Location-then-Price games with Outside goods

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Location-then-Price Games

Classic problem dating back to Hotelling (1929):

- Two firms first choose locations, then prices
  - ⋆ This is a two-stage, continuum action game
- Huge, rich literature on this subject; what's left to be done?
  - ⋆ big contributions to the literature primarily analytical
    - ○ computers used only as an "unfortunate" adjunct
  - ⋆ bias against computational methods is unfortunate
    - ○ two stage, continuum action games are complicated
    - ○ quickly hit a wall in terms of what you can do
    - ○ much of the interesting stuff requires computers
  - ⋆ Hope to show computers and insight aren't incompatible

Extensions to the literature

We'll explore three, intricately related extensions.

- With some exceptions, lit assumes compulsory purchase
  - ⋆ We'll add an outside good
- With some exceptions, lit ignores income variation
  - ⋆ We'll consider heterogeneous tastes and income
- Propose an integrated approach to horiz & vert differentiation
  - ⋆ We'll analyze convex combinations of
    - ○ Hotelling (1929) horizontal model
    - ○ Mussa and Rosen (1986) vertical model

The Hotelling model

The problem that Hotelling posed

- Consumers are located along the unit interval
  - Hotelling assumed a uniform distribution
- Two firms choose a location in this interval
- Once locations are fixed, firms set prices.
- Each consumer buys exactly one unit from some firm
  - choice depends on prices and transportation cost

Hotelling as Horizontal differentiation

Natural to treat location as an hedonic attribute.

- E.g., choice of a wine variety
  - 0 represents Napa Valley Chardonnay
  - 1 represents Carneros Sauvingnon Blanc.
  - in the middle:
    - North Coast Chardonnay
    - North Coast Sauvignon Blanc.
- Looking forward, need an outside good as well:
  - differentiated wine vs beer
  - differentiated private schools vs public school
  - differentiated cars vs public transportation

- Consumer’s location is his *ideal variety*
- Willingness to pay declines quadratically
- Transportation cost notion reinterpreted:
  - ⋆ preferences single-peaked in attribute space
  - ⋆ consumer may buy cheaper, less preferred variety
- Key feature: preferences ideosyncratic
  - ⋆ no attribute “better” than any other
  - ⋆ ordinal rankings purely subjective
    - ○ some drink Chardonnay, others Sav Blanc
- No *notion* of *quality* in this model
  - ⋆ Obviously a huge limitation for practical applications

The Hotelling Specification

Consumer z gets utility $h_{i,z}$ from buying product i:

$$[H^*] \qquad h_{i,z}(x_i, \theta_z, \mathbf{p}) \quad = \quad C - t(x_i - \theta_z)^2 - p_i$$

- x_i is the location of firm i, $i = 1, 2$
- p_i is the price of good i
- θ_z is consumer z's ideal location
- t is the steepness of single peaked preferences
- C is because negative utility is such a depressing idea
- w.l.o.g, $x_2 \geq x_1$.

Consumers are distributed on $[\underline{\vartheta}, \overline{\vartheta}]$ according to pdf f.

- the marginal consumer is defined by the condition

$$t(x_1 - \tilde{\theta}(\mathbf{x},\mathbf{p}))^2 + p_1 = t(x_2 - \tilde{\theta}(\mathbf{x},\mathbf{p}))^2 + p_2$$

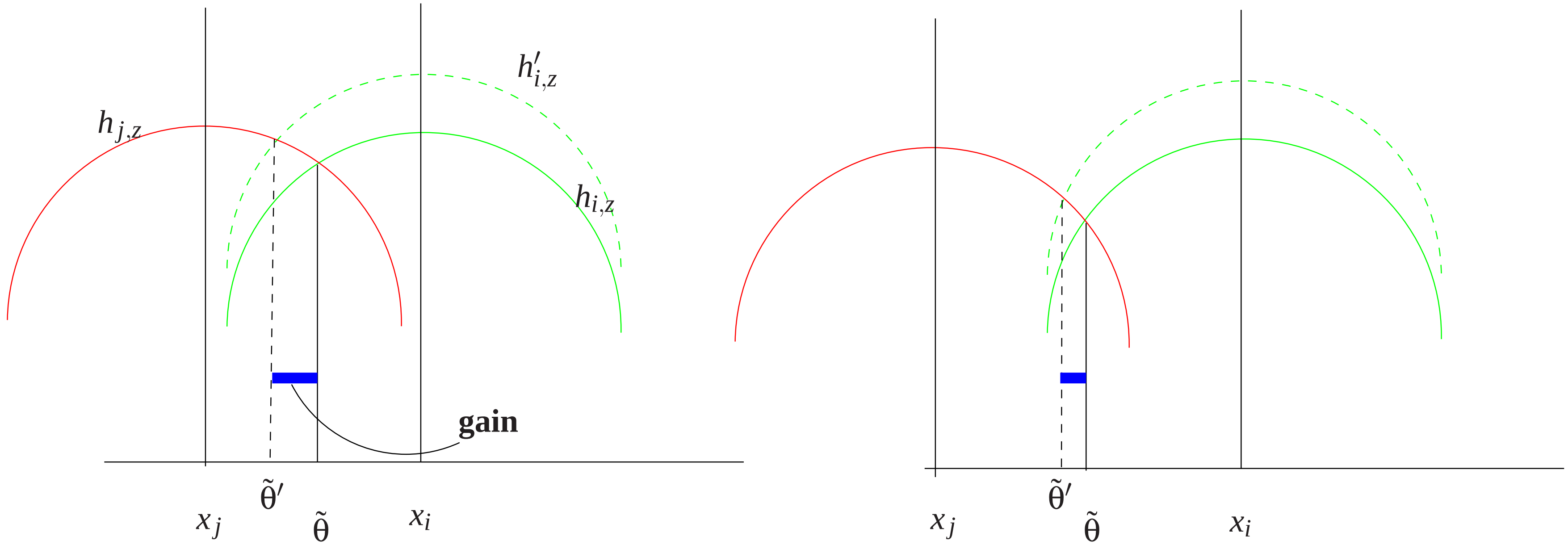
$$\textbf{so that} \qquad \tilde{\theta}(\mathbf{x},\mathbf{p}) = \frac{t(x_2^2 - x_1^2) + (p_2 - p_1)}{2t(x_2 - x_1)}$$

- market shares are then given by:
  - ⋆ consumers in $[\underline{\vartheta}, \tilde{\theta}(\mathbf{x},\mathbf{p})]$ buy from #1
  - ⋆ consumers in $[\tilde{\theta}(\mathbf{x},\mathbf{p}), \overline{\vartheta}]$ buy from #2

Two-stage version of this model quite well understood.

- Anderson, Goeree and Ramer (1997) is a wonderful paper
  - ⋆ Good news &/or bad news: solns easy to characterize
  - ⋆ everything determined shape of f at the median
    - ○ only the marginal consumer matters
  - ⋆ the basic tension:
    - ○ moving closer gains market share
    - ○ moving closer intensifies price competition
  - ⋆ in equilibrium, these forces are balanced

The tension is illustrated by the figure below



- In the left panel
  - ⋆ a unit drop in i's price shifts the marg consumer left
  - ⋆ gain in market share is the blue tile
- In the right panel
  - ⋆ same change in i's price gains a smaller tile
  - ⋆ incentive to compete by price mitigated
- Hence the "principle of maximal differentiation"

Pure vertical differentiation

At the other end of the product differentiation spectrum:

- Ordinal rankings are purely objective
  - ⋆ the *only* distinction between products is quality
  - ⋆ everybody prefers more quality to less
  - ⋆ at equal prices, one firm would supply the entire market
- Most cited specification is Mussa and Rosen (1986)

$$[V^*] \qquad v_z(x_i, \theta_z, \mathbf{p}) = C + 2t x_i \theta_z - p_i$$

- θ_z measures intensity of z's preference for quality
  - ⋆ "discerning" consumers have high θ's

Melding the two frameworks

Both are extreme representations of product differentiation

- Preferences aren't purely ideosyncratic
- Nor are they purely objective
  - Some prefer red wine to white, others the reverse
  - But (virtually) everybody prefers Silver Oak to Gallo
- Natural first step: represent products in two dimensions
  - horizontal characteristic viewed ideosyncratically
  - vertical characteristic represents quality
- To my knowledge, this approach hasn't been explored

A rude shock

Cremer and Thisse (1991): $[H^*]$ can be rewritten as $[V^*]$!

- Let all firms produce at constant marginal cost;
  - ⋆ In $[H^*]$ let this cost be zero
  - ⋆ In $[V^*]$, assume marg cost for i is tx_i^2.
- Firms' strategic variable is *markup* m_i
  - ⋆ in $[H^*]$, $m_i = p_i$.
  - ⋆ in $[V^*]$, $m_i = p_i - tx_i^2$.
- Now rewrite utility specs above as

$$[H^*] \quad h_{i,z}(x_i, \theta_z, \mathbf{m}) = C + 2tx_i\theta_z - tx_i^2 - m_i - t\theta_z^2$$

$$[V^*] \quad v_{i,z}(x_i, \theta_z, \mathbf{m}) = C + 2tx_i\theta_z - tx_i^2 - m_i$$

$$[H^*] \quad h_{i,z}(x_i, \theta_z, \mathbf{m}) \quad = \quad C + 2tx_i\theta_z - tx_i^2 - m_i - t\theta_z^2$$

$$[V^*] \quad v_{i,z}(x_i, \theta_z, \mathbf{m}) \quad = \quad C + 2tx_i\theta_z - tx_i^2 - m_i$$

- h and v differ only by the constant $t\theta_z^2$
  - ⋆ the difference has no impact on z's ordinal rankings.
- Interpretation: in the vertical model, z buys $x_2 > x_1$ iff
  - ⋆ z's quality preference intense enough to offset cost diff
- Cremer and Thisse conclude:

"...the distinction between vertical and horizontal differentiation appears to be merely a matter of interpretation. Formally speaking the Hotelling-type model and the corresponding vertical product differentiation model are equivalent" (p. 384)

The Equivalence Evaporates

$$[H^*] \quad h_{i,z}(x_i, \theta_z, \mathbf{m}) = C + 2tx_i\theta_z - tx_i^2 - m_i - t\theta_z^2$$

$$[V^*] \quad v_{i,z}(x_i, \theta_z, \mathbf{m}) = C + 2tx_i\theta_z - tx_i^2 - m_i$$

- This result has been widely cited
  - ⋆ Nobody seems to have commented on its limitation
  - ⋆ Add an outside good and the constant term matters!
- Add a participation constraint to both models:
  - ⋆ z buys one unit iff $\max_i h_{i,z}$(or $\max_i v_{i,z}$) ≥ 0.
- The set of active participants is quite different: as $C \searrow$:
  - ⋆ In $[H^*]$, consumers on periphery drop out first
  - ⋆ In $[V^*]$, consumers with low θ's drop out first

Our Model

Our model extends the literature in three ways

- Move smoothly between the vertical & horizontal extremes
- Consumers have heterogeneous income levels
- Consumers can purchase neither product
  - ⋆ (Anderson et al. (1992) also has a no-purchase option.
  - ⋆ They study a probabilistic choice model:
  - ⋆ Consumer located at θ purchases good *i* with prob

$$F_i(\theta) = \frac{\exp\left(\left[-p_i - t|x_i - \theta|\right]/\mu\right)}{\exp(V/\mu) + \sum_{\kappa=1}^{2} \exp\left(\left[-p_\kappa - t|x_\kappa - \theta|\right]/\mu\right)}$$

  - ⋆ This approach is too “reduced form” for our tastes)

- Utility specification in our model:

$$[RS^*] \quad u_{i,z}(x_i, \theta_z, \mathbf{m}) = C + 2tx_i\theta_z - tx_i^2 - m_i + y_i - t\left(\beta\theta_z^2 + (1-\beta)E\theta^2\right)$$

- Cf. $[H^*]$ and a slightly modified $[V^*]$

$$[H^*] \quad h_{i,z}(x_i, \theta_z, \mathbf{m}) = C + 2tx_i\theta_z - tx_i^2 - m_i - t\theta_z^2$$

$$[V^*] \quad v_{i,z}(x_i, \theta_z, \mathbf{m}) = C + 2tx_i\theta_z - tx_i^2 - m_i - tE\theta^2$$

- The differences:
  - ⋆ The income term y_i has been added
  - ⋆ The addition to $[V^*]$ is independent of everything
  - ⋆ As $0 \leftarrow \beta \rightarrow 1$, move from $[H^*]$ to $[V^*]$
    - ○ β only affects the set of buyers who participate.

- Consumer participates iff $\max_i u_{i,z}(x_i, \theta_z, \mathbf{m}) \geq \alpha y_z$.
  - ⋆ As α increases, participation constraint tightens
- Distribution of consumers is truncated bivariate normal
  - ⋆ support of y_z is $[\underline{y}, \overline{y}]$
  - ⋆ support of θ_z is $[\underline{\theta}, \overline{\theta}]$
  - ⋆ mean of distribution is (μ_θ, μ_y)
  - ⋆ variance of distribution is $\Sigma(\beta) = \begin{bmatrix} \sigma_\theta & \beta\rho \\ \beta\rho & \sigma_y \end{bmatrix}$
- $\rho > 0$ proxies diminishing marginal utility of income
  - ⋆ Our primitive taste parameter is independent of income
  - ⋆ But $\frac{d^2u}{dy^2} < 0$ implies richer folk pay more for quality

Computation and Smoothing

We nest Matlab's solution engine, fmincon

- This requires a lot of smoothness
  - ⋆ Everything has to be $\mathbb{C}^3$ at least
- But the problem as posed isn't even $\mathbb{C}^1$:
  - ⋆ integration bounds can change abruptly
  - ⋆ so derivative based solution engines can't work.
- We move smoothly between regions using logistic weights
  - ⋆ each integration region is smooth w.r.t. our variables
  - ⋆ logistic weights change smoothly
  - ⋆ so solns to a perturbed version of $[RS^*]$ are obtainable.

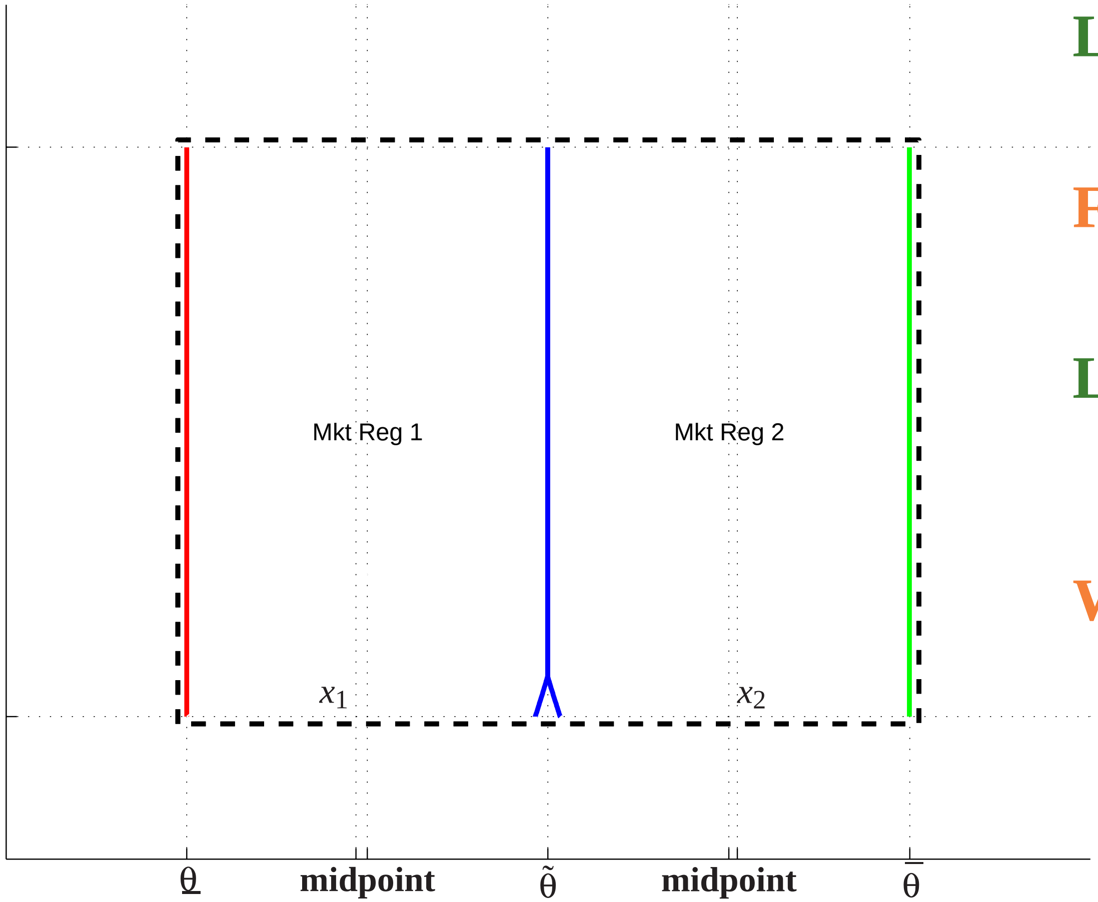
Properties of the Model

The role of participation constraints

- Much more going on now than in the familiar version
  - ⋆ pressure to maximally differentiate as usual
    - ○ centrifugal force
  - ⋆ as firms spread out they lose consumers in middle
    - ○ creates countervailing centripetal force
- effectively, we'll be studying three-firm competition
  - ⋆ intensive margin: compete against other diff product
  - ⋆ extensive margin: compete against outside good
- additional richness means problem is harder to solve
  - ⋆ hopeless to try for analytic solutions

Market regions in equilibrium with loose part. constr.

- The black dotted box is the space of consumers
  - ⋆ Tastes & firm locations on horizontal axis
  - ⋆ Income is on the vertical axis



Lines mark boundaries of market regions

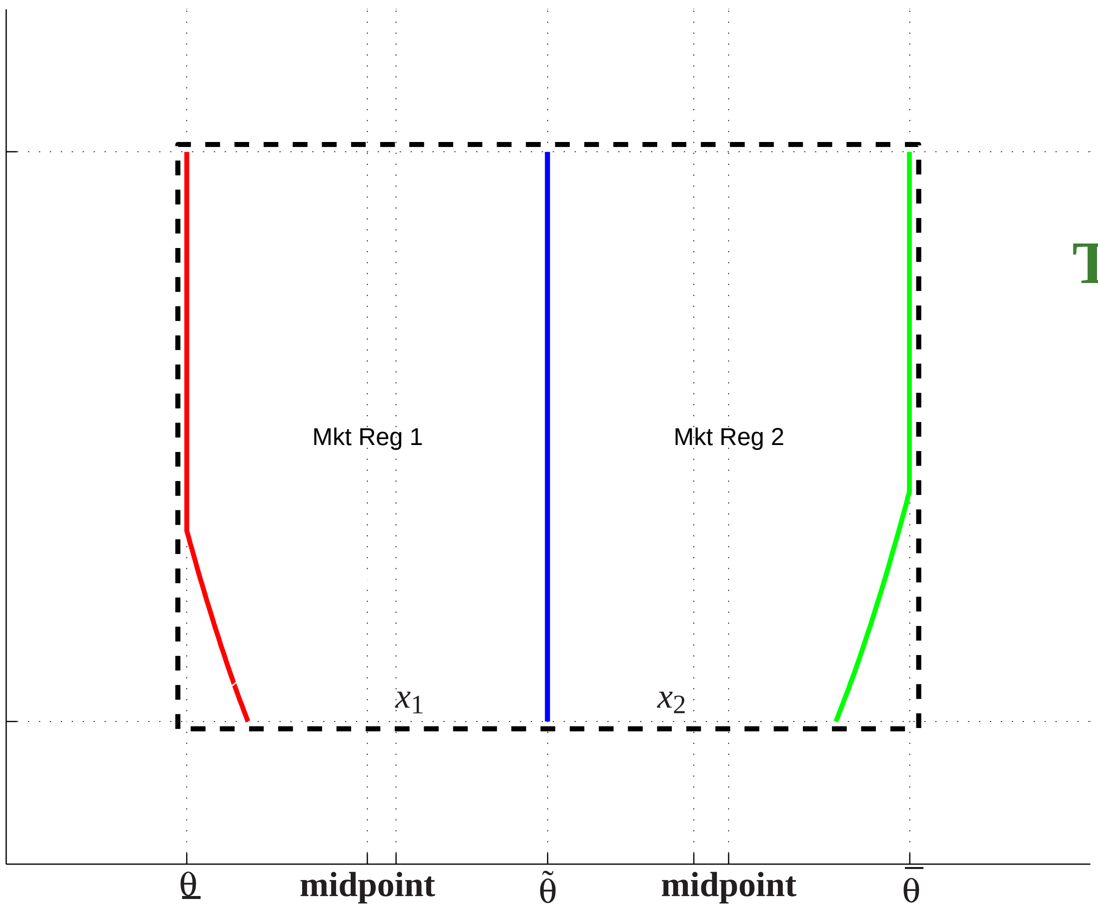
Firms locate a bit closer to extensive boundary

Lowest income levels with $\theta \approx \tilde{\theta}$ drop out. This halts deglomeration

Why no dropouts at outer edges? at midpoint, centripetal force too weak to match the centrifugal force. so firms keep moving out

Market regions in equilibrium with tight part. constr.

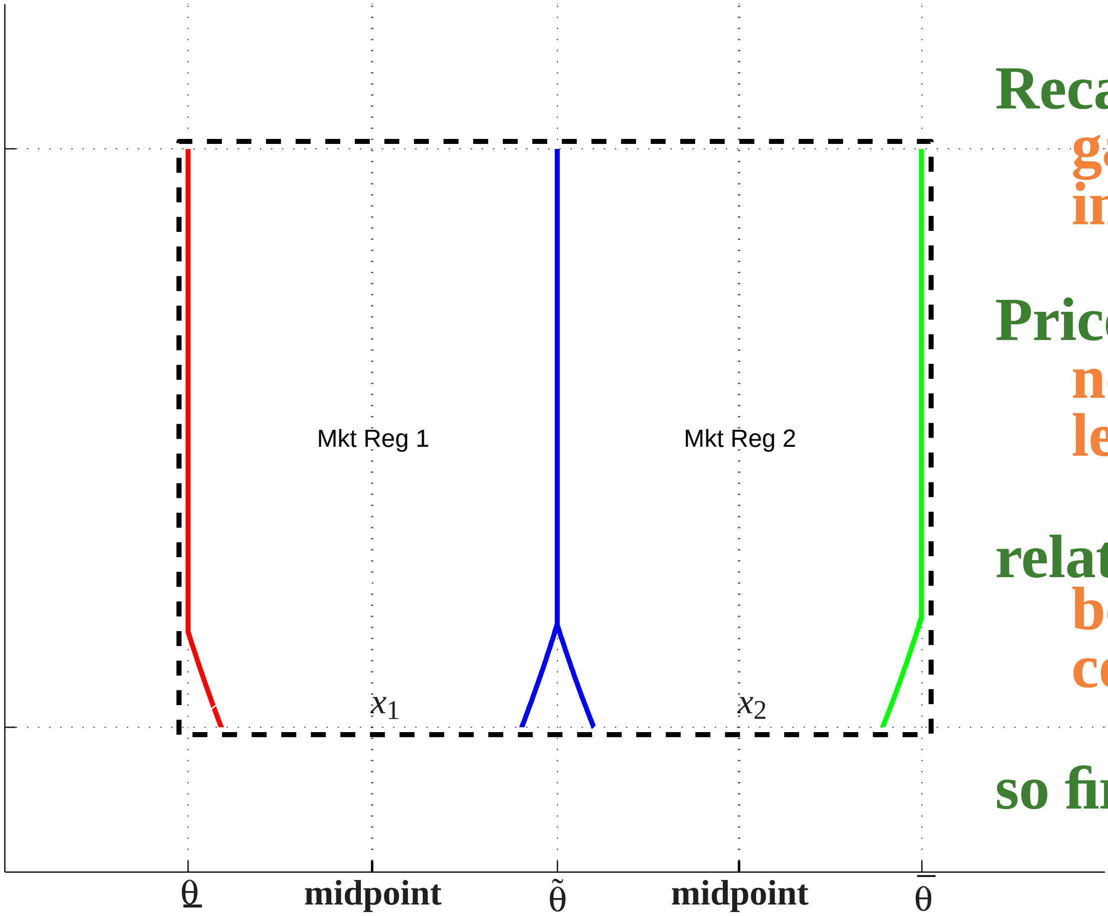
- Note that firms have moved in towards the center
  - ⋆ participation as expected is lower
  - ⋆ but constraints now bind on the *outside*. Why?



To see why, in next slide we'll look at an out-of-equilib picture

Market regions out of equilibrium with tight part. constr.

- Firms are located exactly at mid-point between edges
  - ⋆ participation drops off symmetrically at edges
  - ⋆ at this point, centripital force dominates. Why?
  - ⋆ two factors; one's obvious; the other subtle



Recall: moving to center has 2 effects
  gain market share: good
  intensify price competition: bad

Price competition is for marg con
  now he's partly out of mkt
  less to compete for

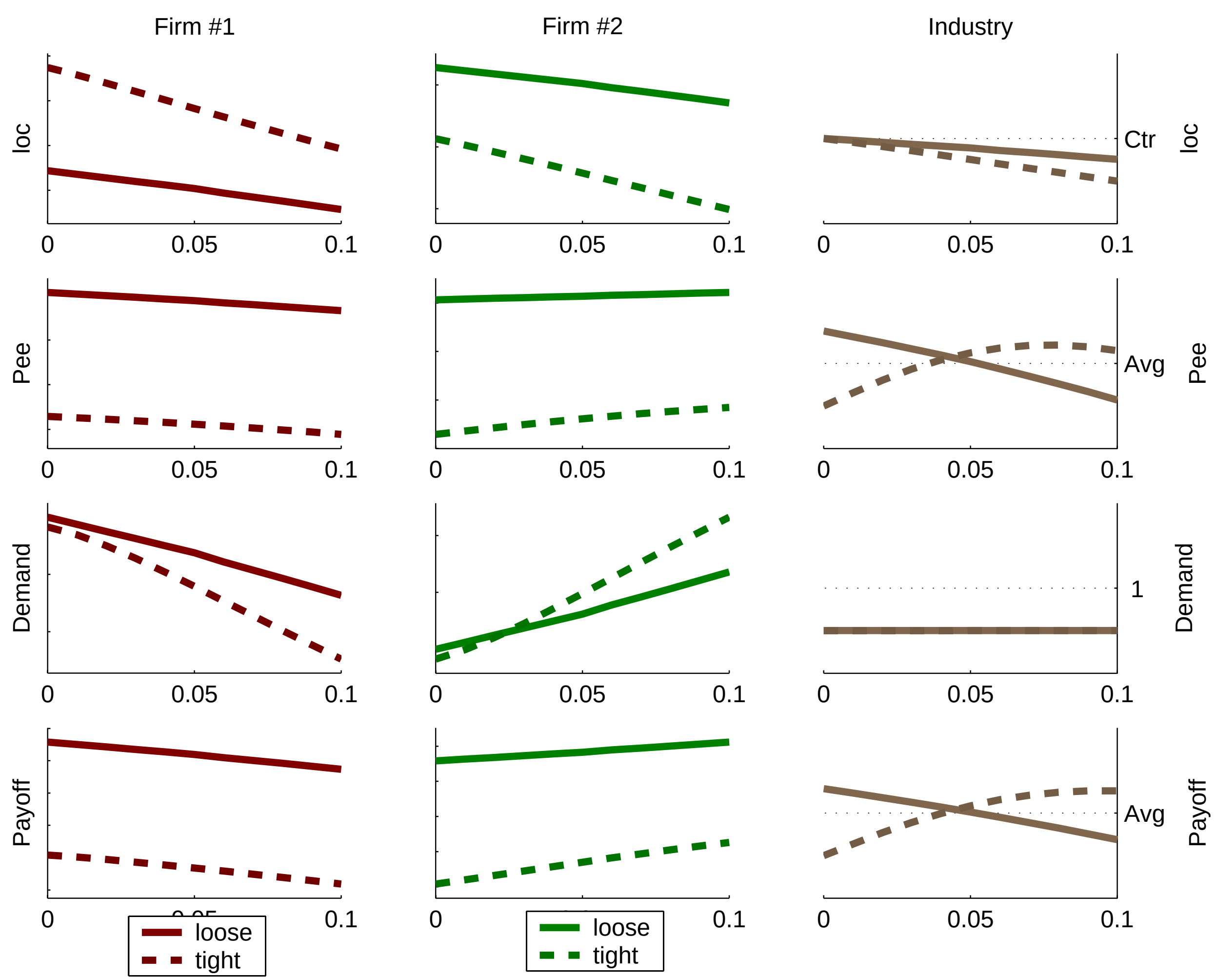
relative to loose constraint
  benefit to moving in stays
  cost of moving in mitigated

so firms move in!

Comparative Statics

What happens as we go from horiz to vertical competition?

- We increase β, comparing loose & tight constraints



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