Two-sided Certification:
The Market for Rating Agencies

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Abstract
Rating agencies tend to offer certification services to both sides of the market: to firms and investors. Conflicts of interest might arise, since firms have an incentive to "optimize" their rating to attract favorably priced financing. We show in a theoretical model, that a credible rating agency will sell its services to both sides of the market to maximize its own profits. Our model shows that observing firms paying the certifiers does not necessarily indicate impure certification results. Furthermore we identify markets in which two-sided certification compared to no certification and one-sided certification has a strong welfare increasing effect.

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1 Introduction

The market for public debt will grow significantly in the upcoming years due to the massive recovery programs which are enacted following the current financial and economic crisis, as politicians revive keynesian theories to stimulate the economy. The question therefore arises, how the debt will be repaid and how investors might value the public debt issuance. Thereby, efficient credit markets are essential to maintain market discipline as proposed by Lane (1993). Financial intermediaries play a key role in the evaluation and increase the efficiency in sovereign credit markets, as it will be shown in the following paper.

The importance of credit ratings grew significantly during the last decades, as the number and complexity of financial products increased. Certification providers seek to overcome the unequal distribution of information in markets to increase efficiency. Since the findings on asymmetric information by Akerlof (1970), a broad literature developed to reduce transaction costs of market participants. Therein the informed party signals (Spence, 1973) its private information to the uninformed party by e.g. building up reputation, issuing warranties or by using third parties, which are credible and certify the information. The uninformed party will build expectations on the quality of the signal and thereafter values the acquired information. Additionally, the uninformed party will try to differentiate the sellers in the market and screen the investment opportunities to identify the best option. This possibility was first described by Rothschild and Stiglitz (1976) and Wilson (1977).

The mechanism is especially prevalent in the world of financial markets. The seller of a financial product possesses private information on the products quality which is unobservable by other market participants, who have expectations on the quality, which might be lower than the actual quality. Therefore, the seller demands a credible signal to maximize the profit of the product. On the other hand, the buyer also seeks to reduce the informational costs to find a suitable investment object and therefore also demands a rating. The literature on the rating industry is not very rich; Strausz (2005) explained the high concentration in the market and Lizzeri (1999) proofed a rather non-intuitive result, that in many circumstances a rating agency will never disclose any information but captures the whole gains from trade.

We contribute to the literature in developing a model which describes the business model of the big rating agencies, which sell their information to both sides of the market -
to investors and firms. The certification industry literature so far solely focused on one-sided certification.

This paper is organized as follows: the next section briefly describes the rating market and the dominant institutions. Section 3 presents three variations of a model describing the market for credit ratings. Thereafter, section 4 discusses the policy implications and Section 5 concludes.

2 The market for rating agencies

Ratings are “summary measures of assessment over the probability that a borrower will default” (Fitch, 2002). Their main purpose is the evaluation of the quality of an investment regarding its repayment likelihood. Typically ratings are grouped into different rating classes, which comprise a specific default probability.

The market for rating agencies is highly concentrated. The two biggest rating agencies, Moody’s and Standard & Poors, share 80 percent of the market and together with the number three, Fitch Ratings, the market share becomes 95 percent. The operating margins of the leading rating agencies is close to 50 percent and relatively stable over the last years, even in the current downturn of the financial markets. Strausz (2005) shows that high profit margins are required to establish credibility and reputation, as rating agencies otherwise cannot withstand capture. Prices therefore can even exceed static monopoly prices. A widespread argument for the high concentration of rating agencies is the rigorous accreditation procedure of rating agencies in the US by the SEC. This is only partly true, since currently 10 Nationally Recognized Statistical Rating Organizations exist in the US. Moreover, also in other regions a high concentration is observable, e.g. in Japan two players share most of the market, namely the Japan Credit Rating Agency and Rating and Investment Information Inc.

Generally, the business model of rating agencies can be summarized with two main pillars. On the one hand, they offer certification services and on the other hand, they offer consultancy, mainly to banks and institutional investors. The combination of the two business areas led to a discussion on conflicts of interest which might evolve, as agencies where consulting banks on the same products as they evaluated in a later stage. The key role in the current financial crisis underlines this major problem.
The business model with respect to the sales model of certification services changed significantly over time. Before 1970, ratings were primarily sold to investors, who paid for the certification service. A subscription was required to obtain information, which were thereafter private information of subscribers. However, after 1970 the rating agencies decided to additionally sell their services to the other side of the market, to firms or issuers. After the firm received a rating, the information was immediately public and could be observed by all market participants. In addition a large number of small rating agencies entered the market, which serve the investor side and sell directly to buyers on a subscription basis.

It is often argued that the current business model of rating agencies is susceptible to bribery, since firms might influence the rating procedure in order to obtain a favorable rating. Rating agencies were accused of being responsible for financial crisis which started in the certification service offered to investors and firms. Primarily the sale of ratings to the originator of product is under steady criticism, as another conflict of interest might evolve.

We show in a theoretical model, that a credible equilibrium exists, where rating agencies sell to both sides of the market to increase their revenues and maximize their profits. Contrary to Lizzeri’s (1999) no revelation result, we show that strong incentives exist for the rating agency to credibly assess the financial product or firm.

3 Model

The model consists of four players: one informed seller, two uninformed buyers and one intermediary. The seller owns a product of quality \( q \) which we assume to be uniformly distributed on the interval \([0, 1]\). The intermediary has no value for the object while the seller has a reservation utility of \( \alpha q \) with \( \alpha \in [0, 1] \). The buyers only know the distribution of the product’s quality and therefore build expectations on the true value. The seller has no possibility to communicate the quality of his product \( q \) directly and credibly to the buyers. The intermediary however owns a technology to evaluate and communicate the product’s quality, which is perfect. If there is demand for an evaluation, by either the seller or by the buyers, the intermediary can determine the quality \( q \) at no cost. In case the seller demands a rating, the intermediary can communicate the quality \( q \) credibly to the market, which is thereafter known to all buyers, hence
public information. If one or both buyers demand an evaluation of the product, the information disclosed by the buyer is private information to the respective buyer.

The game has 4 stages.

(1) The intermediary determines prices $p_f$ and $p_i$ for a rating sold to the seller and for the rating sold to a buyer, respectively.

(2) The seller decides whether to order a rating from the intermediary for the price $p_f$ or whether he declines the offer. If a rating was sold, the information about the quality $q$ is immediately public information.

(3) The buyers decide simultaneously and (independently) whether to order a rating for the seller’s product. Buyers, who decide to order a rating, pay price $p_i$ and receive the information on the quality $q$ as their private information.

(4) The product is sold in a first-price (common value) auction. Finally, payoffs are realized.

Discussion of assumptions:
We assume that the intermediary is honest and uses a perfect information revelation technology. Furthermore we assume that the intermediary has no competitors and so she can exploit her full monopoly power. These assumptions are in line with recent contributions, as e.g. by Strausz (2004). In addition we allow the intermediary to discriminate in prices between sellers and buyers, which is plausible, as different goods are sold to both sides of the market - on seller’s side public information is revealed while on buyer’s side private information is traded. The intermediary acts as a profit-maximizing monopolist.

The seller utility solely depends on the consumption or the sale of a single product. Therefore the seller opts to either sell the product or consumes it by herself. Moreover, all buyers are symmetric ex-ante and no experience or reputation dynamics for selling good or bad quality in the market arise. Furthermore in all parts of the game the information structure is known to the buyers: in the first-price auction they are aware of the information available to their opponent.

The buyers bid for the product in a first-price auction. It seems a natural way to model the selling stage, as initial public offering on financial markets follows this structure. As the academic research is quite silent about picking the ”right” equilibrium
in common value auctions with asymmetric informed bidders we choose a first-price auction (Larson, 2008). One could also apply sequential bidding behavior where the informed/uninformed buyer moves first. Depending on the ordering this might slightly alter the outcomes.

The market is characterized by a single parameter $\alpha \in [0, 1]$. This parameter captures the reservation utility of the seller, which is $\alpha q$ for the quality level $q$. The buyers value the product of quality $q$ with $q$, hence the differences in valuation $(1 - \alpha)q$ generate the possible gains from trade. By applying a market parameter we embed a basic adverse selection framework, where ex-ante no trade occurs.

Finally, we assume that a seller is the first to decide whether to order a rating. This is convincing as it is the seller who initially decides whether to produce a product or not (which is not part of the model).

3.1 The market without the certifier

It is known since Akerlof’s (1970) seminal work on adverse selection, that in markets with $\alpha > \frac{1}{2}$ trades collaps due to the asymmetrically informed parties – we will refer to this market as 'Lemon Market'. The market with $\alpha \leq \frac{1}{2}$ (combined with a first-price-auction) has a unique equilibrium in which all goods are sold for a price of $\frac{1}{2}$ which is the expected quality of the buyers. This means that all possible gains from trade are exploited, even without the intermediary - we refer to such a market as 'Efficient Market'. The implications building on Akerlof’s (1970) lemon model are summarized in the following Proposition:

**Proposition 1** The Market without the Intermediary

(a) In a Lemon Market the optimal bidding strategy for every buyer is to bid 0. All sellers with a positive quality parameter $q$ will not sell their good. There is no welfare generated in a Lemon Market.

(b) For an Efficient Market the optimal bidding strategy for every buyer is to bid the expected quality $q^e = \frac{1}{2}$. Every seller will accept such a bid and the entire welfare $W_{max} = \frac{1-\alpha}{2}$ is exploited.

**Proof** (a) We show that the pair of bidding strategies $(0, 0)$ is an equilibrium. Placing a deviating bid of $b$ leads to winning the object if the bid beats the reservation utility.
The expected quality of such a good is $E[q | \alpha q \leq b]$. As $q$ is uniformly distributed the expected quality is $q^e = \frac{b}{2\alpha}$. Parameter $\alpha$ is greater than $\frac{1}{2}$ and thus $q^e < b$. It is straightforward to show that this equilibrium bids are unique and that as a consequence the market collapses.

(b) It is easy to show that bidding the own valuation is the unique equilibrium in this symmetric first-price auction with common values. As $\alpha \leq \frac{1}{2}$ every seller will accept such a bid as $\frac{1}{2} \geq \alpha q$ for all $q \in [0, 1]$. All products are traded and the full welfare is exploited. q.e.d.

Note: In difference to the classical Lemon model our selling mechanism leads to a unique equilibrium in the case of an Efficient Market. Depending on the beliefs and on firms potentially leaving the market in reaction to that there is a no trade equilibrium in the Efficient Market case, when the buyers have beliefs of $q^e = 0$ and have a willingness to pay corresponding to that of 0. This logic is shown in figure 1.

![Figure 1: Market outcomes in the efficient and the lemon market.](image)

Note: For all models discussed in the following the possible welfare in the whole market to be shared is

$$W_{\text{max}} = \int_{0}^{1} (1 - \alpha)qdq = \frac{1 - \alpha}{2}.$$
3.2 One-sided certification

In this subsection we will develop the results for two different versions of the model. In the first case it is only the sellers who are able to go to the intermediary. Then we present a model where there is only buyer rating.

3.2.1 One-sided seller-certification

The intuition for the model of only the sellers being able to go to the intermediary is quite clear. On the last stage the buyers are symmetrically informed, either both know the quality or both do not. In a Lemon Market they will not bid for the object. In an Efficient Market they will bid their expected valuations. In either case there exists a $\bar{q} \in [0, 1]$ such that all sellers with a quality above $\bar{q}$ will go to the intermediary.

Proposition 2 Bidding behavior for the case of Seller-Rating only

(a) Lemon Market: For uninformed bidders the only equilibrium bid-pair is $(0, 0)$. For informed bidders the only equilibrium is the bid-pair $(q, q)$.

(b) Efficient Market: For uninformed bidders the only equilibrium bid-pair is $(q^e_{\bar{q}}, q^e_{\bar{q}})$ (where $q^e_{\bar{q}}$ denotes the expected quality for a seller who did not go to the intermediary). For informed bidders the only equilibrium is the bid-pair $(q, q)$.

By making the information about the quality parameter $q$ public, the seller can assure that his good will be traded for $q$. In a Lemon Market her good will not be traded if she does not go to the intermediary whereas the good will be traded on expectation in an Efficient Market. As the certifier tries to maximize his profits he will pick such a price that a certain threshold $\bar{q}$ arises above which the firms will order the certification service and for the sellers with a lower quality they will, depending on market structure, be traded on expectation or not at all. The following Theorem states the result for the case of one-sided seller-certification (In the following $\Pi_F$ denotes the expected profits on sellers side and $\Pi_C$ denotes the expected profits for the certifier):

Theorem 1 One-sided seller-certification

If the certifier offers his service only on the seller side the optimal pricing strategy, the critical threshold for quality above which the seller goes to the intermediary, expected profits and expected welfare are the following:
(a) Lemon Market: The optimal strategy for the certifier is a price $p_f = \frac{1 - \alpha}{2}$. It follows $\bar{q} = \frac{1}{2}$, $\Pi_C = \frac{1 - \alpha}{4}$, $\Pi_F = \frac{1 - \alpha}{8}$ and $W = \frac{3}{8}(1 - \alpha)$.

(b) Efficient Market: In this case the optimal strategy for the certifier is a price $p_f = \frac{1}{4}$. It follows $\bar{q} = \frac{1}{2}$, $\Pi_C = \frac{1}{8}$, $\Pi_F = W_{\text{max}} - \frac{1}{8}$ and $W = W_{\text{max}}$.

**Proof** (a) The maximization problem on certifier’s side is the following:

$$\max_{p_f} \Pi(p_f) = (1 - \bar{q})p_f$$

s.t. $(1 - \alpha)q - p_f \geq 0 \quad \forall q \in [\bar{q}, 1]$

There is no possibility to sell the product without going to the intermediary and a rated product will be traded at $q$. From the seller’s condition we get $\bar{q} = \frac{p_f}{1 - \alpha}$. Plugging this into the objective function we get $\Pi(p_f) = p_f(1 - \frac{p_f}{1 - \alpha})$. Taking the derivative with respect to $p_f$ yields $p_f = \frac{1 - \alpha}{2}$, and hence, $\bar{q} = \frac{1}{2}$ with corresponding profit $\frac{1 - \alpha}{4}$. The seller side gets

$$\int_{\frac{1}{2}}^{1} (1 - \alpha)qdq - \frac{1 - \alpha}{4} = \frac{1 - \alpha}{8}$$

in this segment. As the lower segment is not traded a rent of $\frac{1 - \alpha}{8}$ is lost due to the adverse selection problem. Overall welfare is $W = \frac{3}{8}(1 - \alpha)$.

b) The maximization problem on certifier’s side is the following:

$$\max_{p_f} \Pi(p_f) = (1 - \bar{q})p_f$$

s.t. $(1 - \alpha)q - p_f \geq \frac{1}{2}\bar{q} - \alpha q \quad \forall q \in [\bar{q}, 1]$

Hence, the critical quality level of the sellers $\bar{q}$ is determined by $(1 - \alpha)\bar{q} - p_f = \frac{1}{2}\bar{q} - \alpha \bar{q}$. Solving this gives $\bar{q} = 2p_f$. Plugging this into the profit function we get $\Pi(p_f) = p_f(1 - 2p_f)$. Maximizing this gives $p_f = \frac{1}{4}$ and $\bar{q} = \frac{1}{2}$ with corresponding profit $\frac{1}{8}$ for the certifier. The rest of the market is traded without a certificate at a price of $\frac{1}{4}$ and as all products are traded in this market the seller side gets $\Pi_F = \frac{1 - \alpha}{2} - \frac{1}{8}$. As in the market without the certifier $W_{\text{max}}$ is realized.

q.e.d.
3.2.2 One-sided buyer-certification

For this side of the market the objective of the information buying process is fundamentally different from the aims of a seller going to the intermediary. The seller wants to have the information publicly shared. In contrast, a buyer can only profit from the information if she is the only one having this information. In the model the buyers do not know when deciding to order the certification service whether they will be the only one doing so. The only symmetric equilibrium is a mixed-strategy equilibrium in which each buyer decides with a certain probability $\omega$ whether to go to the intermediary or not.

By assumption the buyers know the information structure when entering the first-price auction. The bidding behavior for asymmetrically informed bidders in such a first-price auction is well understood. Beginning with the work of Wilson (1967) and finally solved by Weverbergh (1979) there is a unique equilibrium. The corresponding equilibria for either cases are shown in the next Proposition:

**Proposition 3** Bidding behavior for the case of Seller-Rating only

(a) Lemon Market: For two uninformed bidders the only equilibrium bid-pair is $(0, 0)$. For two informed bidders the only equilibrium is the bid-pair $(q, q)$. If there is only one informed bidder his bidding strategy is $b = \alpha q$ and the uninformed does not bid at all. The expected payoff of a buyer given he is the only one informed in this auction format is $V_{IB} = \frac{1 - \alpha}{2}$.

(b) Efficient Market: For two uninformed bidders the only equilibrium bid-pair is $(q^e, q^e)$ (where $q^e$ denotes the expected quality for a seller which is $\frac{1}{2}$). For two informed bidders the only equilibrium is the bid-pair $(q, q)$. If there is only one informed bidder his bidding strategy is $b = \frac{1}{2} q$. The uninformed mixes on the interval $[0, \frac{1}{2}]$ according to distribution function $F(b) = 2b$. The expected payoff of a buyer given he is the only one informed in this auction format is $V_{IB} = \frac{1}{6}$.

In both cases the expected profit when being uninformed is 0.

**Proof** The bidding equilibria are well-known in the literature. Concerning the different markets there are two potential values of being the only one informed.

Lemon Market: In this case the (expected) payoff of being the only one informed is $q - \alpha q = (1 - \alpha)q$. Formulated as an ex-ante expectation the value of being the only one informed is $(1 - \alpha)q^e = \frac{1 - \alpha}{2}$. 

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Efficient Market: In this case the expected payoff of being the only one informed is $F\left(\frac{1}{2}q\right)(1 - \frac{1}{2})q = \frac{1}{2}q^2$. Formulated as an ex-ante expectation the value of being the only one informed is

$$V_{IB} = \int_0^1 \frac{1}{2}q^2 dq = \frac{1}{6}.$$  

Thus, everything is shown.

As the unique symmetric equilibrium is in mixed strategies the buyers have to be indifferent between ordering a rating and not. Meaning that in expectation they make a profit of 0. From a buyer’s point of view it is just gambling for profits. For the intermediary the most profitable case is to sell her service to both investors. The solution of the whole problem is captured in the following theorem:

Theorem 2 One-sided buyer-certification

If the intermediary offers his service only on the buyer side the optimal pricing strategy depending on market structure, the probability of each buyer going to the intermediary, the expected profits and the expected welfare are the following:

(a) Lemon Market: The optimal strategy for the certifier is to set a price of $p_i = \frac{1}{4} - \alpha$. It follows $\omega = \frac{1}{2}$, $\Pi_C = \frac{1}{4} - \alpha$, $\Pi_F = \frac{1}{8}$ and $W = \frac{3}{8}(1 - \alpha)$.

(b) Efficient Market: In this case the optimal strategy for the certifier is to set a price of $p_i = \frac{1}{12}$. It follows $\omega = \frac{1}{2}$, $\Pi_C = \frac{1}{12}$, $\Pi_F = W_{max} - \frac{1}{12}$ and $W = W_{max}$.

Proof  a) The maximization problem on certifier’s side is the following:

$$\max_{p_i} \Pi(p_i) = \omega^2 2p_i + 2\omega(1 - \omega)p_i$$

s.t. $(1 - w)V_{IB} - p_i = 0$

From the buyers indifference condition one gets $\omega = 1 - \frac{2p_i}{1-\alpha}$. The profit function can be simplified as $\Pi(p_i) = 2\omega p_i$ and by plugging in the value for $\omega$ we get $\Pi(p_i) = 2p_i - \frac{4p_i^2}{1-\alpha}$. Maximizing this function with respect to price $p_i$ yields $p_i = \frac{1-\alpha}{4}$. This leads to $\omega = \frac{1}{2}$ and thus, the certifier’s profit is $\Pi_C = \frac{1-\alpha}{4}$. As $\alpha > \frac{1}{2}$ the firms only make some gains if the information on their quality was known by both investors. In $\frac{1}{4}$ of the cases they make an expected profit of $\frac{(1-\alpha)}{2}$ and hence the overall firm profit is $\frac{1-\alpha}{8}$ and welfare is $W = \frac{3}{8}(1 - \alpha)$.  

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b) The maximization problem on certifier’s side is the following:

$$\max_{p_i} \Pi(p_i) = \omega^2 2p_i + 2\omega(1 - \omega)p_i$$

s.t. $$(1 - w)V_{IB} - p_i = 0$$

Using the expected valuation $V_{IB}$ of $\frac{1}{6}$ leads from the buyers indifference condition to $\omega = 1 - 6p_i$. The profit function can be simplified as $\Pi(p_i) = 2\omega p_i$ and by plugging in the value for $\omega$ we get $\Pi(p_i) = 2p_i - 12p_i^2$. Maximizing this with respect to $p_i$ gives $p_i = \frac{1}{12}, \omega = \frac{1}{2}$ and therefore $\Pi_C = \frac{1}{12}$. Knowing that in such a market all projects are realized it turns out that $\Pi_F = \frac{1 - \alpha}{2} - \frac{1}{12}$.

q.e.d.

3.2.3 Two-sided certification

In the full model we bring the different aims concerning the information about the quality parameter together. A high-quality seller would like both investors to be informed. A buyer can only gain if she is the only one informed. In the Lemon Market and the Efficient Market a seller can hope that both buyers went to the intermediary. In this case he saves the price $p_f$ by having the information on quality shared by the buyers. The certifier may profit from the fact, that he can sequentially partition the market. A high-quality segment ($q \in [\bar{q}, 1]$) which goes to the intermediary and a lower-quality segment in which the buyers order the certification service with a mixed-strategy probability. It is obvious that a buyer will never buy a certificate if the seller already did as the certification process reveals the true quality. In case of an underlying efficient market there is still a market for uncertified goods in which bidding depends on the quality threshold $\bar{q}$ above which a seller orders a certificate on her own.

As in the one-sided buyer-certification there are three possible information outcomes in the bidding stage in two different markets. In this model the distribution of the quality in the rest-market $q \in [0, \bar{q}]$ is determined endogenously. The bidding behavior for either case is summarized in the following Proposition:

**Proposition 4** Bidding behavior for two-sided certification

(a) Lemon Market:

**Proposition 4** If both buyers are informed about the quality $q$ the unique bidding equilibrium is
(q, q).

(a2) If none of the buyers knows quality the unique bidding equilibrium is (0, 0).

(a3) If only one investor is informed about quality \( q \) the unique bidding equilibrium is that the informed buyer bids \( \alpha q \) and the other one does not bid at all. The expected payoff of a buyer given he is the only one informed in this auction format is \( V_{IB} = (1 - \alpha) \frac{q}{2} \).

(b) Efficient Market:

(b1) If both buyers are informed about the quality \( q \) the unique bidding equilibrium is (q, q).

(b2) If none of the buyers knows quality the unique bidding equilibrium is \( (\frac{1}{2} \bar{q}, \frac{1}{2} \bar{q}) \).

(b3) If only one investor is informed about quality \( q \) the unique bidding equilibrium is that the informed buyer bids \( \frac{1}{2} q \). The uninformed mixes on the interval \([0, \frac{1}{2} \bar{q}]\) according to distribution function \( G(b) = \frac{2}{q} b \). The expected payoff of a buyer given he is the only one informed in this auction format is \( V_{IB} = \frac{1}{6} \bar{q} \).

In both cases the expected profit when being uninformed is zero.

**Proof** The bidding equilibria for both cases are shown easily by using the considerations on the auction format so far. We show the results for the expected payoffs of being the only one informed. For non-certified products the quality is distributed according to \( \tilde{F}(q) = \frac{q}{\bar{q}} \).

(a) The probability of winning the auction for the informed bidder is 1. The quality remaining un-certified in the market after the seller-certification is \( \tilde{F}(q) \). In expectation the informed buyer wins an object of quality \( \frac{q}{\bar{q}} \) for a bid of \( \alpha \frac{q}{2} \). This leads to the expected payoff.

(b) Let the object in the auction be of quality \( q \). With a bid of \( \frac{1}{2} q \) the informed buyer wins with a probability of \( G(\frac{1}{2} q) = \frac{q}{\bar{q}} \). If he wins his payoff is \( q - \frac{1}{2} q = \frac{1}{2} q \). Thus, expected payoff ex-ante is determined by

\[
V_{IB} = \int_{0}^{\tilde{q}} q \frac{1}{2} q d\tilde{F}(q) = \int_{0}^{\tilde{q}} q \frac{1}{\bar{q}^2} 2 q dq = \frac{1}{\bar{q}^2} \frac{1}{6} q^3 \bigg|_0^{\tilde{q}} = \frac{1}{6} \bar{q}.
\]
From the analysis so far it is obvious that the expected payoff of being uninformed is zero. q.e.d.

Putting this all together leads us to the optimal behavior of the monopolistic certifier, a seller of product with quality \( q \) and ex-ante uninformed investors.

**Theorem 3**  Two-sided certification

If the intermediary offers his service only on the buyer side the optimal pricing strategy depending on market structure, the probability of each buyer going to the intermediary, the expected profits and the expected welfare are the following:

(a) Lemon Market: The optimal strategy for the certifier is to set prices of \( p_f = \frac{16}{27}(1 - \alpha) \) and \( p_i = \frac{2}{9}(1 - \alpha) \). It follows \( \bar{q} = \frac{2}{3}, \omega = \frac{1}{3}, \Pi_C = \frac{8}{27}(1 - \alpha), \Pi_F = (1 - \alpha)\frac{17}{162} \) and \( W = (1 - \alpha)\frac{65}{162} \).

(b) Efficient Market: The optimal strategy for the certifier is to set prices of \( p_f = \frac{3}{2}(5\sqrt{5} - 11) \) and \( p_i = \frac{1}{4}(7 - 3\sqrt{5}) \). It follows \( \bar{q} = \frac{141 - 63\sqrt{5}}{36 - 16\sqrt{5}}, \Pi_C = \frac{3}{4}(5\sqrt{5} - 11), \Pi_F = \frac{1 - \alpha}{2} - \Pi_C \) and \( W = W_{max} \).

**Proof**  (a) The maximization problem for operating on both sides is the following:

\[
\begin{align*}
\max_{p_f, p_i} \Pi(p_i) &= (1 - \bar{q})p_f + \bar{q}[\omega^22p_i + 2\omega(1 - \omega)p_i] \\
\text{s.t.} \quad (1 - \alpha)q - p_f &\geq \omega^2(1 - \alpha)q \quad \forall q \in [\bar{q}, 1] \\
\text{s.t.} \quad (1 - \omega)(1 - \alpha)\bar{q}^\omega - p_i &= 0
\end{align*}
\]

First, note that the profit function can be simplified to \( \Pi(p_f, p_i) = p_f - \frac{8p_i^3}{(1 - \alpha)(4p_i - p_f)} \). In equilibrium \( \bar{q} \) is determined by the seller’s condition with equality. Using this equation we get \( (1 - \omega^2)(1 - \alpha)\bar{q} = p_f \Leftrightarrow (1 - \omega)(1 - \alpha)\bar{q} = \frac{p_f}{1 + \omega} \). Looking at the buyer’s indifference condition and using \( q^\omega = \frac{1}{2}\bar{q} \) we see that \( \frac{p_f}{2 + \omega} = p_i \Leftrightarrow \omega = \frac{p_f}{2p_i} - 1 \). To get a function \( \bar{q}(p_f, p_i) \) we use \( 1 - \omega^2 = \frac{p_f}{p_i}(1 - \frac{p_f}{4p_i}) \) in the equation given above. This gives \( \bar{q} = \frac{4p_i^2}{(1 - \alpha)(4p_i - p_f)} \). Plugging the two expressions for \( \omega \) and \( \bar{q} \) into the objective function we end up with

\[
\Pi(p_f, p_i) = p_f - \frac{8p_i^3}{(1 - \alpha)(4p_i - p_f)}.
\]

Maximizing this with respect to \( p_f \) and \( p_i \) we finally end up with \( p_f = \frac{16}{27}(1 - \alpha) \) and \( p_i = \frac{2}{9}(1 - \alpha) \). Using the derived functions this implies \( \bar{q} = \frac{2}{3} \) and \( \omega = \frac{1}{3} \). The profit for the certifier is \( \Pi_C = \frac{8}{27}(1 - \alpha) \). In the market below \( \bar{q} \) a share of \( 1 - (\frac{2}{3})^2 = \frac{5}{9} \) is
traded. Hence, the overall welfare determines as

\[ W = \frac{5}{9} \int_0^\frac{2}{3} (1 - \alpha) dq + \int_{\frac{2}{3}}^1 (1 - \alpha) dq = (1 - \alpha) \frac{65}{162}. \]

As the buyers do not make any profit in equilibrium the firm profits are determined by

\[ \Pi_F = W - \Pi_C = (1 - \alpha) \frac{17}{162}. \]

(b) To formulate the maximization problem of the intermediary one needs the expected winning bid for an object of quality \( q \) sold in a first-price auction with asymmetrically informed bidders \( E[b_{\text{win}}|q] \). With a probability of \( \frac{q}{\bar{q}} \) the informed bidder wins with a bid of \( \frac{1}{2}q \). With a probability of \( 1 - \frac{q}{\bar{q}} \) the uninformed wins with an expected bid of \( \frac{1}{2}q + \frac{1}{2}\bar{q} = \frac{1}{4}(q + \bar{q}) \). Thus,

\[ E[b_{\text{win}}|q] = \frac{q}{\bar{q}} \cdot \frac{1}{2}q + (1 - \frac{q}{\bar{q}}) \cdot \frac{1}{4}(q + \bar{q}) = \frac{1}{4}q + \frac{q^2}{4\bar{q}}. \]

As the left hand side of the seller’s constraint is increasing faster than the right hand side we can formulate the maximization problem for the intermediary operating on both sides as the following:

\[
\begin{align*}
\max_{p_f, p_i} \Pi(p_f, p_i) &= (1 - \bar{q})p_f + \bar{q}[\omega^22p_i + 2\omega(1 - \omega)p_i] \\
\text{s.t.} \quad (1 - \alpha)\bar{q} - p_f &= \omega^2(1 - \alpha)q + 2\omega(1 - \omega)(\frac{1}{2} - \alpha)\bar{q} + (1 - \omega)(\frac{1}{2} - \alpha)\bar{q} \\
\text{s.t.} \quad (1 - w)\frac{1}{2}q_{\bar{q}} - p_i &= 0
\end{align*}
\]

This is due to \( E[b_{\text{win}}|q] = \frac{1}{2}q + \frac{q^2}{4\bar{q}} = \frac{1}{2}\bar{q} \). The three parts of the seller’s condition correspond to the profits by being rated twice by investors, rated once, and not being rated at all. In the latter case there will be trade without any information in the rest of the market.

The seller’s condition can be reformulated as \( (1 - \omega^2)(1 - \alpha)\bar{q} - p_f = (1 - \omega^2)(\frac{1}{2} - \alpha)\bar{q} \). Solving the constraints for \( \omega \) and \( \bar{q} \) we get \( \omega = 1 - \frac{6p_i}{q} \) and \( \bar{q} = \frac{2p_f}{1 - \omega^2} \). Replacing \( \omega \) gives \( \bar{q} = \frac{18p_i^2}{6p_i - p_f} \) and by plugging this into the profit function of the certifier we get the
reduced maximization problem:

\[ \max_{p_f, p_i} \Pi(p_f, p_i) = p_f - 6p_i^2 \frac{p_i + p_f}{6p_i - p_f} \]

s.t. \[ 0 \leq \omega, \bar{q} \leq 1. \]

In this version of the model one explicitly has to assure that the constraints for the values of \( \omega \) and \( \bar{q} \) are fulfilled. Using the expressions for these two parameters derived above these constraints are equivalent to

\[ 3p_i \leq p_f \leq 6p_i - 18p_i^2. \]

Solving this problem gives the parameters stated in the Theorem.

q.e.d.

4 Results

The model reveals some striking results. Without a certification service two market outcomes arise: in one market the costs of asymmetric information does not hinder investors and firms from exchanging their products. The reservation utility of the best firm is lower than the expected quality of all firms by the investors and consequently, all products are traded in the market; an efficient market arises. Contrarily, a market in the sense of Akerlof (1970), the asymmetric information problem leads to the collapse of the entire market. No trades will be observed in this ‘lemon market’.

In a lemon market a financial intermediary can partly overcome the asymmetric information problem, as a high proportion of potential trades is realized. In the efficient market, the total welfare was not affected by the introduction of the intermediary. The intermediary in this case receives a high fraction of the rents generated by the market. In an efficient market our results show that a profit-maximizing certifier prefers to operate on seller’s side while she is indifferent in a lemon market. Figure 2 depicts the shares of all parties involved in the market if the certification service is merely offered to the seller side. In the lemon market not the entire welfare can be realized, but a substantial proportion of 75 percent. In both markets, the certifier extracts a high amount of the rents, which rise up to 50 percent of potential welfare in the lemon market. Firms gain in the lemon market by hiring the intermediary, as they extract 25 percent of potential welfare, which could not be realized in an alternative way. In an efficient market the
intermediary does not increase welfare and the sellers do not even want her to operate in the market (ex-ante).

Figure 2: Profit shares with one-sided certification sold to sellers.

If the intermediary decides to merely sell to the investors side, its revenues will shrink from $\frac{1}{8}$ to $\frac{1}{12}$. It is important to notice, that the traded products differ between both sales schemes: if the seller orders a rating, the best half of the firms will demand a certificate, whereas if buyers order ratings, they cannot differentiate between good and bad firms and therefore will select randomly.

Comparing these outcomes with the model in which the intermediary sells its services firstly to the sellers and, if they reject the offer, secondly to the buyers, the welfare in the lemons market increases even further. With two-sided certification, about 70 percent of all products are traded in equilibrium, including the third with highest quality. The welfare loss is down to about 20 percent compared to 100 percent in the case without certification.

The profit for the intermediary is highest in the market with two-sided certification, which is not as intuitive as expected at first sight. By offering the certification service on the secondary market the intermediary faces a negative second-order effect from
sellers hoping to be rated by two buyers. Through the rating offer to the sellers in the lemon market, the sellers with high quality levels \( q \) will except the offer (firms above \( q \)) and the remainder might then be evaluated on a random basis by the investors. Table 1 in the appendix gives a summary report on the equilibrium values of the main variables in the model.

The shares of welfare in the one-sided and the two-sided model are depicted in figure 3. The graph shows the slight increase of the intermediaries' share on welfare for the efficient market. Compared to the 50% jump in profits from offering ratings to the firm’s side instead of operating on investor’s side solely the increase in profits if offering the certification service on both sides in an efficient market is only about 8%.

Figure 3: Profit shares with two-sided certification compared to one-sided.

For the lemon market it shows the increased share of the firms’ profits which is the reduction in lost welfare compared to a one-sided market. The welfare loss due to asymmetric information reduces drastically if it is a two-sided certification intermediary operating in the market.
5 Conclusion

The rating industry is highly concentrated and offers services to both sides of the market. In principle, ratings seek to reduce transaction costs and market inefficiencies which accrue due to information asymmetries of market participants. However, the financing of the intermediaries was under steady criticism, as conflict of interest might arise and the market power could be exploited. The main criticism hereby was the payment of the rating service by the seller side. Therefore we analyze the sales mechanism of financial intermediaries and determine its influence on welfare generation.

Introducing a financial intermediary, which offers its services solely to one side of the market enables trades in a market in the Akerlof sense, which we call the lemon market. Depending on the sales scheme, either selling the certification service merely to the buyer or to the seller, the profit shares of the parties vary. The certifier can maximize its profit by selling to the sellers side.

Furthermore, we show theoretically, that the profit maximizing strategy for the rating agencies is to sell to both sides of the market, as it is done since the 1970s. Furthermore the economic welfare in financial markets increases, as more projects, which are adequately priced, are promoted.

The policy implication of the results of our model is an indirect one. It is not necessarily the case that observing intermediaries being paid by the issuers indicates a cooperation of the two parties or even beautifying the default probability or whatever the quality parameter may be. In a functioning market which fits the world of our model we would rather expect that intermediaries have a tendency to offer their services mainly on seller’s side. A policy implication could be to foster the presence of intermediaries in inefficient markets as the lemon markets as they are able to solve the inefficiency due to asymmetrically distributed information to a certain degree and even increase welfare. It is less clear whether an efficient market needs a certifier. Taking into account that the certifying procedure incorporates some economic costs it may reduce welfare to have such intermediaries.

In recent times it seemed to be a straightforward argument that the financial crisis was provoked by unjustified good ratings arising from the conflicts of interest between certifier and the firms. This paper does not claim that this was not the case. It is rather that one has to look closer at a particular market, the ratings within this market.
and the outcome at the end of the day whether ex-post wrong ratings were disturbed on purpose or not. To look at the agent who paid the certificate is definitely not a sufficient reason for this argument.

## Appendix

### Table 1: Comparing equilibrium outcomes of different model settings

<table>
<thead>
<tr>
<th></th>
<th>Only sellers</th>
<th>Only buyers</th>
<th>Both sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha &gt; \frac{1}{2} ) (lemon market)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>price for seller rating</td>
<td>(\frac{1-\alpha}{2})</td>
<td>-</td>
<td>(\frac{16}{27} (1-\alpha))</td>
</tr>
<tr>
<td>price for buyer rating</td>
<td>-</td>
<td>(\frac{1-\alpha}{4})</td>
<td>(\frac{2}{9} (1-\alpha))</td>
</tr>
<tr>
<td>high-quality threshold</td>
<td>(\frac{1}{2})</td>
<td>-</td>
<td>(\frac{2}{3})</td>
</tr>
<tr>
<td>buyer’s rating probability</td>
<td>-</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td>profit certifier</td>
<td>(\frac{1-\alpha}{4})</td>
<td>(\frac{1-\alpha}{4})</td>
<td>(\frac{8}{27} (1-\alpha))</td>
</tr>
<tr>
<td>profit firm</td>
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<td>(\frac{1-\alpha}{8})</td>
<td>(\frac{17}{162} (1-\alpha))</td>
</tr>
<tr>
<td>welfare</td>
<td>(\frac{3}{8} (1-\alpha))</td>
<td>(\frac{3}{8} (1-\alpha))</td>
<td>(\frac{65}{162} (1-\alpha))</td>
</tr>
</tbody>
</table>

| \( \alpha < \frac{1}{2} \) (efficient market) | \(\frac{1}{4}\) | - | \(\approx 0.27\) |
| price for seller rating | - | \(\frac{1}{12}\) | \(\approx 0.07\) |
| price for buyer rating  | \(\frac{1-\alpha}{2}\) | - | \(\approx 0.573\) |
| high-quality threshold  | - | \(\frac{1}{2}\) | \(\approx 0.24\) |
| buyer’s rating probability | - | \(\frac{1}{2}\) | \(\approx 0.135\) |
| profit certifier        | \(\frac{1}{8}\) | \(\frac{1}{12}\) | \(\approx 0.135\) |
| profit firm             | \(\frac{1-\alpha}{2}\) | \(\frac{1-\alpha}{12}\) | \(\frac{1-\alpha}{2} - 0.135\) |
| welfare                 | \(\frac{1-\alpha}{2}\) | \(\frac{1-\alpha}{2}\) | \(\frac{1-\alpha}{2}\) |
References


Larson, N. (2009), ”Private value perturbartions and informational advantage in common value auctions”, Games and Economic Behavior, in press.


