# Per-Unit Royalty vs Fixed Fee: The Case of Weak Patents<sup>\*</sup>

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#### Abstract

This paper explores a licensor's choice between charging a per-unit royalty or a fixed fee for an innovation covered by a weak patent, i.e. one that is likely to be invalidated by a court if challenged. Using a general (reduced-form) model of competition, we show that, independently of whether the patent holder is an industry insider or outsider, a per-unit royalty scheme dominates a fixed fee if the *strategic* effect of an increase in a potential licensee's unit cost on the equilibrium industry profit is positive. The latter condition is shown to hold in a Cournot (resp. Bertrand) oligopoly with homogeneous (resp. differentiated) products under very general assumptions on the demands faced by firms. As a byproduct of our analysis, we contribute to the oligopoly literature by offering some new insights of independent interest regarding the effects of cost variations on Cournot and Bertrand equilibria.

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### 1 Introduction

Since the seminal contribution by Arrow (1962), analyzing the licensing contract whereby a patent holder sells the right to use a protected technology has become an important topic in the economics of innovation and technology diffusion. Arrow compared the revenues that an outside innovator obtains from licensing a cost-reducing innovation to a competitive industry and to a monopolistic industry. He showed that when a per unit royalty is charged, a perfectly competitive industry generates higher licensing revenues than a monopolistic one.<sup>1</sup> Subsequently, Katz and Shapiro (1985, 1986), Kamien and Tauman (1984, 1986) and Kamien (1992) compared the three licensing policies of fixed fee, auction and per-unit royalty when the buyers are members of an *oligopoly*. The main conclusion one can draw from this literature is that various strategic factors shape the optimal choice of the licensing contract.

For instance, in the homogenous product oligopoly situation, the superiority of the upfront fee and the auction mechanism over the per-unit royalty prevails when the licensor is an outsider and the licensees compete in quantities. The main intuition behind this result is the following: a per-unit royalty increases the marginal cost of production for each licensee, thus lowering the overall surplus that has to be divided between the licensor and the licensees. The fixed fee avoids this drawback. However, when the licensor is an insider to the industry, one reaches the reverse result. The per-unit royalty scheme dominates the fixed fee scheme in this case because it gives the licensor a cost advantage against its competitors, while the use of a fixed fee maintains a level playing field between the licensor and the licensees (Shapiro, 1985; Wang, 1998, 2002; Kamien and Tauman, 2002; Sen, 2002; Sen and Tauman, 2007).<sup>2</sup>

The comparison of different licensing schemes also depends on the strategic variables and the degree of substitution between the competing products. Under price competition with differentiated products, a per-unit royalty contract generates higher licensing revenues than a fixed fee contract when the products are close substitutes or, if not, when the size of the cost-reduction is small (Kamien and Tauman, 1986; Kamien *et al.*, 1987; Muto,1993). If the products are weak substitutes and the innovation is large, licensing through a fixed fee is optimal. The results are quite different under quantity competition: the fixed fee scheme dominates the per-unit royalty scheme when products are sufficiently differentiated (Wang and Yang, 1999; Poddar and Sinha, 2004; Stamatopoulos and Tauman, 2007).

A common feature of all this literature is that patents are viewed as certain or "ironclad" rights, the validity of which is unquestionable. This clearly contradicts what we observe

<sup>&</sup>lt;sup>1</sup>This is a consequence of the well known *replacement effect*, according to which the willingness to pay an innovation is larger for an entrant in a competitive industry than for an incumbent firm.

<sup>&</sup>lt;sup>2</sup>Empirically, royalties seem to be more often used than fixed fees (Taylor and Silberstone, 1973; Rostoker, 1984). However, available data on patent licensing is very limited, because most firms elect not to disclose their private licensing contracts. Most empirical investigations emphasize the factors that affect the likelihood of firms to engage in licensing agreements but are less informative on the licensing scheme (Annand and Khanna, 2000; Vonortas and Kim, 2004, Zuniga and Guellec, 2009)

in practice: about half of the patents that are challenged before US courts are invalidated (Allison & Lemley, 1998).<sup>3</sup> It is now largely recognized that a patent is not a perfectly enforceable right, as are other forms of property. Patents correspond much more to "uncertain or probabilistic rights because they only give a limited right to try to exclude by asserting the patent in court" (Ayres and Klemperer, 1999; Shapiro, 2003; Lemley and Shapiro, 2005). Moreover, this uncertainty is strengthened by the fact that many inventions are granted patent protection by the patent office (PO) even though they do not meet one or several of the statutory requirements: belonging to the patentable subject matters, utility, novelty and non-obviousness (or inventiveness). Such patents are *weak* in the sense that they are likely to be invalidated by a court if challenged by a third party.<sup>4</sup>

The proliferation of *bad-quality* or *weak* patents can be explained by several reasons. First, the major patent offices (USPTO, EPO and JPO) have insufficient resources to ensure an effective review process for the huge and growing number of patent applications (Friebel *et al.*, 2006). Second, mistakes are unavoidable because the patentability requirements, in particular novelty and non-obviousness, are difficult to assess especially for newly patentable subject matters, such as software, business methods and research tools. Third, the incentives provided to the examiners are inadequate for making them fully prosecute and reject the applications that do not meet the standards (Farrell and Merges, 2004; Langinier and Marcoul, 2009). Finally, the continuing debate on patentable subject matters throws additional uncertainty into the validity of many patents (Guellec and van Pottelsberghe, 2007).

The main contribution of this paper is to investigate the optimal licensing scheme from the perspective of a licensor holding a weak patent and to test whether it is affected by the features that matter in the case of an ironclad patent. More precisely, we examine a licensor's decision to charge the users of its technology either a per-unit royalty or a fixed fee when licensing is made prior to the patent's validity assessment and under the shadow of patent litigation. We thus extend three strands of literature:

1. The vast literature that compares various licensing schemes to sell the right to use an invention protected by an unquestionable patent. Our contribution is to extend this comparison to the situation where the invention is protected by a probabilistic right, and more precisely a patent whose probability of being invalidated if challenged is high. We believe that such an

<sup>&</sup>lt;sup>3</sup>This concerns the patent disputes that are not settled prior to the court judgement.

<sup>&</sup>lt;sup>4</sup>The notion of "weak patent" has at least two different meanings in the literature (Ginarte and Park, 1997, van Pottelsberghe de la Potterie, 2010). First, a patent can be said to be weak if it gives its holder a low protection against imitators and other potential infringers, either because the patent's scope is badly defined or because the protection extends to countries in which the enforcement of intellectual property rights is low. Second, a patent can be qualified as weak when it is likely that it does not satisfy at least one of the patentability requirements (patentable subject matter, utility, novelty, non obviousness or inventiveness in the European terminology). In this sense, a weak patent is likely to be invalidated by a court if it is challenged by a third party. This paper focuses on the latter meaning.

extension is warranted in light of the growing proliferation of weak patents.<sup>5</sup> We first provide a sufficient condition under which the holder of a weak patent prefers to charge a per unit royalty rather than a fixed fee. This condition has a very natural economic interpretation: It states that the strategic effect of an increase in an oligopolist's unit cost on the aggregate profit of the industry is positive. This result is very general since the type of competition between the potential users of the technology is not specified in our model except for assuming the existence and uniqueness of a Nash equilibrium to the competition models. Then we show that the sufficient condition above holds and, therefore the per-unit royalty scheme is optimal from the patent holder's perspective, in a wide range of settings.<sup>6</sup> A key difference with the literature considering ironclad patents is that the optimality of a per-unit royalty contract for the licensing of weak patents is independent of the type of downstream competition, the degree of differentiation between products and whether the patent holder is active or not in the downstream market.

2. The burgeoning literature on the licensing of uncertain patents (e.g. Farrell and Shapiro, 2008; Encaoua and Lefouili, 2005, 2009; Choi 2010). This literature has been mostly concerned with the inefficiencies stemming from the low private incentives to litigate a weak patent. As the social harm of weak patents depends on the way they are licensed out<sup>7</sup>, we think it is crucial to get a better understanding of the licensing contracts the holders of those problematic patents would prefer to use.

3. The extensive literature on oligopoly. We contribute to this literature by providing new insights regarding the effects of cost variations on oligopolists' profits (Seade, 1985 and Linnemer and Fevrier, 2004). Since our main result, that the per-unit royalty scheme is preferred by the holder of a weak patent, holds whenever the strategic effect of a unilateral cost increase on industry profits is **positive**, we show that the latter condition holds (i) for Cournot competition with homogenous products under complete generality, and (ii) for Bertrand competition with differentiated products under strategic complementarity. In other words, no restrictions on the demand systems are needed beyond those commonly needed to establish existence of a pure-strategy equilibrium. **An ancillary result is that we provide** 

<sup>&</sup>lt;sup>5</sup>Bessen and Meurer (2008) argue that the inclusion of software and business methods in the patentable subject matters in the US has resulted in an increase in the share of weak patents among all the patents issued by the USPTO.

<sup>&</sup>lt;sup>6</sup>This is a novel justification, based on the uncertainty over patent validity, for the use of per-unit royalties instead of fixed fees in licensing. Various other reasons have been explored in the literature on ironclad rights, including risk aversion (Bousquet et al., 1998), asymmetry of information (Gallini and Wright, 1990; Macho-Stadler and Pérez-Castrillo, 1991; Beggs, 1992; Poddar and Sinha, 2002; Sen 2005a), moral hazard (Macho-Stadler et al., 1996; Choi, 2001), production differentiation in a circular market (Caballero-Sanz et al., 2002), strategic delegation (Saracho, 2002), integer nature of the number of licensees (Sen, 2005b), variation in the quality of innovation (Rockett, 1990).

<sup>&</sup>lt;sup>7</sup>In particular, a licensing contract involving the payment of a fixed fee results in a lower price and therefore a higher consumers' surplus than a contract involving a per-unit royalty because the latter can be passed on (at least partially) to consumers.

#### a lower bound on the effect of a unilateral cost increase on industry profits.

The closest papers to ours are Farrell and Shapiro (2008) and Encaoua and Lefouili (2009) who also deal with the licensing of weak patents in the shadow of patent litigation, though with objectives that are different from the present paper's. In each of those two papers, the licensor is assumed to offer two-part tariff contracts but some of the results are derived under a technical *ad hoc* assumption on the shape of the licensing revenue function which immediately ensures that, for weak patents, pure per-unit royalty licenses are optimal from the patent holder's perspective in the class of two-part tariff licensing contracts (with nonnegative fixed fees) deterring litigation and, therefore, substantially simplifies the analysis.<sup>8</sup> Our main result that the per-unit royalty licensing scheme is preferred over the fixed fee scheme by the holder of a weak patent (in a very wide range of settings) cannot be seen as a mere corollary of the latter optimality statement: In sharp contrast to the former papers, it is obtained under a much weaker condition which has a natural economic interpretation and is shown to hold with broad generality in standard oligopoly models with general demand functions (as are all the assumptions on the equilibrium profit functions made in our reducedform model of competition). Therefore, our result that the licensor of a weak patent is better off charging per-unit royalties rather than fixed fees is arguably robust, especially as it is shown to hold as well in two extensions of the baseline model.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes the optimal per-unit royalty and fixed fee license from the patent holder's perspective. In Section 4 we derive our main result on the comparison between the two licensing schemes for weak patents. In Section 5, we extend the analysis, first by including (small) litigation costs borne by the challenger and, second, by considering a patent holder that is active on the downstream market. In section 6, we show that the general assumptions made on the equilibrium profits in our reduced-form model of competition and the (sufficient) condition under which the per-unit royalty scheme is preferred over the fixed fee scheme by the patent holder (be it an outsider or an insider) hold under general conditions for both a Cournot oligopoly with homogenous goods and a Bertrand oligopoly with differentiated goods. Section 7 concludes.

# 2 The Model

We consider an industry consisting of  $n \ge 2$  symmetric risk-neutral firms producing at a marginal cost  $\bar{c}$  (fixed production costs are assumed to be zero). A firm P outside the industry holds a patent covering a technology that, if used, allows a firm to reduce its marginal cost

<sup>&</sup>lt;sup>8</sup>Farrell and Shapiro (2008) assume that the licensing revenue function where the fixed fee is set to its optimal level (for a given per-unit royalty) is single-peaked in the per-unit royalty while Encaoua and Lefouili (2009) assume that the same function is concave in the per-unit royalty (see assumption A6 in their paper).

from  $\bar{c}$  to  $\bar{c} - \epsilon$ .

We consider the following three-stage game:

First stage: The patent holder P proposes to all firms a licensing contract whereby a licensee can use the patented technology against the payment of a per-unit royalty  $r \in [0, \epsilon]$  or a fixed fee  $F \ge 0.9$ 

Second stage: The *n* firms in the industry simultaneously and independently decide whether to purchase a license. If a firm does not accept the license offer, it can challenge the patent's validity before a court.<sup>10</sup> The outcome of such a trial is uncertain: with probability  $\theta > 0$  the patent is upheld by the court and with probability  $1 - \theta$  it is invalidated. Hence, the parameter  $\theta$  may be interpreted as the patent's quality. If the patent is upheld, then a firm that does not purchase the license uses the old technology, thus producing at marginal cost  $\bar{c}$  whereas a firm that accepted the license offer uses the new technology and pay the per-unit royalty r or the fixed fee F to the patent holder. If the patent is invalidated, all the firms, including those that accepted the license offer can use for free the new technology and their common marginal cost is  $\bar{c} - \epsilon$ .

Third stage: The n firms produce under the cost structure inherited from the second stage. We do not specify the type of competition that occurs. We only assume that there exists a unique equilibrium of the competition game for any cost structure and we set some general assumptions on the equilibrium profit functions.<sup>11</sup> For this purpose, denote  $\pi^e(k, c)$ (respectively  $\pi^i(k, c)$ ) the equilibrium profit function, gross of any potential fixed cost (e.g. fixed license fee), of a firm producing with marginal cost  $c \leq \bar{c}$  (respectively with marginal cost  $\bar{c}$ ) when  $k \leq n$  firms produce at marginal cost c and the remaining n - k firms produce at the marginal cost  $\bar{c}$ .

We now make the following general assumptions for any given n and k = 1, ..., n.

A1. The equilibrium profits of an efficient firm and an inefficient firm, i.e.  $\pi^{e}(k,c)$  and  $\pi^{i}(k,c)$  respectively, are both continuously differentiable in c over the subset of  $[0, \bar{c}]$  in which  $\pi^{i}(k,c) > 0$ . Furthermore, the output function  $c \to q^{e}(n,c)$  is continuously differentiable and strictly positive over  $[0, \bar{c}]$ .

A2. If the firms are symmetric (in terms of efficiency), an identical increase in all firms' marginal costs leads to a decrease in each firm's equilibrium profit:  $\frac{\partial \pi^e}{\partial c}(n,c) < 0$ .

**A3.** An inefficient firm's equilibrium profit is increasing in the efficient firms' marginal cost: If  $\pi^i(k,c) > 0$  then  $\frac{\partial \pi^i}{\partial c}(k,c) > 0$  and if  $\pi^i(k,c) = 0$  then  $\pi^i(k,c') = 0$  for any c' < c.

<sup>&</sup>lt;sup>9</sup>Following Farrell and Shapiro (2008) and Encaoua and Lefouili (2009), we focus on take-it-or-leave-it license offers.

<sup>&</sup>lt;sup>10</sup>For patents granted by the European Patent Office (EPO), the timing is slightly different. Indeed, any patent issued by the EPO can be opposed by a third party and the notice of opposition must be filed in writing at the EPO within nine months from the publication of the mention of the grant of the European patent.

<sup>&</sup>lt;sup>11</sup>A similar general approach was previously adopted by Reinganum (1982) in a strategic search model, and by Amir and Wooders (2000), Boone (2001) and Encaoua and Lefouili (2009) in different R&D models.

**A4.** A firm's profit is decreasing in the number of efficient firms in the industry: for any  $c < \overline{c}$  and any k < n it holds that  $\pi^e(k, c) > \pi^e(k+1, c)$  and  $\pi^i(k, c) \ge \pi^i(k+1, c)$ .

**A5.** A firm's profit increases as it moves from the subgroup of inefficient firms to the subgroup of efficient firms: for any  $c < \bar{c}$  and any k < n it holds that  $\pi^i(k, c) < \pi^e(k+1, c)$ .

As we shall argue in precise detail in Section 6, all these assumptions are satisfied with broad generality in the standard oligopoly models with general demand functions. In particular, these assumptions are clearly satisfied for instance for the widely used settings of Cournot competition with homogeneous goods and linear demand and Bertrand competition with differentiated goods and linear demands.

# 3 Licenses deterring litigation

If litigation occurs then, with probability  $\theta$ , the patent is upheld by the court (thus becoming an ironclad right) and, with probability  $1 - \theta$ , it is invalidated and the technology can be used for free by all firms. Thus, if the patent holder expects its license offer to trigger litigation, it should make an offer that maximizes its revenues should the patent be ruled valid by the court. The patent holder would then essentially act as if the patent were ironclad, and the determination of the terms on which the technology is patented under each licensing scheme would amount to the analysis of licensing offers for ironclad patents, which has already been done extensively in the literature. We therefore consider in what follows only the class of license offers deterring litigation, for which the comparison of the two schemes cannot be derived trivially from that under ironclad patent protection.<sup>12</sup>

Since we look for the subgame perfect equilibria of the game in which litigation is deterred, we start our analysis by determining under each scheme the license offers that do not induce litigation at the second stage of the game.

### 3.1 Per-unit royalty licenses

Let us first examine a firm's incentives to challenge the patent's validity when the patent holder makes a license offer involving the payment of a per-unit royalty r. A firm that decides not to purchase a license is always (weakly) better off challenging the patent's validity: If no other firm challenges the patent's validity it gets a payoff  $\theta \pi^i (n-1, \bar{c}-\epsilon+r)+(1-\theta)\pi^e(n, \bar{c}-\epsilon)$  which is strictly greater than the profit  $\pi^i (n-1, \bar{c}-\epsilon+r)$  it would get by not challenging the patent, and if some other firm challenges the patent's validity then it is indifferent between challenging and not. Thus, a situation where one or more firms do not buy a license and no

 $<sup>^{12}</sup>$ In doing so, we follow Farrell and Shapiro (2008) who also focus on the licenses that are not litigated because they aim to investigate the social costs of the uncertainty over patent validity (which is resolved if litigation occurs). Moreover, there is empirical evidence that the vast majority of patent disputes are settled using licensing agreements before the court decides whether the patent is valid or not (see for instance Allison and Lemley, 1998 and Lemley and Shapiro, 2005).

firm challenges the patent's validity can never be an equilibrium of the second stage subgame. It follows that a license offer deters litigation if and only if it is accepted by all firms.

Assume that the patent holder makes a license offer (in the first stage) involving the payment of a per-unit royalty  $r < \epsilon$ . Let us show that in this case, any outcome with  $k \le n-2$  licensees cannot be an equilibrium. We have already shown that a situation where not all firms buy a license and no firm challenges the patent cannot be an equilibrium so we can focus on situations with  $k \le n-2$  licensees and at least one non-licensee challenging the patent. Any of the other  $n-k-1 \ge 1$  non-licensees has an incentive to unilaterally deviate by buying a license since it would get an expected profit of  $\theta \pi^e(k+1, \bar{c}-\epsilon+r) + (1-\theta) \pi^e(n, \bar{c}-\epsilon)$  instead of the strictly lower expected profit  $\theta \pi^i(k, \bar{c}-\epsilon+r) + (1-\theta) \pi^e(n, \bar{c}-\epsilon)$  if it remains a non-licensee. Therefore, any equilibrium of the second stage subgame involves at least n-1 firms if  $r < \epsilon$ . The latter result extends to the case  $r = \epsilon$  if it is assumed, as will be the case from now on, that a firm which is indifferent between getting a license and not buying a license a license.

Let us now write the condition under which all firms accepting the license offer r is an equilibrium of the second stage subgame. A firm anticipating that all other firms will purchase a license gets a profit equal to  $\pi^e(n, \bar{c} - \epsilon + r)$  if it accepts the license offer. If it does not and challenges the patent's validity then with probability  $\theta$ , the patent is upheld by the court and the challenger gets a profit equal to  $\pi^i(n-1, \bar{c} - \epsilon + r)$  and, with probability  $1 - \theta$ , the challenger gets a profit of  $\pi^e(n, \bar{c} - \epsilon)$  (and so do all other firms). Thus, a firm challenging the patent's validity when all other firms accept the license offer, gets an expected profit of  $\theta \pi^i(n-1, \bar{c} - \epsilon + r) + (1-\theta) \pi^e(n, \bar{c} - \epsilon)$ . Therefore, all firms accepting the license offer is an equilibrium if and only if:

$$\pi^{e}(n,\bar{c}-\epsilon+r) \ge \theta \pi^{i}(n-1,\bar{c}-\epsilon+r) + (1-\theta) \pi^{e}(n,\bar{c}-\epsilon)$$
(1)

The next proposition characterizes the values of the per-unit royalty set by the licensor that induce all firms to buy a license (thus deterring any litigation).

**Proposition 1** Define  $r(\theta)$  as the unique solution in r to the following equation:

$$\pi^{e}(n,\bar{c}-\epsilon+r) = \theta\pi^{i}(n-1,\bar{c}-\epsilon+r) + (1-\theta)\pi^{e}(n,\bar{c}-\epsilon)$$

Then all firms buying a per-unit royalty license is an equilibrium if and and only if  $r \leq r(\theta)$ . Moreover, this is the unique equilibrium of the second stage subgame when  $r \leq r(\theta)$ .

**Proof.** See Appendix.

#### **3.2** Fixed fee licenses

The previous observation that a license offer deters litigation if and only if it is accepted by all firms remains true when the licensor uses a fixed fee scheme. For a license offer involving the payment of a fixed fee F to be accepted by all firms, the following condition must hold:

$$\pi^e(n,\bar{c}-\epsilon) - F \ge \theta \pi^i(n-1,\bar{c}-\epsilon) + (1-\theta) \pi^e(n,\bar{c}-\epsilon)$$

which can be rewritten as:

$$F \le \theta \left[ \pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon) \right] = F(\theta)$$
(2)

The next proposition is the equivalent of Proposition 1 for fixed fee licenses.

**Proposition 2** All firms accepting to pay the fixed fee F to use the patented technology is an equilibrium if and only if  $F \leq F(\theta) = \theta[\pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon)]$ . Moreover, for sufficiently small values of  $\theta$ , i.e. for sufficiently weak patents, all firms accepting the license offer is the unique equilibrium when  $0 \leq F \leq F(\theta)$ .

**Proof.** See Appendix.

#### 3.3 Optimal license under each scheme

The patent holder's licensing revenues are given by  $nrq^e(n, \bar{c} - \epsilon)$  if a per-unit royalty contract is used and by nF if the contract takes the form of a fixed fee payment. The next proposition characterizes the license that maximizes the patent holder's revenues subject to the constraint that all firms decide to purchase a license.

**Proposition 3** For sufficiently weak patents, the optimal per-unit royalty license deterring litigation involves the payment of the royalty rate  $r = r(\theta)$  and the optimal fixed fee license deterring litigation consists of setting  $F = F(\theta)$ .

**Proof.** See Appendix.

### 4 Optimal licensing scheme

Before stating formally our main result let us explain why one could expect the uncertainty over patent validity to increase the attractiveness of per-unit royalties with respect to fixed fees relative to the case of ironclad patents, i.e.  $\theta = 1$ . As shown in Farrell and Shapiro (2008) and Encaoua and Lefouili (2009), the use of per-unit royalties may allow the patent holder to deter litigation while getting licensing revenues which are higher than the expected licensing revenues it would earn if the patent validity issue were resolved before licensing. The reason is that the free riding problem arising from the public good nature of patent invalidation combined with the non-linearity of equilibrium profits with respect to per-unit royalties can make the licensing revenues grow more than proportionally with respect to the patent strength  $\theta$  for small values of this parameter (i.e. for weak patents). Proposition 3 however shows that this cannot happen if a fixed fee license is used instead: the optimal fixed fee deterring litigation is *linear* in the patent strength  $\theta$  which makes the expected licensing revenues the patent holder derives from the optimal fixed fee license deterring litigation also linear in  $\theta$ .

The next proposition provides a sufficient condition for the per-unit royalty scheme to be preferred over the fixed fee scheme by the holder of a weak patent.

**Proposition 4** The optimal per-unit royalty scheme deterring litigation provides higher licensing revenues than the optimal fixed fee scheme deterring litigation for sufficiently weak patents if :

$$\frac{\partial \pi^e}{\partial c}(n,\bar{c}-\epsilon) > -q^e(n,\bar{c}-\epsilon) \tag{3}$$

Moreover, the reverse holds if the reverse strict inequality is satisfied

#### **Proof.** See Appendix.

Condition (3) means that the *strategic* effect of an identical increase in the marginal cost of all (symmetric) firms on the firms' profits is positive.<sup>13</sup> This interpretation results from the following decomposition (where we use the generic variable c instead of  $\bar{c} - \epsilon$  as the second argument of the considered functions) :

$$\frac{\partial \pi^e}{\partial c}(n,c) = \underbrace{-q^e(n,c)}_{direct\ effect} + \underbrace{q^e(n,c)\frac{\partial p^e}{\partial c}(n,c) + (p^e\ (n,c) - c)\frac{\partial q^e}{\partial c}(n,c)}_{strategic\ effect}$$

An increase in all firms' marginal cost c affects their equilibrium profits  $\pi^e(n, c)$  through two channels. First, it yields an increase in each firm's production costs (for a given output). Second, it entails an adjustment of their outputs and/or prices. The first effect, captured by the term  $-q^e(n, c)$ , can be interpreted as a *direct* effect of a common cost increase on equilibrium profits while the second effect, captured by the term  $q^e(n, c) \frac{\partial p^e}{\partial c}(n, c) + (p^e(n, c) - c) \frac{\partial q^e}{\partial c}(n, c)$ , can be interpreted as the *strategic* effect of a cost increase on profits.<sup>14</sup>

<sup>&</sup>lt;sup>13</sup>Another interpretation of the strategic effect may be given in terms of the Lerner index. The strategic effect is positive iff the competition intensity in the industry is such that the Lerner index is below the ratio of the price and quantity elasticities relative to the marginal cost. In this case, the patent holder of a weak patent obtains a higher revenue by licensing through a per-unit royalty rather than a fixed fee.

<sup>&</sup>lt;sup>14</sup>This strategic effect can be further split into a *price effect* captured by the term  $q^e(n,c)\frac{\partial p^e}{\partial c}(n,c)$  and an *output effect* captured by the term  $(p^e(n,c)-c)\frac{\partial q^e}{\partial c}(n,c)$ . In usual models of competition, the former is positive while the latter is negative. Therefore the strategic effect is positive if the price effect outweighs the output effect.

To the best of our knowledge, condition (3) has not been studied in the literature on the effects of cost variations on oligopolists' profits which has mainly focused on the *overall* effect of cost changes on profits (see e.g. Kimmel, 1992; Février and Linnemer 2004). In section 6 we show the mildness of this condition by establishing that it holds under weak assumptions in two of the most usual competition models, namely Cournot competition with homogenous goods and Bertrand competition with differentiated goods.

### 5 Extensions

#### 5.1 Litigation costs

Let us assume in this section that a firm that challenges the patent's validity before a court has to incur some legal costs  $C \ge 0.^{15}$  It is straightforward that the higher those costs the higher the licensing revenues the patent holder can extract from the licensees without triggering litigation. This qualitative observation holds under both schemes. However, we show in what follows that on the quantitative side, the marginal effect of litigation costs on the patent holder's licensing revenues is higher under the per-unit royalty scheme than under the fixed fee scheme if condition (3) holds. This implies that the result in Proposition 1 remains true - and is actually strenghtened - if the model is extended to include (small) legal costs that have to be incurred by a challenger.

Suppose first that the patent holder makes a license offer involving the payment of a perunit royalty  $r \in [0, \epsilon]$ . Note that the inclusion of legal costs in our setting does not affect the fact that the strategy "not buy a license *and* not challenge the patent's validity" is always dominated by the strategy "buy a license". Therefore, the only way a patent holder can deter litigation is to make a license offer that is accepted by all firms. This will be the case if and only if:

$$\pi^{e}(n,\bar{c}-\epsilon+r) \ge \theta \pi^{i}(n-1,\bar{c}-\epsilon+r) + (1-\theta) \pi^{e}(n,\bar{c}-\epsilon) - C$$

It is easily shown the latter constraint is met if and only if  $r \leq r(\theta, C)$  where  $r(\theta, C)$  is the solution in r to the equation:

$$\pi^{e}(n,\bar{c}-\epsilon+r) = \theta\pi^{i}(n-1,\bar{c}-\epsilon+r) + (1-\theta)\pi^{e}(n,\bar{c}-\epsilon) - C$$

and that, for  $\theta$  and C sufficiently small, the optimal per-unit royalty license deterring litigation involves the payment of the royalty  $r(\theta, C)$  (i.e. the constraint is binding). Note also that  $r(\theta, C)$  is strictly increasing in both its arguments.

<sup>&</sup>lt;sup>15</sup> If we consider the opposition procedure at the European Patent Office (EPO) instead of litigation before a court, the cost C can be interpreted as the administrative fee a challenger of a patent has to pay to the EPO for the patent to be reexamined.

Suppose now that the patent holder makes a license offer involving the payment of a fixed fee F. Such a license offer is accepted by all firms if and only if:

$$\pi^e(n,\bar{c}-\epsilon) - F \ge \theta \pi^i(n-1,\bar{c}-\epsilon) + (1-\theta) \pi^e(n,\bar{c}-\epsilon) - C$$

and, therefore, the optimal fixed fee license deterring litigation involves the payment of the fee:

$$F(\theta, C) = \theta \left[ \pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon) \right] + C$$

Let us now compare the licensing revenues derived by the patent holder under the two schemes. Under the optimal per-unit royalty scheme, they are given by

$$\tilde{P}_r(\theta, C) = nr(\theta, C) q^e(n, \bar{c} - \epsilon + r(\theta, C))$$

and under the optimal fixed fee licensing, they are given by:

$$\tilde{P}_F(\theta, C) = nF_n(\theta, C) = n\theta \left[\pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon)\right] + nC$$

Since  $\tilde{P}_r(0,0) = \tilde{P}_F(0,0)$ , a sufficient condition for the existence of  $\tilde{\theta} > 0$  and  $\tilde{C} > 0$  such that the inequality  $\tilde{P}_r(\theta, C) > \tilde{P}_F(\theta, C)$  hold any  $\theta < \tilde{\theta}$  and  $C < \tilde{C}$  is that:

$$\frac{\partial \tilde{P}_r}{\partial \theta} \left( 0, 0 \right) > \frac{\partial \tilde{P}_F}{\partial \theta} \left( 0, 0 \right)$$

and

$$\frac{\partial \tilde{P}_r}{\partial C}\left(0,0\right) > \frac{\partial \tilde{P}_F}{\partial C}\left(0,0\right)$$

The former inequality has already been shown to be equivalent to condition (3). Surprisingly, the latter inequality is equivalent to condition (3) too. Indeed,

$$\frac{\partial \tilde{P}_F}{\partial C}\left(0,0\right) = n$$

and

$$\frac{\partial \tilde{P}_r}{\partial C}(0,0) = n \frac{\partial r}{\partial C}(0,0) q^e(n,\bar{c}-\epsilon)$$

Differentiating with respect to C the equation defining  $r(\theta, C)$  at point  $(\theta, C) = (0, 0)$ , we get:  $\frac{\partial r}{\partial C}(0, 0) = -\frac{1}{\frac{\partial \pi^e}{\partial c}(n, \bar{c} - \epsilon)}$ . Thus,

$$\frac{\partial \tilde{P}_r}{\partial C}(0,0) = -\frac{nq^e(n,\bar{c}-\epsilon)}{\frac{\partial \pi^e}{\partial c}(n,\bar{c}-\epsilon)}$$

Hence

$$\frac{\partial \tilde{P}_r}{\partial C}(0,0) > \frac{\partial \tilde{P}_F}{\partial C}(0,0) \Longleftrightarrow \frac{\partial \pi^e}{\partial c}(n,\bar{c}-\epsilon) > -q^e(n,\bar{c}-\epsilon)$$

Therefore, the result in Proposition 4 is robust - and actually strengthened - in the presence of relatively small legal costs (borne by the challenger).

### 5.2 Internal patentee

Let us consider the case where the patent holder is active in the (downstream) market. More specifically, we assume that one of the *n* firms operating in the market, say firm 1, gets a patent on a technology that lowers the unit production cost from  $\bar{c}$  to  $\bar{c} - \epsilon$ . We assume that  $n \geq 3$  (as we want to have at least two potential licensees beside the patent holder).

Here again, we assume there exists a unique equilibrium of the competition game for any cost structure (with identical profits for firms producing at the same unit cost) and we set some general assumptions on the equilibrium profit functions. We focus on industry cost structures that can emerge following the licensing game, that is, situations in which: one firm - the patent holder - produces at unit cost  $\bar{c} - \epsilon$ , a number  $k \leq n - 1$  of firms - the licensees - produce at a unit cost  $c \in [\bar{c} - \epsilon, \bar{c}]$  and the remaining n - k firms - the non-licensees - produce at unit cost  $\bar{c}$ . We denote by  $\pi^p(k, c)$ ,  $\pi^l(k, c)$  and  $\pi^n(k, c)$  the equilibrium market profits of the patent holder, a licensee producing at an effective unit cost c and a non-licensee respectively.

We replace the assumptions A1-A5 made in our baseline model by the following assumptions: A1'. The equilibrium profits  $\pi^{p}(k, c)$ ,  $\pi^{l}(k, c)$  and  $\pi^{n}(k, c)$  are continuously differentiable in c over  $[0, \bar{c}]$  over the subset of  $[0, \bar{c}]$  in which  $\pi^{n}(c, k) > 0$ . Furthermore, the function  $c \to q^{l}(n, c)$  is continuously differentiable over the subset of  $[0, \bar{c}]$  in which it is strictly positive.

**A2'.** An identical increase in the costs of all firms but the patent holder decreases each one of those firms' equilibrium profit:  $\frac{\partial \pi^l}{\partial c}(n-1,c) < 0.$ 

**A3'.** A non-licensee's equilibrium profit is increasing in the licensees' unit cost: If  $\pi^n(k, c) > 0$ then  $\frac{\partial \pi^n}{\partial c}(k, c) > 0$  and if  $\pi^n(k, c) = 0$  then  $\pi^n(k, c') = 0$  for any c' < c.

**A4'.** A firm's market profit is decreasing in the number of licensees in the industry: for any  $c < \bar{c}$  and any k < n-1 it holds that  $\pi^p(k,c) > \pi^p(k+1,c)$ ;  $\pi^l(k,c) > \pi^l(k+1,c)$  and  $\pi^n(k,c) \ge \pi^n(k+1,c)$ .

**A5'.** A firm's market profit increases as it moves from the subgroup of non-licensees to the subgroup of licensees: for any  $c < \bar{c}$  and any k < n-1 it holds that  $\pi^n(k, c) < \pi^l(k+1, c)$ .

Let us compare the innovator's overall profit, i.e. the sum of its market profit and licensing revenues, under the two licensing schemes.

Under the per-unit royalty scheme, the optimal royalty  $r_I(\theta)$  for sufficiently weak patents is the solution in r to the following equation:

$$\pi^{l}(n-1,\bar{c}-\epsilon+r) = \theta\pi^{n}(n-2,\bar{c}-\epsilon+r) + (1-\theta)\pi^{l}(n-1,\bar{c}-\epsilon)$$

and the patent holder's overall profit is:

$$\tilde{\Pi}_r(\theta) = \pi^p(n-1, \bar{c} - \epsilon + r_I(\theta)) + (n-1)r_I(\theta)q^l(n-1, \bar{c} - \epsilon + r_I(\theta))$$

Under the fixed fee scheme, the optimal fee is given by:

$$F_I(\theta) = \theta \left[ \pi^l (n-1, \bar{c} - \epsilon) - \pi^n (n-2, \bar{c} - \epsilon) \right]$$

and the patent holder's overall profit is then:

$$\tilde{\Pi}_F(\theta) = \pi^p(n-1,\bar{c}-\epsilon) + (n-1)\theta \left[\pi^l(n-1,\bar{c}-\epsilon) - \pi^n(n-2,\bar{c}-\epsilon)\right]$$

Since  $\tilde{\Pi}_{r}(0) = \tilde{\Pi}_{F}(0)$  then  $\tilde{\Pi}_{r}(\theta) > \tilde{\Pi}_{F}(\theta)$  for  $\theta$  sufficiently small if:

$$\frac{d\tilde{\Pi}_{r}\left(\theta\right)}{d\theta}\mid_{\theta=0} > \frac{d\tilde{\Pi}_{F}\left(\theta\right)}{d\theta}\mid_{\theta=0}$$

$$\tag{4}$$

which can be rewritten as:

$$r_I'(0)\left[\frac{\partial \pi^p}{\partial c}\left(n-1,\bar{c}-\epsilon\right)+\left(n-1\right)q^l(n,\bar{c}-\epsilon)\right] > (n-1)\left[\pi^l\left(n-1,\bar{c}-\epsilon\right)-\pi^n\left(n-2,\bar{c}-\epsilon\right)\right]$$

because  $r_I(0) = 0$ . Moreover, differentiating at  $\theta = 0$  the equation defining  $r_I(\theta)$ , we get:

$$r_I'(0) \cdot \frac{\partial \pi^l}{\partial c} \left( n - 1, \bar{c} - \epsilon \right) = \pi^n \left( n - 2, \bar{c} - \epsilon \right) - \pi^l \left( n - 1, \bar{c} - \epsilon \right)$$

which yields:

$$r_I'(0) = \frac{\pi^n \left(n - 2, \bar{c} - \epsilon\right) - \pi^l \left(n - 1, \bar{c} - \epsilon\right)}{\frac{\partial \pi^l}{\partial c} \left(n - 1, \bar{c} - \epsilon\right)}$$

Hence, inequality (4) is equivalent to:  $\frac{\pi^{n}(n-2,\bar{c}-\epsilon)-\pi^{l}(n-1,\bar{c}-\epsilon)}{\frac{\partial\pi^{l}}{\partial c}(n-1,\bar{c}-\epsilon)} \left[\frac{\partial\pi^{p}}{\partial c}\left(n-1,\bar{c}-\epsilon\right)+(n-1)q^{l}(n,\bar{c}-\epsilon)\right] > (n-1)\left[\pi^{e}\left(n-1,\bar{c}-\epsilon\right)-\pi^{i}\left(n-2,\bar{c}-\epsilon\right)\right]$ which can be rewritten as:

$$\frac{\partial \pi^p}{\partial c} \left( n - 1, \bar{c} - \epsilon \right) + \left( n - 1 \right) q^l(n, \bar{c} - \epsilon) > - \left( n - 1 \right) \frac{\partial \pi^l}{\partial c} \left( n - 1, \bar{c} - \epsilon \right)$$

that is,

$$\frac{\partial \pi^p}{\partial c} \left( n - 1, \bar{c} - \epsilon \right) + \left( n - 1 \right) \frac{\partial \pi^l}{\partial c} \left( n - 1, \bar{c} - \epsilon \right) > - \left( n - 1 \right) q^l (n, \bar{c} - \epsilon)$$

Therefore, we get the following result:

**Proposition 5** The optimal per-unit royalty scheme deterring litigation generates higher overall profit for the patent holder than the optimal fixed fee scheme deterring litigation for sufficiently weak patents if:

$$\frac{\partial \pi^p}{\partial c} \left( n - 1, \bar{c} - \epsilon \right) + \left( n - 1 \right) \frac{\partial \pi^l}{\partial c} \left( n - 1, \bar{c} - \epsilon \right) > - \left( n - 1 \right) q^l (n, \bar{c} - \epsilon) \tag{5}$$

The reverse holds if the reverse strict inequality is satisfied.

To see how Condition (5) compares to its counterpart when the patent holder is not active on the market, i.e. Condition (3), let us rewrite both of them with the same notations. For that purpose, let us denote by  $\Pi^*(c_1, c_2, ..., c_n)$  the sum of all firms' equilibrium profits, i.e. the equilibrium aggregate profit, and  $q_i^*(c_1, c_2, ..., c_n)$  firm i's output when each firm j = 1, 2, ..., n produces at unit cost  $c_j$ .

The sufficient condition for the patent holder to prefer the per-unit royalty scheme for sufficiently weak patents when *it is not active in the market* can be rewritten as (here,  $\bar{c} - \epsilon$  is replaced by the generic variable c):

$$\sum_{i=1}^{n} \frac{\partial \Pi^{*}}{\partial c_{i}} \left( c, c, ..., c \right) > -\sum_{i=1}^{n} q_{i}^{*} \left( c, c, ..., c \right)$$
(6)

A sufficient condition for the patent holder, denoted as firm P, to prefer the per-unit royalty scheme for sufficiently weak patents when *it is active in the market* can be rewritten as (here again,  $\bar{c} - \epsilon$  is replaced by the generic variable c):

$$\sum_{\substack{i=1\\i \neq P}}^{n} \frac{\partial \Pi^{*}}{\partial c_{i}} \left( c, c, ..., c \right) > -\sum_{\substack{i=1\\i \neq P}}^{n} q_{i}^{*} \left( c, c, ..., c \right)$$
(7)

Both inequalities have almost the same interpretation: Condition (6) means that the *strategic* effect of an identical increase in all firms' (common) unit cost on the aggregate profit is positive and Condition (7) means that the strategic effect of an increase in the costs of all firms but one on the aggregate profit is positive (firms being equally efficient initially). Note also that both conditions are implied by the following inequality when it holds for any i = 1, 2, ..., n:

$$\frac{\partial \Pi^*}{\partial c_i} \left( c, c, ..., c \right) > -q_i^* \left( c, c, ..., c \right) \tag{8}$$

This condition means that when firms are equally efficient initially, the strategic effect of an increase in one firm's unit cost on the aggregate profit is positive.

We show in what follows that Condition (8) always holds for a Cournot oligopoly (with homogenous products) and a Bertrand oligopoly (with differentiated products), provided existence of pure-strategy equilibrium holds. It then follows that both Condition (6) and Condition (7) hold since they are implied by Condition (8).

# 6 Two standard oligopoly applications

In this section, we provide sufficient conditions of a general nature on the primitives of the two most widely used models of imperfect competition, which lead to Assumptions A1-A5 and A1'-A5' and Condition (8) being verified. Since some of the results below are new to the oligopoly literature, and of some independent interest, we derive them for fully asymmetric versions of the Cournot and Bertrand oligopolies with linear costs. Accordingly, we also change the notation as needed, relative to the other parts of the paper.

#### 6.1 Cournot competition with homogenous products

Consider an industry consisting of n firms competing in Cournot fashion. Firm *i*'s marginal cost is denoted  $c_i$  (fixed production costs are assumed to be zero or otherwise sunk). Suppose the firms face an inverse demand function  $P(\cdot)$  satisfying the following minimal conditions:

**C1**  $P(\cdot)$  is twice continuously differentiable and  $P'(\cdot) < 0$  whenever  $P(\cdot) > 0$ . **C2**  $P(0) > c_i > P(Q)$  for Q sufficiently high, i = 1, 2, ..., n. **C3** P'(Q) + QP''(Q) < 0 for all  $Q \ge 0$  with  $P(\cdot) > 0$ .

These assumptions are quite standard and minimal. **C3** is the familiar condition used by Novshek (1985) to guarantee downward-sloping reaction curves (for any cost function). It is equivalent to the statement that each firm's marginal revenue is decreasing in rivals' output (see Amir, 1996 for an alternative condition).

Firm *i*'s profit function and reaction correspondence are (here,  $Q_{-i} = \sum_{j \neq i} q_j$ )

$$\pi_i(q_i, Q_{-i}) = q_i \left[ P(q_i + Q_{-i}) - c_i \right]$$
 and  $r_i(Q_{-i}) = \arg \max_{q_i \ge 0} \pi_i(q_i, Q_{-i})$ 

The next proposition provides general conditions under which Assumptions **A1-A5** and **A1'-A5'** hold for a Cournot oligopoly.

Proposition 6 Under Assumptions C1-C3, the following holds:

- (a) There exists a unique Cournot equilibrium.
- (b) Firm i's equilibrium profit  $\pi_i^*$  is differentiable in  $c_i$  and in  $c_j$  for any  $j \neq i$ .
- (c) Firm i's equilibrium profit  $\pi_i^*$  is decreasing in  $c_i$  and increasing in  $c_j$  for any  $j \neq i$ .

If in addition, the game is symmetric (with c denoting the unit cost), then

(d) The unique Cournot equilibrium is symmetric.

- (e) The equilibrium output  $q^*$  strictly decreases in c.
- (f) Per-firm equilibrium profit  $\pi^*$  decreases in c.

#### **Proof.** See Appendix.

It is straightforward to relate the different parts of Proposition 6 to Assumptions A1-A5 and A1'-A5'. Part (a) is needed to avoid vacuous statements. Assumptions A1 and A1' are implied by part (b) and the proof of part (e). Assumption A2 follows from part (f) and Assumption A2' follows from combining part (f) with part (c). Assumptions A3 and A3' are implied by part (c). Assumptions A4 and A4' follow from repeated applications of part (c), with one rival firm's cost decreasing at a time. Assumptions A5 and A5' follow from part (c). Note that part (f), though intuitive, actually has a less universal scope that one might think.

Indeed, there is an extensive literature dealing with taxation in oligopolistic industries and one of its key insights is that a common cost increase can lead to some firms benefiting at the expense of others (Seade, 1985, Kimmel, 1992, and Février and Linnemer, 2004). More surprisingly, in a symmetric setting, a cost increase may be beneficial to all firms, when the inverse demand function is sufficiently convex. In light of this result, part (f) may be viewed as giving sufficient conditions for this counter-intuitive effect of taxation not to arise. On the other hand, while the questions addressed in part (c) do not have any direct counterparts in the taxation literature, the results from the latter do suggest that the intuitive conclusions of part (c) will not hold for sufficiently convex inverse demand functions. In this sense, while nevertheless quite general in nature, the most restrictive assumptions made in the present paper are those ruling out the cost paradox. As an instructive illustration, we provide a class of very convex demand functions that violates C3.

**Example**. Consider a duopoly industry with inverse demand  $P(Q) = Q^{-1/b}, \frac{1}{2} < b < 1$ , which clearly fails assumption C3. The profit functions are  $\pi_i(q_i, q_j) = [(q_i + q_j)^{-1/b} - c_i]q_i$ . The equilibrium outputs and profits are, with  $i, j \in \{1, 2\}, i \neq j$ ,

$$q_i^* = \frac{(2b-1)^b}{b^b(c_i+c_j)^{b+1}} [c_i(1-b) + c_j b] \text{ and } \pi_i^*(c_i,c_j) = \frac{(2b-1)^{b-1}}{b^b} \frac{[c_i(1-b) + c_j b]^2}{(c_i+c_j)^{b+1}}$$

It is easily verified that, in violation of our basic assumptions A2 - A5, (i)  $\frac{\partial \pi_i^*(c_i,c_j)}{\partial c_i} > 0$  for some values of the parameters. In particular  $\frac{\partial \pi_i^*(c_i,c_j)}{\partial c_i} |_{c_1=c_2} > 0$  if  $\frac{1}{2} < b < \frac{3}{5}.$ (ii)  $\frac{\partial \pi_i^*(c_i, c_j)}{\partial c_j} < 0$  for some values of the parameters.

(iii) In the *n*-firm symmetric version of this example,  $\frac{\partial \pi^*}{\partial c} > 0$  (see Kimmel, 1992, and Février and Linnemer, 2004).

Note that the above counter-intuitive results pertain to the unique interior Cournot equilibrium. Indeed, this example gives rise to two Cournot equilibria, one of which has each firm producing zero output.

As illustrated by this simple example, an insightful perspective on Proposition (6) is

that, by imposing one of the commonly used conditions to guarantee existence of Cournot equilibrium via the property of strategic substitutes (Novshek, 1985 and Amir, 1996), namely C3, one also obtains as a byproduct that the counter-intuitive results on the effects of uniform or unilateral cost increases (or uniform or individual taxation) on firms' profits do not hold.

#### **Proposition 7** Under Assumptions C1-C3, Condition (8) is verified.

#### **Proof.** See Appendix.

As can easily be seen in the proof, this result actually only requires that total equilibrium output decreases with a unilateral unit cost increase, which holds universally in Cournot competition with linear costs. Note also that this result **provides a lower bound on the effect of a unilateral cost increase on industry profits, an issue not considered in the related literature.** 

We can then state that under the general assumptions C1-C3, the holder of a weak patent prefers to license it out using a per-unit royalty licensing contract rather than a fixed fee contract. This result holds whether the patent holder is active in the (downstream) market or not.

### 6.2 Bertrand competition with differentiated products

Consider an industry consisting of n single-product firms, with constant unit costs  $c_1, c_2, ..., c_n$ . Assume that the goods are imperfect substitutes. Denoting  $D_i(p_1, p_2, ..., p_n)$  the demand for the good produced by firm i, its profit function and reaction correspondence are defined as usual by

$$\pi_i(p_i, p_{-i}) = (p_i - c_i)D_i(p_i, p_{-i}) \text{ and } r_i(p_{-i}) = \arg\max_{p_i} \pi_i(p_i, p_{-i})$$

We will say that the Bertrand oligopoly is symmetric if the demand functions are symmetric and  $c_1 = c_2 = \ldots = c_n \triangleq c$ .

Let  $S_i \triangleq \{(p_1, p_2, ..., p_n) \in \mathbb{R}^n_+ \mid D_i(p_1, p_2, ..., p_n) > 0\}$ . We assume throughout that for every firm *i*:

**B1**  $D_i$  is twice continuously differentiable on  $S_i$ . **B2** (i) $\frac{\partial D_i}{\partial p_i} < 0$ , (ii) $\frac{\partial D_i}{\partial p_j} > 0$  and (iii)  $\sum_{k=1}^n \frac{\partial D_i(p,p,\dots,p)}{\partial p_k} < 0$  over the set  $S_i$ . **B3**  $D_i \frac{\partial^2 \log D_i}{\partial p_j \partial p_i} - \frac{\partial \log D_i}{\partial p_j} \frac{\partial \log D_i}{\partial p_i} > 0$  over the set  $S_i$ , for  $j \neq i$ . **B4**  $\sum_{j=1}^n \frac{\partial^2 D_i(p_1,p_2,\dots,p_n)}{\partial p_i \partial p_j} < 0$  over the set  $S_i$ .

These conditions are quite general, and are commonly invoked for differentiated-good demand systems. They have the following meanings and economic interpretations. For **B2**, part (i) is just the ordinary law of demand; part (ii) says that goods i and j are substitutes; and part (iii) is a dominant diagonal condition for the Jacobian of the demand system, which is required to hold only at equal prices (see e.g., Vives, 1999). It says that, along the diagonal,

own price effect on demand exceeds the total cross-price effects. **B3** says that each demand has (differentiably) strict log-increasing differences in own price and any rival's price. The exact economic interpretation is that the price elasticity of demand strictly increases in any rival's price, which is a very natural assumption (Milgrom and Roberts, 1990). **B4** says that the Hessian of the demand system has a dominant diagonal, which is a standard assumption invoked to guarantee uniqueness of Bertrand equilibrium (Milgrom and Roberts, 1990 or Vives, 1999). **B2(iii)** and **B4** hold that own effects of price changes dominate cross effects, for the level and the slope of demand, respectively.

The following proposition provides sufficient conditions for Assumptions A1-A5 and A1'-A5' to hold in this framework.

#### **Proposition 8** Under Assumptions B1-B4,

(a) The Bertrand game is of strict strategic complements, and has a unique Bertrand equilibrium.

(b) Firm i's equilibrium price  $p_i^*$  is increasing in  $c_j$  for any j.

(c) Firm i's equilibrium profit  $\pi_i^*$  is differentiable in  $c_i$  and  $c_j$  for any  $j \neq i$ .

(d) Firm i's equilibrium profit  $\pi_i^*$  is increasing in  $c_j$  for any  $j \neq i$ .

If in addition, if the game is symmetric, then

- (e) the unique Bertrand equilibrium is symmetric.
- (f) the equilibrium price increases in c.
- (g) per-firm equilibrium profit  $\pi_i^*$  is differentiable in c, and decreasing in c.

**Proof.** See Appendix.

We leave to the reader the task of matching the different parts of Proposition 8 to Assumptions A1-A5 and A1'-A5', as this step is quite similar to the Cournot case.

Anderson, DePalma and Kreider (2001) extends the analysis of the effects of taxation to Bertrand competition with differentiated products, and report analogous findings as in the Cournot case. Since Proposition (8) contains only intuitive results on the effects of cost changes on profits, one concludes that the standard assumptions for existence and uniqueness of Bertrand equilibrium preclude any counter-intuitive effects of taxation.

**Proposition 9** Under Assumptions **B1-B4**, Condition (8) is verified.

#### **Proof.** See Appendix.

As can be seen from the proof, this result only requires that each firm's equilibrium price increases with a unilateral unit cost increase, which holds in Bertrand oligopoly with linear costs whenever the game is supermodular (i.e, **B3** holds). This result also provides a lower bound on the effect of a unilateral cost increase on industry profits, an issue not considered in the related literature. We can then state that under the general assumptions **B1-B4**, the holder of a weak patent prefers to license it out using a per-unit royalty licensing contract rather than a fixed fee contract, both for the cases of an industry insider and outsider.

# 7 Conclusion

This paper shows that the "weakness" of a patent is an alternative justification for the use of a per-unit royalty contract instead of a fixed fee contract for the licensing of an innovation. A sufficient condition under which the holder of a weak patent prefers to license out through a per-unit royalty rather than a fixed fee is provided and shown to be mild in the sense that it holds under weak conditions for a Cournot oligopoly with homogenous goods and a Bertrand oligopoly with heterogenous goods, regardless of whether the patent holder is an outsider or an insider in the industry. A significant difference with respect to the literature on the licensing of ironclad patents is that we get a clear-cut result on the comparison of a patent holder's profits under the two schemes, independent of the type of downstream competition, the degree of differentiation between products and whether the patent holder is active or not in the downstream market, while varying any of these three features can overturn the outcome of the comparison when ironclad patents are considered.

In addition, our model generates some testable predictions that might be worth investigating: First, our results suggest that per-unit royalty licenses should be more prevalent in industries with a significant proportion of firms holding questionable patents, e.g., industries relying on some new patentable subject matter (biotechnology, software, business methods,...). Second, if our predictions are correct then under the presumption that the EPO is more stringent in checking the patentability standards than the USPTO, the use of per-unit royalties should be less prevalent in the EU than in the US.

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# 9 Appendix

#### **Proof of Proposition 1**

All firms accepting the license offer is an equilibrium if and only if:

$$\pi^{e}(n,\bar{c}-\epsilon+r) \ge \theta \pi^{i}(n-1,\bar{c}-\epsilon+r) + (1-\theta) \pi^{e}(n,\bar{c}-\epsilon)$$

which can be rewritten as:

$$g(r,\theta) \equiv \pi^e(n,\bar{c}-\epsilon+r) - \theta\pi^i(n-1,\bar{c}-\epsilon+r) - (1-\theta)\pi^e(n,\bar{c}-\epsilon) \ge 0$$

We have  $g(0,\theta) = \theta \left[\pi^e(n,\bar{c}-\epsilon) - \pi^i(n-1,\bar{c}-\epsilon)\right] \ge \theta \left[\pi^e(n,\bar{c}) - \pi^i(n-1,\bar{c}-\epsilon)\right]$  $\ge \theta \left[\pi^i(n-1,\bar{c}) - \pi^i(n-1,\bar{c}-\epsilon)\right] > 0$  (by **A2** and **A5**) and  $g(\epsilon,\theta) = \pi^e(n,\bar{c}) - \theta\pi^i(n-1,\bar{c}) - (1-\theta)\pi^e(n,\bar{c}-\epsilon) = (1-\theta)(\pi^e(n,\bar{c}) - \pi^e(n,\bar{c}-\epsilon)) < 0$  (by **A2**). Combining this with g being continuous (by **A1**) and strictly decreasing in r (by **A3**) yields: i/ the existence and uniqueness of a solution in r to the equation  $g(r,\theta) = 0$  (within the interval  $[0,\epsilon]$ ), which we denote by  $r(\theta)$ ; ii/ the equivalence between te inequalities  $g(r,\theta) \ge 0$  and  $r \le r(\theta)$ .

#### **Proof of Proposition 2**

We have already showed (in the main text ) that all firms deciding to purchase a license at a fixed fee F is an equilibrium of the second stage subgame if and only if:

$$F \le F(\theta) \equiv \theta \left[ \pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon) \right]$$

All firms but one deciding to purchase a license is an equilibrium of the second stage subgame if the following two conditions hold:

$$\theta \pi^{i}(n-1, \bar{c}-\epsilon) + (1-\theta) \pi^{e}(n, \bar{c}-\epsilon) \ge \pi^{e}(n, \bar{c}-\epsilon) - F$$

and

$$\theta \left[ \pi^e (n-1, \bar{c} - \epsilon) - F \right] + (1 - \theta) \pi^e (n, \bar{c} - \epsilon) \ge \theta \pi^i (n - 2, \bar{c} - \epsilon) + (1 - \theta) \pi^e (n, \bar{c} - \epsilon)$$

Thus, all firms but one deciding to purchase a license is an equilibrium if and only if:

$$F(\theta) \le F \le F_{n-1} \equiv \pi^e(n-1,\bar{c}-\epsilon) - \pi^i(n-2,\bar{c}-\epsilon)$$

Denoting  $F_k \equiv \pi^e(k, \bar{c} - \epsilon) - \pi^i(k - 1, \bar{c} - \epsilon)$  for each k = 1, 2, ..., n, we can further show that for any k = 1, 2, ..., n - 2, a number k of firms accepting the license offer and the other n - kfirms not doing so is an equilibrium if and only if:

$$F_{k+1} \le F \le F_k$$

Moreover, all firms deciding not to buy a license is an equilibrium if and only if:

$$F \ge F_1$$

If we assume that the sequence  $(F_k)_{1 \le k \le n}$  is decreasing, i.e. a firm's willingness to pay for a

license (under ironclad patent protection) decreases with the number of licensees, and that a firm which is indifferent between accepting and refusing the license offer buys a license, then for any  $F \ge 0$ , there is a unique equilibrium to the second stage subgame up to a permutation of firms: all the equilibria of the second stage subgame involve the same number of licensees (which allows to define a "demand function" for licenses which is decreasing in the fixed fee F). However, if  $(F_k)_{1\le k\le n}$  is not decreasing then there might exist some values of F for which there is either no (pure-strategy) equilibrium or multiple equilibria with different number of licensees.

However, if we focus on small values of  $\theta$  and do not care about whether pure-strategy equilibria exist - and which one arises in case they do - if all firms accepting the license offer is not an equilibrium (as in the present paper), then the problem of multiplicity or inexistence of equilibria depicted above does not affect our analysis. The reason is that, to be sure that all firms accepting a license is the unique equilibrium whenever it is an equilibrium, i.e. whenever  $F \leq F(\theta)$ , we only need the inequality  $F(\theta) \leq F_k$  to hold for any k = 1, 2, ..., n - 1, which, given that  $F(\theta) = \theta F_n$ , is true if  $\theta$  is small enough, and more specifically if

$$\theta \le \bar{\theta} = \frac{\min_{1 \le k \le n-1} F_k}{F_n}$$

#### **Proof of Proposition 3**

All firms accepting the payment of a per-unit royalty r is an equilibrium if and only if  $r \leq r(\theta)$ . Furthermore, A1 ensures that the licensing revenue function  $r \to nrq^e (n, \bar{c} - \epsilon + r)$  is strictly increasing in the neighborhood of 0 (its derivative at r = 0 being  $q^e (n, \bar{c} - \epsilon) > 0$ ). Since  $r(\theta)$  is continuous (by the Implicit Function Theorem) and increasing and r(0) = 0, we can conclude that, for  $\theta$  sufficiently small, the function  $nrq^e (n, \bar{c} - \epsilon + r)$  is increasing over  $[0, r(\theta)]$  and, therefore, the optimal per-unit royalty license accepted by all firms involves the payment of the royalty rate  $r(\theta)$ .

The optimal fixed fee license deterring litigation maximizes the patent holder's revenues nFunder the constraint  $F \leq F(\theta)$ . It is straightforward that solution to this constrained maximization program is  $F = F(\theta)$ .

### **Proof of Proposition 4**

The licensing revenues from the optimal per-unit royalty scheme deterring litigation are given by:

$$\tilde{P}_r(\theta) = nr(\theta) q^e(n, \bar{c} - \epsilon + r(\theta))$$

and the licensing revenues from the optimal fixed fee licensing scheme deterring litigation are:

$$\tilde{P}_F(\theta) = nF_n(\theta) = n\theta \left[\pi^e(n,\bar{c}-\epsilon) - \pi^i(n-1,\bar{c}-\epsilon)\right]$$

Since  $\tilde{P}_r(0) = \tilde{P}_F(0)$  then  $\tilde{P}_r(\theta) > \tilde{P}_F(\theta)$  for  $\theta$  sufficiently small if:

$$\frac{d\tilde{P}_r(\theta)}{d\theta} \mid_{\theta=0} > \frac{d\tilde{P}_F(\theta)}{d\theta} \mid_{\theta=0}$$
(9)

which can be rewritten as:

$$nr'(0) q^e(n, \bar{c} - \epsilon) > n \left[ \pi^e(n, \bar{c} - \epsilon) - \pi^i(n - 1, \bar{c} - \epsilon) \right]$$

because r(0) = 0. Moreover differentiating at  $\theta = 0$  the equation defining  $r(\theta)$ , that is,

$$\pi^{e}(n,\bar{c}-\epsilon+r(\theta)) = \theta\pi^{i}(n-1,\bar{c}-\epsilon+r(\theta)) + (1-\theta)\pi^{e}(n,\bar{c}-\epsilon)$$

we get:

$$r'(0) \frac{\partial \pi^e}{\partial c} (n, \bar{c} - \epsilon) = \pi^i (n - 1, \bar{c} - \epsilon) - \pi^e (n, \bar{c} - \epsilon)$$

which yields:

$$r'(0) = \frac{\pi^{i}(n-1,\bar{c}-\epsilon) - \pi^{e}(n,\bar{c}-\epsilon)}{\frac{\partial\pi^{e}}{\partial c}(n,\bar{c}-\epsilon)}$$

Hence, (9) is equivalent to:

$$n\frac{\pi^{i}\left(n-1,\bar{c}-\epsilon\right)-\pi^{e}\left(n,\bar{c}-\epsilon\right)}{\frac{\partial\pi^{e}}{\partial c}\left(n,\bar{c}-\epsilon\right)}q^{e}(n,\bar{c}-\epsilon) > n\left[\pi^{e}\left(n,\bar{c}-\epsilon\right)-\pi^{i}\left(n-1,\bar{c}-\epsilon\right)\right]$$

which can be rewritten as:

$$\frac{\partial \pi^e}{\partial c} \left( n, \bar{c} - \epsilon \right) > -q^e (n, \bar{c} - \epsilon)$$

because  $\pi^i (n-1, \bar{c}-\epsilon) - \pi^e (n, \bar{c}-\epsilon) < 0.$ 

### **Proof of Proposition 6**

(a) follows from the key slope property that every selection of  $r_i$  satisfies (see Amir, 1996, and Amir and Lambson, 2000 for details)

$$-1 < \frac{r_i(Q'_{-i}) - r_i(Q_{-i})}{Q'_{-i} - Q_{-i}} < 0 \text{ for all } Q'_{-i} > Q_{-i}.$$
 (10)

(b) We first show that  $q_i^*$  is continuously differentiable in  $c_i$ . Viewed as a correspondence in the parameter  $c_i$ ,  $q_i^*$  is upper hemi-continuous (or u.h.c.), as a direct consequence of the well-known property of u.h.c. of the equilibrium correspondence for games with continuous payoff functions (jointly in own and rivals' actions), see e.g., Fudenberg and Tirole, 1990. Since  $q_i^*$  is also single-valued in c (from part (b)),  $q_i^*$  must be a continuous function. Then the fact that  $q_i^*$  is continuously differentiable in  $c_i$  follows from the Implicit Function Theorem applied to the first order conditions, and the smoothness of  $P(\cdot)$ . The fact that  $\pi_i^*$  is also continuously differentiable in  $c_i$  follows directly from the fact that  $q_i^*$  has that same property for all *i*.

The proof for the parameter  $c_j, j \neq i$ , follows along the same lines.

(c) Denote firm *i*'s output, profit and its rivals' total outputs at equilibrium by  $q_i^*, \pi_i^*$  and  $Q_{-i}^*$  respectively when the cost vector is  $(c_1, c_2, ..., c_n)$ . Denote the same three variables by  $\hat{q}_i, \hat{\pi}_i$  and  $\hat{Q}_{-i}$  after firm *i*'s cost alone changes to  $\hat{c}_i > c_i$ .

Adding the n first order conditions at the Cournot equilibrium yields

$$nP(Q^*) + Q^*P'(Q^*) = \sum_{k=1}^{n} c_k$$
(11)

Since the LHS of (11) is strictly decreasing in  $Q^*$ , the increase in firm *i*'s cost from  $c_i$  to  $\hat{c}_i$  increases the RHS of (11), which causes the solution to (11) to decrease. In other words,  $\hat{Q} < Q^*$ .

We now show that for any firm  $j \neq i$ , we must have  $\widehat{Q}_{-j} < Q^*_{-j}$ . To this end, first observe that  $\widehat{Q}_{-j} + r_j(\widehat{Q}_{-j}) = \widehat{Q} < Q^* = Q^*_{-j} + r_j(Q^*_{-j})$ . Since (10) holds that  $Q_{-j} + r_j(Q_{-j})$  is increasing in  $Q_{-j}$ , we must have  $\widehat{Q}_{-j} < Q^*_{-j}$ . For firm j,

$$\begin{aligned} \widehat{\pi}_{j} &= \widehat{q}_{j} \left[ P(\widehat{q}_{j} + \widehat{Q}_{-j}) - c_{j} \right] \\ &\geq q_{j}^{*} \left[ P(q_{j}^{*} + \widehat{Q}_{-j}) - c_{j} \right] \text{ by the Cournot property} \\ &> q_{j}^{*} \left[ P(q_{j}^{*} + Q_{-j}^{*}) - c_{j} \right] \text{ since } \widehat{Q}_{-j} < Q_{-j}^{*} \\ &= \pi_{j}^{*} \end{aligned}$$

Henceforth, we consider the case of a symmetric Cournot oligopoly.

(d) Due to the symmetry of the game, asymmetric equilibria, if any, would come in *n*-tuples. Hence, the conclusion follows from part (a) directly.

(e) Let  $q^*$  denote each firm's equilibrium output. Differentiating the first order condition with respect to c, we get:

$$\frac{\partial q^*}{\partial c} \left[ (n+1) P'(nq^*) + nq^* P^*(nq^*) \right] = 1$$
(12)

Using the first order condition and C3, it is easy to see that the term in brackets is strictly negative, it follows that  $\frac{\partial q^*}{\partial c} < 0$ .

We now show that per-firm profit decreases in c. Denote the equilibrium variables by  $q_i^*, \pi_i^*$  and  $Q_{-i}^*$  when the unit cost is c, and by  $q_i', \pi_i'$  and  $Q_{-i}'$  the same variables when the unit cost is c' > c.

Differentiating  $\pi_i^* = q^* \left[ P(nq^*) - c \right]$  with respect to c yields

$$\frac{\partial \pi_i^*}{\partial c} = \frac{\partial q^*}{\partial c} \left[ P(nq^*) - c \right] + q^* \left[ P'(nq^*)n\frac{\partial q^*}{\partial c} - 1 \right]$$

$$= \frac{\partial q^*}{\partial c} (n+1)q^*P'(nq^*) - q^* \text{ by (11)}$$

$$= -q^* \frac{2P'(Q^*) + Q^*P''(Q^*)}{(n+1)P'(Q^*) + Q^*P''(Q^*)} \text{ by (12)}$$

Clearly, **C3** implies that 2P'(Q) + QP''(Q) < 0 for all Q, so the numerator in the above fraction is < 0. It is then easy to see that the denominator is also < 0. Hence  $\frac{\partial \pi_i^*}{\partial c} < 0$ .

### **Proof of Proposition 7**

Let us show that Condition (8) holds (which will imply that both Condition (6) and Condition (7) are satisfied).

Total differentiation w.r.t.  $c_i$  in

$$\Pi^* = (P(Q^*) - c_i) q_i^* + \sum_{j \neq i} (P(Q^*) - c_j) q_j^*$$

yields

$$\frac{\partial \Pi^*}{\partial c_i} = \left[P'(Q^*)\frac{\partial Q^*}{\partial c_i} - 1\right]q_i^* + \left(P(Q^*) - c_i\right)\frac{\partial q_i^*}{\partial c_i} + \sum_{j\neq i}\left[P'(Q^*)\frac{\partial Q^*}{\partial c_i}q_j^* + \left(P(Q^*) - c_j\right)\frac{\partial q_j^*}{\partial c_i}\right]$$

which can be rewritten as:

$$\frac{\partial \Pi^*}{\partial c_i} = -q_i^* + \sum_j \left[ P'(Q^*) \frac{\partial Q^*}{\partial c_i} q_j^* + \left( P(Q^*) - c_j \right) \frac{\partial q_j^*}{\partial c_i} \right]$$

When  $c_i = c_j = c$ , the latter becomes:

$$\begin{aligned} \frac{\partial \Pi^*}{\partial c_i} &= -q_i^* + \sum_j \left[ P'(Q^*) \frac{\partial Q^*}{\partial c_i} q_j^* + (P(Q^*) - c) \frac{\partial q_j^*}{\partial c_i} \right] \\ &= -q_i^* + P'(Q^*) \frac{\partial Q^*}{\partial c_i} \sum_j q_j^* + (P(Q^*) - c) \sum_j \frac{\partial q_j^*}{\partial c_i} \\ &= -q_i^* + P'(Q^*) \frac{\partial Q^*}{\partial c_i} \cdot Q^* + (P(Q^*) - c) \frac{\partial Q^*}{\partial c_i} \\ &= \underbrace{-q_i^*}_{\text{direct effect}} + \underbrace{\frac{\partial Q^*}{\partial c_i} \left[ P'(Q^*) \cdot Q^* + (P(Q^*) - c) \right]}_{\text{strategic effect}} \end{aligned}$$

Adding the n first order conditions at the Cournot equilibrium yields

$$nP(Q^*) + Q^*P'(Q^*) = \sum_{k=1}^n c_k = nc$$

Thus,

$$Q^*P'(Q^*) + (P(Q^*) - c) = \frac{n-1}{n}Q^*P'(Q^*) < 0$$

Moreover, since we have already shown (in the proof of Proposition 6) that  $\frac{\partial Q^*}{\partial c_i} < 0$ ,

$$\frac{\partial Q^*}{\partial c_i} \left[ P'(Q^*) \cdot Q^* + (P(Q^*) - c) \right] > 0$$

which yields:

$$\frac{\partial \Pi^*}{\partial c_i} > -q_i^*$$

#### **Proof of Proposition 8**

First note that for firm *i*, charging a price of  $c_i$  strictly dominates charging any price below  $c_i$ . Hence, we restrict attention to the price space  $[c_i, \infty)$  as the action set for firm i, i = 1, 2, ..., n. Then the transformed profit function  $\log \prod_i (p_i, p_{-i})$  is well defined.

(a) For the proof that the game with log profits as payoffs is of strict strategic complements, observe that, due to **B3**, each payoff  $\Pi_i(p_i, p_{-i})$  satisfies  $\partial^2 \log \Pi_i(p_i, p_{-i})/\partial p_i \partial p_{-i} > 0$ . Hence, by the strong version of Topkis's Theorem (see Amir, 1996 or Topkis, 1998 p. 79), every selection of  $r_i(p_{-i})$  is strictly increasing in  $p_{-i}$ . It follows directly from the property of strategic complements, via Tarski's fixed point theorem, that the Bertrand equilibrium set is nonempty. Uniqueness then follows from a well known argument from **B4** (for details, see Milgrom and Roberts 1990, pp. 1271-1272, or Vives, 1999 pp. 149-150).

(b) To show that the equilibrium price  $p_i^*$  is increasing in  $c_i$ , note that the price game is log-supermodular (from part (a)),  $\log \prod_i (p_i, p_{-i}) = \log(p_i - c_i) + \log D_i(p_i, p_{-i})$  has the increasing differences property in  $(p_i, c_i)$  since  $\partial^2 \log \prod_i (p_i, p_{-i}) / \partial p_i \partial c_i = (p_i - c_i)^{-2} > 0$ , and the constraint set  $[c_i, \infty)$  is clearly ascending in  $c_i$ . So the conclusion follows from Theorem 7 in Milgrom and Roberts (1990).

The fact that the equilibrium price  $p_i^*$  is also increasing in  $c_j$  for any  $j \neq i$ , follows from a similar argument since  $\partial^2 \log \prod_i (p_i, p_{-i}) / \partial p_i \partial c_j = 0$ .

(c) We first show that every equilibrium price  $p_i^*$  is continuously differentiable in  $c_j$ , for all i and j. Viewed as a correspondence in the parameter  $c_j$ ,  $p_i^*$  is u.h.c., by the u.h.c. property of the equilibrium correspondence for games with continuous payoff functions (jointly in own and rivals' actions), see e.g., Fudenberg and Tirole, 1990. Since  $p_i^*$  is also single-valued in  $c_j$  (from part (i)),  $p_i^*$  is a continuous function. Then the fact that  $p_i^*$  is continuously differentiable in  $c_j$  follows from the Implicit Function Theorem. Finally, continuous differentiability of  $\pi_i^*$ 

follows from that of all the  $p_i^{*'s}$ .

(d) Differentiating  $\pi_i^* = (p_i^* - c_i)D_i(p_i^*, p_{-i}^*)$  with respect to  $c_j$ , for  $i \neq j$ , yields

$$\frac{\partial \pi_i^*}{\partial c_j} = \frac{\partial p_i^*}{\partial c_j} D_i(p_i^*, p_{-i}^*) + (p_i^* - c_i) \sum_k \frac{\partial D_i}{\partial p_k} \frac{\partial p_k^*}{\partial c_j}$$
(14)

Using the first order condition  $D_i(p_i^*, p_{-i}^*) + (p_i^* - c_i) \frac{\partial D_i}{\partial p_i} = 0$ , (14) reduces to

$$\frac{\partial \pi_i^*}{\partial c_j} = (p_i^* - c_i) \sum_{k \neq i} \frac{\partial D_i}{\partial p_k} \frac{\partial p_k^*}{\partial c_j} \ge 0$$

since  $\frac{\partial D_i}{\partial p_k} > 0$  (goods are substitutes) and  $\frac{\partial p_k^*}{\partial c_j} \ge 0$  from part (b).

(e) When the Bertrand game is symmetric, the unique Bertrand equilibrium must be symmetric, for otherwise equilibria would come in pairs.

(f) The conclusion follows from the same argument as for part (b) in view of the fact that  $\partial^2 \log \prod_i (p_i, p_{-i}) / \partial p_i \partial c = (p_i - c)^{-2} > 0.$ 

(g) From an argument similar to the proof of part (c),  $p^*$  and thus  $\pi_i^* = (p^* - c)D_i(p^*, p^*, ..., p^*)$  are differentiable with respect to c. We now derive an expression for  $\frac{\partial p^*}{\partial c}$ . The FOC at a Bertrand equilibrium is

$$D_i(p^*, ..., p^*) + (p^* - c)\partial D_i(p^*, ..., p^*) / \partial p_i = 0$$
(15)

Using the Implicit Function Theorem and differentiating the FOC with respect to c yields

$$\left(\frac{\partial D_i}{\partial p_i} + \sum_{k \neq i} \frac{\partial D_i}{\partial p_k}\right) \frac{\partial p^*}{\partial c} + (p_i^* - c_i) \frac{\partial p^*}{\partial c} \sum_k \frac{\partial D_i}{\partial p_k \partial p_i} + (\frac{\partial p^*}{\partial c} - 1) \frac{\partial D_i}{\partial p_i} = 0$$

Hence, using **B2** and **B4**,

$$\frac{\partial p^*}{\partial c} = \frac{\partial D_i / \partial p_i}{2\frac{\partial D_i}{\partial p_i} + \sum_{k \neq i} \frac{\partial D_i}{\partial p_k} + (p_i^* - c_i) \sum_k \frac{\partial D_i}{\partial p_k \partial p_i}} > 0.$$
(16)

We can differentiate  $\pi^* = (p^* - c)D_i(p^*, p^*, ..., p^*)$  with respect to c to obtain

$$\frac{\partial \pi^*}{\partial c} = \left(\frac{\partial p^*}{\partial c} - 1\right) D_i(p^*, ..., p^*) + (p^* - c) \frac{\partial p^*}{\partial c} \sum_k \frac{\partial D_i}{\partial p_k} \\
= D_i(p^*, ..., p^*) \left[ -1 - \frac{\sum_{k \neq i} \frac{\partial D_i}{\partial p_k}}{\partial D_i / \partial p_i} \frac{\partial p^*}{\partial c} \right] \text{ from (15)} \\
= D_i(p^*, ..., p^*) \left[ -1 - \frac{\sum_{k \neq i} \frac{\partial D_i}{\partial p_k}}{2\frac{\partial D_i}{\partial p_i} + \sum_{k \neq i} \frac{\partial D_i}{\partial p_k} + (p^*_i - c_i) \sum_k \frac{\partial D_i}{\partial p_k \partial p_i}} \right] \text{ using (16)} \\
= D_i(p^*, ..., p^*) \left[ -\frac{2\sum_k \frac{\partial D_i}{\partial p_k} + (p^*_i - c_i) \sum_k \frac{\partial D_i}{\partial p_k \partial p_i}}{2\frac{\partial D_i}{\partial p_k} + \sum_{k \neq i} \frac{\partial D_i}{\partial p_k} + (p^*_i - c_i) \sum_k \frac{\partial D_i}{\partial p_k \partial p_i}} \right] < 0 \text{ by } \mathbf{B2} \text{ and } \mathbf{B4}.$$

# **Proof of Proposition 9**

Let us show that Condition (8) holds (which will imply that both Condition (6) and Condition (7) are satisfied).

We have:

$$\Pi^* = (p_i^* - c_i) D_i^* + \sum_{j \neq i} (p_j^* - c_j) D_j^*$$

then:

$$\begin{aligned} \frac{\partial \Pi^*}{\partial c_i} &= \left(\frac{\partial p_i^*}{\partial c_i} - 1\right) D_i^* + (p_i^* - c_i) \frac{\partial D_i^*}{\partial c_i} + \\ &\sum_{j \neq i} \left[\frac{\partial p_j^*}{\partial c_i} D_j^* + (p_i^* - c_i) \frac{\partial D_j^*}{\partial c_i}\right] \end{aligned}$$

which can be rewritten as:

$$\begin{aligned} \frac{\partial \Pi^*}{\partial c_i} &= -D_i^* + \sum_j \left[ \frac{\partial p_j^*}{\partial c_i} D_j^* + (p_j^* - c_j) \frac{\partial D_j^*}{\partial c_i} \right] \\ &= -D_i^* + \sum_j \left[ \frac{\partial p_j^*}{\partial c_i} D_j^* + (p_j^* - c_j) \sum_k \frac{\partial D_j}{\partial p_k} \cdot \frac{\partial p_k^*}{\partial c_i} \right] \\ &= -D_i^* + \sum_j \left[ \frac{\partial p_j^*}{\partial c_i} D_j^* + (p_j^* - c_j) \frac{\partial D_j}{\partial p_j} \cdot \frac{\partial p_j^*}{\partial c_i} + (p_j^* - c_j) \sum_{k \neq j} \frac{\partial D_j}{\partial p_k} \cdot \frac{\partial p_k^*}{\partial c_i} \right] \\ &= -D_i^* + \sum_j \left[ \frac{\partial p_j^*}{\partial c_i} \underbrace{\left( D_j^* + (p_j^* - c_j) \frac{\partial D_j}{\partial p_j} \right)}_{=0} + (p_j^* - c_j) \sum_{k \neq j} \frac{\partial D_j}{\partial p_k} \cdot \frac{\partial p_k^*}{\partial c_i} \right] \\ &= -D_i^* + \sum_j \left[ (p_j^* - c_j) \sum_{k \neq j} \frac{\partial D_j}{\partial p_k} \cdot \frac{\partial p_k^*}{\partial c_i} \right] \end{aligned}$$

We have already shown that  $\frac{\partial p_k^*}{\partial c_i} > 0$  for any k, i (see the proof for part (b) of Proposition 8). Moreover, we have  $\frac{\partial D_j}{\partial p_k} > 0$  for any  $j \neq k$  (from **B2(ii)**). It then follows that:

$$\frac{\partial \Pi^*}{\partial c_i} > -D_i^* = -q_i^*$$

This proof establishes a result which is more general than Condition (8). We actually show that for any  $(c_1, c_2, ..., c_n)$ :

$$\frac{\partial \Pi^*}{\partial c_i} \left( c_1, c_2, ..., c_n \right) > -q_i^* \left( c_1, c_2, ..., c_n \right)$$