# Non Linear Contracting and Endogenous Market Power between Manufacturers and Retailers : Identification and Estimation on Differentiated Products 

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#### Abstract

Résumé We present a methodology allowing to introduce manufacturers and retailers vertical contracting in their pricing strategies on a differentiated product market. We consider in particular some non linear pricing relationships, where resale price maintenance may be used or not with two part tariffs contracts. When agreeing with these two part tariffs contracts, we allow retailers to have some endogenous market power provided by their outside opportunities. However, whether retailers or manufacturers gain in the bargaining over take-it or leave-it multiproduct contract offers is an empirical question. Our contribution allows to recover price-cost margins at the manufacturer and retailer levels from estimates of demand parameters. The methodology developed permits to test between different hypothesis on the contracting and pricing relationships between manufacturers and retailers in the supermarket industry and in particular examine whether wholesale price discrimination or resale price maintenance is used. We apply empirically this method to study the market of bottled water in France. Our empirical evidence shows that manufacturers and retailers use non linear pricing contracts and in particular two part tariffs contracts with resale price maintenance. We also find that the retailers market power is not affected by other manufacturers offers.


Key words : vertical contracts, two part tariffs, double marginalization, collusion, competition, manufacturers, endogenous market power, retailers, differentiated products, water, non nested tests.

[^0]
## 1 Introduction

Vertical relationships between manufacturers and retailers seem to be more and more important in the supermarket industry and in particular in food retailing. Competition analysis and market power estimation on some consumption goods markets should involve the analysis of competition between producers but also between retailers and the whole structure of the industry. Consumer welfare depends crucially on these strategic vertical relationships and the degree of competition between manufacturers and retailers. The aim of this paper is thus to develop a methodology allowing to estimate alternative structural models where the role of manufacturers and retailers is explicit in the horizontal and vertical strategic behaviors. Previous work on these issues generally does not account for the behavior of retailers in the manufacturers pricing strategies. One of the reasons is that information on wholesale prices and marginal costs of production or distribution are generally difficult to obtain. Methods relying on demand side data, where only retail prices are observed, require the structural modelling of vertical contracts between manufacturers and retailers in an oligopoly model. Following Rosse (1970), researchers have thus tried to develop methodologies allowing to estimate price-cost margins, using only data on the demand side, i.e. sales quantities, market shares and retail prices. Empirical industrial organization methods propose to address this question with the estimation of structural models of competition on differentiated products markets (see, for example, Berry, 1994, Berry, Levinsohn and Pakes, 1995, and Nevo, 1998, 2000, 2001, Ivaldi and Verboven, 2001 on markets such as cars, computers, and breakfast cereals). Until recently, most papers in this literature assume that manufacturers set prices and that retailers act as neutral pass-through intermediaries or that they charge exogenous constant margins. However, it seems unlikely that retailers do not use some strategic pricing. Chevalier, Kashyap and Rossi (2003) show the important role of distributors on prices through the use of data on wholesale and retail prices. Actually, the strategic role of retailers has been emphasized only recently in the economics and marketing empirical literatures. Goldberg and Verboven (2001), Mortimer (2004), Sudhir (2001), Berto Villas Boas (2004), Bonnet, Dubois and Simioni (2004) or Villas-Boas and Zhao (2004) introduce retailers' strategic behavior. For instance, Sudhir (2001) considers the strategic interactions between manufacturers and a single retailer on a local market
and focuses exclusively on a linear pricing model leading to double marginalization. These recent developments introducing retailers' strategic behavior consider mostly cases where competition between producers and/or retailers remains under linear pricing. Berto Villas-Boas (2004) extends the Sudhir's framework to multiple retailers and considers the possibility that vertical contracts between manufacturers and retailers make pricing strategies depart from double marginalization by setting alternatively wholesale margins or retail margins to zero. Using recent theoretical developments due to Rey and Vergé (2004) that characterize pricing equilibria in the case of competition under non linear pricing between manufacturers and retailers (namely two part tariffs with or without resale price maintenance), Bonnet, Dubois and Simioni (2004) extend the analysis taking explicitly into account vertical contracts between manufacturers and retailers. Bonnet, Dubois and Simioni (2004) introduce explicit two part tariffs contracts between manufacturers and retailers assuming that the bargaining power between them is exogenously fixed.

However, the consideration of endogenous market power within a vertical relationship has never been taken into account in the empirical industrial organization literature. Linear pricing or non linear pricing has been considered but always with exogenously fixed bargaining power on one side or the other. Here, we allow retailers to benefit from some endogenous market power when facing manufacturers contracts offers. The endogenous market power comes from the available competing offers by other manufacturers that can be used as outside option by retailers but also from the explicit consideration of private label brands owned by retailers. However, the bargaining over take-it or leave-it offers by multiproduct manufacturers implies that retailers may loose against manufacturers when refusing bundling contracts. We show how we can identify and estimate pricecost margins at the retailer and manufacturer levels under the different competition scenarios considered. In particular, we consider two types of non linear pricing relationships with or without endogenous market power of retailers, one where resale price maintenance is used with two part tariffs contracts and one where no resale price maintenance is allowed in two part tariffs. Modelling explicitly optimal two part tariffs contracts (with or without resale price maintenance) allows to recover the pricing strategy of manufacturers and retailers. We do not only recover the total price-cost margins as functions of demand parameters without observing wholesale prices that are estimated but also the division of these margins between manufacturers and retailers.

We then present how to identify and estimate the retailer and manufacturer levels price-cost margins using some identifying assumptions. Using non nested test procedures, we show how to test between the different models using restrictions on marginal costs or exogenous variables that shift the marginal costs of production and distribution. Inference about the true competing forces between manufacturers and retailers is thus drawn as well as inference about wholesale price discrimination, resale price maintenance and other contracting practices in the industry.

We apply this methodology to study the market for retailing bottled water in France and present the first formal empirical estimation of market power of manufacturers and retailers when actors use non linear contracts. This market presents a high degree of concentration both at the manufacturer and retailer levels. It is to be noted that it is actually even more concentrated at the manufacturer level with only three large manufacturers than at the retailer level where we have in France seven large retailing chains. Our empirical evidence shows that, in the French bottled water market, manufacturers and retailers use two part tariffs contracts with resale price maintenance. Moreover, the market power of retailers is not affected endogenously by their outside opportunities because such a case is rejected by the data. It seems that the three main multiproduct manufacturers on this market are big enough for the retailers not being able to refuse offers of one of them. By bundling the two-part tariffs contracts, manufacturers manage to reduce the profitability of refusing contract offers and retailing only other firms' brands.

In section 2, we first present some stylized facts on the market for bottled water in France, an industry where the questions of vertical relationships and competition of manufacturers and retailers seem worth studying. Section 3 presents the main methodological contribution on the supply side. We show how price-cost margins can be recovered with demand parameters, in particular when taking explicitly into account two part tariffs contracts and estimating endogenously the market power of retailers. Section 4 presents the demand model, its identification and the estimation method proposed as well as the testing method between the different models. Section 5 presents the empirical results, tests and simulations. A conclusion with future research directions is in section 6, and some appendices follow.

## 2 Stylized Facts on the Market for Bottled Water in France

The French market for bottled water is one of the more dynamic sector of the French food processing industry : the total production of bottled water has increased by $4 \%$ in 2000 , and its turnover by $8 \%$. Some $85 \%$ of French consumers drink bottled water, and over two thirds of French bottled water drinkers drink it more than once a day, a proportion exceeded only in Germany. The French bottled water sector is a highly concentrated sector, the first three main manufacturers (Nestlé Waters, Danone, and Castel) sharing $90 \%$ of the total production of the sector. Moreover, given the scarcity of natural springs and natural capacity constraints, entry both for mineral or spring water is rather difficult in this market. Compte, Jenny and Rey (2002) comment on the Nestlé/Perrier Merger case that took place in 1992 in Europe and point out that these capacity constraints are a factor of collusion by themselves in addition to the high concentration of the sector. This sector can be divided in two major segments : mineral water and spring water. Natural mineral water benefits from some properties favorable to health, that are officially recognized. Composition must be guaranteed as well as the consistency of a set of qualitative criteria : mineral content, visual aspects, and taste. The mineral water can be marketed if it receives an agreement from the French Ministry of Health. The exploitation of a spring water source requires only a license provided by local authorities (Prefectures) and a favorable opinion of the local health committee. Moreover, the water composition is not required to be constant. The differences between the quality requirements involved in the certification of the two kinds of bottled water may explain part of the large difference that exists between the shelf prices of the national mineral water brands and the local spring water brands. Moreover, national mineral water brands are highly advertised. The bottled water products use mainly two kinds of differentiation. The first kind of differentiation stems from the mineral composition, that is the mineral salts content, and the second from the brand image conveyed through advertising. Actually, thanks to data at the aggregate level (Agreste, 1999, 2000, 2002) on food industries and the bottled water industry, one can remark (see the following Table) that this industry uses much more advertising than other food industries. Friberg and Ganslandt (2003) report an advertising to revenue ratio for the same industry in Sweden, i.e., $6.8 \%$ over the 1998-2001 period. For comparison, the highest advertising
to revenue ratio in the US food processing industry corresponds to the ready-to-eat breakfast cereal industry is of $10.8 \%$. These figures may be interpreted as showing the importance of horizontal differentiation of products for bottled water.

| Year | Bottled Water |  | All Food Industries |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $P C M$ | Advertising/Revenue | $P C M$ | Advertising/Revenue |
| 1998 | $17.38 \%$ | $12.09 \%$ | $6.32 \%$ | $5.57 \%$ |
| 1999 | $16.70 \%$ | $14.91 \%$ | $6.29 \%$ | $6.81 \%$ |
| 2000 | $13.61 \%$ | $15.89 \%$ | $3.40 \%$ | $8.76 \%$ |
| Table : Aggregate Estimates of Margins and Advertising to Sales Ratios. |  |  |  |  |

These aggregate data also allow to compute some accounting price-cost margins ${ }^{1}$ defined as value added ${ }^{2}(V A)$ minus payroll $(P R)$ and advertising expenses $(A D)$ divided by the value of shipments (TR). As emphasized by Nevo (2001), these accounting estimates can be considered as an upper bound to the true price-cost margins.

Recently, the degradation of the distribution network of tap water has led to an increase of bottled water consumption. This increase benefited to the cheapest bottled water, that is to the local spring water. For instance, the total volume of local spring water sold in 2000 reached closely the total volume of mineral water sold the same year. Households buy bottled water mostly in supermarkets : some $80 \%$ of the total sales of bottled water comes from supermarkets. Moreover, on average, these sales represent $1.7 \%$ of the total turnover of supermarkets, the bottled water shelf being one of the most productive. French bottled water manufacturers thus deal mainly their brands through retailing chains. These chains are also highly concentrated, the market share of the first five accounting for $80.7 \%$ of total food product sales. Moreover, these late years, like other processed food products, these chains have developed private labels to attract consumers. The increase in the number of private labels tends to be accompanied by a reduction of the market shares of the main national brands.

We thus face a concentrated market for which the questions of whether or not producers may exert bargaining power in their strategic relationships with retailers is important. The study of competition issues and evaluation of markups, which is crucial for consumer welfare, has then to take into account the possibility that non linear pricing may be used between manufacturers and

[^1]retailers.

## 3 Competition and Vertical Relationships Between Manufacturers and Retailers

Given the structure of the bottled water industry and the retail industry in France, oligopoly models with different vertical relationships can be envisaged. Bonnet, Dubois and Simioni (2004) considered the particular case of competition in two part tariffs without endogenous market power of retailers. Here, we contribute to this analysis by allowing retailers to benefit from some endogenous market power when facing manufacturers' two part tariffs contracts.

We consider $J$ differentiated products defined by the couple brand-retailer corresponding to $J^{\prime}(\leq J)$ national brands and $J-J^{\prime}$ store brands (also called private labels). We suppose that there are $R$ retailers competing in the retail market and $F$ manufacturers competing in the wholesale market. We denote by $S_{r}$ the set of products sold by retailer $r$ and by $F_{f}$ the set of products produced by firm $f$.

### 3.1 Linear Pricing and Double Marginalization

In this model, the manufacturers set their prices first, and retailers follow by setting the retail prices given the wholesale prices. For private labels, prices are chosen by the retailer himself who acts as doing both manufacturing and retailing. We consider Nash-Bertrand competition. We solve this vertical model by backward induction considering the retailer's problem first. The profit $\Pi^{r}$ of retailer $r$ in a given period (we drop the time subscript $t$ for ease of presentation) is given by

$$
\Pi^{r}=\sum_{j \in S_{r}}\left(p_{j}-w_{j}-c_{j}\right) s_{j}(p) M
$$

where $p_{j}$ is the retail price of product $j$ sold by retailer $r, w_{j}$ is the wholesale price paid by retailer $r$ for product $j, c_{j}$ is the retailer's (constant) marginal cost of distribution for product $j, s_{j}(p)$ is the market share of product $j, p$ is the vector of all products retail prices and $M$ is the size of the market. Assuming that a pure-strategy Bertrand-Nash equilibrium in prices exists and that equilibrium prices are strictly positive, the price of any brand $j$ sold by retailer $r$ must satisfy the first-order condition

$$
\begin{equation*}
s_{j}+\sum_{k \in S_{r}}\left(p_{k}-w_{k}-c_{k}\right) \frac{\partial s_{k}}{\partial p_{j}}=0, \quad \text { for all } j \in S_{r} \tag{1}
\end{equation*}
$$

Now, we define $I_{r}$ (of size $(J \times J)$ ) as the ownership matrix of the retailer $r$ that is diagonal and whose elements $I_{r}(j, j)$ are equal to 1 if the retailer $r$ sells products $j$ and zero otherwise. Let $S_{p}$ be the market shares response matrix to retailer prices, containing the first derivatives of all market shares with respect to all retail prices, i.e.

$$
S_{p} \equiv\left(\begin{array}{ccc}
\frac{\partial s_{1}}{\partial p_{1}} & \ldots & \frac{\partial s_{J}}{\partial p_{1}} \\
\vdots & & \vdots \\
\frac{\partial s_{1}}{\partial p_{J}} & \ldots & \frac{\partial s_{J}}{\partial p_{J}}
\end{array}\right)
$$

In vector notation, the first order condition (1) implies that the vector $\gamma$ of retailer $r$ 's margins, i.e. the retail price $p$ minus the wholesale price $w$ minus the marginal cost of distribution $c$, is ${ }^{3}$

$$
\begin{equation*}
\gamma \equiv p-w-c=-\left(I_{r} S_{p} I_{r}\right)^{-1} I_{r} s(p) \tag{2}
\end{equation*}
$$

Remark that for private labels, this price-cost margin is in fact the total price cost margin $p-\mu-c$ which amounts to replace the wholesale price $w$ by the marginal cost of production $\mu$ in this formula.

Concerning the manufacturers' behavior, we also assume that each of them maximize profit choosing the wholesale prices $w_{j}$ of their products $j$ and given the retailers' response (1). The profit of manufacturer $f$ is given by

$$
\Pi^{f}=\sum_{j \in F_{f}}\left(w_{j}-\mu_{j}\right) s_{j}(p(w)) M
$$

where $\mu_{j}$ is the manufacturer's (constant) marginal cost of production of product $j$. Assuming the existence of a pure-strategy Bertrand-Nash equilibrium in wholesale prices between manufacturers, the first order conditions are

$$
\begin{equation*}
s_{j}+\sum_{k \in F_{f}} \sum_{l=1, . ., J}\left(w_{k}-\mu_{k}\right) \frac{\partial s_{k}}{\partial p_{l}} \frac{\partial p_{l}}{\partial w_{j}}=0, \quad \text { for all } j \in F_{f} \tag{3}
\end{equation*}
$$

Consider $I_{f}$ the ownership matrix of manufacturer $f$ that is diagonal and whose element $I_{f}(j, j)$ is equal to one if $j$ is produced by the manufacturer $f$ and zero otherwise.

We introduce $P_{w}$ the $(J \times J)$ matrix of retail prices responses to wholesale prices, containing

[^2]the first derivatives of the $J$ retail prices $p$ with respect to the $J^{\prime}$ wholesale prices $w$.
\[

P_{w} \equiv\left($$
\begin{array}{ccccc}
\frac{\partial p_{1}}{\partial w_{1}} & . . & \frac{\partial p_{J}}{\partial w_{J^{\prime}}} & . . & \frac{\partial p_{J}}{\partial w_{1}} \\
\vdots & & \vdots & & \vdots \\
\frac{\partial p_{1}}{\partial w_{J \prime}} & . . & \frac{\partial p_{J^{\prime}}}{\partial w_{J^{\prime}}} & . . & \frac{\partial p_{J}}{\partial w_{J^{\prime}}} \\
0 & . . & 0 & . . & 0 \\
0 & . . & 0 & . . & 0
\end{array}
$$\right)
\]

Remark that the last $J-J^{\prime}$ lines of this matrix are zero because they correspond to private labels products for which wholesale prices have no meaning.

Then, we can write the first order conditions (3) in matrix form and the vector of manufacturer's margins is ${ }^{4}$

$$
\begin{equation*}
\Gamma \equiv w-\mu=-\left(I_{f} P_{w} S_{p} I_{f}\right)^{-1} I_{f} s(p) \tag{4}
\end{equation*}
$$

The first derivatives of retail prices with respect to wholesale prices depend on the strategic interactions between manufacturers and retailers. Let's assume that the manufacturers set the wholesale prices and retailers follow, setting the retail prices given the wholesale prices. Therefore, $P_{w}$ can be deduced from the differentiation of the retailer's first order conditions (1) with respect to wholesale price, i.e. for $j \in S_{r}$ and $k=1, . ., J^{\prime}$

$$
\begin{gather*}
s_{j}+\sum_{k \in S_{r}}\left(p_{k}-w_{k}-c_{k}\right) \frac{\partial s_{k}}{\partial p_{j}}=0, \quad \text { for all } j \in S_{r} .  \tag{5}\\
\sum_{l=1, . ., J} \frac{\partial s_{j}(p)}{\partial p_{l}} \frac{\partial p_{l}}{\partial w_{k}}-1_{\left\{k \in S_{r}\right\}} \frac{\partial s_{k}(p)}{\partial p_{j}}+\sum_{l \in S_{r}} \frac{\partial s_{l}(p)}{\partial p_{j}} \frac{\partial p_{l}}{\partial w_{k}}+\sum_{l \in S_{r}}\left(p_{l}-w_{l}-c_{l}\right) \sum_{s=1, .,, J} \frac{\partial^{2} s_{l}(p)}{\partial p_{j} \partial p_{s}} \frac{\partial p_{s}}{\partial w_{k}}=0 \tag{6}
\end{gather*}
$$

where $1_{\left\{k \in S_{r}\right\}}=1$ if $k \in S_{r}$ and 0 otherwise. Defining $S_{p}^{p_{j}}$ the $(J \times J)$ matrix of the second derivatives of the market shares with respect to retail prices whose element $(l, k)$ is $\frac{\partial^{2} s_{k}}{\partial p_{j} \partial p_{l}}$, i.e.

$$
S_{p}^{p_{j}} \equiv\left(\begin{array}{ccc}
\frac{\partial^{2} s_{1}}{\partial p_{1} \partial p_{j}} & \cdots & \frac{\partial^{2} s_{J}}{\partial p_{1} \partial p_{j}} \\
\vdots & \cdot & \vdots \\
\frac{\partial^{2} s_{1}}{\partial p_{J} \partial p_{j}} & \cdots & \frac{\partial^{2} s_{J}}{\partial p_{J} \partial p_{j}}
\end{array}\right)
$$

We can write equation (6) in matrix form ${ }^{5}$ :

$$
\begin{equation*}
P_{w}=I_{r} S_{p}\left(I_{r}-\widetilde{I}_{r}\right)\left[S_{p} I_{r}+I_{r} S_{p}^{\prime} I_{r}+\left(S_{p}^{p_{1}} I_{r} \gamma|\ldots| S_{p}^{p_{J}} I_{r} \gamma\right) I_{r}\right]^{-1} \tag{7}
\end{equation*}
$$

where $\gamma=p-w-c, \widetilde{I}_{r}$ is the ownership matrix of private labels of retailer $r$ and $I_{r}-\widetilde{I}_{r}$ thus designates the ownership matrix of national brands by retailer $r$. Equation (4) shows that one can express the manufacturer's price cost margins vector $\Gamma=w-\mu$ as depending on the function $s(p)$

[^3]by replacing the expression (7) for $P_{w}$ in (4). The expression (7) comes from the assumption that manufacturers act as Stackelberg leaders in the vertical relationships with retailers.

### 3.2 Two-Part Tariffs and Endogenous Retail Market Power

We now consider the case where manufacturers and retailers can sign two-part tariffs contracts. We assume that manufacturers make take-it or leave-it offers to retailers and characterize symmetric subgame perfect Nash equilibria as in Rey and Vergé (2004). Rey and Vergé (2004) prove the existence of equilibria under some assumptions on this multiple common agency game. Actually, we assume that manufacturers simultaneously propose two-part tariffs contracts to each retailer. These contracts consist in the specification of franchise fees and wholesale prices but also on retail prices in the case where manufacturers can use resale price maintenance. Thus we assume that, for each product, manufacturers propose the contractual terms to retailers and then, retailers simultaneously accept or reject the offers that are public information.

Contrary to Bonnet, Dubois and Simioni (2004) where it is assumed that if one offer is rejected, then all contracts are refused and retailers set a fixed reservation utility, we allow the possibility that retailers reject a contract offered to them while accepting other offers. Once offers have been accepted, the retailers simultaneously set their retail prices, demands and contracts are satisfied. Assuming that offers of manufacturers are public is a convenient modelling assumption that can however be justified in France by the non-discrimination laws.

Thus, in the case of these two part tariffs contracts, the profit function of retailer $r$ is :

$$
\begin{equation*}
\Pi^{r}=\sum_{s \in S_{r}}\left[M\left(p_{s}-w_{s}-c_{s}\right) s_{s}(p)-F_{s}\right] \tag{8}
\end{equation*}
$$

where $F_{s}$ is the franchise fee paid by the retailer for selling product $s$. The profit function of firm $f$ is equal to

$$
\begin{equation*}
\Pi^{f}=\sum_{k \in F_{f}}\left[M\left(w_{k}-\mu_{k}\right) s_{k}(p)+F_{k}\right] \tag{9}
\end{equation*}
$$

Allowing retailers to enjoy some endogenous market power, we consider the case where retailers may be able to refuse some contracts proposed by manufacturers while accepting other two-part tariffs contracts. Contract offers are simultaneous but the participation constraints of the retailers are such that two-part tariffs contracts offered by a manufacturer $f$ to a retailer $r$ must provide
to the retailer a profit at least as large as the retailer's profit when refusing the proposed contract but accepting all other offers. Moreover, it must be also that the retailers profits are at least larger than some fixed reservation utility level $\bar{\Pi}^{r}$. We thus consider that two-part tariffs contracts are negotiated at the firm level and not by brand, which implies that manufacturers use bundling strategies in their offers to retailers. This is likely to increase the market power of multiproduct manufacturers. Thus, this contracting option takes fully into account the ownership structure of products and brands by multiproduct manufacturers while the previous linear pricing contracts cannot account for it. Retailers can refuse a manufacturer's offers and accept other manufacturers' ones but cannot refuse the two part tariffs contracts offered by one manufacturer on a given brand and accept the offers on other brands of this same manufacturer.

Thus, the manufacturers set the two-part tariffs contracts parameters (wholesale prices and fixed fees) in order to maximize profits as in (9) subject to the following retailers' participation constraints for all $r=1, . ., R$ :

$$
\begin{equation*}
\Pi^{r} \geq \bar{\Pi}^{r} \tag{10}
\end{equation*}
$$

and incentive constraints

$$
\begin{equation*}
\Pi^{r} \geq \sum_{s \in S_{r} \backslash F_{f r}}\left[M\left(\widetilde{p}_{s}^{f r}-w_{s}-c_{s}\right) s_{s}\left(\widetilde{p}^{f r}\right)-F_{s}\right] \tag{11}
\end{equation*}
$$

where $\Pi^{r}$ is the retailer's profit (8) when accepting all the offers, where $\bar{\Pi}^{r}$ is the retailer reservation utility, where $F_{f r}$ is the set of products produced by firm $f$ and distributed by retailer $r$, and $\widetilde{p}^{f r}=\left(\widetilde{p}_{1}^{f r}, . ., \widetilde{p}_{J}^{f r}\right)$ is the vector of retail prices when the products of $F_{f r}$ do not exist. By convention we will have $\widetilde{p}_{i}^{f r}=+\infty$ if $i \in F_{f r}$. Actually, when the retailer $r$ refuses the offers of the manufacturer $f$, he can accept all other offers in which case he sells only all other products. Then, the retailer $r$ sells all products not manufactured by $f$, that is those of the set $S_{r} \backslash F_{f r}$, and the market share $s_{s}\left(\widetilde{p}^{f r}\right)$ of each product of this set corresponds to the market share of product $s$ when all products of manufacturer $f$ retailed by $r$ are absent.

We will examine two cases of interest. The first is the case where the market power of retailers is determined endogenously because of the constraints (11) implying that the bargaining power of a retailer with a given manufacturer is affected by outside opportunities like other manufacturers' offers. The second case is the simple case where constraints (11) do not exist because it is assu-
med that if one offer is rejected then all offers must be rejected. Then, the outside opportunities depend on a fixed exogenous reservation utility and we will say that the market power of retailer is exogenous.

In the general case (following Rey and Vergé (2004) arguments), since the manufacturers can always adjust the fixed fees such that all the constraints (11) will be binding, we have $\forall r=1, . ., R$

$$
\sum_{s \in S_{r}}\left[M\left(p_{s}-w_{s}-c_{s}\right) s_{s}(p)-F_{s}\right]=\sum_{s \in S_{r} \backslash F_{f r}}\left[M\left(\widetilde{p}_{s}^{f r}-w_{s}-c_{s}\right) s_{s}\left(\widetilde{p}^{f r}\right)-F_{s}\right]
$$

In general, if constraints (11) are satisfied, the constraints (10) will be satisfied. The binding constraints (11) imply that the sum of fixed fees paid for the product sold by the manufacturer $f$ to the retailer $r$ is

$$
\sum_{s \in F_{f r}} F_{s}=\sum_{s \in S_{r}} M\left[\left(p_{s}-w_{s}-c_{s}\right) s_{s}(p)-\left(\widetilde{p}_{s}^{f r}-w_{s}-c_{s}\right) s_{s}\left(\widetilde{p}^{f r}\right)\right]
$$

because $s_{s}\left(\widetilde{p}^{f r}\right)=0$ when $s \in F_{f r}$.
Using this expression, one can rewrite the profit of the manufacturer $f$ as

$$
\begin{aligned}
\Pi^{f} & =\sum_{k \in F_{f}}\left[M\left(w_{k}-\mu_{k}\right) s_{k}(p)+F_{k}\right]=\sum_{k \in F_{f}} M\left(w_{k}-\mu_{k}\right) s_{k}(p)+\sum_{r=1}^{R} \sum_{k \in F_{f r}} F_{k} \\
& =\sum_{k \in F_{f}} M\left(w_{k}-\mu_{k}\right) s_{k}(p)+\sum_{r=1}^{R} \sum_{s \in S_{r}} M\left[\left(p_{s}-w_{s}-c_{s}\right) s_{s}(p)-\left(\widetilde{p}_{s}^{f r(s)}-w_{s}-c_{s}\right) s_{s}\left(\widetilde{p}^{f r(s)}\right)\right]
\end{aligned}
$$

where $r(s)$ denotes the retailer of product $s$ and because $\cup_{r=1}^{R} F_{f r}=F_{f}$ (and $F_{f r} \cap F_{f r^{\prime}}=\varnothing$ ). The manufacturer's profit is then

$$
\begin{equation*}
\Pi^{f}=\sum_{k \in F_{f}} M\left(w_{k}-\mu_{k}\right) s_{k}(p)+\sum_{s=1}^{J} M\left[\left(p_{s}-w_{s}-c_{s}\right) s_{s}(p)-\left(\widetilde{p}_{s}^{f r(s)}-w_{s}-c_{s}\right) s_{s}\left(\widetilde{p}^{f r(s)}\right)\right] \tag{12}
\end{equation*}
$$

### 3.2.1 With Resale Price Maintenance

Let's consider the case where manufacturers use resale price maintenance in their contracts with retailers. Then, manufacturers can choose retail prices while the wholesale prices have no direct effect on profit. In this case, the vectors of prices $\widetilde{p}^{f r}$ are such that $\widetilde{p}_{i}^{f r}=p_{i}$ if $i \notin F_{f r}$ and the profit of manufacturer $f$ can then be written as

$$
\Pi^{f}=\sum_{k \in F_{f}} M\left(w_{k}-\mu_{k}\right) s_{k}(p)+\sum_{s=1}^{J} M\left(p_{s}-w_{s}-c_{s}\right)\left[s_{s}(p)-s_{s}\left(\widetilde{p}^{f r(s)}\right)\right]
$$

The first order conditions of the maximization of profit of $f$ with respect to retail prices $\left\{p_{j}\right\} \in F_{f}$ are $: \forall j \in F_{f}$

$$
0=s_{j}(p)+\sum_{k=1}^{J}\left[\left(p_{k}-w_{k}-c_{k}\right)\left(\frac{\partial s_{k}(p)}{\partial p_{j}}-\frac{\partial s_{k}\left(\widetilde{p}^{f r(k)}\right)}{\partial p_{j}}\right)\right]+\sum_{k \in F_{f}}\left(w_{k}-\mu_{k}\right) \frac{\partial s_{k}(p)}{\partial p_{j}}
$$

As Rey and Vergé (2004) argue, a continuum of equilibria exist in this general case, with one equilibrium corresponding to each possible value of the vector of wholesale prices $w$.

As we can re-write the retail margins $(p-w-c)$ as the difference between total margins $(p-\mu-c)$ and wholesale margins $(w-\mu)$, the previous $J-J^{\prime}$ first order conditions can be written in a matrix form as

$$
\begin{equation*}
I_{f}\left(S_{p}-S_{\widetilde{p}}^{f}\right)(\gamma+\Gamma)+I_{f} s(p)-I_{f}\left(S_{p}-S_{\widetilde{p}}^{f}\right) \Gamma+I_{f} S_{p} I_{f} \Gamma=0 \tag{13}
\end{equation*}
$$

where $\Gamma=\left(w_{k}-\mu_{k}\right)_{k=1, \ldots, J}$ is the full vector of wholesale margins and $\gamma+\Gamma$ the vector of total margins.

In the case of private labels products, retailers choose retail prices and bear the marginal cost of production and distribution, maximizing :

$$
\max _{\left\{p_{j}\right\}_{j \in \tilde{S}_{r}}} \sum_{k \in \widetilde{S}_{r}}\left(p_{k}-\mu_{k}-c_{k}\right) s_{k}(p)+\sum_{k \in S_{r} \backslash \widetilde{S}_{r}}\left(p_{k}-w_{k}-c_{k}\right) s_{k}(p)
$$

where $\widetilde{S}_{r}$ is the set of private label products of retailer $r$. Thus, for private label products, additional equations are obtained from the first order conditions of the profit maximization of retailers that both produce and retail these products. The first order conditions give

$$
\sum_{k \in \widetilde{S}_{r}}\left(p_{k}-\mu_{k}-c_{k}\right) \frac{\partial s_{k}(p)}{\partial p_{j}}+s_{j}(p)+\sum_{k \in S_{r} \backslash \widetilde{S}_{r}}\left(p_{k}-w_{k}-c_{k}\right) \frac{\partial s_{k}(p)}{\partial p_{j}}=0 \quad \text { for all } j \in \widetilde{S}_{r}
$$

which can be written

$$
\sum_{k \in S_{r}}\left(p_{k}-\mu_{k}-c_{k}\right) \frac{\partial s_{k}(p)}{\partial p_{j}}+s_{j}(p)-\sum_{k \in S_{r} \backslash \widetilde{S}_{r}}\left(w_{k}-\mu_{k}\right) \frac{\partial s_{k}(p)}{\partial p_{j}}=0 \quad \text { for all } j \in \widetilde{S}_{r}
$$

In matrix notation, these first order conditions are : for $r=1, . ., R$

$$
\begin{equation*}
\left(\widetilde{I}_{r} S_{p} I_{r}\right)(\gamma+\Gamma)+\widetilde{I}_{r} s(p)-\widetilde{I}_{r} S_{p} I_{r} \Gamma=0 \tag{14}
\end{equation*}
$$

where $\widetilde{I}_{r}$ is the ownership matrix of private label products by retailer $r$.

We thus obtain a system of equations with (13) and (14) where $\gamma+\Gamma$ and $\Gamma$ are unknown.

$$
\left\{\begin{array}{c}
I_{f}\left(S_{p}-S_{\tilde{p}}^{f}\right)(\gamma+\Gamma)+I_{f} s(p)-I_{f}\left(S_{p}-S_{\widetilde{p}}^{f}\right) \Gamma+I_{f} S_{p} I_{f} \Gamma=0 \text { for } f=1, . ., F \\
\left(\widetilde{I}_{r} S_{p} I_{r}\right)(\gamma+\Gamma)+\widetilde{I}_{r} s(p)-\widetilde{I}_{r} S_{p} I_{r} \Gamma=0 \text { for } r=1, . ., R
\end{array}\right.
$$

After solving the system (see appendix 7.1), we obtain the expression for the total price-cost margin of all products as a function of demand parameters, of the structure of the industry and the vector $\Gamma$ of wholesale prices :

$$
\begin{aligned}
\gamma+\Gamma= & -\left(\sum_{r} I_{r} S_{p}^{\prime} \widetilde{I}_{r} S_{p} I_{r}+\sum_{f}\left[S_{p}-S_{\widetilde{p}}^{f}\right]^{\prime} I_{f}\left[S_{p}-S_{\widetilde{p}}^{f}\right]\right)^{-1} \\
& \left(\sum_{r} I_{r} S_{p}^{\prime} \widetilde{I}_{r} s(p)-I_{r} S_{p}^{\prime} \widetilde{I}_{r} S_{p} I_{r} \Gamma+\sum_{f}\left[S_{p}-S_{\widetilde{p}}^{f}\right]^{\prime} I_{f}\left(s(p)-\left(S_{p}-S_{\widetilde{p}}^{f}\right) \Gamma+S_{p} I_{f} \Gamma()\right),\right)
\end{aligned}
$$

where the matrix $S_{\widetilde{p}}^{f}$ is

$$
S_{\widetilde{p}}^{f} \equiv\left(\begin{array}{ccc}
\frac{\partial s_{1}\left(\widetilde{p}^{f r(1)}\right)}{\partial p_{1}} & . . & \frac{\partial s_{J}\left(\widetilde{p}^{f r(J)}\right)}{\partial p_{1}} \\
\vdots & & \vdots \\
\frac{\partial s_{1}\left(\widetilde{p}^{f r(1)}\right)}{\partial p_{J}} & . . & \frac{\partial s_{J}\left(\tilde{p}^{f r(J)}\right)}{\partial p_{J}}
\end{array}\right)
$$

This expression shows that the right hand side of equation (15) depends only on demand parameters, on the ownership structure of products and on unknown wholesale margins $\Gamma$.

The particular equilibrium where wholesale prices are such that $w_{s}^{*}=\mu_{s}$ for all $s$, that is $\Gamma=0$, implies that
$\gamma+\Gamma=-\left(\sum_{r} I_{r} S_{p}^{\prime} \widetilde{I}_{r} S_{p} I_{r}+\sum_{f}\left[S_{p}-S_{\widetilde{p}}^{f}\right]^{\prime} I_{f}\left[S_{p}-S_{\widetilde{p}}^{f}\right]\right)^{-1}\left(\sum_{r} I_{r} S_{p}^{\prime} \widetilde{I}_{r}+\sum_{f}\left[S_{p}-S_{\widetilde{p}}^{f}\right]^{\prime} I_{f}\right) s(p)$
When retailers have no endogenous market power :

If retailers have no market power (as in Bonnet, Dubois and Simioni, 2004), we can suppress the incentive constraints (11) and take only into account the participation constraints (10). Then, manufacturers can capture retail profits through the franchise fees and choose retail prices. Appendix 7.2 shows how the profit maximization of the manufacturers leads to the following first order conditions for a manufacturer $j$ and for a given set of equilibrium prices for other manufacturers $\left\{p_{k}, w_{k}\right\}_{k \notin F_{f}}$.

$$
\begin{equation*}
\sum_{k \in F_{f}}\left(p_{k}-\mu_{k}-c_{k}\right) \frac{\partial s_{k}(p)}{\partial p_{j}}+s_{j}(p)+\sum_{k \notin F_{f}}\left(p_{k}-w_{k}-c_{k}\right) \frac{\partial s_{k}(p)}{\partial p_{j}}=0 \quad \text { for all } j \in F_{f} \tag{16}
\end{equation*}
$$

Rewriting (16) as

$$
\sum_{k=1, . ., J}\left(p_{k}-\mu_{k}-c_{k}\right) \frac{\partial s_{k}(p)}{\partial p_{j}}+s_{j}(p)-\sum_{k \notin F_{f}}\left(w_{k}-\mu_{k}\right) \frac{\partial s_{k}(p)}{\partial p_{j}}=0
$$

leads to the following matrix notation for the set of first order conditions of manufacturer $f$

$$
\begin{equation*}
I_{f} S_{p}(\gamma+\Gamma)+I_{f} s(p)-I_{f} S_{p}\left(I-I_{f}\right) \Gamma=0 \tag{17}
\end{equation*}
$$

In the case of private labels products, the first order conditions (14) are also obtained and then all first order conditions (17) and (14) provide a system of equations to be solved in order to find the vector of total price-cost margins $\gamma+\Gamma$

$$
\left\{\begin{array}{c}
I_{f} S_{p}(\gamma+\Gamma)+I_{f} s(p)-I_{f} S_{p} I_{-f} \Gamma=0(f=1, \ldots, F) \\
\left(\widetilde{I}_{r} S_{p} I_{r}\right)(\gamma+\Gamma)+\widetilde{I}_{r} s(p)-\widetilde{I}_{r} S_{p} \Gamma=0(r=1, \ldots, R)
\end{array}\right.
$$

We show in Appendix 7.1 that there is a unique solution that allows to write the vector of total margins $\gamma+\Gamma$ as function of demand parameters, of the structure of the industry and of the vector of wholesale prices $\Gamma$ :

$$
\begin{align*}
\gamma+\Gamma= & -\left(\sum_{r} I_{r} S_{p}^{\prime} \widetilde{I}_{r} S_{p} I_{r}+\sum_{f} S_{p}^{\prime} I_{f} S_{p}\right)^{-1} \\
& \left(\sum_{r} I_{r} S_{p}^{\prime} \widetilde{I}_{r}\left[s(p)-S_{p} I_{r} \Gamma\right]+\sum_{f} S_{p}^{\prime} I_{f}\left[s(p)-S_{p}\left(I-I_{f}\right) \Gamma\right]\right) \tag{18}
\end{align*}
$$

When the equilibrium is such that wholesale prices are equal to the marginal cost of production ( $w_{k}^{*}=\mu_{k}, \forall k$ ), we obtain that total margins are

$$
\begin{equation*}
\gamma+\Gamma=-\left(\sum_{r} I_{r} S_{p}^{\prime} \widetilde{I}_{r} S_{p} I_{r}+\sum_{f} S_{p}^{\prime} I_{f} S_{p}\right)^{-1}\left(\sum_{r} I_{r} S_{p}^{\prime} \widetilde{I}_{r}+\sum_{f} S_{p}^{\prime} I_{f}\right) s(p) \tag{19}
\end{equation*}
$$

Remark that in the absence of private label products, this expression would simplify to the case where the total profits of the integrated industry are maximized, that is

$$
\begin{equation*}
\gamma+\Gamma=-S_{p}^{-1} s(p) \tag{20}
\end{equation*}
$$

because then $\sum_{f} I_{f}=I$.
This shows that when retailers have no endogenous market power, two part tariffs contracts with $R P M$ allow manufacturers to maximize the full profits of the integrated industry if retailers have no private label products. Rey and Vergé (2004) showed that, among the continuum of possible equilibria, the case where wholesale prices are equal to the marginal costs of production is the equilibrium that would be selected if retailers can provide a retailing effort that increases demand. Actually, in this case it is worth for the manufacturer to make the retailer residual claimant of his retailing effort which leads to select this equilibrium wholesale price.

When wholesale prices are such that the retailer's price cost margins are zero $\left(p_{k}^{*}\left(w_{k}^{*}\right)-w_{k}^{*}-c_{k}=\right.$ $0)$, then the first order conditions write as

$$
\sum_{k \in F_{f}}\left(p_{k}-\mu_{k}-c_{k}\right) \frac{\partial s_{k}(p)}{\partial p_{j}}+s_{j}(p)=0 \quad \text { for all } j \in F_{f}
$$

In matrix notations, we get for all $f=1, . ., F$

$$
\begin{equation*}
\gamma_{f}+\Gamma_{f}=(p-\mu-c)=-\left(I_{f} S_{p} I_{f}\right)^{-1} I_{f} s(p) \tag{21}
\end{equation*}
$$

For private label products, denoting $\gamma_{r}^{p l}+\Gamma_{r}^{p l}$ the vector of total price-cost margins of private labels of retailer $r$, we have

$$
\left(\widetilde{I}_{r} S_{p} \widetilde{I}_{r}\right)\left(\gamma_{r}^{p l}+\Gamma_{r}^{p l}\right)+\widetilde{I}_{r} s(p)=0
$$

which gives the following expression for total margins in this case

$$
\gamma_{r}^{p l}+\Gamma_{r}^{p l}=-\left(\widetilde{I}_{r} S_{p} \widetilde{I}_{r}\right)^{-1} \widetilde{I}_{r} s(p)
$$

### 3.2.2 Without Resale Price Maintenance

In the case where manufacturers cannot use Resale Price Maintenance, the retailers prices $\widetilde{p}^{f r}(w)$ are out of equilibrium prices different from the retail prices in equilibrium. The first order conditions of the maximization of the profit of $f(12)$ with respect to wholesale prices $\left\{w_{j}\right\} \in F_{f}$ are then $: \forall j \in F_{f}$

$$
\begin{aligned}
0= & \sum_{i=1}^{J} \sum_{k \in F_{f}}\left(w_{k}-\mu_{k}\right) \frac{\partial s_{k}(p)}{\partial p_{i}} \frac{\partial p_{i}}{\partial w_{j}}+\sum_{k=1}^{J}\left[\frac{\partial p_{k}}{\partial w_{j}} s_{k}(p)-\frac{\partial \widetilde{p}_{k}^{f r(k)}}{\partial w_{j}} s_{k}\left(\widetilde{p}^{f r(k)}\right)\right] \\
& +\sum_{i=1}^{J} \sum_{k=1}^{J}\left[\left(p_{k}-w_{k}-c_{k}\right) \frac{\partial s_{k}(p)}{\partial p_{i}} \frac{\partial p_{i}}{\partial w_{j}}-\left(\widetilde{p}_{k}^{f r(k)}-w_{k}-c_{k}\right) \frac{\partial s_{k}\left(\widetilde{p}^{f r(k)}\right)}{\partial p_{i}} \frac{\partial p_{i}}{\partial w_{j}}\right]
\end{aligned}
$$

In matrix notation, the previous first order conditions give

$$
0=I_{f} P_{w} S_{p} I_{f} \Gamma_{f}+I_{f} P_{w} s(p)-I_{f} \tilde{P}_{w}^{f} s\left(\widetilde{p}^{f}\right)+I_{f} P_{w} S_{p} \gamma-I_{f} P_{w} S_{\widetilde{p}}^{f} \widetilde{\gamma}^{f}
$$

where $\tilde{P}_{w}^{f}$ is the matrix of first order derivatives of retail prices $\widetilde{p}_{j}^{f r(j)}(w)($ for $j=1, . ., J)$ with respect to wholesale prices $w$.

Thus the wholesale margins of products of manufacturer $f$ are

$$
\begin{equation*}
\Gamma_{f}=-\left[I_{f} P_{w} S_{p} I_{f}\right]^{-1}\left(I_{f} P_{w} s(p)-I_{f} \tilde{P}_{w}^{f} s\left(\widetilde{p}^{f}\right)+I_{f} P_{w} S_{p} \gamma-I_{f} P_{w} S_{\widetilde{p}}^{f} \widetilde{\gamma}^{f}\right) \tag{22}
\end{equation*}
$$

where $\gamma$ comes from (2), $\widetilde{\gamma}^{f}=\left(\widetilde{\gamma}_{1}^{f}, . ., \widetilde{\gamma}_{J}^{f}\right)$ where $\widetilde{\gamma}_{k}^{f}$ is the $k^{t h}$ element of vector $-\left(I_{r(k)} S_{\widetilde{p}}^{f} I_{r(k)}\right)^{-1} I_{r(k)} s\left(\widetilde{p}^{f}\right)$.
Remark that out of equilibrium retails prices can be obtained from observed equilibrium retail prices, retail margins at equilibrium and out of equilibrium retail margins using : $\widetilde{p}_{k}^{f r(k)}=\widetilde{\gamma}_{k}^{f r(k)}-$ $\left(p_{k}-w_{k}-c_{k}\right)+p_{k}$ where $\widetilde{\gamma}_{k}^{f r(k)}=\widetilde{p}_{k}^{f r(k)}-w_{k}-c_{k}$. Moreover, $\tilde{P}_{w}^{f}$ can be deduced from the
differentiation of the retailer's first order conditions with respect to wholesale prices. These first order conditions are for all $r=1, . ., R$ and $j \in S_{r}$ :

$$
s_{j}\left(\widetilde{p}^{f r}\right)+\sum_{k \in S_{r} \backslash F_{f r}}\left(\widetilde{p}_{k}^{f r}-w_{k}-c_{k}\right) \frac{\partial s_{k}\left(\tilde{p}^{f r}\right)}{\partial \tilde{p}_{j}^{f r}}=0, \forall j \in S_{r}
$$

which gives for $r=1, . ., R, j \in S_{r}$ and $k=1, . ., J^{\prime}$

$$
\begin{align*}
0= & \sum_{l \in\{1, . ., J\} \backslash F_{f r}} \frac{\partial s_{j}\left(\widetilde{p}^{f r(j)}\right)}{\partial \widetilde{p}_{l}^{f r(j)}} \frac{\partial \widetilde{p}_{l}^{f r(j)}}{\partial w_{k}}-1_{\left\{k \in S_{r}\right\}} \frac{\partial s_{k}\left(\widetilde{p}^{f r(j)}\right)}{\partial \widetilde{p}_{j}^{f r(j)}}+\sum_{l \in S_{r}} \frac{\partial s_{l}\left(\widetilde{p}^{f r(j)}\right)}{\partial \widetilde{p}_{j}^{f r(j)}} \frac{\partial \widetilde{p}_{l}^{f r(j)}}{\partial w_{k}} \\
& +\sum_{l \in S_{r} \backslash F_{f r}}\left[\left(\widetilde{p}_{l}^{f r}-w_{l}-c_{l}\right) \sum_{s \in\{1, \ldots, J\} \backslash F_{f r}} \frac{\partial^{2} s_{l}\left(\widetilde{p}_{p}^{f r(j)}\right)}{\partial \widetilde{p}_{j}^{f r(j)} \partial \widetilde{p}_{s}^{f r(j)}} \frac{\partial \widetilde{p}_{s}^{f r(j)}}{\partial w_{k}}\right] \tag{23}
\end{align*}
$$

Defining $S_{\tilde{p} f}^{p_{j}}$ the $(J \times J)$ matrix of the second derivatives of the market shares with respect to retail prices whose element $(s, l)$ is $\frac{\partial^{2} s_{l}\left(\widetilde{p}^{f r(j)}\right)}{\partial \tilde{p}_{j}^{f(j)} \partial \widetilde{p}_{s}^{f r(j)}}$, i.e.

$$
S_{\tilde{p}_{j}^{f}}^{p_{j}} \equiv\left(\begin{array}{ccc}
\frac{\partial^{2} s_{1}\left(\widetilde{p}^{f r(j)}\right)}{\partial \widetilde{p}_{j}^{f r(j)} \partial \widetilde{p}_{1}^{f r(j)}} & \cdots & \frac{\partial^{2} s_{J}\left(\widetilde{p}^{f r(j)}\right)}{\partial \widetilde{p}_{j}^{f r(j)} \partial \widetilde{p}_{1}^{f r(j)}} \\
\vdots & & \vdots \\
\frac{\partial^{2} s_{1}\left(\widetilde{p}^{f r(j)}\right)}{\partial \widetilde{p}_{j}^{f r(j)} \partial \widetilde{p}_{J}^{f r(j)}} & \cdots & \frac{\partial^{2} s{ }_{J}\left(\widetilde{p}^{f r(j)}\right)}{\partial \widetilde{p}_{j}^{f r(j)} \partial \widetilde{p}_{J}^{f r(j)}}
\end{array}\right)
$$

we can write equation (23) in matrix form to obtain for $r=1, . ., R$

$$
\tilde{P}_{w}^{f}\left[S_{\widetilde{p}}^{f}+I_{r} S_{\widetilde{p}}^{f \prime}+\left(S_{\tilde{p}^{f}}^{p_{1}} I_{r} \widetilde{\gamma}^{f r}|\ldots| S_{\tilde{p}^{f}}^{p_{J}} I_{r} \widetilde{\gamma}^{f r}\right)\right] I_{r}-I_{r} S_{\widetilde{p}}^{f}\left(I_{r}-\widetilde{I}_{r}\right)=0
$$

where $\widetilde{\gamma}^{f r}=\widetilde{p}^{f r}-w-c$.
Denoting the $M_{f r}$ the matrix $\left[S_{\widetilde{p}}^{f}+I_{r} S_{\widetilde{p}}^{f f}+\left(S_{\tilde{p}_{f}}^{p_{1}} I_{r} \widetilde{\gamma}^{f r}|\ldots| S_{\tilde{p}^{f}}^{p_{J}} I_{r} \widetilde{\gamma}^{f r}\right)\right]$ we can solve this system of equation and get the following expression for $\tilde{P}_{w}^{f}$

$$
\tilde{P}_{w}^{f}=-\left(\sum_{r=1}^{R} I_{r} M_{f r}^{\prime} I_{r} S_{\widetilde{p}}^{f}\left(I_{r}-\widetilde{I}_{r}\right)\right)\left(\sum_{r=1}^{R} I_{r} M_{f r}^{\prime} M_{f r} I_{r}\right)^{-1}
$$

Equation (22) shows that one can express the manufacturer's price-cost margins vector as depending on the demand function and the structure of the industry by replacing the expression for $\tilde{P}_{w}^{f}$.

When retailers have no endogenous market power :

If retailers have no market power, we can suppress the constraints (11) and take only into account the constraints (10). Then, as shown in appendix 7.2 , manufacturers only set wholesale prices in the following maximization program

$$
\max _{\left\{w_{k}\right\} \in F_{f}} \sum_{k \in F_{f}}\left(p_{k}-\mu_{k}-c_{k}\right) s_{k}(p)+\sum_{k \notin F_{f}}\left(p_{k}-w_{k}-c_{k}\right) s_{k}(p)
$$

The first order conditions are : for all $i \in F_{f}$,

$$
\sum_{k} \frac{\partial p_{k}}{\partial w_{i}} s_{k}(p)+\sum_{k \in F_{f}}\left[\left(p_{k}-\mu_{k}-c_{k}\right) \sum_{j} \frac{\partial s_{k}}{\partial p_{j}} \frac{\partial p_{j}}{\partial w_{i}}\right]+\sum_{k \notin F_{f}}\left[\left(p_{k}-w_{k}-c_{k}\right) \sum_{j} \frac{\partial s_{k}}{\partial p_{j}} \frac{\partial p_{j}}{\partial w_{i}}\right]=0
$$

which gives in matrix notation

$$
I_{f} P_{w} s(p)+I_{f} P_{w} S_{p} I_{f}(p-\mu-c)+I_{f} P_{w} S_{p}\left(I-I_{f}\right)(p-w-c)=0
$$

This implies that the total price cost margin is such that for all $f=1, . ., F$ :

$$
\begin{equation*}
\gamma_{f}+\Gamma_{f}=\left(I_{f} P_{w} S_{p} I_{f}\right)^{-1}\left[-I_{f} P_{w} s(p)-I_{f} P_{w} S_{p}\left(I-I_{f}\right)(p-w-c)\right] \tag{24}
\end{equation*}
$$

Using (2) to replace $(p-w-c)$ and (7) for $P_{w}$, this allows us to estimate the price-cost margins with demand parameters. Remark again that the formula (2) provides directly the total price-cost margin obtained by each retailer on its private label.

## 4 Differentiated Products Demand

### 4.1 The Random Utility Demand Model

We now describe our demand model for differentiated products. We use a standard random utility model. Denoting $V_{i j t}$ the utility for consumer $i$ of buying good $j$ at period $t$, we assume that it can be represented by

$$
\begin{aligned}
V_{i j t} & =\theta_{j t}+u_{j t}+\varepsilon_{i j t} \\
& =\delta_{j}+\gamma_{t}-\alpha p_{j t}+u_{j t}+\varepsilon_{i j t} \text { for } j=1, ., J
\end{aligned}
$$

where $\theta_{j t}$ is the mean utility of good $j$ at period $t, u_{j t}$ a product-time specific unobserved utility term and $\varepsilon_{i j t}$ a (mean zero) individual-product-period-specific utility term representing the deviation of individual's preferences from the mean $\theta_{j t}$.

Moreover, we assume that $\theta_{j t}$ is the sum of a mean utility $\delta_{j}$ of product $j$ common to all consumers, a mean utility $\gamma_{t}$ common to all consumers and products at period $t$ (due to unobserved preference shocks to period $t$ ) and an income disutility $\alpha p_{j t}$ where $p_{j t}$ is the price of product $j$ at period $t$. Consumers may decide not to purchase any of the products. In this case they choose an outside good for which the mean part of the indirect utility is normalized to 0 , so that $V_{i 0 t}=\varepsilon_{i 0 t}$. Remark that the specification used for $\theta_{j t}$ is such that one could also consider that the mean utility of the
outside good depends also on its time varying price $p_{0 t}$ without changing the identification of the other demand parameters. Actually, adding $-\alpha p_{0 t}$ to the outside good mean utility is equivalent to adding $\alpha p_{0 t}$ to the mean utility of all other goods, which would amount to replace $\gamma_{t}$ by $\gamma_{t}+\alpha p_{0 t}$.

In the bottled water market in France, it seems that customers make a clear difference between two groups of bottled water : mineral water and spring water, such that it makes sense to allow customers to have correlated preferences over such groups ${ }^{6}$. Our demand model incorporates this observation. Indeed, we model the distribution of the individual-specific utility term $\varepsilon_{i j t}$ according to the assumptions of a Generalized Extreme Value (GEV) model (McFadden, 1978) ${ }^{7}$. We assume that the bottled water market can be partitioned into $G$ different groups ( $G=2$ ), each sub-group $g$ containing $J_{g}$ products $\left(\sum_{g=1}^{G} J_{g}=J\right)$. With an abuse of notation, we will also denote $J_{g}$ the set of products belonging to the sub-group $g$. Since products belonging to the same subgroup share a common set of unobserved features, consumers may have correlated preferences over these features. A GEV model allows a general pattern of dependence among the unobserved attributes and yields tractable closed form solutions for the choice probabilities. Assuming that consumers choose one unit of the good that maximizes utility, the distributional assumptions made ${ }^{8}$ yield the following choice probabilities or market shares for each product $j$, as a function of the price vector $p_{t}=\left(p_{1 t}, p_{2 t}, \ldots, p_{J t}\right)$

$$
s_{j t}\left(p_{t}\right)=P\left(V_{i j t}=\max _{l=0,1,,, J}\left(V_{i l t}\right)\right)=s_{j t / g}\left(p_{t}\right) s_{g t}\left(p_{t}\right)
$$

where $s_{g t}\left(p_{t}\right)$ and $s_{j t / g}\left(p_{t}\right)$ denote respectively the probability choice of group $g$ and the conditional probability of choosing good $j$ conditionally on purchasing a good in group $g$. The expressions of

[^4]$$
G(y)=\sum_{g=1}^{G}\left[\sum_{j \in J_{g}} y_{j}^{\frac{1}{1-\sigma_{g}}}\right]^{1-\sigma_{g}} .
$$

The parameter $\sigma_{g}$ associated measures the degree of similarity of the unobserved attributes in subgroup $g$.
these probabilities are given by

$$
\begin{aligned}
s_{j t / g}\left(p_{t}\right) & =\frac{\exp \frac{\theta_{j t}+u_{j t}}{1-\sigma_{g}}}{\sum_{j \in J_{g}} \exp \frac{\theta_{j t}+u_{j t}}{1-\sigma_{g}}} \\
s_{g t}\left(p_{t}\right) & =\frac{\left(\sum_{j \in J_{g}} \exp \frac{\theta_{j t}+u_{j t}}{1-\sigma_{g}}\right)^{1-\sigma_{g}}}{\sum_{g=0}^{G}\left(\sum_{j \in J_{g}} \exp \frac{\theta_{j t}+u_{j t}}{1-\sigma_{g}}\right)^{1-\sigma_{g}}}
\end{aligned}
$$

The conditions required for this model to be consistent with random utility maximization (McFadden's, 1978) are that each similarity index $\sigma_{g}$ belongs to the unit interval $[0,1]$. When $\sigma_{g}$ goes to 1, preferences for products of the same subgroup become perfectly correlated meaning that these products are perceived as perfect substitutes. When $\sigma_{g}$ goes to 0 , preferences for all products become uncorrelated, and the model reduces to a simple multinomial logit model. At the aggregate demand level, the parameter $\sigma_{g}$ allows to assess to which extent competition is localized between products from the same subgroup. This specification is more flexible than a simple multinomial logit specification (since it includes it as a special case). Actually, in the special case where $\sigma_{g}=0$ for $g=1, \ldots, G$, we obtain a simple multinomial logit model which amounts to assume that $\varepsilon_{i j t}$ is i.i.d. with a type I extreme value distribution. Then we have

$$
s_{j t}\left(p_{t}\right)=\frac{\exp \left[\theta_{j t}+u_{j t}\right]}{1+\sum_{j=1, ., J} \exp \left[\theta_{j t}+u_{j t}\right]}
$$

The nested logit model can be interpreted as a special case of the random coefficients logit models estimated by Berry, Levinsohn and Pakes (1995), Nevo (2001), Petrin (2002) and others. McFadden and Train (2000) show that any random utility model can be arbitrarily approximated by a random coefficient logit model. The nested logit model introduces restrictions on the underlying model but they are testable and this model has the advantage to be much more tractable (Berry, 1994, and Berry and Pakes, 2001).

### 4.2 Identification and Estimation of the Demand Model

Our method relies on two structural estimations, first, on the demand model and then on the cost equation. In appendix 7.4, we argue that estimating the model parameters in a single step thanks to the overall price equation would lead to make stronger assumptions.

Following Berry (1994) and Verboven (1996), the random utility model introduced in the pre-
vious section leads to the following equations on aggregate market shares of good $j$ at time $t$ :

$$
\begin{align*}
\ln s_{j t}-\ln s_{0 t} & =\theta_{j t}+\sigma_{g} \ln s_{j t \mid g}+u_{j t} \\
& =\delta_{j}+\gamma_{t}-\alpha p_{j t}+\sigma_{g} \ln s_{j t \mid g}+u_{j t} \tag{25}
\end{align*}
$$

where $s_{j t \mid g}$ is the relative market share of product $j$ at period $t$ in its group $g$ and $s_{0 t}$ is the market share of the outside good at time $t$. In the particular case of the simple multinomial logit model, this equation becomes

$$
\begin{equation*}
\ln s_{j t}-\ln s_{0 t}=\delta_{j}+\gamma_{t}-\alpha p_{j t}+u_{j t} \tag{26}
\end{equation*}
$$

Remark that the full set of time fixed effects $\gamma_{t}$ captures preferences for bottled water relative to the outside good, and can thus be thought of accounting for macro-economic fluctuations (like the weather) that affect the decision to buy bottled water ${ }^{9}$ but also as accounting for the outside good price variation across periods.

The error term $u_{j t}$ captures the remaining unobserved product valuations varying across products and time, e.g. due to unobserved variations in advertising.

The usual problem of endogeneity of price $p_{j t}$ and relative market shares $s_{j t \mid g}$ has to be handled correctly in order to identify and estimate the parameters of these models. Our identification strategy then relies on the use of instrumental variables. Actually, thanks to the collection of data on wages, oil, diesel, packaging material and plastic prices over the period of interest, we construct instruments for prices $p_{j t}$ that are interactions between characteristics of bottled water and these prices (the vector of these instruments is denoted $z_{j t}$ ). The identification then relies on the fact that these input prices affect the product prices because they are correlated with input costs but are not correlated with the idiosyncratic unobserved shocks to preferences $u_{j t}$. For the simple logit model, this set of instrumental variables is sufficient, but for the nested logit model, one has also to take into account the endogeneity of the relative (within group) market shares. For these relative market shares, our strategy relies on the fact that the contemporaneous correlation between $\ln s_{j t \mid g}$ and unobserved shocks $u_{j t}$, which is the source of the endogeneity problem, can be controlled for with some suitable projection of the relative market shares on the hyperplane generated by some observed lagged variables. In order to take into account this

[^5]endogeneity problem, we denote $Z_{j t}=\left(1_{j=1}, . ., 1_{j=J}, \varsigma_{j t-1}, z_{j t}\right)$ the vector of variables on which we project the right hand side endogenous variables (including dummy variables for products), where $\varsigma_{j t-1}$ results form the projection of the lagged variable $\ln s_{j t-1 \mid g}$ on the hyperplane orthogonal to the space spanned by a set of product fixed effects and the variable $\ln s_{j t-2 \mid g} . \varsigma_{j t-1}$ is thus the residual of the regression
$$
\ln s_{j t-1 \mid g}=\pi_{j}+\beta \ln s_{j t-2 \mid g}+\varsigma_{j t-1}
$$

Then, the identification of the coefficients of (25) relies on the following orthogonality condition

$$
E\left(Z_{j t} u_{j t}\right)=0
$$

The identification and estimation of these demand models then permits to evaluate own and cross price elasticities in this differentiated product demand model.

### 4.3 Identification and Tests Across Supply Models

Provided the demand function is identified, let's consider the problem of identification and tests of the supply models with a known demand. The different supply models of section 3 give different restrictions on the supply side. Depending on the model, the implied restrictions do not lead to the same degree of identification or underidentification of price cost margins.

### 4.3.1 Identification within a class of model

Linear pricing models:
In the case of linear pricing between manufacturers and retailers, both manufacturer level and retailer level price-cost margins are identified with (2) and (4).

Non linear pricing models :

In the case of non linear pricing contracts between manufacturers and retailers and in particular of two part tariffs contracts, multiple equilibria may prevent the full identification of price cost margins. Identifying the $J \times T$ retailer level and $J \times T$ manufacturer level price-cost margins implies that $2 \times J \times T$ parameters have to be identified while our structural model generally gives a system of $J T$ equations for the vector of total margins $(\Gamma+\gamma)$ as a function of the vector of wholesale margins $(\Gamma)$ of the form

$$
(\Gamma+\gamma)=H(\Gamma)
$$

where $H($.$) is a known function depending of the class of supply model considered, of the de-$ mand shape and the structure of the industry in terms of products ownership at the retailing and manufacturing levels.

The degree of underidentification is thus at most equal to the dimension of the vector of wholesale prices (or wholesale margins $\Gamma$ ), that is $J T$.

Thus, fixing the vector of wholesale margins is sufficient to get identification of total margins. This is achieved for example when one considers the case of marginal cost pricing of manufacturers that is zero wholesale margins at the manufacturer level $(\Gamma=0)$.

Another identification method consists in making an additional assumption. As products are differentiated by brand and retailer, denoting $C_{j t}=\mu_{j t}+c_{j t}$ the total marginal cost of product $j$ at period $t$ which is the sum of the marginal cost of production and of distribution, the following assumption can be done to obtain identification of margins.

Identification assumption for general two-part tariffs models : The marginal cost of production of a product $j$ depends only on the brand denoted $b(j)$ and the marginal cost of distribution depends only on the retailer identity denoted $r(j)$, that is

$$
\begin{equation*}
C_{j t}=\mu_{j t}+c_{j t}=\mu_{b(j) t}+c_{r(j) t} \text { for all } j=1, . ., J \text { and } t=1, . ., T \tag{27}
\end{equation*}
$$

where $\mu_{b t}$ is the marginal cost of production of brand $b$ and $c_{r t}$ the marginal cost of production of retailer $r$.

This assumption implies restrictions between the $J \times T$ unknown marginal costs $C_{j t}$ and the $(B+R) \times T$ unknown marginal costs $\mu_{b t}, c_{r t}$ (where $B+R<J=B \times R$ and $B$ is the number of brands and $R$ the number of retailers). As retail prices are known, and $H($.$) is known, a one to$ one correspondence between the vector of unknown $J T$ parameters $\Gamma_{j t}$ and the vector of unknown $J T$ marginal costs $C_{j t}$ exist because

$$
C_{j t}=p_{j t}-H(\Gamma) \text { for all } j=1, . ., J \text { and } t=1, . ., T
$$

Thus, the previous identification assumption implies that

$$
p_{j t}-H(\Gamma)=\mu_{b(j) t}+c_{r(j) t} \text { for all } j=1, . ., J \text { and } t=1, . ., T
$$

which reduces the degree of underidentification since it adds $J \times T$ restrictions and only $(B+$
$R) \times T$ additional unknown parameters. The true degree of underidentification will depend on the properties of the non linear function $H($.$) . The identification of margins will thus depend on the$ set of solutions of the following problem.

For product $j$ at time $t$ under model $h$, we denote $\gamma_{j t}^{h}$ the retailer price cost margin, $\Gamma_{j t}^{h}$ the manufacturer price cost margin and $C_{j t}^{h}$ the sum of the marginal cost of production and distribution $\left(C_{j t}^{h}=\mu_{j t}^{h}+c_{j t}^{h}\right)$. Given the unknown vector of wholesale margin $\Gamma$, the marginal cost of production is :

$$
C_{j t}^{h}(\Gamma)=p_{j t}-\left(\Gamma_{j t}^{h}+\gamma_{j t}^{h}\right)(\Gamma)
$$

where the function $\left(\Gamma_{j t}^{h}+\gamma_{j t}^{h}\right)(\Gamma)=H_{h}(\Gamma)$ is known for a given supply model $h$. Denoting by $\left\{\epsilon_{j t}^{h}(\Gamma)\right\}_{j t}$ the projection vector of $\left\{C_{j t}^{h}(\Gamma)\right\}_{j t}$ on the orthogonal space to the space spanned by the $\left\{\left\{1_{b(j)=b}\right\}_{b=1, . ., B},\left\{1_{r(j)=r}\right\}_{r=1, . ., R}\right\}_{j t}$, the set of vectors of wholesale margins $S^{h}$ solutions to the identification restrictions (27) is

$$
S^{h}=\left\{\Gamma \in \mathbb{R}^{J T} \mid \epsilon_{j t}^{h}(\Gamma)=0, \forall j, \forall t\right\}
$$

where $\epsilon_{j t}^{h}(\Gamma)=C_{j t}^{h}(\Gamma)-E\left(C_{j t}^{h}(\Gamma) \mid\left\{1_{b(j)=b}\right\}_{b=1, \ldots, B},\left\{1_{r(j)=r}\right\}_{r=1, \ldots, R}\right)$.
Thus, the degree of underidentification of the supply model depends on $\operatorname{card}\left(S^{h}\right)$. The vector of margins is underidentified if $\operatorname{card}\left(S^{h}\right)>1$, just identified if $\operatorname{card}\left(S^{h}\right)=1$, and overidentified if $S^{h}=\varnothing$.

In practice, we will see that the demand shape is such that we always get overidentification. This result will be obtained by looking at the set $\underset{\Gamma}{\arg \min } \sum_{j=1, \ldots, J ; t=1, \ldots, T} \epsilon_{j t}^{h}(\Gamma)^{2}$ where $\epsilon_{j t}^{h}(\Gamma)$ is obtained from the linear regression

$$
C_{j t}^{h}(\Gamma)=\sum_{b=1}^{B} \mu_{b t}^{h}(\Gamma) 1_{b(j)=b}+\sum_{r=1}^{R} c_{r t}^{h}(\Gamma) 1_{r(j)=r}+\epsilon_{j t}^{h}(\Gamma)
$$

Thus, we will consider the solution

$$
\Gamma^{h *}=\arg \min _{\Gamma} \sum_{j=1, \ldots, J}^{t=1, . ., T} \epsilon_{j t}^{h}(\Gamma)^{2}
$$

as the equilibrium solution.
Then, for any given model $h$ we obtain total price-cost margins $\left(\Gamma_{j t}^{h}+\gamma_{j t}^{h}\right)\left(\Gamma^{h *}\right)$, manufacturer level margins $\Gamma_{j t}^{h *}$ and thus retail level margins $\gamma_{j t}^{h}\left(\Gamma^{h *}\right)=\left(\Gamma_{j t}^{h}+\gamma_{j t}^{h}\right)\left(\Gamma^{h *}\right)-\Gamma_{j t}^{h *}$.

### 4.3.2 Testing between non nested models

We now present how to test between the alternative models once we have estimated the demand model and obtained the different price-cost margins estimates according to their expressions obtained in section 3 .

Denoting by $h$ and $h^{\prime}$ two different models considered, we can obtain estimates of the total marginal costs under both models : $C_{j t}^{h}$ and $C_{j t}^{h^{\prime}}$. Then one can test between these two models using non nested tests using alternatively one of the two following assumptions :

Cost Restriction 1 : The total marginal cost of product $j t$ depends additively on a marginal cost of production of the brand $\mu_{b(j) t}$, on a marginal cost of distribution $c_{r(j) t}$, and a mean zero iid idiosyncratic shock $\epsilon_{j t}^{h}$, that is

$$
\begin{equation*}
C_{j t}^{h}=\mu_{b(j) t}+c_{r(j) t}+\epsilon_{j t}^{h} \text { for all } j=1, . ., J \text { and } t=1, . ., T \tag{28}
\end{equation*}
$$

Cost Restriction 2 : There exist some observable exogenous shocks $W_{j t}$, some unknown timeinvariant product-specific parameters $\omega_{j}^{h}$, and some $i i d$ unobservable random shock $\eta_{j t}^{h}$ such that $\operatorname{corr}\left(\ln \eta_{j t}^{h}, W_{j t}\right)=\operatorname{corr}\left(\ln \eta_{j t}^{h}, \omega_{j}^{h}\right)=0$ and the total marginal cost of production writes

$$
\begin{equation*}
C_{j t}^{h}=\left[\exp \left(\omega_{j}^{h}+W_{j t}^{\prime} \lambda_{h}\right)\right] \eta_{j t}^{h} \text { for all } j=1, . ., J \text { and } t=1, . ., T \tag{29}
\end{equation*}
$$

Using the relationship between retail prices, total marginal cost and estimated margins under model $h, p_{j t}=\Gamma_{j t}^{h}+\gamma_{j t}^{h}+C_{j t}^{h}$, we obtain non nested price equations for models $h$ and $h^{\prime}$.

Under the cost restriction 1, we will then test between the two non nested equations

$$
\left\{\begin{array}{l}
p_{j t}=\Gamma_{j t}^{h}+\gamma_{j t}^{h}+\sum_{b=1}^{B} \mu_{b t}^{h} 1_{b(j)=b}+\sum_{r=1}^{R} c_{r t}^{h} 1_{r(j)=r}+\epsilon_{j t}^{h} \\
p_{j t}=\Gamma_{j t}^{h^{\prime}}+\gamma_{j t}^{h^{\prime}}+\sum_{b=1}^{B} \mu_{b t}^{h^{\prime}} 1_{b(j)=b}+\sum_{r=1}^{R} c_{r t}^{h^{\prime}} 1_{r(j)=r}+\epsilon_{j t}^{h^{\prime}}
\end{array}\right.
$$

that can be estimated using ordinary least squares.
Under the cost restriction 2, we will test between the two non nested equations

$$
\left\{\begin{array}{c}
p_{j t}=\Gamma_{j t}^{h}+\gamma_{j t}^{h}+\left[\exp \left(\omega_{j}^{h}+W_{j t}^{\prime} \lambda_{h}\right)\right] \eta_{j t}^{h} \\
p_{j t}=\Gamma_{j t}^{h^{\prime}}+\gamma_{j t}^{h^{\prime}}+\left[\exp \left(\omega_{j}^{h^{\prime}}+W_{j t}^{\prime} \lambda_{h^{\prime}}\right)\right] \eta_{j t}^{h^{\prime}}
\end{array}\right.
$$

In this case, taking logarithms, one can identify and estimate consistently $\omega_{j}^{h}, \lambda_{g}$, and $\eta_{j t}^{h}$ because

$$
\begin{equation*}
\ln C_{j t}^{h}=\omega_{j}^{h}+W_{j t}^{\prime} \lambda_{h}+\ln \eta_{j t}^{h} \tag{30}
\end{equation*}
$$

and $\operatorname{corr}\left(\ln \eta_{j t}^{h}, W_{j t}\right)=\operatorname{corr}\left(\ln \eta_{j t}^{h}, \omega_{j}^{h}\right)=0$.

Then, we can use in both cases non nested tests (Vuong, 1989, and Rivers and Vuong, 2002) to infer which model is statistically the best. The tests we use consist in testing models one against another. The test of Vuong (1989) applies in the context of maximum likelihood estimation and thus would apply in our case if one assumes normality of $\epsilon_{j t}^{h}$ or $\log$-normality of $\eta_{j t}^{h}$. Rivers and Vuong (2002) generalized this kind of test to a broad class of estimation methods including non linear least squares. Moreover, the Vuong (1989) or the Rivers and Vuong (2002) approaches do not require that either competing model be correctly specified under the tested null hypothesis. Indeed, other approaches such as Cox's tests (see, among others, Smith, 1992) require such an assumption, i.e. that one of the competing model accurately describes the data. This assumption cannot be sustained when dealing with a real data set like ours.

Defining the lack-of-fit criteria in both cases as :

$$
\begin{aligned}
\min _{\mu_{b t}^{h}, c_{r t}^{h}} Q_{n}^{h}\left(\mu_{b t}^{h}, c_{r t}^{h}\right) & =\min _{\mu_{b t}^{h}, c_{r t}^{h}} \frac{1}{n} \sum_{j, t}\left(\epsilon_{j t}^{h}\right)^{2} \\
& =\min _{\mu_{b t}^{h}, c_{r t}^{h}} \frac{1}{n} \sum_{j, t}\left[p_{j t}-\left(\Gamma_{j t}^{h}+\gamma_{j t}^{h}+\sum_{b=1}^{B} \mu_{b t}^{h} 1_{b(j)=b}+\sum_{r=1}^{R} c_{r t}^{h} 1_{r(j)=r}\right)\right]^{2}
\end{aligned}
$$

or

$$
\min _{\lambda_{h}, \omega_{j}^{h}} Q_{n}^{h}\left(\lambda_{h}, \omega_{j}^{h}\right)=\min _{\lambda_{h}, \omega_{j}^{h}} \frac{1}{n} \sum_{j, t}\left(\ln \eta_{j t}^{h}\right)^{2}=\min _{\lambda_{h}, \omega_{j}^{h}} \frac{1}{n} \sum_{j, t}\left[\ln \left(p_{j t}-\Gamma_{j t}^{h}-\gamma_{j t}^{h}\right)-\omega_{j}^{h}-W_{j t}^{\prime} \lambda_{h}\right]^{2}
$$

we can use the following statistical tests detailed with notations of the second case only.
Taking any two competing models $h$ and $h^{\prime}$, the null hypothesis is that the two non nested models are asymptotically equivalent when

$$
H_{0}: \lim _{n \rightarrow \infty}\left\{\bar{Q}_{n}^{h}\left(\bar{\lambda}_{h}, \bar{\omega}_{j}^{h}\right)-\bar{Q}_{n}^{h^{\prime}}\left(\bar{\lambda}_{h^{\prime}}, \bar{\omega}_{j}^{h^{\prime}}\right)\right\}=0
$$

where $\bar{Q}_{n}^{h}\left(\bar{\lambda}_{h}, \bar{\omega}_{j}^{h}\right)\left(\right.$ resp. $\left.\bar{Q}_{n}^{h^{\prime}}\left(\bar{\lambda}_{h^{\prime}}, \bar{\omega}_{j}^{h^{\prime}}\right)\right)$ is the expectation of a lack-of-fit criterion $Q_{n}^{h}\left(\lambda_{h}, \omega_{j}^{h}\right)$ evaluated for model $h$ (resp. $h^{\prime}$ ) at the pseudo true values of the parameters of this model, denoted by $\bar{\lambda}_{h}, \bar{\omega}_{j}^{h}$ (resp. $\bar{\lambda}_{h^{\prime}}, \bar{\omega}_{j}^{h^{\prime}}$ ). The first alternative hypothesis is that $h$ is asymptotically better than $h^{\prime}$ when

$$
H_{1}: \lim _{n \rightarrow \infty}\left\{\bar{Q}_{n}^{h}\left(\bar{\lambda}_{h}, \bar{\omega}_{j}^{h}\right)-\bar{Q}_{n}^{h^{\prime}}\left(\bar{\lambda}_{h^{\prime}}, \bar{\omega}_{j}^{h^{\prime}}\right)\right\}<0
$$

Similarly, the second alternative hypothesis is that $h^{\prime}$ is asymptotically better than $h$ when

$$
H_{2}: \lim _{n \rightarrow \infty}\left\{\bar{Q}_{n}^{h}\left(\bar{\lambda}_{h}, \bar{\omega}_{j}^{h}\right)-\bar{Q}_{n}^{h^{\prime}}\left(\bar{\lambda}_{h^{\prime}}, \bar{\omega}_{j}^{h^{\prime}}\right)\right\}>0
$$

The test statistic $T_{n}$ captures the statistical variation that characterizes the sample values of the lack-of-fit criterion and is then defined as a suitably normalized difference of the sample lack-of-fit criteria, i.e.

$$
T_{n}=\frac{\sqrt{n}}{\hat{\sigma}_{n}^{h h^{\prime}}}\left\{Q_{n}^{h}\left(\widehat{\lambda}_{h}, \widehat{\omega}_{j}^{h}\right)-Q_{n}^{h^{\prime}}\left(\widehat{\lambda}_{h^{\prime}}, \widehat{\omega}_{j}^{h^{\prime}}\right)\right\}
$$

where $Q_{n}^{h}\left(\widehat{\lambda}_{h}, \widehat{\omega}_{j}^{h}\right)\left(\right.$ resp. $\left.Q_{n}^{h^{\prime}}\left(\widehat{\lambda}_{h^{\prime}}, \widehat{\omega}_{j}^{h^{\prime}}\right)\right)$ is the sample lack-of-fit criterion evaluated for model $h$ (resp. $h^{\prime}$ ) at the estimated values of the parameters of this model, denoted by $\widehat{\lambda}_{h}, \widehat{\omega}_{j}^{h}\left(\right.$ resp. $\widehat{\lambda}_{h^{\prime}}, \widehat{\omega}_{j}^{h^{\prime}}$ ). $\hat{\sigma}_{n}^{h h^{\prime}}$ denotes the estimated value of the variance of the difference in lack-of-fit. Since our models are strictly non nested, Rivers and Vuong showed that the asymptotic distribution of the $T_{n}$ statistic is standard normal. The selection procedure involves comparing the sample value of $T_{n}$ with critical values of the standard normal distribution ${ }^{10}$. In the empirical section, we will present evidence based on these different statistical tests.

## 5 Econometric Estimation and Test Results

### 5.1 Data and Variables

Our data were collected by the company SECODIP (Société d'Étude de la Consommation, Distribution et Publicité) that conducts surveys about households' consumption in France. We have access to a representative survey for the years 1998, 1999, and 2000. These data contain information on a panel of nearly 11000 French households and on their purchases of mostly food products. This survey provides a description of the main characteristics of the goods and records over the whole year the quantity bought, the price, the date of purchase and the store where it is purchased. In particular, this survey contains information on all bottled water purchased by these French households during the three years of study. We consider purchases of the seven most important retailers which represent $70.7 \%$ of the total purchases of the sample. We take into account the most important brands, that is five national brands of mineral water, one national brand of spring water, one retailer private label brand of mineral water and one retailer private label spring water. The purchases of these eight brands represent $71.3 \%$ of the purchases of the seven retailers. The national brands are produced by three different manufacturers : Danone, Nestlé and Castel.

[^6]This survey presents the advantage of allowing to compute market shares that are representative of the national French market thanks to a weighting procedure of the available household panel. Then, the market shares are defined by a weighted sum of the purchases of each brand during each month of the three years considered divided by the total market size of the respective month. The market share of the outside good is defined as the difference between the total size of the market and the shares of the inside goods. We consider all other non-alcoholic refreshing drinks as the outside good. Therefore, the market size consists in all non-alcoholic refreshing drinks such as bottled water (including sparkling and flavored water), tea drinks, colas, tonics, fruit drinks, sodas lime. Our data thus allow to compute this market size across all months of the study. It is clearly varying across periods and shows that the market for non-alcoholic drinks is affected by seasons or for example the weather.

We consider eight brands sold in seven distributors, which gives more than 50 differentiated products in this national market. The number of products in our study thus varies between 51 and 54 during the 3 years considered. Considering the monthly market shares of all of these differentiated products, we get a total of 2041 observations in our sample. For each of these products, we compute an average price for each month. These prices are in euros per liter (even if until 2000, the money used was the French Franc). Table 1 presents some first descriptive statistics on some of the main variables used.

| Variable | Mean | Median | Std. dev. | Min. | Max |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Per Product Market share (all inside goods) | 0.005 | 0.003 | 0.006 | $4.10^{-6}$ | 0.048 |
| Per Product Market share : Mineral Water | 0.004 | 0.003 | 0.003 | $10^{-6}$ | 0.048 |
| Per Product Market share : Spring Water | 0.010 | 0.007 | 0.010 | $10^{-5}$ | 0.024 |
| Price in €/liter | 0.298 | 0.323 | 0.099 | 0.096 | 0.823 |
| Price in €/liter : Mineral Water | 0.346 | 0.343 | 0.060 | 0.128 | 0.823 |
| Price in €/liter : Spring Water | 0.169 | 0.157 | 0.059 | 0.096 | 0.276 |
| Mineral water dummy (0/1) | 0.73 | 1 | 0.44 | 0 | 1 |
| Market Share of the Outside Good | 0.71 | 0.71 | 0.04 | 0.59 | 0.78 |

Table 1 : Summary Statistics

We also use data from the French National Institute for Statistics and Economic Studies (INSEE) on the plastic price, on a wage salary index for France, on oil and diesel prices and on an index for packaging material cost. Over the time period considered (1998-2000), the wage salary index always raised while the plastic price index first declined during 1998 and the beginning of 1999 before raising again and reaching the 1998 level at the end of 2000 . Concerning the diesel
price index, it shows quite an important volatility with a first general decline during 1998 before a sharp increase until a new decline at the end of 2000 . Also, the packaging material cost index shows important variations with a sharp growth in 1998, a decline at the beginning of 1999 and again an important growth until the end of 2000 . Interactions of these prices with the dummies for the type of water (spring versus mineral) will serve as instrumental variables as they are supposed to affect the marginal cost of production and distribution of bottled water. Actually, it is likely that labor cost is not the same for the production of mineral or spring water but it is also known in this industry that the plastic quality used for mineral or spring water is usually not the same which is also likely to affect their bottling and packaging costs. Also, the relatively important variations of all these price indices during the period of study suggests a potentially good identification of our cost equations.

### 5.2 Demand Results

We estimate the demand model (25) which is the following

$$
\ln s_{j t}-\ln s_{0 t}=\delta_{j}+\gamma_{t}-\alpha p_{j t}+\sigma_{g} \ln s_{j t \mid g}+u_{j t}
$$

as well as the simple logit demand model (26) using two stage least squares in order to instrument the endogenous variables $p_{j t}$ and $\ln s_{j t \mid g}$. Results are in Table 2. $F$ tests of the first stage regressions show that our instrumental variables are well correlated with the endogenous variables. Moreover, the Sargan test of overidentification validates the exclusion of excluded instruments from the main equation. The price coefficient has the expected sign, the coefficients $\sigma_{g}$ actually belongs to the $[0,1]$ interval as required by the theory. Moreover, since one can reject that parameters $\sigma_{g}$ are zero, it is clear that the nested logit specification is preferred to a simple logit one for this market of bottled water.

| Variable | Nested Logit |
| :--- | :---: |
| Price $(\alpha)$ (Std. error) | $4.11(0.077)$ |
| Mineral water $\sigma_{g}$ (Std. error) | $0.68(0.025)$ |
| Spring water $\sigma_{g}$ (Std. error) | $0.59(0.018)$ |
| Coefficients $\delta_{j}, \gamma_{t}$ not shown |  |
| $F$ test that all $\delta_{j}=0(p$ value $)$ | $55.84(0.000)$ |
| Wald test that all $\gamma_{t}=0(p$ value $)$ | $64.50(0.0034)$ |
| Sargan Test of overidentification $(p$ value $)$ | $8.38(0.08)$ |

Table 2 : Estimation Results of Demand Models

In appendix 7.5 , we present the first stage regression results for the estimation of this demand model using two stage least squares.

Given the demand estimates, it is interesting to note that we find estimates of unobserved product specific mean utilities $\delta_{j}$. Using these parameters estimates, one can look at their correlation with observed product characteristics using ordinary least squares. This is done in Table 3 below.

| Fixed Effects $\delta_{j}$ | Nested Logit |
| :--- | :---: |
| Mineral Water (0/1) (Std. error) | $-0.89(0.08)$ |
| Minerality (Std. error) | $0.63(0.03)$ |
| Manufacturer 1 (Std. error) | $3.89(0.08)$ |
| Manufacturer 2 (Std. error) | $3.57(0.08)$ |
| Manufacturer 3 (Std. error) | $-3.00(0.06)$ |
| Constant (Std. error) | $-2.08(0.04)$ |
| $F$ test $(p$ value) | $3926.94(0.000)$ |

Table $\overline{\overline{3} \text { : Regression of fixed effects on the product characteristics }}$

Table 3 shows that the product specific constant mean utility $\delta_{j}$ is increasing with the minerality of water and that the identity of the manufacturer of the bottled water affects this mean utility. This is probably due to image, reputation and advertising of the manufacturing brands. Remark that if one does not control for the manufacturer identity this mean utility is larger for mineral water rather than spring water but it is not the case anymore when one introduces these manufacturer dummy variables.

Finally, once we obtained our structural demand estimates, we can compute price elasticities of demand for our differentiated products ${ }^{11}$. Table 4 presents the different average elasticities obtained for this nested logit demand model. All of them have the expected sign and the magnitude of ownprice elasticities are much larger than that of cross-price elasticities. Average own price elasticities for mineral water and spring water are almost proportional to average prices of these segments (nearly twice for mineral water than for spring water). As expected, the cross-price elasticities are larger within each segment of product than across segments.

[^7]| Elasticities $\left(\eta_{j k}\right)$ | Mean (Std. Error) |
| :--- | :---: |
| All bottle water |  |
| Own-price elasticity | $-19.95(6.60)$ |
| Cross-price elasticity within group | $0.44(0.34)$ |
| Cross-price elasticity across group | $0.04(0.03)$ |
| Mineral water |  |
| Own-price elasticity | $-23.16(3.85)$ |
| Cross-price elasticity within group | $0.41(0.28)$ |
| Cross-price elasticity across group | $0.04(0.03)$ |
| Spring water |  |
| Own-price elasticity | $-11.14(4.06)$ |
| Cross-price elasticity within group | $0.51(0.44)$ |
| Cross-price elasticity across group | $0.04(0.04)$ |
| Table 4:Summary of Elasticities Estimates |  |

Table 4 : Summary of Elasticities Estimates

These elasticities are quite large but it seems consistent with the fact that our model considers a very precise degree of differentiation. Actually, even for non sparkling spring and natural water, we end up with 56 products as we consider that the brand and the supermarket chain distributor are differentiation characteristics of a bottle of water. It is not surprising to find that these products are importantly substitutable.

However, if one looks at some group level elasticities, one finds much lower absolute values for these elasticities. The Table 5 shows these elasticities for the groups of mineral water or spring water or for different brands or firms (a firm produces several brands on this market). It appears that the total price elasticity of the group of mineral water goes down to - 7.40 instead of an average of -23.16 at the product level and that for spring water it goes down from -11.14 to -3.41 .

| Set of products <br> Group $g$ | Average elasticity <br> $\#$ <br> $\#\{k \in g\}$ <br> $\sum_{k \in g} \eta_{g k}$ |  |
| :---: | :---: | :---: |
| Mineral Water | -0.21 | Total elasticity <br> $\sum_{k \in g} \eta_{g k}$ |
| Spring Water | -0.27 | -7.40 |
| Mineral Water Brand 1 | -0.26 | -3.41 |
| Mineral Water Brand 2 | -0.15 | -1.74 |
| Mineral Water Brand 3 | -0.20 | -1.02 |
| Mineral Water Brand 4 | -0.27 | -1.27 |
| Mineral Water Brand 5 | -0.39 | -1.80 |
| Spring Water Brand 1 | -0.22 | -2.61 |
| Mineral Water Private Label | 0.07 | -1.40 |
| Spring Water Private Label | -0.28 | 0.16 |
| Firm $f$ | $\frac{1}{\#\{k \in f\}} \sum_{k \in f} \eta_{f k}$ | -1.85 |
| Danone | -0.99 | $\sum_{k \in f} \eta_{f k}$ |
| Nestlé | -1.64 | -13.11 |
| Castel | -0.22 | -32.37 |

Table 5: Own-Price Elasticities

### 5.3 Estimation of Price-Cost Margins and Non Nested Tests

Once one has estimated the demand parameters, we can use the formulas obtained in section 3 to compute the price cost margins at the retailer and manufacturer levels, for all products, under the various scenarios considered. We present several models that seem worth of consideration with some variants on manufacturers or retailers behavior. We test between a linear pricing model and several two-part tariffs contracts with or without endogenous market power.

Tables 6 then presents the averages ${ }^{12}$ of product level price cost margins estimates under the different models considered. It is worth noting that price cost margins are generally lower for mineral water than for spring water. As done by Nevo (2001), one could then compare price cost margins with accounting data to evaluate their empirical validity and also eventually test which model provides the most realistic result. However, the lack of data both on retailers or manufacturers margins prevents such analysis. Moreover accounting data only provide an upper bound for price-cost margins. We thus implement further our testing procedure introduced in 4.3. In Table 6, we first consider the case of linear pricing (model 1). In order to save space we do not present other scenarios of linear pricing with variants about the interaction between manufacturers and retailers like assuming collusion between manufacturers and/or retailers or assuming that retailers act as pass-through agents of marginal cost of production because all these models are finally strongly rejected (see Bonnet, Dubois and Simioni, 2004). We then consider several non linear contracting models with exogenous or endogenous market power. Models 2, 3, 4 and 5 correspond to the case where two part tariffs contracts with resale price maintenance are used. We first consider the general case (18) with unrestricted wholesale pricing where we estimate the equilibrium wholesale margins using the method described in 4.3.1. Although the full model is identified, in order to reduce the dimension of parameters to be estimated, we impose a restriction on wholesale margins such that for product $j$ at year $t$, the wholesale margin $\Gamma_{j t}$ depends only on the brand $b(j)$ and the retailer $r(j)$ as

$$
\Gamma_{j t}=\Gamma_{b(j)}+\Gamma_{r(j)}+\Gamma_{t}
$$

[^8]In the case of model 3, we impose no wholesale price discrimination preventing manufacturers to sell a product at different prices to different retailers which implies that the wholesale price of any product $j$ depends only on its brand $b(j)$ and not on the retailers identity $r(j)$. In Model 4 , we assume that wholesale prices are equal to the marginal cost of production. It corresponds to the case of equation (19). Model 5 is the case where the wholesale prices are such that the retailers' margins are zero. Model 6 is the case of two part tariffs contracts without resale price maintenance (24). Models $7,8,9,10$ correspond to the cases where retailers have some endogenous market power. Model 7 is the general case with resale price maintenance (15) and the following models 8 and 9 correspond to case with no wholesale price discrimination and marginal cost pricing of manufacturers. Model 10 is the case of no resale price maintenance. Finally, the case where the total profits of the full industry are maximized corresponds to model 11.

| Price-Cost Margins (\% of retail price $p$ ) | Mineral Water |  | Spring Water |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Mean | Std. | Mean | Std. |
| Linear Pricing (Double Marginalization) |  |  |  |  |
| Model 1 Retailers | 5.51 | 2.02 | 12.19 | 4.34 |
| Manufacturers | 7.07 | 2.52 | 28.92 | 12.21 |
| Total | 12.09 | 3.09 | 26.25 | 20.50 |
| Exogenous Retail Market Power |  |  |  |  |
| Two part Tariffs with RPM |  |  |  |  |
| Model 2 General | 53.05 | 43.70 | 30.91 | 40.11 |
| Model 3 No wholesale price discrimination ( $w_{b(j) t}$ ) | 76.77 | 75.85 | 52.69 | 52.49 |
| Model 4 Manufacturer Marginal cost pricing ( $w=\mu$ ) | 12.71 | 3.09 | 21.42 | 13.59 |
| Model 5 Zero retail margin ( $p=w+c$ ) | 6.88 | 1.81 | 15.48 | 8.33 |
| Two-part Tariffs without RPM |  |  |  |  |
| Model 6 Retailers | 5.07 | 2.44 | 11.18 | 5.36 |
| Manufacturers | 5.64 | 5.29 | 9.57 | 13.06 |
| Total | 11.12 | 5.82 | 21.76 | 16.93 |
| Endogenous Retail Market Power Two part Tariffs with RPM |  |  |  |  |
| Model 7 General | 27.48 | 18.77 | 35.88 | 26.14 |
| Model 8 No wholesale price discrimination ( $w_{b(j) t}$ ) | 23.05 | 14.92 | 33.93 | 25.31 |
| Model $9 \quad$ Manufacturer Marginal cost pricing ( $w=\mu$ ) | 7.02 | 2.16 | 15.60 | 8.36 |
| Two-part Tariffs without RPM |  |  |  |  |
| Model 10 Retailers | 5.51 | 2.02 | 12.19 | 4.34 |
| Manufacturers | 13.87 | 8.85 | 26.46 | 23.74 |
| Total | 19.36 | 9.70 | 38.65 | 27.63 |
| Monopole |  |  |  |  |
| Model 11 Total | 14.66 | 3.84 | 34.07 | 12.80 |

Table 6 : Estimation Results of Price-Cost Margins (averages by groups)

After estimating the different price-cost margins for the models considered, one can recover the marginal cost $C_{j t}^{h}$ and then estimate equations (28) and (29). The empirical results of the estimation of these cost equations are in appendix 7.6. They are useful mostly in order to test which model
fits best the data. We thus performed the non nested tests presented in 4.3. Table 7 presents the Rivers and Vuong tests using the cost restriction 1 and Table 11 in appendix 7.8 presents the tests when using the cost restriction 2. Both tests provide the same inference. The Vuong (1989) tests based on the maximum likelihood estimation of the cost equations under normality draw the same inference about the best model. The statistics of test ${ }^{13}$ show that the best model appears to be the model 5 , that is the case where two part tariffs contracts with resale price maintenance at zero retail margins are used. Also it appears that models with endogenous market power for the retailers are rejected.

Thus, our empirical evidence shows that, in the French bottled water market, manufacturers and retailers use two part tariffs contracts with resale price maintenance. Moreover, the market power of retailers is not affected endogenously by their outside opportunities such case is rejected by the data. It seems that the three main multiproduct manufacturers on this market are big enough for the retailers not being able to refuse offers of one of them. By bundling the two-part tariffs contracts, manufacturers manage to reduce the profitability of retailing only other firms brands.

| $T_{n}=\frac{\sqrt{n}}{\widehat{\sigma}_{n}}\left(Q_{n}^{2}\left(\hat{\Theta}_{n}^{2}\right)-Q_{n}^{1}\left(\hat{\Theta}_{n}^{1}\right)\right) \rightarrow N(0,1)$ |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\backslash$ | $\mathrm{H}_{2}$ |  |  |  |  |  |  |  |  |  |
| $H_{1}$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 1 | 7.67 | 16.42 | -4.83 | -8.05 | -6.68 | 2.52 | 7.20 | -7.42 | -5.73 | -5.57 |
| 2 |  | 2.34 | -29.33 | -28.28 | -27.88 | -9.09 | -7.36 | -27.80 | -26.94 | -18.76 |
| 3 |  |  | -16.60 | -18.12 | -17.53 | -5.65 | -5.40 | -17.94 | -16.80 | -15.21 |
| 4 |  |  |  | -15.59 | -10.58 | 14.48 | 10.81 | -13.84 | -4.57 | 18.61 |
| 5 |  |  |  |  | 9.47 | 18.11 | 13.74 | 5.96 | 11.32 | 18.00 |
| 6 |  |  |  |  |  | 16.55 | 12.35 | -5.53 | 12.99 | 1.07 |
| 7 |  |  |  |  |  |  | -0.42 | -17.70 | -14.96 | -19.01 |
| 8 |  |  |  |  |  |  |  | -12.96 | -11.26 | -13.64 |
| 9 |  |  |  |  |  |  |  |  | 8.56 | 17.45 |
| 10 |  |  |  |  |  |  |  |  |  | -9.49 |

Table 7 : Results of the Rivers and Vuong Test

In this case, Table 6 shows that the average price cost margins are of $6.88 \%$ for mineral water and $15.48 \%$ for spring water. These figures are lower than the rough accounting estimates that one can get from aggregate data (see section 2). As Nevo (2001) remarks the accounting margins only provide an upper bound of the true values. Moreover, the accounting estimates do not take into account the marginal cost of distribution while our structural estimates do. Thus, these empirical

[^9]results seem then quite realistic and consistent with the bounds provided by accounting data. In absolute values, the price-cost margins are on average close for mineral water and for spring water because mineral water is on average more expensive. Actually, the absolute margins are on average of $0.024 €$ for mineral water and $0.022 \in$ for spring water. For our best model, we can look at the average price-cost margins for national brands products versus private labels products. In the case of mineral water, the average price-cost margins for national brands and private labels are not statistically different and about the same with an average of $6.69 \%$ for national brands and of $9.63 \%$ for private labels. However, in the case of natural spring water, it appears that price-cost margins for national brands are larger than for private labels with an average of $23.85 \%$ instead of 7.56\%.

## 6 Conclusion

In this paper, we presented the first empirical estimation of a structural model taking into account explicitly two part tariffs contracts between manufacturers and retailers with or without endogenous market power. We show how to estimate different structural models embedding the strategic relationships between manufacturers and retailers in the supermarket industry. In particular, we presented how one can test whether manufacturers use two part tariffs contracts with retailers. We consider several alternative models of competition between manufacturers and retailers on a differentiated product market and test between these alternatives. We consider in particular several types of non linear pricing relationships with two part tariffs contracts allowing retailers to enjoy some endogenous market power, and where resale price maintenance may be used or not. The method is based on estimates of demand parameters that allow to recover price-cost margins at the manufacturer and retailer levels. We then test between the different models using exogenous variables that are supposed to shift the marginal cost of production and distribution. We apply this methodology to study the market for retailing bottled water in France. Our empirical evidence allows to conclude that manufacturers and retailers use non linear pricing contracts and in particular two part tariffs contracts with resale price maintenance. Moreover, we find that the market power of retailers is not affected endogenously by their outside opportunities because such a case is rejected by the data. It seems that the three main multiproduct manufacturers on this
market are big enough for the retailers not being able to refuse offers of one of them. By bundling the two-part tariffs contracts, manufacturers manage to reduce the profitability of retailing only other firms' brands.

This work calls for further developments and studies about competition under non linear pricing in the supermarket industry. In particular, we need further studies where assumptions of non constant marginal cost of production and distribution would be allowed are needed. Also, it is clear that more empirical work on other markets will be useful for a better understanding of vertical relationships in the retailing industry. Simulation of counterfactual policies as done by Bonnet, Dubois and Simioni (2004) in the particular case of exogenous bargaining power of retailers can also be extended in the current framework.

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## 7 Appendix

### 7.1 Detailed resolution of system of equations

Generically we have systems of equations to be solved of the form

$$
\left\{\begin{array}{c}
A_{f}(\gamma+\Gamma)+B_{f}=0 \\
\text { for } f=1, . ., G
\end{array}\right.
$$

where $A_{f}$ and $B_{f}$ are some given matrices.
Solving this system amounts to solve the following minimization problem

$$
\min _{\gamma+\Gamma} \sum_{f=1}^{G}\left[A_{f}(\gamma+\Gamma)+B_{f}\right]^{\prime}\left[A_{f}(\gamma+\Gamma)+B_{f}\right]
$$

leads to the first order conditions

$$
\left(\sum_{f=1}^{G} A_{f}^{\prime} A_{f}\right)(\gamma+\Gamma)-\sum_{f=1}^{G} A_{f}^{\prime} B_{f}=0
$$

that allow to find the following expression for its solution

$$
(\gamma+\Gamma)=\left(\sum_{f=1}^{G} A_{f}^{\prime} A_{f}\right)^{-1} \sum_{f=1}^{G} A_{f}^{\prime} B_{f}
$$

### 7.2 Detailed proof of the manufacturers profit expression under twopart tariffs

We use the theoretical results due to Rey and Vergé (2004) applied to our context with $F$ firms and $R$ retailers. The participation constraint (10) being binding, we have for all $r \sum_{s \in S_{r}}\left[M\left(p_{s}-\right.\right.$ $\left.\left.w_{s}-c_{s}\right) s_{s}(p)-F_{s}\right]=\bar{\Pi}^{r}$ which implies that

$$
\sum_{s \in S_{r}} F_{s}=\sum_{s \in S_{r}} M\left(p_{s}-w_{s}-c_{s}\right) s_{s}(p)-\bar{\Pi}^{r}
$$

and thus

$$
\begin{aligned}
\sum_{j \in F_{f}} F_{j}+\sum_{j \notin F_{f}} F_{j} & =\sum_{j=1, ., J} F_{j}=\sum_{r=1, ., R} \sum_{s \in S_{r}} F_{s} \\
& =\sum_{r=1, ., R} \sum_{s \in S_{r}} M\left(p_{s}-w_{s}-c_{s}\right) s_{s}(p)-\sum_{r=1, ., R} \bar{\Pi}^{r}=\sum_{j=1,, ., J} M\left(p_{j}-w_{j}-c_{j}\right) s_{j}(p)-\sum_{r=1, ., R} \bar{\Pi}^{r}
\end{aligned}
$$

so that

$$
\sum_{j \in F_{f}} F_{j}=\sum_{j=1, . ., J} M\left(p_{j}-w_{j}-c_{j}\right) s_{j}(p)-\sum_{j \notin F_{f}} F_{j}-\sum_{r=1, ., R} \bar{\Pi}^{r}
$$

Then, the firm $f$ profits are

$$
\begin{aligned}
\Pi^{f} & =\sum_{k \in F_{f}} M\left(w_{k}-\mu_{k}\right) s_{k}(p)+\sum_{k \in F_{f}} F_{k} \\
& =\sum_{k \in F_{f}} M\left(w_{k}-\mu_{k}\right) s_{k}(p)+\sum_{j=1, . ., J} M\left(p_{j}-w_{j}-c_{j}\right) s_{j}(p)-\sum_{j \notin F_{f}} F_{j}-\sum_{r=1, ., R} \bar{\Pi}^{r}
\end{aligned}
$$

Since, producers fix the fixed fees given the ones of other producers, we have that under resale price maintenance :

$$
\begin{aligned}
\max _{\left\{F_{i}, p_{i}\right\}_{i \in F_{f}}} \Pi^{f} & \Leftrightarrow \max _{\left\{p_{i}\right\}_{i \in F_{f}}} \sum_{k \in F_{f}}\left(w_{k}-\mu_{k}\right) s_{k}(p)+\sum_{j=1, . ., J}\left(p_{j}-w_{j}-c_{j}\right) s_{j}(p) \\
& \Leftrightarrow \max _{\left\{p_{i}\right\}_{i \in F_{f}}} \sum_{k \in F_{f}}\left(p_{k}-\mu_{k}\right) s_{k}(p)+\sum_{k \notin F_{f}}\left(p_{k}-w_{k}-c_{k}\right) s_{k}(p)
\end{aligned}
$$

and with no resale price maintenance

$$
\begin{aligned}
\max _{\left\{F_{i}, w_{i}\right\}_{i \in F_{f}}} \Pi^{f} & \Leftrightarrow \max _{\left\{w_{i}\right\}_{i \in F_{f}}} \sum_{k \in F_{f}}\left(w_{k}-\mu_{k}\right) s_{k}(p)+\sum_{j=1, \ldots, J}\left(p_{j}-w_{j}-c_{j}\right) s_{j}(p) \\
& \Leftrightarrow \max _{\left\{w_{i}\right\}_{i \in F_{f}}} \sum_{k \in F_{f}}\left(p_{k}-\mu_{k}\right) s_{k}(p)+\sum_{k \notin F_{f}}\left(p_{k}-w_{k}-c_{k}\right) s_{k}(p)
\end{aligned}
$$

Then the first order conditions of the different two part tariffs models can be derived very simply.

### 7.3 Structural demand equation and instruments

The structural demand model is such that

$$
\begin{aligned}
\ln s_{j t}-\ln s_{0 t} & =\theta_{j t}+\sigma_{g} \ln s_{j t \mid g}+u_{j t} \\
\ln s_{j t}-\ln s_{0 t} & =\delta_{j}+\gamma_{t}-\alpha p_{j t}+\sigma_{g} \ln s_{j t \mid g}+u_{j t}
\end{aligned}
$$

where $s_{j t \mid g}$ is endogenous because $E\left(s_{j t \mid g} . u_{j t}\right) \neq 0$. Taking the log of the expression of the relative market share of good $j$ in group $g$, we have

$$
\ln s_{j t-1 / g}=\frac{\theta_{j t-1}+u_{j t-1}}{1-\sigma_{g}}-\ln \left[\sum_{j \in J_{g}} \exp \frac{\theta_{j t-1}+u_{j t-1}}{1-\sigma_{g}}\right]
$$

Then, with a first order approximation

$$
\ln \left[\sum_{j \in J_{g}} \exp \frac{\theta_{j t-1}+u_{j t-1}}{1-\sigma_{g}}\right] \simeq \frac{\theta_{j^{*} t-1}+u_{j^{*} t-1}}{1-\sigma_{g}}
$$

where $j^{*}$ is such that $\theta_{j^{*} t-1}+u_{j^{*} t-1}>\theta_{j t-1}+u_{j t-1} \forall j \neq j^{*}$. Then,

$$
\ln s_{j t-1 / g} \simeq \frac{\theta_{j t-1}+u_{j t-1}}{1-\sigma_{g}}-\frac{\theta_{j^{*} t-1}+u_{j^{*} t-1}}{1-\sigma_{g}}
$$

Then,

$$
\begin{aligned}
\varsigma_{j t-1} & =\ln s_{j t-1 / g}-E\left(\ln s_{j t-1 / g} \mid\left\{\theta_{j t-1}\right\}_{j=1, \ldots, J}\right) \\
& \simeq \frac{u_{j t-1}}{1-\sigma_{g}}+E\left(\left.\frac{\theta_{j^{*} t-1}+u_{j^{*} t-1}}{1-\sigma_{g}} \right\rvert\,\left\{\theta_{j t-1}\right\}_{j=1, \ldots, J}\right)-\frac{\theta_{j^{*} t-1}+u_{j^{*} t-1}}{1-\sigma_{g}} \\
& \simeq \frac{u_{j t-1}-u_{j^{*} t-1}}{1-\sigma_{g}}
\end{aligned}
$$

Thus, assuming that $\forall j \neq j^{\prime}, \forall t, E\left(u_{j^{\prime} t} \cdot u_{j t-1}\right)=0$ implies that

$$
E\left(u_{j^{\prime} t} \cdot \varsigma_{j t-1}\right) \simeq 0
$$

which justifies the use of $\varsigma_{j t-1}$ in the list of instruments $Z_{t}$.

### 7.4 Identification method of demand and supply parameters

Under a given supply model, for a given product $j$, at period $t$, the total price cost margins $\gamma_{j t}+\Gamma_{j t}$ can be expressed as a parametric function of prices and unobserved demand shocks $u_{t}=\left(u_{1 t}, . ., u_{j t}, . ., u_{J t}\right):$ in the case of two part tariffs with resale price maintenance and no endogenous market power of retailers,

$$
\gamma_{j t}+\Gamma_{j t}=-\left[\left(I_{f} S_{p_{t}} I_{f}\right)^{-1} I_{f} s\left(p_{t}, u_{t}\right)\right]_{j}
$$

where $[.]_{j}$ denotes the $j^{\text {th }}$ row of vector [.].
In the case of cost restriction 2 (it would be similar when using cost restriction 1), the marginal cost can be expressed as a function of observed cost shifter $W_{j t}$, unobserved product specific effects $\omega_{j}$, and unobserved shocks $\eta_{j t}$, we have

$$
C_{j t}=\exp \left(\omega_{j}+W_{j t}^{\prime} \lambda\right) \eta_{j t}
$$

The identification of the price-cost margins relies on the assumption that instruments $Z_{j t}$ satisfy

$$
E\left(Z_{j t} u_{j t}\right)=0
$$

and the identification of the cost function relies on the assumption that

$$
E\left(\ln \eta_{j t} W_{j t}\right)=E\left(\ln \eta_{j t} \omega_{j}\right)=0
$$

However, adding cost and marginal cost equations, one can also get a price equation

$$
p_{j t}+\left[\left(I_{f} S_{p_{t}} I_{f}\right)^{-1} I_{f} s\left(p_{t}, u_{t}\right)\right]_{j}=\exp \left(\omega_{j}+W_{j t}^{\prime} \lambda\right) \eta_{j t}
$$

Identifying the parameters of this price equation would then require the specification of the joint law of unobservable shocks $\left(\eta_{j t}, u_{t}\right)$. Thus, our two-step method has the advantage of providing identification of demand and cost parameters under weaker assumptions. In particular we do not have to make any assumptions on the correlation between unobserved shocks $\left(\eta_{j t}, u_{t}\right)$.

### 7.5 Details on Regressions for Demand Estimates

The first stage regressions for the two stage least squares estimation, presented in Table 9 , are

$$
\begin{aligned}
\ln s_{j t \mid g} & =Z_{j t} \beta^{g}+\xi_{j t}^{g} \text { for } g=1,2 \\
p_{j t} & =Z_{j t} \beta^{p}+\xi_{j t}^{p}
\end{aligned}
$$

| First stage regressions |  | Dependent Variable |  |
| :--- | :---: | :---: | :---: |
| Explanatory variables |  |  |  |
| $Z_{j t}$ | Price $p_{j t}$ | $\ln s_{j t \mid g}$ (Spring) | $\ln s_{j t \mid g}$ (Mineral) |
| $z_{j t}$ | (wage) $w_{t}^{1} 1_{(j \in \text { Mineral })}$ | $0.00757(0.0243)$ | $-0.0186(0.0252)$ |
| (wage) $w_{t}^{1} 1_{(j \in \text { Spring })}$ | $0.0533(0.0285)$ | $-1.36 \mathrm{e}-14(0.039)$ |  |
|  | $0.00453(0.01)$ | $-0.0178(0.0295)$ | $0.0265(0.0461)$ |
| (plastic) $w_{t}^{2} 1_{(j \in \text { Mineral })}$ | $0.00129(0.0117)$ | $0.0178(0.0121)$ | $-6.51 \mathrm{e}-15(0.016)$ |
| (plastic) $w_{t}^{2} 1_{(j \in \text { Spring })}$ | $0.0165(0.0189)$ |  |  |
| (diesel) $w_{t}^{3} 1_{(j \in \text { Mineral })}$ | $-0.00317(0.0048)$ | $0.00907(0.0049)$ | $8.66 \mathrm{e}-15(0.0077)$ |
| (diesel) $w_{t}^{3} 1_{(j \in \text { Spring })}$ | $0.00149(0.0056)$ | $-0.00907(0.0058)$ | $0.0027(0.00909)$ |
| (oil) $w_{t}^{4} 1_{(j \in \text { Mineral })}$ | $0.00671(0.0061)$ | $-0.0121(0.00635)$ | $-1.06 \mathrm{e}-14(0.010)$ |
| (oil) $w_{t}^{4} 1_{(j \in \text { Spring })}$ | $-0.00551(0.0071)$ | $0.0121(0.00743)$ | $-0.00293(0.0116)$ |
| (packaging) $w_{t}^{5} 1_{(j \in \text { Mineral })}$ | $-0.00185(0.0070)$ | $0.00571(0.0073)$ | $-1.45 \mathrm{e}-15(0.011)$ |
| (packaging) $w_{t}^{5} 1_{(j \in \text { Spring })}$ | $-0.00618(0.0082)$ | $-0.00571(0.0085)$ | $-0.0111(0.0133)$ |
| $\varsigma_{j t-1}$ (mineral water) | $-0.0471(0.0279)$ | $0.535(.0289)$ | $2.65 \mathrm{e}-15(0.045)$ |
| $\varsigma_{j t-1}$ (spring water) | $0.0311(0.0328)$ | $-0.535(.034)$ | $0.209(0.053)$ |
| Product fixed effects not shown |  |  |  |
| $F(53,1808)$ test, (p-value) | $122.18(0.00)$ | $298.30(0.00)$ | $202.06(0.00)$ |

Table 9 : First Stage Regressions for the Demand Estimation

### 7.6 Estimates of Cost Equations

Here, we present the empirical results of the estimation of the cost equation (30) for $h=1, \ldots, 13$
that is

$$
\ln C_{j t}^{h}=\omega_{j}^{h}+W_{j t} \lambda_{g}+\ln \eta_{j t}^{h}
$$

where variables $W_{j t}$ include time dummies $\delta_{t}$, wages, oil, diesel, packaging material and plastic price variables interacted with the dummy variable for spring water $(S W)$ and mineral water $(M W)$.

|  | Coefficients (Std. err.) |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\ln C_{j t}^{h}$ | salary $S W$ | salary $M W$ | plastic $S W$ | plastic $M W$ | packaging $S W$ | packaging $M W$ |
| Model 1 | $0.005(0.025)$ | $0.026(0.019)$ | $0.015(0.012)$ | $-0.005(0.010)$ | $-0.002(0.008)$ | $0.001(0.006)$ |
| Model 2 | $-0.259(0.082)$ | $-0.125(0.061)$ | $0.051(0.039)$ | $0.030(0.032)$ | $0.107(0.026)$ | $-0.071(0.021)$ |
| Model 3 | $0.021(0.099)$ | $0.020(0.065)$ | $0.088(0.043)$ | $-0.025(0.034)$ | $-0.015(0.028)$ | $-0.045(0.022)$ |
| Model 4 | $-0.017(0.018)$ | $0.035(0.013)$ | $0.005(0.008)$ | $0.006(0.007)$ | $0.004(0.006)$ | $-0.002(0.004)$ |
| Model 5 | $0.010(0.013)$ | $0.036(0.010)$ | $0.005(0.006)$ | $0.006(0.006)$ | $-0.000(0.004)$ | $-0.003(0.003)$ |
| Model 6 | $0.007(0.015)$ | $0.035(0.012)$ | $0.005(0.007)$ | $0.005(0.006)$ | $-0.000(0.004)$ | $-0.002(0.004)$ |
| Model 7 | $-0.132(0.045)$ | $0.011(0.033)$ | $0.125(0.021)$ | $0.054(0.017)$ | $0.060(0.015)$ | $0.048(0.011)$ |
| Model 8 | $-0.169(0.060)$ | $0.104(0.040)$ | $0.132(0.027)$ | $0.012(0.021)$ | $0.024(0.018)$ | $0.062(0.013)$ |
| Model 9 | $0.008(0.014)$ | $0.035(0.011)$ | $0.006(0.007)$ | $0.004(0.005)$ | $-0.000(0.004)$ | $-0.002(0.003)$ |
| Model 10 | $0.005(0.016)$ | $0.034(0.012)$ | $0.005(0.008)$ | $0.003(0.006)$ | $0.000(0.005)$ | $-0.002(0.004)$ |
| Model 11 | $-0.017(0.018)$ | $0.035(0.013)$ | $0.005(0.008)$ | $0.006(0.007)$ | $0.004(0.005)$ | $-0.003(0.004)$ |

Table 10 : Cost Equations for the Nested Logit Model

|  | Coefficients (Std. err.) |  |  |  |  |  |  |  |  |  | All $\delta_{t}=0$ | All $\omega_{j}^{g}=0$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\ln C_{j t}^{h}$ | diesel $S W$ | diesel $M W$ | oil $S W$ | oil $M W$ | $F$ test $(p$ val. $)$ | $F$ test $(p$ val. $)$ |  |  |  |  |  |  |
| Model 1 | $-0.009(0.005)$ | $0.001(0.004)$ | $0.013(0.007)$ | $-0.003(0.006)$ | $1.22(0.190)$ | $194.80(0.000)$ |  |  |  |  |  |  |
| Model 2 | $-0.053(0.018)$ | $-0.017(0.014)$ | $0.070(0.025)$ | $0.034(0.021)$ | $6.78(0.000)$ | $123.41(0.000)$ |  |  |  |  |  |  |
| Model 3 | $-0.001(0.021)$ | $0.010(0.015)$ | $-0.015(0.027)$ | $-0.036(0.022)$ | $3.02(0.000)$ | $119.28(0.000)$ |  |  |  |  |  |  |
| Model 4 | $-0.009(0.004)$ | $-0.004(0.003)$ | $0.011(0.005)$ | $0.004(0.004)$ | $1.81(0.004)$ | $296.68(0.000)$ |  |  |  |  |  |  |
| Model 5 | $-0.003(0.003)$ | $-0.002(0.002)$ | $0.004(0.004)$ | $0.001(0.003)$ | $1.78(0.005)$ | $423.03(0.000)$ |  |  |  |  |  |  |
| Model 6 | $-0.003(0.003)$ | $-0.002(0.002)$ | $0.003(0.006)$ | $0.004(0.005)$ | $1.56(0.025)$ | $355.90(0.000)$ |  |  |  |  |  |  |
| Model 7 | $-0.055(0.010)$ | $-0.055(0.008)$ | $0.065(0.013)$ | $0.051(0.011)$ | $19.17(0.200)$ | $89.58(0.000)$ |  |  |  |  |  |  |
| Model 8 | $-0.022(0.013)$ | $-0.019(0.010)$ | $0.001(0.018)$ | $0.004(0.014)$ | $8.10(0.190)$ | $64.20(0.000)$ |  |  |  |  |  |  |
| Model 9 | $-0.005(0.003)$ | $-0.003(0.002)$ | $0.006(0.004)$ | $0.002(0.003)$ | $1.84(0.003)$ | $325.57(0.000)$ |  |  |  |  |  |  |
| Model 10 | $-0.002(0.003)$ | $-0.002(0.003)$ | $0.000(0.005)$ | $0.001(0.004)$ | $1.35(0.095)$ | $350.51(0.000)$ |  |  |  |  |  |  |
| Model 11 | $-0.009(0.004)$ | $-0.005(0.003)$ | $0.012(0.005)$ | $0.004(0.004)$ | $1.81(0.004)$ | $296.68(0.000)$ |  |  |  |  |  |  |

Table 10 (continued) : Cost Equations for the Nested Logit Model

### 7.7 Nested Logit Demand Formulas

In the case of the nested logit model, the price elasticity of product $j$ market share with respect
to price of product $k$ :

$$
\eta_{j k} \equiv \frac{\partial s_{j}}{\partial p_{k}} \frac{p_{k}}{s_{j}}= \begin{cases}\frac{\alpha}{1-\sigma_{g}} p_{k}\left[\sigma_{g} s_{j / g}+\left(1-\sigma_{g}\right) s_{j}-1\right] & \text { if } j=k \text { and }\{j, k\} \in g \\ \frac{\alpha}{1-\sigma_{g}} p_{k}\left[\sigma_{g} s_{k / g}+\left(1-\sigma_{g}\right) s_{k}\right] & \text { if } j \neq k \text { and }\{j, k\} \in g \\ \alpha p_{k} s_{k} & \text { if } j \in g \text { and } k \in g^{\prime} \text { and } g \neq g^{\prime}\end{cases}
$$

Price elasticities of group $g$ market share with respect to product $k$ :

$$
\eta_{g k} \equiv \frac{\partial s_{g}}{\partial p_{k}} \frac{p_{k}}{s_{g}}== \begin{cases}\alpha p_{k} s_{g^{\prime}} s_{k / g^{\prime}} & \text { if } \mathrm{k} \in g^{\prime} \text { and } g \neq g^{\prime} \\ \alpha p_{k} s_{k / g}\left(s_{g}-1\right) & \text { if } \mathrm{k} \in g\end{cases}
$$

Price elasticities of firm $f$ manufacturer's total market share with respect to product $k$ :

$$
\eta_{f k} \equiv \frac{\partial s_{f}}{\partial p_{k}} \frac{p_{k}}{s_{f}}= \begin{cases}\frac{\alpha}{1-\sigma_{g}} p_{k}\left[\sigma_{g} s_{k / g}+\left(1-\sigma_{g}\right) s_{k}\right]-\frac{\alpha}{1-\sigma_{g}} \frac{s_{k}}{s_{F_{f}}} p_{k} & \text { if } k \in F_{f} \\ \frac{\alpha}{1-\sigma_{g}} p_{k}\left[\sigma_{g} s_{k / g}+\left(1-\sigma_{g}\right) s_{k}\right] & \text { if } k \notin F_{f} \text { and }\left\{F_{f}, k\right\} \in g \\ \alpha p_{k} s_{k} & \text { if } k \notin F_{f} \text { and } F_{f} \in g \text { and } k \in g^{\prime}\end{cases}
$$

In the case where the retailer $r$ refuses the manufacturer $f$ 's contract offers, the products aren't
sold and the market shares derivatives with respect to prices become
$\frac{\partial s_{k}}{\partial p_{j}}\left(\widetilde{p}^{f r(k)}\right)= \begin{cases}\frac{\alpha}{1-\sigma_{g}} s_{k}\left[\sigma_{g} s_{k / g}\left(\widetilde{p}^{f r(k)}\right)+\left(1-\sigma_{g}\right) s_{k}\left(\widetilde{p}^{f r(k)}\right)-1\right] & \text { if } j=k \text { and }\{j, k\} \in g \\ \frac{\alpha}{1-\sigma_{g}} s_{k}\left(\widetilde{p}^{f r(k)}\right)\left[\sigma_{g} s_{j / g}\left(\widetilde{p}^{f r(k)}\right)+\left(1-\sigma_{g}\right) s_{j}\left(\widetilde{p}^{f r(k)}\right)\right] & \text { if } j \neq k \text { and }\{j, k\} \in g \\ \alpha s_{j}\left(\widetilde{p}^{f r(k)}\right) s_{k}\left(\widetilde{p}^{f r(k)}\right) & \text { if } j \in g \text { and } k \in g \prime \text { and } g \neq g^{\prime}\end{cases}$
where $\widetilde{p}^{f r(k)}$ is the vector of prices in case the products manufactured by $f$ are not sole to retailer and $r(k)$ denotes the retailer identity of product $k$.

### 7.8 Additional non nested tests

Non nested tests using cost restriction 2 :

| $T_{n}=\frac{\sqrt{n}}{\widehat{\sigma}_{n}}\left(Q_{n}^{2}\left(\hat{\Theta}_{n}^{2}\right)-Q_{n}^{1}\left(\hat{\Theta}_{n}^{1}\right)\right) \rightarrow N(0,1)$ |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\backslash$ | $\mathrm{H}_{2}$ |  |  |  |  |  |  |  |  |  |
| $H_{1}$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 1 | 9.07 | 17.75 | -3.25 | -5.86 | -4.85 | 4.66 | 18.29 | -5.38 | -4.16 | -4.47 |
| 2 |  | 3.13 | -30.42 | -32.94 | -31.42 | -8.21 | -5.32 | -31.92 | -30.95 | -15.61 |
| 3 |  |  | -18.12 | -20.38 | -19.21 | -6.74 | -4.89 | -19.73 | -18.43 | -12.23 |
| 4 |  |  |  | -15.89 | -11.83 | 15.55 | 20.01 | -14.99 | -5.95 | 14.14 |
| 5 |  |  |  |  | 10.16 | 19.45 | 22.99 | 4.67 | 12.61 | 16.04 |
| 6 |  |  |  |  |  | 17.70 | 21.60 | -6.13 | 13.14 | 1.70 |
| 7 |  |  |  |  |  |  | 5.52 | -18.83 | -16.45 | -17.01 |
| 8 |  |  |  |  |  |  |  | -22.36 | -20.60 | -14.64 |
| 9 |  |  |  |  |  |  |  |  | 9.29 | 14.44 |
| 10 |  |  |  |  |  |  |  |  |  | -7.91 |

Table 11: Results of the Rivers and Vuong Test


[^0]:    * University of Toulouse (INRA)
    ${ }^{\dagger}$ University of Toulouse (INRA, IDEI) and CEPR
    $\ddagger$ We especially thank B. Jullien, T. Magnac, V. Réquillart, P. Rey for useful discussions. Any remaining errors are ours.

[^1]:    ${ }^{1}$ The underlying assumptions in the definition of these price-cost margins are that the marginal cost is constant and is equal to the average variable cost (see Liebowitz, 1982).
    ${ }^{2}$ Value added is defined as the value of shipments plus services rendered minus cost of materials, supplies and containers, fuel, and purchased electrical energy.

[^2]:    ${ }^{3}$ Remark that in all the following, when we use the inverse of non invertible matrices, it means that we consider the matrix of generalized inverse which means that for example $\left[\begin{array}{ll}2 & 0 \\ 0 & 0\end{array}\right]^{-1}=\left[\begin{array}{cc}1 / 2 & 0 \\ 0 & 0\end{array}\right]$.

[^3]:    ${ }^{4}$ Rows of this vector that correspond to private labels are zero.
    ${ }^{5}$ We use the notation $(a \mid b)$ for horizontal concatenation of $a$ and $b$.

[^4]:    ${ }^{6}$ Friberg and Ganslandt (2003) observe the same structure for bottled water demand in Sweden.
    ${ }^{7}$ Recent papers (Slade, 2004, and Benkers and Verboven, 2004 ) make the same assumption when modeling the demand side of the markets they analyze.
    ${ }^{8}$ The cumulative distribution function of the vector of the individual-specific utility terms $\varepsilon_{i j t}$ for individual $i$ at time $t$ is given by $F(\varepsilon)=\exp \left(-G\left(e^{-\varepsilon_{i 1 t}}, \ldots, e^{-\varepsilon_{i J t}}\right)\right)$ where the function $G$ is defined as follows

[^5]:    ${ }^{9}$ Similarly, in all the regressions they perform, Friberg and Ganslandt (2003) include also a dummy for the high demand season, i.e. summer.

[^6]:    ${ }^{10}$ If $\alpha$ denotes the desired size of the test and $t_{\alpha / 2}$ the value of the inverse standard normal distribution evaluated at $1-\alpha / 2$. If $T_{n}<t_{\alpha / 2}$ we reject $H_{0}$ in favor of $H_{1}$; if $T_{n}>t_{\alpha / 2}$ we reject $H_{0}$ in favor of $H_{2}$. Otherwise, we do not reject $H_{0}$.

[^7]:    ${ }^{11}$ Formulas of the different elasticities are given in appendix 7.7.

[^8]:    ${ }^{12}$ Note that the average price-cost margin at the retailer level plus the average price-cost margin at the manufacturer level do not sum to the total price cost margin because of the private labels products for which no price cost margin at the manufacturer level is computed, the retailer price cost margin being then equal to the total price cost margin.

[^9]:    ${ }^{13}$ Recall that for a $5 \%$ size of the test, we reject $H_{0}$ in favor of $H_{2}$ if $T_{n}$ is lower than the critical value -1.64 and that we reject $H_{0}$ in favor of $H_{1}$ if $T_{n}$ is higher than the critical value 1.64.

