Two-sided Certification
The Market for Rating Agencies

Erik R. Fasten  Dirk L. Hofmann

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Research Question & Motivation

- **Research Question:**
  To which side of the market would an honest certifier offer his service - to Seller-Side, to Buyer-Side or to both Sides?

- **Relevance:**
  - Asymmetric distribution of information, e.g. in financial markets
  - Role of rating agencies in the current financial crisis
  - Regulation of the business model of rating agencies required?

- **Certification industry more general:**
  - TüV
  - Stiftung Warentest
  - Food labels (Öko-test, Bio-Siegel, etc.)
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Ratings and Rating markets

What is a rating?

Definition: "Ratings are summary measures of assessment over the probability that a borrower will default." (Fitch, 2002)
Different default probabilities are grouped into rating classes: e.g. Moody’s: Aaa to C
Players: combined market share of Moody’s and S&P: 80 %

How to construct a rating?

Private information of firms is accumulated and announced
Public information is accumulated and condensed

Why do ratings exist?

Information asymmetries between market participants
Reduction of risk premia and volatility
Ease of risk sharing

Who pays for the rating?

Before 1970: primarily paid by investors - private information
Thereafter: mainly paid by firms - public information
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Related Literature

- Stahl, K., R. Strausz, 2009, ”Certification and Exchange in Vertically Concentrated Markets”, *work in progress*
Results from our Model

- A certifier can reduce welfare losses due to asymmetric information (Lemon Markets).
- A certifier can even make profits in efficient markets which would work without the certification service.
- If the certifier is solely able to sell to one side of the market he will offer his service to sellers.
- Independent of market type a profit maximizing certifier will sell to both sides of the market.
- In a Lemon Market two-sided certification increases welfare.
- In case of two-sided certification the bigger share (about 80%) of the certifier’s revenues are generated by selling on seller side.
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In case of two-sided certification the bigger share (about 80%) of the certifier’s revenues are generated by selling on seller side.
The model includes 4 players of 3 different kinds:

- **The certifier**
  Objective: Maximize profits by selling her certification service to a seller and/or the buyers.

- **One seller**
  Objective: Maximize profits by selling her product at a high price (above her reservation utility) to one of the buyers

- **Two buyers**
  Objective: Maximize utility by buying the product below own willingness to pay
Assumption: The quality of a product $q$ is distributed according to the uniform distribution on the interval $[0,1]$.

Quality $q$ is private information on seller’s side and is not credibly communicable.

The buyers’ willingness to pay for a product of quality $q$ is $q$ and the reservation utility of the seller is $\alpha q$, $\alpha \in [0,1]$.

Quality $q$ can be certified by an intermediary for a certain price; the quality is announced truthfully; the certifier is able to discriminate between sellers and buyers.
The Model: Parameters & Variables

- Assumption: The quality of a product q is distributed according to the uniform distribution on the interval [0,1].
- Quality q is private information on seller’s side and is not credibly communicable.
- The buyers’ willingness to pay for a product of quality q is q and the reservation utility of the seller is $\alpha q$, $\alpha \in [0,1]$.
- Quality q can be certified by an intermediary for a certain price; the quality is announced truthfully; the certifier is able to discriminate between sellers and buyers.
Market structure depending on parameter $\alpha$

Depending on parameter $\alpha$ two fundamentally different kinds of markets appear.

- For values $\alpha > \frac{1}{2}$ we get a Lemon market in the sense of Akerlof (1970).
- For $\alpha \leq \frac{1}{2}$ the market clears.

Note: Maximal welfare $W_{max}$ exploitable is $\int_{0}^{1} (1 - \alpha)q dq = \frac{1 - \alpha}{2}$. 
The Model: Timing

1. Certifier decides on prices for certification $p_s, p_b$.
2. Seller decides to pay the certifier $p_s$ or not, if so quality $q$ will be credibly announced ($q$ is public information afterwards).
3. Buyers decide simultaneously whether to pay the certifier $p_b$ (true quality $q$ is then private information for the buyer).
4. Bidding stage for the product, modeled by a first-price-auction with common values; the seller’s reservation utility $\alpha q$ serves as an (unknown) reserve price. (Assumption: The information structure among the bidders is known.) Payoffs are realized.
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Three different Models - A comparison

We will consider three different models:

Sell certification service to seller’s side only:

1 → 2 → 4

Sell certification service to buyers’ side only:

1 → 3 → 4

Sell certification service to both sides:

1 → 2 → 3 → 4
Ratings on Seller Side only - Bidding Behavior

Each price $p_s$ of the certifier induces a quality threshold $\bar{q}$ above which sellers order a rating.

The model is solved by backwards induction:

- **Lemon Market:**
  Uninformed buyers bid 0 and informed buyers bid $q$.

- **Efficient Market:**
  Uninformed buyers bid $q^e_\bar{q}$ and informed buyers bid $q$, where $q^e_\bar{q}$ is the expected quality in the unrated market.
Ratings on Seller Side only - Bidding Behavior

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\[
\text{Indifference conditions for the sellers:}
\]

\begin{align*}
\text{Lemon Market: } (1 - \alpha)\bar{q} - p_s &= 0 \\
\text{Efficient Market: } (1 - \alpha)\bar{q} - p_s &= \frac{1}{2} \bar{q} - \alpha \bar{q}
\end{align*}

- The quality thresholds are

\begin{align*}
\text{Lemon Market: } \bar{q} &= \frac{p_s}{1-\alpha}, \ p_s \in [0, (1 - \alpha)], \\
\text{Efficient Market: } \bar{q} &= 2p_s, \ p_s \in [0, \frac{1}{2}].
\end{align*}
Ratings on Seller Side only - Certifier Behavior

- Maximization problem of the certifier:

\[
\max_{p_s} \Pi(p_s) = (1 - \tilde{q}(p_s))p_s,
\]

given the corresponding functions \( \tilde{q}(p_s) \) depending on market parameter \( \alpha \).

- Results:

  Lemon Market \((\alpha > \frac{1}{2})\):

  \[
p_s = \frac{1 - \alpha}{2}, \quad \tilde{q} = \frac{1}{2}, \quad \Pi_C = \frac{1 - \alpha}{4}, \quad \Pi_S = \frac{1 - \alpha}{8}, \quad W = \frac{3}{8}(1 - \alpha)
  \]

  Efficient Market \((\alpha \leq \frac{1}{2})\):

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p_s = \frac{1}{4}, \quad \tilde{q} = \frac{1}{2}, \quad \Pi_C = \frac{1}{8}, \quad \Pi_S = \frac{1 - \alpha}{2} - \frac{1}{8}, \quad W = W_{\text{max}} = \frac{1 - \alpha}{2}
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Ratings on Buyer Side only - Bidding Behavior

- Recall: Two buyers compete to buy the product
- Each buyer decides whether to order a rating for a given certification price $p_b$

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The distribution function of bids for a single uniformed buyer is given by $F(b) = 2b$.

- The buyer’s decision depends on the profit from ordering a rating compared to the profit for staying uninformed
- The symmetric equilibrium requires a mixed strategy, $\omega$ denotes the probability of ordering a rating
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- The symmetric equilibrium requires a mixed strategy, $\omega$ denotes the probability of ordering a rating
There is no equilibrium in pure strategies.

...therefore buyers apply a mixed-strategy of buying private information.

The value of being exclusively informed $V_{ib}$ depends on market structure:

- **Lemon Market**: Expected payoff is $V_{ib}^L = \frac{1-\alpha}{2}$.
- **Efficient Market**: Expected payoff is $V_{ib}^E = \frac{1}{6}$.

Each price $p_b$ of the certifier induces a certain probability $\omega$ for the mixed strategy equilibrium:

- **Lemon Market**: $\omega = 1 - \frac{2p_b}{1-\alpha}$, $p_b \in [0, V_{ib}^L]$.
- **Efficient Market**: $\omega = 1 - 6p_b$, $p_b \in [0, V_{ib}^E]$. 
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Ratings on Buyer Side only - Information Acquisition

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Ratings on Buyer Side only - Certifier Behavior

Maximization problem of the certifier:

\[
\max_{p_b} \Pi_C(p_b) = \omega(p_b)^2 2p_b + 2\omega(p_b)(1 - \omega(p_b))p_b,
\]

given the corresponding functions \(\omega(p_b)\) depending on market parameter \(\alpha\).

Results:

Lemon Market \((\alpha > \frac{1}{2})\):

\[
p_b = \frac{1 - \alpha}{4}, \quad \omega = \frac{1}{2}, \quad \Pi_C = \frac{1 - \alpha}{4}, \quad \Pi_S = \frac{1 - \alpha}{8}, \quad W = \frac{3}{8}(1 - \alpha)
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Efficient Market \((\alpha \leq \frac{1}{2})\):

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Ratings on Buyer or Seller Side: A comparison

- Certifier prefers to sell to seller side in an efficient market.
- Certifier is indifferent to which side to sell in a Lemon Market.
- For markets with $\alpha > \frac{1}{2}$ certification has a strong welfare increasing effect.
Ratings on Seller and Buyer Side - Bidding behavior

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The distribution function of bids for a single uniformed buyer is given by \(F_{\tilde{q}} = \frac{2}{q} b\). In this case: \(q_{q}^e = \frac{q}{2}\).
The value of being exclusively informed $V_{ib}$ depends on market structure:

Lemon Market: Expected payoff is $V_{ib}^L = (1 - \alpha) \bar{q}^2$,

Efficient Market: Expected payoff is $V_{ib}^E = \frac{1}{6} \bar{q}$. 
Each price-combination \((p_s, p_b)\) of the certifier induces a certain quality threshold \(\bar{q}\) and a probability \(\omega\) for the mixed strategy equilibrium:

Indifference conditions for the sellers and the buyers:

Lemon Market:

\[
(1 - \alpha)\bar{q} - p_s = \omega^2 (1 - \alpha)\bar{q} \quad \text{and} \quad (1 - \omega) V_{ib}^L(\bar{q}) - p_b = 0
\]

Efficient Market:

\[
(1 - \omega^2)\frac{\bar{q}}{2} - p_s = 0 \quad \text{and} \quad (1 - \omega) V_{ib}^E(\bar{q}) - p_b = 0
\]

The quality thresholds and the rating probabilities are:

Lemon Market:

\[
\bar{q} = \frac{4p_b^2}{(1-\alpha)(4p_b - p_s)} \quad \text{and} \quad \omega = \frac{p_s}{2p_b} - 1 \quad \text{with} \quad p_s \text{ and } p_b \quad \text{s.t.} \quad 0 \leq \bar{q}, \omega \leq 1,
\]

Efficient Market:

\[
\bar{q} = \frac{18p_b^2}{6p_b - p_s} \quad \text{and} \quad \omega = \frac{p_s}{3p_b} - 1, \quad \text{with} \quad p_s \text{ and } p_b \quad \text{s.t.} \quad 0 \leq \bar{q}, \omega \leq 1.
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Ratings on Seller and Buyer Side - Induced Rating Probability and Induced quality threshold

- Each price-combination \((p_s, p_b)\) of the certifier induces a certain quality threshold \(\bar{q}\) and a probability \(\omega\) for the mixed strategy equilibrium:

- Indifference conditions for the sellers and the buyers:
  - Lemon Market:
    \[
    (1 - \alpha)\bar{q} - p_s = \omega^2 (1 - \alpha)\bar{q} \text{ and } (1 - \omega)V_{ib}^L(\bar{q}) - p_b = 0
    \]
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    (1 - \omega^2)\bar{q}^2 - p_s = 0 \text{ and } (1 - \omega)V_{ib}^E(\bar{q}) - p_b = 0
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- The quality thresholds and the rating probabilities are:
  - Lemon Market: \(\bar{q} = \frac{4p_b^2}{(1-\alpha)(4p_b - p_s)}\) and \(\omega = \frac{p_s}{2p_b} - 1\) with \(p_s\) and \(p_b\) s.t. \(0 \leq \bar{q}, \omega \leq 1\),
  - Efficient Market: \(\bar{q} = \frac{18p_b^2}{6p_b - p_s}\) and \(\omega = \frac{p_s}{3p_b} - 1\), with \(p_s\) and \(p_b\) s.t. \(0 \leq \bar{q}, \omega \leq 1\).
Maximization problem of the certifier:

\[
\max_{p_s, p_b} \Pi_C(p_s, p_b) = (1 - \bar{q})p_s + \bar{q}[\omega^2 2p_b + 2\omega(1 - \omega)p_b]
\]

given the corresponding functions \(\bar{q}(p_s, p_b)\) and \(\omega(p_s, p_b)\) depending on market parameter \(\alpha\).

Results:
Lemon Market: \(p_s = \frac{16}{27}(1 - \alpha), \ p_b = \frac{2}{9}(1 - \alpha), \ \bar{q} = \frac{2}{3}, \ \omega = \frac{1}{3}, \ \Pi_C = \frac{8}{27}(1 - \alpha), \ \Pi_S = (1 - \alpha)\frac{17}{162}\) and \(W = (1 - \alpha)\frac{65}{162} \neq W_{max}\).
Efficient Market: \(p_s = \frac{3}{4}(5\sqrt{5} - 11), \ p_b = \frac{1}{4}(7 - 3\sqrt{5}), \ \bar{q} = \frac{141 - 63\sqrt{5}}{36 - 16\sqrt{5}}, \ \Pi_C = \frac{3}{4}(5\sqrt{5} - 11), \ \Pi_F = \frac{1-\alpha}{2} - \Pi_C\) and \(W = W_{max}\).
Maximization problem of the certifier:

\[
\max_{p_s, p_b} \Pi_C(p_s, p_b) = (1 - \bar{q})p_s + \bar{q}[\omega^2 2p_b + 2\omega(1 - \omega)p_b]
\]

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A comparison of the three models

Table: Comparing equilibrium outcomes of different model settings

<table>
<thead>
<tr>
<th></th>
<th>Only sellers</th>
<th>Only buyers</th>
<th>Both sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha &gt; \frac{1}{2} ) (lemon market)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>price for seller rating</td>
<td>( \frac{1-\alpha}{2} )</td>
<td>-</td>
<td>( \frac{16}{27} (1 - \alpha) )</td>
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<tr>
<td>price for buyer rating</td>
<td>-</td>
<td>( \frac{1-\alpha}{4} )</td>
<td>( \frac{2}{9} (1 - \alpha) )</td>
</tr>
<tr>
<td>high-quality threshold</td>
<td>( \frac{1}{2} )</td>
<td>-</td>
<td>( \frac{2}{3} )</td>
</tr>
<tr>
<td>buyer’s rating probability</td>
<td>-</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>profit certifier</td>
<td>( \frac{1-\alpha}{4} )</td>
<td>( \frac{1-\alpha}{4} )</td>
<td>( \frac{8}{27} (1 - \alpha) )</td>
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<tr>
<td>profit seller</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{17}{162} (1 - \alpha) )</td>
</tr>
<tr>
<td>welfare</td>
<td>( \frac{3}{8} (1 - \alpha) )</td>
<td>( \frac{3}{8} (1 - \alpha) )</td>
<td>( \frac{65}{162} (1 - \alpha) )</td>
</tr>
</tbody>
</table>

\( \alpha < \frac{1}{2} \) (efficient market)

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<tr>
<td>price for seller rating</td>
<td>( \frac{1}{4} )</td>
<td>-</td>
<td>( \approx 0.27 )</td>
</tr>
<tr>
<td>price for buyer rating</td>
<td>-</td>
<td>( \frac{1}{12} )</td>
<td>( \approx 0.07 )</td>
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<tr>
<td>high-quality threshold</td>
<td>( \frac{1}{2} )</td>
<td>-</td>
<td>( \approx 0.573 )</td>
</tr>
<tr>
<td>buyer’s rating probability</td>
<td>-</td>
<td>( \frac{1}{4} )</td>
<td>( \approx 0.24 )</td>
</tr>
<tr>
<td>profit certifier</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{1}{12} )</td>
<td>( \approx 0.135 )</td>
</tr>
<tr>
<td>profit seller</td>
<td>( \frac{1-\alpha}{2} - \frac{1}{8} )</td>
<td>( \frac{1-\alpha}{2} - \frac{1}{12} )</td>
<td>( \frac{1-\alpha}{2} - 0.135 )</td>
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<td>welfare</td>
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A comparison of the three models III

- Efficient Market:
  Certifier’s Share on Welfare increases from the range of 25%-50% to the range of 27% – 54% (increase in profit 8%).

- Lemon Market:
  Certifier’s Share on Welfare increases from 50% to 60% (increase in profit 18.5%).

- Distribution of Revenues (Seller- vs. Buyer-Side):
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  Empirically for the Market of Rating Agencies: 4:1
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Certifiers profit from entering each kind of market.
If a Certifier had to decide he would choose to sell on seller’s side.
Certifiers increase their profit by selling to both sides of the market.
Welfare losses are reduced in inefficient markets.
Traded volumes increase through the existence of rating agencies.
Policy Implications

- Depending on the market structure it is counterproductive to prohibit the sale to both sides.
- For inefficient markets it is even desirable to have a certifier (a Rating Agency).
- The observed revenue shares in the real world are not a sufficient reason to argue that conflicts of interest distort the quality of ratings.
- Concerning regulation: Better try to assess the quality of the predictions made by the Rating Agencies, than dictating where they sell their service.
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