An Economic Analysis of Product Differentiation under Latent Separability

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Outline

- Demand System: AIDS, Q-AIDS, Translog, Rotterdam, Discrete Choice
- The Concept of Latent Separability
- Use of Quadratic Almost Ideal Demand System (Q-AIDS)
- Procedure to Determine the Number of Latent Groups
- Procedure to Transform from Latent Characteristics Space to Observable Products
- Estimation Results
- Concluding Comments

Demand Systems

Traditional / Representative Consumer (AIDS, Q-AIDS, Translog)

1. **D:** Assumption of Representative Consumer

   D: Curse of Dimensionality

   A: Stronger Theoretical Underpinnings

   A: Avoid assumptions like unit purchase and exogenous characteristics

Discrete Choice Models (Logit, Nested logit, Random Coefficient Discrete Choice Models)

1. **D:** Unit Purchase Assumption

   D: Exogenous Characteristics (Except Price)

   A: Smaller Dimension, Intuitively Appealing

Concept of Latent Separability

- Gorman: Hedonic pricing

- Lancaster: Activity based characteristics / Physical characteristics

- **Blundell and Robin (2000):** Preferences are separable into different groups based on their latent characteristics

- Similar to the idea of hedonic characteristics

Q-AIDS and Latent Characteristics

- Rank 3 Demand System:

$$w = \alpha + \Gamma \ln p + \beta[\ln M - \ln a(p)] + \frac{\tau}{c(p)}[\ln M - \ln a(p)]^2$$

where: $\ln a(p) = \alpha_0 + \alpha^T \ln p + 1/2 * (\ln p)^T \Gamma (\ln p)$, $\ln c(p) = \beta^T \ln p$ and $d(p) = \tau^T \ln p$

- The system is good for capturing any non-linear effect of expenditure/ income on demand.

- Latent Demand System:

$$w = \Pi^T[\widetilde{\alpha} + \widetilde{\Gamma} \ln b + \widetilde{\beta}[\ln M - \ln \widetilde{a}(b)] + \frac{\widetilde{\tau}}{\widetilde{c}(b)}[\ln M - \ln \widetilde{a}(b)]^2]$$

where Π is the transformation matrix

Latent Groups

- Π is a $(m \times n)$ matrix; m: demand dimension in latent space and n: demand dimension in product space

- m latent groups with price aggregator $\ln b(p) = \Pi \ln p$

- Segments/Groupings: Depends on the rank of the demand system

  \- Rank of the Parameter space = Product Groups

- Let: $rank\left(\underline{\overline{\Theta|}}\right) \equiv rank\left(\begin{bmatrix}\overline{B|}\\ \underline{\Gamma|}\end{bmatrix}\right) \leq n-1$.

- Let $\Pi| = \left[\Lambda \quad \Psi|\right] = \begin{bmatrix}\Lambda_1 & \Psi_1\\ \Lambda_2 & \Psi_2\end{bmatrix}$

- Define $\Phi \equiv [\Lambda_1 - 1_{m-1}\Lambda_2]^{-1}[\Psi_1 - 1_{m-1}\Psi_2]$.

  Then $\overline{B|} = \overline{\widetilde{B}}\Pi| = \overline{\widetilde{B}}\begin{bmatrix}\Lambda & \Psi|\end{bmatrix}$ can be written as:

$$\overline{B|} = \begin{bmatrix}B_1 & B_2\end{bmatrix} = \begin{bmatrix}B_1 & B_1\Phi\end{bmatrix}$$

  And $\underline{\Gamma|} = \begin{bmatrix}\Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22}\end{bmatrix}$ can be stated as $\underline{\Gamma|} = \Pi|^T\widetilde{\Gamma}\Pi| = \begin{bmatrix}\Lambda\widetilde{\Gamma}\Lambda & \Lambda\widetilde{\Gamma}\Psi| \\ \Psi|^T\widetilde{\Gamma}\Lambda & \Psi|^T\widetilde{\Gamma}\Psi|\end{bmatrix}$ such that:

$$\underline{\Gamma|} = \begin{bmatrix}\Gamma_{11} & \Gamma_{11}\Phi \\ \Phi^T\Gamma_{11} & \Phi^T\Gamma_{11}\Phi\end{bmatrix}$$

- Reduced form relationship:

$$vec\begin{bmatrix}B_2 \\ \Gamma_{12} \\ \Gamma_{22}\end{bmatrix} = \begin{bmatrix}I_{n-m} \otimes \begin{bmatrix}B_1 \\ \Gamma_{11} \\ \Gamma_{21}\end{bmatrix}\end{bmatrix} vec(\Phi) + u$$

- Rank Test (Cragg and Donald 1996/1997): LU decomposition of a matrix with stochastic elements

Population: $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ then test statistic:

$$T_1 = vec(\widetilde{A}_{22} - \widetilde{A}_{21}\widetilde{A}_{11}^{-1}\widetilde{A}_{12})^T[\widetilde{S}\widetilde{W}\widetilde{S}^T]^{+}vec(\widetilde{A}_{22} - \widetilde{A}_{21}\widetilde{A}_{11}^{-1}\widetilde{A}_{12})$$

- Equivalent Rank Test Statistic:

$$T_2 = \left\{ vec \begin{bmatrix} B_2 \\ \Gamma_{12} \\ \Gamma_{22} \end{bmatrix} - [I_{n-m} \otimes \begin{bmatrix} B_1 \\ \Gamma_{11} \\ \Gamma_{21} \end{bmatrix} vec(\Phi^e) \right\}^T$$

$$V^{+} \left\{ vec \begin{bmatrix} B_2 \\ \Gamma_{12} \\ \Gamma_{22} \end{bmatrix} - [I_{n-m} \otimes \begin{bmatrix} B_1 \\ \Gamma_{11} \\ \Gamma_{21} \end{bmatrix} vec(\Phi^e) \right\}$$

Identification

- Restrictions: $\Pi 1_n = 1_m$. Then $\Pi = [\Lambda, \Psi]$ where Λ and Ψ are matrices of dimensions $(m \times m)$ and $m \times (n - m)$

- Exact Identification is based on exclusivity restrictions such that Λ is diagonal.

- We can recover Λ and Ψ from the estimated $vec(\Phi)$ and other estimated parameters.

Data

- IRI scanner data [Q1:1988 to Q4:1992]: Carbonated Soft Drinks [Diet 7 brands, Regular 9 brands]

- 46 US cities 20 quarters

- Regular: Coke, Pepsi, 7-Up, Mountain Dew, Sprite, RC Cola, Dr. Pepper, Private label, and an aggregate All-Other brand

- Diet: diet Coke, diet Pepsi, diet 7-Up, diet Sprite, diet Dr. Pepper, diet Private Label, and diet All-Other brand

- diet Mt. Dew and diet RC Cola dropped

- Market share: Highest-regular Coke; Lowest-diet Private label

Empirics

- Estimated System: 15 demand equation, 16 price equation, 1 expenditure equation. Method: FIML

- *The Demand System:* $w_{ilt} = \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln(p_{jlt}) + \beta_i \ln\left(\frac{M_{lt}}{P_{lt}}\right) + \frac{\tau_i}{\prod\limits_{i=1}^{N} p_{ilt}^{\beta_i}} \ln\left(\frac{M_{lt}}{P_{lt}}\right)^2$

Demographic Translating: $\alpha_i = \alpha_{0i} + \sum_{k=1}^{K} \lambda_{ik} Z_{klt}$ and $\alpha_{0i} = \sum_{r=1}^{9} d_{ir} Dr$

Reduced Form Price Equations: $p_{ilt} = \theta_{i0} + \theta_{i1} UPV_{ilt} + \theta_{i2} MCH_{ilt} + \theta_{i3} PRD_{ilt} + \theta_{i4} CR_{lt}^{4}$

Reduced Form Expenditure Equations: $M_{lt} = \eta TR_t + \sum_{r=1}^{9} \delta_r D_r + \phi_1 INC_{lt} + \phi_2 INC_{lt}^2$

- Demographic Translating Variables: 9 Regional Binaries, % Hispanic Population, Median HH Size, Median HH Age, % Earning less than 10K, % Earning More than 50K.

- No. of parameters estiamted: 467 parameters (263 of them significant); Demand side: 375 parameters (205 parameters Significant).

- Latent groupings/segments: 9 [at 1% level of significance]

- We estimate price and expenditure elasticities

- Unique brands by segments is based on rank test

Latent Segments

- Only-cola drinks [4]: diet Pepsi, regular Coke, regular Pepsi, regular R.C. Cola.
- Teen oriented cola drinks [1]: regular Dr. Pepper.
- Teen oriented clear drink [1]: regular Mt. Dew.
- Budget CSD [1]: regular Private label.
- Combinations of orange, cherry and other flavored CSD [2]: diet All-Others; Regular All-Others.
- Rest of the brands are expressed as extension.
- A different set of brands can be used to recover information on latent variables.

Elasticities

- Price elasticities: All own price elasticities are significant and negative

- All expenditure elasticities: Positive and significant

- Latent separability restrictions decrease variance of the parameter estimates

- Efficiency gain [Absoluate Percentage Deviation]:

  Variance of price elasticities: 110% gain

  Variance of expenditure elasticities: 119% gain

Own Price Elasticities

Brand	Unrestricted	Latent Separability
Diet Pepsi (n)	-2.83 *(0.34)*	-3.20 *(0.02)*
Diet Coke (n)	-3.36 *(0.44)*	-2.88 *(0.10)*
Diet 7-Up	-2.53 *(0.42)*	-1.13 *(0.26)*
Diet Sprite	-4.61 *(0.46)*	-1.84 *(0.37)*
Diet Dr. Pepper	-4.65 *(0.60)*	-1.46 *(0.57)*
Diet Private Label	-4.84 *(0.77)*	-2.29 *(0.65)*
Diet All-Other	-2.69 *(0.36)*	-2.89 *(0.00)*
Reg. 7-Up (n)	-3.27 *(0.37)*	-1.95 *(0.04)*
Reg. Coke (n)	-5.50 *(0.53)*	-5.60 *(0.01)*
Reg. Dr. Pepper (n)	-4.78 *(1.03)*	-4.56 *(0.20)*
Reg. Mt. Dew (n)	-7.77 *(1.06)*	-7.66 *(0.02)*
Reg. Pepsi	-4.30 *(0.62)*	-4.32 *(0.01)*
Reg. R.C. Cola (n)	-10.05 *(1.43)*	-11.63 *(0.51)*
Reg. Sprite (n)	-5.07 *(0.55)*	-2.59 *(0.15)*
Reg. Private Label (n)	-2.88 *(0.56)*	-2.90 *(0.01)*
Reg. All-Other	-2.37 *(0.31)*	-1.72 *(0.45)*

Expenditure Elasticities

Brand	Unrestricted	Latent Separability
Diet Pepsi	1.10 (0.09)	1.05 (0.01)
Diet Coke	0.91 (0.08)	0.75 (0.01)
Diet 7-Up	0.54 (0.15)	0.73 (0.01)
Diet Sprite	0.87 (0.13)	0.95 (0.32)
Diet Dr. Pepper	2.18 (0.38)	1.49 (0.31)
Diet Private Label	1.13 (0.28)	1.49 (0.31)
Diet All-Other	0.90 (0.04)	0.91 (0.00)
Reg. 7-Up	0.90 (0.12)	0.76 (0.04)
Reg. Coke	1.21 (0.11)	1.23 (0.00)
Reg. Dr. Pepper	0.41 (0.32)	0.34 (0.03)
Reg. Mt. Dew	1.36 (0.25)	1.18 (0.02)
Reg. Pepsi	1.16 (0.11)	1.18 (0.02)
Reg. R.C. Cola	1.12 (0.39)	0.73 (0.32)
Reg. Sprite	1.09 (0.13)	1.58 (0.06)
Reg. Private Label	1.75 (0.25)	1.81 (0.03)
Reg. All-Other	0.68 (0.11)	0.85 (0.05)

Ranking of groups by valuation

Groups/Brands	Shadow Value
Reg. Coke	1.151
Reg. Dr Pepper	1.061
Diet. Pepsi	1.024
Reg. Pepsi	1.023
Diet. All-Other	1.007
Reg. Mt. Dew	1.001
Reg. R.C. Cola	0.999
Reg. Private Label	0.999
Reg. All-Other	0.998

Concluding Comments

- Concept of latent separability can help us to overcome some of the shortcomings of representative consumer based demand systems

- Improve efficiencies of parameter estimates

- Can help us recover information on segments based on latent characteristics

- We can recover information on shadow prices of latent characteristics