ESTIMATING DYNAMIC PRICING DECISIONS IN OLIGOPOLISTIC MARKETS:

AN EMPIRICAL APPROACH USING MICRO- AND MACRO-LEVEL DATA [1]

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May 10, 2004

(First Draft: June 15, 2002)

[1] We thank the participants at the Cornell Pricing Conference (Aug 2002) and at the Cowles Conference on Economic Dynamics at Yale (Oct 2002), and seminar participants at the University of Toronto (March 2003), Stanford University (May 2003), University of California at Berkeley (Oct 2003), INSEAD (Nov 2003) and University of Texas at Dallas (Nov 2003) for useful feedback on earlier drafts of this paper. In particular, we thank Dan Ackerberg, Steve Berry, Hai Che, Tulin Erdem, Avi Goldfarb, Sanjog Misra, V. Padmanabhan, Ashutosh Prasad, Ram C. Rao, Miklos Sarvary, K. Sudhir, and J. Miguel Villas-Boas for their insightful comments.

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ABSTRACT

It is well documented in the marketing literature that *demand dynamics* usually exist in households' brand choices within categories of inexpensive, frequently consumed packaged goods. In this paper, we study demand dynamics that arise due to *state dependence* effects, i.e., a household's brand choice in the current period depends on the household's previous brand choices. Such effects arise out of households' tendencies to repeat-purchase brands out of habit ("inertia") or switch brands in a desire for variety ("variety-seeking"). When state dependence effects are present, the share of a brand in the current period will be a function of not just the current marketing activities of firms, but also lagged shares of the brand in previous periods. To the extent that state dependence effects characterize the evolution of brands' market shares over time, firms must dynamically compete in prices over time taking into account inter-temporal linkages in demand for their brands. In this study, we model and estimate such dynamic pricing policies of firms.

Since state dependence effects are difficult to identify using aggregate sales data, we propose an estimation technique that combines the use of both micro- and macro-level data. First, we use micro-level scanner panel data on a product category to estimate a household-level brand choice model with state dependence. We then use the estimates of this brand choice model, along with observed inter-purchase times in the product category, to construct a predictive model for market-level brand sales. This brand-sales model serves as an input into a dynamic pricing model of firms, which is based on the idea that firms compete on prices in an infinite-period, repeated dynamic game with discounting. We estimate this dynamic pricing game adopting a recently proposed estimation technique (Berry and Pakes 2000). We benchmark the model against other models under the assumption that firms are not forward-looking ("myopic" model), and that firms assume that there is no state dependence in demand ("static" model). Comparing these three sets of estimates to marginal costs in the relevant industry allows us to understand whether price-competition is dynamic, myopic or static in the industry under study. Using scanner panel data and store scanner data on the cola product category we find that households are strongly inertial in their brand choices. Estimating the proposed dynamic pricing model after taking into account the effects of such inertia generate intuitively reasonable estimates of costs, and hence profit margins, of brands in the product category. Our study is the first in the marketing literature to estimate a fully structural, dynamic pricing model on actual data, and is useful to understand both demand dynamics and the cost structure of competing firms in an oligopoly.

Key-words: Dynamic Pricing , State Dependence, Dynamic Controls, Dynamic Optimization.

1. INTRODUCTION

The importance of pricing within a firm's marketing mix cannot be underemphasized. While pricing its product, a firm must take into account not only its cost structure and its customers' willingness to pay, but also the effects of competition from other firms within the product category (see, for example, Moorthy 1985, Iyer and Seetharaman 2004). A large number of oligopolistic models of pricing have been accordingly developed in normative and empirical domains over the past few decades (for informative reviews, see Rao 1993, Vives 2001, Nagle and Holden 2002). In addition to incorporating the effects of competitive prices, a firm must also account for the effects of *demand dynamics*. One source of demand dynamics is *state dependence* effects, which have been shown to be particularly important for inexpensive, frequently purchased categories of packaged goods. For example, conditional on preferences and marketing activities, the probability that a given household is likely to buy Coke or Pepsi on a visit to the store is partly a function of which cola brand the household bought on its previous purchase. One household may buy Coke on consecutive purchase occasions "out of habit" (even if Pepsi were on sale at the second purchase occasion), while another household may switch from Coke to Pepsi (even if Coke were on sale at the second purchase occasion) just to "try something different." Both households do not simply respond to marketing activities (such as price) in making their brand choices. While the first household exhibits positive state dependence ("inertia") in their brand choices, the second household exhibits negative state dependence ("variety-seeking"). When such state dependence effects characterize household-level brand choices, it is likely that such state dependence will characterize market-level brand shares as well. For example, if Coke experiences an increase in its market share on a given week, the effects of such increased share are likely to persist in the future if households are predominantly inertial in the cola category. Conversely, Pepsi's future market share is likely to benefit from the current increased share of Coke if households are predominantly variety seeking in the category. Given such inter-temporal dependencies in brands' market shares, manufacturers must account for their effects while determining optimal pricing policies for their brands. For example, in the presence of inertia, manufacturers must recognize the long-term benefits of a price promotion, since households that buy their brand on promotion in the current period are likely to "stick around" with the same brand in the future. Conversely, in the presence of variety seeking, manufacturers must recognize the long-term pitfalls of promoting their brands, since households that buy their brand on promotion in the current period are likely to defect to competing brands in the future. If there is a heterogeneous mix of households, some of whom are inertial and others are variety seeking, the pricing implications for manufacturers may be even more complicated. In short, it is important for manufacturers to recognize and respond to the effects of demand dynamics, such as due to state dependence, while making pricing decisions for their brands. In this paper, we estimate an oligopolistic pricing model using empirical data from the cola product category that is characterized by significant demand dynamics.

The existing literature on oligopolistic pricing of brands in packaged goods categories shows a surprising dearth of papers that incorporate the effects of demand dynamics, although there is an extensive empirical literature that documents the effects of state dependence in households' brand choices using scanner panel data (for some recent papers, see Erdem 1996, Seetharaman, Ainslie and Chintagunta 1999, Seetharaman 2003).[3] This paper addresses this gap in the pricing literature. Klemperer (1987a) derives the normative pricing implications of demand inertia in a duopoly using a two-period framework, and shows that the non-cooperative pricing equilibrium in the second period may be the same as the collusive outcome in an otherwise identical market without inertia. However, he also shows that the pricing power that firms gain in the second period leads to vigorous price competition in the first period, which may more than dissipate firms' extra monopolistic returns from the second period. Klemperer (1987b) extends this framework to include rational (i.e. "forward-looking") consumers, and shows that the first-period prices may be less competitive as well because consumers who realize that firms with higher market shares will charge higher prices in the future will be less price-elastic compared to naïve consumers[4]. Beggs and Klemperer (1992) extend these results for the infinite-period case, and show, over a wide range of parametric assumptions, that firms obtain higher prices and profits compared to those in the absence of inertia. Wernerfelt (1991) and Chintagunta and Rao (1996) develop the normative pricing implications of state dependence effects in demand using continuous-time models, and show that in the presence of demand inertia, the firm with the higher baseline preference level charges the higher price in steady state. Kopalle, Assuncao and Rao (1996) develop the normative pricing implications of reference-price effects (i.e., the effects of past prices on current demand) in an oligopoly. Freimer and Horsky (2001) develop an analytical model of duopolistic pricing to demonstrate that if each manufacturer responds to demand dynamics by alternating their brand's prices over time, they will reap greater profits than if they adopted a constant price over time. All of these *analytical* efforts motivate our *empirical*

[3] There is an extensive body of work in marketing on dynamic pricing in *monopolistic* markets (for a recent example, see Krishnan, Bass and Jain 2000).

effort to understand whether observed prices in oligopolistic markets are indeed consistent with dynamic pricing practices of firms. Unlike previous work, we do not limit our attention to a duopoly and/or steady-state behavior. More importantly, we explicitly *estimate* a dynamic, structural model of pricing using the first-order conditions of the dynamic pricing game on scanner data.

Our proposed dynamic pricing model is based on the idea that firms compete on prices in an infinite-horizon, dynamic, Bertrand-Nash game with discounting. Therefore, firms are forward-looking while determining current optimal prices for their brands. The dynamics in the pricing game arise on account of state dependence effects in each firm's demand function, i.e. a firm's demand in a given period is a function of lagged demand of all firms in the product category, taking into account pair-wise inter-product similarities between brands (as in Seetharaman, Feinberg and Chintagunta 2001, and Che, Seetharaman and Sudhir 2003). The parameters of these demand functions are calibrated using *micro-level* scanner panel data on households' inter-purchase times and conditional brand choices. The calibrated demand equation serves as an input for the dynamic pricing model, whose parameters – specifically, marginal costs of firms - are then estimated using *macro-level* scanner data on brand sales and prices, using a recently proposed estimation technique (Berry and Pakes 2000). Our study is unique in combining micro- and macro-level data to empirically calibrate the parameters of the pricing model.

We benchmark our cost estimates both against those obtained under the assumption that firms are not forward-looking and maximize profits for the current period only ("myopic" pricing model), and against those obtained under the (traditional) assumption that firms ignore state dependence in demand ("static" pricing model). Comparing these three sets of estimates to actual marginal costs in the relevant industry allows us to understand whether observed prices in the industry are consistent with dynamic-, myopic- or static- price competition. Another study that investigates dynamic pricing policies of firms in the presence of state dependence effects is Che, Seetharaman and Sudhir (2002). However, we propose a *structural* model of pricing that is based on an infinite-period, dynamic game played between firms, while their study takes a *reduced-form* approach by looking at two-period and three-period pricing models only. [5]

Our dynamic pricing model is in the same spirit as the dynamic choice models of Pakes (1986), Rust (1987), Keane and Wolpin (1994) and Erdem and Keane (1996), in that it requires the solution of a discrete-

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[4] Villas-Boas (2003) also obtains this result using a flexible model of consumer learning about product attributes, which captures the effects of demand inertia, among other effects.

time, stochastic dynamic optimization problem from the agent's standpoint, except that our agent is the firm instead of the consumer. However, our model is more complicated then these dynamic choice models in the following sense: 1. It has *continuous controls*, i.e. prices can take values from a continuum (instead of *discrete controls*, i.e., consumers choosing between a discrete number of actions in the dynamic choice models), which greatly increases the dimensionality of the agent's decision problem, and 2. It has *multiple agents simultaneously making decisions*, i.e. multiple firms solving not only their own dynamic optimization problems but also their best responses to each other's dynamic optimization problems. In terms of these two features, our model is directly in line with the recent work of Pakes (1994), Pakes and McGuire (1994), and Pakes and McGuire (2001). The estimation procedure that we employ, recently developed by Berry and Pakes (2000), imaginatively circumvents having to analytically or numerically solve for the equilibrium of the dynamic pricing game using the *rational expectations* assumption (as will be explained later). We believe that our work is the first in the marketing literature to propose an empirically estimable dynamic pricing model for an oligopoly. [6] Also, the fact that we use both micro- and macro-level data in the empirical estimation is a unique aspect of our work. Unlike dynamic optimization models of households' brand choices, which are *paramorphic* representations of agents' decision-making, we believe that our dynamic optimization model of firms' pricing decisions is an *isomorphic* representation of agents' decision-making. This is because complicated normative pricing computations are now not only possible, given the recent advent of computational power and pricing decision-support software, but also profitable from the firm's standpoint.

The remainder of the paper is organized as follows. In section 2, we develop a firm's dynamic pricing problem as a discrete time, stochastic dynamic optimization problem. Section 3 presents the brand sales model, while section 4 presents the brand choice model. In section 5 we discuss the estimation procedure for the brand choice model using micro-level data. In section 6, we develop the estimation procedure for the dynamic pricing model using macro-level data. Section 7 presents the empirical results from the cola product category. In section 8, we conclude with a brief summary and directions for future research.

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[5] However, their study models the strategic behavior of the retailer while ours does not.

[6] Berry and Pakes (2000) demonstrate the viability of their estimation approach using simulated data. Ours is the first empirical validation of their approach using real-world data.

2. MANUFACTURER PRICING MODEL

Our pricing model is based on the assumption that brands' prices are the outcome of an infinitely repeated Nash-Bertrand pricing game between manufacturers. This dynamic pricing game is one of *complete information*, i.e., each manufacturer's profit function is assumed to be common knowledge among all the manufacturers. The dynamic pricing game also involves *imperfect information* in that at each stage of the game, the manufacturers are assumed to make their pricing decisions simultaneously rather than sequentially. Therefore, the relevant analytical solution concept for this dynamic pricing game is *sub-game perfection*[7]. We ignore the strategic role of the retailer in determining the price of a brand, as in Chintagunta and Rao (1996) and Freimer and Horsky (2001).[8]

Manufacturers are assumed to choose a pricing policy for their brand in order to maximize the net present value of the expected lifetime profits for their brand (also called the *optimal return* function), as shown below.

$$V_j(\Sigma_t) = \sup_{\{p_{jt}\}} E\left[\sum_{s=t}^{\infty} g^{s-t} p_{js} \mid \Sigma_t\right], \tag{1}$$

where $V_j(.)$ stands for the net present value of the expected lifetime profits[9] of brand *j*, g is a discount factor[10], Σ_t is the information set available to manufacturers at time t, and p_{js} is a single-period profit function (also called *single-period return* function) given by

$$p_{js} = (p_{js} - c_{js})Z_{js}, \tag{2}$$

where p_{js} is the price of brand *j* at time *s*, c_{js} is the marginal cost associated with brand *j* at time s, and Z_{js} stands for the sales of brand *j* at time *s*. The objective of the manufacturer, as embodied in equation (1), is to find the optimal pricing policy that has value equal to the optimal return function. Σ_t defines the continuous *state space* of the firms' discrete-time, dynamic optimization problems. What renders this dynamic optimization

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[7] In other words, the backwards induction solution for the dynamic game entails the solution for a real game rather than just the solution for a single-person optimization.

[8] Explicitly modeling and estimating the retailer's separate role within the proposed dynamic pricing game is a non-trivial challenge. We leave this for future research.

[9] In the case of manufacturers with multiple brands, our assumption will be that profits are maximized over the product line.

[10] This is fixed at 0.95 in the empirical estimation.

problem *stochastic* is the fact that the information at time t is defined by lagged sales of all brands in the product category (as will be explained in the next two sections), i.e., $\Sigma_t = \left(Z_{t-1}, Z_{t-2}, \ldots, Z_{t-U}\right)$. The sales vector $Z_t = \left(Z_{1t}, Z_{2t}, \ldots, Z_{Jt}\right)'$ evolves from one period to the next in a probabilistic manner, given the continuous decision variable $p_t = \left(p_{1t} p_{2t} \ldots p_{Jt}\right)'$, according to a *transition law*. We describe this next.

3. BRAND SALES MODEL

Since firms typically deal with aggregate sales (or market shares) of their brands and their competitors' brands, and not individual households' brand choices, they require a predictive model for brand sales in order to make their optimal pricing decisions. We assume a simple function of brand *j*'s sales in period *t*, as shown below.

$$Z_{jt} = q_{jt} * m_t + h_{jt}, \tag{3}$$

where Z_{jt} stands for the sales of brand *j* at time *t*, q_{jt} stands for the expected market share of brand *j* at time *t*, m_t stands for the expected product category sales at time *t*, and h_{jt} is the brand-specific estimation error. We assume that $h_{jt} \sim iid\, \mathrm{N}(0, s_j^2)$.[11]

First, we specify the expected product category sales at time *t*, m_t, as follows.

$$m_t = W_t' a, \tag{4}$$

where W_t is a vector of explanatory variables driving aggregate demand in the product category. In our empirical application this vector includes six indicator variables for the seven stores in the data, and an indicator variable each for holiday (i.e., Thanksgiving and Christmas) and summer. Note that m_t, may be considered as the potential market size, and is assumed to be independent from past and current marketing decisions.

Existence of state dependence effects implies that households' brand choices in the past will affect sales of different brands in the current period. Households' brand choices in the past are reflected in the

[11] For simplification we assume that h_{jt} is *iid* over time. It is easy to generalize it to handle autocorrelations.

aggregate sales for different brands in the past. Their influence on current brand sales comes through q_{jt} in (3), as explained below.

Allowing for unobserved heterogeneity among households, we let p_r stand for the prior probability of a household belonging to segment *r* (i.e., support *r* of the unobserved discrete heterogeneity distribution), and $P_{r(k \to j)t}$ stand for the *conditional* (on the household's location in the heterogeneity distribution) transition probability characterizing a household's transition from buying brand *k* at its previous purchase occasion to buying brand *j* at the current purchase occasion *t, given* that the household belongs to segment *r* (Note: The specification of these transitional probabilities is discussed in detail in the next section). The *unconditional* (on the household's location in the heterogeneity distribution) transition probability of a household that bought brand *k* at its previous purchase occasion for buying brand *j* at purchase occasion *t* is $\sum_{r=1}^{R} p_r * P_{r(k \to j)t}$.

Let q_{ks} stand for the (observed) market share of brand *k* at time *s*. Suppose that there are *J* alternatives. Using the above-mentioned logic to compute *unconditional transition probabilities* for all households who made their most recent category purchase at time *s*, the *unconditional brand choice probabilities* of these households for brand *j* at purchase occasion *t* is equal to $\sum_{k=1}^{J} \left(q_{ks} * \sum_{r=1}^{R} p_r * P_{r(k \to j)t} \right)$.

Next we assume that all households purchase the product at least once in *N* weeks, and a fraction of households, f_u ($u=1, \ldots, N$), buys the product once in every *u* weeks. We assume that f_u is independent of households' brand choices and marketing variables. In other words, we assume that households exogenously decide when to buy the product category.[12] Armed with these assumptions, we can now map the lagged (observed) market shares of brands, i.e., q_{ks} , to the current (predicted) market shares of brands, i.e. q_{jt} as follows: From the standpoint of the current period *t*, all the potential category purchasers can be divided into *N* groups: those who made their most recent category purchase a week ago (f_1), those who made their most recent category purchase two weeks ago (f_2) and so on. Among the f_1 households who bought a week ago, the fraction

[12] We recognize that the exogeneity assumption may a strong one. However, we invoke it to keep the analytical framework for the dynamic pricing problem tractable. Unless there is inventory stockpiling in the data – which does not seem to occur in the soft drinks category that we use in our empirical analysis – we believe that this assumption will not adversely affect our findings about households' brand choice parameters.

that will purchase brand j at week t is equal to $\sum_{k=1}^{J}\left(q_{kt-1} * \sum_{r=1}^{R} p_r * P_{r(k\to j)t}\right)$. Among the f_2 households who bought two weeks ago, the fraction that will purchase brand j at week t is equal to $\sum_{k=1}^{J}\left(q_{kt-2} * \sum_{r=1}^{R} p_r * P_{r(k\to j)t}\right)$. Summing these brand choice probabilities over the N household types yields the following expression, that relates the current market share of brand j to the lagged market shares of *all* brands in the category over the past N weeks.

$$q_{jt} = \sum_{u=1}^{N} f_u * \left\{\sum_{k=1}^{J}\left(q_{kt-u} * \sum_{r=1}^{R} p_r * P_{r(k\to j)t}\right)\right\}, \tag{5}$$

Therefore, the temporal evolution in brand sales (Z_{jt}) in (3) is a function of the temporal evolution in brand market shares (q_{jt}), as shown above, which in turn is a function of the temporal evolution in a household's brand choice probabilities, captured by the transition probabilities $P_{r(k\to j)t}$. We specify these transition probabilities next.

4. BRAND CHOICE MODEL

The Seetharaman, Feinberg and Chintagunta (2001; henceforth SFC) model, an extension of the model of Lattin and McAlister (1985), is a first-order Markov model of brand choice which rests on the following premise: households switching among products in response to their variety-seeking needs are, on a given purchase occasion, more likely to buy a product that is dissimilar to the one purchased on the most recent purchase occasion; households switching among products in response to inertial tendencies are, on a given purchase occasion, more likely to buy a product that is similar to the one purchased on the most recent purchase occasion. The SFC model presumes that, in the absence of state dependence, an individual household belonging to segment r goes about its purchases in multinomial logit fashion, with an *unconditional* probability of purchasing brand i given by $P_{rit} = \exp(X_{it} b_r) / \sum_{k=1}^{k=K} \exp(X_{kt} b_r)$, where $\mathbf{X_{it}}$ is the vector of marketing variables characterizing product i at time t, and $\mathrm{b}_{\mathbf{r}}$ is the corresponding vector of response parameters for households belonging to segment r (assumed to be common across products). However, for a household exhibiting state dependence, products similar to that just purchased are less or more attractive depending on whether the household seeks variety or inertia respectively. This is modeled as

$$P_{r(j\to i)t} = \frac{P_{rit} - V_r S_{rjit}}{1 - V_r \sum_{k=1}^{K} S_{rjkt}} , \qquad (6)$$

where $P_{r(j\to i)t}$ stands for the household's probability of switching from product *j* to product *i* at time *t* (i.e., the household's *conditional* probability of buying product *i* given that product *j* was purchased at the previous purchase occasion), $V_r \in [-\infty, 1]$ denotes the household's state dependence parameter, with positive values representing variety-seeking and negative values representing inertia, and S_{rjit} measures the degree of similarity between products *j* and *i*, as perceived by a consumer in segment *r*, and is given by

$$S_{rjit} = c_{ji}\ min(P_{rjt}, P_{rit}) \qquad (7)$$

where $c_{ji} \in [0,1]$, $c_{ij} = c_{ji}$, and $c_{ii} = 1$.

As per equation (6), the more positive the value of V_r, the greater the degree to which a household seeks variety, with $V_r = 1$ corresponding to maximal variety seeking (achieved, for example, by switching back and forth between a pair of dissimilar products). Similarly, the more negative the value of V_r, the greater the degree to which a household exhibits inertia, with $V_r = -\infty$ corresponding to maximal inertia (achieved, for example, by switching back and forth between a pair of similar products). $V_r = 0$ indicates an absence of state dependence in the household's brand choices and implies that $P_{r(j\to i)t} = P_{rit}$, corresponding to a zero-order multinomial logit model. The parameter c_{ji} is a measure of inter-product similarity between products *j* and *i*, so that $c_{ji} = 0$ indicates a lack of similarity (that is, no shared features), and $c_{ji} = 1$ corresponds to complete similarity (only shared features, in that the smaller-share brand of the pair offers no unique features beyond its larger-share competitor). The appealing feature of the SFC model is that it not only models the effects of inertia and variety seeking on household choice behavior (through the parameter V_r), but also allows such effects to depend on inter-product similarities in a parsimonious manner (through the parameters c_{ji}).

5. ESTIMATION OF BRAND CHOICE MODEL USING MICRO-LEVEL DATA

In this section we will specify the parameters to be estimated using micro-level data. First, the SFC brand choice model is estimated by maximizing the following sample likelihood function.

$$L = \prod_{h=1}^{H} \left[\sum_{r=1}^{R} p_r \prod_{t=1}^{T_h} \prod_{i=1}^{J} \left(\frac{P_{rit} - V_r S_{rj_{h(t-1)}i}}{1 - V_r \sum_{k=1}^{K} S_{rj_{h(t-1)}k}} \right)^{d_{hit}} \right], \tag{8}$$

where δ_{hit} is an indicator variable that takes the value *1* if product *i* is purchased by household *h* at time *t*, $j_{h(t-1)}$ is the product purchased by household *h* on the previous purchase occasion, *H* is the number of households, T_h is the number of purchases made by household *h*, and all other variables are as defined previously (see equations 5 and 6). We address the *initial conditions* problem by assigning the MNL choice probability P_{ri0} for each household's first purchase (at time $t = 0$).

We calibrate the parameters f_u ($u=1, \ldots, N$) using the empirical distribution of inter-purchase times in the product category. We set $N = 5$ in our empirical application involving the cola category. The values of f_u are presented in Table 1.

Plugging the estimated values of $P_{r(k \to j)t}$, p_r and f_u in equation (5), one can predict the temporal evolution of brands' market shares q_t. It is important to note that f_u and p_r cannot be separately identified using macro-level data. This is one reason for our using micro-level scanner panel data to estimate the temporal evolution of brands' market shares[13].

Last, but not the least, we address the issue of potential endogeneity of brands' prices in the brand choice model (see Villas-Boas 2002, Che, Seetharaman and Sudhir 2003). Such endogeneity may arise out of unaccounted (in the model) marketing activities of brands that render the observed price variables to be correlated with the stochastic error structure of the brand choice model. Since our model is not linear in the parameters, the usual techniques for endogeneity correction for linear regression models - such as two-stage least squares, instrumental variable methods etc. – do not apply. However, in order to prevent potential

[13] The other reason is that it may not be possible, in general, to uniquely identify first-order dependencies in brands' market shares without micro-level brand choice data.

endogeneity concerns, we include the following variables as additional controls within the brand choice model (i.e. by including them in the vector X_{jt}).

1. National advertising expenditures of a brand at time t (ADVERTISING_{jt}),
2. Total number of coupons redeemed for a brand at time t (COUPONS_{jt}),
3. Holiday indicator variable (HOLIDAY = 1 for Christmas and Thanksgiving, 0 otherwise),
4. Seasonality indicator variable (SEASON = 1 for summer, 0 otherwise).

Variables 1 and 2 above are different across brands, and are assumed to be associated with a common coefficient across brands in the brand choice model (just as is the case for the price, display and feature variables). Variables 3 and 4, being common across brands, are assumed to be associated with different coefficients for different brands. Having included these additional variables as covariates in the brand choice model, we believe that we would have mitigated, if not eliminated, the effects of endogeneity problems vis-à-vis brands' prices.[14] Since these four variables are included only to proxy for unobserved marketing activities of brands (that are ostensibly equally observed by all households within the market), we restrict their coefficients to be homogeneous across households.

6. ESTIMATION OF PRICING MODEL USING MACRO-LEVEL DATA

Structural parameters in equations (1), (2), (3) and (4), other than the demand parameters, will be estimated using macro-level data. First, given demand parameters for q_{jt} in equation (5), we can obtain consistent estimates $(\hat{a}', \hat{s}_j)$ in equations (3) and (4) directly using store-level brand sales data. Now we will discuss how to estimate the parameters in the value function in equations (1) and (2).

Given that a brand's expected demand in a given period is a function of its own lagged demand and the lagged demand of its competitors' brands (as shown in equation 5), the price chosen by a brand in the current period will affect not only its own current demand and the current demand of its competing brands, but also its own future demand and the future demand of competing brands. Therefore, manufacturers' pricing

[14] If one had access to valid instruments for prices, it would be useful to correct for price endogeneity using the contraction mapping procedure outlined in Berry, Levinsohn and Pakes (1995).

policies have to take the future into account. This is the intuition for our dynamic pricing model, also called a *stochastic accumulation model* (Leonard and Long 1992).

The technique to solve for the optimal pricing policy that yields the firm its optimal return function, as given in equation (1), is to take derivatives of the firm's value function and set them to zero. Since this is difficult to do, we can use *Blackwell's Condition* and recast this problem using a contraction mapping, whose fixed point is the solution to the Bellman equation, as shown below.

$$V_j(\Sigma_t) = \sup_{\{p_{jt}\}} \left\{ E p_{jt}(p_{jt}, \Sigma_t) + g \int V_j(\Sigma') dF(\Sigma' | \Sigma_t, p_{jt}) \right\} \tag{9}$$

where $F(.|.,p)$ is the Markov transition kernel for $\{\Sigma_t\}$ conditional on the action p. As discussed in the previous section, lagged sales of brands from the previous N weeks will affect current and future profits for brand j. Hence, we can write $\Sigma_t = (Z_{t-1}, Z_{t-2}, \ldots, Z_{t-N})$, where $Z_{t-u} = \left(Z_{1t-u}, Z_{2t-u}, \ldots, Z_{Jt-u}\right)'$ is a J-dimensional vector of observed sales of the J brands at time t-u. Under the assumption in equation (3) we may write

$$\begin{aligned} & F(\Sigma' | \Sigma_t, p_t) \\ & = F(Z_t | Z_{t-1}, \ldots, Z_{t-N}, p_t) \\ & = N(m_t \cdot q_t, \Omega), \end{aligned}$$

where Ω is a $J´J$ variance-covariance matrix with diagonal elements equal to $(s_1^2, s_2^2, \ldots, s_J^2)$ and off-diagonal elements equal to zero.

It is well known that looking at the fixed point of this functional equation (9) is a lot easier than taking derivatives of equation (1). However, this is still difficult to do using conventional dynamic optimization techniques. Specifically, for a given set of parameters, one has to compute the solution to the Bellman equation for each firm and then compute the pricing equilibrium across firms (assuming that it is unique). This procedure needs to be repeated until one can locate an optimal set of parameters that "explains" the observed data well. This is very difficult to implement even by existing computational standards (Pakes 1994, Pakes and McGuire 1994, Pakes and McGuire 2001). Further, there may exist a multitude of equilibria in the repeated pricing game, in which case figuring out which of these equilibria will indeed be observed becomes an additional source of difficulty. In order to get around these problems, we adopt a recently proposed estimation technique that is based on the optimality conditions for dynamic controls (Berry and Pakes 2000). This technique relies on two assumptions:

1. *Rational Expectations*, i.e. $\sum_{s=t}^{\infty} g^{s-t} p_{js} = V_j(\Sigma_t) + e_{jt}$, and $E(e_{jt} \mid (\Sigma_t)) = 0$. In other words, the actual lifetime profits can be written as a sum of expected lifetime profits plus a random error whose mean is zero conditional on the information set.

2. *Smoothness*, i.e. $F(\Sigma' \mid \Sigma_t, p)$ has support independent of *p*. Its density function *f(.|.,p)* is differentiable in p for all Σ_t.

Assumption 2 is automatically satisfied if equation (3) holds. However, assumption 1 may be restrictive in that it may not be satisfied in many real-world applications. For example, it does not allow for "unexpected" permanent changes in the market structure or costs of production so that actual future profits of firms are very different from what is expected at time *t*. However, in a business context involving mature brands competing in a stable marketing environment, such an assumption may be quite reasonable. This is the line of reasoning that we employ to justify using the proposed solution technique in our empirical application.

From equation (9) the first-order conditions for each j and t can be written as follows.

$$\begin{aligned} 0 &= \frac{\partial Ep_{jt}(\Sigma_t, p_{jt})}{\partial p_{jt}} + g \cdot \int V_j(\Sigma_{t+1}) \frac{\partial f(\Sigma_{t+1} \mid \Sigma_t, p_{jt})}{\partial p_{jt}} d\Sigma_{t+1} \\ &= \frac{\partial Ep_{jt}(\Sigma_t, p_{jt})}{\partial p_{jt}} + g \cdot \int V_j(\Sigma_{t+1}) \frac{\partial \ln f(\Sigma_{t+1} \mid \Sigma_t, p_{jt})}{\partial p_{jt}} \cdot f(\Sigma_{t+1} \mid \Sigma_t, p_{jt}) d\Sigma_{t+1} \\ &= \frac{\partial Ep_{jt}(\Sigma_t, p_{jt})}{\partial p_{jt}} + g \cdot E[V_j(\Sigma_{t+1}) \frac{\partial \ln f(\Sigma_{t+1} \mid \Sigma_t, p_{jt})}{\partial p_{jt}} \mid \Sigma_t] \end{aligned} \tag{10}$$

Using assumption 1 (i.e., rational expectations), this can be rewritten as follows.

$$\begin{aligned} 0 &= \frac{\partial Ep_{jt}(\Sigma_t, p_{jt})}{\partial p_{jt}} + \sum_{s=t+1}^{\infty} g^{s-t} \cdot p_{js} \cdot \frac{\partial \ln f(\Sigma_{t+1} \mid \Sigma_t, p_{jt})}{\partial p_{jt}} + x_{jt} \\ &= (p_{jt} - c_{jt}) \cdot \frac{\partial q_{jt}(\Sigma_t, p_{jt})}{\partial p_{jt}} \cdot m_t + q_{jt}(\Sigma_t, p_{jt}) \cdot m_t + \sum_{s=t+1}^{\infty} g^{s-t} \cdot p_{js} \cdot \frac{\partial \ln f(\Sigma_{t+1} \mid \Sigma_t, p_{jt})}{\partial p_{jt}} + x_{jt} \end{aligned} \tag{11}$$

where $q_{jt}(\Sigma_t, p_{jt})$ is defined in equation (5) and,

$$x_{jt} = g \cdot E[V_j(\Sigma_{t+1})\frac{\partial \ln f(\Sigma_{t+1}|\Sigma_t, p_{jt})}{\partial p_{jt}}|\Sigma_t] - \sum_{s=t+1}^{\infty} g^{s-t} \cdot p_{js} \cdot \frac{\partial \ln f(\Sigma_{t+1}|\Sigma_t, p_{jt})}{\partial p_{jt}} \tag{12}$$

where $E[x_{jt}|S_t] = 0$ based on the assumption of rational expectations. The second line in (11) is derived from differentiating the profit function p_{jt} in equation (2) with respect to price p_{jt} and using equation (3). As explained above, $\Sigma_{t+1} = Z_t \sim N(m_t \cdot q_t, \Omega)$, with density function

$f(m_t \cdot q_t, \Omega) = (2p)^{-J/2} |\Omega|^{-1/2} \exp(-\frac{1}{2}(Z_t - m_t \cdot q_t)'\Omega^{-1}(Z_t - m_t \cdot q_t))$. Since parameters *(a', s_j)* are estimated separately, what remains to be estimated are the parameters in the marginal cost function c_{jt}.

Indexing all functions in (10) with the parameter vector of interest β, and using x_{jt} to denote the observed data at period t for j, we have

$$E[x(x_{jt}, b)|\Sigma_t] = E[\frac{\partial Ep_{jt}(\Sigma_t, p_{jt}; b)}{\partial p_{jt}} + \sum_{s=t+1}^{\infty} g^{s-t} \cdot p_{js}(b) \cdot \frac{\partial \ln f(\Sigma_{t+1}|\Sigma_t, p_{jt}; b)}{\partial p_{jt}}|\Sigma_t] = 0 \tag{13}$$

This equation underlies the estimation of β. It implies that we can use the market shares of all brands in previous periods as well as current marketing variables as our instruments in model estimation. In this sense, as in Hansen and Singleton (1982), our instruments are derived from the information structure of the model and not arbitrarily assumed.

Let h(Σt;β) denote a rich function of $(\Sigma_t;\beta)$, and T denote the total number of observations. We can form moment conditions based on (12) as follows.

$$G_T(b) = \frac{1}{T}\sum_{j,t} m(x_{jt}, b) \equiv \frac{1}{T}\sum_{j,t} x(x_{jt}, b)h(\Sigma_t, b) \tag{14}$$

An estimator of β can be obtained by minimizing a norm of $G_T(\beta)$. This estimator will be consistent and asymptotically normal.

The key simplification of this estimation technique comes from the *rational expectations* assumption, which says that future prices and shares can be assumed to be generated by a process that is consistent with the firm's beliefs at the time that they are making their pricing decision. The researcher is agnostic about what equilibrium the firm is in, and only relies on the assumption that the firm knows which equilibrium the firm is in.[15] In equation (9), the firm's competitors' equilibrium actions are all incorporated in $V_j(\Sigma')$ and $F(\Sigma' | \Sigma_t, p_{jt})$. This technique avoids the "curse of dimensionality" problem associated with conventional solution techniques for the dynamic pricing game. The computational advantages of this technique are three-fold: first, there is no need to search for a fixed-point for $V(\Sigma_t)$ as required in conventional dynamic optimization algorithms; second, we do not need to numerically compute the equilibrium in the Nash-Bertrand pricing game; third, when multiple equilibria exist, we need to neither subjectively pick an equilibrium nor assign probabilities for each possible equilibrium. Instead, we just let the data tell us which equilibrium is obtained and, by assumption, first-order conditions have to be satisfied for each observation.

While the computational simplification afforded by the Berry and Pakes (2000) technique is significant, the data requirements for implementing the technique are strong. In particular, in order to accurately approximate the value function $V_j(\Sigma')$, a long time-series of observations is required. In addition, the rational expectations assumption may be quite restrictive in some applications. As explained earlier, the rational expectations assumption does not hold under "unexpected" permanent changes in market structure, when realized future sales of firms may be very different from what is expected at time t. Therefore, we note that our empirical technique is applicable to pricing and sales data from mature product categories as long as one has a long time series of observations as well as no major structural changes happen in the industry during the sample period.

We estimate the pricing model using weekly data on prices and market shares of brands at the store-level over a period of 104 weeks, from a set of seven stores in the same metropolitan market in which the IRI scanner panel data was collected. The estimates of the demand function obtained using our micro-level (i.e., household-level) dataset will be appropriate to predict market shares in our macro-level dataset since both datasets pertain to the same geographic market ("single-source data"). We pool the weekly observations across stores to obtain 728 usable observations. We use sales in the first 5 weeks (altogether $5 \times 7 = 35$ observations) as our "initial" sample to construct the expected market share in week 6. We also need to use observed data to construct the value function $V(S_t)$. Because prices and sales in periods following the sample period are unobserved, we cannot compute the value function for the final weeks in the data. Therefore, prices and sales associated with the last 26 weeks (altogether $26 \times 7 = 182$ observations) are used only to construct value

[15] This implies that we do not have to explicitly solve for the sub-game perfect equilibrium of the dynamic pricing

functions for the previous weeks, but are truncated from model estimation. As a result, we only use 511 observations in model estimation. Employing such a "truncation sample" is not unlike constructing an "initialization sample" for distributed lag models, where one needs to observe past data for all periods in the estimation sample, including the first period (Seetharaman 2003a). We also assume the marginal cost function for a brand to be as follows.

$$c_{jt} = c_{0,j} + SEASON_t{}' c_1 + c_2 \cdot YEAR2_t \tag{15}$$

where $SEASON_t$ is a vector of seasonal dummies (one each for Summer, Fall and Winter) for week t, and $YEAR2_t$ is an indicator variable that takes the value 1 if week t belongs to the second year of the sample period. $(c_{0,j}, c_1{}', c_2)$ is a vector of cost parameters to be estimated.

Next, we compare the proposed dynamic pricing model, given in equation (11), with two other pricing models, which will serve as benchmark models. One model, which we call the *static model*, ignores state dependence in the demand function, assumes market shares of brands to take the multinomial logit functional form, and therefore assumes one-period profit maximization for manufacturers. In other words, manufacturer j's pricing problem is now given by.

$$\max_{\{p_{jt}\}} E[\boldsymbol{p}_{jt} \mid \Sigma_t] \tag{16}$$

From (11) we can derive the first-order condition as follows.

$$\begin{aligned} 0 &= \frac{\partial E\boldsymbol{p}_{jt}(\Sigma_t, p_{jt})}{\partial p_{jt}} + \boldsymbol{x}_{jt} \\ &= (p_{jt} - c_j) \cdot \frac{\partial \boldsymbol{q}_{jt}(\Sigma_t, p_{jt})}{\partial p_{jt}} \cdot m_t + \boldsymbol{q}_{jt}(\Sigma_t, p_{jt}) \cdot m_t + \boldsymbol{x}_{jt} \end{aligned} \tag{17}$$

where $\theta_{jt}(.)$ is the expected market share of brand j at time t and given by the (heterogeneous) MNL brand choice probability.

Another pricing model, which we may call the *myopic model*, does not ignore state dependence in the demand function, but assumes that manufacturers maximize current-period profits only. The objective function and first-order condition for this model are still given by equations (16) and (17) respectively, except that $\boldsymbol{q}_{jt}(\Sigma_t, p_{jt})$ now accounts for state dependence, and looks as in equation (5).

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game, far less decide which of multiple equilibria is the relevant one to use for estimation purposes!

Assuming that all other parameters in the demand function remain the same, the absolute magnitude of $\frac{\partial q_{jt}(\Sigma_t, p_{jt})}{\partial p_{jt}}$ will decrease in the presence of inertia and increase in the presence of variety-seeking. This implies that optimal prices under the myopic pricing model, compared to those under the static pricing model, will be higher in the presence of inertia and lower in the presence of variety-seeking. The intuition for this result, which is consistent with Klemperer (1987a,b), is that if households are inertial (variety-seeking), they will be less (more) price responsive which makes it more profitable for firms to charge a higher (lower) price.

Compared to equation (17), there is an additional term given by $-\sum_{s=t+1}^{\infty} g^{s-t} \cdot p_{js} \cdot \frac{\partial \ln f(\Sigma_{t+1} | \Sigma_t, p_{jt})}{\partial p_{jt}}$, in equation (11), i.e. the proposed dynamic model of pricing. This term represents the effects of current prices on future market shares, and hence future profits. If firms take the effects of current prices on future profits into account, they will tend to charge lower prices, which will decrease their price-cost margins. This tendency is more significant in the presence of inertia because the effects of current prices on future profits will be much stronger than in the case of variety seeking. This implies that optimal prices under the proposed dynamic pricing model, compared to those under the myopic pricing model, will be lower in the presence of inertia and higher in the presence of variety-seeking. This is consistent with the pricing implications derived by Beggs and Klemperer (1992).

7. EMPIRICAL RESULTS

We employ IRI's scanner panel database on household purchases in a metropolitan market in a large US city. For our analysis, we pick the cola product category. [16] The dataset covers a period of two years from June 1991 to June 1993 and contains shopping visit information on 494 panelists across seven different stores (two in an urban and five in a suburban market). Choosing households that bought at least twice in the product category (since we need to exclude the first purchase of each household while estimating the brand choice model, in order to compute households' transition probabilities starting from their second purchase) yields 484

[16] We also estimate the proposed model on a second product category: coffee. Since the substantive findings obtained from the second product category are identical to those obtained from cola, we have suppressed these results in the paper. These results are available from the authors.

households. The dataset contains information on marketing variables – price, in-store displays, newspaper feature advertisements and coupon activity – at the SKU-level for each store/week. The product category contains four brands: Coke, Pepsi, Royal Crown and Private Label. Descriptive statistics, based on the micro-level scanner panel data, are given in Table 1. We supplement this micro-level dataset with LNA data on quarterly national advertising expenditures for the four brands under study. For all weeks within a given quarter, national advertising for a brand is assumed to be equal (given by the quarterly advertising expenditure for the brand divided by twelve weeks).

(Insert Table 1)

Coke is the most expensive brand (77 cents per regular package size), while the Private Label is the least expensive brand (54 cents per regular package size). Pepsi enjoys much higher display activity (32%) and feature activity (40%) than the other brands, and also has the highest share in the category (47.9%) over the period of study. Descriptive statistics, based on the macro-level store scanner data, are given in Table 2.

(Insert Table 2)

There is good agreement between Tables 1 and 2, especially in terms of brands' market shares, which is satisfactory from the standpoint of using the micro-level parameter estimates with the macro-level data in the estimation of the supply-side pricing model. During the early nineties, i.e. the time period of our study, colas accounted for 60 – 70 % of carbonated beverage sales in the US, with 35 % of their national distribution occurring through food stores (Yoffie 2002). Differences across the five suburban stores in cola category sales are displayed in Figure 1. Temporal price patterns for the four cola brands at one of the five suburban stores are displayed in Figure 2.

(Insert Figures 1-2)

It is clear from Figure 1 that the baseline sales for colas are different across the five stores in the dataset, which justifies our employing store-specific intercepts while modeling cola sales at the store-level. It is hard to discern any obvious patterns of price competition between cola brands simply by visually inspecting Figure 2 (and the same is true of the price patterns in the remaining stores), which further justifies the explicit estimation of a pricing model using the macro-level data. The estimated model parameters, along with their standard errors, for the brand choice model based on micro-level data, are given in Tables 3-5.

(Insert Tables 3-5)

In terms of intrinsic brand preferences, the Private Label is the least preferred brand, which is consistent with the observed market shares in Tables 1 and 2. The magnitude of the estimated price parameters are larger under the MNL model than under the SFC model, which suggests that households' responsiveness to prices will be overstated if one does not model the effects of state dependence. The opposite is true of the estimated display and feature parameters, i.e. the magnitudes of the estimated display and feature parameters are smaller under the MNL model than under the SFC model, which suggests that households' responsiveness to displays and features will be understated if one does not model the effects of state dependence. There is a significant degree of inertia uncovered in the product category, with the estimated state dependence coefficients (*V*) being –2.79, -4.93, -0.69 and -2.11 for the four supports of the heterogeneity distribution. [17] In terms of the estimated similarity parameters, Coke and Pepsi are perceived to be much more similar, along unobserved product attributes, than other pairs of cola brands (with a similarity coefficient of 0.6624). Conversely, Coke and the Private Label are perceived as being least similar to each other (with a similarity coefficient of 0.0032).[18] National advertising is estimated to have a positive effect on a household's brand choice probability, with the estimated magnitude of the effect being smaller when state dependence effects are ignored (0.67 instead of 0.84).

Under the state dependence model, the coefficient associated with summer is estimated to be negative for Pepsi, i.e. a household's probability of choosing Pepsi decreases during summer. To the extent that the holiday and seasonality indicators are included in the model to account for the effects of unobserved marketing activities of brands, we interpret this estimate as a proxy for unobserved variables associated with Pepsi during summer that decreased its brand choice probability among households in the market. By and large, the estimated coefficients associated with the holiday and seasonality indicator variables are insignificant, which indicates that potential endogeneity of brands' price variables may not be a big concern in our data (unless, of course, that these indicator variables are poor proxies for other unobserved drivers of brand choices). In terms of model fit (using the Schwarz Bayesian Criterion, i.e. SBC), the proposed state dependence model of Seetharaman, Feinberg and Chintagunta (SFC) is observed to clearly outperform the Multinomial Logit (MNL) model, which indicates that state dependence effects are significant in the cola category. These results are consistent with previous studies that have documented strong effects of inertia in consumer choices among

17 The four-support heterogeneous solution was found to be optimal in terms of SBC fit.

packaged goods (see, for example, Erdem 1996, Seetharaman, Ainslie and Chintagunta 1999). Next, we present the results of estimating the brand sales model using macro-level data.

(Insert Table 6)

From Table 6, we can see that the baseline store and store 1 have larger baseline category sales of colas compared to the remaining five stores. Further, cola sales are observed to increase during holidays and summer, which makes intuitive sense. Next, we present the results of the estimating supply-side pricing model using macro-level data in Tables 7 and 8.

(Insert Tables 7-8)

Upon inspecting the supply-side estimates from the heterogeneous model (see Table 8), we find the estimated costs, and therefore margins, to be plausible. For example, the national brands – Pepsi, Coke and Royal Crown – have higher estimated costs [19] (~ 45 to 54 cents) than the private label (~ 37 cents). This agrees remarkably well with the cost estimate of 47 cents (that includes cost of sales of 12 cents, selling and delivery costs of 1 cent, advertising and marketing costs of 28 cents, general and administration costs of 6 cents) on a selling price of 71 cents for a typical brand in the cola category, as discussed in Yoffie (2002). The private label is estimated to pass through a more-than-commensurate amount of the cost-savings in the form of a reduced price to the marketplace that renders its margins lower (~ 7 %) than the national brands' (~ 19 – 20 %). However, since the estimated margins are for the entire channel (i.e., concentrate producer plus bottler plus retailer), it is not clear if the *retailer's margin* is lower for the private label compared to the national brand. If the private label were a store brand, for example, it is possible that the retailer's margin on the brand is actually *higher* than that on the national brands. Given our data constraints, investigating whether this is indeed the case is beyond the scope of this paper. However, the institutional realism of our cost estimates lends face validity to our proposed estimation procedure.

Compared to the estimated costs in the dynamic pricing model, the estimated costs from the myopic model and from the static model are severely understated for all brands. In other words, the margins are severely overstated. For example, the estimated price-cost margin for the private label is only 7.2 % under the

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[18] As in Multidimensional Scaling (MDS), we assume that *brand perceptions*, reflected in the similarity parameters, are constant across consumers. It is difficult to separately identify differences in the similarity parameters from differences in the state dependence parameter across consumers.

[19] Note that costs refer not only to manufacturing costs but also include advertising, sales promotions, distribution and all relevant economic costs incurred in the supply chain.

dynamic model, but 99.6 % under the myopic model, and 98.8 % under the static model! Given the popular wisdom in the grocery industry, it appears that the cost estimates from the dynamic pricing model are more believable than those from the myopic and static models.

The intuition for lower estimated costs under the static model is as follows: if firms are not forward-looking and respond to zero-order market shares of brands, optimal prices will be higher than those in the presence of inertia (because in the latter case, firms have an incentive to lower price in order to benefit from the effects of inertia in the future). Since the observed prices used in the estimation are the same under both the dynamic and static specifications, the empirical consequence of the higher optimal price under the static model is a lower estimated cost. This is what is observed in Tables 7 and 8. Comparing Tables 7 and 8, we find that the supply-side estimates for cola under the dynamic pricing model are quite robust to whether or not we incorporate unobserved heterogeneity in the parameters of the demand-side model. This is probably because market-level price elasticities for cola brands are reasonably well estimated even if one ignores unobserved heterogeneity across households in terms of their responsiveness to prices (which may not be true for all product categories in general [20]).

8. CONCLUSIONS

In this paper, we propose and estimate a dynamic pricing model in which manufacturers' current prices have an impact not only on their current demand, but also on the future demand for all brands. Such temporal dynamics arise on account of state dependence in households' brand choices in the product category. We estimate a dynamic brand choice model, recently proposed by Seetharaman, Feinberg and Chintagunta (2001), using micro-level scanner panel data, and then use the estimated parameters of this model to build a predictive model of aggregate demand for brands, and finally estimate the parameters of a dynamic pricing model using macro-level store scanner data from stores in the same geographic market. Demand-side estimation proceeds using likelihood-based techniques. The state dependence model is observed to fit observed brand choices better than the multinomial logit model. Supply-side estimation entails the solution of dynamic optimization problems of firms, as well as the solution of equilibrium pricing strategies between firms. We use

[20] In fact, in the coffee category, we find substantial differences in the estimated price elasticities between the homogeneous and heterogeneous demand specifications.

an estimation technique, recently proposed by Berry and Pakes (2000), to handle the computational difficulties associated with the estimation of the supply-side equation. We find that the observed data are consistent with the proposed dynamic pricing model in terms of the reasonableness of the estimated cost parameters. The cost parameters recovered under alternative (myopic and static) pricing models look unrealistically low. The proposed technique will be of value not only to firms (possibly new entrants) hoping to understand marginal costs of competitors in their industry, but also to regulators who want to characterize the cost-structure in industries involving frequently purchased products.

There are several interesting areas for future research. First, we assume that the micro-level scanner panel data contain a representative sample of the households that together constitute the macro-level store data. The predictive demand model used in our supply-side analysis is predicated on this assumption. Explicitly investigating whether panel data provide representative inferences about store-level effects will be an interesting area of future research (see Gupta, Chintagunta, Kaul and Wittink 1996). Second, we pool aggregate shares and prices across the seven stores in our dataset to estimate the supply-side pricing equations. Explicitly investigating differences across stores and/or retail competition would be an interesting area of future research, although this would require sufficiently long time series of observations for each store in the market to enable the identification of store-specific parameters (for an structural model of store competition, see Villas-Boas 2001). Third, we use a probabilistic model of brand choices in our study. Testing whether our inferences are robust to a random utility model of brand choices would be an interesting avenue for future research. Given that a recent paper by Seetharaman (2003b) documents that probabilistic and random utility models of state dependence yield very similar inferences about the estimated price elasticities and steady-state market shares of brands, we believe that an alternative model of brand choices, based on random utility, would yield insights that are similar to those obtained in this study. Fourth, we model state dependence as a *first-order* phenomenon. Investigating whether state dependence is of higher-order, and if so, its consequences on firms' pricing decisions, is an issue that warrants study. Fifth, we do not endogenously account for the effects of the outside good in the estimation of our demand model. In other words, even though we account for the effects of heterogeneous inter-purchase times across households, we do not allow such inter-purchase times to be determined by brand-specific marketing variables (in order to keep the supply-side model tractable). Explicitly relaxing this assumption is an important avenue for future research, especially since that would enable the testing of alternative pricing structures - such as collusive pricing – using a menu approach as in

Che, Seetharaman and Sudhir (2003). Sixth, one can demonstrate the managerial implications of the proposed estimation approach by computing predicted optimal competitive pricing policies of oligopolistic firms under alternative assumptions about market structure, using the recently proposed technique of Pakes and McGuire (2001). Seventh, in our proposed framework it would be useful to investigate the consequences of allowing consumers to be rational in the sense that they understand firms' incentives to price dynamically, and therefore anticipate future prices of brands (as in Beggs and Klemperer 1992).

The primary contribution of this paper is in introducing an empirical methodology that enables the estimation of dynamic pricing games using store-level scanner data. Although there exists an emerging literature on the estimation of dynamic optimization models of demand (see Erdem and Keane 1996, for example), estimating dynamic optimization models of supply has remained a neglected area of research inquiry on account of the computational challenges involved. This paper is a first effort at addressing this gap. It is important to extend the proposed framework to estimate more fully specified pricing models that incorporate the effects of the outside good, the strategic role of the retailer etc. Such models can be used to distinguish between alternative explanations for the observed prices of brands in our datasets. In fact, our dynamic pricing framework can serve as a useful blueprint to enable re-investigations of research questions pertaining to channel issues that have been raised in a static framework within the recently emerging *New Empirical Industrial Organization* (NEIO) paradigm (for a review of this literature, see Kadiyali, Sudhir and Rao 2001, Dube et al. 2002).

TABLE 1: Descriptive Statistics from Micro-Level Scanner Panel Data

A. Average Values of Marketing Variables and Market Shares of Brands over Study Period

Number of households = 484
Number of purchases = 8454

Brand	Price ($/unit)	Display	Feature	Share
Private Label	0.5397	0.10	0.09	10.5 %
Pepsi	0.6899	0.32	0.40	47.9 %
Coke	0.7660	0.18	0.26	26.1 %
Royal Crown	0.7309	0.13	0.14	15.5 %

B. Frequency Distribution of Average Inter-Purchase Times across Households

Number of households = 484
Number of purchases = 8454

Average Inter-Purchase Time in Weeks	Percentage of Households
1	31.2
2	17.2
3	10.5
4	8.2
5	6.1
≥ 6	26.8

TABLE 2: Descriptive Statistics from Macro-Level Data

A. Average Values of Marketing Variables and Market Shares of Brands over Study Period

Number of stores = 7, Number of weeks = 104

Brand	Price ($/unit)	Display	Feature	Share
Private Label	0.4329	0.19	0.12	4.5 %
Pepsi	0.7482	0.36	0.27	55.0 %
Coke	0.7130	0.39	0.32	26.2 %
Royal Crown	0.6147	0.32	0.34	14.4 %

TABLE 3: Parameter estimates of Homogeneous Brand Choice Models Estimated on Micro-Level Data

MNL: Multinomial Logit, SFC: Seetharaman, Feinberg and Chintagunta (2001)

Parameter	MNL	SFC
V	Na	-5.0537
$C_{PL\text{-}Pepsi}$	Na	0.0498
$C_{PL\text{-}Coke}$	Na	0.0074
$C_{PL\text{-}RC}$	Na	0.0025
$C_{Pepsi\text{-}Coke}$	Na	0.2681
$C_{Pepsi\text{-}RC}$	Na	0.1591
$C_{Coke\text{-}RC}$	Na	0.0929

TABLE 3: Parameter estimates of Homogeneous Brand Choice Models Estimated on Micro-Level Data (contd…)
(Standard Errors in Parentheses, *ns* means not significant at the 0.05 significance level)

MNL: Multinomial Logit, SFC: Seetharaman, Feinberg and Chintagunta (2001)

Parameter	MNL	SFC
a_{PL}	-1.0728 (0.0572)	-0.6886 (0.0810)
a_{Pepsi}	0.8079 (0.0398)	0.6678 (0.2341)
a_{Coke}	0.3275 (0.0626)	0.1681 (0.3261) ^{ns}
a_{RCCola}	0	0
Price	-4.0596 (0.1176)	-3.9462 (0.1664)
Display	0.8599 (0.0502)	1.1730 (0.0670)
Feature	0.5678 (0.0481)	0.6784 (0.0645)
Advertising	0.7332 (0.1560)	0.6707 (0.5895) ^{ns}
Coupons	-0.0465 (0.0181)	-0.0191 (0.0227) ^{ns}
Holiday_{PL}	-0.2377 (0.2588) ^{ns}	-0.2701 (0.2882) ^{ns}
Summer_{PL}	0.1798 (0.0874)	0.0965 (0.0922) ^{ns}
Holiday_{Pepsi}	0.1176 (0.2380) ^{ns}	0.1059 (0.2924) ^{ns}
Summer_{Pepsi}	-0.1433 (0.0318)	-0.1324 (0.0896) ^{ns}
Holiday_{Coke}	0.2423 (0.2522) ^{ns}	0.3039 (0.2589) ^{ns}
Summer_{Coke}	-0.0351 (0.0253) ^{ns}	0.0692 (0.1314) ^{ns}
Holiday_{RCCola}	-0.1232 (0.3645) ^{ns}	-0.1418 (0.3330) ^{ns}
Summer_{RCCola}	-0.0014 (0.0186) ^{ns}	-0.0129 (0.0050)
Log-Lik.	-8195	-6616
SBC	16534	13439

TABLE 4: Parameter estimates for Heterogeneous MNL Model Estimated on Micro-Level Data
(Standard Errors in Parentheses, *ns* means not significant at the 0.05 significance level)

Parameter	Support 1	Support 2	Support 3	Support 4
a_{PL}	-3.80 (0.19)	-1.86 (0.41)	-1.76 (0.14)	1.33 (0.13)
a_{Pepsi}	-0.78 (0.12)	1.32 (0.24)	2.03 (0.11)	0.34 (0.16)
a_{Coke}	-0.96 (0.13)	2.83 (0.25)	0.49 (0.13)	0.79 (0.16)
a_{RCCola}	0	0	0	0
Price	-5.68 (0.40)	-2.92 (0.44)	-4.03 (0.24)	-3.92 (0.33)
Display	1.06 (0.13)	1.11 (0.17)	0.96 (0.11)	0.69 (0.14)
Feature	0.25 (0.13)	-0.09 (0.07) ^{ns}	0.53 (0.10)	1.36 (0.13)
Advertising	0.67 (0.22)	Same	Same	Same
Coupons	-0.01 (0.02) ^{ns}	Same	Same	Same
$Holiday_{PL}$	-0.30 (0.15)	Same	Same	Same
$Summer_{PL}$	0.19 (0.13) ^{ns}	Same	Same	Same
$Holiday_{Pepsi}$	0.14 (0.12) ^{ns}	Same	Same	Same
$Summer_{Pepsi}$	-0.13 (0.09) ^{ns}	Same	Same	Same
$Holiday_{Coke}$	0.31 (0.10)	Same	Same	Same
$Summer_{Coke}$	-0.01 (0.03) ^{ns}	Same	Same	Same
$Holiday_{RCCola}$	-0.15 (0.18) ^{ns}	Same	Same	Same
$Summer_{RCCola}$	-0.05 (0.11) ^{ns}	Same	Same	Same
Support prob.	0.21	0.13	0.44	0.22
Log-Lik.	-6335			
SBC	13004			

TABLE 5: Parameter estimates for Heterogeneous SFC Model Estimated on Micro-Level Data
(Standard Errors in Parentheses, *ns* means not significant at the 0.05 significance level)

Parameter	Support 1	Support 2	Support 3	Support 4
a_{PL}	-1.25 (0.35)	-1.20 (0.12)	-3.90 (0.25)	0.66 (0.11)
a_{Pepsi}	0.70 (0.15)	2.22 (0.15)	-0.92 (0.15)	0.25 (0.12)
a_{Coke}	2.17 (0.01)	0.28 (0.15)	-1.69 (0.17)	0.68 (0.14)
a_{RCCola}	0	0	0	0
Price	-2.58 (0.53)	-4.09 (0.31)	-5.02 (0.51)	-5.42 (0.36)
Display	1.46 (0.22)	1.35 (0.14)	1.26 (0.15)	1.01 (0.13)
Feature	0.06 (0.09) ^{ns}	0.68 (0.12)	0.18 (0.03)	1.18 (0.13)
Advertising	0.84 (0.25)	Same	Same	Same
Coupons	-0.01 (0.02) ^{ns}	Same	Same	Same
$Holiday_{PL}$	-0.32 (0.48) ^{ns}	Same	Same	Same
$Summer_{PL}$	0.11 (0.12) ^{ns}	Same	Same	Same
$Holiday_{Pepsi}$	0.16 (0.45) ^{ns}	Same	Same	Same
$Summer_{Pepsi}$	-0.17 (0.04)	Same	Same	Same
$Holiday_{Coke}$	0.27 (0.47) ^{ns}	Same	Same	Same
$Summer_{Coke}$	-0.04 (0.13) ^{ns}	Same	Same	Same
$Holiday_{RCCola}$	-0.11 (0.38) ^{ns}	Same	Same	Same
$Summer_{RCCola}$	0.01 (0.05) ^{ns}	Same	Same	Same
Support prob.	0.13	0.40	0.13	0.34
Log-Lik.	-5736			
SBC	11896			

TABLE 5: Parameter estimates of Heterogeneous SFC Model (contd…)

Parameter	Support 1	Support 2	Support 3	Support 4
V	-2.79	-4.93	-0.69	-2.11
$C_{PL\text{-}Pepsi}$	0.3395	Same	Same	Same
$C_{PL\text{-}Coke}$	0.0032	Same	Same	Same
$C_{PL\text{-}RC}$	0.0378	Same	Same	Same
$C_{Pepsi\text{-}Coke}$	0.6624	Same	Same	Same
$C_{Pepsi\text{-}RC}$	0.4342	Same	Same	Same
$C_{Coke\text{-}RC}$	0.1610	Same	Same	Same

TABLE 6: Parameter estimates of Brand Sales Model Estimated on Macro-Level Data

Parameter	Estimate
Intercept	21802
Store 1	26021
Store 2	-9135
Store 3	-19334
Store 4	-19371
Store 5	-19901
Store 6	-7012
Holiday	3896
Summer	2570

TABLE 7: Supply-Side Cost Estimates from Macro-Level Data (Based on the Homogeneous Demand Model)

A. Dynamic Model (i.e. State Dependent Demand Model, Forward-Looking Pricing Model)

Brand	Estimated Cost (c_j) & Std. Error	Price (P_j)	Margin
Private Label	$ 0.3869 ($ 0.0124)	$ 0.4367	5.5 %
Pepsi	$ 0.5365 ($ 0.0060)	$ 0.7270	20.2 %
Coke	$ 0.5240 ($ 0.0060)	$ 0.6907	19.2 %
Royal Crown	$ 0.4583 ($ 0.0076)	$ 0.6111	19.1 %
Summer	$ 0.0255 ($ 0.0083)	Na	Na
Fall	$ 0.0126 ($ 0.0091)	Na	Na
Winter	$ 0.0098 ($ 0.0131)	Na	Na
Year 2	-$0.0050 ($ 0.0054)	Na	Na

B. Myopic Model (i.e. State Dependent Demand Model, Current-Period Pricing Model)

Brand	Estimated Cost (c_j) & Std. Error	Price (P_j)	Margin
Private Label	$ 0.0000 ($ 0.0289)	$ 0.4367	99.9 %
Pepsi	$ 0.0128 ($ 0.0205)	$ 0.7270	98.1 %
Coke	$ 0.0076 ($ 0.0201)	$ 0.6907	98.9 %
Royal Crown	$ 0.0172 ($ 0.0193)	$ 0.6111	97.1 %
Summer	$ 0.0000 ($ 0.0223)	Na	Na
Fall	$ 0.0000 ($ 0.0248)	Na	Na
Winter	$ 0.0000 ($ 0.0235)	Na	Na
Year 2	$0.0000 ($ 0.0143)	Na	Na

TABLE 7 (contd…)

C. Static Model (i.e. Multinomial Logit Demand Model, Current-Period Pricing Model)

Brand	**Estimated Cost (c_j) & Std. Error**	**Price (P_j)**	**Margin**
Private Label	$ 0.0484 ($ 0.0146)	$ 0.4356	88.7 %
Pepsi	$ 0.2117 ($ 0.0083)	$ 0.7229	69.3 %
Coke	$ 0.2602 ($ 0.0080)	$ 0.6924	61.0 %
Royal Crown	$ 0.1732 ($ 0.0098)	$ 0.6005	69.9 %
Summer	-$ 0.0071 ($ 0.0056)	Na	Na
Fall	$ 0.0253 ($ 0.0074)	Na	Na
Winter	$ 0.0223 ($ 0.0062)	Na	Na
Year 2	-$0.0143 ($ 0.0046)	Na	Na

TABLE 8: Supply-Side Cost Estimates From Macro-Level Data (Based on the Heterogeneous Demand Model)

A. Dynamic Model (i.e. State Dependent Demand Model, Forward-Looking Pricing Model)

Brand	**Estimated Cost (c_j) & Std. Error**	**Price (P_j)**	**Margin**
Private Label	$ 0.3727 ($ 0.0158)	$ 0.4367	7.2 %
Pepsi	$ 0.5391 ($ 0.0132)	$ 0.7270	18.8 %
Coke	$ 0.5211 ($ 0.0128)	$ 0.6907	18.6 %
Royal Crown	$ 0.4477 ($ 0.0134)	$ 0.6111	19.7 %
Summer	$ 0.0296 ($ 0.0139)	Na	Na
Fall	$ 0.0146 ($ 0.0135)	Na	Na
Winter	$ 0.0300 ($ 0.0151)	Na	Na
Year 2	-$0.0037 ($ 0.0053)	Na	Na

TABLE 8 (contd…)

B. Myopic Model (i.e. State Dependent Demand Model, Current-Period Pricing Model)

Brand	**Estimated Cost (c_j) & Std. Error**	**Price (P_j)**	**Margin**
Private Label	$ 0.0000 ($ 0.0205)	$ 0.4367	99.6 %
Pepsi	$ 0.0000 ($ 0.0197)	$ 0.7270	99.8 %
Coke	$ 0.4378 ($ 0.0211)	$ 0.6907	33.9 %
Royal Crown	$ 0.2929 ($ 0.0187)	$ 0.6111	49.5 %
Summer	$ 0.0041 ($ 0.0206)	Na	Na
Fall	$ 0.0000 ($ 0.0212)	Na	Na
Winter	$ 0.0000 ($ 0.0199)	Na	Na
Year 2	$0.0000 ($ 0.0130)	Na	Na

C. Static Model (i.e. Multinomial Logit Demand Model, Current-Period Pricing Model)

Brand	**Estimated Cost (c_j) & Std. Error**	**Price (P_j)**	**Margin**
Private Label	$ 0.0060 ($ 0.0209)	$ 0.4356	98.8 %
Pepsi	$ 0.1095 ($ 0.0099)	$ 0.7229	84.2 %
Coke	$ 0.1894 ($ 0.0084)	$ 0.6924	71.7 %
Royal Crown	$ 0.1552 ($ 0.0111)	$ 0.6005	73.1 %
Summer	$ 0.0000 ($ 0.0054)	Na	Na
Fall	$ 0.0142 ($ 0.0069)	Na	Na
Winter	$ 0.0000 ($ 0.0057)	Na	Na
Year 2	-$0.0060 ($ 0.0040)	Na	Na

FIGURE 1: STORE-LEVEL CATEGORY SALES FOR COLAS AT SUBURBAN STORES

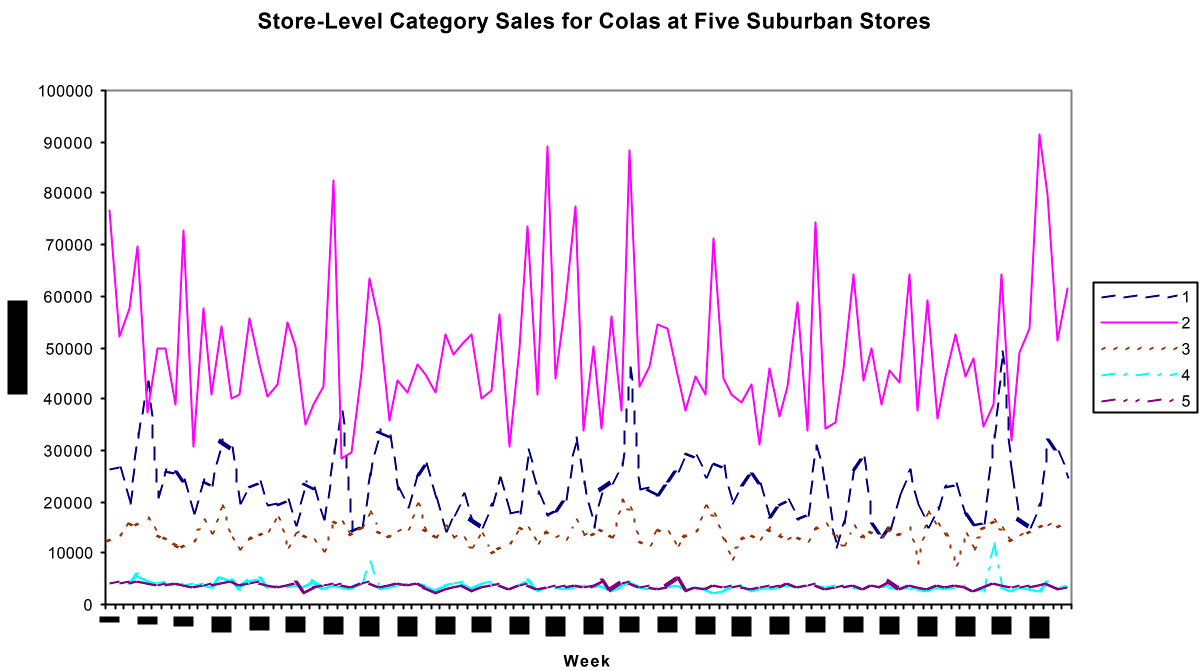
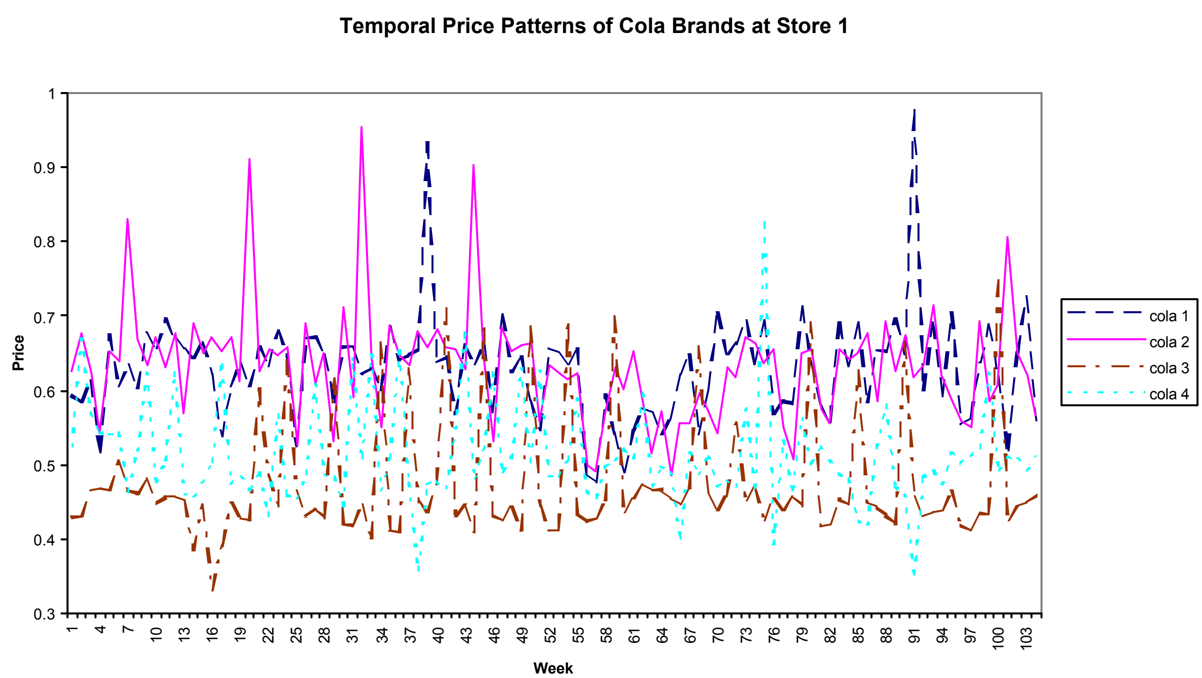


FIGURE 2: TEMPORAL PRICE PATTERNS OF COLA BRANDS AT STORE 1



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