## Mixed Markets in the Food Processing Industry


#### Abstract

The food processing industry in Western countries operates in markets that usually are highly concentrated and that often consist of a few cooperatives and investor-owned firms in intense competition. The literature is divided in the view of whether this mixed market structure is a stable equilibrium or not. Some suggest that the cooperatives eventually will crowd out all investor-owned firms. To analyze this divergence in predictions, we develope a model generalizing the family of models of mixed markets. It is shown that a mixed market equilibrium may occur under quite general conditions. Also, it is shown that an investor-owned firm may serve as a yardstick of production to a cooperative, contributing to increased payoff for farmers relative to a market with a single operating cooperative.


Keywords: cooperative; endogenous membership; investor-owned firm; mixed market; yardstick of production

JEL: L 11, L 13, P12, P13, Q 13

## Introduction

The agricultural food processing industry is highly concentrated in most western countries. Structural oligopsonies and/or oligopolies have been, and still are the industry norm (Rogers and Sexton). Some of the processing agents are organized as regular investor-owned firms while others are organized as various types of cooperatives. Frequently the two types of firms coexist in so-called mixed markets (Cotterill; Fulton; Sexton; Bergman). Unfortunately, the extensive literature on this subject does not provide us with unambiguous predictions about the market outcome. While some studies suggest that the cooperatives eventually will crowd out the investor-owned firms (Rhodes; Albæk and Schultz); others argue that a mixed market equilibrium exists (Sexton; Tennbakk; Karantininis and Zago; Hendrikse). One reason for the diverging predictions may be explained by the highly specific assumptions the authors lay down for the cooperatives regarding both their objectives and bylaws and the competitive environment they face. This diversity limits the applicability of the predictions put forward.

The main purpose of this article is to generalize the family of models of mixed markets and to use the generalized model to demonstrate that the mixed market structure we observe is indeed a market equilibrium. The general conditions for this equilibrium to evolve are identified. The analysis shows that in the case of an open membership coop, the iof may serve as a "yardstick of production" for the coop. It helps the member farmers to achieve an increased payoff relative to a situation with a single coop by offering an outside option. In the case of diseconomies of scale it will help reduce the problem of excess production.

The results are important for proper understanding of the role cooperatives play in the food processing industry. They are especially relevant for the anti-trust authorities and for our understanding of the impact of e.g. the Capper-Volstead Act.

The article is structured as follows: first a general model is developed. The alternative objectives for the cooperative are then discussed and the maximization problems solved. The payoff functions for the investor-owned firm and the open membership cooperative are derived, and the equilibrium market structure is derived and analyzed. The results are then discussed before some concluding remarks are presented in the final section.

## The Model

Consider a market where $N$ identical farmers are producing a homogenous raw product (intermediary input) that has to be further processed before being sold to consumers. The commodity is assumed to be bulky and/or perishable causing economies of scope and scale in transport and processing. Hence, there are only a few processors available for the individual farmer, a so-called spatial oligopsony (Rogers and Sexton). In the subsequent analysis, the number of processors is restricted to two. One is a marketing cooperative (coop) and the other a conventional investor-owned firm (iof).

The farmers are assumed to be rational. Each farmer can sell his output to the iof at a uniform commodity price, or join and supply the coop being paid according to the coop's payment scheme. Hence, of the $N$ farmers, a subset $n_{c} \in[0, N]$ chooses membership in the coop, whereas the other subset, $n_{f} \in[0, N]$, chooses to supply the iof. All farmers are assumed to be active, so that $N=n_{c}+n_{f} .{ }^{1}$

The set up constitutes a two-stage game. In stage one, each farmer chooses the processor to supply: in stage two, each processor pursues his objective given the number of suppliers he attracts in stage one of the game. The output is then determined and the payoff offered to each supplier is derived. The two-stage game is solved the standard way by backward induction.

Costs
The total production cost for a farmer $i$ who supplies a quantity $q_{i}$ of the raw commodity is defined as
(1) $T C_{i}=g\left(q_{i}\right) \quad, \quad g(0)=0, \quad g^{\prime}\left(q_{i}\right)>0, \quad g^{\prime \prime}\left(q_{i}\right) \geq 0, \quad g^{\prime \prime \prime}\left(q_{i}\right) \geq 0, \quad i=1, \ldots, N$

This is an ordinary increasing and convex cost function. ${ }^{2}$ The fixed costs in the primary production are assumed to be zero (sunk costs). This is a strict simplification, but does not qualitatively alter the analysis.

Both processor's are assumed to have an identical cost structures consisting of a production scale-independent part $b_{j}$, and a scale-dependent part $c_{j}(\cdot)$. Subscript $j=c, f$ denotes the type of processor where $c$ indicates the coop, and $f$ the iof. Without loss of generality, the processors are assumed not to use any other input than the fixed capital equipment and the intermediary input supplied by the farmers. The units are normalized so that one unit of input is transformed into one unit of the final commodity. Firm $j$ 's total processing costs can then be expressed as

$$
\begin{equation*}
T P C_{j}=b_{j}+c_{j}\left(Q_{j}\right), \quad c_{j}(0)=0, \quad c_{j}^{\prime}\left(Q_{j}\right)>0, \quad c_{j}^{\prime \prime}\left(Q_{j}\right) \geq 0, \quad j=c, f \tag{2}
\end{equation*}
$$

The marginal cost is assumed positive and increasing in quantity due to limited processing capacity in each period.

The scale elasticity, denoted by $\varepsilon_{j}$, plays a crucial role in the subsequent analysis. As long as the use of input is efficient, it is defined by the long-run average cost divided by the long-run marginal cost, $\varepsilon_{j} \equiv A C_{j} / M C_{j}$. There is economies of scale if $\varepsilon_{j}>1$, and diseconomies of scale if $\varepsilon_{j}<1$. To simplify the analysis, it is convenient to use a special parameter $\alpha_{j}$ as an indicator for the degree of (dis)economies of scale in processing. It is defined by

$$
\begin{equation*}
\alpha_{j} \equiv\left(c_{j}^{\prime}\left(Q_{j}\right)-\frac{b_{j}+c_{j}\left(Q_{j}\right)}{Q_{j}}\right)=M C_{j}-A C_{j}=\left(1-\varepsilon_{j}\right) M C_{j} \quad, \quad j=c, f \tag{3}
\end{equation*}
$$

It follows that $\alpha_{j}$ will be negative in situations where there are economies of scale in processing, positive under diseconomies of scale, and zero under constant returns to scale.

The processed good is assumed to be less perishable and easier to handle and to transport than the raw commodity. This is reflected in the model by assuming perfect competition in the market for the final good. The demand is completely elastic and represented by the exogenous output price, $P$. This strict assumption ensures that the analysis focuses on the processors’ competition for input. ${ }^{3}$ It requires that each processor's output is small relative to the total output of the processed commodity. This is most likely to occur for (homogenous) bulk commodities. If the processors differentiate their products, imperfect competition will occur. This option is addressed in the Discussion section.

## Stage Two: The Output Decision

In the second stage of the game, the quantity produced is determined and the payoff to farmers derived based on the distribution of farmers between the two processors determined in Stage one. The behavior of the players in this stage of the game is modeled below.

The Investor-owned Firm:
The iof is assumed to maximize profit by choosing the quantity $Q_{f}$ of the intermediary input to process and market.

The $n_{f}$ atomistic farmers who decided to supply the iof in stage one act as price takers and are offered a price $\omega_{f}$ per unit. Each farmer i maximizes profit by choosing his output $q_{i}$ so as to solve the following problem.

$$
\begin{equation*}
\max _{q_{i}}\left\{\omega_{f} q_{i}-g\left(q_{i}\right)\right\}, \quad i=1, \ldots, n_{f} \tag{4}
\end{equation*}
$$

The first-order condition for maximum defines the inverse supply function

$$
\begin{equation*}
\omega_{f}=g^{\prime}\left(q_{i}\right), \quad i, \ldots, n_{f} \quad \text { where } \quad q_{i}>0 \quad \text { as long as } \quad \omega_{f}>g^{\prime}(0) \tag{5}
\end{equation*}
$$

For any number of suppliers determined in stage one, $n_{f}$, the iof chooses the total quantity to process and market, $Q_{f}$, so as to maximize profits $\Pi_{f}\left(Q_{f} ; n_{f}\right)$. This is done under the restriction that it at least must break even, $\Pi_{f}\left(Q_{f} ; n_{f}\right) \geq 0$. It is assumed
to close down if it earns negative profit. Since all the $n_{f}$ producers are assumed to be identical, they deliver identical quantities $q_{f}$ to the iof in equilibrium and total quantity supplied is defined as $Q_{f} \equiv n_{f} q_{f}$. Inserting this definition and equation (5) into the iof's objective function gives the following problem to solve

$$
\begin{equation*}
\max _{Q_{f}}\left\{P Q_{f}-b_{f}-c_{f}\left(Q_{f}\right)-g^{\prime}\left(\frac{Q_{f}}{n_{f}}\right) Q_{f}\right\} \text { s.t. } \Pi_{f}\left(Q_{f} ; n_{f}\right) \geq 0 \tag{6}
\end{equation*}
$$

The first order condition for the unrestricted problem reads

$$
\begin{equation*}
P-c_{f}^{\prime}\left(Q_{f}\right)-g^{\prime}\left(\frac{Q_{f}}{n_{f}}\right)=g^{\prime \prime}\left(\frac{Q_{f}}{n_{f}}\right) \frac{Q_{f}}{n_{f}} \tag{7}
\end{equation*}
$$

The right hand side of equation (7) is positive per definition. It follows that the output price is higher than total marginal costs in optimum, $\left(P>T M C_{f}\right)$, and consequently that production and processing will be lower than what is socially optimal. We have the classical underproduction problem caused by a firm exercising market power.

Implicit derivation of $\omega_{f}$ with respect to $n_{f}$ in the inverse supply function $\omega_{f}=g^{\prime}\left(Q_{f} / n_{f}\right)$ holding $Q_{f}$ constant, yields $\partial \omega_{f} / \partial n_{f}=-g^{\prime \prime}\left(Q_{f} / n_{f}\right) Q_{f} / n_{f}^{2}<0$. Hence, the unit price paid to the suppliers, $\omega_{f}$, decreases with the number of suppliers $n_{f}$. It follows that the fewer the farmers supplying the iof (which is exogenous to the iof in stage two of the game), the lower is the firm's margin. A minimum number of suppliers $\bar{n}_{f}$ is required for the iof to obtain non-negative profit and stay in business. It is defined by the break-even condition evaluated in optimum, $\Pi_{f}\left(Q_{f} ; \bar{r}_{f}\right)=0$.

The payoff that the iof 's offer the suppliers in stage one of the game is a function with properties presented in Lemma 1.

Lemma 1 (Properties of the iof payoff): Denote by $N$ the total number of producers, and let $\bar{n}_{f}$ denote the minimum number of suppliers necessary for the iof to break even, defined by $\Pi_{f}\left(Q_{f} ; \bar{n}_{f}\right)=0$.
a) The payoff $\pi_{f}^{*}\left(n_{f}\right)$ is a function of the number iof suppliers, $n_{f}$. It is zero for all $n_{f} \in\left[0, \bar{n}_{f}\right)$ and positive and strictly decreasing for all $n_{f} \in\left[\bar{n}_{f}, N\right]$.

$$
\pi_{f}^{*}\left(n_{f}\right)\left\{\begin{array}{l}
=0 \forall n_{f} \in\left[0, \bar{n}_{f}\right) \\
>0 \forall n_{f} \in\left[\bar{n}_{f}, N\right]
\end{array} \text { and } \frac{\partial \pi_{f}^{*}\left(n_{f}\right)}{\partial n_{f}}<0 \forall n_{f} \in\left[\bar{n}_{f}, N\right]\right.
$$

b) An additional supplier will cause the individual output to decrease for all existing suppliers while the aggregate output will increase.

$$
\frac{\partial q_{f}}{\partial n_{f}}>0 \forall n_{f} \in\left[\bar{n}_{f}, N\right] \quad \text { and } \quad \frac{\partial Q_{f}}{\partial n_{f}}>0 \quad \forall n_{f} \in\left[\bar{n}_{f}, N\right]
$$

- Proof: See Appendix 1

It follows from Lemma 1 that the more suppliers the iof attracts in stage one, the stronger is its market power towards the suppliers. As a result, the payoff the iof offers decreases with the number of suppliers as shown by the curve marked $a$ in Figure 1. Even if the individual output of the iof's suppliers decreases with an additional supplier, the aggregate output increases.


Figure 1: Payoff function for the investor-owned firm.

## The Open Membership Cooperative:

The coop is owned and operated by the members who produce and supply the intermediary input. It is assumed to practice an open membership policy, letting farmers join or resign from the organization without costs. ${ }^{4}$ To be considered a cooperative it is assumed that at least three farmers must be associated, $n_{c} \geq 3$.

The maximization of joint profit (JPM) and of net average revenue product (NARP) are the two most frequently objectives employed in analyses of marketing cooperatives. Both are examined below.

Joint Profit Maximization: The objective of a JPM coop is to maximize the joint profit $\Pi_{c}\left(Q_{c} ; n_{c}\right)$ from the primary production of all farmers and from the processing activity (Enke; Taylor). Hence, the coop coordinates each member's production quantity. The coop will close down if it cannot secure the members a non-negative profit. The maximization problem then reads

$$
\begin{equation*}
\max _{Q_{c}}\left\{P Q_{c}-b_{c}-c_{c}\left(Q_{c}\right)-n_{c} g\left(\frac{Q_{c}}{n_{c}}\right)\right\} \text { s.t. } \Pi_{c}\left(Q_{c} ; n_{c}\right) \geq 0 \tag{8}
\end{equation*}
$$

The assumption that all coop farmers are identical and provides identical output in equilibrium, $q_{c}=Q_{c} / n_{c}$, results in a first order condition for the unrestricted problem stating that price equals total marginal costs in optimum.

$$
\begin{equation*}
P-c_{c}^{\prime}\left(Q_{c}\right)-g^{\prime}\left(\frac{Q_{c}}{n_{c}}\right)=0 \tag{9}
\end{equation*}
$$

It follows that the solution is socially optimal. The second order condition is assumed to be satisfied for all defined quantities.

Lemma 2 (Properties of the JPM coop): Denote by $N$ the total number of suppliers. Further, let $\hat{n}_{c}$ denote the minimum numbers of members necessary for the coop to break even, defined by $P-\left[b_{c}+c_{c}\left(\hat{n}_{c} q_{c}\right)\right] / \hat{n}_{c} q_{c}=0$. Then, $\bar{n}_{c} \equiv \max \left[3, \hat{n}_{c}\right]$ denotes the minimum number of members necessary for the coop to stay in business.
$\alpha_{c} \equiv c_{c}^{\prime}\left(n_{c} q_{c}\right)-\left[b_{c}+c_{c}\left(n_{c} q_{c}\right)\right] / n_{c} q_{c}$ is a parameter capturing the degree of economies of scale in processing.

The cooperative maximizing the joint profit (JPM), offers a payoff to the members, $\pi_{c}^{*}\left(n_{c}\right)$, that is zero for $n_{c} \in\left[0, \bar{n}_{c}\right)$ and positive for all $n_{c} \in\left[\bar{n}_{c}, N\right]$ and that holds the following properties:
a) The payoff is i) strictly increasing in $n_{c}$ under economies of scale in processing $\alpha_{c}<0$, ii) strictly decreasing under diseconomies of scale in processing, $\alpha_{c}>0$, and iii) constant under constant returns to scale in processing, $\alpha_{c}=0$.

$$
\pi_{c}^{*}\left(n_{c}\right)\left\{\begin{array} { l l } 
{ = 0 , } & { n _ { c } \in [ 0 , \overline { n } _ { c } ) } \\
{ > 0 , } & { n _ { c } \in [ \overline { n } _ { c } , N ] }
\end{array} \quad \text { and } \quad \frac { \partial \pi _ { c } ^ { * } ( n _ { c } ) } { \partial n _ { c } } \left\{\begin{array}{lll}
>0 & \text { if } & \alpha_{c}<0 \\
=0 & \text { if } & \alpha_{c}=0 \\
<0 & \text { if } & \alpha_{c}>0
\end{array} \quad \forall n_{c} \in\left[\bar{n}_{c}, N\right]\right.\right.
$$

b) An additional member will cause the individual output to decrease while the aggregate output increases.

$$
\frac{\partial q_{c}}{\partial n_{c}}<0 \quad \text { and } \quad \frac{\partial Q_{c}}{\partial n_{c}}>0 \quad \forall n_{c} \in\left[\bar{n}_{c}, N\right]
$$

■ Proof: See Appendix 2

A production (or processing) function exhibiting increasing returns to scale for low output levels and decreasing returns to scale as output increases is referred to by Frisch (1965) as the ultra-passum law of production. Above it is established that the aggregate output $Q_{c}$ increases with the number of members. Hence, in a situation with ultrapassum law in processing, the scale parameter $\alpha_{c}$ will increase with increasing number of members, $\partial \alpha_{c} / \partial n_{c}>0$. It will start out being negative and eventually take on
positive values. The payoff function associated with this type of processing technology is shown in figure 2. In the same figure, the payoff for overall increasing and overall decreasing returns to scale are demonstrated.


Figure 2: Payoff function for the cooperative. Curve a describes the payoff under ultra-passum production technology in processing. Curve $\boldsymbol{b}$ describes the payoff under overall economies of scale, and c under overall diseconomies of scale.

It is shown that a new member causes an increase in the total output even if the individual output of each farmer is reduced. It follows that the new member produces more than the aggregate reduction in production of all existing farmers. This property is independent of the scale properties in production.

NARP Maximization: The NARP coop (Helmberger and Hoos) is assumed to be obliged to process and market all input the members choose to supply. Consequently it has no control over the production of intermediary input and the quantity it has to process. It lacks the ablility of coordination found in the JPM-coop. The average net return per unit of input, also called net average revenue product (NARP) is in the present set-up defined by ${ }^{5}$

$$
\begin{equation*}
\omega_{c}=P-\frac{b_{c}+c_{c}\left(Q_{c}\right)}{Q_{c}} \quad(\equiv N A R P) \tag{10}
\end{equation*}
$$

In this static model with only one single output sold in a competitive market, the coop has no direct influence on the $N A R P$, but acts as a mere price clearing central for the members.

The maximization approach of Albæk and Schultz is employed where each member $i$ chooses his quantity $q_{i}$ so as to maximize his own profit. The member takes as given both the number of other coop members, $n_{c}-1$, and the production level they choose, $Q_{c-i}=\sum_{k=1}^{n_{c}} q_{j}, k \neq i$. The individual member then seeks to maximize

$$
\begin{equation*}
\max _{q_{i}}\left\{\left(P-\frac{b_{c}+c_{c}\left(q_{i}+Q_{c-i}\right)}{q_{i}+Q_{c-i}}\right) q_{i}-g\left(q_{i}\right)\right\} \quad \text { s.t. } \quad \pi_{c}\left(q_{i} ; n_{c} Q_{c-i}\right) \geq 0, i=1, \ldots, n_{c} \tag{11}
\end{equation*}
$$

Each farmer is assumed to require a non-negative profit and internalizes only the effect that his output decision has on his own profit. The effect imposed on the other members is ignored.

Assume that all $n_{c}$ suppliers in the coop are identical and produce the same output $q_{c}$ in equilibrium. The first order condition for maximum in the unrestricted problem when using $Q_{c-i}=\left(n_{c}-1\right) q_{c}$ and $q_{i}=q_{c}$ in equilibrium, is then given by

$$
\begin{equation*}
P-c_{c}^{\prime}\left(Q_{c}\right)-g^{\prime}\left(\frac{Q_{c}}{n_{c}}\right)=-\frac{\left(n_{c}-1\right)}{n_{c}} \underbrace{\left(c_{c}^{\prime}\left(Q_{c}\right)-\frac{b_{c}+c_{c}\left(Q_{c}\right)}{Q_{c}}\right)}_{\alpha_{c}} \tag{12}
\end{equation*}
$$

The solution will only be socially optimal (price equal to marginal costs) when there are constant returns to scale in processing ( $\alpha_{c}=0$ ). Economies of scale ( $\alpha_{c}<0$ ) give underproduction since the price exceeds marginal costs in equilibrium and overproduction occurs under diseconomies of scale ( $\alpha_{c}>0$ ).

Lemma 3 (Properties of the NARP coop payoff): Denote by $N$ the total number of suppliers. Further, let $\hat{n}_{c}$ denote the minimum numbers of members necessary for the coop to break even, defined by $P-\left[b_{c}+c_{c}\left(\hat{n}_{c} q_{c}\right)\right] / \hat{n}_{c} q_{c}=0$. Then, $\bar{n}_{c} \equiv \max \left[3, \hat{n}_{c}\right]$ denotes the minimum number of members necessary for the coop to stay in business. The parameter capturing the degree of economies of scale in processing is denoted
$\alpha_{c} \equiv c_{c}^{\prime}\left(n_{c} q_{c}\right)-\left[b_{c}+c_{c}\left(n_{c} q_{c}\right)\right] / n_{c} q_{c}$. Since $\alpha_{c}$ is increasing in $Q_{c}$, it follows that it also is increasing in $n_{c}$. Let $\delta \equiv\left(\frac{n_{c} q_{c}}{n_{c}-1}\right)\left(c_{c}^{\prime \prime}\left(n_{c} q_{c}\right)+g^{\prime \prime}\left(q_{c}\right)\right)$ denote an "upper threshold" for the economies of scale such that $\alpha_{c}>-\delta$ and let $\beta \equiv\left(\frac{n_{c}}{n_{c}-2}\right) q_{c} c^{\prime \prime}\left(n_{c} q_{c}\right)>0$. Assume that $\gamma \equiv\left(n_{c}-2\right) g^{\prime \prime}\left(q_{c}\right)-c_{c}^{\prime \prime}\left(n_{c} q_{c}\right)>0$.
a) The cooperative maximizing net average revenue product (NARP) offers a positive payoff, $\pi_{c}^{*}\left(n_{c}\right)$, to all members if $n_{c} \in\left[\bar{n}_{c}, N\right]$ and zero payoff for all $n_{c} \in\left[0, \bar{n}_{c}\right)$. This implies that i) the payoff will be strictly increasing in the number of members, $n_{c}$, for situations with economies of scale in processing, $\alpha_{c}<0$, where a limit $\delta$ is set to the economies of scale so that $\delta<\alpha_{c}$, ii) The payoff will be strictly decreasing under diseconomies of scale, $\alpha_{c}>0$, and iii) the payoff will be constant under constant returns to scale, $\alpha_{c}=0$.

$$
\pi_{c}^{*}\left(n_{c}\right)\left\{\begin{array} { l l } 
{ = 0 , } & { n _ { c } \in [ 0 , \overline { n } _ { c } ) } \\
{ > 0 , } & { n _ { c } \in [ \overline { n } _ { c } , N ] }
\end{array} \quad \text { and } \quad \frac { \partial \pi _ { c } ^ { * } ( n _ { c } ) } { \partial n _ { c } } \left\{\begin{array}{llll}
>0 & \text { if } & \alpha_{c} \in(-\delta, 0) \\
=0 & \text { if } & \alpha_{c}=0 \\
<0 & \text { if } & \alpha_{c} \in(0, \infty)
\end{array} \quad \forall n_{c} \in\left[\bar{n}_{c}, N\right]\right.\right.
$$

b) The production from an additional member will make the aggregated quantity increase in the number of members as long as $\gamma \geq 0$ and $n_{c} \in\left[\bar{n}_{c}, N\right]$. The members' individual output will i) increase for all existing members under economies of scale, $\alpha_{c} \in(-\delta, 0)$, ii) decrease for all existing members under diseconomies of scale, $\alpha_{c} \in(0, \infty)$, and iii) remain constant for all existing members under constant returns to scale, $\alpha_{c}=0$.

$$
\begin{aligned}
& \text { For } \begin{array}{l}
n_{c} \in\left[\bar{n}_{c}, N\right] \text {, } \\
\frac{\partial q_{c}}{\partial n_{c}}\left\{\begin{array}{lll}
>0 & \text { if } & \alpha_{c} \in(-\delta,-\beta) \\
=0 & \text { if } & \alpha_{c}=-\beta \\
<0 & \text { if } & \alpha_{c} \in(-\beta, \infty)
\end{array} \quad \text { and } \quad \frac{\partial Q_{c}}{\partial n_{c}}>0 \forall \alpha_{c} \in(-\delta, \infty) \text { if } \quad \gamma \geq 0\right.
\end{array}
\end{aligned}
$$

■ Proof: See Appendix

The properties of the payoff functions in the two different types of cooperatives are shown to be analogous. Figure 2 will hence apply also to the NARP coop. However, comparison of the two first order conditions given in equations (9) and (12) shows that only the JPM coop entails a Pareto optimal solution. The reason is that the individual farmer in the NARP coop does not take into account the impact his action has on the outcome of all other farmers; it leads to too low production in a situation with economies of scale in processing, and too high production in a situation with diseconomies of scale. It follows that the payoff in the NARP coop always will be lower than the one of the JPM coop for any given number of suppliers, except for constant returns to scale, $\alpha_{c}=0$ where they are equal. This is due to lack of coordination in the NARP coop. It follows that under an ultra-passum technology in processing, maximum payoff will occur for a lower number of members in a JPM coop than in a NARP coop, all other things equal. If the farmers could freely decide the objective of the coop they should obviously choose the JPM-objective. However, the objective of the NARP coop may be seen as the outcome of a former constitution game in the organization, and as such, difficult to change. It is often a result of restrictions made by the Government and/or the Anti-trust authorities.

Lemmas 2 and 3 cover a wide range of firms and situations since they are based on quite general functional forms.

## Stage 1: Farmers' Choice of Processor - Market Game

The farmers are assumed to be rational and will hence seek to maximize utility. In this analysis, utility conveniently is assumed to be identical to profit.

In the two preceding sections we have derived the payoff functions for the coop, $\pi_{c}^{*}\left(n_{c}\right)$, and for the iof, $\pi_{f}^{*}\left(n_{f}\right)$. These two functions represent the maximum payoff obtainable for the individual supplier of the coop and the iof respectively, given the number of members/suppliers of each firm. Whenever there is a higher economic return through the coop, producers will switch from the iof to the coop and vice versa. ${ }^{6}$ Farmer $i$ will hence always choose the processor that offers him the highest profit.

$$
\begin{equation*}
\max \left[\pi_{c}^{*}\left(n_{c}\right), \pi_{f}^{*}\left(n_{f}\right)\right] \tag{13}
\end{equation*}
$$

To simplify the analysis it is assumed that $\pi_{c}(N)>0$ and $\pi_{f}(N)>0$. In other words, the payoff will be positive so that all the $N$ farmers will produce under all possible market structures. The producers will keep on switching between the two processors until an equilibrium distribution of producers $\left(n_{c}^{*}, n_{f}^{*}\right)$ is obtained where all receive an identical payoff $\pi^{*}\left(n_{c}^{*}, n_{f}^{*}\right) .{ }^{7}$ If no producer has the incentive to alter his own choice when the choices of all the other producers are revealed to him, this distribution constitutes a sub game perfect Nash equilibrium.

Possible outcomes of the market game are a single coop with the payoff $\pi^{*}(N, 0) \equiv \pi_{c}^{*}(N)$, a single iof with payoff $\pi^{*}(0, N) \equiv \pi_{f}^{*}(N)$, or a mixed market equilibrium where both firms coexist, offering the farmers identical profit:
$\pi^{*}\left(n_{c}^{*}, n_{f}^{*}\right) \equiv \pi_{c}^{*}\left(n_{c}^{*}\right) \equiv \pi_{f}^{*}\left(n_{f}^{*}\right) \quad$ s.t. $\quad n_{c}+n_{f}=N$.
Based on the characteristics of the payoff functions of the iof and the coop described in Lemmas 1, 2 and 3 respectively, the Nash-equilibria for the input market can be found. By using the definition $n_{f} \equiv N-n_{c}$ in the iof's payoff function, the payoff functions of both processors can conveniently be inserted into a common diagram and the equilibrium can be demonstrated and discussed graphically. The number of members in the coop is measured on the $x$-axis from the left to the right. The total number of
farmers $N$ is the maximum. Evidently, the number of suppliers of the iof will be displayed from right to left. The characteristics of the equilibria are presented in Proposition 1-3 below. They refer to the crossing or non-crossing of the two firms’ payoff-functions in the common diagram

PROPOSITION 1 (No Crossing of Payoff Functions): The payoff functions of both the coop and the iof are shown to be continuous over their range. It follows that there exists a market structure with a single firm if no crossing of the two payoffs take place.
Depending on the costs, the market will consist of $a$ :
a) Single coop defined by, $\left(n_{c}^{*}, n_{f}^{*}\right)=(N, 0)$ with the payoff $\pi^{*}\left(n_{c}^{*}, n_{f}^{*}\right)=\pi_{c}^{*}(N)$ if

$$
\pi_{c}^{*}\left(n_{c}\right)>\pi_{f}^{*}\left(N-n_{c}\right) \quad \forall \quad n_{c} \in\left[\bar{n}_{c},\left(N-\bar{n}_{f}\right)\right]
$$

b) Single iof defined by $\left(n_{c}^{*}, n_{f}^{*}\right)=(0, N)$ with the payoff $\pi^{*}\left(n_{c}^{*}, n_{f}^{*}\right)=\pi_{f}^{*}(N)$ if

$$
\pi_{c}^{*}\left(n_{c}\right)<\pi_{f}^{*}\left(N-n_{c}\right) \quad \forall \quad n_{c} \in\left[\bar{n}_{c},\left(N-\bar{n}_{f}\right)\right]
$$

■ Proofs follow directly from Lemmas 1, 2 and 3.
The two equilibrium solutions from Proposition 1 are described in figures 3 panel a and b respectively. These market outcomes are likely to occur if one of the firms has very low processing costs relative to the other.


Figure3 a: No crossing of payoffs where the coop payoff surpasses the iof payoff for all $\left(n_{c}, n_{t}\right)$. The equilibrium solution results in a single cooperative (corner solution).


Figure $3 \mathbf{b}$ : No crossing of payoffs where the iof payoff surpasses the coop payoff for all $\left(n_{c}, n_{t}\right)$. The equilibrium solution results in a single investor-owned firm (corner solution).

PROPOSITION 2 (Single Crossing of Payoff Functions): The payoff functions of both the coop and the iof are shown to be continuous over their range. The payoffs will cross once and only once in a point where the distribution of producers is denoted $\left(n_{c}^{k}, n_{f}^{k}\right)$ under the following conditions:
a) Assume that $\pi_{c}^{*}\left(\bar{n}_{c}\right)<\pi_{f}^{*}\left(N-\bar{n}_{c}\right)$ and $\pi_{c}^{*}\left(N-\bar{n}_{f}\right)>\pi_{f}^{*}\left(\bar{n}_{f}\right)$. From Lemma 1-3 we have that $\partial \pi_{c}^{*}\left(n_{c}\right) / \partial n_{c}>0$ and $\partial \pi_{f}^{*}\left(N-n_{c}\right) / \partial n_{c}>0$, evaluated for the distribution of producers, $\left(n_{c}^{k}, n_{f}^{k}\right)$.
i) If $\pi_{c}^{*}\left(n_{c}^{k}\right)>\pi_{c}^{*}(N)$, we will have a mixed market equilibrium defined by $\left(n_{c}^{*}, n_{f}^{*}\right)=\left(n_{c}^{k}, n_{f}^{k}\right)$ where $\bar{n}_{c} \leq n_{c}^{k} \leq\left(N-\bar{n}_{f}\right)$ and $\bar{n}_{f} \leq n_{f}^{k} \leq\left(N-\bar{n}_{c}\right)$. The equilibrium payoff will be $\pi^{*}\left(n_{c}^{*}, n_{f}^{*}\right)=\pi_{c}^{*}\left(n_{c}^{*}\right)=\pi_{f}^{*}\left(n_{f}^{*}\right)$. (See figure 4 , panel a)
ii) If $\pi_{c}^{*}\left(n_{c}^{k}\right) \leq \pi_{c}^{*}(N)$, we will have a single coop in equilibrium defined by $\left(n_{c}^{*}, n_{f}^{*}\right)=(N, 0)$. The equilibrium payoff will be $\pi^{*}\left(n_{c}^{*}, n_{f}^{*}\right)=\pi_{c}^{*}(N)$. (See figure 4, panel b)
iii) Let $\pi_{c}^{*}\left(\bar{n}_{c}\right)>\pi_{f}^{*}\left(N-\bar{n}_{c}\right)$ and $\pi_{c}^{*}\left(N-\bar{n}_{f}\right)<\pi_{f}^{*}\left(\bar{n}_{f}\right)$. Here $\partial \pi_{c}^{*}\left(n_{c}\right) / \partial n_{c}<0$ and $\partial \pi_{f}^{*}\left(N-n_{c}\right) / \partial n_{c}>0$ evaluated in $\left(n_{c}^{k}, n_{f}^{k}\right)$. These conditions yield a stable mixed market equilibrium defined by $\left(n_{c}^{*}, n_{f}^{*}\right)=\left(n_{c}^{k}, n_{f}^{k}\right)$, where $\bar{n}_{c} \leq n_{c}^{k} \leq\left(N-\bar{n}_{f}\right)$ and $\bar{n}_{f} \leq n_{f}^{k} \leq\left(N-\bar{n}_{c}\right)$. The resulting payoff will be $\pi^{*}\left(n_{c}^{*}, n_{f}^{*}\right)=\pi_{c}^{*}\left(n_{c}^{k}\right)=\pi_{f}^{*}\left(n_{f}^{k}\right)$. (See figure 4 panel c)

- Proof follows directly from Lemmas 1, 2 and 3.

If the coop has overall economies of scale it will crowd out the iof under "single crossing" of payoffs. This situation is described in Figure 4a. However, if it has an ultra-passum technology in processing and single crossing, the equilibrium solution will depend on whether the single firm payoff is higher or lower than the mixed market. A situation where a mixed market is the equilibrium is described in Figure 4b.

If the coop has overall diseconomies of scale and there exists a single crossing of payoffs, a mixed market will emerge. This situation is described in figure 4 panel c.


Figure 4 a: Single crossing of payoffs where the coop payoff is increasing at the crossing point. The mixed equilibrium will be Pareto-dominated by the single cooperative equilibrium (corner solution).


Figure 4 b: Single crossing of payoffs where the coop payoff is increasing at the crossing point. The single cooperative equilibrium (corner solution) will be Pareto-dominated by the mixed equilibrium.


Figure 4 c: Single crossing of payoffs where the coop payoff is decreasing at the crossing point. A mixed market will be the equilibrium solution.

PROPOSITION 3 (Double Crossing of Payoff Functions): The payoff functions of both the coop and the iof are shown to be continuous over their range. Assume that $\pi_{c}^{*}\left(\bar{n}_{c}\right) \leq \pi_{f}^{*}\left(N-\bar{n}_{c}\right)$ and $\pi_{c}^{*}\left(N-\bar{n}_{f}\right) \leq \pi_{f}^{*}\left(\bar{n}_{f}\right)$. It follows then from Lemmas 1-3 that the payoff functions will cross twice. Let $\left(n_{c}^{k}, n_{f}^{k}\right)$ and $\left(n_{c}^{l}, n_{f}^{l}\right)$ denote the distribution of producers in the two crossings of the payoff functions and let $n_{c}^{k}<n_{c}^{l}$.

We have that $\partial \pi_{f}^{*}\left(N-n_{c}\right) / \partial n_{c}>0 \quad \forall \quad n_{c} \in\left[\bar{n}_{c},\left(N-\bar{n}_{f}\right)\right]$. It follows that $\pi^{*}\left(n_{c}^{k}, n_{f}^{k}\right)<\pi^{*}\left(n_{c}^{l}, n_{f}^{l}\right)$ and we will have a mixed market equilibrium defined by $\left(n_{c}^{*}, n_{f}^{*}\right)=\left(n_{c}^{l}, n_{f}^{l}\right)$ with the equilibrium payoff $\pi^{*}\left(n_{c}^{*}, n_{f}^{*}\right)=\pi_{c}^{*}\left(n_{c}^{l}\right)=\pi_{c}^{*}\left(n_{f}^{l}\right)$.

- Proof follows directly from Lemmas 1,2 and 3.

The mixed market described in Proposition 3 is illustrated in figure 5. In this setting the coop is assumed to have an ultra passum processing technology.


Figure 5: Double-crossing of payoffs where the mixed market equilibrium a will be Paretodominated by equilibrium $b$.

A two-tier market has been analyzed where only two potential processors, an open membership coop and a regular iof, operate in the input market. They are price takers in the final market. It is proved that all types of market structures, including a mixed market, are possible equilibria in the market game. Depending on the cost structure and cost level of the processors, we may expect a single cooperative, a single investor-owned firm or a mixed market consisting of both processors to emerge.

## Discussion

Since a coop has maximum profit or maximum price to the member producers as objective, one may think that it will benefit the producers if the coop is the only processor in the market. However, according to the analysis performed, this is not always the case. Situations will exist where a mixed market is superior to a single coop for all producers. The reason is diseconomies of scale in the coop's processing activity. It results in decreasing payoff when the number of members increases because each new member causes the total output to increase. The problem is impossible to avoid as long as the coop practices an open membership policy. It is proved that the payoff for a coop pursuing the NARP objective always will be lower than the one offered by a JPM coop, except for a situation with constant returns to scale in processing. This is due to the problem of non-optimal quantity produced by the NARP coop members under both economies and diseconomies of scale in processing. The individual producer does not take into account the loss his marginal increase in production inflicts upon all the other members. As a result, the likelihood of a mixed market will be highest if the coop maximizes NARP. A JPM coop is more efficient and hence more likely to crowd out the iof.

The output market is assumed to be perfectly competitive. It follows that the payoff to the coop members will be decreasing in the number of members only under diseconomies of scale in processing. Diseconomies of scale are claimed to be rare in the food processing industry. However, we may experience diseconomies of scale when a cooperative grows into a "large" organization with a "loose" management. A reduction in efficiency may also be caused by increasingly diverging objectives of the members as the organization grows. It can also be shown that the payoff will decline if the cooperative offers its output in a market with a downward-sloping demand curve. Hence, imperfect competition in the output market may cause decreasing payoff in the
coop for an additional supplier even if no diseconomies of scale in processing exist. This makes a mixed market more likely to evolve and become an equilibrium solution.

This brings us back to the article of Albæk and Schultz. They analyze a duopoly in the output market and suggest that the coop will always be superior to the iof with regard to the payoff, eventually driving the iof out of the market. The result is based on the two firms producing identical output and competing in a Cournot manner in the output market. Hence, the price of output is linking the two firms' strategies together in the second stage of the game. The cooperative is shown to always produce more than the iof, forcing the price of output down for both firms. Eventually, the iof will be driven out of the market. Differentiating the output can help moderate this extreme situation since it will bring a higher degree of independence between the players at the second stage of the game. The iof will be less sensitive to the coop's strategy and may earn enough profit to remain in the market.

The introduction of heterogeneous farmers, as in the contribution of Sexton, will also reduce the dependency in the input market. This may advance a mixed market equilibrium.

The model of Karantininis and Zago turns out to be a special case of the general model developed in this article. They model a situation where the coop is of the NARP type and exhibits diseconomies of scale for all quantities processed. There are no fixed costs so the iof has no binding break-even restriction. It follows that they always will find a mixed market equilibrium.

Sexton formalizes the notion of the salutary effect the coop has on its rivals' behavior as a "yardstick of competition" for the possible exercise of oligopsony power in a structural oligopsony. The present article shows that in the case of an open membership coop, the iof may serve as a "yardstick of production" for the coop. It helps the member farmers to achieve an increased payoff relative to a situation with a single coop by offering an outside option. In the case of diseconomies of scale it will help
reduce the problem of excess production. Competition with another cooperative would have ensured the same or even a better outcome. However, because of the common ownership of capital in cooperatives it can be a slow process to set up a new one. Probably, the formation of an investor-owned firm will be faster and an iof will be more likely to enter the market first. One thing to notice is that even competition from an iof may enhance the situation for members of a single coop with open membership.

## Concluding Remarks

It has been argued that mixed markets consisting of investor-owned processors and open membership cooperatives cannot be a long term stable equilibrium. Further, it is suggested that the cooperative eventually will crowd out the investor-owned firm leading to a single cooperative in the market. However, we observe many mixed markets in the agricultural sector.

This article has proven that a mixed market may exist as a stable equilibrium under quite general conditions. Diseconomies of scale in processing cause the payoff to producers to decrease with an additional coop member (crowding). In such a situation a single coop will experience excessive supply, causing too low returns for the producers. The problem originates in the open membership policy. The inability to restrict production in the NARP coop (lack of coordination) aggravates the problem. In this situation the entry of an iof into the market will serve as a "yardstick of production"; it offers an outside option to the farmers that eventually may result in higher profits for all farmers, independent of which one of the processors they choose to supply.

A task for further research is to explore the possible effect of differentiating the output in an imperfect output market. Also, empirical work regarding the predictions made in this article would be useful. More insight is needed regarding different market types and their dependence on economies of scale in processing.

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## Appendix

## Proof of Lemma 1

The envelope theorem is used on the first order condition for maximum in equation (7) to first find the effect of an additional supplier on total output.
(14) $\frac{\partial Q_{f}}{\partial n_{f}}=\left[\frac{2 g^{\prime \prime}\left(\frac{Q_{f}}{n_{f}}\right)+g^{\prime \prime \prime}\left(\frac{Q_{f}}{n_{f}}\right) \frac{Q_{f}}{n_{f}}}{n_{f} c_{f}^{\prime \prime}\left(Q_{f}\right)+2 g^{\prime \prime}\left(\frac{Q_{f}}{n_{f}}\right)+g^{\prime \prime \prime}\left(\frac{Q_{f}}{n_{f}}\right) \frac{Q_{f}}{n_{f}}}\right] \frac{Q_{f}}{n_{f}}>0 \quad \forall Q_{f}>0, \bar{n}_{f} \leq n_{f} \leq N$

Due to the assumption that $g^{\prime \prime \prime}\left(q_{f}\right) \geq 0$, total output is strictly increasing in the number of suppliers, $n_{f}$.

The effect on the individual supplier's quantity of an additional supplier is found by substituting the equilibrium condition $Q_{f}=n_{f} q_{f}$ into the first order condition (7) and then using the envelope theorem

$$
\begin{equation*}
\frac{\partial q_{f}}{\partial n_{f}}=\frac{-c_{f}^{\prime \prime}\left(n_{f} q_{f}\right) q_{f}}{n_{f} c_{f}^{\prime \prime}\left(n_{f} q_{f}\right)+2 g^{\prime \prime}\left(q_{f}\right)+g^{\prime \prime \prime}\left(q_{f}\right) q_{f}}<0 \quad \forall \bar{n}_{f} \leq n_{f} \leq N \tag{15}
\end{equation*}
$$

The net payoff offered, is obtained by inserting the optimal quantity from equation (7) into the iof supplier's profit function, equation( 4). It is a function of the number of suppliers from stage one of the game.

$$
\begin{equation*}
\pi_{f}^{*}\left(n_{f}\right)=\omega q_{f}\left(n_{f}\right)-g\left(q_{f}\right)=g^{\prime}\left(q_{f}\right) q_{f}-g\left(q_{f}\right) \quad \text { where } \quad q_{f}=q_{f}\left(n_{f}\right) \tag{16}
\end{equation*}
$$

From equation (16) and (15) it follows that

$$
\begin{equation*}
\frac{\partial \pi_{f}^{*}\left(n_{f}\right)}{\partial n_{f}}=g^{\prime \prime}\left(q_{f}\right) q_{f} \frac{\partial q_{f}}{\partial n_{f}}<0 \tag{17}
\end{equation*}
$$

## Proof of Lemma 2

From equation (9) we have

$$
\begin{equation*}
\frac{\partial q_{c}}{\partial n_{c}}=-\frac{c_{c}^{\prime \prime}\left(n_{c} q_{c}\right) q_{c}}{c_{c}^{\prime \prime}\left(n_{c} q_{c}\right) n_{c}+g^{\prime \prime}\left(q_{c}\right)}<0, \quad \forall q_{c}>0 \tag{18}
\end{equation*}
$$

The joint profit is distributed symmetrically so the individual farmer's profit (payoff) takes the form of $\pi_{c}^{*}\left(n_{c}\right)=\Pi_{c}^{*} / n_{c}$. By using the envelope theorem we find that

$$
\begin{equation*}
\frac{\partial \pi_{c}^{*}\left(n_{c}\right)}{\partial n_{c}}=-\frac{Q_{c}}{n_{c}^{2}} \alpha_{c} \tag{19}
\end{equation*}
$$

Inserting $Q_{n}=n_{c} q_{c}$ into the first order condition (9) yields

$$
\begin{equation*}
\frac{\partial Q_{c}}{\partial n_{c}}=\frac{Q_{c}}{n_{c}}\left[1-1 /\left(1+\frac{g^{\prime \prime}\left(Q_{c} / n_{c}\right)}{c_{c}^{\prime \prime}\left(Q_{c}\right) n_{c}}\right)\right]>0, \quad \forall \bar{n}_{c} \leq n_{c} \leq N \tag{20}
\end{equation*}
$$

## Proof of Lemma 3

The profit for the individual member in equation (11), is a sum of concave and strictly concave functions in $q_{c}$. It follows that it must be strictly concave and that the second order condition for maximum will be satisfied for all positive quantities $q_{c}$, that is

$$
\begin{equation*}
\frac{\partial^{2} \pi_{c}^{*}}{\left(\partial q_{c}\right)^{2}}=-\left(\frac{2}{n_{c}}\right)\left(\frac{n_{c}-1}{n_{c} q_{c}}\right) \underbrace{\left(c_{c}^{\prime}\left(n_{c}\right)-\frac{b_{c}+c_{c}\left(n_{c} q_{c}\right)}{n_{c} q_{c}}\right)}_{\alpha_{c}}-\left(\frac{1}{n_{c}} c_{c}^{\prime \prime}\left(n_{c} q_{c}\right)+g^{\prime \prime}\left(q_{c}\right)\right)<0 \quad \forall q_{c} \tag{21}
\end{equation*}
$$

By using the envelope theorem on the first order condition from equation (12) where $Q_{c} \equiv n_{c} q_{c}$, we find that

$$
\begin{equation*}
\frac{\partial q_{c}\left(n_{c}\right)}{\partial n_{c}}=-\frac{q_{c}}{n_{c}}\left[\frac{\left(\frac{n_{c}-2}{n_{c} q_{c}}\right) \alpha_{c}+c_{c}^{\prime \prime}\left(n_{c} q_{c}\right)}{\left(\frac{n_{c}-1}{n_{c} q_{c}}\right) \alpha_{c}+c_{c}^{\prime \prime}\left(n_{c} q_{c}\right)+g^{\prime \prime}\left(q_{c}\right)}\right] \tag{22}
\end{equation*}
$$

This expression is negative under diseconomies of scale in processing, $\alpha_{c}>0$, when $n_{c}>2$. For $\alpha_{c}<0$, the expression will depend upon the sign of the nominator relative to the denominator. In the subsequent analysis it is assumed that $\alpha_{c}>-\frac{n_{c} q_{c}}{n_{c}-1}\left[c_{c}^{\prime \prime}\left(n_{c} q_{c}\right)+g^{\prime \prime}\left(q_{c}\right)\right] \equiv-\delta$ to assure that the denominator in equation ( 22) remains positive. The interpretation of this assumption is that an upper limit is set for economies of scale in processing. By using the envelope theorem on the first order condition and that $Q_{c}=n_{c} q_{c}$ we find how the total quantity changes when the number of members increases.

$$
\begin{equation*}
\frac{\partial Q_{c}\left(n_{c}\right)}{\partial n_{c}}=\frac{1}{n_{c}}\left[\frac{\alpha_{c}+g^{\prime \prime}\left(\frac{Q_{c}}{n_{c}}\right) Q_{c}}{\left(\frac{n_{c}-1}{Q_{c}}\right) \alpha_{c}+c_{c}^{\prime \prime}\left(Q_{c}\right)+g^{\prime \prime}\left(\frac{Q_{c}}{n_{c}}\right)}\right] \tag{23}
\end{equation*}
$$

Again, for $\alpha_{c}>0$ (diseconomies of scale), total quantity increases with the number of members. Rearranging the denominator that is assumed to be positive entails the following condition

$$
\begin{equation*}
\alpha_{c}>-Q_{c} g^{\prime \prime}\left(q_{c}\right)-\frac{Q_{c}}{n_{c}-1}\left(c_{c}^{\prime \prime}\left(Q_{c}\right)-\left(n_{c}-2\right) g^{\prime \prime}\left(q_{c}\right)\right) \tag{24}
\end{equation*}
$$

It follows that the nominator will be non-negative for $\alpha_{c}<0$ if $c_{c}^{\prime \prime}\left(n_{c} q_{c}\right) \leq\left(n_{c}-2\right) g^{\prime \prime}\left(q_{c}\right)$, which is assumed in the subsequent analysis. ${ }^{8}$ The interpretation of this condition is that the marginal cost of processing the optimal quantity for any number of members, is increasing less than $\left(n_{c}-2\right)$ times the marginal cost of production of the member farmers. It follows that the total quantity always will increase with an increasing number of members under the given conditions.

Next the derivate of the payoff $\pi_{c}^{*}\left(n_{c}\right)$ with respect to the number of members and insert the result from equation (22)

$$
\begin{equation*}
\frac{\partial \pi_{c}^{*}\left(n_{c}\right)}{\partial n_{c}}=\alpha_{c} \frac{q_{c}}{n_{c}}\left[\frac{-\frac{2}{n_{c}}\left(\frac{n_{c}-1}{n_{c} q_{c}}\right) \alpha_{c}-\left(\frac{1}{n_{c}} c_{c}^{\prime \prime}\left(n_{c} q_{c}\right)+g^{\prime \prime}\left(q_{c}\right)\right)}{\left(\frac{n_{c}-1}{n_{c} q_{c}}\right) \alpha_{c}+\left(c_{c}^{\prime \prime}\left(n_{c} q_{c}\right)+g^{\prime \prime}\left(q_{c}\right)\right)}\right] \tag{25}
\end{equation*}
$$

The numerator in the large brackets is the expression from the second order condition in equation (21) and thus negative for all $\alpha_{c}$. The sign of the denominator is already assumed to be positive. Hence, the sign of the expression will be equal to the sign of $\alpha_{c}$. Payoff will thus increase for an increasing number of members as long as there are economies of scale in processing. When production reaches a level where an extra member causes diseconomies of scale, the payoff will decrease with an increase in members. For constant returns to scale, the payoff will be insensitive to the number of members.

The NARP includes the coop's fixed cost, $b_{c}$. A certain number of members is hence required to generate a sufficient volume of production to cover this fixed cost. This reflects the existence of economies of scale in the market. By inserting the optimal quantity from the first order condition in equation (12) into equation (10), the optimal NARP is found. The minimum number of members required for the coop to break even, $\bar{n}_{c}$, is defined by $P-\left[b_{c}+c_{c}\left(\bar{n}_{c} q_{c}\right)\right] / \bar{n}_{c} q_{c}=0$

## Footnotes

${ }^{1}$ Notice that the set-up includes both a single coop, $n_{c}=N$ and a single iof, $n_{f}=N$.
${ }^{2}$ To ensures that the second order condition for maximum profit is satisfied, it is assumed that $g^{\prime \prime \prime}\left(q_{f}\right) \geq 0$ (sufficient condition). The assumption signifies an increasing rate of change in the convex curvature of the inverse supply function. A rationale for the assumption is a limited production capacity for each producer. It holds true for all power and exponential functions.
${ }^{3}$ This set-up is similar to that used by Helmberger in his seminal article on cooperatives. It deviates however from that of Albæk and Schultz where the two firms compete in the output market, and not for input. Hence, it is not straightforward to compare the results.
${ }^{4}$ Normally, the coop requires new members to subscribe a small premium for the entry of new members. This premium (investment) often pays no or low dividend or interest. For the purpose of the present analysis the premium may be ignored without loss of generality.
${ }^{5}$ This is a simplified version of the NARP since units are chosen so that one unit of input yields one unit of output. The quantities of both are hence conveniently denoted by $Q_{c}$.
${ }^{6}$ This is in accordance with Helmberger who states: "The cooperative will be seeking new members so long as net returns are less than the maximum amount obtainable [...]. If the profit seeking firms fail to match the cooperative's price, their suppliers will seek, presumably with success, cooperative membership."
${ }^{7}$ This has parallels to the Theory of Clubs and the "voting with the feet equilibrium" (Buchanan; Tiebout).
${ }^{8}$ This is a sufficient, but not a necessary condition.

