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Certification of quality, demand uncertainty and supply commitment

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Abstract

This paper analyses the trade-off between the Certification and the Brand strategy under monopoly and duopoly in a context vertical product differentiation. We consider an extended game with observable delay. Facing demand uncertainty, firms decide whether to commit to quality effort and output early or wait until the uncertainty has been resolved. We show how the level of demand uncertainty and the production constraints demarcate the equilibrium outcomes. Moreover, we extend the analysis to the traditional trade-off between commitment and flexibility and raise the question whether a first mover is determined by the firm's trade-off.

Key Words: Demand Uncertainty, Commitment, Flexibility, Vertical Product Differentiation, Observable Delay, Endogenous first mover, Certification of Quality.

Jel Codes: *D80, L13, C70*

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1. Introduction.

In Agriculture, the certification of quality is very often associated with a control of supply within the production process. For example, the quality wines produced in delimited areas (vqprd) system is based on the production area delimitation, which involves in the exclusion of some producers and limits the certified wine potential production, as well as on specific production restrictions (for example the maximum production yield per hectare). The quality cheese production (for example the *Parmigiano Reggiano* in Italy or the *Appellation Camembert de Normandie* in France) relies more and more on similar rules. Moreover, the supply restriction is the main requirement imposed by all sorts of quality labels concerning the animal production (as example, the chicken *Label Rouge* in France or the *Prosciutto di Parma* in Italy). Thus, in the agricultural sector, there is a great number of empirical cases in which quality and quantity interact.

The rarity of the production factors implies that the quality depends on the production location and, beyond a defined threshold, the quality and the quantity are negatively correlated. Thus, the producers who adhere to a quality certification system, have to commit to specific product characteristics and quantity restrictions. This commitment involves several economic difficulties. As the production quantities are constrained, the producers face a strong inflexibility concerning the certified product *ex-post* supplied quantities, whereas the demand is characterized by structural fluctuations (according to the consumer's tastes evolution) and conjunctural ones (for example the fruits and vegetables demand varies considerably according to the meteorology).

It can be useful to appeal to the mechanisms of the international competition on the wine's market. The Certification of Origin's system (developed in the traditionally producing countries as France or Italy) competes with the industrial production of new exporting countries as the Australia, the U.S.A. or the Chile (the " New World wines" countries). In these latter, the large firms (Jacob's Creek, Gallo, Southcorp, etc.) develop a whole series of brands, easily identified by consumers, thanks to a great volume of commercialisation and notoriety. Considerable investments in promotion are associated with these brands and the firm efficiency is based on its capacity for scale economies, which allows it to meet market volume requirements. Thus, the pure private brand strategy is advantageous for the firms, because it allows speedier adjustments to market conditions, particularly changing in this field of the agrifood consumption. Indeed, whereas the wine consumption is nowadays stagnating in the countries with the highest wine production (and consumption) as France, Italy or Spain, on the other hand, it is not the same in the U.S.A, in the United Kingdom and in the Asian countries, as China or Japan, where the competition between the vqprd wines and the industrial ones is very strong and leads to several strategic difficulties for the producers, in a context of an uncertain evolution of the world wine consumption.

The question that we raise is that of the producer's incentive to commit to a quality certification system, in a context of demand uncertainty. We have interpreted the certification strategy as a commitment constraining the firm's strategic choices. The analysis of the producers' gain associated to the certification starts from this idea: if the firm chooses the certification, it commits to specific production requirements. As a result, it is constrained in terms of quantity and gives up a part of its strategic flexibility.

In exchange, the producer benefits from a collective reputation related to the certification (for example the adhesion to a certification of origin as "Chianti" or "Champagne" allows the wine producers to benefit from the notoriety of these collective brands). Thus, this mimimum quality is the starting point of the producer's strategy. A brand producer gives up to this collective reputation to be not constrained as regards to his strategic choices. The trade-off between these two strategies becomes particularly bitter in a context of structural demand uncertainty.

As the producer chooses to commit to the certification, then he has problems in meeting market requirements as well as in adapting himself to the evolution of the competition on the international markets. So, he has to take into account the competitive advantage related to the collective brand and anticipate the market competitive conditions, in particular the competition coming from the private brands.

As regards the agricultural economics literature, there exists a great number of papers related to *i)* the certification systems' effectiveness, *ii)* the consumers' willingness to pay for Certifications of Origin and brands[1], *iii)* the Certification of Origin economic organization[2], *iv)* the public policy requirements as regards the European Competition Policy[3] and the defence of the Appellations of Origin on the international markets[4]. Nevertheless, there exists a restricted number of papers studying the strategic aspects of the producer's trade-off between the commitment to the certification system and the brand strategy and the related consequences as regards consumers'

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[1] See for example Steiner (1999,2004), Loureiro, McCluskey (2000), Van Ittersum, Candell, Torelli (1999), Skuras, Vakrou (1999), Combris, Lecocq, Visser (1997), Lecocq, Magnac, Pichery, Visser (2004), Bazoche, Combris, Giraud-Héraud (2005).
[2] See for example Barjolle, Mesplou (1997), Barjolle, Lehmann (1997), Canali (1997), Ménard (1997,2000) for a neo-institutional approach and Boccaletti (1992), Canali (1992) for an analysis of the competitiveness and marketing strategies concerning the Appellations of Origin.
[3] See for example Raynaud, Valceschini (1998), Raynaud (1999) for and analysis of the quality policy requirements and the principles of free competition.
[4] See for example Bendekgey, Mead (1992), Brody (1994), Mahé (1997), Romain-Prot (1995,1997).

surplus. Let us consider the papers of Chambolle and Giraud-Héraud (2003, 2005) in a context where *(i)* the quality construction is not a fixed costs affair, but mainly concerns variable production costs and *(ii)* the demand is uncertain and influences producer's commitment possibilities.

At these conditions, our research's theoretical framework is that of strategic flexibility in industrial economics. In a non strategic context, the firm's technological choice results from the trade-off between the flexibility's cost and the possibility for the firm to adapt itself to the demand fluctuations[5]. Boyer and Moreaux (1989) have shown the conditions at which a monopolist chooses a more flexible technology the more uncertain the demand function is.

In an oligopoly context, the firms face the trade off between the possibility to commit and influence rivals' behaviour and the possibility to act in a context of perfect information. Some papers consider the trade-off between the commitment and the flexibility in the construction of entry barriers[6]. Other papers analyze the commitment – flexibility trade-off in duopoly games, in which firms have the possibility to commit or delay their strategic choices. Some authors consider the flexibility as a technological choice. Boyer and Moreaux (1995) analyze a two stage game, in which the firms choose technology (flexible or inflexible) and then compete *à la Cournot*. They characterize the simultaneous move equilibria of the game according to the level of demand uncertainty and expected market size.

There exist also a whole part of the "commitment and flexibility" literature which focuses on the flexibility as a timing decision. An important question is whether an endogenous first mover is determined by the firm's choice between commitment and flexibility, i.e. whether the firms commit more for aggressive than for defensive reasons[7].

Spencer and Brander (1992) have considered an extended game with "observable delay" *à la* Hamilton and Slutsky (1990) in a context of demand uncertainty, in which each firm decides, at the first stage, whether to commit its output before uncertainty is resolved or not. Then the two firms play a two-stage Cournot game, the timing decision being observed by both firms. They show that *i)* no pure strategy equilibria emerges, in which one firm acts before uncertainty and the other after with initially symmetric firms and *ii)* for low levels of uncertainty firms are trapped in the committed regime, but they would prefer the flexible one. There are several papers which analyze extended games with "action commitment" *à la* Hamilton and Slutsky, in which the decision to commit and the choice as to the level of output to produce under commitment are compressed into a

[5] See for example Oi (1961) and Tisdell (1963) for an analysis of flexibility in a perfect competition context.
[6] See for example Yildizoğlu (1994), Henry (1993), Lecostey (1994), Gabszewicz, Poddar (1997).
[7] Many authors have analyzed whether each player prefers to move first or to move second comparing profits of leaders and followers, see for example Gal-Or (1985), Dorwick (1986) and Boyer and Moreaux (1987). In their extended game, Hamilton and Slutsky (1990) endogenously determine who move first.

single stage. The commitment – flexibility trade-off is thus affected by uncertainty as to the nature of the rival's action as well as by the demand uncertainty itself. Thus the Stackelberg equilibrium can endogenously arise with symmetric firms, as in Sadanand et Sadanand (1996) or in Dewit and Leahy (2001). Consideration of the random shock as the only source of uncertainty leads to a more transparent formulation of the effects of the random shock itself.

Our paper extends the literature in the following directions. First, we consider an extended game with observable delay *à la* Hamilton and Slutsky, in a context of demand uncertainty *and* vertical product differentiation. Second, we model the peculiarities of the certification of quality in agriculture. The paper consists of two sections. In the first one, we analyze the monopoly context. In the second one, we analyze the duopoly context.

For each section, we consider at first the trade-off between the brand and the certification strategy. We formalize the certification of quality as a commitment to a constraining output, which allows firms to benefit from a collective reputation. Second, we consider the trade-off between flexibility and commitment assuming that *i)* the collective reputation is null and *ii)* the committment output level is high enough that it is not constraining anymore.

Our purpose is to define the conditions at which a producer is incentivated to commit to a certification of quality, with particular attention to the question whether an endogenous first mover is determined by the firm's choice between commitment and flexibility.

We obtain the following results.

In the analysis of the trade-off between flexibility and commitment, we characterize the simultaneous move game equilibria of the game according to the level of demand uncertainty and expected market size. For low levels of uncertainty, a prisoner's dilemma arises such that both firms decide to commit at the equilibrium, but they would instead earn higher profits if they were both flexible. For intermediate levels of uncertainty, there are no pure strategy equilibria such that one firm acts before uncertainty and the other after. Thus, firms commit for defensive reasons. For high levels of uncertainty both firms decide to be flexible at the equilibrium.

In the analysis of the trade-off between the brand and the certification strategy, we characterize three incentives for the producers to adopt a certification strategy. At first, the stronger is the implicit control of the production process as well as of the firm's market strategies, the more a soft level of quantity constraint is necessary so that the producer decides to submit himself to the certification. Secondly, the stronger is the required commitment to the production process, the more the producer adopts the certification only in a context of low demand uncertainty. These two incentive arise both in the monopoly and in the duopoly context. Finally, the duopoly context's analysis shows that an important incentive is the level of competition on the market, in particular the fact that no other producers have already adopted an identical certification strategy. Thus, a firm tends to commit to the certification system more for aggressive than for defensive reasons.

In particular, we characterize the simultaneous move game's equilibria according to the level of uncertainty and quantity constraint.

For low levels of demand uncertainty and quantity constraint, both firm choose the Certification strategy at the equilibrium. The equilibrium allocation is Pareto optimal.

For intermediate levels of both uncertainty and quantity constraint, two asymmetric Nash equilibria arise despite the fact that firms are *ex-ante* identical and only the firms' exogenous characteristics can anticipate which equilibrium will be selected. Thus, firms tends to commit more for aggressive than for defensive reasons, that is more to gain a first-mover advantage than to avoid becoming the follower.

As a high level of demand uncertainty is associated to a strong quantity constraint, both firm choose the Brand strategy. In this context, we show that the prisoner's dilemma *can* arise, in which the firms would prefer the Certification regime, but instead are trapped in the low level Brand Equilibrium. This is clearly a case in which inefficiency arises through competitive incentives to be flexible. Moreover, we specify how to eliminate any inefficient allocation from the set of equilibria.

2. The Monopoly context.

We firstly present the general version of the model. We consider a market of size M and assume uncertainty in demand in the following sense. The market size is assumed to be a random variable, which can assume the value $\underline{M}$ or $\overline{M}$ (with $\underline{M} < \overline{M}$), with probability ½. The probability distribution is assumed to be common knowledge.

We denote β the expected marked size and V the volatility, given by the following expression[8] :

(1.1) $$V = \frac{(\overline{M} - \underline{M})^2}{4}$$

[8] The assumption that the market size is always positive ($\underline{M} < 0$) implies a maximum level of demand uncertainty $V < \overline{V}(\beta) = \beta^2$.

The product's quality on the differentiated market is represented by the parameter $\mu \geq 0$. Following Mussa and Rosen (1978), consumers are distinguished by a taste one-dimensional parameter θ, expressing the intensity of an individual's preference for quality and uniformly distributed over the interval $[0,t]$ according to a density $f(\theta)=1/t$. Each consumer is assumed to either buy one unit of the good or nothing. Consumer θ's surplus is given by $S=\theta\mu-p$ if he buys one unit of the good of quality μ at price *p*. This formulation expresses the difference between a reservation price $\theta\mu$ and the purchase price *p*. The quality μ is not bought by a consumer θ if $S(\theta,\mu,p)\leq 0$ (only the consumers between p/μ and *t* buy the quality μ's good).

In this section we suppose there is only one firm who offers a unique product of quality μ on the differentiated market. The total demand *q* is then given by:

(1.2) $$q = M\int_{p/\mu}^{t} f(\theta)d\theta = \frac{M}{t}(t-\frac{p}{\mu})$$

By inverting equation (1.2) it is also possible to consider that for any quantity *q* of the μ quality good, the price for the consumer is determined by :

(1.3) $$p(q)=\frac{t}{M}\mu(M-q)$$

We verify that the price *p* increases in the quality μ and decreases in the quantity *q*.

We define below *i)* the trade-off between the Brand and the Certification strategy and *ii)* the trade-off between the flexibility and the commitment.

2.1. The trade-off between the Brand and the Certification strategy.

2.1.1. *The Brand Strategy*.

This strategy corresponds to the traditional way to consider the competition in the product differentiation models.

In order to produce one unit of quality μ's good, the firm has to make an effort δ. The cost of the effort is assumed to be given by the following:

(1.4) $$c(\delta) = c\delta^2$$

The producer does not benefit from an *a priori* reputation. Then, the quality supplied to the consumers is expressed by $\mu = \delta$[9]. Both the effort δ and the quantity *q* are assumed to be set after uncertainty is resolved. Thus, the brand producer benefits from a maximal flexibility to adapt himself to the real market size. His profit is given by:

(1.5) $$\pi(\delta, q) = [p(q) - c(\delta)]q$$

where $p(q)$ is given by (1.3). We maximize the profit $\pi(\delta, q)$ according to the quality effort δ and the quantity q and we obtain the producer's optimal strategy in the case of the Brand Strategy, which is given by the following:

(1.6) $$\left|\begin{array}{l} \delta^b = \dfrac{t}{3c} \\ q^b(M) = \dfrac{M}{3} \end{array}\right.$$

We verifiy that the monopolist's effort δ^b is increasing in the average consumers' willingness to pay *t*, but it does not depend on *M*. As the quantity is an increasing function of *M*, we can say that a perfect information on the market size is important in the producer's decisional process.

Using π^b to denote the realized monopolist's profit in the case of the Brand strategy and E the expected value, the expected profit is given by:

(1.7) $$E[\pi^b] = \frac{\beta t^2}{27c}$$

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[9] The specification of quality μ choice with a quadratic marginal cost $c(\mu) = \mu^2$ (used in the vertical product differentiation models) is equivalent, but our specification will be more realistic in a context of demand uncertainty.

We verify that the expected profit is an increasing function of the expected market size β and an increasing quadratic function of the average consumer's willingness to pay *t*. As the realized monopolist's profit π^b is a linear function of the market size M, uncertainty does not have any effect on the expected profit in the case of Brand Strategy.

2.1.2. *The Certification Strategy*.

As we explained in introduction, we interpret the certification strategy as a commitment to a constraining output, which allows the producer to benefit from a collective reputation, in the sense of an exogenous improvement of the consumers' willingness to pay.

Formally, we assume that if the producer chooses the certification strategy he limits the quantity to a level *z* and benefits from an *a priori* reputation $s \geq 0$. Then, if the effort of the producer is δ (making a private brand starting from the collective reputation *s*), the quality supplied will be $\mu = s + \delta$.

The certified producer limits his production to the level *z* and, at the same time, chooses the effort's level δ, to improve his good's quality as regards to the collective reputation *s*. The effort is assumed to be set before uncertainty is resolved. As we assume risk neutrality, the monopolist maximises his expected profit according to the effort δ.

Using $\pi(M)$ to denote the monopolist's profit for a market size *M* and E to denote expected value, the producer has the following maximisation problem:

(1.8) $$\underset{\delta}{Max}\, E[\pi(\delta, z)] = \frac{1}{2}\pi(\delta, z, \overline{M}) + \frac{1}{2}\pi(\delta, z, \underline{M})$$

In order to simplify the analysis we resolve the monopolist's maximisation problem according to the following assumpions.

Firstly, we interprete the collective reputation *s* as an exogenous improvement of the consumer's willingness to pay[10].

We assume that the *s* value is constant and equal to the quality chosen by the monopolist in the case of Brand strategy, then:

[10] This assumption is supported by a large part of econometric analysis of consumer's willingness to pay for Certifications of Origin. See for example Loureiro and McCluskey (2000) and Steiner (1999) for an hedonic price approach and Bazoche, Combris, Giraud-Héraud (2005) for an experimental study.

(H1) $$s = \frac{t}{3c}$$

The assumption (H1), which we hold along the paper, allow us to simplify the analytic expressions without changing the economic consideration resulting from this model. In the competitive context, the assumption (H1) will allow us to characterize only the case in which the equilibrium always exists, such that certified producer is the high quality good's one.

Secondly, the commitment to the quantity level *z*, required from the certification, is interpreted as a constraint imposed by the public authority. In order to make this production restriction really constraining it is necessary to assume that $z < q^b(\underline{M})$.

Thus, for any possible level of the market size (i.e. if $M = \underline{M}$ or if $M = \bar{M}$), the quantity restriction z limits the supplied quantity as regards to the optimal quantity supplied by the monopolist in the case of Brand strategy.

We easily show that the hypothesis $z < q^b(\underline{M})$ is equivalent to a maximum value of the volatility $\bar{V}(z)$ given by the following expression:

(H2) $$V \leq \bar{V}(z) = (\beta - 3z)^2$$

Thus, for $V \leq \bar{V}(z)$ the certified producer is really constrained in terms of quantity. Following the assumptions (H1) and (H2) and (1.8) we maximize the monopolist's expected profit according to the quality effort δ and obtain the monopolist's optimal strategy given by:

(1.9) $$\delta^c = \frac{t[\beta(\beta - z) - V]}{2c(\beta^2 - V)}$$

We verify that the certification strategy's effort in terms of quality is higher than the brand strategy's one ($\delta^c - \delta^b > 0$) and decreases in the demand uncertainty ($\partial\delta^c / \partial V < 0$).

Substituting (1.9) into (1.8) and using π^c to denote the monopolist's profit in the case of the Certification strategy, the expected profit is given by:

(1.10) $$E[\pi^c] = \frac{t^2 z[\beta(\beta - z) - V][\beta(7\beta - 3z) - 7V]}{12c(\beta^2 - V)^2}$$

As the monopolist's realized profit π^c in the case of Certification Strategy is a concave function of the market size *M*, the randomness in demand reduces expected profit. Thus, the expected profit decreases in demand uncertainty ($\partial E[\pi^c] / \partial V < 0$).

Furthermore, it increases as the quantity constraint becomes softer ($\partial E[\pi^c] / \partial z > 0$).

2.1.3. *Results.*

Let us denote by Δ^c the relative value of the Certification strategy obtained by the following expression:

(1.11) $$\Delta^c = E[\pi^c] - E[\pi^b]$$

Using equations (1.7) and (1.10), we verify that the relative value of the Certification strategy Δ^c is a decreasing function of the demand uncertainty ($\partial \Delta^c / \partial V < 0$) and an increasing function of the quantity constraint ($\partial \Delta^c / \partial z > 0$).

Proposition I .

There exists a function $\hat{V}(z)$, increasing in z (and a function $\hat{z}(V)$, increasing in V), such that the monopolist chooses the Certification strategy if and only if $V < \hat{V}(z)$ and $z > \hat{z}(V)$.

As uncertainty becomes more important the relative value of the Certification strategy falls and turns negative at the « switching volatility »$\hat{V}(z)$. The « switching volatility » is increasing in z. Thus, the higher is z, the more the firm stands a higher volatility before switching to the Brand strategy.

As the Certification system becomes less constraining, the relative value of the Certification strategy increases and turns positive at the « switching constraint » $\hat{z}(V)$. This increases in *V*. Thus, the higher is the volatility, the more a soft quantity constraint is required so that the firm switches from the Brand to the Certification strategy.

If $V > \hat{V}(z)$ ($z < \hat{z}(V)$), the producer chooses the Brand Strategy (Figure 1).

2.2. The trade-off between the flexibility and the commitment.

In the case of flexibility, we refer to the analysis of the Brand Strategy (2.1.1). Thus the monopolist's optimal strategy in terms of quality effort and quantity is given by (1.6) and the related expected profit is given by (1.7). In this section, as we formalize the commitment in the traditional way, we assume s to be zero. Thus the quality will be equal to the quality effort ($\mu = \delta$). Furthermore, z is assumed to be high enough ($z > q^b(\bar{M})$) that it is nomore constraining and the monopolist commits to its optimal output level. Using $\pi(M)$ to denote the monopolist's profit for a market size M and E to denote expected value, the producer has the following maximisation problem:

(1.12) $$\underset{\delta,q}{Max}\, E[\pi(\delta,q)] = \frac{1}{2}\pi(\delta,q,\bar{M}) + \frac{1}{2}\pi(\delta,q,\underline{M})$$

We maximize the expected profit $E[\pi(\delta,q)]$ according to the quality effort δ and the quantity q and obtain the monopolist's optimal strategy given by:

(1.13) $$\left|\begin{array}{l} \delta^c = \dfrac{t}{3c} \\ q^c(M) = \dfrac{\beta^2 - V}{3\beta} \end{array}\right.$$

We verify that *i)* the committed monopolist's quality effort is equal to the flexible monopolist's one and *ii)* the optimal quantity is decreasing in the demand uncertainty ($\partial\delta^c/\partial V < 0$). Substituting (1.13) into (1.12) and using π^c to denote the committed monopolist's profit, the expected profit is given by:

(1.14) $$E[\pi^c] = \frac{t^2}{27\beta c}(\beta^2 - V)$$

As the committed monopolist's realized profit π^c is a concave function of the market size *M*, the randomness in demand reduces expected profit. Thus, the expected profit decreases in demand uncertainty ($\partial E[\pi^c]/\partial V < 0$). Using equation (1.7) and (1.14), we easily verify that the relative value of the commitment is always negative.

3. The duopoly context.

We first present the general version of the model. In the following paragraphs, we consider *(i)* the trade-off between the Brand and the Certification strategy and *(ii)* the trade-off between the flexibility and the commitment.

We allow both firms the possibility to commit before demand uncertainty is resolved. We assume that $\underline{M}$ is high enough that both firms would always produce positive outputs. The only question is whether to commit or delay. As we explained in the introduction, we consider an extended game with observable delay in a context of demand uncertainty. There are three stages in the game, as represented in Fig.2.

In stage 1, each firm decides whether to commit or to retain the flexibility to set the quality effort and the output after the market size is revealed. The outcome of the "timing" decision is then observed by both firms.

In stage 2, if either firm has decided to commit, it then commits to the quality effort and output in a context of demand uncertainty. In the case *(i)* the committed firm sets the quality effort starting from a collective reputation and commits to a constraining output level. Then, the market size is revealed.

In stage 3, if either firm does not have committed in stage 2, it then sets the quality effort and the quantity in a context of perfect information.

One firm supplies the quantity q_h of the high quality good μ_h and the other the quantity q_l of the low quality good μ_l. The surplus of a θ's consumer, when he buys the good of quality μ_i and price p_i is denoted by $S = \theta\mu_i - p_i(\mu_i)$. The market is covered only on the segment $[\underline{\theta}, t]$ ($\underline{\theta} = p_l / \mu_l$).

The consumer indifferent between the high and the low quality good is characterized by the parameter $\hat{\theta} = \dfrac{(p_h - p_l)}{(\mu_h - \mu_l)}$.

The demand functions of the low and high quality good are respectively given by:

$$
(2.1) \qquad \left|\begin{array}{l} q_l = M\int_{\underline{\theta}}^{\hat{\theta}} f(\theta)d\theta = \dfrac{M}{t}(\hat{\theta} - \dfrac{p_l}{\mu_l}) \\ q_h = M\int_{\hat{\theta}}^{t} f(\theta)d\theta = \dfrac{M(t-\hat{\theta})}{t} \end{array}\right.
$$

Following (2.1), the inverse demand curves can be written as:

(2.2) $$\left| \begin{aligned} & p_l(\mu_l, q_l, q_h) = \frac{t}{M}\mu_l(M - q_h - q_l) \\ & p_h(\mu_h, q_h, \mu_l, q_l) = \frac{t}{M}(M\mu_h - q_h\mu_h - q_l\mu_l) \end{aligned} \right.$$

We verify that the low quality good's price depends on the high quality μ_h level only through the high quality good's output q_h.

3.1. The trade-off between the Brand and the Certification strategy.

We develop in the following paragraphs the three possibles cases: *i)* both firms choose the Brand strategy (*Brand Regime*), *ii)* both firms choose the Certification strategy (*Certification Regime*), *iii)* only one firm chooses the Certification strategy (*Asymmetric Regime*).

We denote by *c* the commitment strategy and by *b* the brand strategy.

3.1.1. *The Brand Regime*.

Each firm chooses simultaneously the effort δ_i and the quantity $q_i\,(i = l, h)$, after uncertainty is resolved. The cost of the quality effort for the firm *i* is given by the following:

(2.3) $$c_i(\delta) = c\delta_i^2$$

As no firm benefits from the *a priori* collective reputation *s*, the supplied quality is given by $\mu_i = \delta_i$. The low and the high quality firm's profits are respectively given by the following:

(2.4) $$\left| \begin{aligned} & \pi_l(\delta_l, q_l, q_h) = [p_l(\delta_l, q_l, q_h) - c_l(\delta_l)]q_l \\ & \pi_h(\delta_h, q_h, \delta_l, q_l) = [p_h(\delta_h, q_h, \delta_l, q_l) - c_h(\delta_h)]q_h \end{aligned} \right.$$

where p_i $(i = l, h)$ is given by (2.2).

Firm *i* maximizes its profit function according to the quality effort δ_i and the quantity q_i $(i = l, h)$.

The equilibrium efforts and quantities selected by the low and the high quality firm are, respectively given by :

(2.5)
$$\left|\begin{array}{l} \delta_l = \dfrac{6t}{23c} \\ q_l(M) = \dfrac{6M}{23} \\ \delta_h = \dfrac{9t}{23c} \\ q_h(M) = \dfrac{5M}{23} \end{array}\right.$$

We verify that *i)* the firm *i*'s quality effort δ_i is an increasing function of the average consumers' willingness to pay *t*, but it does not depend on *M* and *ii)* the quantity is an increasing function of the market size *M*. The two products are strictly differentiated at the Nash equilibrium. Gal-Or…The high quality firm supplies the lower quantity. Let us denote by π_i the profit of the firm *i*. We then obtain the low and the high quality firm profits, respectively given by:

(2.6)
$$\left|\begin{array}{l} \pi_l = \dfrac{216Mt^2}{12167c} \\ \pi_h = \dfrac{225Mt^2}{12167c} \end{array}\right.$$

Since the high quality firm earns higher profits than the low quality firm, each firm has the incentive to produce the high quality good. Lehmann-Grube… We assume that each firm benefits from the quality leader advantage with probability ½. Denoting $\pi^{b,b}$ the firm *i*'s realized profit in the Brand Regime and E the expected value, the firm *i*'s expected profit is given by:

(2.7)
$$E[\pi^{b,b}] = \frac{441\beta t^2}{24334c}$$

As the firm *i*'s realized profit $\pi^{b,b}$ is a linear function of the market size *M*, the demand uncertainty does not have any effect on the expected profit.

3.1.2. *The Certification Regime*.

Each firm commits to the quality effort δ_i and to the quantity constraint *z* in stage 1. The quantity restriction *z* is assumed to be constraining as regards to the lowest of the Brand equilibrim quantities, i.e. $z < Min\{q_l(\underline{M}), q_h(\underline{M})\}$. This hypothesis is equivalent to a maximum value of volatility $\bar{V}(z)$:

$$V \leq \bar{V}(z) = \frac{(5\beta - 23z)^2}{25} \tag{2.8}$$

As both firm benefit from the collective reputation *s*, given by the assumption (H1), the quality μ_i supplied by the firm *i* is given by $\mu_i = s + \delta_i$. Using $\pi_i(M)$ to denote the firm *i*'s profit if the market size is *M* (*i*=*l*,*h*) and *E* the expected value, the low quality firm's maximization problem is given by the following:

$$\max_{\delta_l} E[\pi_l(\delta_l, z)] = \frac{1}{2}\pi_l(\delta_l, z, \bar{M}) + \frac{1}{2}\pi_l(\delta_l, z, \underline{M}) \tag{2.9}$$

The high quality firm's maximization problem is given by the following:

$$\max_{\delta_h} E[\pi_h(\delta_h, z, \delta_l)] = \frac{1}{2}\pi_h(\delta_h, z, \delta_l, \bar{M}) + \frac{1}{2}\pi_h(\delta_h, z, \delta_l, \underline{M}) \tag{2.10}$$

Solving simultaneously the problems (2.9) and (2.10) the equilibrium levels of effort δ_i and output q_i (*i*=*l*,*h*) are obtained as following:

$$\left| \begin{array}{l} \delta_l = \dfrac{t[\beta(\beta - 2z) - V]}{2c(\beta^2 - V)} \\ \delta_h = \dfrac{t[\beta(\beta - z) - V]}{2c(\beta^2 - V)} \end{array} \right. \tag{2.11}$$

We verify that the firm *i*'s quality effort is decreasing in the demand uncertainty. Furthermore, the more constraining is *z* the higher is the quality effort. We then obtain the low and the high quality firm's expected profit, respectively given by:

(2.12) $$\begin{cases} E[\pi_l] = \dfrac{t^2 z[\beta(\beta-2z)-V][\beta(7\beta-6z)-7V]}{12c(\beta^2-V)^2} \\ E[\pi_h] = \dfrac{t^2 z[7V^2-2V\beta(7\beta-10z)+\beta^2(15z^2-20\beta z+7\beta^2)]}{12c(\beta^2-V)^2} \end{cases}$$

As in the Brand Regime, we verify that the high quality firm earns the higher expected profit. We hold the assumption such that each firm has probability ½ to be the high quality good's producer.

Denoting $\pi^{c,c}$ the firm *i*'s realized profit in the Certification Regime and *E* the expected value, the expected profit of each firm is obtained as following:

(2.13) $$E[\pi^{c,c}] = \frac{t^2 z[14V^2-4V\beta(7\beta-10z)+\beta^2(27z^2-40\beta z+14\beta^2)]}{24c(\beta^2-V)^2}$$

As the firm *i*'s realized profit $\pi^{c,c}$ is a concave function of the market size *M*, the randomness in demand reduces expected profit. Thus, the firm *i*'s expected profit decreases in demand uncertainty ($\partial E[\pi^{c,c}]/\partial V < 0$).

Furthermore, we verify that the firm *i*'s expected profit increases as the quantity constraint becomes softer ($\partial E[\pi^{c,c}]/\partial z > 0$).

3.1.3. *The Asymmetric Regime.*

In stage 1, only one firm decides to commit to the certification system. The other chooses quality effort and output in stage 3. The certified firm is assumed to benefit from the high quality advantage[11]. Let us then denote by the l the flexible firm and by h the certified firm.

The brand firm maximises its profit function $\pi_l(\delta_l, q_l, z)$ according to the effort δ_l and the quantity q_l. Using (2.4) and solving the first-order conditions yields the brand firm's reaction function for δ_l and q_l as functions of the quantity constraint *z* and the random variable *M*:

[11] Following the hypothesis (H1), we verify *ex-post* that the low quality firm does not have any interest in deviating by leapfrogging in qualities, i.e. in investing in a higher quality level than ist rival. Thus, we characterize a context in which the equilibrium always exists, such that the certified firm is the high quality good's one.

(2.14) $$\begin{cases} \delta_l(z,M) = \dfrac{t}{3Mc}(M-z) \\ q_l(z,M) = \dfrac{(M-z)}{3} \end{cases}$$

We verify that for more constraining quantity restrictions, the brand firm's best reaction's quantity and quality effort increase. The certified firm chooses its optimal strategy according to the problem:

(2.15) $$\underset{(\delta_h)}{Max}\, E[\pi_h(\delta_h, z, \delta_l, q_l)] = \frac{1}{2}\pi_h(\delta_h, z, \delta_l, q_l, \overline{M}) + \frac{1}{2}\pi_h(\delta_h, z, \delta_l, q_l, \underline{M})$$

Using (2.15) and given the brand firm's reaction function given by (2.14), solving the first-order condition yields the high quality firm's optimal quality effort δ_h as a function of *z* and *V*:

(2.16) $$\delta_h = \frac{t[\beta(\beta - z) - V]}{2c(\beta^2 - V)}$$

As in (2.11), we verify that the certified firm's quality effort is a decreasing function of the demand uncertainty and increases as *z* become more constraining. Let us denote by $\pi_i^{c,b}$ the firm *i*'s realized profit in the asymmetric regime ($i = l, h$) and *E* the expected value. Substituting (2.14) and (2.16) into (2.15), we obtain the expected profit of the certified firm:

(2.17) $$E[\pi_h^{c,b}] = \frac{t^2 z[17V^2 + \beta^2(\beta - z)(17\beta - 5z) - V(4z^2 - 22\beta z + 34\beta^2)]}{36c(\beta^2 - V)^2}$$

As the certified firm's realized profit $\pi_h^{c,b}$ is a concave function of the random variable *M*, the certified firm's expected profit decreases in demand uncertainty ($\partial E[\pi_h^{c,b}]/\partial V < 0$). Furthermore, it increases as *z* becomes less constraining, ($\partial E[\pi_h^{c,b}]/\partial z > 0$). Then we substitute (2.14) into the profit function of the brand firm, as indicated in (2.4), and calculate its expected value, as in (2.9). The brand firm's expected profit is obtained as following:

(2.18) $$E[\pi_l^{c,b}] = \frac{t^2[V^2(\beta - 3z) + V(\beta - z)(z^2 + 4\beta z - 2\beta^2) + \beta^2(\beta - z)^3]}{27c(\beta^2 - V)^2}$$

As the flexible firm's realized profit $\pi_l^{c,b}$ is a convex function of the random variable *M*, the flexible firm expected profit increases in demand uncertainty ($\partial E[\pi_l^{c,b}]/\partial V > 0$). Furthermore, it increases as *z* becomes more constraining ($\partial E[\pi_l^{c,b}]/\partial z < 0$).

3.1.4. *Results.*

The duopoly context's analysis allows us to take into account the effects of the strategic interaction on the firm's trade-off. We study firstly the best reply functions at the timing decision stage. We then characterize the simultaneous move equilibria of the game, according to the level of demand uncertainty and quantity constraint[12]. Using (2.7), (2.13), (2.17), (2.18) we construct the game's pay-off matrix (Table 1) indicating the expected profit of each firm according to the chosen strategy (Brand or Certification). The first entry in each cell is firm 1's expected profit.

		Firm 2	
		C	**B**
Firm 1	**C**	$E[\pi^{c,c}], E[\pi^{c,c}]$	$E[\pi_h^{c,b}], E[\pi_l^{c,b}]$
	B	$E[\pi_l^{c,b}], E[\pi_h^{c,b}]$	$E[\pi^{b,b}], E[\pi^{b,b}]$

Table 1 : pay-off matrix.

3.1.4.1.The Best Reply Functions at the timing decision stage.

As the matrix is symmetric, we consider the best response of firm 2, given the choice of firm 1, without loss of generality. Suppose that the firm 1 chooses the Certification strategy. Let us denote by $\Delta^{c,c}$ the relative value of the certification strategy for firm 2, given by:

(2.19) $$\Delta^{c,c} = E[\pi^{c,c}] - E[\pi_l^{c,b}]$$

[12] The trade-off can be also analyzed according to the level of demand uncertainty and expected market size at a given level of quantity constrainty. Despite, it is more significant for our purposes to allow the expected market size to be fixed and take into account the effect of *z* on the trade-off (without loss of generality).

The certification strategy is the best response to the certification strategy if the following condition holds:

(C1) $\Delta^{c,c} > 0 \Leftrightarrow c = BR(c)$

Using (2.13) and (2.18), we verify the following.

Lemma 1. The relative value of replying with the certification strategy to the certification strategy $\Delta^{c,c}$ *decreases in the demand uncertainty and increases as z becomes less constraining.*

There exist a function $\hat{V}'(z)$ *increasing in z (and a function* $\hat{z}'(V)$ *increasing in V), such that the condition (C1) holds if and only if* $V < \hat{V}'(z)$ *(*$z > \hat{z}'(V)$*).*

Suppose now that the firm 1 chooses the Brand Strategy. Let us denote by $\Delta^{b,b}$ the relative value of the brand strategy for firm 2, given by:

(2.20) $\Delta^{b,b} = E[\pi^{b,b}] - E[\pi_h^{\ c,b}]$

The brand strategy is the best response to the brand one if the following condition holds:

(C2) $\Delta^{b,b} > 0 \Leftrightarrow b = BR(b)$

Using (2.7) and (2.17), we verify the following.

Lemma 2. The relative value of replying with the brand strategy to the brand strategy $\Delta^{b,b}$*, increases in the demand uncertainty and increases as z becomes more constraining.*

There exist a function $\hat{V}(z)$ *increasing in z (and a function* $\hat{z}(V)$ *increasing in V), such that* $\Delta^{b,b} > 0$ *if and only if* $V > \hat{V}(z)$ *(*$z < \hat{z}(V)$*).*

Lemma 3 (i). The indifference locus given the rival's brand strategy is higher than the indifference locus given the rival's certification strategy ($\hat{V}(z) > \hat{V}'(z)$ *and* $\hat{z}(V) < \hat{z}'(V)$*).*

Lemma 3 (ii). The indifference locus given the rival's brand strategy rises much faster in z than the indifference locus given the rival's certification strategy ($\partial\hat{V}(z)/\partial z > \partial\hat{V}'(z)/\partial z$*).*

Following Lemma 1-2, the relative value of replying with the certification strategy to the certification one *i)* decreases in *V* and turns negative at the switching volatility $\hat{V}'(z)$ and *ii)* increases in *z* and turns positive at $\hat{z}'(V)$. The relative value of replying with the brand strategy to the brand one *i)* increases in V and turns positive at the switching volatility $\hat{V}(z)$ and *ii)* increases as z becomes more constraining and turns positive at the quantity restriction $\hat{z}(V)$.

Fig.3 represents the effects of the demand uncertainty on the best reply functions of each firm, given the other firm's strategy. In the zone (1), each firm chooses systematically the brand strategy, whatever the rival's choice is. Thus, the brand strategy is the dominant one. In the zone (3), the certification strategy is the dominant one. Lemma 3 *(i)* suggests that "defensive commitment" tends to have a relatively low value compared to the "aggressive commitment" to get the high quality leadership. It is for this reason that for intermediate levels of uncertainty and quantity constraint, in the zone (2), each firm differentiates its strategy from the rival's one. Thus, for a given level of volatility, the quantity constraint – requested to switch to the certification strategy – is lower *if* the rival adopts the brand strategy. Furthermore, both the indifference loci are increasing in z. Thus, the higher is z, the higher is the switching volatility. Following Lemma 3 *(ii)*, this effect of z on the trade-off is more important *if* the rival adopts the brand strategy.

In addition, Fig. 3 represents the asymptotic effects of the functions $\hat{V}(z)$ and $\hat{V}'(z)$. In fact, in some contexts, the uncertainty has no effect on the trade-off. On one hand, if the quantity restriction is too strong ($z \leq \underline{z}$), both firms choose the brand strategy, whatever the uncertainty is. On the other hand, if the quantity restriction is too soft ($z \geq \overline{z}$), both firms always choose the certification strategy.

3.1.4.2.The simultaneous move equilibria at the timing decision stage.

We are now able to characterize the Nash Equilibria of the game, represented in Fig.4.

Proposition I.

Following Lemma 1-3 the Nash Equilibria of the game are characterized as following:

(i) *if* $V < \hat{V}'(z)$ *(and* $z > \hat{z}'(V)$*), the unique Nash equilibrium is the Certification regime and it is also a dominant strategy equilibrium (NE=(c,c));*

(ii) *if* $\hat{V}'(z) < V < \hat{V}(z)$ *(and* $\hat{z}(V) < z < \hat{z}'(V)$*), two asymmetric Nash equilibria arise (*$NE \in \{(c,b),(b,c)\}$*);*

(iii) *if* $V > \hat{V}(z)$ *(*$z < \hat{z}(V)$*), the brand regime is the unique Nash equilibrium and is also a dominant strategy equilibrum (NE=(b,b)).*

High levels of uncertainty and strong quantity restriction lead firms to choose the Brand strategy, while low levels of uncertainty and soft quantity restriction lead firms to commit to the certification system before uncertainty is resolved. For intermediate levels of uncertainty and quantity constraint, no firm has a dominant strategy. Two asymmetric Nash Equilibria arise, in which one firm acts before uncertainty and the other after. We then hold the following results.

Lemma 4.

(i) There exists a function $\tilde{V}(z)$ *increasing in z (and a function* $\tilde{z}(V)$ *increasing in V), such that if* $V < \tilde{V}(z)$ *(* $z > \tilde{z}(V)$ *), then* $E[\pi^{c,c}] - E[\pi^{b,b}] > 0$.

(ii) There exists a function $\tilde{V}'(z)$ *increasing in z (and a function* $\tilde{z}'(V)$ *increasing in V), such that if* $V > \tilde{V}'(z)$ *(* $z < \tilde{z}'(V)$ *), then* $E[\pi_l^{c,b}] - E[\pi_h^{c,b}] > 0$. *We verify that* $\min[\tilde{V}(z), \hat{V}(z)] > \max[\tilde{V}'(z), \hat{V}'(z)]$ *(* $\max[\tilde{z}(V), \hat{z}(V)] < \min[\tilde{z}'(V), \hat{z}'(V)]$ *).*

Lemma 5. There exists a function $\tilde{V}''(z)$ *decreasing in z (and a function* $\tilde{z}''(V)$ *decreasing in V), such that if* $V < \tilde{V}''(z)$ *(* $z < \tilde{z}''(V)$ *), then* $E[\pi_h^{c,b}] - E[\pi^{c,c}] < 0$. *We verify that, if* $V < \tilde{V}''(z)$ *(* $z < \tilde{z}''(V)$ *), then* $\tilde{V}'(z) < \hat{V}'(z) < \hat{V}(z) < \tilde{V}(z)$ *(* $\tilde{z}'(V) > \hat{z}'(V) > \hat{z}(V) > \tilde{z}(V)$ *).*

Lemma 6. The condition $E[\pi_l^{c,b}] - E[\pi^{b,b}] > 0$ *always holds.*

Following *Lemma 4-6*, we are able to characterize the game's allocations' ranking in each zone of the Fig.5. The results are presented in the Table 2.

	Zone 1 $NE = (b,b)$	Zone 2 $NE \in \{(c,b),(b,c)\}$	Zone 3 $NE = (c,c)$
a	$E[\pi_l^{c,b}] > E[\pi^{b,b}] > E[\pi^{c,c}] > E[\pi_h^{c,b}]$	$E[\pi_l^{c,b}] > E[\pi^{c,c}] > E[\pi_h^{c,b}] > E[\pi^{b,b}]$	$E[\pi^{c,c}] > E[\pi_l^{c,b}] > E[\pi_h^{c,b}] > E[\pi^{b,b}]$
b	$E[\pi_l^{c,b}] > E[\pi^{c,c}] > E[\pi^{b,b}] > E[\pi_h^{c,b}]$	$E[\pi_h^{c,b}] > E[\pi_l^{c,b}] > E[\pi^{c,c}] > E[\pi^{b,b}]$	$E[\pi^{c,c}] > E[\pi_h^{c,b}] > E[\pi_l^{c,b}] > E[\pi^{b,b}]$
c	$E[\pi_l^{c,b}] > E[\pi^{b,b}] > E[\pi_h^{c,b}] > E[\pi^{c,c}]$	$E[\pi_l^{c,b}] > E[\pi_h^{c,b}] > E[\pi^{c,c}] > E[\pi^{b,b}]$	$E[\pi_h^{c,b}] > E[\pi^{c,c}] > E[\pi_l^{c,b}] > E[\pi^{b,b}]$
d		$E[\pi_l^{c,b}] > E[\pi_h^{c,b}] > E[\pi^{b,b}] > E[\pi^{c,c}]$	

Table 2 : game's pay-off ranking.

In the zone *(1)*, the unique Nash equilibrium is (*b,b*). In the zone (*1b*), (*b,b*) is the only Pareto inefficient allocation. In the zone *(2)*, two Asymmetric Equilibria arise, which are Pareto efficient. In the zone *(3)*, the unique Nash equilibrium is (*c,c*) and it is Pareto efficient.

Following *Proposition I* and *Lemma 4-5*, the following results holds.

Proposition II

If (i) $V > \tilde{V}''(z)$ $(z > \tilde{z}''(V))$ *and (ii)* $\hat{V}'(z) < V < \tilde{V}'(z)$ $(\tilde{z}'(V) < z < \hat{z}'(V)$, *then two asymmetric Nash Equilibria arise, in which the high quality firm earns higher profits than the low quality one.*

If (i) $\max[\tilde{V}'(z), \hat{V}'(z)] < V < \hat{V}(z)$ $(\hat{z}(V) < z < \min[\tilde{z}'(V), \hat{z}'(V)]$*), then two asymmetric Nash Equilibria arise, in which the low quality firm earns higher profits than the high quality one.*

Proposition II shows that the first mover advantage (i.e. the high quality advantage) does not necessarily lead to earn higher profits.

Proposition III

If (i) $V < \tilde{V}''(z)$ $(z < \tilde{z}''(V))$ *and (ii)* $\hat{V}(z) < V < \tilde{V}(z)$ $(\tilde{z}(V) < z < \hat{z}(V)$, *then the unique Nash Equilibrium is the Brand Regime and it is Pareto dominated by the Certification Regime.*

In the zone (*1b*) in the Fig.4, the unique Nash Equilibrium is (*b,b*) and it is the only Pareto inefficient. A *prisoner's dilemma* arises, in which both firms are trapped in the Brand Regime because of the incentive to be tough on the market, but would instead prefer the Certification Regime.

Thus, we show that *i)* two pure strategy equilibria emerge, in which one firm acts before uncertainty and the other after despite the fact that firms are *ex-ante* identical and *ii)* for high levels of uncertainty firms are trapped in the brand regime, but would instead prefer the certification regime.

Proposition IV.

If an exogenous incentive I is associated to the adoption of the certification strategy, such that the following condition holds:

$$E[\pi^{b,b}] - E[\pi_h^{c,b}] < I < E[\pi_l^{c,b}] - E[\pi^{c,c}]$$

any inefficient allocation is eliminated from the set of equilibria.

3.2. The trade-off between the flexibility and the commitment.

We develop in the following sections the three possibles cases: *i)* both firms choose the flexibility (Flexible Regime), *ii)* both firms choose the commitment (Committed Regime), *iii)* only one firm chooses the commitment (Asymmetric Regime). We denote by c the commitment strategy and by f the flexibility strategy. In the case of the Flexible regime, we refer to the analysis of the Brand Regime (1.1.1). Thus the firm i's optimal strategy in terms of quality effort and quantity is given by (2.5) and the related expected profit is given by (2.7) and denoted this time by $E[\pi^{f,f}]$. In this section, we allow the collective reputation s to equal zero and the firm i's quality to equal the quality effort ($\mu_i = \delta_i$). Furthermore, z is high enough that it is nomore constraining ($z > q^b(\bar{M})$) and the firm i commits to its optimal output level.

3.2.1. *The Committed Regime.*

Using $\pi_i(M)$ to denote the firm i's profit if the market size is M (i=l,h) and E the expected value, the low quality firm's maximization problem is given by the following:

(2.21) $$\max_{\delta_l, q_l} E[\pi_l(\delta_l, q_l)] = \frac{1}{2}\pi_l(\delta_l, q_l, \bar{M}) + \frac{1}{2}\pi_l(\delta_l, q_l, \underline{M})$$

The high quality firm's maximization problem is given by the following:

(2.22) $$\max_{\delta_h, q_h} E[\pi_h(\delta_h, q_h, \delta_l, q_l)] = \frac{1}{2}\pi_h(\delta_h, q_h, \delta_l, q_l, \bar{M}) + \frac{1}{2}\pi_h(\delta_h, q_h, \delta_l, q_l, \underline{M})$$

Solving simultaneously the problems (2.21) and (2.22) the equilibrium levels of effort δ_i and output q_i (i=l,h) are given by the following:

(2.23) $$\left| \begin{aligned} \delta_l &= \frac{6t}{23c} \\ q_l &= \frac{6(\beta^2 - V)}{23\beta} \\ \delta_h &= \frac{9t}{23c} \\ q_h &= \frac{5(\beta^2 - V)}{23\beta} \end{aligned} \right.$$

We verify that the demand uncertainty has no effect on the firm *i*'s quality effort, while the output level is a decreasing function of the demand uncertainty. Using (2.21)-(2.23), we then obtain the low and the high quality firm's expected profit, respectively given by:

$$\text{(2.24)} \qquad \begin{cases} E[\pi_l] = \dfrac{216t^2(\beta^2 - V)}{12167\beta c} \\ E[\pi_h] = \dfrac{225t^2(\beta^2 - V)}{12167\beta c} \end{cases}$$

We verify that the high quality firm earns the higher expected profit. We hold the assumption such that each firm has probability ½ to be the high quality producer.

Denoting $\pi^{c,c}$ the firm *i*'s realized profit in the Committed regime and *E* the expected value, the expected profit of each firm is obtained as following:

$$\text{(2.25)} \qquad E[\pi^{cc}] = \frac{441t^2(\beta^2 - V)}{24334\beta c}$$

As the firm *i*'s realized profit $\pi^{c,c}$ is a concave function of the market size *M*, the randomness in demand reduces expected profit. Thus, the firm *i*'s expected profit decreases in demand uncertainty ($\partial E[\pi^{c,c}] / \partial V < 0$).

Furthermore, it increases in the expected market size β.

3.2.2. *The Asymmetric Regime*.

In the stage 1, only one firm decides to commit. The other sets quality effort and quantity in stage 3. We hold the assumption that the committed firm benefits from the high quality advantage. Let us then denote by the l the flexible firm and by h the committed firm.

The follower maximises its profit function $\pi_l(\delta_l, q_l, q_h)$ according to the effort δ_l and the quantity q_l. Using (2.4) and solving the first-order conditions yields the following reaction function for δ_l and q_l as functions of the leader's quantity q_h and the random variable *M*:

(2.26) $$\begin{cases} \delta_l(q_h, M) = \dfrac{t}{3Mc}(M - q_h) \\ q_l(q_h, M) = \dfrac{(M - q_h)}{3} \end{cases}$$

We verify that the follower's best reaction's quantity and quality decrease in the leader's output. The leader chooses his optimal strategy according to the problem :

(2.27) $$\underset{\delta_h, q_h}{Max}\, E[\pi_h(\delta_h, q_h, \delta_l, q_l)] = \frac{1}{2}\pi_h(\delta_h, q_h, \delta_l, q_l, \bar{M}) + \frac{1}{2}\pi_h(\delta_h, q_h, \delta_l, q_l, \underline{M})$$

Using (2.27) and given the follower's reaction function given by (2.26), the leader's optimal effort δ_h and quantity q_h are respectively obtained as following:

(2.28) $$\begin{cases} \delta_h(\beta, V) = \dfrac{t[\beta(5\beta + \sqrt{25\beta^2 + 60V}) - 12V]}{6c(5\beta^2 - 4V)} \\ q_h(\beta, V) = \dfrac{(\beta^2 - V)(10\beta - \sqrt{25\beta^2 + 60V})}{3(5\beta^2 - 4V)} \end{cases}$$

We verify that the leader's output decreases in demand uncertainty and increases in the expected market size, while the leader's quality effort increases in V and decreases in β.

Let us denote by $\pi_i^{c,f}$ the firm i's realized profit in the asymmetric regime ($i = l, h$) and E the expected value.

Substituting (2.26) and (2.28) into (2.27), we obtain the expected profit of the leader:

(2.29) $$E[\pi_h^{c,f}] = \frac{5t^2(\beta^2 - V)[25\beta^3 - 180\beta V + \sqrt{5}(5\beta^2 + 12V)^{\frac{3}{2}}]}{486c(5\beta^2 - 4V)^2}$$

As the leader's realized profit $\pi_h^{c,f}$ is a concave function of the random variable M, the leader's expected profit decreases in demand uncertainty ($\partial E[\pi_h^{c,f}]/\partial V < 0$)[13]. Furthermore, it

[13] The demand uncertainty's effect on the committed firm's output and expected profit is a consequence of the choice of the random variable. In the models where the randomness does not affect the slope of the demand function (see for example Spencer and Brander, 1992, Boyer and Moreaux, 1995), the demand uncertainty has no effect on the

increases in the expected market size β. Then we substitute (2.26) and (2.28) into the profit function of the low quality firm, as indicated in (2.4), and calculate its expected value, as in (2.20).

The follower's expected profit is obtained as follow:

$$(2.30)\ E[\pi_l^{c,f}] = \frac{t^2[6552V^3\beta - 7250V^2\beta^3 + 225V\beta^5 + 500\beta^7 + (\beta^2 - V)\sqrt{60V + 25\beta^2}(492V^2 + 25V\beta^2 + 100\beta^4)]}{729c(5\beta^2 - 4V)^3}$$

As the follower's realized profit $\pi_l^{c,f}$ is a convex function of the random variable *M*, the expected profit increases in demand uncertainty ($\partial E[\pi_l^{c,f}]/\partial V > 0$). Furthermore, it is a concave function of the expected market size β.

3.2.3. *Results.*

We study firstly the best reply functions at the timing decision stageand then charcterize the simultaneous move equilibria of the game according to the level of demand uncertainty and expected market size.

Using (2.7), (2.25), (2.29), (2.30) we construct a pay-off matrix (Table 3) indicating the expected profit of each firm according to the chosen strategy (Commitment of Flexibility). The first entry in each cell is firm 1's expected profit.

		Firm 2	
		C	**F**
Firm 1	**C**	$E[\pi^{c,c}], E[\pi^{c,c}]$	$E[\pi_h^{c,f}], E[\pi_l^{c,f}]$
	F	$E[\pi_l^{c,f}], E[\pi_h^{c,f}]$	$E[\pi^{f,f}], E[\pi^{f,f}]$

Table 3 : pay-off matrix.

committed firm's output and expected profit. On the contrary, in this paper we choose the random variable in order to be more realistic and formalize the effect of the randomness on the committed firm's expected profit.

3.2.3.1.The Best Reply Functions at the timing decision stage.

As the matrix is symmetric, we consider the best response of firm 2, given the choice of firm 1, without loss of generality. Suppose that the firm 1 chooses the commitment. Let us denote by $\Delta^{c,c}$ the relative value of the commitment strategy for firm 2, given by:

(2.31) $$\Delta^{c,c} = E[\pi^{c,c}] - E[\pi_l^{c,f}]$$

The commitment is the best reply to the commitment if the following condition holds:

(C3) $$\Delta^{c,c} > 0 \Leftrightarrow c = BR(c)$$

Using (2.25) and (2.30), we verify the following.

Lemma 7. The relative value of the commitment $\Delta^{c,c}$ is a decreasing function of the demand uncertainty and increases in the expected market size β.

There exist a function $\hat{V}(\beta)$ increasing in β (and a function $\hat{\beta}(V)$ increasing in V), such that the condition (C3) holds if and only if $V < \hat{V}(\beta)$ (and $\beta > \hat{\beta}(V)$).

Suppose now that the firm 1 chooses the flexibility. Let us denote by $\Delta^{f,f}$ the relative value of the brand strategy for firm 2, given by:

(2.32) $$\Delta^{f,f} = E[\pi^{f,f}] - E[\pi_h^{c,f}]$$

The flexibility is the best reply to the flexibility if the following condition holds:

(C4) $$\Delta^{f,f} > 0 \Leftrightarrow f = BR(f)$$

Using (2.7) and (2.29), we verify the following.

Lemma 8. The relative value of the flexibility $\Delta^{f,f}$ is an increasing function of the demand uncertainty and decreases in the expected market size β. There exist a function $\hat{V}'(\beta)$ increasing in β (and a function $\hat{\beta}'(V)$ increasing in V), such that the condition (C4) holds if and only if $V > \hat{V}'(\beta)$ (and $\beta < \hat{\beta}'(V)$).

Lemma 9 (i). The indifference locus given the rival's flexibility is lower than the indifference locus given the rival's commitment ($\hat{V}'(\beta) < \hat{V}(\beta)$ and $\hat{\beta}(V) < \hat{\beta}'(V)$).

Lemma 9 (ii). The indifference locus given the rival's flexibility rises slowly in β than the indifference locus given the rival's commitment.

Following Lemma 7-8, the relative value of replying with the commitment to the commitment *i)* decreases in *V* and turns negative at the switching volatility $\hat{V}(\beta)$ and *ii)* increases in the expected market size and turns positive at $\hat{\beta}(V)$. The relative value of replying with the flexibility to the flexibility *i)* increases in *V* and turns positive at $\hat{V}'(\beta)$ and *ii)* decreases in β and turns negative at $\hat{\beta}'(V)$.

Fig.6 represents the effects of the demand uncertainty and the expected market size on the best reply function of each firm, given the other firm's strategy. In the zone (1), a firm chooses systematically the flexibility, whatever the rival's choice is. Thus, the flexibility is the dominant strategy. In the zone (3), the commitment is the dominant strategy.

Lemma 9 *(i)* suggests that a firm tends to commit more for defensive than for aggressive reasons. Thus, in the intermediate zone (2), a firm chooses systematically the same strategy as its rival. Thus, for a given level of volatility, the expected market size – required to switch to the commitment strategy – is lower *if* the rival's committed.

Furthermore, both the indifference loci increase in β, thus the higher is the expected market size, the higher is the swiching volatility. Following Lemma 9 *(ii)*, this effect of β on the trade-off is more important *if* the rival's committed.

3.2.3.2.The simultaneous move equilibria at the timing decision stage.

We are now able to characterize the Nash Equilibria of the game, represented in Fig.7.

Proposition V

Following Lemma 7-9, the Nash Equilibria of the game are characterized as following:

(i) if $V < \hat{V}'(\beta)$ (and $\beta > \hat{\beta}'(V)$), the unique Nash equilibrium is the committed regime and it is also a dominant strategy equilibrium ($NE = (c,c)$);

(ii) if $\hat{V}'(\beta) < V < \hat{V}(\beta)$ (and $\hat{\beta}(V) < \beta < \hat{\beta}'(V)$), two symmetric Nash equilibria arise ($NE \in \{(c,c),(f,f)\}$);

(iii) if $V > \hat{V}(\beta)$ *(* $\beta < \hat{\beta}(V)$ *), the unique Nash equilibrium is the flexible regime and is also a dominant strategy equilibrum (* $NE = (f, f)$ *).*

High levels of uncertainty lead firms to choose the flexibility, while low levels of uncertainty lead firms to commit to the certification system before uncertainty is resolved.

For intermediate levels of uncertainty, no firm has a dominant strategy and two symmetric Nash Equilibria arise, in which both firms acts at the same time.

We then hold the following results.

Lemma 10.

(i) There exists a function $\tilde{V}(\beta)$ *increasing in* β *(and a function* $\tilde{\beta}(V)$ *increasing in V), such that if* $V > \tilde{V}(\beta)$ *(* $\beta < \tilde{\beta}(V)$ *), then* $E[\pi_l^{c,f}] - E[\pi_h^{c,f}] > 0$.

(ii) There exists a function $\tilde{V}'(\beta)$ *increasing in* β *(and a function* $\tilde{\beta}'(V)$ *increasing in V), such that if* $V > \tilde{V}'(\beta)$ *(* $\beta < \tilde{\beta}'(V)$ *), then* $E[\pi^{f,f}] - E[\pi_l^{c,f}] < 0$.

(iii) There exists a function $\tilde{V}''(\beta)$ *increasing in* β *(and a function* $\tilde{\beta}''(V)$ *increasing in V), such that if* $V > \tilde{V}''(\beta)$ *(* $\beta < \tilde{\beta}''(V)$ *), then* $E[\pi^{c,c}] - E[\pi_h^{c,f}] > 0$.

We verify that the following relations always hold $\hat{V}'(\beta) < \hat{V}(\beta) < \tilde{V}(\beta) < \tilde{V}'(\beta) < \tilde{V}''(\beta)$ and $\hat{\beta}'(V) > \hat{\beta}(V) > \tilde{\beta}(V) > \tilde{\beta}'(V) > \tilde{\beta}''(V)$.

Proposition VI.

The following condition always holds $E[\pi^{c,c}] - E[\pi^{f,f}] < 0$.

If $V < \hat{V}'(\beta)$ *(and* $\beta > \hat{\beta}'(V)$ *), then the unique Nash equilibrium is the committed regime and it is Pareto dominated by the flexible regime.*

If $\hat{V}'(\beta) < V < \hat{V}(\beta)$ *(and* $\hat{\beta}(V) < \beta < \hat{\beta}'(V)$ *), two symmetric Nash equilibria arise (* $NE \in \{(c,c),(f,f)\}$ *), such that the flexible regime Pareto dominates the committed regime.*

In the zone (*3*) the unique Nash Equilibrium is (*c,c*) and it is Pareto inefficient. As in the model of Spencer and Brander, for low levels of demand uncertainty, a *prisoner's dilemma* arises, in which both firms are trapped in the committed regime, but would instead prefer the flexibility.

In the zone (2), the game has the particular structure of the coordination game, in which two symmetric equilibria arise, such that both firms would make more profits if they were to choose flexibility rather than commitment. As in Spencer and Brander (1992), we show that *i)* no pure strategy equilibria emerges, in which one firm acts before uncertainty and the other after *if* the firms are initially symmetric and *ii)* for low levels of uncertainty firms are trapped in the committed regime, but they would prefer the flexible one.

Following *Lemma 10* and Proposition VI, we are now able to characterize the game's allocations' ranking in each zone of the Fig.8. The results are presented in the Table 4.

In the zone *(1)*, the unique Nash equilibrium is (*f,f*) and it is the only Pareto efficient allocation. In the zone *(2)*, two symmetric Nash Equilibria arise. Only the Flexible Regime is Pareto efficient. In the zone *(3)*, the unique Nash equilibrium is (*c,c*) and it is Pareto inefficient.

<table>
<tr><th></th><th>Zone 1 $NE=(f,f)$</th><th>Zone 2 $NE\in\{(c,c),(f,f)\}$</th><th>Zone 3 $NE=(c,c)$</th></tr>
<tr><td>a</td><td>$E[\pi_l^{c,f}]>E[\pi^{f,f}]>E[\pi^{c,c}]>E[\pi_h^{c,f}]$</td><td rowspan="4">$E[\pi^{f,f}]>E[\pi_h^{c,f}]>E[\pi^{c,c}]>E[\pi_l^{c,f}]$</td><td rowspan="4">$E[\pi_h^{c,f}]>E[\pi^{f,f}]>E[\pi^{c,c}]>E[\pi_l^{c,f}]$</td></tr>
<tr><td>b</td><td>$E[\pi_l^{c,f}]>E[\pi^{f,f}]>E[\pi_h^{c,f}]>E[\pi^{c,c}]$</td></tr>
<tr><td>c</td><td>$E[\pi^{f,f}]>E[\pi_l^{c,f}]>E[\pi_h^{c,f}]>E[\pi^{c,c}]$</td></tr>
<tr><td>d</td><td>$E[\pi^{f,f}]>E[\pi_h^{c,f}]>E[\pi_l^{c,f}]>E[\pi^{c,c}]$</td></tr>
</table>

Table 4: payoff's ranking

4. Conclusions and extensions.

We have analyzed the trade-off between commitment and flexibility in a duopoly setting, in a context of vertical product differenciation.

We have show that the degree of demand uncertainty and quantity constraint play a crucial role in determining equilibrium outcomes. For low levels of demand uncertainty and quantity constraint, the unique Nash equilibrium is the Certification regime and it is Pareto optimal. For intermediate levels of both uncertainty and quantity constraint, two asymmetric Nash equilibria arise for initially symmetric firms. Thus, certified firms commit more for aggressive than for defensive reasons. As an important level of uncertainty is associated to a strong quantity constraint, the Brand Equilibrium occurs and inefficiency can arise through the competitive incentive to be tough on the market.

There are several avenues along which our analysis could be extended. First, our results have been confined to interior solutions, implying that both firms always produce positive quantities. However some firms may not enter when demand turns out to be low. This raises the issue of firms engaging in probabilistic entry deterrence. A second interesting extension would be to allow for asymmetric information, assuming that the uncertainty is initially private information. Finally, on could analyze the role of the cost or size asymmetry in determining the order of moves.

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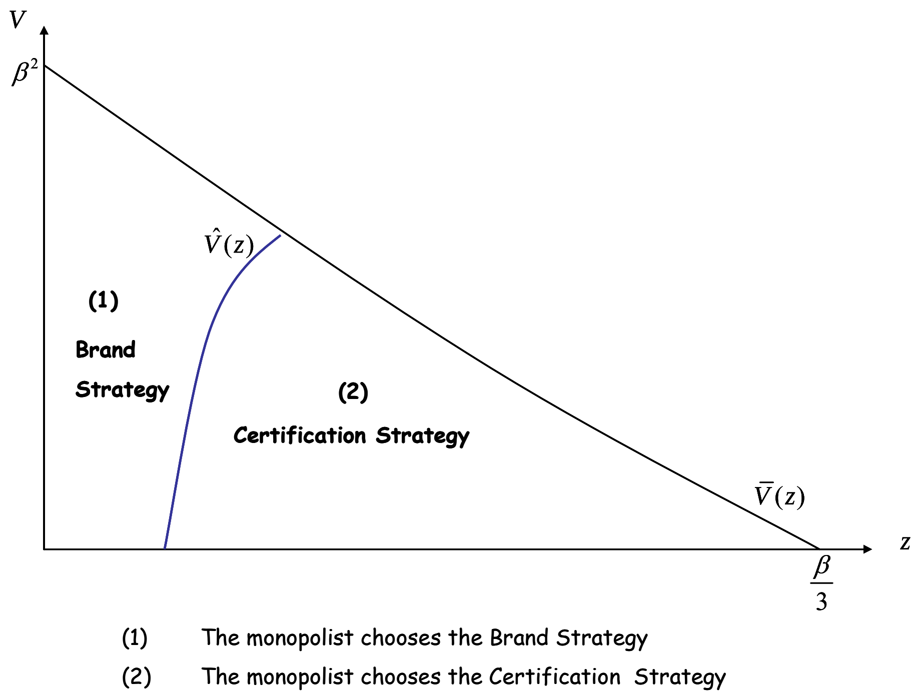


Fig. 1: the monopolist's trade-off between the Brand and the Certification strategy.

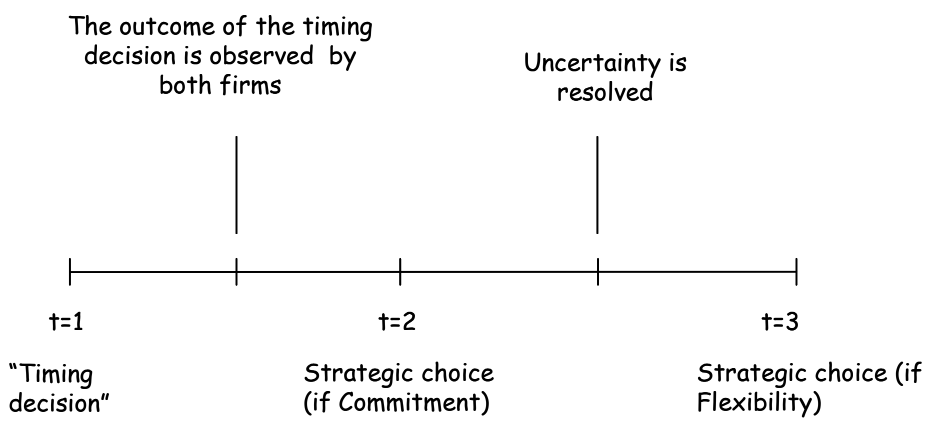


Fig. 2: the structure of the extended game with observable delay in a context of demand uncertainty.

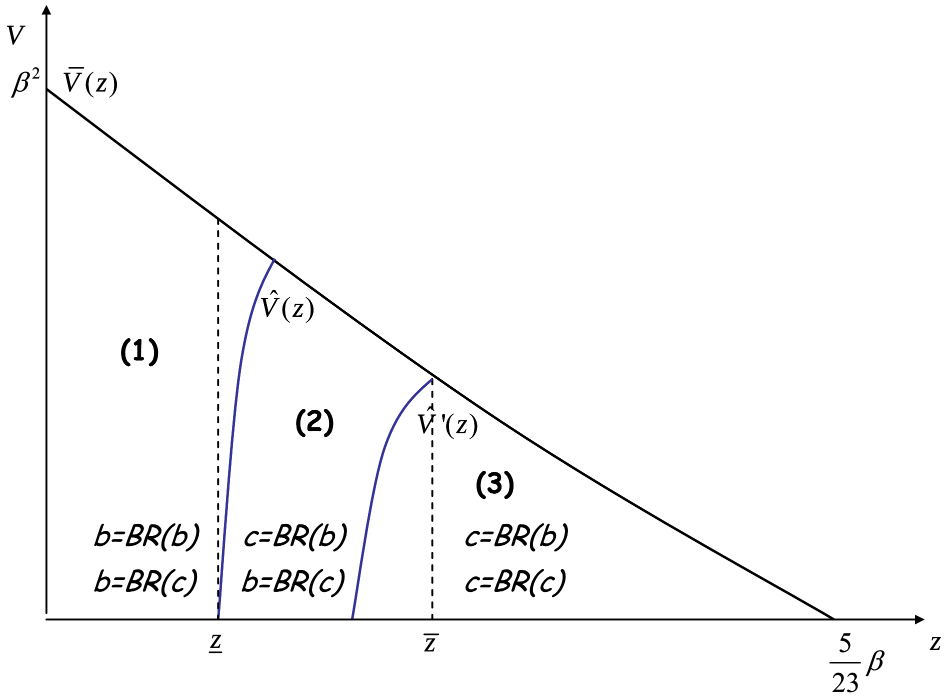
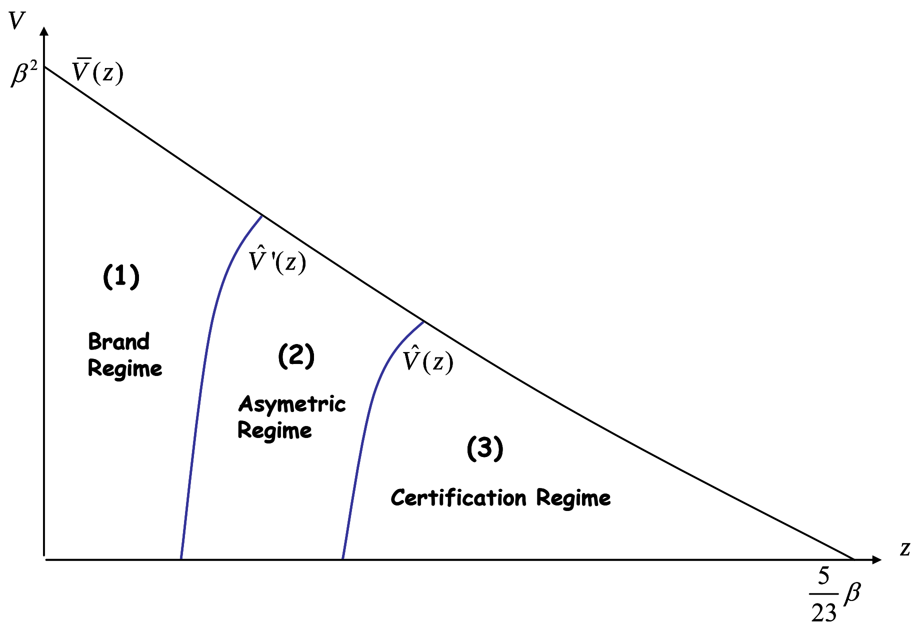


Fig. 3: the best reply functions at the timing decision stage.

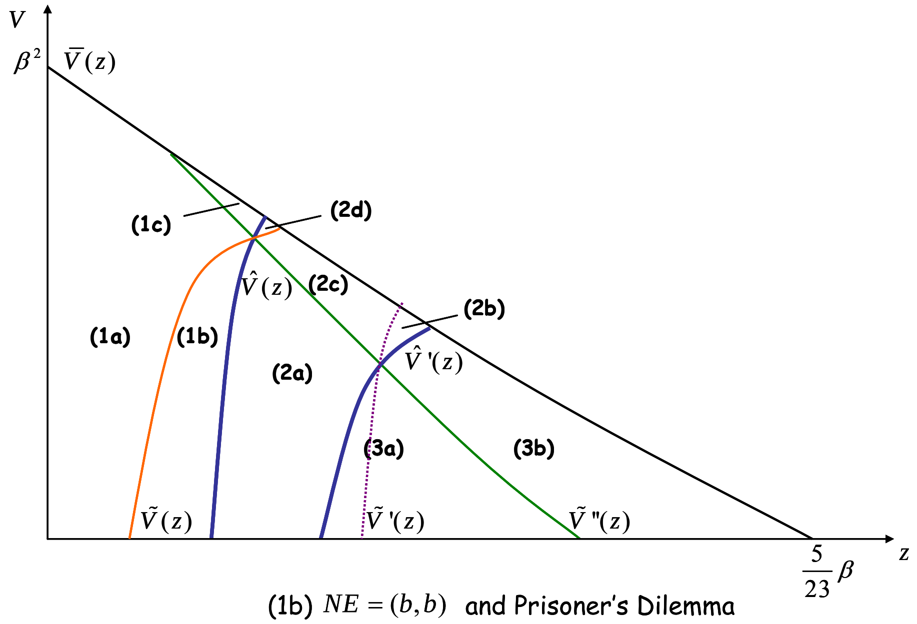


(1) $NE = (b,b)$

(2) $NE \in \{(c,b),(b,c)\}$

(3) $NE = (c,c)$

Fig. 4: the simultaneous move game equilibria.



(1b) $NE = (b,b)$ and Prisoner's Dilemma

(2) Coordination Game structure.

Fig. 5: simultaneous move game equilibria and Pareto optimality.

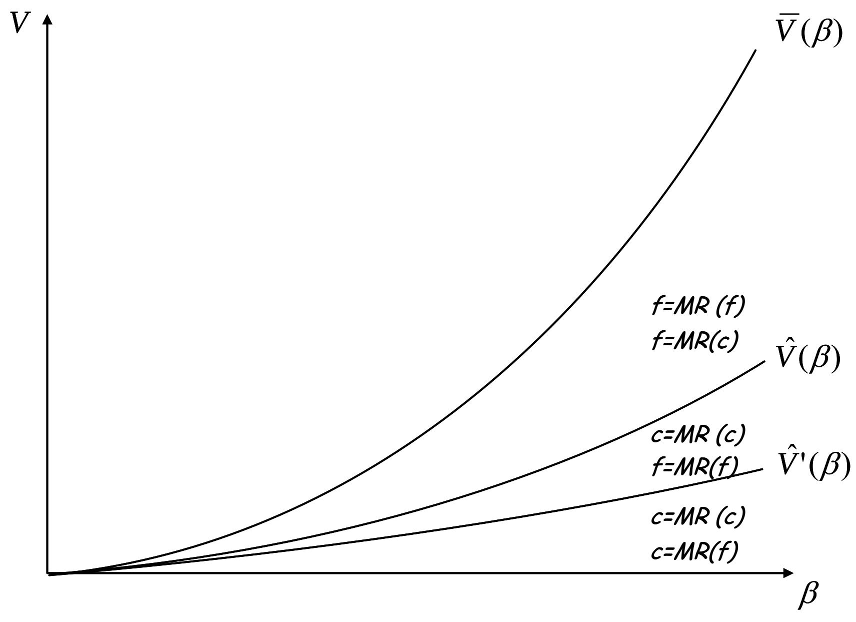
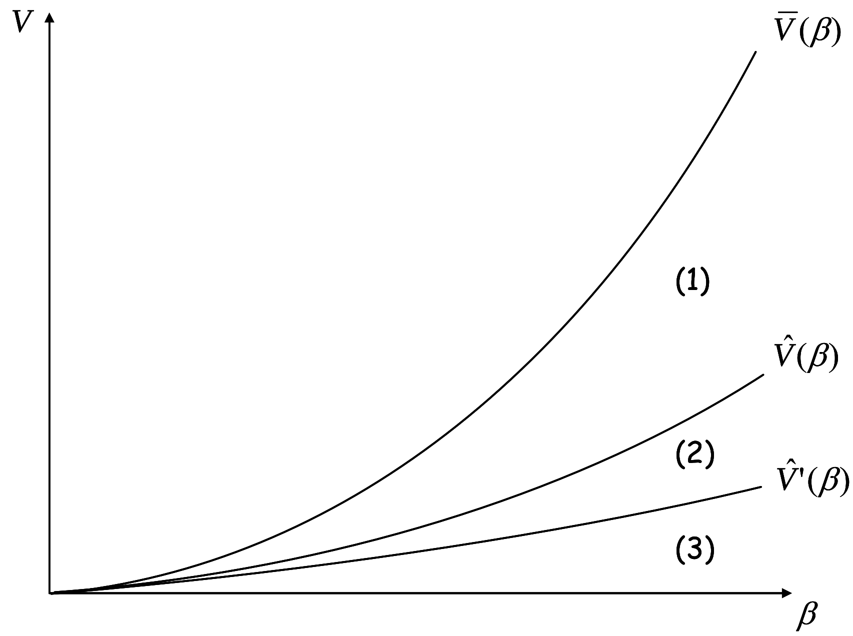


Fig. 6: the best reply functions at the timing decision stage.



(1) $NE = (f, f)$

(2) $NE \in \{(c,c),(f, f)\}$

(3) $NE = (c,c)$

Fig. 7: the simultaneous move game equilibria.

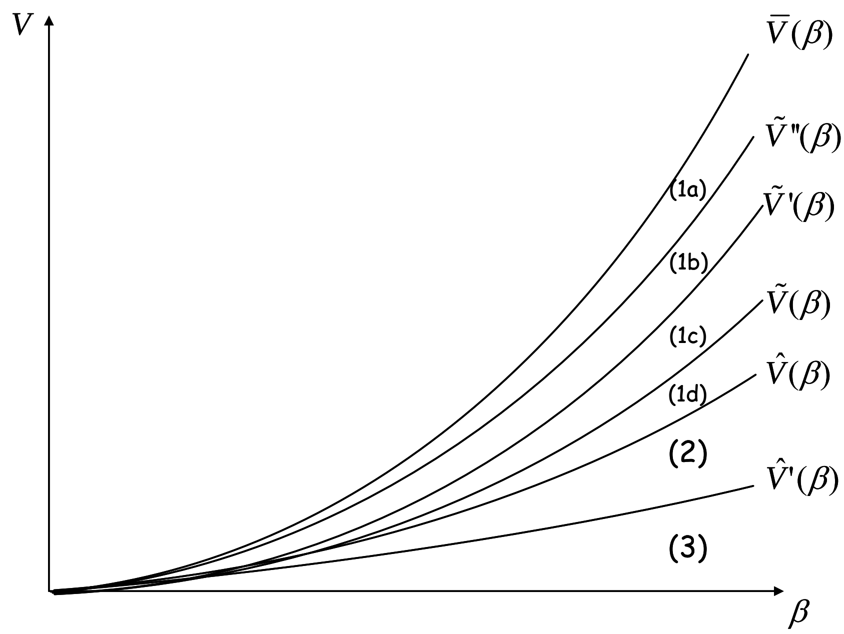


Fig. 8: simultaneous move game equilibria and Pareto optimality.

Proposition de titres:

- Commitment and Flexibility under oligopoly: does a first mover endogenously emerge?
- Commitment and Flexibility in a context of vertical product differentiation: conditions for asymmetric equilibria.

- Certification of Quality versus Brand under duopoly: fighting over uncertain demand.
- Certification of Quality versus Brand in a context of demand uncertainty: reasons to commit.
- Certification of Quality versus Brand in a context of demand uncertainty: when do firms differentiate?