([w_1^c(\delta, N), w_2^c(\delta, N)] \subset (0, p^m)\). For \(\delta \geq (N - 1)/N\), both non-deviation constraints hold for any \(w^c \in [0, p^m]\), hence any wholesale price between 0 and \(p^m\) is optimal.

As \(\hat{\delta}(p^c, N)\) is strictly increasing in \(p^c\) for \(p^c \in (0, p^m)\) (by step 2), and \(\hat{\delta}(p^c, N) \in (0, (N - 1)/N)\)
for all \( p^c \in (0, p^m] \), there exists a unique \( \delta^0 (N) \in (0, (N - 1)/N) \) such that some collusion is sustainable if and only if \( \delta > \delta^0 (N) \). For any \( \delta \in \left( \delta^0 (N), \delta^1 (N) \right) \), the solution of \( P \) is uniquely defined by the following system of equations: \( \delta = \frac{1}{N} \ln \left( \frac{1}{\phi (p^c (\delta, N), N) \phi (w^c (\delta, N))} \right) \) and \( w^c (\delta, N) = w (p^c (\delta, N)) \), which, using (A3), can be rewritten as:

\[
\phi (p^c (\delta, N)) = (1 - \delta) N \phi (w^c (\delta, N)),
\]
\[
w^c (\delta, N) = w (p^c (\delta, N)).
\]

The fact that \( \delta (p^c, N) \) is strictly increasing in \( p^c \) for \( p^c \in (0, p^m] \) implies that \( \delta^0 (N) \) is implicitly defined by \( p^c (\delta, N) = 0 \).

**Proof of Proposition 3.** Collusion on price \( p^c \in (0, p^m] \) is sustainable if and only if there is a contract \( (F^c, w^c) \) such that both (12) and (13) hold at price \( p^c \). The price deviation constraint in (12) can be rewritten as:

\[
F^c \leq \frac{1}{N} D (p^c) (p^c - w^c) - \frac{1 - \delta}{\delta} \max \left\{ \frac{N - 1}{N} D (p^c) (p^c - w^c), \frac{1}{N} D (p^c) (w^c - p^c) \right\}.
\]

(A9)

Because the contract deviation constraint in (13) is easier to satisfy the larger \( F^c \), collusion is sustainable if and only if (13) holds when \( F^c \) is equal to the upper bound in (A9), which amounts to the following condition:

\[
\frac{\phi (p^c)}{N (1 - \delta)} \geq \phi \left( \min \{w^c, p^m\} \right) + \frac{1}{\delta} \max \left\{ \frac{N - 1}{N} D (p^c) (p^c - w^c), \frac{1}{N} D (p^c) (w^c - p^c) \right\}.
\]

(A10)

There are three possible ranges of \( w^c \): (i) \( w^c = p^c \), (ii) \( w^c < p^c \), (iii) \( w^c > p^c \). We consider each case in turn:

(i) If \( w^c = p^c \), then (A10) simplifies to:

\[
\frac{\phi (p^c)}{N (1 - \delta)} \geq \phi (p^c),
\]

which holds if and only if \( \delta \geq \frac{N - 1}{N} \). This completes the proof of the *if* statement in the proposition.

Any price \( p^c \in (0, p^m] \) can be sustained by means of the contract offers \((0, p^c)\) if \( \delta \geq \frac{N - 1}{N} \).

(ii) If \( w^c < p^c \), then (A10) becomes:

\[
\frac{\phi (p^c)}{N (1 - \delta)} \geq \phi (w^c) + \frac{N - 1}{\delta N} D (p^c) (p^c - w^c).
\]

(A11)

If \( \delta \leq \frac{N - 1}{N} \), then the right-hand-side of (A11) is at least \( \phi (p^c) + w^c (D (w^c) - D (p^c)) > \phi (p^c) \), whereas the left-hand side of (A11) is at most \( \phi (p^c) \). Hence, (A11) is violated for any \( \delta \leq \frac{N - 1}{N} \).
If \( w^c > p^c \), then (A10) can be rewritten as:

\[
\frac{\phi(p^c)}{N(1 - \delta)} \geq \phi(\min \{w^c, p^m\}) + \frac{D(p^c)(w^c - p^c)}{N - 1}.
\]  

(A12)

As \( \phi(\min \{w^c, p^m\}) > \phi(p^c) \) and \( D(p^c)(w^c - p^c) > 0 \) if \( w^c > p^c \), (A12) is violated whenever \( N(1 - \delta) \geq 1 \Leftrightarrow \delta \leq \frac{N-1}{N} \).

Because (A10) never holds if \( \delta < \frac{N-1}{N} \), we can conclude that collusion is sustainable only if \( \delta \geq \frac{N-1}{N} \).

\[\blacksquare\]

Proof of Proposition 5. Recall from Proposition 2 that \( 0 < N\phi(w(p^m)) - \phi(p^m) \) (otherwise \( \delta_1 < 0 \)). Define the price \( \tilde{p} \in (0, w(p^m)) \) as follows:

\[
\tilde{p} : D(\tilde{p})(w(p^m) - \tilde{p}) = N\phi(w(p^m)) - \phi(p^m) - \Delta,
\]  

(A13)

where \( \Delta \in (\max \{0, N\phi(w(p^m)) - \phi(p^m) - D(0)w(p^m)\}, N\phi(w(p^m)) - \phi(p^m)) \). As the left-hand side of (A13) is strictly decreasing in \( \tilde{p} \) and goes from \( D(0)w(p^m) \) to 0 as \( \tilde{p} \) goes from 0 to \( w(p^m) \), (A13) has a unique solution in \((0, w(p^m))\).

Consider the following penal code:\(^{35}\)

- If a retailer deviates to a contract with \( \bar{w} \in [\tilde{p}, w^c) \) in period \( \tau \), then
  - at the price-setting stage \((T = 2)\) of period \( \tau \), all retailers charge \( \tilde{p} \) and each sells share \( sD = \frac{1}{N} \) of total demand,
  - the game reverts to the collusive path from period \( \tau + 1 \) onwards.

- If a retailer deviates to a contract with \( \bar{w} < \tilde{p} \) in period \( \tau \), then:
  - at the price-setting stage \((T = 2)\) of period \( \tau \), all retailers charge \( \tilde{p} \), the deviator serves a share:
    \[
sD = \frac{1}{N} \frac{\phi(p^m)}{\phi(\tilde{p})} - \varepsilon_D
    \]
    of total demand, where \( \varepsilon_D > 0 \) is an arbitrarily small number, and the other retailers split the remaining demand equally.
  - the game reverts to the collusive path from period \( \tau + 1 \) onwards.

- If a retailer deviates in price (from either the collusive path or a punishment in play), then the game reverts to the symmetric competitive equilibrium with zero profits for all firms from the next period onwards.

\(^{35}\)Since deviations to supply contracts with \( \bar{w} > w^c \) are unprofitable even if the other retailers ignore the deviation and continue to charge the collusive price and offer the collusive supply contract in future periods, they will be ignored.
The proof will proceed by showing that all sustainability conditions for collusion on the monopoly price \( p^m \) are slack at \( \delta = \delta_1 (N) \) if \( \varepsilon_D \) is small enough and \( w^c \) is chosen appropriately, which implies that full collusion is sustainable for some \( \delta < \delta_1 (N) \).

Collusion is sustainable if and only if the following four conditions are met: (i) no retailer has an incentive to deviate in price from the collusive path, (ii) no retailer has an incentive to deviate in contract from the collusive path, (iii) no punisher has an incentive to deviate from a punishment in play, (iv) the deviator has no incentive to deviate from its own punishment. Because the punishments of contract deviations involve reversion to the collusive path in the following period and price deviations are punished by reversion to a static equilibrium, (iii) and (iv) boil down to ruling out deviations from punishments of contract deviations at the price-setting stage of the deviation period.

There are no profitable price deviations from the collusive path (condition i) if and only if:

\[
\frac{\phi(p^m)}{N(1-\delta)} \geq D(p^m) \left( p^m - \frac{N-1}{N}w^c \right).
\]  

(A14)

As shown in the proof of Proposition 2, (A14) is binding if \( p^c = p^m \), \( w^c = w(p^m) \), and \( \delta = \delta_1 (N) \). From now onwards let:

\[ w^c = w(p^m) + \varepsilon_C, \]

where \( \varepsilon_C > 0 \) is small. Because the right-hand-side of (A14) is decreasing in \( w^c \), (A14) is slack at \( \delta = \delta_1 (N) \) for any \( w^C > w(p^m) \). Hence, (A14) will hold for some \( \delta < \delta_1 (N) \).

Next, we rule out price deviations in the contract deviation period (conditions iii and iv). As \( \hat{p} < w(p^m) \), the punishers are selling below cost at the price-setting stage of the deviation period. Hence, their best deviation is to raise price to earn zero instead of a negative profit, but this triggers reversion to the static equilibrium with zero profits in all future periods. Hence, the punishers are willing to comply with the punishment in play if and only if:

\[
\frac{1-s_D}{N-1} D(\hat{p}) (\hat{p} - (w^m + \varepsilon_C)) + \frac{\delta}{1-\delta} \frac{\phi(p^m)}{N} \geq 0,
\]

which can be rewritten as:

\[
s_D \geq 1 - \frac{\delta}{1-\delta} \frac{N-1}{N} \frac{\phi(p^m)}{D(\hat{p}) (w^m + \varepsilon_C - \hat{p})}, \]  

(A15)

where \( s_D \) denotes the market share of the deviator. At \( \delta = \delta_1 (N) \), (A15) is:

\[
s_D \geq 1 - \frac{N-1}{N} \frac{N\phi(w^m) - \phi(p^m)}{D(\hat{p}) (w^m + \varepsilon_C - \hat{p})}.
\]  

(A16)

For \( \varepsilon_C \) close enough to 0, (A13) implies:

\[
D(\hat{p}) (w^m + \varepsilon_C - \hat{p}) < N\phi(w^m) - \phi(p^m),
\]  

(A17)
which implies that the right-hand side of (A16) lies strictly below \( \frac{1}{N} \). For \( \varepsilon_D \) close enough to 0, \( s_D \geq \frac{1}{N} \) for any \( \tilde{w} \) in the above-described penal code, hence (A16) is slack.

The nature of the deviator’ optimal deviation from its own punishment at the price-setting stage \((T = 2)\) of the deviation period depends on \( \tilde{w} \). For \( \tilde{w} \in [\hat{p}, w^c) \), the most profitable deviation is to raise price to make zero instead of negative short-term profits. This deviation is unprofitable if and only if:

\[
\frac{1}{N} D (\hat{p} \tilde{p} - \tilde{w}) + \frac{\delta}{1 - \delta} \frac{\phi(p^m)}{N} \geq 0. \tag{A18}
\]

For \( \delta = \delta_1(N) \), (A18) is equivalent to:

\[
1 \leq \frac{N \phi(w^m) - \phi(p^m)}{D (\hat{p} \tilde{p} - \tilde{w})}. \tag{A19}
\]

(A19) holds for all \( \tilde{w} < w^C = w^m + \varepsilon_C \) if it holds for \( \tilde{w} = w^m + \varepsilon_C \). Because (A17) holds for \( \varepsilon_C \) close enough to zero, it follows that (A19) is slack for all \( \tilde{w} \in [\hat{p}, w^c) \) provided \( \varepsilon_C \) is small enough.

For \( \tilde{w} < \hat{p} \), the deviator’s best deviation from its own punishment is to undercut \( \hat{p} \) in order to serve the entire demand, which is unprofitable if and only if:

\[
\left(1 - \frac{1}{N} \frac{\phi(p^m)}{\phi(\hat{p})} + \varepsilon_D \right) D (\hat{p} \tilde{p} - \tilde{w}) \leq \frac{\delta}{1 - \delta} \frac{\phi(p^m)}{N}. \tag{A20}
\]

If (A20) holds for \( \tilde{w} = 0 \), then it will hold for any \( \tilde{w} < \hat{p} \). At \( \delta = \delta_1(N) \) and \( \tilde{w} = 0 \), (A20) becomes:

\[
\phi(\hat{p}) (1 + \varepsilon_D) \leq \phi(p^m). \tag{A21}
\]

As \( \phi(\hat{p}) < \phi(p^m) \), (A21) is slack for \( \varepsilon_D \) close enough to 0.

It remains to show that the condition that rules out contract deviations from the collusive path is slack for \( \delta = \delta_1(N) \). The maximum slotting fee that a retailer can obtain by deviating to a wholesale fee \( \tilde{w} < w^c \) is:

\[
s_D D (\hat{p} \tilde{w}).
\]

Hence, the condition that rules out contract deviations becomes:

\[
V^C = \frac{1}{1 - \delta} \frac{\phi(p^m)}{N} \geq s_D (\hat{p} \tilde{p}) + \frac{\delta}{1 - \delta} \frac{\phi(p^m)}{N} \tag{A22}
\]

\[
\frac{\phi(p^m)}{N} \geq s_D \phi(\hat{p}). \tag{A23}
\]

Because the penal code prescribes either \( s_D = \frac{1}{N} \) or \( s_D \leq \frac{1}{N} \phi(\hat{p}) - \varepsilon_D \), (A23) is always slack. ■

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References


[37] Rey, P., and Whinston, M. “Does Retailer Power lead to Exclusion?” IDEI working paper no. 666, Institute of Industrial Economics, University of Toulouse 1, 2011.


Figure 1: Optimal cartel agreement in a duopoly with linear demand