Bundling and Competition for Slots: On the Portfolio Effects of Bundling*

Doh-Shin Jeon† and Domenico Menicucci‡

February 11, 2009

Abstract

We consider competition among n sellers when each of them sells a portfolio of distinct products to a buyer having limited slots (or shelf space). We study how bundling affects competition for slots. When the buyer has k number of slots, efficiency requires the slots to be allocated to the best k products among all products. We first find that without bundling, equilibrium often does not exist and hence the outcome is often inefficient. Bundling changes competition between individual products into competition between portfolios and reduces competition from rival products. Therefore, each seller has an incentive to bundle his products. Furthermore, under bundling, an efficient equilibrium always exists. In particular, in the case of Digital goods, all equilibria are efficient if firms do not use slotting contracts. However, inefficient equilibria can exist if firms use slotting contracts. In the case of physical goods, pure bundling also can generate inefficient equilibria. Finally, we identify portfolio effects of bundling and analyze the consequences on horizontal merger.

Key words: Bundling, Portfolios, Slots (or Shelf Space), Pure Bundling, Slotting Contracts

JEL Code: D4, K21, L13, L41, L82

*We thank the participants of seminars at Georgia Institute of Technology, Korea University, Université de Cergy-Pontoise, Université Paris 10, Université Toulouse 1 and of presentations at EARIE08, Net Institute Conference 08, PET08, SED 08. We also thank the comments from Jung-Hyun Ahn, Jay Pil Choi, Denis Gromb, Bruno Jullien, Byung-Cheol Kim, Patrick Rey and Régis Renault. Jeon gratefully acknowledges the financial support from the Spanish government under SEJ2006-09993/ECON.

†Universitat Pompeu Fabra, IESE (SP-SP) and Toulouse School of Economics doh-shin.jeon@upf.edu

‡Università degli Studi di Firenze, Italy. domenico.menicucci@dmd.unifi.it

1 Introduction

There are many situations in which sellers with different portfolios of products compete for limited slots (or shelf space) of a buyer who wants to build up her own portfolio of distinct products. In this situation, sellers may employ bundling as a strategy to win the competition for slots. Even though bundling has been a major antitrust issue and a subject of intensive research, to the best of our knowledge, the literature seems to have paid little attention to competition among portfolios of distinct products and, in particular, no paper seems to have studied how bundling affects portfolios' competition for slots. In this paper, we attempt to provide a new perspective on bundling by addressing this issue.

Examples of situations we described above are abundant both among digital products and among physical products. For instance, in the movie industry, each movie distributor has a portfolio of distinct movies and buyers (either movie theaters or TV stations) have limited slots. More precisely, the number of movies that can be projected in a season (or in a year) by a theater is constrained by time and the number of projection rooms. Likewise, the number of movies that a TV station can show during prime time of a season (or year) is also limited. Actually, allocation of slots in movie theaters has been one of the main issues raised in the movie industry during the last presidential election in France[1]. Furthermore, bundling in the movie industry (known as block booking[2]) was declared illegal in two supreme court decisions in U.S.: *Paramount Pictures* (1948), where blocks of films were rented for theatrical exhibition, and *Loew's* (1962), where blocks of films were rented for television exhibition. In addition, recently in *MCA Television Ltd. v. Public Interest Corp.* (11th Circuit, April 1999), the court of appeals reaffirmed the *per se* illegal status of block booking.

A different situation we have in mind is that of manufacturers' competition for retailers' shelf space. Manufacturers having a large portfolio of products may practice bundling (often called full-line forcing) to win the competition for slots[3] and there has been antitrust cases

[1] Cahiers du Cinema (April, 2007) proposes to limit the number copies per film since certain movies by saturating screens limits other films' access to screens and asks each presidential candidate's opinion about the policy proposal.

[2] Block booking refers to "the practice of licensing, or offering for license, one feature or group of features on the condition that the exhibitor will also license another feature or group of features released by distributors during a given period" (Unites States v. Paramount Pictures, Inc., 334 U.S. 131, 156 (1948)).

[3] For instance, Procter and Gamble uses 'golden-store' arrangement such that to be considerd a golden store, a retailer must agree to carry 40 or so P&G items displayed together. See "P&G has big plans for the shelves of tiny stores in emgering nations", Wall Street Journal, July 17, 2007.

related to this practice[4]. For instance, the French Competition Authority fined Société des Caves de Roquefort for using selectivity or exclusivity contracts with supermarket chains.[5] Furthermore, slotting arrangements, the payment by manufacturers for retail shelf space, have become increasingly important and have been the subject of recent antitrust litigations[6] and the focus of Federal Trade Commission studies.[7]

In our model, we assume away buyer's private information, which allows us to depart from the existing literature on bundling that usually considers a framework of second-degree price discrimination and to identify what seems to us a first-order effect of bundling associated with the buyer's slot constraint. Actually, in the case of movie industry, Kenney and Klein (1983) point out that second-order price discrimination explanation of bundling is inconsistent with the facts of *Paramount* and *Loew's* since the prices of the blocks varied a great deal across markets. Furthermore, in the Digital era, the prices are more and more tailored to buyers' characteristics as in the case of pricing of academic journals (Edlin and Rubinfeld 2004, Jeon and Menicucci 2006).

We consider a simultaneous pricing game among n sellers (or firms) who sell their products to a buyer having $k(> 0)$ number of slots. Each seller i has a portfolio of n_i distinct products. The buyer has a unit demand for each product. In our setting, a product needs to occupy a slot to generate a value. Products have heterogenous values and the values are independent.[8] Therefore, in the absence of the slot constraint, there is no competition among the sellers. Social efficiency requires the slots to be allocated to the best k products among all products. In this setup, we study how the outcome of competition depends on the nature of the products (digital goods versus physical goods) and different contractual arrangements between each seller and the buyer.

Given a portfolio of products belonging to a firm, we define bundling as a contract that specifies a price for every subset of the portfolio. A particular class of bundling contracts

[4]Procter & Gamble / Gillette, DG Competition case COMP/M.3732; Société des Caves de Roquefort, Conseil de la Concurrence, Decision 04-D-13, 8th April 2004.

[5]Société des Caves de Roquefort's market share in the Roquefort cheese market was 70% but, through the contract, could occupy eight among all nine brands that Carrefour, a supermarket chain, sold.

[6]See, for example, R.J. Reynolds Tobacco Co. v. Philip Morris, Inc., 199 F. Supp. 2d 363 (M.D.N.C. 2002), aff'd per curiam, 67 Fed. Appx. 810 (4th Cir. 2003); American Booksellers Ass'n, Inc. v. Barnes & Noble, Inc., 135 F. Supp. 2d 1031 (N.D. Cal. 2001); Intimate Bookshop, Inc. v. Barnes & Noble, Inc., 88 F. Supp. 2d 133 (S.D.N.Y. 2000); FTC v. H.J. Heinz Co., 116 F. Supp. 2d 190 (D.C.C. 2000), rev'd, 246 F.3d 708 (D.C. Cir. 2001).

[7]See FTC Report (2001) and FTC Study (2003).

[8]In other words, the value that a product generates does not depend on the set of the other products that occupy the slots. This assumption is for simplicity and our results hold at least for simple substitutions among products (see the end of section 5).

is what we call "independent pricing plus a fixed fee". A strategy in this class consists of a fixed fee for the right to buy products in the portfolio and one individual price for each product. There are three interesting special cases of this class. Individual pricing corresponds to the case with zero fixed fee; pure bundling corresponds to the case with zero individual prices; a "technology-renting" strategy is the case in which each individual price is equal to the cost of production.

Interestingly, the change from independent pricing to bundling opens a new contractual dimension, i.e. contracting on slots. Note that under independent pricing, the buyer will purchase only those products that would occupy a slot and therefore slotting contracts are redundant. In contrast, under bundling, for instance, if all firms offer pure bundles, the buyer may end up buying more products than the slots and hence we need to distinguish bundling with slotting contracts and bundling without slotting contracts. If a seller sells a bundle with a slotting contract, upon accepting the deal, the buyer must allocate a slot to each product in the bundle. Therefore, the contractual space increases as we move from independent pricing to bundling without slotting contracts and from bundling without slotting contracts to bundling with the permission of slotting contracts.

Our main results are the following. First, under independent pricing, equilibrium (in pure strategy) often does not exist and hence the outcome is often inefficient. Second, each firm has an incentive to use bundling instead of independent pricing since bundling reduces competition from rival products. Third, when bundling is allowed, there always exists an efficient equilibrium where each firm uses a technology-renting strategy, regardless of whether or not firms can use slotting contracts. Our technology-renting equilibrium generalizes the marginal cost pricing result[9] in the literature on competition in non-linear pricing (Armstrong-Vickers, 2001, 2008 and Rochet-Stole 2002) to a situation in which any number of firm can sell any number of products. Furthermore, in the case of digital good, all equilibria are efficient if slotting contracts are prohibited. In the case of physical good, all equilibria are efficient if individual prices cannot be smaller than costs. However, if sellers use slotting contracts, inefficient equilibria can arise even in the case of digital good. Fourth, we identify portfolio effects of bundling and analyze the implications on horizontal merger. By portfolio effects of bundling, we mean that even though two firms end up selling products of identical values to the buyer, they can realize different profits if their portfolios are different in terms of the products that are not sold. We show that because of the portfolio effects, firms have an incentive to merge. Furthermore, when we endogenize the merger by considering the auction of a given product, we find that the firm with the

[9] Charging the variable price(s) equal to the (constant) marginal cost(s) is equivalent to renting the production technology.

strongest portfolio wins the auction, suggesting a tendency of increasing concentration.

One interesting theoretical result is that there is an intermediate level of contractual space such that decreasing or increasing contractual space beyond this level can hurt efficiency. For instance, in the case of digital good, all equilibria are efficient if bundling is allowed and slotting contracts are forbidden. On the one hand, if bundling is forbidden, efficient equilibria may not exist. On the other hand, if bundling and slotting contracts are allowed, there can exist inefficient equilibria.

To illustrate the incentive to practice bundling, consider a simple example in which firm 1 produces two products of value 3 at zero cost, firm 2 produces one product of value 2 and another product of value zero at zero cost and the buyer has two slots. Suppose that firm 1 wants to sell both products. Then, under independent pricing, each product of firm 1 faces competition from the best product of firm 2 and hence firm 1 realizes a total profit of 2. Consider now bundling. Throughout the paper, in order to determine the price of a given bundle, we first consider *the best alternative portfolio* that the buyer can build up without buying the bundle and ask how much extra value the buyer can get by improving the portfolio with the purchase of the bundle.[10] Then, without buying the bundle, the best alternative portfolio is composed of only firm 2's products. Instead, if the buyer buys the bundle, she can replace firm 2's products with firm 1's products. This implies that firm 1 can realize a total profit of 4. This example shows that bundling reduces competition from rival products by changing competition between individual products into competition between portfolios. More precisely, in our example, under independent pricing, each product of firm 1 faces competition form the best product of firm 2 but, under bundling, one product of firm 1 faces competition from the second best product of firm 2. Up to now, we assumed that firm 1 wants to sell both products. However, under independent pricing, if firm 2 chooses zero prices, it is optimal for firm 1 to sell only one product in order to relax the slot constraint, which in turn changes the best response of firm 2: firm 2 now charges price equal to 2 to the first product. Then, it is again optimal for firm 1 to sell both products. We show in section 3 that because of this circular argument, equilibrium in pure strategy often does not exist without bundling.

To give the intuition about efficiency under bundling, consider digital products (i.e. zero cost[11]) and assume that slotting contracts are forbidden. Consider a firm owning one product belonging to the k best among all products in the industry. Then the best

[10] More precisely, Lemma 1 shows that for any given strategy profile of rivals, a firm can find a best response in the set of technology-renting strategies.

[11] We assume that the fixed cost of production is already incurred: for instance, movies are already produced.

alternative portfolio that the buyer can build without the product includes a product inferior to the product. Since the buyer can increase her payoff by replacing the inferior product with this product, the firm can always sell it at a strictly positive price. Therefore, all equilibria are efficient. However, if firms use slotting contracts, inefficient equilibria can arise since if the buyer is bound by slotting contracts, the buyer may not be able to replace the inferior product with the superior one.

Finally, in order to explain portfolio effects of bundling, consider the efficient equilibrium where each firm uses a technology-renting strategy. Hence, in the equilibrium, the buyer builds up the portfolio composed of the k best products and the fixed fee that each firm charges is the difference between the value generated by the equilibrium portfolio and the value generated by the best alternative portfolio without buying his bundle. In general, as a firm's portfolio becomes stronger, the best alternative portfolio that the buyer can build without the firm's portfolio is weaker. This statement applies as well to the portfolio of the products which do not belong to the k best as long as they affect the best alternative portfolio. This is why two firms end up selling products of identical values can realize different profits depending on the portfolios of the products that are not sold.

There are only a few papers on block booking. According to the leverage theory, on which the Supreme Court's decisions were based, block booking allows a distributor to extend its monopoly power in a desirable movie to an undesirable one. This theory was criticized by Chicago School (see e.g. Bowman 1957, Posner 1976, Bork 1978) since the distributor is better off by selling only the desirable movie at a higher price. As an alternative, Stigler (1968) proposed a theory based on second-degree price discrimination. However, Kenney and Klein (1983) point out that simple price discrimination explanation is inconsistent with the facts of *Paramount* and *Loew's* and argue that block booking mainly prevents exhibitors from oversearching, (i.e. from rejecting films revealed ex post to be of below-average value).[12]

Most papers on bundling study bundling of two (physical) goods in the context of second-degree price discrimination and focus on either surplus extraction (Schmalensee, 1984, McAfee et al. 1989, Salinger 1995 and Armstrong 1996, 1999) in a monopoly setting or entry deterrence (Whinston 1990, Choi-Stefanadis 2001, Carlton-Waldman 2002, and Nalebuff 2004)[13] in a duopoly setting.[14] Armstrong (1999) and Bakos and Brynjolfsson

[12] Their hypothesis is empirically tested in a recent paper by Hanssen (2000) but the author finds little support for the hypothesis. But Kenny and Klein (2000) do not agree with Hanseen's analysis.

[13] Choi-Stefanadis (2001) and Carlton-Waldman (2002) do not use the framework of second-degree price discrimination.

[14] Armstrong-Vickers (2008) is a little bit closer to our paper in that each firm can practice bundling in their model: they study competition between two symmetric firms producing two horizontally differentiated

(1999) are an exception, in that they study bundling of a large number of goods, but they maintain the second-degree price discrimination framework: they show that bundling allows a monopolist to extract more surplus (since it reduces the variance of average valuations by the law of large numbers) and thereby unambiguously increases social welfare.[15] In our paper as well, each seller can bundle any number of goods. However, since we assume complete information (and hence full surplus extraction is possible under the monopoly setting), the rent extraction issue does not arise and there is no use in applying the law of large number. It is also important to remind that we consider a static simultaneous pricing game and do not address the entry deterrence issue.

In Jeon-Menicucci (2006), we take a framework similar to the one in the current paper to study bundling electronic academic journals. More precisely, publishers owning portfolios of distinct journals compete to sell them to a library who has a fixed budget to allocate between journals and books. We find that bundling is a profitable strategy both in terms of surplus extraction and entry deterrence. Conventional wisdom says that bundling has no effect in such a setting and this is true in the absence of the budget constraint. However, when the budget constraint binds, we find that each firm has an incentive to adopt bundling but bundling reduces social welfare by reducing the library's consumption of journals and books. In the current paper, instead of focusing on the budget constraint, we focus on the slot constraint. Another difference is that Jeon-Menicucci (2006) focus on products (journals) of homogeneous value while in the current paper we consider products of heterogenous values. In spite of similarities of the frameworks, the result we obtain here is completely opposite to the one in the previous paper since we find that the allocation under bundling is efficient while the allocation under independent pricing is not necessarily efficient.

Shaffer (1991)[16] considers an upstream monopolist selling two directly substitutable products with variable quantity and finds that brand specific two-part tariffs alone do not allow the monopolist to capture the maximum rent from the downstream firm but full-line forcing (equivalent to bundling) does. We consider products of independent values and hence the rent extraction issue Shaffer considers does not arise in a monopoly setting.

Finally, to some extent, our efficiency result of bundling is related to Bernheim and Whinston (1985, 1998) and O'Brien and Shaffer (1997) who show that when two single-product firms simultaneously offer non-linear tariffs to a common retailer, the vertically-

products (i.e. consumers are located in a two-dimensional hotelling space). They find that compared to linear pricing, non-linear pricing has the benefit of efficient variable prices (i.e. marginal cost pricing) but the cost of excessive brand loyalty.

[15] Bakos and Brynjolfsson (2000) apply their first paper to entry deterrence.

[16] See also Vergé (2001) who performs the social welfare analysis in the setup of Shaffer (1991).

integrated outcome is obtained.[17] However, they do not consider the slot constraint: if there is no slot constraint in our model, all firms fully extract the buyer's surplus regardless of whether they practice bundling or not. We consider competition among n firms when each firm can bundle any number of different products and study how different contractual arrangements affect competition depending on whether they sell digital goods or physical goods.

In what follows, section 2 reviews the Chicago School Criticism of leverage theory with a simple model and explains our contribution with respect to it. Section 3 illustrates the key results with a simple example. Section 4 presents the model. Section 5 presents the main results when firms do not use slotting contracts. Section 6 studies the situation when firms can use slotting contracts. Section 7 identifies portfolio effects of bundling and studies the implications on horizontal mergers. Section 8 derives policy implications and suggests issues to be studied in the future.

2 Chicago School Criticism of Leverage Theory

According to the leverage theory of tying (or bundling), a multiproduct firm with monopoly power in one market can monopolize a second market using the leverage provided by its monopoly power in the first market. The theory, however, was largely discredited as a result of criticisms originating in the Chicago School (see e.g. Bowman 1957, Posner 1976, Bork 1978). In this section, we review the Chicago School Criticism of leverage theory with a simple model and explain our framework and contribution with respect to it.

Consider two independent products (1, 2) and two sellers (A, B). A is the monopolist of product 1 and A and B compete in the market for product 2. There is a single customer, called C, who has a unit demand for each product. Assume that the cost of production is $c(>0)$ for all products. C's willingness to pay for product 1 is $u_A^1(>c)$: C's willingness to pay for product 2 produced by A (or B) is $u_A^2 > c$ ($u_B^2 > c$). Assume $c > u_B^2 - u_A^2$, which means that when $u_B^2 > u_A^2$, once C buys product 2 from A, B cannot induce C to buy his product without making a loss. In addition, we assume $u_A^1 + u_A^2 > u_B^2$, which implies that by bundling the two products, A can force C to buy both products from A.

In the absence of bundling, seller $i(=A, B)$ simultaneously chooses a price for product $j(=1,2)$ $p_i^j \in \mathbf{R}_+$. In equilibrium, A always sells product 1 at $p_A^1 = u_A^1$ and sells product 2 at $p_A^2 = c + u_A^2 - u_B^2$ if and only if $u_A^2 \geq u_B^2$. Hence, A's profit without bundling is given by $p_A^1 + p_A^2 = u_A^1 - c + \max\{0, u_A^2 - u_B^2\}$. Note that under independent pricing, the outcome

[17] O'Brien-Shaffer (2005) show that this result also holds under simultaneous Nash bargaining for the case of N single-product firms.

is always socially efficient.

Suppose now that A bundles both products and charge P_A for the bundle. Then, in equilibrium, A succeeds in selling the bundle at $P_A = u_A^1 + c + u_A^2 - u_B^2$, realizing a profit of $u_A^1 - c + u_A^2 - u_B^2$. Note that under bundling, the outcome is socially inefficient if $u_A^2 < u_B^2$.

Comparing A's profit without bundling with its profit with bundling shows that bundling does not affect the profit if A is more efficient than B in product 2 (i.e. $u_A^2 \geq u_B^2$) and decreases it otherwise. This shows that A never has the incentive to practice bundling for the purpose of monopolizing the tied product market. Furthermore, a laissez-faire policy always achieves social efficiency since firm A's private incentive to practice bundling is aligned with the social incentive.

However, we notice that Chicago School's criticism is a *weak argument* in a double sense: a social planner never has any strict incentive to favor bundling (since outcome is always socially efficient without bundling but it can be inefficient with bundling) and sellers never have any strict incentive to practice bundling (since a seller can never strictly increase its profit with bundling).

In our paper, we consider competition among any number of sellers when each of them sells any number of distinct products to a buyer. We assume that all products are independent but the buyer has a limited number of slots. This slot constraint creates competition among products. In this setting, we find *a strong argument* for laissez-faire regarding bundling: we show that (i) the outcome of competition among portfolios is efficient in general under bundling (for instance, an efficient equilibrium always exists), but can be inefficient (an equilibrium in pure strategy can fail to exist) without bundling (ii) each seller has an incentive to practice bundling since bundling reduces competition from rival products. In the next section, we illustrate these results with a simple example.

3 Illustration with a simple example

We here give a simple example to illustrate some main results. There are two sellers, A and B. A has two products of value $(u_A^1, u_A^2) = (4, 3)$ and B has one product of value $u_B^1 = 2$: u_i^j means the value that the customer, C, obtains from the j-th best product among firm i's products. We assume that the values of the three products are independent and hence there is no direct competition among them. However, C has only two slots, which generates competition among them. The production cost is zero for all products. We note that social efficiency requires that the two slots be occupied by only A's products.

3.1 Without bundling: non-existence of equilibrium

Consider a simultaneous pricing game without bundling: seller $i(=A,B)$ simultaneously chooses a price for product $j(=1,2)$ $p_i^j \in \mathbf{R}$.[18] We show below that this game has no equilibrium in pure strategy. We assume as a tie-breaking rule that if C is indifferent among several products, C buys the products with the highest (gross) values.[19] Without loss of generality, we can assume that A chooses prices such that $4-p_A^1 \geq \max\{0, 3-p_A^2\}$: the net surplus that C makes from buying A's best product is positive and larger than the one it makes from buying A's second best product.

First, there is no equilibrium in which A sells only its best product (i.e. there is no equilibrium with $p_A^2 > 1$). Suppose first that A charges $p_A^2 > 3$. Then, B's best response is $p_B^1 = 2$. Hence, in the candidate equilibrium, A charges $p_A^1 = 4$ and hence achieves a profit equal to 4. This cannot be an equilibrium since A can deviate and charge for instance $p_A^{2\prime} = 3$ and $p_A^{1\prime} = 4$. Then A sells both products and realizes a profit equal to 7. Suppose now that A charges $p_A^2 \in (1,3]$. Then, B can sell its product by charging $p_B^1 = p_A^2 - 1 - \varepsilon$ with $\varepsilon(>0)$ small enough. $4 - p_A^1 \geq \max\{0, 3-p_A^2\}$ implies that in the candidate equilibrium, A charges $p_A^1 = 1 + p_A^2$ and hence A's profit is $1 + p_A^2$. Consider now A's deviation in which A charges $p_A^{2\prime} = p_A^2 - \varepsilon$ and $p_A^{1\prime} = p_A^1 - \varepsilon$. Then A sells both products and realizes a profit equal to $1 + 2(p_A^2 - \varepsilon)$, which is larger than $1 + p_A^2$.

Second, there is no equilibrium in which A sells both products (i.e. there is no equilibrium with $p_A^2 \leq 1$). Note first that $p_A^2 \leq 1$ together with $4 - p_A^1 \geq \max\{0, 3-p_A^2\}$ implies that $p_A^1 \leq 1 + p_A^2$ and therefore A's profit cannot be larger than 3. However, A can realize a profit equal to 4 by choosing $p_A^1 = 4$ and $p_A^2 = 3$ regardless of B's strategy.

Therefore, we have a circular argument, which explains the non-existence of equilibrium. On the one hand, if A occupies only one slot, A can extract full surplus from his best product. But then, B's best response is to charge a monopoly price, which triggers A's deviation to occupy both slots. On the other hand, if A occupies both slots, each of A's product faces competition from B's product such that A's total profit is lower than the profit from selling only one profit.

[18] Equilibrium does not exist even though we allow firms to be able to charge negative prices.

[19] This tie-breaking rule is standard. For instance, if two firms producing a homogenous good with different marginal costs compete in prices and the cost differential is small, in equilibrium, both firms charge the price equal to the highest marginal cost and the tie is broken by assuming that all consumers buy the good from the firm with the lowest marginal cost.

3.2 Bundling

Consider now that A sells a bundle of both products and charges a price $P_A \in \mathbf{R}_+$. For notational consistency, let $P_B \in \mathbf{R}_+$ denote the price that B charges for its product. Consider the simultaneous pricing game. Then, the unique equilibrium is $P_A = 5$ and $P_B = 0$. In the equilibrium, C buys A's bundle and hence the outcome is socially efficient. It is easy to see why this is an equilibrium. A has no incentive to charge a higher price; then C prefers buying B's product instead of A's bundle. Given that B's profit is zero, $P_B = 0$ is a best response.

Although the example is simple, it generates useful insights. First, it shows that each firm has an incentive to use bundling since bundling reduces competition from rival firms' products. To explain this, we first remind that conditional on $p_B^1 = 0$, without bundling, A's best response is to sell only the best product, which generates a profit of 4. A has no interest in selling both products even though both of them are superior to B's one since then the slot constraint binds and *each* of A's product faces competition from B's product. Bundling changes competition between individual products into competition between portfolios. In particular, when A sells only the bundle of both products, C has no option of buying only one of them. To make the intuition more precise, assume that B has a second product of value $u_B^2 = 0$, bundles both products and charges $P_B = 0$. Then, competition between the two bundles is equivalent to the situation in which A's first product competes with B's first product and at the same time A's second product competes with B's second product. This clearly shows that bundling allows A to reduce competition that A faces from B's product(s). This also explains that A can make a profit of $2 + 3 = 5$ under bundling.

The intuition for why bundling restores efficiency is that under competition among bundles, every seller having a superior product can make a positive profit by inducing C to replace an inferior product with the superior one. As an illustration, consider the following example $(u_A^1, u_A^2) = (4, 2)$ and $u_B^1 = 3$. In this example either, equilibrium in pure strategy does not exist under independent pricing. However, under bundling, there is a unique equilibrium in pure strategy: $P_A = 4$ and $P_B = 1$, which is efficient. Contrary to the leverage theory, even though C buys A's bundle, B can realize a profit by inducing C to replace A's second product with B's one.

4 The setting

There are n firms (or sellers), denoted by $i = 1, ..., n$, and a buyer; we use "he" for each firm and "she" for the buyer; we also use $i = 0$ to represent the buyer. Each firm i (> 0)

has a portfolio of n_i distinct products. We use ij to denote firm i's j-th best product (for instance, 12 represents firm 1's 2nd best product) and $\mathcal{B}_i = \{i1, ..., in_i\}$ represents i's portfolio of products; let $\mathcal{B} \equiv \mathcal{B}_1 \cup ... \cup \mathcal{B}_n$. The buyer has a unit demand for each product and has k (≥ 1) number of slots. A product needs to occupy a slot to generate a value.[20] The buyer can be for instance a movie theater having a limited number of projection rooms, or a retailer with limited shelf space. Let u_i^j be the value that the buyer obtains from allocating a slot to product ij; thus $u_i^1 \geq u_i^2 \geq ... \geq u_i^{n_i} > 0$ for $i = 1, ..., n$. We assume that the values are independent: we show at the end of Section 5 that our results hold for substitutes as well. In the case in which $n_i \geq k$, it is straightforward that only the k best products of firm i matter in our setting. In the case of $n_i < k$, we define $u_i^{n_i+1} = ... = u_i^k = 0$. In this way we can think, without loss of generality, that each firm's portfolio consists of k products. Even though we assume one buyer, we have in mind a situation with many buyers in separate markets; they do not compete and each seller can discriminate them in terms of prices as in the case of distribution of movies or TV series to different countries. Therefore, we assume that a prototype of each product is already produced and the cost of (re)production is $c \geq 0$ for every product $ij \in \mathcal{B}$[21]; for instance, in the case of Digital good, the first copy is already produced and the cost of reproduction c is zero. Assume for simplicity that no cost is incurred by the buyer.[22] The buyer's payoff is given by the sum of the values obtained from the purchased products minus the prices paid.

Let u^j denote the value that the buyer obtains from the j-th best product among all products in $\mathcal{B}$; thus $u^1 \geq u^2 \geq ... \geq u^{nk}$. We assume $u^k > \max\{c, u^{k+1}\}$, so that the set of the k best products, denoted with B^{FB}, is unique. It is socially optimal to occupy all slots with the products in B^{FB}. For any $B \subseteq \mathcal{B}$, let $U(B)$ represent the total value that the buyer obtains from allocating k slots to the best k products in B: obviously, if B has less than k number of products, the total value is computed by allocating one slot to each product. In particular, we define $U \equiv U(B^{FB}) = u^1 + ... + u^k$. Let $B_i^{FB} \equiv B^{FB} \cap \mathcal{B}_i$ and let q_i^{FB} represent the cardinality of B_i^{FB} (hence, $q_1^{FB} + ... + q_n^{FB} = k$) while $U_i^{FB} \equiv U(B_i^{FB}) = u_i^1 + ... + u_i^{q_i^{FB}}$. Without loss of generality, we assume that there exists n^{FB} between 1 and n such that $q_i^{FB} \geq 1$ (that is, $B_i^{FB} \neq \emptyset$) for $i = 1, ..., n^{FB}$, and $q_i^{FB} = 0$ (that is, $B_i^{FB} = \emptyset$) for

[20] By assuming unit demand, we assume for simplicity that a product can occupy at most one slot in that the value generated from occupying a second slot is zero. This assumption can be relaxed without changing the main results.

[21] The assumption of homogeneous cost is made without loss of generality: at the end of Section 5 we show that the results are (qualitatively) unaffected if this assumption is relaxed.

[22] If instead the buyer bears cost $\gamma_i^j \geq 0$ to generate a value from product ij, then we can consider $u_i^j - \gamma_i^j$ as the buyer's gross value and the following analysis applies.

$i = n^{FB} + 1, ..., n$.

In this setup, we study how bundling affects the set of products occupying the buyer's slots. Precisely, we are interested in knowing when the slots are occupied by the products in B^{FB}. We say that an equilibrium is (socially) efficient if all slots are allocated to the products in B^{FB}: then the first-best outcome is realized, which is the reason why use the superscript FB.

4.1 Contracts and game

In this section, we first describe the bilateral contracts that seller i can propose to the buyer in our model and then introduce the timing of the game that we study.

4.2 Bilateral contracts without slotting contracts

- Menu of bundles[23]

In the absence of slotting contracts (that will be defined later on), the most general contract between seller i and the buyer is that firm i offers a menu of bundles with prices $\{P_i(B_i)\}_{B_i \subseteq \mathcal{B}_i}$: firm i chooses $P_i(B_i) \geq 0$ for each $B_i \subseteq \mathcal{B}_i$, with $P_i(\emptyset) = 0$. Then, if the buyer buys bundle B_1 from firm 1, .., bundle B_n from firm n (some of these sets may be empty),[24] then she pays $P_1(B_1) + ... + P_n(B_n)$. Let $s_i = \{P_i(B_i)\}_{B_i \subseteq \mathcal{B}_i}$ denote a generic strategy of firm i and S_i be the strategy space for firm i.[25]

- Independent pricing plus a fixed fee

A particular class of menu of bundles is the strategy which is composed of individual prices $(p_{i1}, ..., p_{ik})$ and a fixed fee $F_i \geq 0$ such that $P_i(B_i) = F_i + \sum_{ij \in B_i} p_{ij}$ for any (non-empty) $B_i \subseteq \mathcal{B}_i$. In this case, if the buyer wants to buy at least one product from firm i, she must first pay F_i for the right to buy, and then she pays the individual prices of the products that she selects to buy. Let $IF_i \subset S_i$ be the set of "independent pricing plus

[23] Our definition of menu of bundles generalizes the notion of mixed bundling used in the context of two goods. In this case, mixed bundling means that the seller charges a price for each good and another price for the bundle of both goods.

[24] In what follows, we simply write that the buyer buys $B_1 \cup ... \cup B_n$.

[25] In fact, for some $s_i \in S_i$, the buyer may want to buy more than one bundle from firm i (for instance, if buying two small bundles is cheaper than buying a big bundle composed of the two small ones). However, none of our arguments or proofs below depends on the assumption that the buyer buys at most one bundle from each firm. Thus, for the sake of simple notation, we make this assumption in the rest of the paper.

a fixed fee" strategies and let $if_i \in IF_i$ an element of the set. Three particular cases of "independent pricing plus a fixed fee" strategies are of great interest:

$*$ Independent pricing: Independent pricing is an extreme case with $F_i = 0$, thus $P_i(B_i) = \sum_{ij \in B_i} p_{ij}$ for any $B_i \subseteq \mathcal{B}_i$.

$*$ Pure bundling: Pure bundling is another extreme case with $p_{ij} = 0$ for each $ij \in \mathcal{B}_i$ such that $P_i(B_i) = F_i$ for any $B_i \subseteq \mathcal{B}_i$. In other words, pure bundling is a deal of all-or-nothing.

$*$ Technology-renting: A technology renting strategy consists of two elements: firm i rents its production technology to the buyer by charging $p_{ij} = c$ for each $ij \in \mathcal{B}_i$, and extracts the buyer's surplus by levying a fixed rental fee F_i. Let $TR_i \subset IF_i$ be the set of technology-renting strategies, and $tr_i \in TR_i$ an element of the set.

4.3 Bilateral contracts with slotting contracts

In what follows, we will distinguish two cases depending on whether slotting contracts are used or not. If firm i does not use any slotting contract, the buyer has full freedom in allocating the slots among all products she purchased. In contrast, if the buyer buys from firm i a bundle B_i with a slotting contract (and $q_i = \#B_i$ is the number of products in B_i), the buyer must allocate q_i number of slots to the products in B_i. Note that under independent pricing, slotting contracts are redundant since the buyer will not buy any product that will not occupy a slot. In section 5 we study competition among bundles without slotting contracts, and in section 6 we allow for slotting contracts.

4.4 Timing

In sections 5 and 6, we consider a two-stage simultaneous pricing game in which

- at stage one, each firm i simultaneously makes a contract offer;
- at stage two, the buyer observes all firms' offers and chooses among them to decide the products to buy and allocates the slots to the products.

At stage two, as a tie-breaking rule, we assume that in case the buyer is indifferent among different combinations of products, she chooses the combination that maximizes her (gross) value.[26]

[26] This tie-breaking rule is standard in that it is basically equivalent to the following rule applied to two firms producing a homogenous good with different marginal costs. In Bertrand equilibrium, if the cost differential is not large, both firms charge the price equal to the highest marginal cost and the tie is broken by assuming that all consumers buy the good from the firm with the lowest marginal cost.

5 Bundling without slotting contracts

In this section, we study competition among sellers when bundling is allowed but slotting contracts are prohibited. In section 5.1, we first show that each firm has an incentive to practice bundling. In section 5.2, we describe an efficient Nash equilibrium (NE) for any $c \geq 0$. Section 5.3 shows that any NE is efficient if c is small and identifies a sufficient condition to make all equilibria efficient for any c. Section 5.4 gives results on profits. Section 5.5 performs robustness checks by introducing heterogeneous costs or substitutions.

5.1 Incentive to bundle

We first describe an important property of the technology-renting strategies in the following lemma.

Lemma 1 *For any profile (s_i, s_{-i}), let $\pi_i \geq 0$ denote the profit of firm i given (s_i, s_{-i}). Then, firm i can make profit π_i also by playing a technology-renting technology $tr_i \in TR_i$ instead of s_i such that the fixed fee F_i associated with tr_i is equal to π_i.*

Proof. See Appendix. ■

Lemma 1 says that no firm i loses anything by restricting attention to strategies in TR_i *regardless of the strategies used by other firms.* We will use often this result in the proofs of our propositions. The lemma also suggests that each firm has at least a weak incentive to practice bundling. We now provide an example to illustrate a case in which a firm, without bundling, cannot achieve the profit that he can achieve with a technology renting strategy.

Example 1 *Assume $n = 2$, $c = 0$, $k = 3$. Firm 1 has three products with value $(5, 3, 0)$ and firm 2 has three products with value $(4, 1.9, 1)$. Suppose that firm 2 uses a technology-renting strategy and charges $F_2 = 4$. Then, from Lemma 1, one of firm 1's best responses is to use a technology-renting strategy and to charge $F_1 = 5.1$. Instead, if firm 1 switches to independent pricing, each of his two best products faces the competition from product 22, and the best firm 1 can do is to sell 11 and 12 at prices 3.1 and 1.1 respectively, with a profit of 4.2 which is inferior to 5.1.*

The reason why independent pricing gives a smaller profit than a technology-renting strategy in the above example is the following. Under independent pricing, each product of firm 1 faces competition from firm 2's second-best product, which does not occupy any slot. This is because, under independent pricing, the buyer has the option of buying (and paying) only one product from firm 1, and thus the firm cannot induce the buyer to buy both 11 and 12 if he charges prices higher than $5-1.9$ and $3-1.9$, respectively. In contrast, under bundling (or technology-renting), such an option does not exist: without paying the fixed fee, no product of firm 1 is available while after paying the fixed fee, the buyer gets both products of firm 1 at the same time (even though the buyer can use only one of them in this case, it does not allow her to save any payment). Therefore, under bundling, the two products of firm 1 compete with the second-best and the third-best products of firm 2, which allows firm 1 to realize a higher profit: 5.1 is derived from $5+3-1.9-1$. In other words, bundling allows firm 1 to reduce competition from rival products.

Lemma 1 and Example 1 together imply

Proposition 1 *(incentive to bundle) Each firm has at least a weak (and sometimes a strict) incentive to practice bundling instead of independent pricing.*

5.2 An efficient equilibrium

In this section, we show that an efficient equilibrium exists for any $c \geq 0$; in this equilibrium each firm i uses a technology-renting strategy, and thus we can think from Lemma 1 that the strategy space for each firm i is given by the set of possible values of F_i in $[0, +\infty)$.

Let $\mathcal{B}_{-i}$ represent the set $\cup_{h \neq i} \mathcal{B}_h$ of products in the portfolios of firms different from i, where $-i$ represents all firms except firm i. In order to understand the equilibrium value of F_i, we need to know *the best alternative portfolio* of products that the buyer can build up to occupy the slots when she does not buy any product from firm i (for $i = 1, ..., n^{FB}$). More precisely, we suppose that the buyer has already rented the technologies of all other firms, and is considering whether to rent also the technology of firm i; thus we can view i as the marginal seller. In this context we determine the highest F_i firm i can charge to induce the buyer to rent i's technology.

For this purpose, let u_{-i}^j represent the value of the j-th best product among the products in $\mathcal{B}_{-i}$. For instance, if $c = 0$ the best alternative portfolio when the buyer does not buy any product from i is made by the products with values $(u_{-i}^1, ..., u_{-i}^k)$. However, if $c > 0$, $u_{-i}^q < c$ may occur for some $q \leq k$. Then, the best alternative portfolio is composed of less than k number of products since the buyer will not buy any product with value smaller than c. We below describe the best alternative portfolio for any c.

Let $B_{-i}^{FB} \equiv B^{FB} \backslash B_i^{FB}$ denote the set of the first best products in $\mathcal{B}_{-i}$. Obviously, B_{-i}^{FB} will be included in the best alternative portfolio since every product in B^{FB} has a value larger than c. Note also that B_{-i}^{FB} includes exactly $k - q_i^{FB}$ number of products. Now let B_{-i}^{SB} represent the subset of the best products in $\mathcal{B}_{-i} \backslash B_{-i}^{FB}$ such that the value of each product in B_{-i}^{SB} is not smaller than c and the cardinality of B_{-i}^{SB}, denoted by q_{-i}^{SB}, is not larger than q_i^{FB}. Therefore q_{-i}^{SB} is the number in $\{0, ..., q_i^{FB}\}$ with the following property:

- If $c > u_{-i}^{k-q_i^{FB}+1}$, then $q_{-i}^{SB} = 0$;
- If $u_{-i}^{k} \geq c$, then $q_{-i}^{SB} = q_i^{FB}$;
- If $u_{-i}^{k-q_i^{FB}+1} \geq c > u_{-i}^{k}$, then q_{-i}^{SB} is the number in $\{1, ..., q_i^{FB}-1\}$ such that $u_{-i}^{k-q_i^{FB}+q_{-i}^{SB}} \geq c > u_{-i}^{k-q_i^{FB}+q_{-i}^{SB}+1}$.

From the definition of B_{-i}^{SB}, when the buyer does not buy anything from i, the best alternative portfolio is given by $B_{-i}^{FB} \cup B_{-i}^{SB}$. Then $U_{-i} \equiv u_{-i}^{1} + ... + u_{-i}^{k-q_i^{FB}+q_{-i}^{SB}}$ represents the total (gross) value from the best alternative portfolio and $U_{-i}^{SB} \equiv u_{-i}^{k-q_i^{FB}+1} + ... + u_{-i}^{k-q_i^{FB}+q_{-i}^{SB}}$ is the total (gross) value from B_{-i}^{SB}.

Let tr_i^* denote the technology renting strategy of firm i in the equilibrium we are describing. Then, the fixed fee associated with tr_i^*, denoted by F_i^*, is given by:

$$\begin{aligned} F_i^* &= U - ck - [U_{-i} - c(k - q_i^{FB} + q_{-i}^{SB})] \\ &= U_i^{FB} - cq_i^{FB} - (U_{-i}^{SB} - cq_{-i}^{SB}) \qquad \text{for} \quad i = 1, ..., n^{FB} \\ F_i^* &= 0 \qquad \text{for} \quad i = n^{FB}+1, ..., n. \end{aligned} \tag{1}$$

The fee F_i^* is equal to the difference between the total value of the best portfolio (net of the cost of producing it) and the total value of the best alternative portfolio (net of the cost of producing it). For instance, in the case of Digital goods (i.e., $c = 0$), F_i^* is simply equal to $U - U_{-i} = U_i^{FB} - U_{-i}^{SB}$. More precisely, when i chooses $F_i = F_i^*$, i supposes that all the other production technologies are rented already to the buyer (and thus all products in $\mathcal{B}_{-i}$ are already available to the buyer at cost) and his production technology is the marginal (i.e. the last) one that the buyer considers about renting. Under this assumption, F_i is set to make the buyer indifferent between renting i's technology or not. On the one hand, without renting i's technology, the net value that the buyer obtains from the best alternative portfolio is $U_{-i} - c(k - q_i^{FB} + q_{-i}^{SB}) - \sum_{h \neq i} F_h^*$. On the other hand, if the buyer rents i's technology, she obtains a net value from the best portfolio equal to $U - ck - \sum_{h \neq i} F_h^* - F_i$. Then F_i is set equal to F_i^* in order to make the two payoffs equal, which induces the buyer to buy B_i^{FB}, while a value of F_i higher than F_i^* would induce the buyer not to rent i's technology, which justifies i's presumption that his technology is the marginal one. Hence, if each firm i sets $F_i = F_i^*$, the buyer buys all products in B^{FB} and the outcome is efficient.

The next proposition establishes that the profile $(tr_1^*, ..., tr_n^*)$ is a Nash equilibrium: We call this equilibrium the technology-renting equilibrium.

Proposition 2 *(technology-renting equilibrium) For any $c \geq 0$, there exists a NE in which each firm i uses the technology-renting strategy tr_i^* and this equilibrium is efficient. In this NE, firm i's profit is F_i^* for $i = 1, ..., n$, while the buyer's payoff is $\sum_{i=1}^{n^{FB}} (U_{-i}^{SB} - cq_{-i}^{SB}) \equiv \pi_0^*$.*

Proof. See Appendix for the proof. We here provide a sketch of the proof. We first show that given $(tr_1^*, ..., tr_n^*)$, the buyer buys B_i^{FB} for $i = 1, ..., n^{FB}$. And then we prove that there is no profitable deviation for any firm. To check the deviation, from Lemma 1, it is enough to consider firm i's deviation in the set TR_i of technology-renting strategies. Obviously, firm i has no incentive to decrease F_i below F_i^*; firm i has no incentive to increase F_i above F_i^* since we show that then the buyer will not buy any product from firm i. ■

Our technology-renting equilibrium to some extent generalizes the marginal cost pricing result in the literature on competition in non-linear pricing (Armstrong-Vickers, 2001, 2008 and Rochet-Stole 2002) to a situation in which each firm can produce any number of products. In the technology-renting equilibrium, the buyer builds up the first best portfolio B^{FB} and fills the slots with it. This generates a social surplus equal to $U - ck$ which is split among the firms and the buyer as follows: firm i's profit is F_i^* for $i = 1, ..., n$, and the buyer's payoff is $\sum_{i=1}^{n^{FB}} (U_{-i}^{SB} - cq_{-i}^{SB})$. A consequence of Proposition 2 is that when firms $-i$ play tr_{-i}^*, firm i has no incentive to deviate from tr_i^* by using another strategy, such as pure bundling or any other menu of bundles. The following example illustrates the technology-renting equilibrium.

Example 2 *(Illustration of the technology-renting equilibrium) Consider the case in which $k = 2$, $n = 2$, $c = 3$, and*

$$(u_1^1, u_1^2) = (12, 8), \qquad (u_2^1, u_2^2) = (10, 5)$$

Then $p_{ij} = 3$, $F_1^* = (12 - 3) - (5 - 3) = 7$ and $F_2^* = (10 - 3) - (8 - 3) = 2$.

5.3 Efficiency

In this subsection, we first illustrate a case in which an inefficient equilibrium arises for $c > 0$ because of pure bundling. And then we show that all equilibria are efficient for c small enough and that the same result holds for any c if firms are restricted to marginal prices which are not smaller than c.

5.3.1 An inefficient equilibrium

The game we are considering may have many NE different from $(tr_1^*, ..., tr_n^*)$, and some of them can be inefficient as the following example illustrates.

Example 3 *(pure bundling and inefficiency) Consider the setting of Example 2:* $k = 2$, $n = 2$, $c = 3$ *and*

$$(u_1^1, u_1^2) = (12, 8), \qquad (u_2^1, u_2^2) = (10, 5)$$

In the following NE, each firm i *plays a pure bundling strategy and the buyer buys* $\{11, 12\}$ *rather than* $B^{FB} = \{11, 21\}$*:*

$$\begin{aligned} s_1 &\in S_1 \quad \text{is such that} \quad F_1 = 11, \quad p_{11} = p_{12} = 0; \\ s_2 &\in S_2 \quad \text{is such that} \quad F_2 = 6, \quad p_{21} = p_{22} = 0. \end{aligned}$$

In the above example, given (s_1, s_2), the buyer buys $\mathcal{B}_1$ and gets a payoff of 9. Under pure bundling, firm 1 induces the buyer to buy product 12 even though this product does not belong to B^{FB}, and firm 2 is unable to sell the superior product 21 for the two following reasons. First, from the all-or-nothing deal, given that the buyer buys 11, her marginal cost of getting product 12 is $p_{12} = 0$. Second, in order not to make a loss, firm 2 must charge a price for 21 at least equal to $c = 3$ while the buyer's gain from replacing 12 with 21 is 2.

5.3.2 Efficiency for c small

The previous reasoning regarding example 3 also suggests that the result may be different (i.e. the inefficient equilibrium may disappear) if c were smaller than 2, as this makes $10 - c$ larger than 8 and therefore firm 2 could induce the buyer to replace 12 with 21 and make a positive profit. Indeed, we can prove that if $c < u^k - u^{k+1}$ then in any NE the buyer buys B^{FB}. This makes the issue of multiplicity not very serious from the point of view of social welfare.

Proposition 3 *(efficiency) All Nash equilibria are efficient (i.e., in any NE, the buyer buys the set* B^{FB}*) if* $c < u^k - u^{k+1}$.

Proof. See Appendix. ■

Proposition 3 holds because of the argument made above with reference to Example 3. Suppose a profile of strategies $(s_1, ..., s_n)$ induces the buyer to buy $B = B_1 \cup ... \cup B_n$ such that product ij belongs to B^{FB} but not to B, while product $i'j'$ belongs to B and not to B^{FB}. Then firm i can induce the buyer to buy ij for an extra outlay of $c + \varepsilon$ and to give ij the slot previously assigned to $i'j'$, since the buyer increases her profit by $u_i^j - c - \varepsilon - u_{i'}^{j'}$, and $u_i^j - u_{i'}^{j'} > c$ because $u^k - u^{k+1} > c$. Notice that this argument does not require that the buyer stop buying product $i'j'$, since $P_{i'}(B_{i'} \setminus \{i'j'\})$ may be larger than $P_{i'}(B_{i'})$.[27]

Next, we give a simple corollary of Proposition 3:

Corollary 1 *In the case of Digital good (that is, $c = 0$), all equilibria are efficient and therefore a policy of laissez-faire regarding bundling achieves efficiency.*

Although the corollary is an obvious consequence of Proposition 3, it has an important policy implication. The example in Section 3 considers the case of Digital good and shows that when bundling is prohibited, an equilibrium in pure strategies fails to exist and hence the outcome of competition is not efficient. In contrast, when bundling is allowed, the corollary says that all equilibria are efficient and Proposition 2 says that at least one equilibrium exists. In addition, we know from proposition 1 that each firm does have an incentive to practice bundling. Therefore, we can conclude that in the case of digital goods, a policy of laissez-faire achieves efficiency.

5.3.3 Achieving efficiency for c not small

Example 3 shows that inefficient equilibria can exist for c large. In this section, we show that efficiency holds in any equilibrium if firms are prohibited from setting the marginal price[28] of any product below its cost c. Precisely, we consider the following restriction for firm i's strategies:

$$\begin{array}{c} \text{for any } ij \in \mathcal{B}_i \text{ and any } B_i \subset \mathcal{B}_i \text{ such that } ij \notin B_i, \\ P_i(B_i \cup \{ij\}) - P_i(B_i) \geq c \quad \text{for } i = 1, ..., n. \end{array} \tag{2}$$

The meaning of (2) is that as the number of objects in a bundle of firm i increases, the price of the bundle needs to increase at least by the cost of the additional products in the bundle. As a consequence, if firm i is interested in selling a particular bundle B_i for a certain price P^*, condition (2) forces him to make each subset of B_i available at a price strictly (weakly) smaller than P^* if $c > 0$ (if $c = 0$) such that the buyer can save at least c by cancelling a product within B_i. In particular, the condition makes it impossible for a firm to use the pure bundling strategy (i.e. to propose only a single bundle by charging only a fixed fee). Note that an "independent pricing plus fixed fee" strategy satisfies (2)

[27] But here we are using the free disposal assumption, which makes sense in this section where there is no slotting contract. In the next section where the buyer can sign slotting contracts, free disposal is still assumed as long as it does not violate some slotting contracts signed by the buyer.

[28] By the marginal price we mean the increase in the price of a bundle when an additional product is added to the bundle.

if and only if $p_{ij} \geq c$ for any ij (i.e. each individual price is larger than the cost) and in particular every technology-renting strategy satisfies (2). Thus, from Lemma 1 we know that firm i can always find a best response to any s_{-i} in the set of strategies satisfying (2). Note also that in the case of digital good, (2) is equivalent to $P_i(B_i \cup \{ij\}) \geq P_i(B_i)$: the price of a bundle does not decrease as it includes more products.

Under the restriction (2), we can show that for any c, all the NE are efficient.[29]

Proposition 4 *(efficiency) Suppose that each firm must satisfy (2). Then, all equilibria are efficient for any $c \geq 0$.*

Proof. See Appendix. ■

The intuition for the efficiency result of Proposition 4 is somewhat linked to the intuition for Proposition 3. Suppose that a profile of strategies $(s_1, ..., s_n)$ induces the buyer to buy $B = B_1 \cup ... \cup B_n$ such that product ij belongs to B^{FB} but not to B while product $i'j'$ belongs to B and not to B^{FB}. When c is small (Proposition 3) firm i can increase his profit by inducing the buyer to replace $i'j'$ with ij, because $u_i^j > u_{i'}^{j'} + c$, and this does not require that the buyer tries to save money by purchasing $B_{i'} \backslash \{i'j'\}$ because $B_{i'} \backslash \{i'j'\}$ is not necessarily less expensive than $B_{i'}$. When instead (2) holds, firm i can increase his profit by inducing the buyer to replace $i'j'$ with ij (with the latter product priced marginally at $c + \varepsilon$) because cancelling $i'j'$ allows the buyer to save at least c, and thus the buyer earns at least $u_i^j - (c + \varepsilon) - (u_{i'}^{j'} - c) > 0$ from replacing $i'j'$ with ij.

Remark 1: In the practice of competition policy, firms' charging prices below cost have been discussed in the context of predation: Areeda and Turner (1975) were the first to propose to use pricing below costs to identify predation. Our model does not deal with predation but interestingly Proposition 4 shows that prohibiting firms from charging individual prices (or marginal prices) below costs makes all equilibria efficient in our static pricing game.

5.4 Profits

In this subsection, we study the sellers' payoffs in the case of duopoly, that is when $n = 2$. Then we have:

Proposition 5 *(profits) If $n = 2$,*
(i) in any NE the profit of firm i is not larger than F_i^*, the profit in the technology-renting equilibrium, for $i = 1, 2$.

[29] Not surprisingly, condition (2) is violated by the strategy of firm 1 in example 3.

(ii) when each firm must satisfy (2), in any NE the profit of firm i is equal to F_i^*, for $i = 1, 2$.

Proof. See Appendix. ■

The intuition for proposition 5(i) is that when $n = 2$, F_1^* makes the buyer indifferent between buying B_1^{FB} from 1 (at the marginal price cq_1^{FB}), and buying B_{-1}^{SB} from 2 (at the marginal price cq_{-1}^{SB}). Then, if 1 attempts to make a profit larger than F_1^*, 2 can increase his profit by inducing the buyer to buy B_{-1}^{SB} at a price a bit higher than cq_{-1}^{SB} . Furthermore, when each firm is required to charge a marginal price larger than or equal to c, we can pin down the equilibrium profit of each firm and show that it is equal to F_i^*.

Proposition 5(i) implies

Corollary 2 *If $n = 2$,*

(i) the technology-renting equilibrium Pareto dominates any other equilibrium in terms of sellers' payoffs.

(ii) when each firm must satisfy (2), in all equilibria, the outcome is identical to that of the technology-renting equilibrium (in terms of allocation of slots and each player's payoff).

Not surprisingly, each seller's profit is lower in the inefficient equilibrium of example 3 than in the technology-renting equilibrium of example 2.

Remark 2: Corollary 2(i) is similar to the finding (Proposition 1) of Bernheim and Whinston (1998) that undominated equilibria maximize the joint payoffs of all sellers and the (single) buyer.

Remark 3: For $n > 2$, it is more difficult to pin down each firm's equilibrium profit. When all firms are required to satisfy (2), we can show that firm i can realize at least a profit equal to F_i^* (in fact, this is proven in the proof of Proposition 5(ii) for an arbitrary $n \geq 2$), but we have been unable to establish that F_i^* is also an upper bound for the profit of firm i.

5.5 Robustness

In this subsection, we perform two robustness checks.

5.5.1 The case of heterogenous costs

We can show that our notation can be modified to extend all the previous results in this section to the case in which production costs are heterogenous, that is the production cost of firm i for product ij is $c_i^j \geq 0$. For this purpose, we define $v_i^j = u_i^j - c_i^j$ as the value of product ij *net of production cost* and order products according to their net values such that $v_i^1 \geq v_i^2 \geq ... \geq v_i^k$; we suppose that $v^k > \max\{0, v^{k+1}\}$. Now B^{FB} includes the k products with the highest net value and $B_i^{FB} \equiv B^{FB} \cap \mathcal{B}_i$. For $B \subseteq \mathcal{B}$, let $V(B)$ represent the total net value that the buyer obtains from allocating the slots to the k products with highest net value in B (if $\#B \geq k$), when she pays the production cost for each such product. Let $V \equiv V(B^{FB})$ and $B_{-i}^{FB} \equiv B^{FB} \backslash B_i^{FB}$, a set which includes exactly $k - q_i^{FB}$ number of products. Now let B_{-i}^{SB} represent the subset of the best products in $\mathcal{B}_{-i} \backslash B_{-i}^{FB}$ such that the net value of each product in B_{-i}^{SB} is not smaller than 0 and the cardinality of B_{-i}^{SB}, denoted by q_{-i}^{SB}, is not larger than q_i^{FB}. Therefore q_{-i}^{SB} is the number in $\{0, ..., q_i^{FB}\}$ with the following property:

- If $0 > v_{-i}^{k-q_i^{FB}+1}$, then $q_{-i}^{SB} = 0$;
- If $v_{-i}^k \geq 0$, then $q_{-i}^{SB} = q_i^{FB}$;
- If $v_{-i}^{k-q_i^{FB}+1} \geq 0 > v_{-i}^k$, then q_{-i}^{SB} is the number in $\{1, ..., q_i^{FB} - 1\}$ such that $v_{-i}^{k-q_i^{FB}+q_{-i}^{SB}} \geq 0 > v_{-i}^{k-q_i^{FB}+q_{-i}^{SB}+1}$.

A technology renting strategy for firm i is defined as $p_{ij} = c_i^j$ for $j = 1, ..., k$ together with F_i, and Lemma 1 holds. Proposition 2 holds with $F_i^* = V - V_{-i}$ where $V_{-i} \equiv V(B_{-i}^{FB} \cup B_{-i}^{SB})$. Proposition 3 holds if $v^k > \max_{ij \in \mathcal{B} \backslash B^{FB}} u_i^j$ and Proposition 4 holds under the restriction $P_i(B_i \cup \{ij\}) - P_i(B_i) \geq c_i^j$.

5.5.2 The case of substitutes

In this subsection we consider substitution among products. Since it is hard to describe all possible substitutions for every possible subset of $\mathcal{B}$, we consider a simple case of substitution and show that our results are robust. Precisely, we assume that if the buyer buys the set of products $B \subseteq \mathcal{B}$ with $q = \#(B)$, then there is a number $s > 0$ such that her gross utility from the products in B is given by $U(B) - (q-1)s$. With this subadditive utility function we represent a negative synergy which has a constant magnitude as the number of products used by the buyer increases.

Our previous results in this section hold to this setting provided that we replace u_i^j with $\tilde{u}_i^j \equiv u_i^j - s$ for each i and j. Indeed, let $\tilde{U}(B)$ be defined as $U(B)$ in which $\tilde{u}_i^j$ replaces u_i^j. Then it is simple to see that $U(B) - (q-1)s = \tilde{U}(B) + s$. In other words, the buyer's gross utility from bundle B is given by $\tilde{U}(B)$, except for the constant s. Then everything happens as if the value of each product has been reduced by s, and thus the technology renting equilibrium still exists, albeit with different fixed fees. As a general result we find that both the firms and the buyer make (weakly) lower profits in this equilibrium than when $s = 0$; the driving force for this result is that the social value which is generated by trade is lower because of the substitution. In particular, the buyer's profit is strictly reduced unless she made a zero profit (i.e., $\pi_0^* = 0$) when $s = 0$. The profit of each firm $i = 1, ..., n^{FB}$ is also reduced, unless $q_i^{FB} = q_{-i}^{SB}$ and $u_{-i}^k - s \geq c$. Finally, the results about efficient equilibria are unchanged by the negative synergy.

6 Bundling with slotting contracts

In this section we study a setting in which each firm i can use slotting contracts. When firm i uses slotting contracts, buying a bundle B_i requires the buyer to allocate slots to all products in B_i; therefore, if all firms use slotting contracts, the buyer can buy $B_1 \cup ... \cup B_n$ only if $\#(B_1 \cup ... \cup B_n) \leq k$. By using slotting contracts, it may be possible (and profitable) for firm i to induce the buyer to buy a bundle bigger than B_i^{FB}, in order to make sales of the rival firms difficult. In extreme cases, i may succeed in occupying all slots with his own products by setting $P_i(B_i)$ very high for each $B_i \neq \mathcal{B}_i$ and choosing $P_i(\mathcal{B}_i)$ to induce the buyer to buy $\mathcal{B}_i$, as it occurs in the following example.

Example 4 *(slotting contracts and inefficiency) Suppose that $n = 2$, $k = 3$, $c = 0$, and*

$$(u_1^1, u_1^2, u_1^3) = (10, 7, 6); \qquad (u_2^1, u_2^2, u_2^3) = (9, 8, 1)$$

Here $B^{FB} = \{11, 21, 22\}$, so that $q_1^{FB} = 1$ and $q_2^{FB} = 2$. However, there exists an inefficient NE in which $P_i(B_i)$ is high enough for each $B_i \neq \mathcal{B}_i$, for $i = 1, 2$, and $P_1(\mathcal{B}_1) = 5$, $P_2(\mathcal{B}_2) = 0$. In words, each firm i offers only $\mathcal{B}_i$ through a slotting contract, and Bertrand competition between $\mathcal{B}_1$ and $\mathcal{B}_2$ determines the above prices. In this NE, firm 1 occupies the three slots even though products 21 and 22 are both better than 12 and 13.

This example shows that even in the case of digital goods, inefficient equilibria exist when firms can use slotting contracts; this contrast with Proposition 3, which shows that efficiency is always achieved for small values of c without slotting contracts. The reason for why such a result does not hold with slotting contracts is that a firm i with a product $ij \in B^{FB}$ may not be able to induce the buyer to modify her portfolio by replacing an inferior product of a rival firm with product ij, even though firm i charges a very small price, in case the rival firm uses a slotting contract. Indeed, all three products of firm 1 are bounded with the slotting contract such that replacing for instance product 13 with product 21 implies that the buyer cannot use any product of firm 1, which the buyer cannot afford. On the contrary, without slotting contracts, after buying $\mathcal{B}_1$, the buyer can freely dispose of any product in $\mathcal{B}_1$ to replace it with a superior product.

In spite of Example 4, we can show that Lemma 1 holds and the technology-renting equilibrium described by Proposition 2 is a NE also under slotting contracts and thus an efficient NE always exists in this setting. Furthermore, also Propositions 4 and 5 hold in this environment.

Corollary 3 *Regardless of whether each firm uses slotting contracts or not (hence, this setting includes the extreme case in which all firms use slotting contracts),*

(i) a firm can find a best response among technology-renting strategy without using slotting contracts

(ii) there exists a technology-renting equilibrium; the profile $(tr_1^*, ..., tr_n^*)$ *described in (1) is a NE for any* $c \geq 0$*;*

(iii) every NE is efficient under condition (2);

(iv) if $n = 2$*, the technology-renting equilibrium Pareto dominates any other NE.*

Proof. For the proof of (i) we can follow the proof of Lemma 1 to show that – given a profile of strategies of other firms – firm i does not lose anything from using a suitable technology renting strategy without the clause of slotting contracts. For the proof of (ii)-(iv), we notice that in the proofs of Proposition 2, 4 and 5(i) we never use the possibility that the buyer does not allocate a slot to a product she has purchased. In other words, in the equilibria of Proposition 2, 4 and 5(i), the buyer buys only the products that she will use. This differs from the proof of Proposition 3, which indeed does not apply under slotting contracts, as Example 4 proves. ■

Corollary 3(iii) suggests that if firms are prohibited from charging individual (or marginal) prices below costs, all equilibria are efficient regardless of whether they can use slotting contracts. This result is not surprising since, as we said in subsection 5.3.3, condition (2) makes pure bundling impossible and, when a firm offers a bundle, forces each firm to offer a complete subset of the bundle as well. This in turn allows to be able to buy only the products that would occupy a slot and makes slotting contracts redundant.

Although corollary 3(iv) holds for duopoly, the following example shows that there can be a Pareto undominated inefficient equilibrium if there are more than two firms.

Example 5 *(slotting contracts and pareto undominated inefficient equilibria). Suppose that* $n = 3$*,* $k = 3$*,* $c = 0$*, and*

$$(u_1^1, u_1^2, u_1^3) = (10, 7, 6); \qquad (u_2^1, u_2^2, u_2^3) = (u_3^1, u_3^2, u_3^3) = (9, 8, 1)$$

Here $B^{FB} = \{11, 21, 31\}$ and $q_1^{FB} = 1$, $F_1^* = 2$. However, there exists an inefficient equilibrium in which each firm i proposes only the bundle $\mathcal{B}_i$ of all his products and uses the slotting contract. Prices are $P_1(\mathcal{B}_1) = 5$, $P_2(\mathcal{B}_2) = 0$, $P_3(\mathcal{B}_3) = 0$. In this equilibrium, firm 1 occupies all slots and his profit is 5 ($> F_1^*$).

7 Portfolio effects of Bundling

In this section, we identify portfolio effects of bundling and analyze the consequences on a horizontal merger. For this purpose, we focus our discussion on the technology-renting equilibrium in which each seller i makes a profit of F_i^*.

7.1 Portfolio effects

By portfolio effects of bundling, we mean that under bundling, two firms who end up selling products with the same values can make different profits. Consider the digital good for simplicity and suppose that firms 1 and 2 are such that $q_1^{FB} = q_2^{FB}$ and $u_1^j = u_2^j$ for $j = 1, ..., q_1^{FB}$, but $u_1^j > u_2^j$ for $j = q_1^{FB} + 1, ..., k$. Then, we have $U_1^{FB} = U_2^{FB}$ and $U_{-1}^{SB} \leq U_{-2}^{SB}$, which implies

$$F_1^* = U_1^{FB} - U_{-1}^{SB} \geq F_2^* = U_2^{FB} - U_{-2}^{SB}.$$

In addition, if $B_{-1}^{SB} \cap \mathcal{B}_2 \neq \phi$ then $U_{-1}^{SB} < U_{-2}^{SB}$ and thus $F_1^* > F_2^*$: the buyer ends up purchasing products of identical values from both firms but pays a higher price to firm 1. This is because in equilibrium each firm i extracts with F_i^* the surplus that the buyer obtains by replacing the products belonging to B_{-i}^{SB} in the best alternative portfolio (that the buyer constitutes without buying any product from i) with the products belonging to B_i^{FB}. Although $U_1^{FB} = U_2^{FB}$, the best alternative portfolio is not the same: it is weaker when the buyer does not buy any product from 1 than when she does not buy any product from 2. This explains why firm 1 can extract a higher surplus than firm 2.

In other words, for digital good, in the technology-renting equilibrium, we have

$$F_i^* = U - U_{-i}.$$

In the equation, U is the same for all firms while U_{-i} is smaller for firms with better portfolio. In contrast, in the case of independent pricing, there is no such portfolio effect; if an equilibrium (in pure strategy) exists under independent pricing, all products of identical value must be sold at the same prices.

Corollary 4 *(portfolio effects) Bundling generates portfolio effects, which do not exist under independent pricing. More precisely, suppose that firms* 1 *and 2 are such that* $q_1^{FB} = q_2^{FB}$ *and* $u_1^j = u_2^j$ *for* $j = 1, ..., q_1^{FB}$*, but* $u_1^j > u_2^j$ *for* $j = q_1^{FB} + 1, ..., k$*. Then, in the technology-renting equilibrium, the buyer buys products of identical values from both firms (i.e.,* B_1^{FB} *from 1 and* B_2^{FB} *from 2) but pays* $F_1^* \geq F_2^*$*. In addition, if* $B_{-1}^{SB} \cap \mathcal{B}_2 \neq \phi$*, we have* $F_1^* > F_2^*$*.*

7.2 Portfolio effects and horizontal merger

A natural and important consequence of the portfolio effects is that it creates incentives for a horizontal merger. We will first consider a merger between two given firms and then endogeneize the merger.

Consider the merger of any two firms i and h (with $i \neq h$) and let the two firms after the merger be denoted by $i + h$.

Proposition 6 *(exogenous merger) Consider the merger of any two firms* i *and* h*.*
(i) The merger affects neither social welfare nor any other firm's profit.
(ii) The merger weakly increases the merging firms' profit, and hence weakly decreases the buyer's payoff.
In case $q_i^{FB} \geq 1$ *and* $q_h^{FB} \geq 1$*, the merger strictly increases the merging firms' profit, and hence strictly decreases the buyer's profit, unless* $\max\{u_{-i}^{k-q_i^{FB}+1} - c, 0\} = \max\{u_{-h}^{k-q_h^{FB}+1} - c, 0\} = \max\{u_{-(i+h)}^{k} - c, 0\}$*.*
In case $q_i^{FB} \geq 1$ *and* $q_h^{FB} = 0$*, the merger strictly increases the merging firms' profit if and only if* $u_h^1 > \max\{c, u_{-i}^{k+1}\}$*.*

Social welfare is not affected by the merger since the buyer buys the products in B^{FB} regardless of the market structure of the sellers. The profit of a firm i' different from i and h is not affected by the merger since the profit of i' is the difference between the value of the best portfolio B^{FB} and the value of the best alternative portfolio $B_{-i'}^{FB} \cup B_{-i'}^{SB}$ that the buyer can build up without buying any product from firm i'. Since the composition of both $B_{-i'}^{FB}$ and $B_{-i'}^{SB}$ is not affected by the merger, the merger does not change the profit of any third firm.

The proposition shows that in general the merger is profitable and decreases the buyer's payoff since to some extent, increasing the portfolio through the merger protects more the products sold from competition. Before the merger, if the buyer does not buy any product from i, the best alternative portfolio included products from h. In contrast, after the merger, the best alternative portfolio does not include products from h, which implies that the merger allows firm i to command a higher price. The same logic applies to firm h as well.

The conditions in the proposition under which the merger does not increase the profits of the merging firms are very stringent. For instance, when $q_i^{FB} \geq 1$ and $q_h^{FB} \geq 1$, then the condition holds only in the two following cases. The first case occurs when $u_{-i}^{k-q_i^{FB}+1} \leq c$ and $u_{-h}^{k-q_h^{FB}+1} \leq c$. Then $u_{-(i+h)}^{k-q_i^{FB}-q_h^{FB}+1} \leq c$ holds and thus neither firm i nor firm h faces any competition, implying that firm $i+h$ does not face any competition either. Thus $U_i^{FB} - cq_i^{FB}$ and $U_h^{FB} - cq_h^{FB}$ are the profits of i and h before the merger while $i+h$ earns $U_{i+h}^{FB} - cq_{i+h}^{FB} = U_i^{FB} + U_h^{FB} - c(q_i^{FB} + q_h^{FB})$. The second case arises when $u_{-i}^{k-q_i^{FB}+1} = u_{-h}^{k-q_h^{FB}+1} = u_{-(i+h)}^{k} = u$ for some $u \geq c$. In this case, $i+j$ faces as fierce competition as i and h face separately, and the profits of i, h and $i+h$ are respectively $U_i^{FB} - uq_i^{FB}$, $U_h^{FB} - uq_h^{FB}$, and $U_{i+h}^{FB} - uq_{i+h}^{FB} = U_i^{FB} + U_h^{FB} - u(q_i^{FB} + q_h^{FB})$.

We now endogenize the merger in the following way. Suppose that a given firm h sells a product in his portfolio with value $u > c$ for some exogenous reasons. We determine the value each firm $i(\neq h)$ attaches to acquiring this product. From proposition 6, as any merger does not affect a third firm's profit, if a firm different from firm i buys the product, then firm i's profit is unchanged with respect to the profit before the sale. If instead firm i buys the product, then his profit increases only when the product belongs to $B_{-i}^{FB} \cup B_{-i}^{SB}$: in this case, the profit of i increases by $u - \max\{u_{-i}^{k+1}, c\}$. For instance, if $u_{-i}^{k+1} \geq c$, i's purchase of h's product will modify the best alternative portfolio that the buyer can build up when she does not buy any product from i such that in the portfolio, the product bought from h is replaced by a product with value u_{-i}^{k+1} and hence i can increase his profit by $u - u_{-i}^{k+1}$. If $u_{-i}^{k+1} < c$, the product bought from h simply disappears in the best alternative portfolio and therefore i's profit increases by $u - c$.

If we assume that firm h uses a second price auction, we get the following result.

Proposition 7 *(endogenous merger) Suppose that firm h sells a product in his portfolio with value $u(> c)$ through a second-price auction.*
(i) The only undominated bid b_i of firm i is

$$b_i = \begin{cases} u - \max\{u_{-i}^{k+1}, c\}, & \text{if } u > u_{-i}^{k+1}; \\ 0, & \text{otherwise.} \end{cases}$$

(ii) If there is a firm (say firm 1) whose portfolio dominates each other firm's one in the sense that $q_1^{FB} \geq q_i^{FB}$ and $u_1^{q_1^{FB}+j} \geq u_i^{q_i^{FB}+j}$ for $j = 1, 2, ..., k - q_1^{FB}$ and for $i = 2, ..., n$, then $b_1 \geq b_i$ for $i = 2, ..., n$. Therefore, there is a tendency of increasing concentration.

Proof. We only need to prove (ii). For this, we prove that $u_{-i}^{k+1} \geq u_{-1}^{k+1}$. From $u_1^{q_1^{FB}+j} \geq u_i^{q_i^{FB}+j}$ for $j = 1, 2, ..., k - q_1^{FB}$, it follows that $u_{-i}^{k-q_i^{FB}+j} \geq u_{-1}^{k-q_1^{FB}+j}$ for $j = 1, 2, ...$ Hence, taking $j = q_i^{FB} + 1$ we find $u_{-i}^{k+1} \geq u_{-1}^{k-q_1^{FB}+q_i^{FB}+1} \geq u_{-1}^{k+1}$ where the last inequality follows from $-q_1^{FB} + q_i^{FB} \leq 0$. ■

Proposition 7(ii) suggests that there is a tendency of increasing concentration since the dominating firm has a higher willingness to pay for the product in auction. In order to give the intuition, we suppose $h \neq 1$ and $h \neq 2$ and compare 1's bid with 2's bid. Note that from the definition of the dominance, the best alternative portfolio when the buyer does not buy any product from 1 is worse than the one when the buyer does not buy anything from 2. Therefore, only two cases may arise: either h's product on sale belongs to both portfolios or it belongs only to the best alternative portfolio when the buyer does not buy anything from 1. In the second case, it is clear that 1 makes a positive bid while 2 makes zero bid. In the first case, we need to think which product is going to replace h's product in the best alternative portfolio. Since the best alternative portfolio when the buyer does not buy anything from 1 is inferior to the one when the buyer does not buy anything from 2, the product replacing h's product is worse in the first portfolio than in the second. This implies that 1 gains more than 2 from purchasing the product of firm h.

Remark 4: Our proposition 6 is similar to the results that O'Brien and Shaffer find (Propositions 4-6) when they study a horizontal merger in a Nash bargaining setup in which n single product firms sell products to a buyer: in their model, the products are substitutes and there is no slot constraint. However, they do not endogeneize the merger as we do in Proposition 7.

8 Policy implications and concluding remarks

Our results have interesting policy implications. First, bundling such as offering a menu of bundles or "individual prices plus a fixed fee" is socially desirable and should be allowed. Second, regarding pure bundling (or full-line pricing), in the case of digital goods, the technology-renting strategy is identical to pure bundling and therefore pure bundling of all products belonging to a firm achieves efficient allocation and is socially desirable. In contrast, in the case of physical goods, full-line forcing can create inefficient equilibria and hence competition authority should be careful. Third, regarding slotting contracts, in the case of digital goods, our analysis implies that prohibiting slotting contracts is socially desirable since then all equilibria are efficient. In contrast, in the case of physical goods, inefficient equilibria can arise either because of charging prices below costs or because of slotting contracts. Although we have shown that prohibiting firms from charging individual prices below costs makes all equilibria efficient in the case of physical goods, in practice, it would be difficult to monitor whether firms charge prices below costs (De la Mano-Durand, 2005). Then, prohibiting only slotting contracts may not be enough to achieve efficient allocations.

As challenging issues for future studies, it would be interesting to explore dynamic implications of the portfolio effects in a setting in which we endogeneize the portfolio of each firm. We can also model the buyer as a downstream firm and study the firm's pricing with respect to final consumers. Even in a setting with a monopoly downstream firm, we can study the interaction between bundling at upstream level and bundling at downstream level. Of course, it would be more interesting to extend this monopoly setting to competition between downstream firms, which is very relevant for cable or digital TV.

References

[1] Armstrong, Mark (1996). "Multiproduct Nonlinear Pricing." *Econometrica*, 64, 51-75

[2] Armstrong, Mark (1999). "Price Discrimination by a Many-Product Firm." *Review of Economic Studies*, 66, 151-168.

[3] Armstrong, Mark and John Vickers (2001). "Competitive Price Discrimination." 32(4): 579-605, *RAND Journal of Economics*

[4] Armstrong, Mark and John Vickers (2008). "Competitive Nonlinear Pricing and Bundling." forthcoming in *Review of Economic Studies*

[5] Areeda, P and D. T. Turner (1975). "Predatory Pricing and Related Practices under Section 2 of the Sherman Act." 88: 697-733, *Harvard Law Review*

[6] Bakos, Yannis and Eric Brynjolfsson (1999). "Bundling Information Goods: Pricing, Profits and Efficiency." *Management Science*, 45, 1613-1630.

[7] Bakos, Yannis and Eric Brynjolfsson (2000). "Bundling and Competition on the Internet: Aggregation Strategies for Information Goods." *Marketing Science*, 19, 63-82.

[8] Bernheim, Douglas and Michael Whinston (1985). "Common Marketing Agency as a Device for Facilitating Collusion." *RAND Journal of Economics*, 15, 269-281.

[9] Bernheim, Douglas and Michael Whinston (1998). "Exclusive Dealing." *Journal of Political Economy*, 106, 64-103.

[10] Bork, Robert H. (1978). *The Antitrust Paradox: A Policy at War with Itself.* New York, New York: Basic Books.

[11] Bowman, Ward (1957). "Tying Arrangements and the Leverage Problem," *Yale Law Journal*, 67, pp.19-36.

[12] Carlton, W. Dennis and Michael Waldman (2002). "The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries." *RAND Journal of Economics*, 33(2): 194-220

[13] Choi, Jay Pil (2006). "Antitrust Analysis of Tying Arrangements," J. P. Choi (ed.), *Recent Developments in Antitrust: Theory and Evidence*, MIT Press.

[14] Choi, Jay Pil and Stefanadis, Chris (2001). "Tying, Investment, and the Dynamic Leverage Theory", *RAND Journal of Economics*, pp. 52-71.

[15] De la Mano, Miguel and Benoît Durand (2005). "A Three-Step Structured Rule of Reason to Assess Predation under Article 82." Discussion Paper, Office of the Chief Economist. DG Competition. European Commission.

[16] Edlin, Aaron S., and Daniel L. Rubinfeld (2004). "Exclusion or Efficient Pricing: The 'Big Deal' Bundling of Academic Journals." *Antitrust Law Journal*, 72, 119-157.

[17] Federal Trade Commission Staff Report (2001). "Report on the Commission Workshop on Slotting Allowances and Other Marketing Practices in the Grocery Industry."

[18] Federal Trade Commission Staff Study (2003). "Slotting Allowances in the Retail Grocery Industry: Selected Case Studies in Five Product Categories."

[19] Hanssen, F. Andrew (2000). "The Block booking of Films Reexamined." *Journal of Law and Economics*, 43 (2): 395-426

[20] Jeon, Doh-Shin and Domenico Menicucci (2006). "Bundling Electronic Journals and Competition among Publishers." *Journal of the European Economic Association*, vol.4 (5), September 2006, 1038-83

[21] Kenney, Roy W. and Klein, Benjamin (1983). "The Economics of Block Booking." *Journal of Law and Economics*, 26 (3): 497-540

[22] Kenney, Roy W. and Klein, Benjamin (2000). "How Block booking Facilitated Self-Enforcing Films Contracts." *Journal of Law and Economics*, 43 (2): 427-35

[23] McAfee, R. Preston, John McMillan, Michael D. Whinston (1989). “Multiproduct Monopoly, Commodity Bundling, and Correlation of Values.” *Quarterly Journal of Economics*, 114, 371-384.

[24] Nalebuff, Barry (2004). “Bundling as an Entry Barrier.” *Quarterly Journal of Economics*, 119, 159-188.

[25] O’Brien, Daniel and Greg Shaffer (1997). “Non-Linear Supply Contracts, Exclusive Dealing, and Equilibrium Market Foreclosure”, *Journal of Economics and Management Strategy*, 6, 755-785.

[26] O’Brien, Daniel and Greg Shaffer (2005). ”Bargaining, bundling, and clout: the portfolio effects of horizontal mergers.” *RAND Journal of Economics*, 36, 573-595

[27] Posner, Richard A. (1976). *Antitrust Law: An Economic Perspective*, Chicago: University of Chicago Press.

[28] Rochet, Jean-Charles and Lars Stole (2002). ”Non-linear Pricing and Random Participation.” 69(1): 277-311, *Review of Economics Studies.*

[29] Salinger, Michael A. (1995). “A Graphical Analysis of Bundling.” *Journal of Business*, 68, 85-98.

[30] Schmalensee, Richard L. (1984). “Gaussian Demand and Commodity Bundling.” *Journal of Business* 57. S211-S230.

[31] Shaffer, Greg (1991). ”Capturing Strategic Rent: Full-Line Forcing, Brand Discounts, Aggregate Rebates, and Maximum Resale Price Maintenance.” Journal of Industrial Economics 39 (5): 557-575

[32] Stigler, George J. (1968). “A Note on Block Booking.” in *The Organization of Industry,* edited by George J. Stigler. Homewood, Illinois: Richard D. Irwin.

[33] Vergé, Thibaud (2001). ”Multi-product Monopolist and Full-line Forcing: The Efficiency Argument Revisited.” *Economics Bulletin,* 12(4):.1-9.

[34] Whinston, Michael D. (1990). “Tying, Foreclosure and Exclusion.” *American Economic Review*, 80, 837-59.

Appendix

Proof of Lemma 1

Consider any arbitrary profile of strategies (s_i, s_{-i}) and let π_i be the profit of firm i given (s_i, s_{-i}). We show that i can achieve the same profit π_i by playing $tr_i \in TR_i$ such that $F_i = \pi_i$. This fact is obvious if $\pi_i = 0$ and therefore we consider the case of $\pi_i > 0$. In order to prove this result, it suffices to show that the buyer buys at least one product from i when i plays tr_i. We find that, (i) given (tr_i, s_{-i}), the buyer can make the same payoff that she makes with (s_i, s_{-i}) since she can buy the same products, with the same outlay; (ii) given (tr_i, s_{-i}), the buyer cannot realize a higher payoff than with (s_i, s_{-i}) without buying at least one product of firm i, because otherwise she would not buy anything from i given (s_i, s_{-i}), and this contradicts $\pi_i > 0$.

Proof of Proposition 2

We split the proof into two steps.

Step 1 When the upstream firms play $(tr_1^*, ..., tr_n^*)$, the buyer buys the products in B^{FB} and thus each firm i's profit is F_i^* while the buyer's payoff is $\sum_{i=1}^{n^{FB}} (U_{-i}^{SB} - cq_{-i}^{SB})$.
We start by proving by contradiction that the buyer buys at least some products from (say) firm 1; the same argument applies for firm $i = 2, ..., n^{FB}$. Suppose that the buyer buys nothing from firm 1, and that she buys $\bar{q}$ products from firms -1, with values $w_{-1}^1, ..., w_{-1}^{\bar{q}}$ such that $w_{-1}^1 \geq ... \geq w_{-1}^{\bar{q}}$. Obviously, $\bar{q} \leq k$ since buying more than k products implies that some of them would not be used: then the buyer may reduce her outlay without reducing her gross payoff. Actually, however, $\bar{q} \leq k - q_1^{FB} + q_1^{SB}$ must hold since $p_{ij} = c$ for any $i = 1, ..., n$ and any $j = 1, ..., k$ and there are only $k - q_1^{FB} + q_1^{SB}$ products in the portfolios of firms -1 with values not lower than c. But then we can prove that for any $\bar{q} \leq k - q_1^{FB} + q_1^{SB}$, the buyer can increase her payoff by buying from 1 the products in B_1^{FB}. First, consider $\bar{q} \leq k - q_1^{FB}$. Then, it is obvious that the buyer can increase her payoff by simply buying the products in B_1^{FB} in addition to buying from firms -1 the products of values $w_{-1}^1, ..., w_{-1}^{\bar{q}}$. Second, consider $k - q_1^{FB} < \bar{q} \leq k - q_1^{FB} + q_1^{SB}$. Then, we show below that the buyer can increase her payoff by buying the products in B_1^{FB} and not buying from firms -1 products of values $w_{-1}^{k--q_1^{FB}+1}, ..., w_{-1}^{\bar{q}}$. This changes the buyer's payoff by

$$\begin{aligned}
\Delta\pi_0 &= -F_1^* - cq_1^{FB} + U_1^{FB} - \sum_{j=1}^{\bar{q}-k+q_1^{FB}} (w_{-1}^{k-q_1^{FB}+j} - c) \\
&= u_{-1}^{k-q_1^{FB}+1} + ... + u_{-1}^{k-q_1^{FB}+q_1^{SB}} - cq_1^{SB} - [w_{-1}^{k-q_1^{FB}+1} + ... + w_{-1}^{\bar{q}} - c(\bar{q} - k + q_1^{FB})] \\
&\geq u_{-1}^{k-q_1^{FB}+1} + ... + u_{-1}^{k-q_1^{FB}+q_1^{SB}} - cq_1^{SB} - [u_{-1}^{k-q_1^{FB}+1} + ... + u_{-1}^{\bar{q}} - c(\bar{q} - k + q_1^{FB})]
\end{aligned}$$

We note that the set $\{u_{-1}^{k-q_1^{FB}+1}, ..., u_{-1}^{\bar{q}}\}$ is a subset of $\{u_{-1}^{k-q_1^{FB}+1}, ..., u_{-1}^{k-q_1^{FB}+q_1^{SB}}\}$, and the set $\{u_{-1}^{k-q_1^{FB}+1}, ..., u_{-1}^{k-q_1^{FB}+q_1^{SB}}\} \backslash \{u_{-1}^{k-q_1^{FB}+1}, ..., u_{-1}^{\bar{q}}\}$ has cardinality $q_1^{SB} - (\bar{q} - k + q_1^{FB}) \geq 0$. As $u_{-1}^{k-q_1^{FB}+q_1^{SB}} \geq c$, $\Delta\pi_0$ is non-negative.

In this way we have proved that the buyer buys at least one product of firm 1, and then pays the fixed fee F_1^*. This reveals that the buyer buys at least all the products in B_i^{FB}, $i = 1, ..., n^{FB}$. But since the buyer will not buy more than k products, it must be the case that she buys $B_1^{FB} \cup ... \cup B_{n^{FB}}^{FB}$ and buys nothing from firm $i = n^{FB} + 1, ..., n$.

Step 2 When each firm i plays tr_i^*, firm j cannot make a profit larger than F_j^*. We prove this claim for firm 1, and the same argument applies for $i = 2, ..., n^{FB}$. From Lemma 1, it is enough to consider firm 1's deviation in the set of technology-renting strategy TR_1. Obviously, firm 1 has no incentive to decrease F_1 from F_1^*. We now prove that firm 1 has no incentive to increase F_1 from F_1^*. Note first that from the fact F_i^* makes the buyer indifferent between renting i's technology or not, the buyer can achieve the payoff equal to π_0^* without buying any product from firm i for any given $i = 1, ..., n^{FB}$. Suppose now that firm 1 chooses $F_1 = F_1^* + \varepsilon$ for $\varepsilon > 0$. We need to prove that the buyer will not buy any product from 1. The buyer can make a profit of π_0^* by buying only from firms -1, and she cannot make a profit $\pi \geq \pi_0^*$ by buying one or more products from 1 (given $F_1 = F_1^* + \varepsilon$), because if she could then she would make at least profit $\pi + \varepsilon > \pi_0^*$ before the deviation of 1: a contradiction.

Proof of Proposition 3

Suppose that a NE $(s_1, ..., s_n)$ exists such that the buyer buys a portfolio of products $B = B_1 \cup ... \cup B_n$ and, for instance, B_1 does not include all the products in B_1^{FB}; that is, $B_1^{FB} \backslash B_1 \neq \emptyset$. Let π_1 denote the profit of firm 1 and let π_0 denote the buyer's payoff in this NE. Consider now the strategy $tr_1 \in TR_1$ of firm 1 with $F_1 = \pi_1 + \varepsilon$ for $\varepsilon > 0$ and small. We below prove that the buyer buys at least one product from firm 1, and therefore 1's profit increases to $\pi_1 + \varepsilon$. In order to prove this, we first note that if the buyer does not buy any product from 1, she cannot make a payoff higher than π_0 [otherwise she would not buy B given $(s_1, ..., s_n)$]. Then it suffices to show that the buyer can earn more than π_0 by purchasing $B \cup \{1j\}$, with $1j \in B_1^{FB} \backslash B_1$, which includes at least one product offered by firm 1. Consider first the case in which $\#(B) < k$. Then, the buyer's payoff from buying $B \cup \{1j\}$ is equal to $\pi_0 + u_1^j - c - \varepsilon$, which is larger than π_0 since $1j \in B^{FB}$ implies $u_1^j > c$. Consider now the case in which $\#(B) = k$. In this case, the buyer's payoff from $B \cup \{1j\}$ is $\pi_0 + u_1^j - \varepsilon - c - u_h^{j'}$, where hj' denotes the lowest valued product in B, which the buyer removes from one slot to make room for product $1j$. We know that $u_1^j \geq u^k$ since $1j \in B^{FB}$ and $-u_h^{j'} \geq -u^{k+1}$ since $hj' \notin B^{FB}$, hence $u_1^j - \varepsilon - c - u_h^{j'} \geq u^k - u^{k+1} - \varepsilon - c > 0$ holds given that $u^k - u^{k+1} > c$.

Proof of Proposition 4

The proof is similar to the proof of Proposition 3. Suppose that a NE exists such that the buyer buys $B = B_1 \cup ... \cup B_n$ and, for instance, B_1 does not include all the products in B_1^{FB}; there exists a product $1j \in B_1^{FB} \backslash B_1$. Let π_1 denote the profit of firm 1 in this NE, while π_0 represents the buyer's payoff in the NE. Consider the strategy of firm 1 in TR_1 such that $F_1 = \pi_1 + \varepsilon$ (for $\varepsilon > 0$ and small). In order to prove that this is a profitable deviation for 1, it suffices to show that the buyer buys at least one product from firm 1, as this yields 1 a profit of $\pi_1 + \varepsilon$ which is larger than π_1. Note first that in the case in which the buyer does not buy any product from 1, she can make at most a profit equal to π_0. Consider first when $\#(B) < k$. Then, the buyer can buy $B \cup \{1j\}$ and gets a payoff equal to $\pi_0 + u_1^j - c - \varepsilon$, which is larger than π_0 since $1j \in B^{FB}$ implies $u_1^j > c$. Consider now the case of $\#(B) = k$. In this case, the buyer can buy $B \cup \{1j\} / \{hj'\}$ where hj' denotes the lowest valued product in B: the buyer replaces hj' with $1j$. Then, the change in the buyer's payoff is $u_1^j - c - \varepsilon - u_h^{j'} + [P_h(B_h) - P_h(B_h \backslash \{hj'\})]$, which is at least as large as $u_1^j - \varepsilon - u_h^{j'} > 0$ since $P_h(B_h) - P_h(B_h \backslash \{hj'\}) \geq c$ by (2). Thus, after 1's deviation, the buyer will buy at least one product of 1.

Proof of Proposition 5

(i) The proof is by contradiction. Suppose there exists a NE such that 1 makes a profit higher than $F_1^* = U_1^{FB} - cq_1^{FB} - (U_{-1}^{SB} - cq_{-1}^{SB})$. Precisely, suppose that 1 sells a bundle $B_1 \subset \mathcal{B}_1$ such that $q_1 = \#B_1$, and 2 sells a bundle $B_2 \subset \mathcal{B}_2$ such that $q_2 = \#B_2$. Then 1's revenue $P_1(B_1)$ is larger than $F_1^* + cq_1$, while 2's revenue is $\bar{P}_2 \equiv P_2(B_2)$. The payoff of the buyer is

$$\alpha \equiv \sum_{1j \in B_1} u_1^j + \sum_{2j \in B_2} u_2^j - P_1(B_1) - \bar{P}_2$$

and it is clear that this is smaller than

$$\beta \equiv \sum_{1j \in B_1} u_1^j + \sum_{2j \in B_2} u_2^j - (F_1^* + cq_1) - \bar{P}_2$$

since $P_1(B_1) > F_1^* + cq_1$. We prove that the latter inequality implies the existence of a profitable deviation for firm 2.

Let 2 deviate by using a technology renting strategy with $F_2 = \bar{P}_2 - cq_2 + \varepsilon$, with $\varepsilon > 0$ and small. Since in the candidate NE considered in the beginning the profit of 2 is equal to $\bar{P}_2 - cq_2$, this deviation of 2 is profitable if and only if the buyer buys at least one object from 2. In order to prove that this is the case, we notice that if the buyer does not buy anything from firm 2, then she buys only bundles offered by 1, and they cannot yield the buyer a payoff larger than α otherwise we obtain a contradiction with the fact that the initial candidate is a NE. Let $\tilde{B}_2$ denote the bundle of 2 which includes his best $k_2 = q_2^{FB} + q_{-1}^{SB}$ products. We show below that the buyer's profit if she buys only $\tilde{B}_2$ is larger than α, and thus we infer that she will definitely buy at least one product from firm 2.

Suppose for the moment that $\varepsilon = 0$. Then the payoff of the buyer from buying only $\tilde{B}_2$ is

$$\gamma \equiv \sum_{j=1}^{k_2} u_2^j - \bar{P}_2 - c(k_2 - q_2)$$

and we below prove $\gamma \geq \beta$; then, since $\beta > \alpha$, we obtain $\gamma > \alpha$ for the case in which $\varepsilon = 0$, which implies that $\gamma > \alpha$ holds for $\varepsilon (> 0)$ small. The inequality $\gamma \geq \beta$ is equivalent to

$$\sum_{j=1}^{k_2} u_2^j - c(k_2 - q_2) \geq \sum_{1j \in B_1} u_1^j + \sum_{2j \in B_2} u_2^j - F_1^* - cq_1$$

and, recalling that $F_1^* = U_1^{FB} - cq_1^{FB} - (U_{-1}^{SB} - cq_{-1}^{SB})$, we can write the equivalent condition:

$$\begin{aligned} \sum_{j=1}^{k_2} u_2^j - c(q_2^{FB} + q_{-1}^{SB} - q_2) \quad &\geq \quad \sum_{1j \in B_1} u_1^j + \sum_{2j \in B_2} u_2^j - [U_1^{FB} - cq_1^{FB} - (U_{-1}^{SB} - cq_{-1}^{SB})] - cq_1 \\ \text{or} \qquad U_1^{FB} - cq_1^{FB} + U_2^{FB} - cq_2^{FB} \quad &\geq \quad \sum_{1j \in B_1} u_1^j - cq_1 + \sum_{2j \in B_2} u_2^j - cq_2 \end{aligned}$$

This inequality is obviously satisfied by definition of q_1^{FB}, q_2^{FB}.

(ii) Here we prove that when firms are required to satisfy (2), in any NE the profit of firm i is at least F_i^*. This result holds for any $n \geq 2$, but in the case of $n = 2$ [jointly with (i)] it implies the statement in Proposition 5(ii).[30]

This claim is obvious for $i = n^{FB} + 1, ..., n$, as $F_i^* = 0$ for these firms. About firm $i = 1, ..., n^{FB}$, we show that if firm i plays (1), then (regardless of the strategies followed by the other firms), the buyer buys B_i^{FB}; hence, firm i can get a profit equal to F_i^*; this establishes that his equilibrium profit is not lower than F_i^*. The proof is by contradiction and is written for firm 1. Suppose that 1 plays tr_1^* and the buyer buys nothing from firm 1, while she buys $\bar{q}$ products from firms -1, with values $w_{-1}^1, ..., w_{-1}^{\bar{q}}$ such that $w_{-1}^1 \geq ... \geq w_{-1}^{\bar{q}}$. Obviously, $\bar{q} \leq k - q_1^{FB} + q_1^{SB}$ because of (2) and because the best alternative portfolio without buying any product from 1 can not have more than $k - q_1^{FB} + q_1^{SB}$ products from the definition of q_1^{FB} and q_1^{SB}. But then the buyer can increase her payoff by buying from 1 the products in B_1^{FB}. To prove this, consider first the case of $\bar{q} \leq k - q_1^{FB}$. Then, it is obvious that the buyer can

[30] In particular, the proof of Proposition 5(i) applies *verbatim* also to the setting in which firms must satisfy (2), since the deviation which is proposed for firm 2 uses a technology renting strategy.

increase her payoff by simply buying the products in B_1^{FB} in addition to buying from firms -1 the products of values $w_{-1}^1, ..., w_{-1}^{\bar{q}}$. Second, consider $k - q_1^{FB} < \bar{q} \le k - q_1^{FB} + q_1^{SB}$. Then, we show below that the buyer can increase her payoff by buying the products in B_1^{FB} and not buying from firms -1 products of values $w_{-1}^{k--q_1^{FB}+1}, ..., w_{-1}^{\bar{q}}$. This changes the buyer's payoff by

$$\Delta\pi_0 = -F_1^* - cq_1^{FB} + U_1^{FB} - \sum_{t=1}^{\bar{q}-k+q_1^{FB}} w_{-1}^{k-q_1^{FB}+t} + E$$

where E is the money that the buyer saves by not buying the products with values $w_{-1}^{k--q_1^{FB}+1}, ..., w_{-1}^{\bar{q}}$. Because of (2), E is not smaller than $c(\bar{q} - k + q_1^{FB})$ and therefore we have:

$$\Delta\pi_0 \ge -F_1^* - cq_1^{FB} + U_1^{FB} - \sum_{t=1}^{\bar{q}-k+q_1^{FB}} (w_{-1}^{k-q_1^{FB}+t} - c)$$

Then we can argue like in the proof of Step 1 in the proof of Proposition 1 to prove that the latter term is non negative.

Proof of Proposition 6

We take care of only part (ii) when $q_i^{fb} \ge 1$ and $q_h^{fb} \ge 1$ since the proof of part (i) is given just after the statement.

Before the merger, the profits of firms i and h are $U_i^{FB} - cq_i^{FB} - (U_{-i}^{SB} - cq_{-i}^{SB})$ and $U_h^{FB} - cq_h^{FB} - (U_{-h}^{SB} - cq_{-h}^{SB})$, respectively. After the merger, the profit of $i+h$ is $U_{i+h}^{FB} - cq_{i+h}^{FB} - (U_{-(i+h)}^{SB} - cq_{-(i+h)}^{SB})$. Since $q_{i+h}^{FB} = q_i^{FB} + q_h^{FB}$ and $U_{i+h}^{FB} = U_i^{FB} + U_h^{FB}$, the inequality

$$U_{i+h}^{FB} - cq_{i+h}^{FB} - (U_{-(i+h)}^{SB} - cq_{-(i+h)}^{SB}) \ge U_i^{FB} - cq_i^{FB} - (U_{-i}^{SB} - cq_{-i}^{SB}) + U_h^{FB} - cq_h^{FB} - (U_{-h}^{SB} - cq_{-h}^{SB})$$

is equivalent to

$$U_{-i}^{SB} - cq_{-i}^{SB} + U_{-h}^{SB} - cq_{-h}^{SB} \ge U_{-(i+h)}^{SB} - cq_{-(i+h)}^{SB}. \tag{3}$$

We prove that (3) holds with equality if $\max\{u_{-i}^{k-q_i^{FB}+1} - c, 0\} = \max\{u_{-h}^{k-q_h^{FB}+1} - c, 0\} = \max\{u_{-(i+h)}^k - c, 0\}$ and with inequality otherwise.

Step 1 When $q_{-i}^{SB} < q_i^{FB}$ and/or $q_{-h}^{SB} < q_h^{FB}$, (3) holds with equality if $\max\{u_{-i}^{k-q_i^{FB}+1} - c, 0\} = \max\{u_{-h}^{k-q_h^{FB}+1} - c, 0\} = 0$, otherwise it holds with strict inequality.

Proof. From the definition of q_{-i}^{SB}, it is clear that if $q_{-i}^{SB} < q_i^{FB}$ (respectively, $q_{-h}^{SB} < q_h^{FB}$), then $q_{-(i+h)}^{SB} \le q_{-i}^{SB}$ (respectively, $q_{-(i+h)}^{SB} \le q_{-h}^{SB}$) and $U_{-i}^{SB} - cq_{-i}^{SB} \ge U_{-(i+h)}^{SB} - cq_{-(i+h)}^{SB}$ (respectively, $U_{-h}^{SB} - cq_{-h}^{SB} \ge U_{-(i+h)}^{SB} - cq_{-(i+h)}^{SB}$). Hence, (3) holds with strict inequality unless $U^{SB}_{-i} - cq^{SB}_{-i} = U^{SB}_{-h} - cq^{SB}_{-h} = 0$, and the latter condition is satisfied if and only if $u^{k-q^{FB}_i+1}_{-i} \leq c$ and $u^{k-q^{FB}_h+1}_{-h} \leq c$.

Step 2 When $q^{SB}_{-i} = q^{FB}_i$ and $q^{SB}_{-h} = q^{FB}_h$, (3) holds with strict inequality unless $\max\{u^{k-q^{FB}_i+1}_{-i} - c, 0\} = \max\{u^{k-q^{FB}_h+1}_{-h} - c, 0\} = \max\{u^k_{-(i+h)} - c, 0\}$.

Proof. Given that $q^{SB}_{-i} = q^{FB}_i$ and $q^{SB}_{-h} = q^{FB}_h$, (3) reduces to

$$\begin{aligned}(u^{k-q^{FB}_i+1}_{-i} - c) + ... + (u^k_{-i} - c) + (u^{k-q^{FB}_h+1}_{-h} - c) + ... + (u^k_{-h} - c) \\ \geq \max\{u^{k-q^{FB}_i-q^{FB}_h+1}_{-(i+h)} - c, 0\} + ... + \max\{u^k_{-(i+h)} - c, 0\}\end{aligned} \tag{4}$$

Since $u^{k-q^{FB}_i+j}_{-i} \geq u^{k-q^{FB}_i-q^{FB}_h+j}_{-(i+h)}$ for $j = 1, ..., q^{FB}_i$ and $u^{k-q^{FB}_h+j}_{-h} \geq u^{k-q^{FB}_i-q^{FB}_i+j}_{-(i+h)} \geq u^{k-q^{FB}_h+j}_{-(i+h)}$ for $j = 1, ..., q^{FB}_h$, we infer that the equality holds in (4) if and only if $u^{k-q^{FB}_i+j}_{-i} - c = \max\{u^{k-q^{FB}_i-q^{FB}_h+j}_{-(i+h)} - c, 0\}$ for $j = 1, ..., q^{FB}_i$ and $u^{k-q^{FB}_h+j}_{-h} - c = \max\{u^{k-q^{FB}_i-q^{FB}_i+j}_{-(i+h)} - c, 0\} = \max\{u^{k-q^{FB}_h+j}_{-(i+h)} - c, 0\}$ for $j = 1, ..., q^{FB}_h$. This occurs if and only if $u^{k-q^{FB}_i+1}_{-i} - c = u^{k-q^{FB}_h+1}_{-h} - c = \max\{u^k_{-(i+h)} - c, 0\}$, and this condition is equivalent to $\max\{u^{k-q^{FB}_i+1}_{-i} - c, 0\} = \max\{u^{k-q^{FB}_h+1}_{-h} - c, 0\} = \max\{u^k_{-(i+h)} - c, 0\}$ since $q^{SB}_{-i} \geq 1$ and $q^{SB}_{-h} \geq 1$ imply $u^{k-q^{FB}_i+1}_{-i} - c = \max\{u^{k-q^{FB}_i+1}_{-i} - c, 0\}$ and $u^{k-q^{FB}_h+1}_{-h} - c = \max\{u^{k-q^{FB}_h+1}_{-h} - c, 0\}$.