Informational opacity and honest certification

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Abstract

This paper studies the interaction of disclosure rules and reputational concerns in certification markets. We argue that by revealing less information a certifier substantially reduces the threat of capture. Opaque disclosure rules may reduce profits but also constrain feasible bribes. If quality is binary, noisy disclosure rules dominate full information revelation. For three or more specifications of quality, a cut-off disclosure rule where several qualities are bundled into one certificate will improve certifier credibility. Our results have important policy implications regarding certification markets in general and rating agencies in particular.

Keywords: Certification; Collusion; Bribery; Reputation

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1 Introduction

The important effects of information asymmetries on the allocation and distribution of resources are well established in the literature. Concerning product quality those effects may be particularly severe as producers may choose to underinvest in quality which eventually results in a breakdown of trade. Consequently, a large body of literature identifies ways to bypass those problems, reputation and warranties are two examples. In many cases those represent only partial solutions and the lack of credible communication between informed and uninformed parties can result in the emergence of a particular type of market institution: certification intermediaries. Certifiers inspect products whose characteristics are private information to agents and fully or partially reveal this information to third parties, e.g. consumers. Examples of certifiers are laboratories that test consumer products, auditors that validate the accounts of firms, rating agencies that assign credit ratings for issuers of debt obligations and schools that certify the ability of students.

Certifiers themselves, however, might be tempted to accept bribes for releasing favored certificates. This behavior, which we shall henceforth call capture, enables the certifier to extract payments other than the certification fee and may relieve him from the need to spend resources on determining product quality. In this paper we focus on a situation where consumers are aware of those threats of capture. In particular, their ability to learn whether a certifier has been captured makes the certifier face a classical reputation dilemma, where he has to decide whether the short-run gain from capture outweighs the future profit losses from losing credibility.

This paper investigates the role of reputation as a safeguard against capture. For this purpose we study a model of moral hazard where producers first have to make an investment choice which in turn determines the probability distribution of his product’s quality. The payoffs assigned to each quality outcome thus determine incentives to invest. The certifier has at his disposal two instruments to influence producer behavior, a fixed certification fee and the disclosure rule. Our major finding is that to maintain credibility, a certifier also makes use of the second tool. More precisely, we show that under certain circumstances honest certification requires partial disclosure of quality.

If quality can only take on two different specifications, full disclosure yields maximal static certifier profits. But for medium discount factors and sufficiently low screening costs, a noisy disclosure rule improves certifier credibility. We focus on disclosure rules that allow for unambiguous deviation detection after consumption, namely such where the assignment of valuable certificates will guarantee high quality levels. If faced with more than two quality levels, credibility can also be maintained by clustering different quality levels into subclasses,
thereby revealing only partial information.

The intuition behind our results is the following. The short term gain from being captured consists in the payment of a bribe as well as the savings made on product screening. Because the amount of the highest bribe the certifier can demand directly depends on the publicly announced disclosure rule, this difference is largest for full disclosure and can be substantially reduced by revealing less precise information. On the other hand, static profits are maximized under full disclosure, and revealing only partial information will typically reduce the long-term loss from losing credibility. The first effect can exceed the latter only if more quality levels are certified, but this will necessarily increase screening costs which in turn reduces certifier profits. Credibility can thus only be improved as compared to full disclosure if these costs do not surmount a certain level.

The paper closest to ours is Strausz (2005) which in a pure adverse selection setting analyzes the effects of the threat of capture on certification prices. Allowing only for full disclosure, he finds that in order to maintain credibility, a certifier will be required to set prices above the static monopoly price for low discount factors. The results resemble our main findings, in that a higher certification fee in Strausz’ model implies less information being disclosed. Our paper extends the model in Strausz (2005) along two dimensions, on the production side by introducing moral hazard and on the certification side by endogenizing the choice of the disclosure rule. Our results suggest that the latter is of importance when it comes to reputational concerns of certifiers.

Apart from that, the analysis of this article will touch on issues that are related to those in the literature on the disclosure of private information and the literature on reputation building under asymmetric information.

Regarding the literature on disclosure of private information our paper is related to Lizzeri (1999) and Albano and Lizzeri (2001). Both papers address the question of optimal information revelation by certification intermediaries. Lizzeri (1999) finds that it is optimal for a monopolistic certifier in a static adverse selection environment to reveal almost no information. Our model is more general in that it endogenizes the distribution of quality and takes into account bribery and reputational concerns of the certifier. Albano and Lizzeri (2001) study optimal disclosure rules in a static model of both moral hazard and adverse selection. As opposed to this paper, they assume that certifiers do not incur any observation costs, that producers are heterogeneous in production costs and they produce goods of particular quality. Some of our findings are similar, but we also get different results which we shall discuss in detail throughout the paper. Faure-Grimaud et al. (2009) study contracts between firms and rating agencies in a static adverse selection model where quality cannot fully be observed by the seller. They identify conditions under which the ownership of certification results is
left to firms. Their work as well as Lizzeri (1999) is related to ours in that it addresses the question of the utility of reduced information revelation in certification markets.

In the literature on reputation building our framework has similarities with some articles, e.g. Shapiro (1983), Biglaiser (1993). The question of information revelation however is addressed in none of them. Levin (2003) extends the standard moral hazard setting to situations where contractual agreements are enforceable only to a certain degree and where reciprocal relations are long-term. He finds that effort levels induced are lower as compared to the standard setting. Some driving factors in his model can be found in our setting as well.

There is also a large literature on reputational concerns of rating agencies, e.g. Mathis et al. (2009). The standard approach assumes that a certifier has an exogenously given type - honest or opportunistic - and examines conditions under which an opportunistic agency actually follows honest certification.

Lastly, we provide a novel explanation for why certifiers often choose to only partially reveal their information. Certifiers can often be observed to choose not to reveal all information they have at hand, examples are rating agencies, bio-labels or teachers grading their students. We already mentioned Lizzeri (1999). Farhi et al. (2009) explain this effect by assuming that sellers are information-averse and Dubey and Geanakoplos (2010) offer an explanation by analyzing a model in which a grading scheme, i.e. the disclosure rule of a performance measure, is chosen to induce higher levels of effort.

The paper is organized as follows. Section 2 presents the model. Section 3 analyzes the case of two levels of quality. Section 4 treats the general case of three or more quality specifications. Section 5 concludes. All proofs are relegated to the appendix.

2 The setup

We consider a dynamic framework in discrete time. Each period $t = 1, 2, \ldots, \infty$ a short-lived monopolistic producer is born. He produces a single unit of quality $q_t \in \{q^0, q^1, \ldots, q^n\}$, where we assume quality levels to be ordered, i.e. $0 = q^0 < q^1 < \ldots < q^n \leq 1$. Prior to production, a producer chooses some investment level $e_t \in [0, 1]$ which influences the quality he may produce. In particular quality is stochastic and drawn from a probability distribution $\text{Prob}(q_t \leq q | e_t) = F(q | e_t)$, which does not depend on time, i.e. quality levels are independent across time. The family of distributions $\{F(q | e) | e \in [0, 1]\}$ is assumed to satisfy first-order stochastic dominance, i.e. $F(q | e) \leq F(q | \tilde{e})$ for all $q$ and $e > \tilde{e}$. For simplicity we assume $F(0 | 0) = \text{Prob}(q_t = 0 | e_t = 0) = 1$, hence without investment quality $q^n \leq 1$ is assumed to ensure interior solutions for some of our specified probability distributions.
producer chooses $e_t$
good is produced
begin of
period $t$

begin of
period $t$ + 1

producer learns $q_t$
good is sold in auction
consumers learn $q_t$

Figure 1: Timing in one period without certification

is at the lowest level with certainty. Investment is costly and the cost function $k(\cdot)$ is twice continuously differentiable and satisfies $k'(e) > 0, k''(e) > 0$ for all $e \neq 0$ and $k'(0) = 0$ as well as $k'(1) \geq 1$. The good’s quality represents the reservation price of consumers. Both investment level and product quality are a producer’s private information, i.e. the market exhibits informational asymmetries. Consumers observe the product’s quality only after consumption and never observe the investment decision of producers. All other components of the model are common knowledge.

Each producer is short-lived and leaves the market after having offered his good in a second price auction\(^2\). Figure 1 summarizes the timing in one period $t$.

Without any further economic institutions being present, the producer cannot persuade consumers of believing his product to be of high quality, and the market price cannot be made contingent on the good’s quality. The market outcome can be computed with standard tools. Consumers form a belief $q_t^c$ about the offered quality. In equilibrium, this belief has to be consistent with the actual expected quality $E(q_t|e_t)$. Given any belief, the producer’s optimal choice of investment will be $e_t = 0$, as he maximizes $q_t^c - k(e_t)$. But since $E(q_t|0) = 0$, the unique equilibrium must be one where producers choose $e_t = 0$ in every period and the quality of the good is zero in each period. Since the marginal cost of investment at $e = 0^3$ is zero whereas the value of investment is strictly positive due to first order stochastic dominance, the outcome is inefficient. We summarize this finding in the following Lemma.

**Lemma 1** Without certification, producers choose $e_t = 0$ in each period. The good is always of (sure) quality $q_t = 0$ and sold at a price 0.

Due to asymmetric information, the price a producer gets in the second price auction is independent of the good’s actual quality. Producers would benefit from finding a way to assure consumers of having invested a positive amount in order to realize larger profits. Moral hazard however prevents them from doing so.

In this paper we focus on certification as a remedy for this kind of market failure. Assume

\(^2\)The second price auction results in a standard monopoly price which equals consumers’ valuations. It circumvents signalling issues, e.g. letting the informed party take a publicly observed action that might be interpreted as a signal.

\(^3\)We omit subscripts where possible.
that there is an infinitely long lived certifier, who offers to disclose some information about a good’s quality to consumers, prior to the good being sold. At the beginning of the game, in period $t = 0$, the certifier announces a fee $f \geq 0$ and a disclosure rule $D = (C, \mathcal{A})$. The fee is to be paid by any producer who wishes to have her product tested. The disclosure rule consists of set $C = \{C_1, \ldots, C_m\}$ of potential certificates, a product may be awarded with, and a set of probability vectors $\mathcal{A} = \{\alpha^0, \ldots, \alpha^n\}$, where the $k$-th entry of vector $\alpha_i$ reflects the probability a product of quality $q^i$ is awarded certificate $C_k$ whenever tested. Upon testing a product, the certifier incurs a personal cost of $c \geq 0$. The timing of the game with certification is illustrated in figure 2.

We make the following implicit assumptions on the certification process: First, in our setting we allow the certifier to learn the true quality of a product when testing it. In particular, this is irrespective of the amount of information actually needed with regard to the disclosure rule. More information does not hurt the certifier, it however might lead to further deviation possibilities. We will discuss this in section 5. Secondly, our model assumes the certification decision of a producer being unobservable. With probability $1 - \sum_k \alpha^i_k$ a product of quality $q^i$ remains uncertified and is sold under the same "label" as products which were not even tested. This assumption is also not crucial and we will discuss it further in section 5.

Possible disclosure rules encompass for example full disclosure, where $C = \{q^0, \ldots, q^n\}$ and $\alpha^i$ is the $i$-th unit vector, no disclosure, where $C = \{C\}$ and $\alpha^i = (1)$ or cut-off disclosure, where $C = \{C\}$ and $\alpha^i = (1)$ for all $i \geq \kappa$ and $\alpha^i = (0)$ for all $i < \kappa$, in particular all quality levels above $q^\kappa$ receive a certificate with certainty, whereas all other quality levels are never being certified.

Before starting to analyze the certification game, we introduce some additional notation. Let $V_{C_k}$ be the value of a product with certificate $C_k$. This value not only depends on the disclosure rule, but also on the producer’s decision whether to certify a given quality level. Let $d(q) = 1$ if a producer in equilibrium attends the certifier when having a product of
quality $q$ and $d(q) = 0$ otherwise. Bayesian updating yields

$$V_{C_k} = \sum_{j=1}^{n} \frac{d(q_j)\alpha_k^j P(q_j|e)}{\sum_i d(q_i)\alpha_k^i P(q_i|e)} q^j$$

(1)

With slight abuse of notation we will use $V_k$ for $V_{C_k}$ whenever the context allows for unambiguous interpretation. The value of an uncertified product is denoted $V^{un}$ and is derived with a similar formula as (1). Furthermore, denote $\pi^{i,c} = \sum_k \alpha^i_k V_k + (1 - \sum_k \alpha^i_k) V^{un} - f$ the expected profit of a producer when certifying a product of quality $q^i$ under the disclosure rule $D$. The payoff from not certifying the product simply is $\pi^{i,n} = V^{un}$.

3 Binary quality

We first consider the case where quality takes on only two specifications. Let $q^0 = 0$ and $q^1 = q$ denote low and high quality respectively and let $\nu := q - 0$ be the difference in qualities. Furthermore, let $Prob(q|e) = p(e) = e^5$. It is instructive to first analyze the certifier's problem statically, where reputational concerns do not play a role and certificates are always assigned in accordance with the announced disclosure rule. We investigate optimal static disclosure rules in the following section and thereafter continue to analyze the dynamic game. Splitting the analysis this way allows us to disentangle the question of profit maximizing disclosure rules from the capture problem. In the binary case it is always optimal from a pure profit perspective to use a full disclosure rule, whereas it might be impossible to sustain honest certification using such a rule. The absence of a full disclosure rule then is precisely due to reputational concerns and not due to myopic profit maximizing behavior.

3.1 Static solution

With two quality levels, there are only two deterministic disclosure rules the certifier could possibly choose: full disclosure and no disclosure. Clearly, no disclosure will never be optimal from the certifier’s perspective since it does not give any investment incentive to the producer. All products will therefore be of low quality and no producer would be willing to pay a positive fee to get his product certified. The profit of the certifier can never be greater than zero under no disclosure.

As a benchmark, we discuss the case of full disclosure in detail here. The analysis is useful

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Footnotes:

4We set $V_{C_k} = 0$, whenever none of the products, eligible for certificate $C_k$, are being tested.

5Allowing for more general functions $p(\cdot)$ does not add value in the binary quality case, as changes of this kind can be processed by adequately adjusting the producer’s cost function $k(\cdot)$. 

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for an evaluation of allocative effects of certification fees. Let $f \geq 0$ be the fee set by the certifier. A low quality producer always has the option to sell his product uncertified which yields a non-negative price. Certification however results in a price equal to zero and with the certification fee being non-negative, low quality producers never attend the certifier. High quality sellers attend the certifier whenever $v - f \geq 0$. The producer’s expected payoff from investing $e$ is given by

$$ e(v - f) + (1 - e) \cdot 0 - k(e) $$

The optimal level of investment is thus given by the first-order condition

$$ (v - f) = k'(e) $$

Differentiating (3) w.r.t. $f$ and rearranging we get $\frac{de}{df}|_{f=\hat{f}} = -\frac{1}{k''(e(\hat{f}))}$ which has negative sign by our assumptions. Demanding a large fee therefore has two opposing effects on certifier’s profits. On the one hand, profits increase since payments are larger. On the other hand payments are less frequently made, since high quality is less likely. In particular if the certifier intends to capture all rents, i.e. $f = v$, those rents shrink to zero. Anticipating producer’s optimal investment $e(f)$ and only high quality producers applying for a certificate, the certifier’s profit with the fee $f$ is

$$ \Pi(f) = e(f)(f - c) $$

Maximizing (4) with respect to $f$ yields the optimal certification fee under full disclosure $f^{FD}$. From the envelope-theorem the certifier’s profit decreases with $c$. By repeatedly applying the implicit function theorem, we find that the optimal certification fee under full disclosure increases with $c$ and therefore the investment level of producers decreases. The more costly it is for the certifier to test products, the less efficient the market will be, as the efficient investment level is given by $v = k'(e^*)$. The following Lemma summarizes.

**Lemma 2** With a full disclosure rule, the certifier optimally sets a certification fee $f^{FD}$ that solves $f^{FD} - c = e(f^{FD}) \cdot k''(e(f^{FD}))$. The certifiers profit and the equilibrium investment level are decreasing in $c$, whereas the optimal certification fee increases with $c$.

All other possible disclosure rules are necessarily noisy, i.e. the certification process is a lottery over the set of available certificates $C = \{C_1, \ldots, C_m\}$. Let $\alpha^1 = (\alpha^1_1, \ldots, \alpha^1_m)$ and $\alpha^0 = (\alpha^0_1, \ldots, \alpha^0_m)$ be the respective probability vectors for high and low quality products. If the certifier demands a certification fee $f$, producers owning a product of quality $q^i$ demand
Fees can be chosen such that in equilibrium all producers certify or that only high quality sellers certify. Denote $\Pi^D(f)$ the certifier’s profit for $f \geq 0$ and $D = (C, \alpha^1, \alpha^0)$. The following proposition shows that this profit never exceeds that under full disclosure.

**Proposition 1** For any disclosure rule $D = (C, \alpha^1, \alpha^0)$ and any fee $f \geq 0$, the profit of the certifier in the resulting subgame is lower than the largest profit under full disclosure, i.e. $\Pi^D(f) \leq \Pi^{FD}$. The inequality is strict whenever $c > 0$.

The argument of the proof is as follows: first, if in equilibrium only high quality producers demand certification all certificates are worth $v$. A positive probability of not granting a certificate to high quality producers increases the value of uncertified products but also reduces investment incentives. Effectively, for a given fee $f$, lower investment is induced and therefore the certifier’s profit is lower than under full disclosure where a high quality producer always receives a certificate. Second, equilibria where all producers apply for certification necessarily require low fees. A certifier may increase the fee by raising $\pi^0,D$, but this reduces the difference $\pi^{1,D} - \pi^{0,D}$ in payoffs for high and low quality sellers. Reducing $\pi^{1,D} - \pi^{0,D}$ however reduces investment incentives and thereby both $\pi^{0,D}$ and $\pi^{1,D}$. The proposition shows that this effect necessarily reduces the certifiers profits, although the demand for certification is larger.

Proposition 1 contrasts the findings in Albano and Lizzeri (2001). In their model, full disclosure is only optimal with a non-linear pricing scheme, whereas with a flat fee noisy disclosure strictly dominates full disclosure. One difference to their model is the cost of certification $c$ and our Proposition 1 suggests that a certifier’s cost plays a significant role for the choice of optimal disclosure rules. For $c = 0$ we find noisy disclosure rules that yield the certifier a profit of $\Pi^{FD}$. However, with only two quality levels a flat fee for a full disclosure rule can also be interpreted as a non-linear pricing scheme in the sense of Albano and Lizzeri (2001, Proposition 4) and our results reflect their findings in that special case.

### 3.2 The capture problem

So far we assumed that the certifier sticks to the disclosure rule he announced, in particular that he conducts the lottery honestly and grants the respective certificate. Yet, there exists
pressure from producers to award their product better certificates, for instance award a sure 
certificate instead of some lottery if a noisy disclosure rule is applied. In addition, the cer-
tifier may simply announce some certificate without expending resources to determine the 
actual quality of the product. In this section we address these problems by introducing the 
possibility of capture.

We follow Strausz (2005) in modelling the possibility of capture, using the framework of 
enforceable capture as initiated by Tirole (1986). This framework assumes that the certifier 
and the producer can write an enforceable side-contract with transfers. Consumers are fully 
aware of the possibility of these side-contracts, but cannot observe them.

We extend our model as follows: After a producer with a product of quality $q$ enters, the 
certifier, without observing $q$, may make an offer $(C, b)$ to the producer. The offer consists 
of a certificate $C$, issued in case of acceptance, and a financial transfer $b$ to be paid by the 
producer. The certifier thus offers to ”sell” the sure certificate $C$ at the price $b$, and thereby 
avoids to spend resources on determining the true quality of the product. A producer how-
ever can reject this offer and insist on honest certification by paying the fee $f$. This last 
assumption is motivated following Kofman and Lawarrée (1993) in assuming that the certi-
fier cannot forge certification without the help of the producer.

Observe that the choice of the disclosure rule puts some limits on the set of feasible capture 
offers. For a general disclosure rule $D = \{C, \alpha\}$ only offers of the form $(C, b)$ with $C \in C$ 
are feasible. With full disclosure this comprises offers of the type $(q, b)$ with $q \in \{0, ..., q^n\}$.

With a cut-off disclosure rule, where all products whose quality exceed a certain threshold 
receive a certificate which we denote $C^{co}$, only offers of the form $(C^{co}, b)$ are feasible.

Within the framework presented here, capture may subvert honest certification for two rea-
sons. First, producers with low quality products are willing to side-contract with the certifier 
in order to obtain a higher certification and sell their product at a higher price. Second, a 
certifier can save the cost $c$ when being captured. Hence, by allowing collusion before the 
certifier lays out $c$ and learns the actual quality of the product, we may analyze these two 
threats simultaneously.

We assume that consumers trust certificates as long as they do not detect deviation. A 
certifier who anticipates this behavior may be prevented from succumbing to the temptation 
of becoming captured because he knows that loosing credibility will result in zero demand 
in future periods. As discussed earlier, the issuing of certificates that are not covered by the 
previously announced disclosure rule is interpreted by consumers as a sign of dishonesty and, 
subsequently, makes them believe that the product is of the lowest quality. Also, certificates 
which are not issued in equilibrium will be treated in the same manner.

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6See also Laffont and Tirole (1991) and the survey in Tirole (1992) and Khalil and Lawarrée (1995)
In order to make consumer beliefs more precise, let \( h_t = (n_t, C_t, q_t) \) denote the certification outcome in period \( t \), where \( n_t \in \{0, 1\} \) indicates whether certification in period \( t \) took place, \( C_t \) represents the certificate issued by the certifier and \( q_t \) is the actual quality observed after consumption. If certification in period \( t \) did not take place, then \( n_t = 0 \) and \( C_t = \emptyset \). Now let \( H_t = (h_1, \ldots, h_{t-1}) \) summarize the history of certification at the beginning of period \( t \). Finally, we denote \( q_t^e(n_t, C_t, H_t) \) a consumer’s belief in period \( t \) when faced with a product that is assigned a certificate \( C_t \) and when having observed history \( H_t \). The consumer’s behavior may then be comprehended by the following assumption about beliefs.

**Assumption 1** The consumers’ beliefs \( q_t^e(n_t, C_t, H_t) \) satisfy \( q_t^e(1, C_t, H_t) = V_{C_t} \) whenever \( C_t \in \mathcal{C} \) and \( \{ \tau < t | n_\tau = 1 \land \text{Prob}(C = C_\tau | q = q_\tau) = 0 \} = \emptyset \). Moreover \( q_t^e(1, C_t, H_t) = 0 \) whenever \( \{ \tau < t | n_\tau = 1 \land \text{Prob}(C = C_\tau | q = q_\tau) = 0 \} \neq \emptyset \) or \( C_t \notin \mathcal{C} \).

The Assumption states that consumers trust the certifier whenever he announces an available certificate (from the set \( \mathcal{C} \)) and cheating has not been detected in previous periods. On the other hand, if consumers are certain that capture has taken place or if the certifier announces some certificate outside of \( \mathcal{C} \), consumers will always believe the product to be of the lowest quality 0. Assumption 1 therefore captures the intuitive idea that consumers trust the certifier whenever they have no reason to distrust him. In equilibrium beliefs have to be rational, i.e. they have to be confirmed, hence beliefs following our assumption lead to honest equilibria, in the sense that capture takes place with probability zero.

It is instructive to first abstract from the certifier’s choice concerning the disclosure rule. Consequently we proceed by analyzing the capture problem under a full disclosure rule first and afterwards investigate the impact of the certifier’s ability to alter information revelation. Our setting with two levels of quality allows for two types of bribing offers: \((q, b)\) and \((0, b)\). Obviously, an offer \((0, b)\) is turned down by all types of producers, as it is worth nothing. Hence, in the following we focus on offers \((q, b)\) and with slight abuse of notation talk of a bribe \( b \), rather than \((q, b)\). An offer \( b \) is accepted by high quality producers whenever \( b < f \). Low quality producers accept any bribe \( b < v \), as acceptance will yield positive profits compared to zero profits for rejection. Given equilibrium play by producers, i.e. by the time producers have to make an investment decision they do not expect being offered a bribe they would be willing to accept, the quality of a product will be high with probability \( p(e(f)) \) and the acceptance probability of a bribing offer \( b \) at a certification fee \( f \) is

\[
\alpha(b|f) = \begin{cases} 
1, & b < f \\
1 - e(f), & f \leq b < v \\
0, & b \geq v
\end{cases}
\]
Using this probability, one may calculate the certifier’s expected payoff $\Pi(b|f)$ from a bribing offer $b$. Let $\Pi^h(f) = \frac{1}{1-\delta} \Pi(f)$ be the certifier’s expected long term profit from honest certification and $\delta$ the discount rate. For $b < f$ all producer types will accept the bribe, but only for low quality producers this is perceived as cheating. Then the resulting expected profit from capture is $\Pi(b|f) = b + e(f)\delta \Pi^h(f)$. For $f \leq b < v$, only low quality producers accept the bribe and the profit is $\Pi(b|f) = (1 - e(f))b + e(f)(f - c + \delta \Pi^h(f))$. Whenever $b \geq v$, all producers reject the bribe and the certifier obtains $\Pi(b|f) = \Pi^h(f)$. Due to the positive cost $c > 0$, the expected profit from an offer $b$ features a jump at $b = f$ and one at $b = v$. As the function is increasing in $b$ both on the interval $[0,f)$ and $[f,v)$ this profit is maximized either for $b \to f$ or for $b \to v$.

The payoff $\Pi(b|f)$ represents the certifier’s expected payoff from making the bribing offer $b$. If it exceeds the certifier’s payoff from honest certification $\Pi^h(f)$ the certifier is actually better off becoming captured with the associated probability $\alpha(b|f)$. We say that certification at a price $f$ is capture proof if and only if

$$\Pi^h(f) \geq \Pi(b|f)$$

for all $b$. An analysis of condition (6) yields the following result:

**Proposition 2** In the case of full disclosure and the certification fee being $f$, an equilibrium satisfying Assumption 1 is capture proof. It exists if and only if $f \geq c$ and

$$\delta \geq \begin{cases} \tilde{\delta}(f) \equiv \frac{v}{\tilde{v} + \Pi(f)}, & c \leq c(f) \\ \tilde{\delta}(f) \equiv \frac{f - \Pi(f)}{f - \Pi^h(f)}, & c > c(f) \end{cases}$$

with $c(f) = \min\{f, \frac{(v-f)(1-e(f))}{e(f)}\} > 0$.

The proposition highlights the crucial role the discount factor plays for the existence of honest, i.e. capture proof, equilibria. Consider the case of low cost levels, when the first case applies. Here the intuition behind Proposition 2 is straightforward. For low cost levels, as in standard repeated games, the critical discount factor determines the relative weights of the short run gain - the bribe $b$ - and the long run loss of capture - forgone future profits from certification. To see this, note that all bribes $b < v$ are accepted with some positive probability, thus the largest possible short-run gain equals $v$. In the long, a certifier fears for his per-period profits $\Pi(f)$. The certification fee only enters via the per-period profit. This is the case, because for low costs of certification $c$, it turns out to be the low quality producers who exert largest pressure on the certifier’s honesty and those who accept a large bribe $b \approx v$ are exactly those who anyway do not certify.
For larger costs $c$ this last point is not true anymore. Rather then offering a bribe which is going to be accepted only by low quality producers, a certifiers profit $\Pi(b|f)$ is larger if offering a sure certificate to all potential producers and thereby only saving the certification costs. In this case, the largest possible bribe is $f$, but relative to what would be earned honestly the (expected) short run gain is $f - \Pi(f)$. The (expected) long run loss is $(1 - e(f))\Pi(f)$, since the certifier only forgoes future profits if the bribe is actually detected, i.e. if collusion with a low quality producer takes place.

The two curves $\hat{\delta}(f)$ and $\bar{\delta}(f)$ are shown in figure 3. As formally proven in the appendix the curve $\hat{\delta}(f)$ is convex on $[c, v]$ and exhibits a unique minimum. Also the curve $\bar{\delta}(f)$ exhibits a unique minimum, though not being convex on the whole interval $[c, f]$. Honest certification with a fee $f$ and full disclosure therefore is an equilibrium for some $\delta$, whenever $\delta \geq \max\{\hat{\delta}(f), \bar{\delta}(f)\}$. These properties are summarized in the following Proposition.

**Proposition 3** For any discount factor $\delta \geq \delta^{FD}$ there exists at least one fee $f$ which sustains honest certification under full disclosure, where

$$\delta^{FD} = \begin{cases} \hat{\delta}(f^{FD}(c)) = \frac{v}{v + \Pi^{FD}(c)}, & c < c_1 \\ \hat{\delta}(\tilde{f}(c)) = \bar{\delta}(\tilde{f}(c)), & c_1 \leq c \leq c_2 \\ \bar{\delta}^{FD}(c), & c > c_2 \end{cases}$$

where $\tilde{f}(c) = \arg\min_f \bar{\delta}(f)$ and $\tilde{f}(c) < \tilde{f}(c) < f^{FD}(c)$ is given in the proof.

For any discount factor $\delta < \delta^{FD}$ no capture-proof equilibrium can be sustained.
An immediate consequence from the last two propositions is that the static monopoly fee $f^{FD}$ is the only fee that can sustain honest certification for all discount factors $\delta \geq \delta^{FD}$, whenever the certifier’s costs are not too high. This result stands in stark contrast to the finding in Strausz (2005), where the certifier, although using a full disclosure rule, might have to set a fee higher than the optimal static one in order to signal honesty. Consequently, in our setting adjusting the certification fee cannot serve as a tool to credibly signal honesty. In Strausz (2005) the fee has a direct effect on the maximal bribe, which will be accepted by producers who do not intend to certify otherwise. Like in our model the maximum threat of capture stems from such an offer. However, with only two quality specifications, the largest acceptable bribe will always be $v$, independent of the fee charged by the certifier. To be more precise, the largest possible bribe is determined by the difference in values of the highest certificate, i.e. the certificate which results in the largest price in the auction where the product is sold, and the worst certificate, which results in the lowest market price for the product. If in equilibrium some producers abstain from certification, the lowest certificate corresponds to selling a product uncertified. For full disclosure this difference equals $v$, irrespective of the certification fee which explains the results we obtained so far.

This paper proposes that alternative disclosure rules can improve the certifier’s credibility by reducing the maximal bribe accepted and thereby the short-run gain from capture. We stick with the restriction of out-of-equilibrium beliefs, formulated in Assumption 1. This restriction implies, that we do not address the question of finding the lowest possible discount factor, which still allows for capture-proof equilibria with possibly some other beliefs. Instead we want to argue, that alternative disclosure rules may enlarge the set of discount factors for which honest equilibria exist. Other belief specifications would necessarily lead to some opportunistic behavior by the certifier also on the equilibrium path, as unambiguous deviation detection becomes impossible. We will discuss these issues later in Section 5.

As we allow for noisy disclosure in general, consumers might not be able to detect certain deviations allowed by noisy disclosure rules\(^7\). In fact only two types of disclosure rule allow for capture-proof equilibria under Assumption 1, namely

(A) $\mathcal{C} = \{C_1, C_2\}$\(^8\), $\alpha^1 = (1 - \alpha, \alpha)$ and $\alpha^0 = (1, 0)$. In words: the certifier awards two different certificates, where one certificate ($C_2$) implies that the product is of high quality whereas the other ($C_1$) is granted to both types of products.

In equilibrium all producers demand certification.

\(^7\)E.g. let $\mathcal{C} = \{C_1, C_2\}$ and $\alpha_1^1 \neq 0 \neq \alpha_1^2$ as well as $\alpha_2^1 \neq 0 \neq \alpha_2^2$, i.e. both certificates are granted with positive probability to both types of products. W.l.o.g. let $V_{C_1} > V_{C_2}$. It is profitable to sell the sure certificate $C_1$ to low quality producers. Consumers cannot detect this with certainty.

\(^8\)A larger set of certificates would be possible, important is that there is only one which is granted to low quality products and no product remains uncertified.
(B) $\mathcal{C} = \{C\}$, $\alpha^1 = \alpha$ and $\alpha^0 = 0$. In words, there is only one certificate which is only granted to high quality products with probability $\alpha \in (0, 1]$.9

In equilibrium only high quality producers demand certification10.

Note, that for case (A) it would be impossible to detect an accepted capture offer $(C^1, b)$ but such an offer would not be accepted by any producer unless $b < f$ which makes such a bribe unprofitable for the certifier, as he could always get $f$ when staying honest.11

In order to investigate the threat of capture for all excluded disclosure rules, one has to resort to the theory of repeated games with imperfect monitoring (e.g. Fudenberg et al. (1994)). To show our main points this merely generates an unnecessary complication.

We continue by analyzing the two before-mentioned types of disclosure rules under the threat of capture. Starting point is type (B). The effect of such a disclosure rule is to increase the value of an uncertified product and thereby reduce the largest bribe that is still going to be accepted with positive probability. Low quality producers only accept a bribe $(C, b)$ when $v - b > V^\text{un} \iff b < v - V^\text{un}$. Whenever $\alpha < 1$ we have $V^\text{un} > 0$ and therefore $b < v - V^\text{un} < v$. That is, the short-run gain from capture is unambiguously reduced which facilitates the existence of capture-proof equilibria. But those disclosure rules also reduce the certifiers per period profit, as is shown in Proposition 1. Setting $\alpha < 1$ has an distortive effect on the final allocation as it typically reduces a producer’s investment. This reduces the per period profit and lowers the long-run loss from capture. But it also increases the short-run gain as bribes are accepted with higher probability, since more producers will have a low quality product. In our next proposition we show that this will make it even harder to sustain honest equilibria.

Lemma 3 Let $D$ be a disclosure rule of type (B). For any fee $f$, such that in the static equilibrium all high quality producers demand certification no honest equilibrium can be sustained for any $\delta < \delta^{FD}$.

Let us now consider disclosure rules of type (A). In an equilibrium where all producers demand certification all low quality producers receive certificate $C_1$. High quality producers can receive both certificates, thus the value of certificate $C_1$ is strictly positive. The optimal fee a certifier may set in this case makes the low quality producer just indifferent between paying for the certificate and not certifying. An uncertified good is identified as being of

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9 For $\alpha = 1$ the outcome of this disclosure rule corresponds to full disclosure.

10 Actually a larger set of certificates would be possible, crucial is that only high quality producers demand certification such that any certificate is worth $v$ and whenever consumers learn a certified product is of low quality they know capture has taken place with certainty. Important however is that high quality products might not receive any certificate with positive probability.

11 We shall give a formal proof of this statement in the proofs of Lemma 3 and Proposition 4.
low quality. As certificate $C_2$ is worth $v$, the actual difference between the best and worst possible outcome is $v$, as in the case of full disclosure. So how could such a disclosure rule be beneficial to the certifier against the threat of capture? Any collusion with a low quality producer leads to a loss in profit in the current period of $f - c$ as those producers would have otherwise paid for certification. The short term gain from any capture $b$ is reduced by this amount, which makes it less attractive. But also per period profits are lower than under full disclosure, so the overall effect is again ambiguous. As we shall show in the next Proposition, capture-proof equilibria can be sustained for lower discount if the costs of certification are low.

**Proposition 4** For $c < c^*$ there exists $\delta(c) < \delta^{FD}$ such that a capture proof equilibrium can be sustained for all $\delta \geq \delta(c)$ with some disclosure rule $D$ of type (A) and fee $f$, such that in equilibrium all producers demand certification. $c^*$ is given in the proof.

The idea of the proof is as follows: It is possible to implement the same level of investment as under full disclosure with such a disclosure rule. Hence the allocation is not distorted and the acceptance probability of a bribe $(C_2, v)$, that exerts the largest threat of capture, is the same as for an offer $(v, v)$ under full disclosure. This unambiguously reduces the short-run gain, since with noisy disclosure also low quality producers would certify in equilibrium. For low cost levels, the reduction in per period profits is low and outweighed by the short-run gain. Overall the payoff from being captured is lower, which makes it possible to sustain honest equilibria also for low discount factors.

We illustrate our findings with an example.

**Example 1** Let $v = 1, p(e) = e$ and $k(e) = e^2/2$. Optimal full disclosure requires $f = \frac{1+c}{2}$ and profits are $\Pi^{FD} = (\frac{1-c}{2})^2$. By Proposition 3 honest equilibria exist if and only if $\delta \geq \delta^{FD} = 1/(1 + (1-c)^2/4)$. Now consider the following disclosure rule: The certifier offers two different certificates, $C_1$ and $C_2$. If a high quality product is to be certified, it is assigned $C_2$ with probability $\alpha \in (0, 1)$ and $C_1$ with probability $1 - \alpha$. Low quality products always get $C_1$ assigned. The certification fee is $f \geq 0$. It can be shown, that the per period profit for such a disclosure rule is maximized for $f = 1/4$ and $\alpha = 2/3$, the resulting profit is $\Pi = 1/4 - c$. With those values, honest equilibria can be sustained for all $\delta \geq 3/4 + c$. It is now easy to see that $3/4 + c < 1/(1 + (1-c)^2/4)$ for small values of $c$, namely for $c < c^* \approx 0.07$.

## 4 Three and more quality levels

We continue by considering the case of three or more different levels of quality. We restrict ourselves to the case $c = 0$ here, which still allows us to replicate the insights from the binary
case, but simplifies the analysis by avoiding the case distinctions for large cost levels, that we have already seen in the last section.

Besides full disclosure and no disclosure, a certifier now has several more deterministic disclosure rules available. Consider for example the case of three different quality levels $q^0, q^1$ and $q^2$. One potential rule would be to issue two certificates $C_1$ and $C_2$, where $C_1$ is granted to products of quality $q^0$ and $q^1$, whereas $C_2$ is only granted to products of quality $q^2$. Another type of disclosure rule, which we shall call cut-off disclosure, has only one certificate that is granted to all products whose quality exceeds a certain pre-specified threshold $\kappa$. The effect of cut-off disclosure is to lower the value of the best certificate a producer might obtain. If the cut-off is such that at least two different qualities may exceed it, the value of the certificate is the average of those quality levels and hence lower than the value of being identified as of the best quality. If the cut-off equals the under full disclosure endogenously determined quality type that is just willing to certify, then value of not having a certificate remain constant, whereas the type of the highest quality will loose under a cut-off disclosure rule. As we have seen in the previous section, the difference between the best and the worst possible outcome play a crucial role for the existence of honest equilibria. We will show in this section that for low discount factors the certifier might use a cut-off disclosure instead of a full disclosure. Though equivalent from a per period profit perspective, the cut-off disclosure only allows for lower bribes and therefore makes it easier for the certifier to resist the threat of capture.

In order to formalize the ideas just presented, we make a further assumption on the distribution of qualities, which allows us make explicit derivations.

**Assumption 2** There exist values $\gamma_1, \ldots, \gamma_n$ such that $\sum \gamma_i = 1$ and $p(q^i|e) = \gamma^i e$ for all $i = 1, \ldots, n$ and $p(q^0|e) = 1 - e$.

It is a tedious job to determine the optimal fee the certifier sets under full disclosure. For every fee there will be a critical type $\bar{q}(f)$, such that all producer types above or equal $\bar{q}(f)$ are willing to pay for being certified and all other types not. Say this critical type is the $k$-th quality level, i.e. $\bar{q}(f) = q^k$. Then we must have $q^k - f \geq V^{un} > q^{k-1} - f$. The value $V^{un}$ endogenously depends on the investment choice, which in turn depends on who certifies in equilibrium. The conditions for the critical type of the subgame, after the certifier
announced full disclosure and the fee $f \geq 0$ can be summarized by the following

$$q^k - f \geq V^{un} > q^{k-1} - f$$

(7)

$$V^{un} = \sum_{i=1}^{k-1} q^i \frac{\gamma^i e}{1 - \sum_{j=k}^{n} \gamma^j e}$$

(8)

$$e \in \arg\max \sum_{j=k}^{n} \gamma^j \tilde{e}(q^j - f) + (1 - \sum_{j=k}^{n} \gamma^j \tilde{e})V^{un} - \tilde{e}^2/2$$

(9)

Define $k(f)$ this critical type. The certifiers per period profit in the subgame is given by

$$\Pi^{FD}(f) = \sum_{i=k(f)}^{n} \gamma^i e(f)(f - c)$$

where $e(f)$ denotes the equilibrium level of investment. We first want to show, that the same profit can be reached with a cut-off disclosure rule.

**Lemma 4** Let $\Pi^{FD}(f)$ be the profit with a full disclosure rule and a fee $f \geq 0$. Then a cut-off disclosure rule with cut-off $q^{k(f)}$ and the same fee leads to exactly the same outcome in the subgame, i.e. in the same certifier profit, in the same investment level of producers and the same producer types certify.

The idea behind the proof of Lemma 4 is the following. To reach the same profits it is sufficient to get the same types certifying, which is easily checked to be true. The value for not being certified remains the same and as the fee also does not change all types who do not certify under full disclosure also refrain from doing so under cut-off disclosure. Furthermore, the value of the certificate $V^{\kappa}$ is no less than $q^{k(f)}$ since it is the average of all types above $q^{k(f)}$. Hence all types who are allowed to demand a certificate actually do so. The argument so far has a caveat: if the investment level changes all expected values do change as well. In the proof we show that this is not the case, i.e. the equilibrium investment level stays the same.

Does the converse also hold, i.e. starting with a cut-off $\kappa$ and a fee $f$, can one find a fee $\tilde{f}$ which enables the certifier to obtain a larger profit? The cut-off allows for more freedom in setting fees for the certifier. To have type $q^k$ certifying it is not necessary to have $q^k - f \geq V^{un}$, only the weaker condition $V - f \geq V^{un}$ has to be satisfied. This would in principle allow for larger profits, as the same participation can be guaranteed with larger fees, i.e. larger revenues. But the fee also affects the investment decisions, a larger fee generally lowers investment incentives for producers and thereby lowers the probability that a product of high quality is going to be produced. This lowers the certifier’s revenue. We show that this
decrease outweighs the gains from a higher fee.

**Proposition 5** Let \((\kappa, f)\) be a cut-off disclosure rule and \(\Pi^{CO}(\kappa, f)\) be the certifier’s per period profit in the subgame. Then there exists a fee \(\tilde{f}\) such that \(\Pi^{FD}(\tilde{f}) \geq \Pi^{CO}(\kappa, f)\).

Proposition 5 implies, that both disclosure rules are equivalent in the absence of a threat of capture. The optimal per period profits must be the same.

We will show in the remainder that the equivalence fails to hold if capture is an urgent problem. Let us start considering the capture problem for full disclosure. Let \(f\) be the certification fee and \(k(f)\) the critical type. Then the acceptance probability of a bribe \((q^n, b) = b\), which clearly has the largest threat, is given by

\[
\alpha(b|f) = \begin{cases} 
1, & b < f \\
1 - e(f) + \sum_{i=1}^{l} \gamma^i e(f), & q^n - (q^{l+1} - f) \leq b < q^n - (q^l - f); \ l \in \{k(f), \ldots, n\} \\
1 - e(f) + \sum_{i=1}^{k(f)-1} \gamma^i e(f), & q^n - (q^{k(f)+1} - f) \leq b < q^n - V^{un} \\
0, & b \geq q^n - V^{un}
\end{cases}
\]

As in section 3 one can show that the the largest threat stems from bribes which are accepted only by non-certifying producers, in particular from bribes \(b\) that approach \(q^n - V^{un}\). The next Lemma proves this fact in detail.

**Lemma 5** With full disclosure and a fee \(f \geq 0\), an honest equilibrium can be sustained if and only if the discount factor satisfies

\[
\delta \geq \delta(f) \equiv \frac{q^n - V^{un}}{q^n - V^{un} + \Pi^{FD}(f)}
\]

The Lemma highlights again the trade-off for the certifier. Picking the fee \(f\), such that per period profits are maximized, might lead to a large spread in possible outcomes for producers. A manipulation of the fee on the one hand reduces profits, on the other hand it might decrease the outcome gap \(q^n - V^{un}\).

We have discussed already, that cut-off disclosure rules can replicate the profit of any full disclosure rule. But such a disclosure rule also lowers the difference in outcomes. The best possible outcome is \(V^\kappa\) instead of \(q^n\), therefore the outcome gap is also lower. This immediately reduces the threat of capture as we will show now. A bribing offer with a cut-off disclosure rule can only be of the type \((V^\kappa, b) = b\), where a sure certificate is given
against a payment of $b$. Given a fee $f$, the acceptance probability is

$$
\alpha(b|f) = \begin{cases} 
1, & b < f \\
1 - e(f) + \sum_{i=1}^{n-1} \gamma^i e(f), & f \leq b < V - V^{un} \\
0, & b \geq V - V^{un}
\end{cases}
$$

Compared to full disclosure, lower bribes are accepted with positive probability, since $V^\kappa - V^{un} \leq q^n - V^{un}$. The condition for the critical discount factor however is similar. Let $\Pi^\kappa(f)$ be the certifiers per period profit with a cut-off disclosure rule, cut-off $\kappa$ and fee $f$. Then we can show

**Lemma 6** With a cut-off disclosure rule, cut-off $\kappa$ and a fee $f$, such that all producers with $q \geq q^\kappa$ certify, an honest equilibrium can be sustained if and only if the discount factor satisfies

$$
\delta \geq \delta(f) \equiv \frac{V^\kappa - V^{un}}{V^\kappa - V^{un} + \Pi^\kappa(f)}
$$

The last two Lemmata imply that with cut-off disclosure rules it is easier to sustain honest equilibria, in the sense that the critical discount factor becomes lower. By Lemma 4 and Proposition 5 both disclosure rules lead to similar profits. However, with a full disclosure rule the largest possible bribe is $q^n - V^{un}$, whereas it is $V^\kappa - V^{un} < q^n - V^{un}$ for cut-off disclosure. The certifier gains by lowering the value of his best certificate, while leaving his profits on the same level. This unambiguously reduces the threat of capture, as only the short-run gain is affected - it is lowerd - while the long-run loss stays constant. We state this formally in the following Proposition.

**Proposition 6** Given a fee $f$, the critical discount factor that makes it possible to sustain honest equilibria is lower for a cut-off disclosure rule than for full disclosure.

In order to illustrate our finding, we give an example with three different quality levels.

**Example 2** Let $q \in \{0; \frac{1}{2}; 1 \}$ and $p(\frac{1}{2}|e) = p(1|1) = \frac{\epsilon}{2}$. With a full disclosure rules, honest equilibria can be sustained for all $\delta \geq \delta^{FD} \approx 0.8256$. Here, the certifier sets a fee $f^{FD} = \frac{17}{40}$ and obtains per period profits of $\Pi^{FD} = 169/800^{12}$. The value of remaining uncertified is zero. Applying instead a cut-off disclosure rule with $\kappa = 1$, i.e. a product may receive a certificate whenever its quality is at least $q^1 = \frac{1}{2}$, honest equilibria exist for all $\delta \geq \delta^{CO} \approx 0.7802$. In the optimum, a certifier sets the same fee as under full disclosure, i.e. $f^{CO} = f^{FD}$, and also the per period profits are equal. The difference stems from the fact, that the maximal bribe is $3/4$ with a cut-off rule and $1$ with full disclosure.

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12A larger fee $f$, such that only producers of quality $q^2 = 1$ are willing to pay for certification, does not yield lower critical discount factors in this example.
5 Discussion

In this article, we have analyzed the effects of reputational concerns on optimal disclosure rules from the point of view of a monopolistic certifier. Our main finding is that a certifier may, if capture is an issue, announce a disclosure rule that constraints the set of feasible bribes and thereby lowers the maximal possible bribe. In particular, full disclosure might make it impossible to sustain honest certification whereas this might still be possible with noisy or coarse disclosure rules. For the case of three or more quality levels, as analyzed in section 4, a cut-off disclosure rule turns out to be a valuable tool for honesty. Compared to full disclosure it does not change the profit of the certifier but substantially reduces the set of feasible bribes and also the largest possible one. Our results give a novel explanation for the widespread usage of coarse disclosure schemes, such as exam grades (A,B,C,... instead of 100,99,...) or credit ratings. As for only two levels of quality cut-off disclosure is not an option, we showed in section 3 that a certifier coarsens information by adding noise. Optimally there are two certificates issued, one for only high quality products and one to both types of quality. The second certificate is vague, in the sense that it only a partially reveals the true quality of the product.

Note that we restricted our analysis to showing that revealing less information can reduce the threat of capture in certification markets. We did not derive the globally minimal discount factor that enables the certifier to sustain honest certification. For the case of two quality specifications, we have focused on "practical" disclosure rules, namely those that allow consumers to unambiguously detect deviations. To analyze whether other noisy disclosure rules further lower the critical discount factor, one would have to resort to the theory of repeated games with imperfect public information. Furthermore, for more than two quality specifications, we exclusively treated the effects of cut-off disclosure rules as compared to full disclosure. More specifically, we did not compare such coarse information revelation to noisy disclosure rules, a topic which is worth future research.

Our results from section 3 allow for a different interpretation. Certifiers are usually not announcing noisy disclosure rules in the sense of stating that higher qualities are being granted a certificate with a certain probability only. Instead, they are building a reputation for granting valuable certificates only in rare cases. This could be due to an imperfect testing technology but also to strategic considerations as is being proposed in this article. Put differently, a certifier might not always have an incentive to improve his imperfect testing technology. Furthermore, if we interpret the discount factor as a reputational rent, our analysis suggests that certifiers tend to remain vague if only few products are being certified.
Lastly, our findings have some important policy implications. Regarding the current financial crisis, it is often claimed that forcing rating agencies to issue more precise information might help. Our results suggest that doing so may lead to severe consequences for the functioning of those markets, as it might get more difficult to build up a reputation and resist capture if certificates are required to be very precise.

References


### A Proofs

**Proof of Proposition 1.** We distinguish two cases. In the subgame, following the certifier’s choice of a disclosure rule and a fee, either all producer types certify or only high quality producers certify their product. A third case, where the fee is such that no producer is ever willing to pay it in order to get her product certified trivially leads to lower profits than full disclosure.

**Case 1:** Let us start with the case where only high quality producers certify in equilibrium. Rational behavior by consumers dictates that every certificate is worth $v$, as only high quality products receive certificates. Uncertified products however can be of either high or low quality and have value $V^\text{un} < v$. The investment decision of a producer is

$$
\max_e e \left( \sum_k \alpha_k^1 v + (1 - \sum_k \alpha_k^1) V^\text{un} - f \right) + (1 - e) V^\text{un} - k(e)
$$

which the first-order condition for the optimal level of investment

$$
\left( \sum_k \alpha_k^1 (v - V^\text{un}) - f \right) = k'(e)
$$

Rewriting this constraint yields $(v - f - (1 - \sum_k \alpha_k^1)(v - V^\text{un}) - V^\text{un}) = k'(e)$. This implies, that for the same fee $f$ equilibrium effort is lower than under full disclosure, whenever
\[ \sum_k \alpha_k^1 < 1 \footnote{The case \( \sum_k \alpha_k^1 = 1 \) coincides with full disclosure and therefore profits are the same.}. \]

Furthermore, if high quality sellers demand certification under disclosure rule \( D \) and fee \( f \), they would also demand certification under full disclosure and fee \( f \), since \( v - f \geq \sum_k \alpha_k^1 v + (1 - \sum_k \alpha_k^1) V^{un} - f \geq V^{un} \geq 0 \).

We thus have
\[ 
\Pi^D(f) = e(f, D) \cdot (f - c) < e(f) \cdot (f - c) \leq \Pi^{FD}
\]

**Case 2:** Now assume all producers are willing to pay \( f \) in order to get their product certified. The profits for the certifier in this case are
\[ 
\Pi(f) = f - c
\]

W.l.o.g. assume \( \sum_i \alpha_i^1 = \sum_i \alpha_i^0 = 1 \), i.e. products always receive some certificate. Otherwise simply define a new certificate \( C_{n+1} \), which is granted with the respective probabilities to former uncertified products. Adding \( C_{n+1} \) neither changes the investment decision of producers nor does it change the decision for certification. To guarantee that a low quality producer demands certification, the fee cannot be too large, i.e.
\[ 
f \leq \sum_i \alpha_i^0 V_i \quad (10)
\]

A producer’s investment decision is
\[ 
\max_e e \left( \sum_i \alpha_i^1 V_i \right) + (1 - e) \left( \sum_i \alpha_i^0 V_i \right) - f - k(e)
\]

which yields the first-order condition
\[ 
\left( \sum_i (\alpha_i^1 - \alpha_i^0) \cdot V_i \right) = k'(e) \quad (11)
\]

Furthermore, from Bayesian Updating we have
\[ 
V_i = v \cdot \frac{e\alpha_i^1}{e\alpha_i^1 + (1 - e)\alpha_i^0} \quad (12)
\]

Using (12) we get \( e\alpha_i^1 V_i + (1 - e)\alpha_i^0 V_i = ve\alpha_i^1 \) and summing over \( i \) yields
\[ 
e \sum_i \alpha_i^1 V_i + (1 - e) \sum_i \alpha_i^0 V_i = ev \quad (13)
\]
Therefore

\[ \Pi^D(f) = f - c \leq \sum_i \alpha_i^0 V_i - c = e \left( v - \sum_i (\alpha_i^1 - \alpha_i^0) V_i \right) - c = e(v - k'(e)) - c \quad (14) \]

where the first inequality follows from (10), the first equality from (13) and the last equality follows from (11).

Now consider the case of full disclosure, from the first order condition for investment we get

\[ f = v - k'(e) \]

and with this the profit rewrites into

\[ \Pi^{FD}(e) = e(v - k'(e)) - c \quad (15) \]

Clearly, for \( c = 0 \) expressions (14) and (15) coincide, so maximal profits also coincide.

As \( c \) increases, profits under full disclosure decrease at rate \( e < 1 \), whereas profits under a disclosure rule that makes all producer types demand certification decrease faster, at rate 1. Therefore, for any \( c > 0 \) we have \( \Pi^D(f) < \Pi^{FD} \).

**Proof of Proposition 2.** Observe that the certifiers profit from deviating \( \Pi(b|f) \) is increasing both in the interval \([0, f)\) and \([f, v)\). Thus, the largest threat of capture is either exposed by a bribe \( b \rightarrow f \) or \( b \rightarrow v \). Let us compare the certifiers profit from deviation at these limits. We have that \( \lim_{b \rightarrow f} \Pi(b|f) \leq \lim_{b \rightarrow v} \Pi(b|f) \) if and only if

\[ (v - f)(1 - e(f)) - e(f)c \geq 0 \quad (16) \]

This inequality certainly holds true for \( c = 0 \), since \( f < v \) and \( e(f) \in [0, 1] \).

Holding \( f \) constant, the LHS in (16) is strictly decreasing in \( c \), hence there exists a critical \( c(f) > 0 \) such that for any \( c \leq c(f) \) we have \( \lim_{b \rightarrow f} \Pi(b|f) \leq \lim_{b \rightarrow v} \Pi(b|f) \) and the reverse inequality holds for any \( c \geq c(f) \).

Now consider the first case, \( c \leq c(f) \). We have that \( \Pi(b|f) \leq \Pi^h(f) \) for all \( b \) if and only if \( \lim_{b \rightarrow v} \Pi(b|f) \leq \Pi^h(f) \). This is equivalent to

\[ \delta \geq \delta(f) \equiv \frac{v}{v + e(f) \cdot (f - c)} \]

In the other case, \( c \geq c(f) \) we have that \( \Pi(b|f) \leq \Pi^h(f) \) for all \( b \) if and only if \( \lim_{b \rightarrow f} \Pi(b|f) \leq \Pi^h(f) \), which in turn is equivalent to

\[ \delta \geq \delta(f) \equiv \frac{f - e(f) \cdot (f - c)}{f - e(f) \cdot e(f) \cdot (f - c)} \]
Proof of Proposition 3. Since $\Pi(f) = e(f)(f - c)$ is strictly concave on $[c, v]$ by our assumptions, the function $\delta(f)$ is strictly convex on $[c, v]$ and exhibits a unique minimum, which is attained at $f = f^{FD}$.

Step 1: There exists $c_1 \in (0, v]$ such that whenever $c < c_1$ honest equilibria can be sustained for all $\delta \geq \delta^{FD} = \delta(f^{FD}(c))$.

For this to hold, we must have (16) satisfied at $f = f^{FD}(c)$. As already discussed, (16) strictly holds for $c = 0$ and any $f \in (c, v)$, so also in particular for $f = f^{FD}(0)$. Furthermore, for $c = v$ we have $f^{FD}(v) = v$ and $e^{FD}(v) = 0$, therefore (16) is satisfied with equality. Define the function $\theta(c) := (v - f^{FD}(c))(1 - e(f^{FD}(c))) - ce(f^{FD}(c)) = \Pi^{FD}(c) + v - f^{FD}(c) - ve^{FD}(c)$, where $\Pi^{FD}(c)$, $f^{FD}(c)$ and $e^{FD}(c)$ denote the respective equilibrium values according to Proposition 2. We have $\theta'(0) < 0$ and $\theta''(c) \geq 0$, hence $\theta(\cdot)$ is convex on $[0, v]$. Together with $\theta(0) > 0$ and $\theta(v) = 0$, consequently there exists $c_1 \in (0, v]$ such that $\theta(c) > 0$ for all $c < c_1$ and $\theta(c) \leq 0$ for all $c \geq c_1$. This implies, that for all $c < c_1$ honest equilibria can be sustained for any $\delta \geq \delta^{FD} = \delta(f^{FD}(c))$.

Step 2: There exists $c_2 \in (0, v]$ such that whenever $c > c_2$ honest equilibria can be sustained for all $\delta \geq \delta^{FD} = \delta(f^{FD}(c))$.

Denote $\hat{f}(c) := \arg \min \delta(f|c)$. For the claim to hold, we need to show $\delta(\hat{f}(c)) \geq \delta(\hat{f}(c))$ for all $c$ above some threshold $c_2$.

First of all, we have $\hat{f}(c)$ increasing in $c$. Second consider the function $\theta(f, c) = (v - f)(1 - e(f)) - ce(f)$. We have $\theta_c = -e(f) < 0$ and $\theta_{ef} = -e'(f) > 0$. Therefore, whenever $\theta(\hat{f}(c), c) \leq 0$ it follows that $\theta(\hat{f}(c), c) \leq 0$ for all $\hat{c} > c$. Lastly, $\theta(\hat{f}(v), v) = 0$, such that there exists $c_2 \in (0, v]$ such that $\theta(\hat{f}(c), c) \leq 0$ and for any such $c$ our claim holds true.

Step 3: In the last step we show $c_1 < c_2$ and that in this range there exists an intersection with $\delta(f) = \delta(f)$ which yields the critical discount factor. Furthermore we have $\hat{f}(c) < f^{FD}(c)$ for all $c \in [c_1, c_2]$.

Proof of Lemma 3. First of all, the producer’s optimal investment, given $\alpha$ and $f$ is given by $k'(e) = \alpha(v - V^{un}) - f$. Again, denote $e(f)$ this optimal decision, for some $f$. Throughout, we hold $\alpha$ constant. Only offers of the type $(C, b) = b$ are feasible. The value of the certificate is $v$ and the acceptance probability of bribe $b$ is given by

$$\alpha(b|f, D) = \begin{cases} 1, & b < v - (\pi^{1,D} - f) \\ p_L, & v - (\pi^{1,D} - f) \leq b < v - \pi^{0,D} \\ 0, & \text{otherwise} \end{cases}$$

where $p_L$ is the probability of low quality in equilibrium. The expected profit from offering a
bribe $b$ is
\[
\Pi(b|f, D) = \begin{cases} 
    b + (1 - p_L)\delta \Pi^h(f, D), & b < v - (\pi^{1,D} - f) \\
    p_L b + (1 - p_L)(f - c + \delta \Pi^h(f, D)), & v - (\pi^{1,D} - f) \leq b < v - \pi^{0,D} \\
    \Pi^h(f, D), & \text{otherwise}
\end{cases}
\]

Profits are increasing in $b$ on the intervals $[0, v - (\pi^{1,D} - f))$ and $[v - (\pi^{1,D} - f), v - \pi^{0,D})$, such that the maximal threat stems from bribes at one of the boundaries. A certifier will resist making offer $b$, whenever $\Pi^h(f, D) \geq \Pi(b|f, D)$. For bribes in $[0, v - (\pi^{1,D} - f))$ this condition rewrites into
\[
\delta \geq \frac{b - \Pi^D(f)}{b - (1 - p_L)\Pi^D(f)}
\]
whereas for $b \in [v - (\pi^{1,D} - f), v - \pi^{0,D})$ it rewrites into
\[
\delta \geq \frac{p_L b + (1 - p_L)(f - c) - \Pi^D(f)}{p_L b + (1 - p_L)(f - c) - (1 - p_L)\Pi^D(f)}
\]
Observe that $\Pi^D(f) = e(f - c) = (1 - p_L)(f - c)$ and (18) is increasing in $b$. It is therefore maximized at $b = v - \pi^{1,D}$ and simplifies to
\[
\delta \geq \delta^A \equiv \frac{v - \pi^{0,D}}{v - \pi^{0,D} + \Pi^D(f)}
\]
Also (17) is increasing in $b$ and reaches its maximum at the corner $b = v - (\pi^{0,D} - f)$, which gives
\[
\delta \geq \delta^B \equiv \frac{v - (\pi^{1,D} - f) - \Pi^D(f)}{v - (\pi^{1,D} - f) - (1 - p_L)\Pi^D(f)}
\]
From the proof of Proposition 1 follows $1 - p_L = e = \pi^{1,D} - f - \pi^{0,D}$ and we find that $\delta^A > \delta^B$ whenever
\[1 - v + f - c + \pi^{0,D} > 0\]
which is true since $v \leq 1$. Hence, the largest threat from capture stems from a bribe which is only accepted by low quality producers and is maximal, i.e. $b = v - \pi^{0,D}$. Since low quality producers do not certify, we have $\pi^{0,D} = V^{\text{un}}$. The critical discount for noisy disclosure is
\[
\delta^A \equiv \frac{v - V^{\text{un}}}{v - V^{\text{un}} + \Pi^D(f)}
\]
Increasing $V^{\text{un}}$ leads to a reduction in profits $\Pi^D(f)$. We will show that $\delta^A$ increases in $V^{\text{un}}$, taking into account the profit reduction. Clearly, holding $V^{\text{un}}$ constant, the discount
is factor is lower for larger profits, that is for the profit maximizing choice of $f$ and $\alpha$ that yields a fixed value of being uncertified $V^{un}$. The program for finding this profit is

$$\max_{\alpha, f} \ e \cdot f$$

s.t. \quad \begin{align*}
e &= \alpha (v - V^{un} - f) \quad (\lambda) \\
V^{un} &= \frac{e(1 - \alpha)}{1 - \alpha e} \quad (\mu) \end{align*}$$

Denote $\Pi(V^{un})$ the solution to this problem. We have

$$\frac{\partial}{\partial V^{un}} \Pi(V^{un}) = \lambda \alpha + \mu$$

where $\lambda$ and $\mu$ are the respective Lagrange multipliers of the constraints. From the first-order conditions to the problem above we get $\lambda = -e/\alpha$ and $\mu = \frac{(f - e/\alpha)(1 - \alpha e)^2}{1 - \alpha e}$. Using that $f = (v - V^{un})/2$ and $e = \alpha (v - V^{un})/2$ in equilibrium, we get

$$\frac{\partial}{\partial V^{un}} \Pi(V^{un}) = -\alpha \frac{v - V^{un}}{2}$$

Furthermore

$$\frac{\partial \delta^A}{\partial V^{un}} = \frac{-\Pi(V^{un}) - (v - V^{un}) \frac{\partial \Pi(V^{un})}{\partial V^{un}}}{(v - V^{un} + \Pi(V^{un})^2} = \frac{\alpha (v - V^{un})^2}{4} \frac{v - V^{un} + \Pi(V^{un})^2}$$

where we used $\Pi(V^{un}) = (v - V^{un})^2/4$ in equilibrium. Now $\frac{\partial \delta^A}{\partial V^{un}}$ is obviously positive, thus increasing and minimized at $V^{un} = 0$ which corresponds to the case of full disclosure.

**Proof of Proposition 4.** First, note that a producer’s investment decision is given by $k'(e) = \alpha (v - V^2)$, where $V^2$ is the equ. value of certificate $C_2$.\(^{14}\) The acceptance probability of bribe $(C_1, b) = b$ is

$$\alpha(b|f, D) = \begin{cases} 
1, & b < f + (1 - \alpha)(v - V^2) \\
p_L, & f + (1 - \alpha)(v - V^2) \leq b < f + v - V^2 \\
0, & \text{otherwise}
\end{cases}$$

where $p_L$ denotes the probability of producer being of low type on the equilibrium path. The

\(^{14}\)Recall, that $V^1 = v$!
The expected profit from offering a bribe $b$ is

$$
\Pi(b|f, D) = \begin{cases} 
    b + (1 - p_L)\delta \Pi^h(f, D), & b < f + (1 - \alpha)(v - V^2) \\
    p_L b + (1 - p_L)(f - c + \delta \Pi^h(f, D)), & f + (1 - \alpha)(v - V^2) \leq b < f + v - V^2 \\
    \Pi^h(f, D), & \text{otherwise}
\end{cases}
$$

Profits are increasing in $b$ on the intervals $[0, f + (1 - \alpha)(v - V^2))$ and $[f + (1 - \alpha)(v - V^2), f + v - V^2)$, such that the maximal threat stems from bribes at one of the boundaries.

A certifier will resist making offer $b$, whenever

$$
\Pi^h(f, D) \geq \Pi(b|f, D)
$$

We have $\Pi(f + (1 - \alpha)(v - V^2)|f) \leq \Pi(f + v - V^2|f)$ if and only if

$$
(v - V^2 + c)(1 - p_L) \leq \alpha(v - V^2)
$$

(21)

Now recall that $\alpha(v - V^2) = k'(e) = k'(1 - p_L)$, which certainly is independent of $f$, only that we must have $c \leq f \leq V^2$. Knowing this, condition (21) rewrites into

$$
(v - V^2 + c)e \leq k'(e)
$$

(22)

Note that this condition only depends on $\alpha$, the parameter of the disclosure rule, but not on $f$. Taking $e$ as the equilibrium level $e(\alpha)$, it folows that for any $\alpha$ it is either the bribe that is accepted by all producer types $b_1 = f + (1 - \alpha)(v - V^2)$ that poses the largest threat on the certifier, or it is the bribe $b_2 = f + (v - V^2)$, which is only accepted by low quality producers. Both $b_1$ and $b_2$ depend on $\alpha$. Rearranging the condition $\Pi(b|f) = \Pi^h(f)$, we get the following further characterization of the two described cases. Whenever (22) holds true, a capture-proof equilibrium using fee $f$ can be sustained whenever

$$
\delta \geq \frac{f + v - V^2 - (f - c)}{f + v - V^2}
$$

This is clearly minimized for the maximal fee which is $f = V^2$, hence the critical discount factor becomes $\frac{v - V^2 + c}{v - V^2 + c + (V^2 - c)}$. This expression only depends on the disclosure via $V^2$, it is minimized for the maximal $V^2$, which also the profit-maximizing choice.

If (22) however does not hold, we can sustain capture-proof equilibria using fee $f$, whenever

$$
\delta \geq \frac{f + (1 - \alpha)(v - V^2) - (f - c)}{f + (1 - \alpha)(v - V^2) - (f - c)(1 - p_L)}
$$
Again, this expression is minimized for $f = V^2$, the profit maximizing fee under the corresponding disclosure rule, such that capture-proof equilibria can be sustained for

$$\delta \geq \frac{(1-\alpha)(v-V^2) + c}{(1-\alpha)(v-V^2) + c(1-p_L) + p_L V^2}$$

**Proof of Lemma 4.** Let $k(f)$ be the critical type under full disclosure and set $\kappa = k(f)$. We show that the investment choice $e(f)$ and all certification decisions from the equilibrium of the subgame with full disclosure also form an equilibrium in the subgame with cut-off disclosure. As $q^{k(f)} - f \geq V^{\text{un}}$ by assumption we also find $V - f > q^{k(f)} - f \geq V^{\text{un}}$. Given the investment choice, $V^{\text{un}}$ is unaffected, since all other producer types would not receive a certificate, even if they would pay for it. The optimal investment level with a cut-off rule $e^{CO}$ is given by

$$\max_e \sum_{i=\kappa}^{n} \gamma_i e V + (1 - \sum_{i=\kappa}^{n} \gamma_i e)V^{\text{un}} - e^2/2$$

The first order condition is $e = \sum_{i=\kappa}^{n} (V - f - V^{\text{un}})\gamma_i$. The full disclosure rule has the equilibrium investment choice $e = \sum_{i=k(f)}^{n} (q^i - f - V^{\text{un}})\gamma_i$. Now observe

$$\sum_{i=\kappa}^{n} V\gamma_i = \sum_{i=\kappa}^{n} \left( \sum_{j=\kappa}^{n} \frac{q^j e}{\sum_{j=\kappa}^{n} \gamma_j e} \right) \gamma_i = \sum_{i=\kappa}^{n} \gamma_i q^i$$

and since $k(f) = \kappa$ the claim is proven.

**Proof of Lemma 5.** The proof is analogue to Lemma 2. Like there it is easy to show that the largest threat stems from a bribe which is accepted only by those producers who do not intent to certify. The profit of such a bribe is $(1-p_c)b + p_c(f - c + \delta \Pi^h)$, where $p_c$ denotes the probability that a producer demands certification. Again we show $\Pi^h(f) \geq \Pi(b|f)$ whenever

$$\delta \geq \frac{b}{b + \Pi^{FD}(f)}$$

which is maximized for $b \rightarrow q^n - V^{\text{un}}$. This proves the claim.

**Proof of Lemma 6.** The proof as similar as the proofs in section 2. The profit from
offering bribe \( b \) is

\[
\Pi(b|f) = \begin{cases} 
  b + p_c \delta \Pi^h, & b < f \\
  b(1 - p_c) + p_c(f - c + \delta \Pi^h), & f \leq b < V^\kappa - V^\text{un} \\
  \Pi^h, & b \geq V^\kappa - V^\text{un}
\end{cases}
\]

where \( p_c \) denotes the probability a producer demands certification in equilibrium. For \( b < f \) we have \( \Pi^h \geq \Pi(b|f) \) whenever

\[
\delta \geq \frac{b - \Pi^\kappa(f)}{b - p_c \Pi^\kappa(f)}
\]

which is increasing in \( b \), such that the threat of bribes \( b \in [0, f) \) is largest for \( b \to f \), which yields the critical discount factor \( \delta_l(f) = \frac{f - \Pi^\kappa(f)}{f - p_c \Pi^\kappa(f)} \).

For \( f \leq b < V^\kappa - V^\text{un} \) we find \( \Pi^h \geq \Pi(b|f) \) whenever

\[
\delta \geq \frac{b}{b + \Pi^\kappa(f)}
\]

which is again increasing in \( b \) and thus maximal for \( b \to V^\kappa - V^\text{un} \). This results in a critical discount factor \( \delta_h(f) = \frac{V^\kappa - V^\text{un}}{V^\kappa - V^\text{un} + \Pi^\kappa(f)} \).

Now \( \delta_l(f) > \delta_h(f) \) whenever

\[
(V^\kappa - V^\text{un})(f - p_c \Pi^\kappa(f)) > (f - \Pi^\kappa(f))(V^\kappa - V^\text{un} + \Pi^\kappa(f))
\]

which simplifies to

\[
(1 - p_c)(V^\kappa - V^\text{un}) - f + \Pi^\kappa(f) > 0
\]

which is true, since \( V^\kappa - V^\text{un} - f > 0 \). The relevant critical discount factor is therefore \( \delta_h(f) \).