Downstream Competition, Exclusive Dealing and Upstream Collusion

Marie-Laure Allain[*], Claire Chambolle[†] and Clémence Christin[‡]

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Abstract

This paper analyses the impact of exclusive dealing contracts on the scope for upstream collusion in a framework with upstream and downstream imperfect competition. We consider a repeated game of vertical contract choice in a double duopoly. We show that exclusive dealing contracts may either facilitate or deter upstream collusion, depending on the strength of competition at both levels. We show that when interbrand competition is soft, collusion is easier to sustain when producers can offer exclusive dealing contracts to retailers than when they cannot, whereas when interbrand competition is fierce, exclusive dealing contracts deter collusion. In the latter case exclusive dealing can surprisingly improve consumer welfare.

Keywords: Collusion, Vertical Contracts.
JEL Codes: L13, L41.

1 Introduction

Buyer power is usually seen by Competition authorities as reducing the risk of upstream collusion. The European Commission's Merger Guidelines, for instance, state that "special consideration is given to the possible impact of [countervailing power] on the stability of coordination"[1]: "Even firms with very high market shares may not be in a position, post-merger, to significantly impede effective competition [if their customers] possess countervailing buyer power. Countervailing buyer power should be understood as the bargaining

[*]CNRS, Laboratoire d'Econométrie de l'Ecole Polytechnique (1 rue Descartes 75 005 Paris, France, email : marie-laure.allain@shs.polytechnique.fr) and CREST-LEI.
[†]INRA (ALISS: 65 bld de Brandebourg, 94205 Ivry sur Seine, email: chamboll@ivry.inra.fr) and Ecole Polytechnique.
[‡]Laboratoire d'Econométrie de l'Ecole Polytechnique (1 rue Descartes 75 005 Paris, France, email : clemence.christin@shs.polytechnique.fr, tel: 01 55 55 82 13.)
[1]"Guidelines on the assessment of horizontal mergers under the Council Regulation on the control of concentrations between undertakings", Official Journal of the European Union, 2004/C 31/03, § 57.

strength that the buyer has vis-à-vis the seller in commercial negotiations due to its size, its commercial significance to the seller and its ability to switch to alternative suppliers." It is often believed that if the demand is fragmented due to the presence of many small or medium sized buyers, this can make market sharing agreements among suppliers easier (Inderst and Shaffer, 2007).

The role of demand structure and vertical relations between suppliers and strategic buyers in collusion and anticompetitive agreements is thus a relevant economic question. However, rather little literature has been devoted to this topic. Most of the theoretical literature on collusion focuses indeed on industries in which firms sell their products to consumers directly. Jullien and Rey (2007) focus on the effect of resale Price Maintenance contracts (RPM) on the stability of collusion among producers. Nocke and White (2007) study the effect of vertical integration on the stability of upstream collusion. Snyder (1996) considers the effect of the size of buyers on upstream collusion and shows that large buyers are more likely to deter collusion among the suppliers than small ones: his result is consistent with the analysis of competition authorities

In this paper we study upstream collusion in a model where two upstream firms sell differentiated goods through two differentiated downstream firms. Following the traditional analysis introduced by Friedman (1971) on tacit collusion, we thus determine horizontal collusive equilibria that can be sustained by trigger strategies and study the effect of collusion on up- and downstream profits as well as on consumer's surplus. We analyse the effect of exclusive dealing contracts on the stability of collusion.

We show that if interbrand competition is soft enough, upstream collusion is easier to sustain when exclusive dealing contracts are available than when they are not. On the contrary, when interbrand competition is fierce enough, allowing for exclusive dealing contracts between producers and retailers hinders upstream collusion. The consumers' surplus analysis is symmetric: when interbrand competition is soft enough, exclusive dealing contracts should not be made available to firms, because they harm consumers both when collusion is possible and when producers are forced to compete. On the contrary, when interbrand competition is fierce enough, exclusive dealing contracts may benefit consumers by hindering collusion, compared to a case where no exclusive dealing contract can be signed.

The paper is structured as follows. Section 2 introduces the model and develops assumptions on the game. Section 3 analyzes collusion in the benchmark model without exclusive dealing contracts. Section 4 introduces exclusive dealing contracts and studies collusion in this endogenous market structure. Section 5 focuses on welfare effects and Section 6 concludes.

2 The Model

Consider two firms A and B producing differentiated goods at the same marginal cost, normalized to 0. They cannot sell their products directly on the final market, but have to use two differentiated downstream firms, called 1 and 2, as intermediaries, or, say, retailers. A downstream firm's initial marginal cost of retailing is normalized to 0.

As upstream and downstream firms are differentiated, there are potentially four goods available for the consumers to purchase. The final consumers' inverse demand function for good K sold by firm i (denoted good Ki) would then be:

$$p_{Ki}(q_{Ki}, q_{Kj}, q_{Li}, q_{Lj}) = 1 - q_{Ki} - aq_{Li} - bq_{Kj} - abq_{Lj}, \quad \text{with} \quad i \neq j \quad \text{and} \quad K \neq L \qquad (1)$$

where q_{Ki} is the quantity of good K sold by downstream firm i on the final market, and similarly for q_{Kj}, q_{Li}, q_{Lj}. Parameter $a \in [0,1]$ represents the degree of substitution between goods A and B, *i.e.* the level of inter-brand competition, whereas $b \in [0,1]$ represents the degree of substitution between downstream firms, *i.e.* the level of intra-brand competition. Note that if $a = b = 0$, all goods are independent, whereas $a = b = 1$ corresponds to a homogenous good. Besides, we assume for simplicity that the substitution between one good sold in a given shop and the other good sold in the other shop is a combination of intra- and inter-brand competition.[2] We denote by $D^{Ki}(p_{A1}, p_{A2}, p_{B1}, p_{B2})$ the final demand for good Ki.

Producers simultaneously offer non-discriminatory linear tariffs to the retailers: to ensure that firm i ($i = 1, 2$) sells good K ($K = A, B$), producer K has to offer a linear tariff denoted by w_K taking into account the participation constraint of each retailer.

There is an infinite number of periods. At each period, the timing of the stage game is as follows:

Stage 1 Choice of market structure: Producers wishing to use exclusive dealing contracts (if available) simultaneously and secretly offer exclusive clauses to one or two retailers. Each retailer that has received such an offer accepts or rejects it.

Stage 2 Exclusive dealing contracts are made public and producers simultaneously make take-it-or-leave-it offers to retailers: Producer K offers retailer i a linear tariff w_{Ki}, unless retailer i has an exclusive dealing contract with K's rival. Tariffs are common knowledge.

Stage 3 Retailers then compete in price on the final market: each retailer sets a price for each good she sells.[3]

[2] For a discussion of these assumptions, see Allain and Chambolle [2007] or Dobson and Waterson [2007].
[3] Note that the demand for a given good at a given outlet may be zero.

In the repeated game, both upstream firms have the same discount factor denoted $\delta \in [0, 1]$. Downstream firms are assumed to change at each period, so that they are neutral to upstream collusion and have no incentive to hinder collusion among the producers in order to improve their own future profits.[4] Finally, each producer K maximizes his continuation profit $\Pi_K = \sum_{t=0}^{\infty} \delta^t \pi_K(t)$, where $\pi_K(t)$ is K's immediate profit at date t.

Before studying the stability of collusion in this game, we analyze a benchmark situation where exclusive dealing contracts are not available.

3 Benchmark: no exclusive dealing contracts

In this section we determine the critical discount factor above which collusion can be sustained through classical trigger strategies. At each period, firms play a two-stage game as stage 1 vanishes in the absence of exclusive dealing contracts. We determine in turn the competitive equilibria, punishment strategies and finally the collusive equilibrium.

3.1 The competitive equilibrium.

The stage game is solved with backward induction in the Appendix.

Lemma 1. *The stage game has a unique Nash equilibrium such that (i) the two producers offer the same wholesale tariff to the two retailers $w^*_{IR} = \frac{1-a}{2-a}$, (ii) the two retailers set the same price for each good, $p^*_{IR} = 1 - \frac{1}{(2-a)(2-b)}$ and (iii) the four goods are sold on the final market.*

Proof. See Appendix 7.1.1. □

The competitive wholesale tariff w^*_{VS} decreases with upstream competition (a). Besides, when a goes to 0, w^*_{VS} goes to the monopoly price (local monopolies), whereas it goes to 0 when a goes to 1 (perfect competition). Besides, the competitive final price p^*_{VS} is decreasing when downstream competition increases. It goes to w^*_{VS} when b goes to 0.

Finally, the retailers' equilibrium profits π^*_D is increasing in a and decreasing in b: the fiercer upstream competition and the softer downstream competition, the larger the part of the total profit retailers can capture. Upstream profits π^*_U decrease with a. Downstream competition has however an ambiguous impact on upstream profits: due to the variation of downstream prices and thus of quantities sold, and although the wholesale tariff w^*_{VS} does not depend on b, π^*_U decreases with b when $b \leq 1/2$, and increases with b otherwise.

[4] This technical assumption allows us to focus on upstream collusion. It is used by Jullien and Rey (2007) and Nocke and White (2007).

3.2 Joint profit maximizing strategies

We turn now to the optimal collusive strategies: Upstream firms maximize the upstream industry's aggregate profit, taking into account the retailers' participation constraints. In other words, in stage 1 of the one-period game, each upstream firm chooses the tariff that maximizes $\sum_{K=A,B,i=1,2} w_{Ki} D_{Ki}(p^*_{A1}(w), p^*_{A2}(w), p^*_{B1}(w), p^*_{B2}(w))$, where w is the vector of wholesale tariffs: $w = (w_{Ki})_{K\in\{A,B\},i\in\{1,2\}}$. The joint profit maximizing tariffs are symmetric and equal to $w^C_{VS} = 1/2$. Final price is thus $p^C_{IR} = 1 - \frac{1}{2(2-b)}$.

As in the competitive case, a producer's collusive profit π^C_U is decreasing in a and a retailer's collusive profit, π^C_D, is decreasing in b; besides π^C_U is decreasing in b when $b < 1/2$ and increasing in b otherwise. However, contrary to the competitive case, π^C_D is now decreasing in a too. Indeed, in the competitive equilibrium, the wholesale price w^*_{VS} decreases with upstream competition faster than the final price p^*_{VS}; it results that a retailer's unitary margin $(p^*_{VS} - w^*_{VS})$ increases with a, and thus that her competitive profit is increasing in a. On the contrary, in the collusive situation, the wholesale price w^C_{IR} does not depend on a: producers are not competing on the upstream level and can thus set the monopoly wholesale price, that is 1/2. Moreover, the final price p^C_{IR} does not depend on a either: collusion at the upstream level is enough to annihilate the effect of inter-brand competition on downstream choices: a is only present in the expression of the quantity, $q^C_{IR} = \frac{1}{2(1+a)(2-b)(1+b)}$, which is naturally decreasing in a. Indeed, only the direct effect, such that for a given price, demand for a good decreases with the substitution degree, is present here. The consequence is that, contrary to the competitive equilibrium, in collusion retailers do not benefit from inter-brand competition at all. Hence, their profit no longer increase with their buyer power.

3.3 Deviations from the collusive path

We now determine a producer's optimal symmetric deviation. We compute which tariff an individualistic producer, say A, should offer if he takes as given that his rival will offer w^C_{IR}.

Lemma 2. *A producer's optimal deviation strategy depends on both upstream and downstream competition:*

(i) When upstream competition is soft enough, the optimal deviation is such that all goods are carried on the final market;

(ii) When upstream and downstream competition levels are intermediary, the producer's optimal deviation is to set asymmetric prices such that his rival's good is carried by only one retailer;

(iii) When upstream competition is fierce enough, the optimal deviation is such that the rival good is completely excluded from the market.

Proof. See Appendix 7.1.2. □

When interbrand competition is soft enough, A offers the same tariff to the two retailers. It has no incentive to cut prices so as to exclude B from the market. As a increases however, it becomes more and more easy to exclude B because the increased competition means that for a given drop in A's tariffs, A's market share increases more. As a consequence, there exists a value of a above which it becomes profitable for A to offer a lower tariff to one of the retailer, in order to exclude B from this outlet. Finally, when upstream competition is fierce enough, A sets symmetric tariffs again, such that B's goods are not sold in any outlet. As a increases, goods become closer substitutes and price such that quantities of $B1$ and $B2$ are zero increases. It goes to w_{IR}^C when a tends towards 1.

3.4 The repeated game

We now determine the conditions under which the trigger collusive strategy in which producers offer w_{IR}^C as long as no deviation has been observed, and w_{IR}^* otherwise can sustain a subgame-perfect equilibrium. The condition such that the trigger collusive strategy is stable can be written as follows:

$$\pi_K^{Dev} + \frac{\delta}{1-\delta}\pi_U^* \leq \frac{1}{1-\delta}\pi_U^C \quad K \in \{A, B\} \tag{2}$$

with π_K^{Dev} K's deviation profit. We denote by δ_{IR}^* the discount factor threshold above which collusion can be sustained.

Proposition 1. *When the industry is vertically separated, trigger collusive strategies sustain a subgame-perfect equilibrium if and only if*

$$\delta \geq \delta_{IR}^* = \begin{cases} \frac{(2-a)^2}{8-8a+a^2} & \textit{if the deviation market structure is } (AB, AB), \\ 1 + \frac{8(1-a)a^4\left(8-6b^2-b^4-4a^2b^2+3a^2b^4\right)}{D} & \textit{if the deviation market structure is } (AB, A), \\ 1 - \frac{a^4}{a^4-(2-a)^2(1-a-a^2)} & \textit{if the deviation market structure is } (A, A), \end{cases}$$

where D depends on a and b.

The threshold δ_{IR}^* is increasing in inter-brand competition a as long as $a \leq \frac{2\sqrt{10}-4}{3}$ and decreasing in inter-brand competition otherwise. This results from the fact that it is only possible for a deviating producer to exclude his rival when upstream competition is fierce enough.

4 Exclusive dealing

Consider now that exclusive dealing contracts are available: at stage 1 of the stage-game, each producer can offer exclusive dealing contracts to retailers. We assume that rejecting a producer's exclusive dealing offer at that stage implies giving up the possibility of selling his good: If a retailer rejects the exclusive dealing contract offered by a given producer, she can carry only his rival's good. As in the benchmark case, we determine the competitive equilibria, the collusive path and optimal deviations from this path. This allows us to find collusive equilibria.

4.1 Competition

Lemma 3. *The unique competitive equilibrium is such that each producer signs an exclusive dealing contract with one retailer and sets the wholesale tariff* $w_{ED}^* = \frac{(1-ab)(2+ab)}{4-ab-2a^2b^2}$.

Proof. See appendix 7.2.1. □

The competitive wholesale tariff is then decreasing in inter- and intrabrand competition. It goes to 0 when ab goes to 1 (perfect competition with exclusive dealing) and to the monopoly wholesale tariff 1/2 when ab goes to 0 (local monopolies). Final prices are symmetric and equal to $p_{ED}^* = \frac{2(1-ab)(3-a^2b^2)}{(2-ab)(4-ab-2a^2b^2)}$, which decreases with a and b.

Producers' competitive profit decreases with competition (both upstream and downstream). Besides, if inter-brand competition is soft enough, this profit is lower than their profit when exclusive dealing contracts are not available, whereas it is higher otherwise. Indeed, the fiercer competition among producers, the more a producer has an incentive to offer an exclusive dealing contract to a retailer. Moreover, the price difference between exclusive dealing and no exclusive dealing becomes high enough to offset the quantity loss when producers sell to only one retailer. As a consequence, when inter-brand competition is soft enough, producers are worse off in competition when exclusive dealing contracts are available than when they are not. Indeed, in that case, when choosing to which retailer they offer exclusive dealing contracts, producers face a prisoner's dilemma: on the one hand, they would both prefer a situation where all goods are carried (because they would then have access to twice as many consumers as with exclusive dealing), but on the other hand, each producer has an incentive to offer an exclusive dealing contract to one retailer when his rival offers no exclusive dealing contract.

4.2 Joint-profit maximizing strategies

We now turn to the optimal collusive strategies. Taking the market structure as given, producers set wholesale tariffs that maximize the upstream joint profit. Given these wholesale tariffs, each retailer always has an incentive to sell all the goods available to her. As a consequence, when producers are colluding, the only way to have a retailer carry only one good is to have her sign an exclusive dealing contract with the concerned producer. Then, the upstream joint-profit is maximized when no exclusive dealing contract is sold with any retailer. As a consequence, producers set the same wholesale tariff as in the benchmark case: $w_{IR}^C = 1/2$, and the four goods are sold on the final market. The collusive path is thus exactly the same as when exclusive dealing contract are not available.

4.3 Deviation from the collusive path

We consider that producer A wants to deviate from the collusive path. Allowing for exclusive dealing contracts broadens the scope for deviation. Indeed, in addition to the deviation strategies available in the benchmark case, that imply only setting different wholesale tariffs than the collusion wholesale tariff at stage 2 of the stage-game, producer A can now deviate in stage 1 by offering exclusive dealing contracts to one or two retailers. With such a strategy, the exclusion of B from (part of) the market is made independent of wholesale tariffs: A can then exclude B without cutting prices. However, such a deviation (say by A) is detected by the rival (B) at the end of stage 2. As a consequence, producer B can react to this deviation right from the price setting stage, by switching to its competitive strategy earlier than it would have otherwise. If the benefit from exclusive dealing contract offsets the loss due to earlier punishment, then A has an incentive to deviate right from stage 1.

Lemma 4. *When exclusive dealing contracts are available, a producer's optimal deviation is such that:*

(i) When upstream competition is soft enough, he signs an exclusive dealing contract with one of the retailers.

(ii) Otherwise, he does not offer any exclusive dealing contract to retailers and sets prices as in the benchmark case.

Proof. See Appendix 7.2.2. □

Indeed, the fiercer competition among producers, the more agressive B's reaction when he finds out that A has signed an exclusive dealing contract with one of the retailers. For high values of a, the benefit A gets from excluding B from one half of the market is thus offset by his loss due to the price war that follows.

Note that producer A can never deviate by signing exclusive dealing contracts with both retailers. Indeed, retailers then anticipate that if they accept, A will set monopoly wholesale prices at the next stage, since he cannot commit to offering lower prices. As a consequence, each retailer has an incentive to refuse the exclusive dealing contract if she anticipates that her rival was offered the same contract.

4.4 The repeated game

We now determine the threshold discount factor above which collusion is stable, and compare this threshold to the benchmark case. The condition such that the collusive trigger strategy is stable is very similar as that given by Equation (2), although deviation and punishment profits are different. Indeed, the punishment profit is now the competitive profit when each producer has signed an exclusive dealing contract with one retailer, and thus much lower than in the benchmark case. On the contrary, the deviation profit is higher than in the benchmark case. The resulting threshold discount factor is noted δ^*_{ED}.

Proposition 2. *When interbrand competition is soft enough, $\delta^*_{ED} < \delta^*_{IR}$: collusion is easier to sustain when producers can offer exclusive dealing contracts to retailers than when they cannot.*

On the contrary, when interbrand competitions is fierce enough, $\delta^*_{ED} > \delta^*_{IR}$: collusion is then easier to sustain when exclusive dealing contracts are not available.

This result is mostly due to the form of the punishment profits. Indeed, in the competitive equilibrium, producers are worse off with exclusive dealing contracts when inter-brand competition is soft enough, and better off otherwise. Note that the effect of punishment completely offsets the effect of the deviation profit: although deviation is more profitable with exclusive dealing contract than without for low enough values of a, collusion is still easier to sustain with exclusive dealing contracts than without.

5 Welfare Effects

In this section, we analyse the effect of upstream collusion on consumers' surplus.

Note first that in the stage-game, allowing producers and retailers to sign exclusive dealing contracts harms consumers. Indeed, since at the competitive equilibrium, firms indeed sign exclusive dealing contracts, the diversity of the goods offered is reduced. Besides, it softens competition between the goods sold, since each brand is sold in only one outlet. As a consequence, final prices are higher than in the benchmark case.

Now focusing on the repeated game, it is first interesting to see that, since making exclusive dealing contracts available does not change the collusive path, it does not affect collusive final prices, and hence the effect of collusion on consumers' surplus. As a consequence, since competitive prices decrease with upstream competition, whereas the collusive price is constant with upstream competition, consumers are all the more worse off in collusion than in competition, that inter-brand competition is fierce.

However, competitive prices with exclusive dealing are so high that in that case, when inter-brand competition is soft enough, the collusive price is actually lower than the competitive price. Therefore, when inter-brand competition is soft enough, exclusive dealing contracts should not be available, for they harm consumers through two effects. First, they facilitate collusion. Second, even when collusion is not stable, they induce much higher prices on the final market than prices that would be chosen without exclusive dealing contracts.

On the contrary, when inter-brand competition is fierce enough, allowing for exclusive dealing contracts may benefit consumers. Indeed, in that region, collusion is easier when such contracts are not available than when they are. As a result, when the producers' discount factor δ is in the interval $[\delta^*_{ED}, \delta^*_{IR}]$, then the possibility of signing exclusive dealing contracts benefits consumers because it forces producers to remain in the competitive equilibrium, whereas they would have a collusive behaviour otherwise. Since this is also a region where competitive prices with exclusive dealing are lower than collusive prices, allowing for exclusive dealing contracts indeed reduces prices.

6 Conclusion

In this paper, we analyse the impact of exclusive dealing contracts on the scope for collusion between the suppliers, and the overall effect of exclusivity and collusion on welfare. We show first that exclusive dealing contracts may either facilitate or hinder collusion depending on upstream and downstream competition. We show that when interbrand competition is soft enough, collusion is easier to sustain when producers can offer exclusive dealing contracts to retailers, but when interbrand competition is fierce, exclusive dealing contracts tend to deter collusion. The overall effect of exclusive dealing on consumer welfare is usually negative, but it can surprisingly become positive when inter-brand competition is fierce enough: In that case, exclusive dealing deters collusion among the producers, and final prices are lower in the competitive equilibrium under exclusive dealing contracts than in the collusive equilibrium without exclusion. Note that it is the producer's decision to offer exclusive dealing contracts: in that case, the endogenous choice of vertical contracts hinders welfare-detrimental collusion. Finally, exclusive dealing contracts, which are usually viewed as reducing buyer power, may in some cases be pro-competitive as they reduce the scope for collusion.

References

[1] M.L. Allain and C. Chambolle. Anti-competitive effects of resale-below-cost laws. CREST Working Paper 2007-29, 2007.

[2] J. Crémer and M.H. Riordan. On governing multilateral transactions with bilateral contracts. *RAND Journal of Economics*, 18:436–451, 1987.

[3] J.W. Friedman. A non-cooperative equilibrium for supergames. *Review of Economic Studies*, 38:1–12, 1971.

[4] J.K. Galbraith. Countervailing power. *American Economic Review*, 44:1–6, 1954.

[5] R. Inderst and Shaffer G. The role of buyer power in merger control. *chapter for the ABA Antitrust Section Handbook, Issues in Competition Law and Policy (W.D. Collins, ed., in preparation).*

[6] R. Inderst and Mazzarotto N. Buyer power in distribution. *Draft chapter for the ABA Antitrust Section Handbook, Issues in Competition Law and Policy (W.D. Collins, ed., in preparation).*

[7] R. Inderst and C. Wey. Bargaining, mergers, and technology choice in bilaterally oligopolistic industries. *RAND Journal of Economics*, 34:1–19, 2003.

[8] B. Jullien and P. Rey. Resale price maintenance and collusion. *RAND Journal of Economics*, 2007.

[9] C. Milliou and E. Petrakis. Upstream horizontal mergers, vertical contracts, and bargaining. *International Journal of Industrial Orgainzation*, 25:963–987, 2007.

[10] V. Nocke and L. White. Do vertical mergers facilitate upstream collusion? *American Economic Review*, 97:1321–1339, 2007.

[11] D. O'Brien and G. Shaffer. Vertical control with bilateral contracts. *RAND Journal of Economics*, 23:299–308, 1992.

[12] D. O'Brien and G. Shaffer. Nonlinear supply contrats, exclusive dealing and equilibrium market foreclosure. *Journal of Economics & Management Strategy*, 6:755–785, 1997.

[13] P. Rey and T. Vergé. Bilateral control with vertical contracts. *RAND Journal of Economics*, 35:728–746, 2004.

[14] C.M. Snyder. A dynamic theory of countervailing power. *RAND Journal of Economics*, 27:747–769, 1996.

7 Appendix

7.1 No exclusive dealing contracts

7.1.1 The Competitive Equilibrium.

Consider stage 3 of the game, given a vector of wholesale tariffs $(w_{Ki})_{K\in\{A,B\},i\in\{1,2\}}$. Assume that retailers both decide to sell the two available goods. Retailer $i's$ profit is thus (for $i=1,2$):

$$\Pi_i = (p_{Ai}-w_{Ai})D_{Ai}^{AB,AB}(p_{A1},p_{A2},p_{B1},p_{B2}) + (p_{Bi}-w_{Bi})D_{Bi}^{AB,AB}(p_{A1},p_{A2},p_{B1},p_{B2})$$

In this subgame, the equilibrium prices are (provided that all demands are positive):

$$p_{Ki}^*(w_{Ki},w_{Kj}) = \frac{b(1+b)-2(1+w_{Ki})-bw_{Kj}}{4-b^2}$$

In stage 2, the producers thus choose the following symmetric wholesale tariffs:

$$w_{A1}=w_{A2}=w_{B1}=w_{B2}=\frac{1-a}{2-a}$$

It is straightforward that no retailer has an incentive to stop selling one of the goods (which would increase his demand for the other good, but not enough to be profitable). Besides, we also check that no retailer has an incentive to price one good low enough for his competitor to choose not to sell this good.

Finally, we consider possible deviations from the producers involving a change in market structure. We develop here the analysis of one case, the proofs of the other cases being very similar. We assume that A tries to deviate from the symmetric equilibrium candidate by excluding B's product from one retailer's shelf.

Assume for simplicity that B has set $w_{Bi} = w^*$ for $i = 1, 2$ and that A deviates offering (w^d_{A1}, w^d_{A2}) in order to exclude the product B from retailer 2' shelf. We prove here that to induce the retailer 2 to sell only the product A, it is necessary for A to offer an advantage in input price on product A towards his rival ($w^d_{A1} > w^d_{A2}$) sufficient to annihilate the demand for product B at the retailer 2. The corresponding deviation price is:

$$w^d_{A2} = \frac{(2-b-b^2)(w^*-1+a) + abw^d_{A1}}{a(2-b^2)}$$

At this deviating price, the demand for good $B2$ is zero and the resulting demand functions are:

$$\begin{aligned}
D^{AB,A}_{A1}(p_{A1}, p_{A2}, p_{B1}) &= \frac{(1-a)(1-b)(1-ab) - (1-a^2b^2)p_{A1} + b(1-a^2)p_{A2} + a(1-b^2)p_{B1}}{(1-a^2)(1-b^2)} \\
D^{AB,A}_{B1}(p_{A1}, p_{A2}, p_{B1}) &= \frac{1-a+ap_{A1}-p_{B1}}{1-a^2} \\
D^{AB,A}_{A2}(p_{A1}, p_{A2}, p_{B1}) &= \frac{1-b+bp_{A1}-p_{A2}}{1-b^2}
\end{aligned}$$

Corresponding optimal prices are:

$$\begin{aligned}
p^*_{A1}(AB, A) &= \frac{2(1+w^d_{A1}) + bw^d_{A2} - b(1+b)}{4-b^2} \\
p^*_{B1}(AB, A) &= \frac{(4-b^2)(1+w_{B1}) - ab(2+b-bw^d_{A1} - 2w^d_{A2})}{2(4-b^2)} \\
p^*_{A2}(AB, A) &= \frac{2(1+w^d_{A2}) + bw^d_{A1} - b(1+b)}{4-b^2}
\end{aligned}$$

Replacing in producer A's profit and maximizing towards w^d_{A1}, we obtain the optimal deviation in input prices, $(w^{d*}_{A1}, w^{d*}_{A2})$ and the corresponding deviation profit denoted $\pi^{AB,A}_A$. We first check that retailer 1 has no incentive to stop selling A nor to stop selling B. Comparing $\pi^{AB,A}_A$ to $\pi^{AB,AB}_A$ when A does not deviate, we find that deviation is never profitable when it is possible.

7.1.2 No exclusive dealing contracts: Deviation from the collusive path

Let us assume that A wants to deviate from the collusive path. He then takes as given that B sets the tariff w^C_{IR}. He wants to maximize his individual profit. He can either set the symmetric tariff such that B's product is still sold, that is maximize:

$$\pi_A = \sum_{i=1,2} w_{Ai} D_{Ai}^{AB,AB}(w_{A1}, w_{A2}, w^C, w^C)$$

provided that all quantities are indeed positive. He can also offer asymmetric tariffs such that one of the retailers (say 2) decides not to sell good B, that is maximize:

$$\pi_A = \sum_{i=1,2} w_{Ai} D_{Ai}^{AB,A}(w_{A1}, w_{A2}, w^C) \tag{3}$$

provided that retailer 2 has indeed an incentive not to sell B and all other quantities are positive. Finally, A can maximize his profit when the good B is never sold, provided that both retailers have an incentive not to sell B. In that case, we have:

$$\pi_A = \sum_{i=1,2} w_{Ai} D_{Ai}^{A,A}(w_{A1}, w_{A2})$$

Other possible strategies of A are not relevant.

Producer A thus maximizes the three profits given above, respecting the specific constraints on quantities and on retailers' profits in each case. The first deviation (such that B sells his good to both retailers) leads A to set symmetric wholesale tariffs:

$$w_A 1 = w_A 2 = w_{Dev}^{AB,AB} = \frac{2-a}{4}$$

This deviation is possible (that is all quantities are positive with this deviation) is and only if $a < \sqrt{3} - 1$. Otherwise, only one of the two other deviations can occur.

In order to completely exclude B from the market, A sets wholesale tariffs such that each retailer is indifferent between selling both goods and selling only good A if her rival sells only A. In other words, for the third deviation, A solves the equation:

$$\begin{aligned}(p_{A1}^{A,A} - w_{A1}) D_{A1}^{A,A}(p_{A1}^{A,A}, p_{A2}^{A,A}) &= (p_{A1}^{AB,A} - w_{A1}) D_{A1}^{AB,A}(p_{A1}^{AB,A}, p_{A2}^{AB,A}, p_{B1}^{AB,A}) \\ &+ (p_{B1}^{AB,A} - w^C) D_{B1}^{AB,A}(p_{A1}^{AB,A}, p_{A2}^{AB,A}, p_{B1}^{AB,A})\end{aligned}$$

where $p_{Ki}^{A,A}$ (respectively $p_{Ki}^{AB,A}$) is the equilibrium final price of good Ki when the market structure is (A, A) (resp. (AB, A)). This gives w_{A1}, and similarly we find w_{A2} by solving the parallel equation for retailer 2. Finally, A sets symmetric wholesale tariffs:

$$w_{A1} = w_{A2} = w_{Dev}^{A,A} = 1 - \frac{1}{2a}$$

Finally, we determine the deviation tariffs such that B sells only through one retailer, say 1. A must first make sure that 2 is indifferent between selling only A and selling the two goods, taking as given that 1 sells both goods. In other words, A sets w_{A2} to solve the following equation:

$$\begin{aligned}(p_{A2}^{AB,A} - w_{A2}) D_{A2}^{AB,A}(p_{A1}^{AB,A}, p_{A2}^{AB,A}, p_{B1}^{AB,A}) &= (p_{A2}^{AB,AB} - w_{A2}) D_{A2}^{AB,AB}(p_{A1}^{AB,AB}, p_{A2}^{AB,AB}, p_{B1}^{AB,AB}, p_{B2}^{AB,AB}) \\ &+ (p_{B2}^{AB,AB} - w^C) D_{B2}^{AB,AB}(p_{A1}^{AB,AB}, p_{A2}^{AB,AB}, p_{B1}^{AB,AB}, p_{B2}^{AB,AB})\end{aligned}$$

* We thus find an expression of w_{A2} as a function of w_{A1}:

$$w_{A2}^{Dev}(w_{A1}) = 1 - \frac{ab(1-w_{A1}) + (2-b)(1-w^C)}{2a}$$

We replace w_{A2} by this expression in A's profit function given by equation (3). The optimal wholesale tariff for $A1$ is the tariff that maximizes this profit. Finally, we then have an asymmetric deviation such that:

$$\begin{aligned} w_{A1}^{AB,A} &= \frac{4a(2-a) + ab^2(5a-4a^2-6) - 2b^2(1-a)^2(1+a) + b^4(1-2a)(1-2a^2)}{2a(4-3b^2)(2-a^2b^2) - b^4} \\ w_{A2}^{AB,A} &= 1 - \frac{ab(1-w_{A1}^{AB,A}) + (2-b)(1-w^C)}{2a} \end{aligned}$$

This deviation is only possible if quantities of goods $A1$, $A2$ and $B1$ are positive, which is true only for $a \in [2-\sqrt{2}, \hat{a}]$, with $\hat{a} \approx 0,76$, and $b \in [\underline{b}(a), \overline{b}(a)]$, where $\underline{b}$ and $\overline{b}$ are two functions from $(0,1)$ to $(0,1)$.

Eventually, we compare the profits obtained by A with these three deviations and find a decreasing function of a $\hat{b} : a \mapsto \hat{b}(a)$ and an increasing function of a $\tilde{b} : a \mapsto \tilde{b}(a)$ such that A's optimal deviation leads to the market structure (AB, AB) when:

- $a \leq 0,67$ or
- $a \in (0,67, \sqrt{3}-1]$ and $b < \hat{b}(a)$ or $b > \tilde{b}(a)$

It leads to the market structure (AB, A) when

- $a \in (0,67, \sqrt{3}-1]$ and $b \in [\hat{b}(a), \tilde{b}(a)]$ or,
- $a \in (\sqrt{3}-1, \hat{a}]$ and $b > \frac{2-2a-a^2}{2a^2(1-2a)} + \frac{\sqrt{4-8a+16a^2-44a^3+25a^4+16a^5}}{2a^2(2a-1)}$.

It leads to the market structure (A, A) otherwise.

7.2 Exclusive dealing

7.2.1 The competitive equilibrium

When exclusive dealing contracts are available, no equilibrium holds without exclusivity: if producer B chooses not to offer an exclusivity clause to any retailer, his competitor A is better off offering exclusivity to one retailer as it increases it final demand.

We now consider what A's best reply is when B offers an exclusive dealing contract to one retailer, say 2. In that case, A can offer an exclusive dealing contract to 2 too. 2 is then indifferent between the two contracts and chooses one, whereas 1 does not sell any good. As a consequence, one of the producers' profit is 0. A can also offer no exclusive dealing contract to any retailer, or offer an exclusive dealing contract to 1 only. Note that he is always better off in the second case, since when he offers no exclusive dealing contract, good B is sold in the two outlets and downstream competition is fiercer than it would be if B was sold only in 2. As a consequence, we focus on what happens when A offers an exclusive dealing contract to 1. If each retailer accepts the contract offered, then only two goods are sold on the final market: $A1$ and $B2$. The demand function for good $A1$ is:

$$D_{A1}^{A,B}(p_{A1}, p_{B2}) = \frac{1 - ab - p_{A1} + p_{B2}}{1 - a^2b^2}$$

and the demand function for $B2$ is symmetric. Retailer 1's profit is $\pi_1 = (p_{A1} - w_{A1}) D_{A1}^{A,B}(p_{A1}, p_{B2})$ and 2's profit is symmetric. Each retailer wants to maximize her profit, which leads to the following equilibrium prices in stage 3: $p_{A1}^{A,B}(w_{A1}, w_{A2}) = \frac{2 - a^2b^2 + 2w_{A1} - ab(1 - w_{B2})}{4 - a^2b^2}$, and p_{B2} is symmetric.

In stage 2, producer A maximizes his profit $\pi_A = w_{A1} D_{A1}^{A,B}(w_{A1}, w_{B2})$, and similarly for B. The equilibrium wholesale tariffs are symmetric and equal to $w^{A,B} = \frac{(1-ab)(2+ab)}{4-ab-2a^2b^2}$. No producer has any incentive to deviate from this situation: exclusive dealing is an equilibrium.

As a consequence, the unique competitive equilibrium is such that each producer signs an exclusive dealing contract with one retailer and sets the wholesale tariff $w_{ED}^* = \frac{(1-ab)(2+ab)}{4-ab-2a^2b^2}$. The producers' profit is thus

$$\Pi_{ED}^* = \frac{(2+ab)(2 - 2ab - a^2b^2 + a^3b^3)}{(2-ab)(1+ab)(4-ab-2a^2b^2)^2}$$

7.2.2 The optimal deviation

The deviation studied in the benchmark case are still available here. However, a producer can now deviate by offering an exclusive dealing contract to one or two retailer(s).

Let us assume that A takes as given that B offers no exclusive dealing contract to any retailer (*i.e.* respects the collusive strategy). Does he have an incentive to offer an exclusive dealing contract to one retailer, say 2? If he does so, then B discovers the deviation at the beginning of stage 2. Therefore, B then switches back to teh competitive strategy at stage 2, given that he cannot sell his good through retailer 2. We thus have three goods available: $A1$, $A2$ and $B1$.

At stage 3, retailers maximize their profits and set the following final prices:

$$\begin{aligned}
p_{A1} &= \frac{1-b}{2-b} + \frac{2w_{A1} + bw_{A2}}{4-b^2} \\
p_{A2} &= \frac{1-b}{2-b} + \frac{bw_{A1} + 2w_{A2}}{4-b^2} \\
p_{B1} &= \frac{1+w_{B1}}{2} - \frac{ab(2(1-w_{A1}) + b(1-w_{A2})}{2(4-b^2)}
\end{aligned}$$

In stage 2, producers maximize their individual profits, and we thus find the wholesale tariffs:

$$\begin{aligned} w_{A1} &= \frac{(1-a)(4+a(2+b^2)}{8-2a^2-3a^2b^2} \\ w_{A2} &= \frac{1}{2}\left(1-\frac{ab(2+a)}{8-2a^2-3a^2b^2}\right) \\ w_{B1} &= \frac{(1-a)(4+2a-a^2b^2)}{8-2a^2-3a^2b^2} \end{aligned}$$

This deviation is optimal if retailer accept it and A earns a higher profit with this deviation than with any "classical" deviation (that is with no exclusive dealing contract). this is true for a low enough.