Large Shareholders, Monitoring, and Ownership Dynamics: Toward Pure Managerial Firms?

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Abstract

We study ownership dynamics in a framework where the manager and the large shareholder, both risk neutral, simultaneously choose effort and monitoring level respectively to serve their non-congruent interests, and where the large shareholder's instantaneous net returns decrease in her fraction of ownership of the firm. At the Markov-perfect equilibrium, in order to elicit more effort from the manager, the large shareholder divests her shares. In the case where the incongruence of their interests is mild, divestment is drastic: all her shares are sold in one go. In the reverse case, where their interests diverge sharply, the divestment is gradual in order to prevent a sharp fall in share price. In the limit the firm becomes purely managerial, with a diverse ownership, and no monitoring by shareholders.
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1 Introduction

The dynamic theory of the firm has a long history. Beginning with Roos (1925, 1927), the theory has made significant advances, with contributions ranging from the dynamic monopoly models of Hotelling (1931), Coase (1972), and dynamic duopoly models of Fershtman and Kamien (1987), and Boyer, Lasserre and Moreaux (2010). Another aspect of firm dynamics concerns the evolution of the ownership structure. Our paper is a modest step in this direction.

This paper develops a dynamic model of divestment by a large shareholder of a firm where her interest and that of the manager are not perfectly congruent. In our model, all shareholders, large and small, are risk neutral and have perfectly congruent objectives; however only the large shareholder monitors the manager while the small investors free ride on her monitoring effort.[1] We show how the degree of divergence of interests between the manager and the large shareholder affects the process of divestment. We demonstrate that when their interests diverge sharply, the divestment is gradual in order to prevent a sharp fall in share price. In the limit the firm becomes purely managerial, with a diverse ownnership, and no monitoring by shareholders. This paper thus serves to highlight a mechanism that lies behind the tendency for corporate governance to move gradually from concentrated to dispersed ownership, a pattern that has been observed over more than a century in major capitalist economies (such as Great Britain and the USA), and also more recently in countries such as Brazil. The key to our explanation is that the large shareholder cannot resist the temptation to sell shares when small investors' marginal benefit flow is greater than her own. While reducing her ownership (which entails a decrease in her monitoring effort) adversely affects the dividend flow to all investors, it does elicit more effort from the manager.

Berles and Means (1932) pointed to the transition to dispersed ownership

[1] These assumptions are also made in the seminal contribution of Burkart, Gromb, and Panunzi (1997, p. 697), who however do not consider the dynamic process of divestment by the large shareholder. We build our dynamic model using the key elements of their static model.

in the US. Recent empirical work confirms this tendency.[2] For the U.K., the same tendency was reported in Scott (1990), and Franks et al. (2004), among others. Gorga (2009) documented a similar trend in Brazil from 1997 to 2002. Various reasons have been offered to explain the tendency for reduced concentration of ownership. Subrahmanyam and Titman (1999) argue that it becomes advantageous for firms to have a more dispersed ownership when informational asymmetries between insiders and external investors are less important. Roe (1994) and LaPorta et al. (1999) attribute the dispersion of ownership in the US to the specific US laws and policies that discourage onwership concentration.

In this paper, we explain the tendency toward dispersed ownership by modelling, on the one hand, the trade-off between the gains from monitoring by a large shareholder and those from managerial initiatives, and on the other hand, the incentives for the large shareholder to divest (gradually, in typical cases) when her marginal valuation of ownership is below the small investors' valuation of the dividend stream that would arise on the assumption that she does not divest. The former aspect was investigated in an elegant static model by Burkart, Gromb, and Panunzi (1997). The latter aspect is built on the literature concerning the Coase conjecture. Coase (1972) argued that when a monopolist producing a durable good at constant marginal cost cannot commit, rational expectations by potential buyers, and his ability to sell repeatedly, would result in only one possible equilibrium outcome: he can only charge the price that would prevail under perfect competition, and the market demand is satisfied instantaneously.[3] In our model, where the large shareholder corresponds to the Coasian monopolist, we show that Coase's conjecture holds if the divergence of interests between the large shareholder and the manager is mild; in constrast, if this divergence is very strong, the

[2] See, e.g., Demsetz and Lehn (1985), Mikkelson et al. (1997), LaPorta et al. (1999), Urosevic (2001).

[3] Coase's conjecture was confirmed by Stokey (1981), Gul et al. (1986), and others, under the assumptions that marginal cost is constant and the interval between successive sales shrinks to zero. Coase's conjecture fails if there is increasing marginal cost (Kahn, 1987), or depreciation (Karp, 1996).

Coase conjecture fails, and the large shareholder will divest only gradually, with share price falling slowly over time, converging only in the long run to the competitive price. There is also an intermediate case, in which at first the large shareholder undertakes a massive sale of shares, to be followed by a slow process of divestment of the remaining shares.

The intuition behind our results is simple. In all cases, the divestment is caused by the fact that small shareholders perceive that, under the assumption that the large shareholder would not divest, their dividend stream per share is worth more than the large shareholder's marginal returns on a share (as she has to incur the monitoring cost). This wedge in marginal valuations implies that equilibrium must involve share trading. When the divergence of interests between the manager and the large shareholder is mild, her total instantaneous payoff (net of monitoring cost) is a strictly concave and increasing function of her fraction of ownership. Therefore the revenue she would obtain from selling her shares at the competitive share price strictly dominates the present value of the stream of her instantaneous payoff obtained from maintaining her initial stock. Hence her optimal policy is to sell off all her shares in one go. In the reverse case, the strong divergence of interests implies that her total instantaneous payoff is a strictly convex and increasing function of her fraction of ownership. The equilibrium share price function must in this case equal the large shareholder's capitalised marginal instantaneous payoff, which increases in her shareholding. Selling shares too quickly would cause a drastic fall in share price. So it is optimal for her to sell gradually.[4]

Our paper is related to a strand of literature which deals with the dynamic process of adjustment of shareholding based on the insight from the literature on the Coase conjecture. Unlike our model specification which places emphasis on the conflict between the manager and the large shareholder, Gomes

[4] In the intermediate case, which involves weak congruence of interests, coupled with a large stake, the large shareholder's instantaneous payoff function is S shaped: it is convex (concave) when her fraction of ownership is small (large). Then the large shareholder's optimal strategy is to make an initial lumpy sale of a fraction of her shares, to be followed by a time path of gradual sale of the remaining fraction.

(2000) assumes that the large shareholder is also the manager of the firm. In that model, the owner-manager is playing a share-selling game against the collection of small investors. The gains from trade arises because by selling her shares, the owner-manager can diversify idiosyncratic risks with investors. The investors perceive that the owner-manager may be of one type or another. Although the owner-manager knows her type, investors know only the probability distribution of types. At each period, the owner-manager moves by choosing her new fraction of equity ownership and her effort level which is unobservable. Investors update their belief about the owner-manager's type, and they price shares in the market accordingly. Gomes shows that when outside investors face this adverse selection problem, the owner-manager's equilibrium strategy involves divesting her shares gradually over time (in contrast to the perfect information benchmark, where the owner-manager would sell all her shares in the first period). This gradualism is necessary for the entrepreneur to develop a reputation for treating minority shareholders well.

Gomes's conclusion that a risk-averse owner-manager would divest shares gradually over time is re-inforced by DeMarzo and Urošević (2006) who show (in a model with moral hazard instead of adverse selection) that if moral hazard is weak enough, the large shareholder trades immediately to the competitive price-taking allocation. With strong moral hazard, however, she will adjust her stake gradually. DeMarzo and Urošević (2006) emphasize the large shareholder's tradeoff between risk diversification (which calls for a small shareholding) and her incentives and ability to improve the firm's performance (which increases with her fraction of ownership of her firm). DeMarzo and Urošević assume that the utility function of the large shareholder exhibits constant absolute risk aversion. Her wealth consists of a risk-free account and risky shares in her firm. Her sale strategy is motivated by consumption smoothing and risk diversification. When she sells her shares, investors anticipate a decrease in her effort. Hence, when reducing her stake, she is likely to generate a decrease in share price.[5] Edelstein et al. (2007) generalize the model of DeMarzo and Urošević (2006) to a setting with multiple strategic insiders. They show that the aggregate stake of the insiders decreases gradually over time, and that the long run equilibrium aggregate stake of the insiders are greater for firms with a larger number of insiders.

In contrast, in our model, all agents (the large shareholder, the small investors, and the manager) are risk neutral: their utility function is linear in income. Moreover, we focus on the divergence of interests between the large shareholder and the manager (who does not own shares): in each period, there is an agency problem occuring between the large shareholder and the manager.[6] The separation of management (by the manager) and control (by the large shareholder) is a major driving force behind the dynamics of share sales. By divesting, the large shareholder can influence the time path of the equilibrium effort level of the manager, as well as the time path of her own level of monitoring of his action. The wedge between her marginal valuation of a share and that of the atomistic investor indeed arises from a strategic effect, namely managerial effort being negatively decreasing in the fraction of shares held by the large shareholder. This strategic effect is absent in DeMarzo and Urošević (2006). While both our paper and that of DeMarzo and Urošević (2006) show that total divestment is the ultimate outcome, the mechanisms driving the dynamic process in the two models are quite different.

The paper is organized as follows. Section 2 describes the basic framework, drawn from Burkart, Gromb, and Panunzi (1997). Section 3 deals with the commitment benchmark. Section 4 turns to time-consistent strategies and characterizes the Markov-perfect equilibrium corresponding to different

[5] The authors also noted that the time-inconsistency problem, raised by Coase, also applies to their model. They consider both the benchmark case where the large shareholder can commit ex ante to an ownership policy, and the case where commitment is not possible.

[6] Our setting is based on the seminal paper of Burkart, Gromb, and Panunzi (1997), who restricted attention to a static setting. Our dynamic analysis has added some interesting features about the equilibrium divestment strategies, as outlined above.

regions of the parameter space. Section 5 concludes.

2 The Model

Our model is a dynamic extension of the static model proposed by Burkart, Gromb, and Panunzi (1997), or BGP for short.

2.1 The basic static setting

Consider a firm run by a manager, denoted by M. He manages the firms and owns no shares. There is a large shareholder, denoted by S. She owns a fraction α of shares but does not manage the firm. Let there be n shares. The large shareholder owns αn shares. She has a strong incentive to monitor the manager when α is large. The remaining fraction $1-\alpha$ is owned by a continuum of atomistic shareholders who free ride on the monitoring effort of S. All agents are risk neutral.

In each period, the firm must choose one project to carry out. The projects it faces come in four known types, denoted by $i = \{0, 1, 2, 3\}$. A type i project, if carried out, will yield a pair of benefits (Π^i, b^i) where Π^i is verifiable and accrues to the shareholders, while b^i is non-verifiable and accrues to the manager. Assume that $\Pi^0 = b^0 = 0$ and that both Π^1 and b^1 are large negative numbers, say $\Pi^1 = b^1 = -k\Pi$ where $\Pi > 0$ and k is a large positive number. The pair of benefits associated with a type 2 project is $(\Pi, b) \gg (0, 0)$ if the state of nature is A, and is $(\Pi, 0)$ if the state of nature is not A. Each type 3 project yields the pair (Π, b) if the state of nature is A, and the pair $(0, b)$ if the state of nature is not A.[7] State A occurs with probability $\lambda < 1$. The state of nature occurs before the firm chooses its project. However this information is not revealed to the manager unless he exercises effort, in which case he will obtain the information with some probability. The numbers Π, b, k and λ are common knowledge. The properties of type 2 and type 3 projects are

[7] BGP's specification of the payoffs is slightly different from ours. However the analysis of the static model is essentially the same for both specifications.

summarized in Table 1.

Type 2	Type 3	Probability
(Π, b)	(Π, b)	$\lambda \in (0,1)$
$(\Pi, 0)$	$(0, b)$	$1-\lambda$

Table 1

For simplicity, assume that each period the firm is presented with exactly four projects, one from each type. Assume that everyone knows which of these four projects is a type 0 project. The remaining three projects, however, are presented to the firm as named projects (γ, μ, θ). Everyone knows that in period t there is a one-to-one mapping ϕ_t from the set $\{\gamma, \mu, \theta\}$ to the set of project types $\{1, 2, 3\}$. However, if the manager does not spend some effort $e > 0$ in information-seeking activities, this mapping will not be revealed to the manager (nor the the shareholders).

If M exercises effort level e in period t, where $e \in [0, 1]$, then with probability e, he will be completely informed, i.e., he will discover both (i) the mapping ϕ_t and (ii) whether the state of nature in period t is A or not A. By investing e in the information-seeking activities, then, with probability e, all the uncertainty is eliminated for the manager, but with probability $1-e$, he will remain completely uninformed.

The large shareholder, S, does not observe the manager's choice of effort level e. She chooses her monitoring effort level E, where $E \in [0, 1]$, to attempt to find out the information that the manager has obtained. If M remains completely uninformed, then S learns that M knows nothing. If M is completely informed, then S will find out that M is informed, and with probability E she captures all of his information (about the mapping ϕ_t and the state of nature) while with probability $1-E$ she obtains no information. It is assumed that neither e nor E is verifiable, and that M and S must make their choice (e, E) simultaneously.

Notice that the parameter λ is a measure of the congruence of interests between the manager and the shareholders. In the polar case where $\lambda = 1$, there would be perfect congruence of interests. In what follows, we assume that $0 < \lambda < 1$.

After the choices (e, E) have been made, there are three possible cases. First, if both parties remain uninformed, they will rationally agree that the firm should choose the type 0 project.[8] Second, if M is the only informed party, then S knows that M will assure himself the payoff b (since he knows both the true mapping ϕ_t and the state of nature), which implies that, from S's vantage point, the income accruing to the shareholders is Π/n per share with probability λ and zero per share with probability $1-\lambda$. Third, if both parties are informed, S, knowing which of (γ, μ, θ) is the type 2 project, will exercise her control right and require M to undertake the type 2 project. Carrying out the type 2 project implies that M's payoff is b with probability λ and 0 with probability $1-\lambda$.

Let us reproduce below BGP's derivation of the Nash equilibrium pair (e, E).

M's effort cost is $(1/2)e^2$. Given E, he chooses $e \in [0,1]$ to maximize his expected payoff

$$-\frac{1}{2}e^2 + (1-e) \times 0 + e \times \{E[\lambda b + (1-\lambda) \times 0] + (1-E)b\}$$

The first order condition yields M's *downward-sloping* reaction function:

$$e = \min\{1, b[1-(1-\lambda)E]\}. \tag{1}$$

This implies that the large shareholder's monitoring will reduce the manager's incentives to exercise effort. Taking into account the fact that $E \in [0,1]$ and assuming that $0 < b < 1$, we deduce that manager's chosen effort level is at least λb and at most b.

Denote by $D(e, E)$ the expected aggregate dividends for the shareholders. Then

$$D(e, E) \equiv \{(1-e) \times 0 + e \times [E\Pi + (1-E)\lambda\Pi]\} = e\Pi[\lambda + (1-\lambda)E] \tag{2}$$

[8] Recall that k is a large positive number, therefore the expected payoff from a random choice of projects is negative.

Note that for a fixed E, an increase in the manager's effort raises the expected dividends

$$\frac{\partial D}{\partial e} = \Pi(\lambda + (1-\lambda)E) > 0 \tag{3}$$

Similarly, for a fixed $e > 0$, an increase in the shareholder's monitoring effort raises the expected dividends:

$$\frac{\partial D}{\partial E} = e(1-\lambda)\Pi > 0. \tag{4}$$

Assume that S's effort cost is $(1/2)E^2$. Given e, the large shareholder S will choose $E \in [0,1]$ to maximize her expected payoff,

$$-\frac{1}{2}E^2 + \alpha D(e,E) \tag{5}$$

Her first order condition is

$$\alpha D_E = E \tag{6}$$

i.e. the large shareholder equates her marginal cost of monitoring to the marginal increase in her expected dividend income that results from increased monitoring.

This condition yields S's *upward-sloping* best-reply function:

$$E = \min\{1, \alpha\Pi(1-\lambda)e\} \tag{7}$$

To ensure an interior Nash equilibrium, the following assumption is made.

Assumption A: $0 < \lambda < 1$, $0 < b < 1$, and

$$0 < b\Pi < \frac{1}{\lambda(1-\lambda)}.$$

It follows from Assumption A and equations (1) and (7) that the Nash equilibrium is interior:

$$E(\alpha) = \frac{\alpha\Pi b(1-\lambda)}{1+\alpha\Pi b(1-\lambda)^2} < 1 \text{ and } e(\alpha) = \frac{b}{1+\alpha\Pi b(1-\lambda)^2} < 1 \tag{8}$$

Let us define $\Omega \equiv \Pi b$. The parameter Ω may be regarded as an index of the "stake" of the game between the manager and the large shareholder. Given α, the Nash equilibrium monitoring effort $E(\alpha)$ depends on the two parameters Ω and λ.[9] Note that

$$E'(\alpha) = \frac{\Omega(1-\lambda)}{\left[1+\alpha\Omega(1-\lambda)^2\right]^2} > 0 \text{ and } e'(\alpha) = \frac{-b\Omega(1-\lambda)^2}{\left[1+\alpha\Omega(1-\lambda)^2\right]^2} < 0. \tag{9}$$

Denote the expected aggregate dividends (or *equity value*) by $W(\alpha) \equiv D(e(\alpha), E(\alpha)) = e(\alpha)\Pi\left[\lambda + (1-\lambda)E(\alpha)\right]$. From (8) and (2),

$$W(\alpha) \equiv D(e(\alpha), E(\alpha)) = \frac{\Omega\left[\lambda + \Omega\alpha(1-\lambda)^2(1+\lambda)\right]}{\left(1+\Omega\alpha(1-\lambda)^2\right)^2}. \tag{10}$$

Equity value, $W(\alpha)$, is in general a *non-monotone* function of the ownership fraction α of the large shareholder. To see this formally, let us denote by F the set of all admissible couples (λ, Ω) that satisfies Assumption A:

$$F \equiv \left\{(\lambda, \Omega) \in \mathbb{R}^2_+ | \quad 0 < \Omega < \frac{1}{\lambda - \lambda^2} \text{ and } 0 < \lambda < 1\right\} \tag{11}$$

The upper boundary of F is the U-shaped curve $\Omega = \frac{1}{\lambda(1-\lambda)} \equiv f(\lambda)$, where $\lim_{\lambda\to 0} f(\lambda) = \lim_{\lambda\to 1} f(\lambda) = \infty$. Since

$$W'(\alpha) = \frac{\Omega^2(1-\lambda)^3\left(1-\alpha\Omega(1-\lambda^2)\right)}{\left(1+\Omega\alpha(1-\lambda)^2\right)^3} \tag{12}$$

we conclude that $W'(\alpha) > 0$ *for all* $\alpha \in (0,1)$ if and only if the admissible couple (λ, Ω) belongs to the set Q defined below:

$$Q \equiv \left\{(\lambda, \Omega) \in \mathbb{R}^2_+ | \quad 0 < \Omega < \frac{1}{1 - \lambda^2} \text{ and } 0 < \lambda < 1\right\} \subset F. \tag{13}$$

Note that Q is a proper subset of F. The upper boundary of region Q is the curve $\Omega = \frac{1}{1-\lambda^2} \equiv q(\lambda) < f(\lambda)$. Along this curve, as $\lambda \to 0$, $\Omega \to 1$, and as $\lambda \to 1, \Omega \to \infty$. Clearly, if $(\lambda, \Omega) \in Q$, then equity value $W(\alpha)$ is maximized at the corner $\alpha = 1$.

[9] The greater is Ω, the greater is the equilibrium monitoring effort level of the large shareholder. However, the Nash equilibrium level of monitoring $E(\alpha; \Omega, \lambda)$ is not monotone in λ. For values of λ close to unity, a marginal increase λ (i.e. a higher degree in congruence on interests) leads to lower equilibrium monitoring.

It is also easy to verify that if $(\lambda, \Omega) \in F - Q$ then there exists a unique value $\widehat{\alpha} \in (0,1)$, which depends on λ and Ω, such that (i) $W(\alpha)$ is maximized at $\alpha = \widehat{\alpha}$, where

$$\widehat{\alpha} \equiv \frac{1}{\Omega(1-\lambda^2)} < 1 \text{ for } (\lambda, \Omega) \in F - Q.$$

(ii) $W'(\alpha) > 0$ if $\alpha < \widehat{\alpha}$, and (iii) $W'(\alpha) < 0$ if $\alpha > \widehat{\alpha}$.

Define α_2^* to be the fraction of shares owned by the large shareholder that would maximize equity value:

$$\alpha_2^* \equiv \arg \max_{0 \leq \alpha \leq 1} W(\alpha)$$

Then

$$\alpha_2^* = \min \left\{ 1, \frac{1}{\Omega(1-\lambda^2)} \right\}.$$

Consider the large shareholder's net income (after subtracting her effort cost):

$$R(\alpha) \equiv \alpha W(\alpha) - \frac{1}{2} [E(\alpha)]^2 . \tag{14}$$

It can be verified that $R'(\alpha)$ is strictly positive for all $\alpha \in (0,1)$:

$$R'(\alpha) = \frac{\Omega\lambda + \alpha\Omega^2(1-\lambda-\lambda^2+\lambda^3)}{(1+\alpha\Omega(1-\lambda)^2)^3} = \frac{\Omega\lambda + \alpha\Omega^2(1-\lambda)^2(1+\lambda)}{(1+\alpha\Omega(1-\lambda)^2)^3} > 0 \text{ for all } \lambda \in (0,1). \tag{15}$$

The sum of the instantaneous payoffs to the large shareholder and the collection of small shareholders is called "*net equity value,*" defined as

$$V(\alpha) = R(\alpha) + (1-\alpha)W(\alpha) = W(\alpha) - \frac{1}{2}[E(\alpha)]^2$$

BGP showed that net equity value is maximized at[10]

$$\alpha_1^* = \frac{1}{\frac{1}{1-\lambda} + \Omega(1-\lambda^2)} < 1.$$

[10] See their Proposition 1. Clearly, $\alpha_1^* < \alpha_2^*$.

Remark 1: BGP did not report an interesting fact, which we state below as fact 1:

Fact 1: *The marginal value of a share to the large shareholder is smaller than its value to an atomistic shareholder. That is, $R'(\alpha)/n < W(\alpha)/n$.*

The proof is as follows. From (14),

$$R'(\alpha) = W(\alpha) + \alpha W'(\alpha) - E(\alpha)E'(\alpha) \tag{16}$$

Therefore[11]

$$R'(\alpha) - W(\alpha) = \alpha W'(\alpha) - E(\alpha)E'(\alpha) = \alpha D_e \frac{de(\alpha)}{d\alpha} < 0 \tag{17}$$

More explicitly, using (15) and (10),

$$R'(\alpha) - W(\alpha) = -\frac{\alpha \left(\lambda + (1+\lambda)\alpha\Omega(1-\lambda)^2\right)\Omega^2(1-\lambda)^2}{\left(1+\alpha\Omega(1-\lambda)^2\right)^3} \leq 0 \text{ for all } \alpha \in [0,1] \tag{18}$$

with equality only at $\alpha = 0$.[12]

Fact 1 suggests the following conjecture: if share trading is allowed to take place at each point of time, the only time-consistent equilibrium outcome is that the large shareholder will sell her shares, either in a lumpy fashion, or gradually, or both. This conjecture will be shown to be correct (see Section 4).

For the analysis of the dynamic adjustment process in the following sections, it is important to determine whether $R(\alpha)$ is strictly convex, or strictly concave, or neither. It turns out that this depends on the value taken by the couple (λ, Ω). The following Lemma is useful.

Lemma 1 *Consider the following subsets of F, denoted by X, T and A,*

$$X \equiv \left\{ (\lambda, \Omega) \in \mathbb{R}_+^2 | \quad 0 < \lambda < \frac{1}{2} \text{ and } 0 < \Omega < \left(\frac{1-2\lambda}{2-2\lambda}\right)\frac{1}{1-\lambda^2} \right\} \tag{19}$$

[11] From the definition of $W(\alpha)$ and the first order condition (6), $\alpha W'(\alpha) \equiv \alpha \left[D_e \frac{de(\alpha)}{d\alpha} + D_E \frac{dE(\alpha)}{d\alpha} \right] = \alpha D_e \frac{de(\alpha)}{d\alpha} + E(\alpha) \frac{dE(\alpha)}{d\alpha}$.

[12] The inequality $R'(\alpha) < W(\alpha)$ for all $\alpha < 1$ reflects the fact that the small shareholders are free-riding on the monitoring effort of the large shareholder, who incurs monitoring costs without being compensated. (These costs are non-verifiable.)

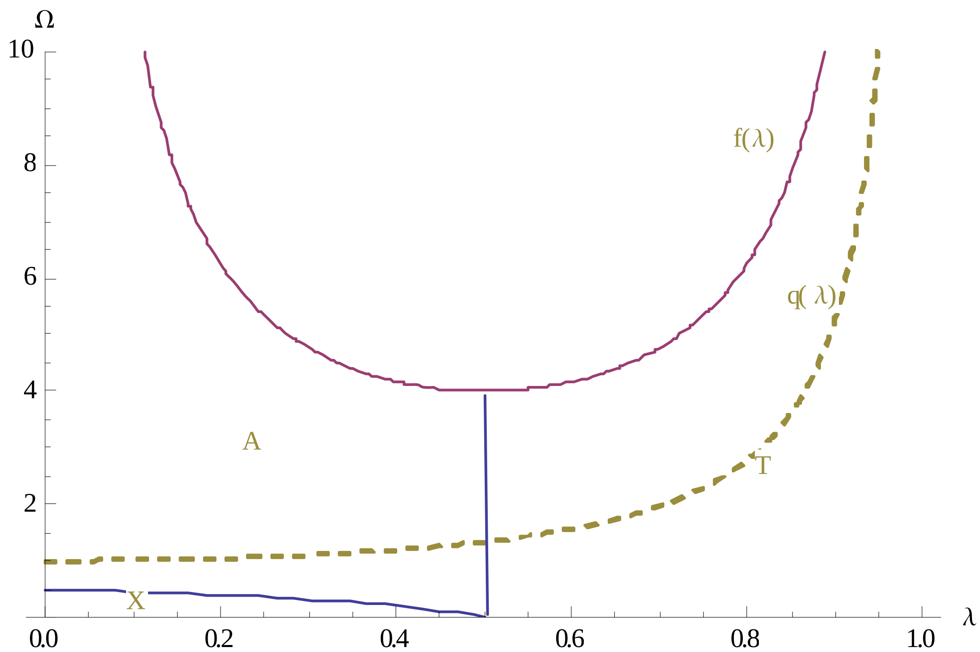

Figure 1: The three cases.

$$T \equiv \left\{ (\lambda, \Omega) \in \mathbb{R}_{+}^{2} | \quad \frac{1}{2} < \lambda < 1 \text{ and } 0 < \Omega < \frac{1}{\lambda - \lambda^2} \right\} \tag{20}$$

$$A \equiv \left\{ (\lambda, \Omega) \in \mathbb{R}_{+}^{2} | \quad 0 < \lambda \leq \frac{1}{2} \text{ and } \left(\frac{1-2\lambda}{2-2\lambda} \right) \frac{1}{1-\lambda^2} < \Omega < \frac{1}{\lambda - \lambda^2} \right\} F \tag{21}$$

Then $R(\alpha)$ is (i) strictly convex in α for all $\alpha \in (0,1)$ if $(\lambda, \Omega) \in X$, (ii) strictly concave in α for all $\alpha \in (0,1)$ if $(\lambda, \Omega) \in T$, and (iii) is S-shaped (convex for all α in the open interval $(0, \widetilde{\alpha})$ and convex for all α in the open interval $(\widetilde{\alpha}, 1)$ if $(\lambda, \Omega) \in A$, where

$$\widetilde{\alpha} \equiv \frac{1-2\lambda}{2\Omega(1-\lambda)(1-\lambda^2)}. \tag{22}$$

Proof From (15),

$$R''(\alpha) = \frac{\Omega^2 (1-\lambda)^2 \left[(1-2\lambda) - 2\alpha\Omega(1-\lambda)^2(1+\lambda) \right]}{(1+\alpha\Omega(1-\lambda)^2)^4} \tag{23}$$

(i) If (λ, Ω) is in X, then

$$2\Omega < \frac{1-2\lambda}{(1-\lambda)(1-\lambda^2)}$$

implying $2\Omega\alpha(1-\lambda)(1-\lambda^2) < 1-2\lambda$ hence $R'' > 0$ for all $\alpha \in [0,1]$.

(ii) If (λ, Ω) is in T, then $(1-2\lambda) < 0$, hence $R'' < 0$ for all $\alpha \in [0,1]$.

(iii) If (λ, Ω) is in A, $(1-2\lambda) - 2\alpha\Omega(1-\lambda)^2(1+\lambda)$ can be of either sign. Then define $\widetilde{\alpha}(\lambda, \Omega)$ by

$$0 < \widetilde{\alpha}(\lambda, \Omega) \equiv \frac{1-2\lambda}{2\Omega(1-\lambda)(1-\lambda^2)} < 1 \text{ for } (\lambda, \Omega) \in A,$$

we can see that $R''(\alpha) > 0$ for $0 < \alpha < \widetilde{\alpha}(\lambda, \Omega)$ and $R''(\alpha) < 0$ for $\widetilde{\alpha}(\lambda, \Omega) < \alpha < 1$. ■

Remark: The upper boundary of region X is the curve

$$\Omega = \frac{1-2\lambda}{2(1-\lambda)^2(1+\lambda)} = \left(\frac{1-2\lambda}{2-2\lambda}\right)\frac{1}{1-\lambda^2} \equiv h(\lambda) \text{ for } 0 < \lambda < \frac{1}{2}$$

Along this curve, as $\lambda \to 1/2, \Omega \to 0$. As $\lambda \to 0, \Omega \to 1/2$. This negatively-sloped curve lies below the curve $\Omega = \frac{1}{1-\lambda^2} \equiv q(\lambda)$.

3 A dynamic version of the model: the commitment benchmark

Let us now assume that the projects mentioned above last for only one period (or, more precisely, since we use continuous time, for an arbitrarily small time interval). Assume that at each instant t, a new set of projects become available. At t the manager exercises effort level $e(t)$ and the large shareholder chooses her monitoring level $E(t)$. If the large shareholder's ownership fraction at t is $\alpha(t)$, her equilibrium instantaneous payoff is her expected dividends minus her effort costs,

$$R(\alpha(t)) \equiv \alpha W(\alpha(t)) - \frac{1}{2}\left[E(\alpha(t))\right]^2. \tag{24}$$

Suppose the large shareholder contemplates reducing her ownership of shares at the rate $-\dot{\alpha}(t)n$ at time t. (We allow $\dot{\alpha}(t)$ to be of either sign.)

Let $p(t)$ be the market price of a share at time t. Recall that there are n shares, where n is hold constant. We assume that investors have rational expectations, so that the share price at t is simply the value of the discounted stream of expected dividends:

$$p(t) = \int_t^\infty \exp(-r(\tau - t)) \frac{W(\alpha(\tau))}{n} d\tau. \tag{25}$$

where r is the interest rate. Differentiating (25) with respect to t yields

$$\dot{p}(t) = rp(t) - \frac{W(\alpha(\tau))}{n}. \tag{26}$$

This equation is the usual non-arbitrage condition in a competitive asset market: the return to holding an asset (i.e. the sum of capital gains and dividends) is just equal to the opportunity cost, $rp(t)$, of foregone interest income. The payoff to the large shareholder is then

$$J^c(\alpha_0) = \int_0^\infty \exp(-rt) \left[R(\alpha(t)) - \dot{\alpha}(t) np(t) \right] dt \tag{27}$$

where $-\dot{\alpha}(t)np(t)$ is flow of cash receipts generated by her divesting rate $-\dot{\alpha}(t)n$, subject to $\alpha(0) = \alpha_0$ and $1 \geq \alpha(t) \geq 0$.

What is her optimal divesting strategy? The answer to this question depends on whether S can commit to a time path of sale of her shares. Let us begin with the the benchmark case where S is able to commit.

Suppose that the large shareholder is able to commit to a whole time path of her shareholding $\alpha(t)$. Her objective function (27) can then be simplified as follows. Let us write

$$\phi(t) \equiv \exp(-rt)np(t) = \int_t^\infty \exp(-r\tau) W(\alpha(\tau)) d\tau$$

Since W is bounded, it is clear that $\lim_{t \to \infty} \phi(t) = 0$. Then

$$\int_0^\infty \exp(-rt) np(t) \dot{\alpha}(t) dt = \int_0^\infty \phi(t) \dot{\alpha}(t) dt =$$

$$= [\alpha(\infty)\phi(\infty) - \alpha(0)\phi(0)] - \int_0^\infty \dot{\phi}(t) \alpha(t) dt =$$

$$-\alpha(0)\int_0^\infty \exp(-rt)W(\alpha(t))dt+\int_0^\infty \alpha(t)\exp(-rt)W(\alpha(t))dt$$

Thus the objective function of S becomes

$$\max_{0\leq\alpha(t)\leq 1}\int_0^\infty \exp(-rt)\left[R(\alpha(t))+(\alpha_0-\alpha(t))\,W(\alpha(t))\right]dt$$

Obviously, the solution is to choose the same value for $\alpha(t)$ for all $t\in(0,\infty)$. It is optimal to make an immediate jump in the state variable (an impulse control) to some optimal *committed level* $\alpha(0^+)=\alpha^c$, and after this initial jump, $\alpha(t)$ will be kept constant at α^c for ever, where α^c is the value of α that maximizes $Z(\alpha;\alpha_0)\equiv(R(\alpha)/r)-(\alpha-\alpha_0)(W(\alpha)/r)$ subject to $\alpha\in[0,1]$. That is, the large shareholder chooses her immediate net acquision, $n\alpha-n\alpha_0$, of shares to maximize the capitalised value of her time-invariant dividend flow (net of her effort cost), $R(\alpha)/r$, minus the cost of acquision (i.e., the price of $W(\alpha)/(nr)$ per share, multiplied by the number of shares acquired, $n\alpha-n\alpha_0$). Note that in principle $n\alpha-n\alpha_0$ can be positive (acquision of shares) or negative (sale of shares).

We now show that $Z(\alpha;\alpha_0)$ is strictly quasi-concave in α, attaining its maximum at some $\alpha^c\in(0,\alpha_0)$. Using the definition of Z, we obtain the derivative

$$Z_\alpha(\alpha;\alpha_0)\equiv\frac{1}{r}R'(\alpha)-\frac{1}{r}\left[W(\alpha)-(\alpha_0-\alpha)\,W'(\alpha)\right]$$

The term $\frac{1}{r}W(\alpha)-\frac{1}{r}(\alpha_0-\alpha)W'(\alpha)$ is the marginal revenue from divesting, while the term $\frac{1}{r}R'(\alpha)$ is her marginal cost of divesting (it is measured as the fall in the capitalised value of her dividend flow (net of her effort cost) caused by a marginal reduction in her share ownership). Let α^c be the value of α such that the two terms are equalized. It is easy to verify that given any $\alpha_0>0$, α^c is in the interior of the interval $[0,\alpha_0]$. Futhermore, the smaller is α_0, the smaller is α^c. The proof is as follows.

From the definition of $R(\alpha)$, i.e.(24), we have the identity

$$R'(\alpha)\equiv\alpha W'(\alpha)+W(\alpha)-E(\alpha)E'(\alpha)\text{ for all }\alpha\in[0,1]\,.$$

So the first order condition for maximizing $Z(\alpha;\alpha_0)$ reduces to

$$rZ_\alpha(\alpha;\alpha_0) \equiv \alpha_0 W'(\alpha) - E(\alpha)E'(\alpha) = 0$$

This condition gives[13]

$$\alpha^c = \frac{\alpha_0(1-\lambda)}{1+\alpha_0(1-\lambda)^2\Omega(1+\lambda)} \equiv \chi(\alpha_0)$$

$$= \frac{1}{\frac{1}{\alpha_0(1-\lambda)} + (1-\lambda)(1+\lambda)\Omega} \in (0,1)$$

Note that α^c is smaller than α_0 :

$$\frac{\alpha^c}{\alpha_0} = \frac{(1-\lambda)}{1+\alpha_0(1-\lambda)^2\Omega(1+\lambda)} = \frac{1}{\frac{1}{1-\lambda} + \alpha_0(1-\lambda)\Omega(1+\lambda)} < 1.$$

Proposition 1 (Optimal asset sale strategy under commitment). *If the large shareholder can make a binding commitment on her time path of share holding, her optimal policy is to reduce her shareholding immediately from her initial holding α_0 to a committed level α^c where*

$$\alpha^c = \frac{\alpha_0(1-\lambda)}{1+\alpha_0(1-\lambda)^2\Pi b(1+\lambda)} \equiv \chi(\alpha_0) < \alpha_0$$

and afterward she retains her remaining shares for ever.

The large shareholder is willing to commit not to sell more shares thereafter because she wants to elicit a higher initial share price. Rational buyers would not be willing to pay this price if they think she would go on reducing her shares and consequently her monitoring effort level.

The solution described in Proposition 1 displays the property of time-inconsistency. The commitment strategy of holding $\alpha(t) = \chi(\alpha_0)$ for all $t \in (0,\infty)$ implies that at any time $t_1 > 0$, if the large shareholder would be released from her original commitment, she would again want to sell immediately some more shares (because at t_1 the relevant initial holding would be α_{t_1}) and thus the share price would fall below the initial price, $W(\alpha^c)/(rn)$.

[13] The SOC is satisfied at α^c.

This action would inflict capital losses to the previous buyers of shares, because they have been fooled into believing that the large shareholder would sell assets only once. Solutions that display time-inconsistency are generally regarded as unacceptable (Coase, 1972). Therefore we must look for time-consistent solutions.

4 Markov perfect equilibrium

In this section, we seek solutions that have the time-consistent property, and, in addition, that would be robust to perturbation. More precisely, we are insisting on a stronger property than time-consistency, namely Markov perfect equilibrium.[14] In a Markov perfect equilibrium, the large shareholder uses a Markovian strategy ω and the market has a Markovian price function, or expectation rule, ρ (which we will explain in more detail below) such that (i) given ρ, the Markovian strategy ω maximizes S's payoffs, for all possible starting (date, state) pairs (t, α_t), and (ii) given ω, the Markovian price function ρ is consistent with rational expectations.[15]

Assume that the atomistic agents all have a common Markovian price expectation function $p(t) = \rho(\alpha(t))$, where ρ is a function of the state variable α. The price expectations function must be rational, in the sense that the share price must equal the capitalized value of the future dividend stream:

$$\rho(\alpha(t)) = \frac{1}{n}\int_t^{\infty} \exp(-r(\tau - t))W(\alpha(\tau))d\tau, \tag{28}$$

where $\{\alpha(.)\}_t^{\infty}$ is the time path of the state variable α induced by the strategy ω of the large shareholder, from time t, when the state variable takes the value α_t.

A strategy ω of the large shareholder is a specification of (i) a collection of disjoint intervals $I_1, I_2, ..., I_m$ where $I_i \equiv [a_i, b_i] \subset [0, 1]$, (ii) a lumpy sale

[14] For an exposition of the concepts of time-consistency and Markov perfect equilibrium, and a proof that Markov perfect equilibria are time-consistent, see Dockner et al. (2000). Long (2010) provides some simple examples.

[15] For some examples of Markovian price function in the industrial organization literature, see Karp (1996), Driskill and McCafferty (2001), and Laussel et al. (2004).

function $L_i(.)$ that specifies a downward jump in the state variable, such that $\alpha - 1 \leq L_i(\alpha) \leq \alpha$, (if $L_i(\alpha)$ is negative, it signifies a lumpy purchase of shares), and (iii) a gradual sale function $g(.)$ defined for all $\alpha \notin I_i$, such that

$$\dot{\alpha}(t) = -g(\alpha(t)) \text{ for } \alpha \notin I_i$$

where $g(\alpha) \in (-\infty, \infty)$.

The payoff to the large shareholder, given (t, α_t), is

$$\int_t^{\infty} \exp(-r(\tau - t)) \left[R(\alpha(\tau)) - \dot{\alpha}(\tau) np(\tau)\right] d\tau$$

where $p(\tau) = \rho(\alpha(\tau))$.

Definition: *A Markov-perfect equilibrium is a pair (ρ, ω) such that, (i) given the price function ρ, the strategy ω maximizes the large shareholder's payoff, starting at any (date,state) pair (t, α_t), and (ii) given ω and (t, α_t), the price function ρ satisfies the rational expectation properties (28).*

Remark: Equation (28) yields the usual non-arbitrage condition (26).

4.1 Markov perfect equilibrium in the parameter region X (convex $R(\alpha)$)

By Lemma 1, in region X the large shareholder's instantaneous returns function $R(\alpha)$ is strictly convex for all $\alpha \in (0, 1)$. Furthermore, since $W(0) = R'(0)$, it follows that $R(\alpha) > \alpha W(0)$ for all $\alpha > 0$. This means that, starting with α_0, if the large shareholder were to sell all her α_0 instantaneously, her share would be sold at the price $p = \frac{1}{nr} W(0)$, and her payoff (revenue from sales) would be $\frac{\alpha_0}{r} W(0)$, which is strictly smaller than $\frac{1}{r} R(\alpha_0)$, her payoff if she does not offer to sell her shares. This suggests that selling her shares gradually would be better than selling them off in one go. The following proposition confirms this intuition.

Proposition 2: *If (λ, Ω) is in the set X (defined by (19)), so that the shareholder's net returns function $R(\alpha)$ is convex, then the large shareholder's equilibrium strategy is to sell her shares gradually, such that $\alpha(t) \to 0$*

asymptotically as $t \to \infty$, and the atomistic investors' equilibrium price function is

$$\rho(\alpha) = \frac{1}{nr}\left[\frac{\Omega\left(\lambda + \alpha\Omega(1-\lambda)^2(1+\lambda)\right)}{\left(1+\alpha\Omega(1-\lambda)^2\right)^3}\right] = \frac{1}{nr}R'(\alpha) \tag{29}$$

where the equilibrium price is increasing in the fraction of shares held by the large shareholder:

$$\rho'(\alpha) = \frac{1}{nr}R''(\alpha) > 0 \text{ for all } \alpha \in (0,1) \text{ and for all } (\lambda, \Omega) \in X.$$

The large shareholder's optimal rate of sale at time t is $-\dot{\alpha}(t)$, where

$$\frac{-\dot{\alpha}(t)}{\alpha(t)} = \frac{\left(\lambda + \alpha\Omega(1-\lambda)^2(1+\lambda)\right)\Omega^2(1-\lambda)^2}{\left(1+\alpha\Omega(1-\lambda)^2\right)^3 N\rho'(\alpha)} > 0 \tag{30}$$

Along the path of of disinvestment, the share price falls monotonically, converging asymptotically to $\rho(0) = \frac{1}{rn}W(0) = \frac{1}{rn}R'(0)$.

Proof:

The Hamilton-Jacobi-Bellman (HJB) equation for the large shareholder is

$$rJ(\alpha) = \max_{\dot{\alpha}}\left\{R(\alpha) - \dot{\alpha}n\rho(\alpha) + J'(\alpha)\dot{\alpha}\right\} \tag{31}$$

Since this equation is linear in $\dot{\alpha}$, the optimal $\dot{\alpha}$ is finite only if

$$n\rho(\alpha) = J'(\alpha) \text{ for all } \alpha \in (0,1) \tag{32}$$

Substituting this into the HJB equation (31), we obtain

$$J(\alpha) = \frac{1}{r}R(\alpha) \text{ for all } \alpha \in (0,1)\,. \tag{33}$$

Thus, the value function evaluated at α is just equal to the discounted stream of net returns that would be obtained if α were kept constant for ever.[16]

Then the equilibrium price function is

$$\rho(\alpha) = \frac{1}{n}J'(\alpha) = \frac{1}{rn}R'(\alpha) \tag{34}$$

[16] When the function $R(\alpha)$ is strictly convex, the large shareholders gains nothing by selling gradually as compared with keeping α for ever, but she must sell gradually in the Markov perfect equilibrium. For, if she instead held on to her α_0, the expected price of shares would be constant for ever at $W(\alpha_0)/nr$, which would of course induce her to sell.

All the necessary conditions for an equilibrium are satisfied. Let us verify that this is indeed better than selling off all of α in one go. The latter action would yield a return of $\frac{1}{r}R(0)+\frac{1}{r}\alpha_0 W(0)=\frac{1}{r}\alpha_0 W(0)$.In region X, the function $R(\alpha)$ is strictly convex in α. This strict convexity and $R(0)=0$ implies that $R(\alpha)>R'(0)\alpha$ for all $\alpha>0$. But from (18), $R'(0)=W(0)$. Therefore $R(\alpha)>W(0)\alpha$ for all $\alpha>0$.This shows that selling gradually is better than selling all α off in one go.

Finally let us characterize the selling strategy and the time path of sales. Because of (18), $\rho(\alpha)<W(\alpha)/rn$.[17][18]

Recall that $p(t)=\rho(\alpha(t))$. Since we require that the no-arbitrage condition (26) holds, we must have

$$\rho'(\alpha)\dot{\alpha}=r\rho(\alpha)-\frac{W(\alpha)}{n}$$

i.e.,

$$n\rho'(\alpha)\dot{\alpha}=rn\rho(\alpha)-W(\alpha)=R'(\alpha)-W(\alpha)$$

$$\dot{\alpha}(t)=-\frac{\alpha\left(\lambda+\alpha\Omega(1-\lambda)^2(1+\lambda)\right)\Omega^2(1-\lambda)^2}{\left(1+\alpha\Omega(1-\lambda)^2\right)^3 n\rho'(\alpha)}$$

Using (23) to substitute for $\rho'(\alpha)n$,

$$\frac{\dot{\alpha}}{\alpha}=\frac{r\left(\lambda+\alpha\Omega(1-\lambda)^2(1+\lambda)\Omega^2(1-\lambda)^2\right)\left(1+\alpha\Omega(1-\lambda)^2\right)}{\left[-1+2\lambda+2\alpha\Omega(1-\lambda)^2(1+\lambda)\right]\Omega^2(1-\lambda)^2}<0 \text{ in region } X.$$

[17] To interpret this inequality, use (34) (16), (17) to obtain

$$\rho(\alpha)=\frac{R'(\alpha)}{nr}=\frac{1}{nr}\left[W(\alpha)+\left(\alpha D_e\frac{de(\alpha)}{d\alpha}\right)\right]<\frac{W(\alpha)}{nr}$$

where the term $\alpha D_e\frac{de(\alpha)}{d\alpha}$ is negative. This equation says that the equilibrium price of a share when the large shareholder's fraction of ownnersip is α must equal the expected discounted stream of dividend per share (if this fraction were to remain constant for ever) minus a term which reflects the disincentive effect on managerial effort due to the large shareholder's monitoring activities.

[18] Why is the price lower than the current level of dividend per share? The key to the answer lies in the fact that in region X, $W'(\alpha)>0$.Therefore, as the large shareholder sells more and more shares, the dividend per share falls, and investors know this. They would not pay a price equal to the present value of a constant stream of current dividend.

Note that as $\alpha \to 0$, $\dot{\alpha}/\alpha$ tends to a negative constant. Thus $\alpha(t) \to 0$ asymptotically.

We must show that given the price rule (29), no deviation from the trading strategy (30) can increase the large shareholder's expected discounted profits above $J(\alpha) = R(\alpha)/r$. Consider a given lumpy sale L_i deviation taking α to $\alpha - L_i$ where L_i is in the interior of the interval $(\alpha - 1, \alpha)$.[19] After the deviation, the large shareholder uses the equilibrium trading rule, the value of the continuation game is $J(\alpha - L_i)$, which implies, by (34), that $J'(\alpha - L_i) = n\rho(\alpha - L_i)$. So the expected payoff obtained from this deviation is $J(\alpha - L_i) + nL_i\rho(\alpha - L_i)$. The best among such interior deviations is found by differentiating $J(\alpha - L_i) + nL_i\rho(\alpha - L_i)$ with respect to L_i. This gives the necessary condition for the best L_i is $-J'(\alpha - L_i) - nL_i\rho'(\alpha - L_i) + n\rho(\alpha - L_i) = 0$. Since $J'(\alpha - L_i) = n\rho(\alpha - L_i)$, we have $J''(\alpha - L_i) = n\rho'(\alpha - L_i)$, so the above necessary condition reduces to $L_i J''(\alpha - L_i) = 0$. But $J'' > 0$. It follows that $L_i^* = 0$, i.e. the best interior deviation is zero deviation. Let us now consider a jump to $\alpha = 1$. This deviation yields the payoff $\frac{1}{r}\left[R(1) - (1 - \alpha)W(1)\right]$, while sticking to the candidate equilibrium strategy yields $\frac{1}{r}R(\alpha)$. Now the strict convexity of $R(\alpha)$ in region X implies that $R(\alpha) - R(1) > R'(1)(\alpha - 1)$. Thus $R(\alpha) > R(1) - (1-\alpha)R'(1) > R(1) - (1-\alpha)W(1)$, because $W(1) > R'(1)$ by (18). We conclude that there is no profitable discontinuous deviation.■

To summarize, when parameter values are in region X, the equilibrium outcome is that the large shareholder sells her shares gradually. If she were to sell them off all in one go, her payoffs would be lower. Since the share price function $\rho(\alpha)$ is increasing in α, as the large shareholder divests, the price declines. If she were to divest all in one go, the share price would fall too sharply.

Interestingly, the fraction of shares held by large shareholder never vanishes in finite time. Starting from any initial fraction α_0, the time it takes to reduce her holding to a given fraction $\underline{\alpha} > 0$ is increasing in λ and de-

[19] The case of deviation that causes a jump to 0 has been examined above. Deviation to 1 is examined separately below.

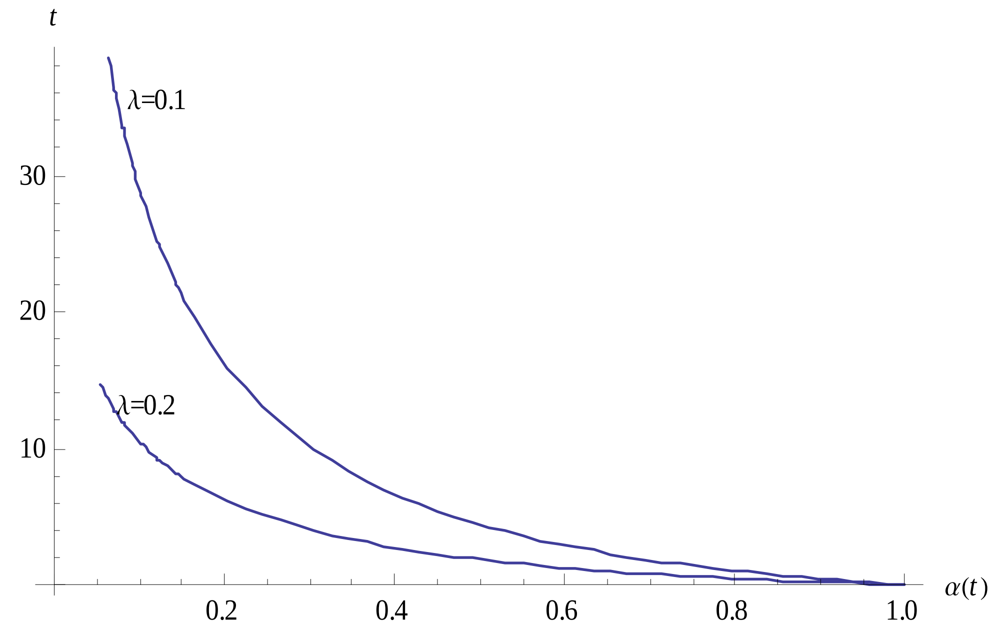

Figure 2: How long it takes to reach $\alpha = 0$.

creasing in r. Suppose for instance that she initially holds $\alpha = 1$, $\Omega = 0.2$ and $r = 0.05$. If $\lambda = 0.1$, the time it takes for α to falls to 0.1 is 29 years. If $\lambda = 0.2$, the corresponding time is 10.5 years. Figure 2 plots the time paths under these parameter values.

4.2 Markov perfect equilibrium in the parameter region T (concave $R(\alpha)$)

In region T, the instantaneous returns function $R(\alpha)$ is strictly concave for all $\alpha \in (0,1)$. Furthermore, since $W(0) = R'(0)$, it follows that $R(\alpha) < \alpha W(0)$ for all $\alpha > 0$. This means that, starting with α_0, if the large shareholder were to sell all her α_0 instantaneously, her share would be sold at the price $p = \frac{1}{nr}W(0)$, and her payoff (revenue from sales) would be $\frac{1}{nr}W(0)(n\alpha_0)$, which is strictly greater than $\frac{1}{r}R(\alpha_0)$, which is her payoff if she does not offer to sell her shares. This suggests that selling her shares gradually would be worse than selling them all off in one go. The following result confirms this intuition.

Lemma 2 *When $R(\alpha)$ is concave, the policy of gradual sales/purchases cannot be an equilibrium.*

Proof Suppose we were to try the HJB equation (31) as in the preceding subsection, then, under the assumption of gradualism, we would have come up with the implication that $\rho(\alpha) = \frac{1}{nr}R'(\alpha)$ and that $\dot{\alpha}/\alpha > 0$, because $\frac{1}{nr}R''(\alpha) < 0$ in region T. This would implies that, given α_0, the large shareholder will purchase additional shares gradually. In particular, as $\alpha \to 1$, the equation reduces to

$$\frac{\dot{\alpha}}{\alpha} = \frac{r\left(\lambda + \Omega(1-\lambda)^2(1+\lambda)\Omega^2(1-\lambda)^2\right)\left(1+\Omega(1-\lambda)^2\right)}{\left[-1+2\lambda+2\Omega(1-\lambda)^2(1+\lambda)\right]\Omega^2(1-\lambda)^2}$$

Since $\lambda > 1/2$ in region T, the right hand side is a constant, implying that α will reach 1 at some finite t_1. We now show that the implied share price path $p(t)$ would not satisfy the rational expectations requirement (28). Indeed, for all $t < t_1$, we would have

$$p(t) = \rho(\alpha(t)) = \frac{1}{nr}R'(\alpha(t))$$

Therefore

$$\lim_{t \uparrow t_1} p(t) = \frac{1}{nr}R'(1).$$

But, from (28) and (18),

$$p(t_1) = \frac{1}{n}\int_t^{\infty} \exp(-r(t-\tau))W(1)d\tau = \frac{W(1)}{nr} > \frac{1}{nr}R'(1).$$

This implies an upward jump in shares price at time t_1, which is not consistent with rational expectations. (Atomistic investors, expecting such an upward jump, would refuse to sell their shares before t_1, defeating the large shareholder's gradual purchase scheme.)■

Proposition 3 *When the parameter vector (λ, Ω) is in region T, the unique Markov perfect equilibrium consists of the price function $\rho(\alpha) = \frac{1}{nr}W(0)$ for all $\alpha \in [0,1]$ and the lumpy sale strategy $L(\alpha) = \alpha$ for all $\alpha \in [0,1]$. The payoff to the large shareholder is*

$$J(\alpha) = \frac{\alpha W(0)}{r} > \frac{R(\alpha)}{r} \tag{35}$$

where the inequality is strict for all $\alpha \in (0,1]$.

Proof

(i) Given the lumpy sale strategy $L(\alpha) = \alpha$ (i.e., given that the large shareholder sells off all her shares at the initial instant), rational expectations imply that the atomistic traders must hold linear price function $\rho(\alpha) = \frac{1}{nr}W(0)$.

(ii) Given the linear price $\rho(\alpha) = W(0)/(nr)$, the large shareholder's problem is to maximize, given any $\alpha_0 \in [0,1]$,

$$J(\alpha_0) = \max \int_0^\infty \exp(-rt)\left[R(\alpha(t)) - n\dot{\alpha}(t)\rho(\alpha(t))\right] dt = \int_0^\infty \exp(-rt)\left[R(\alpha(t)) - \frac{1}{r}\dot{\alpha}(t)W(0)\right] dt$$

Integration by parts, noting that $\alpha(t)$ is bounded, yields

$$\int_0^\infty \dot{\alpha}(t)\left[\frac{1}{-r}\exp(-rt)\right] dt = \frac{\alpha(0)}{r} - \int_0^\infty \alpha(t)\exp(-rt)dt = \int_0^\infty \left[\alpha_0 - \alpha(t)\right]\exp(-rt)dt$$

Thus

$$J(\alpha_0) = \max \int_0^\infty \exp(-rt)\left[R(\alpha(t)) + (\alpha_0 - \alpha(t))\, W(0)\right] dt$$

Since $R(\alpha) + (\alpha_0 - \alpha)W(0)$ is strictly concave in α when the parameter vector (λ, Ω) is in region T, the solution is trivially to set $R'(\alpha(t)) = W(0)$, i.e. $\alpha(t) = 0$ for all t. This means a downward jump in α at time zero.

It follows that $J(\alpha_0) = (\alpha_0/r)W(0)$. Since $R(\alpha)$ is strictly concave and $R'(0) = W(0)$ by equation (18), we obtain (35).

(iii) gradual sales/purchases cannot be an equilibrium, as showed in Lemma 2.■

The intuition behind Proposition 3 is as follows. In Region T, the interests of the manager and the shareholders diverge only mildly. The overall tendency is to reduce the large shareholder's stake in the firm. Since $R(\alpha)$ is concave, if she were to sell her shares gradually, the required share price function would be decreasing in α, which would imply that it would pay to reduce α to zero as quickly as possible so as to get the highest possible price.

4.3 Markov perfect equilibrium when $R(\alpha)$ is S-shaped

When the vector of parameter (λ, Ω) is in region A, the instantaneous returns function $R(\alpha)$ is convex in the range $[0, \widetilde{\alpha}]$ and concave in the range $(\widetilde{\alpha}, 1]$ where $\widetilde{\alpha}$ is defined by (22).In this region, the interests of the manager and the shareholders diverge sharply, as in region X, but the absolute possible benefits for both parties are larger than in region X. From our analysis in the two preceding sub-sections, it becomes clear that the Markov perfect equilibrium in this case would be for the large shareholder to make an initial lumpy sale of part of her stock, if $\alpha_0 > \widetilde{\alpha}$. Once this asset position $\widetilde{\alpha}$ is reached, she will start a gradually sale policy, liquidating her shares asymptotically. We formalize this in Proposition 4.

Proposition 4 *When the vector of parameter (λ, Ω) is in region A, then if $\alpha > \widetilde{\alpha}$ the large shareholder will divest immediately in a lumpy fashion the fraction of her stock in excess of $\widetilde{\alpha}$. Afterwards, she gradually divests the remaining shares. The atomistic investors hold the following price expectation rule*

$$\rho(\alpha) = \begin{cases} \frac{1}{nr} R'(\widetilde{\alpha}) & \text{if } \alpha \in [\widetilde{\alpha}, 1] \\ \frac{1}{nr} R'(\alpha) & \text{if } \alpha \in [0, \widetilde{\alpha}] \end{cases} . \tag{36}$$

The value function of the large shareholder is

$$J(\alpha) = \begin{cases} \frac{1}{r} R(\widetilde{\alpha}) + (\alpha - \widetilde{\alpha}) \frac{U'(\widetilde{\alpha})}{r} & \text{if } \alpha \in [\widetilde{\alpha}, 1] \\ \frac{1}{r} R(\alpha) & \text{if } \alpha \in \alpha \in [0, \widetilde{\alpha}] \end{cases} .$$

The lumpy sale function is $L(\alpha) = \alpha - \widetilde{\alpha}$ for all $\alpha \in [\widetilde{\alpha}, 1]$. The gradual sale rule is

$$g(\alpha) = -\dot{\alpha} = \frac{\alpha \left(\lambda + \alpha\Omega(1-\lambda)^2(1+\lambda)\right) \Omega^2 (1-\lambda)^2}{\left(1 + \alpha\Omega(1-\lambda)^2\right)^3 N \rho'(\alpha)} > 0 \text{ for } \alpha \in \alpha \in [0, \widetilde{\alpha}] \tag{37}$$

Proof

(i) Given that $L(\alpha) = \alpha - \widetilde{\alpha}$ for all $\alpha \in [\widetilde{\alpha}, 1]$, rational expectations on the part of atomistic investors imply that $\rho(\alpha) = \rho(\widetilde{\alpha})$ for all $\alpha \in [\widetilde{\alpha}, 1]$. Given $g(\alpha)$ defined by (37), for all α in $[0, \widetilde{\alpha}]$, the same argument as that used in the proof of Proposition 2 applies to show that the price rule $\rho(\alpha) = \frac{1}{nr} R'(\alpha)$ satisfies the rational expectations requirement.

(ii) Given (36), any deviation by the large shareholder inplying a discontinuous variation in α in the interval $[0, \widetilde{\alpha}]$ can be ruled out, as was shown in the proof of Proposition 2. Given that $\rho(\alpha)$ is constant in $[\widetilde{\alpha}, 1]$, any deviation implying a jump in α from one value to another value $\widehat{\alpha} \neq \widetilde{\alpha}$ is ruled out by an argument similar to that used in part (ii) of the proof of Proposition 3.

(iii) Given (36), any deviation by the large shareholder implying an upward jump from some $\alpha' < \widetilde{\alpha}$ to some $\alpha'' > \widetilde{\alpha}$ would yield a present value equal to $\frac{1}{r}[R(\alpha'') - R'(\widetilde{\alpha})(\alpha'' - \alpha')]$ whereas sticking to the candidate equilibrium yields $\frac{1}{r}R(\alpha')$. We can show that this *is* not profitable, because, from the convexity of R for $\alpha < \widetilde{\alpha}$, it holds that $\frac{1}{r}R(\alpha') > \frac{1}{r}[R(\widetilde{\alpha}) - R'(\widetilde{\alpha})(\widetilde{\alpha} - \alpha')]$, while from the concavity of R for $\alpha > \widetilde{\alpha}$, it holds that $\frac{1}{r}[R(\widetilde{\alpha}) - R'(\widetilde{\alpha})(\widetilde{\alpha} - \alpha')] > \frac{1}{r}[R'(\alpha'') - R'(\widetilde{\alpha})(\alpha'' - \alpha')]$.

Finally, consider a deviation that implies a downward jump from some $\alpha' > \widetilde{\alpha}$ to some $\alpha'' < \widetilde{\alpha}$. This would yield a value $J(\alpha'') + (\alpha' - \alpha'')J'(\alpha'')$, which, given the convexity of J for values of $\alpha \leq \widetilde{\alpha}$, is lower than the value $J(\widetilde{\alpha}) + (\alpha' - \widetilde{\alpha})J'(\widetilde{\alpha})$ obtained by following the equilibrium path. These arguments also rule out any other candidate equilibrium.■

5 Conclusions

We have shown that a large shareholder divests her shares of because, in the absence of share trading, there would exist a wedge between her marginal returns on holding these assets and the atomistic investors' valuation of a share. This wedge arises because the atomistic investors free ride on her monitoring effort which is aimed at reducing the manager's opportunistic behavior (such as choosing projects that are more advantageous to him than to the shareholders). As she divests, the manager increases his effort, but in general this is to the detriment of the firm's profit stream. (We show that $W'(\alpha) > 0$ for $\alpha < \alpha_2^*$). This evolution toward a pure managerial firm, in which the owners do not monitor the manager, can be gradual or immediate, depending on the degree of incongruence of the manager's interest to that of the owners.

One may wonder whether this tendency for disperse ownership would disappear if the owners can propose incentive contracts to the manager. Burkart, Gromb and Panaunzi (1997) have already answered this question by showing that, depending on parameter values, there nevertheless remains in that case some scope for monitoring and a negative relationship between the manager's effort and the large shareholder's stake in the firm.

Another question is why the large shareholder does not manage the firm herself instead of hiring a manager, for then she would have no incentive to divest her shares.[20] A possible answer is that she may lack managerial skills, or she may not have time.

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