Forbidding Below Cost Pricing for Retailers: a Strategic Inflationary Mechanism

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Abstract

This paper explores the indirect inflationary mechanism allowed by loss leaders banning laws. In a double duopoly model where producers compete to sell differentiated products through differentiated retailers, we show that the ban of resale at a loss can be pro-collusive: colluding producers can use this rule strategically to set higher wholesale prices, and pay the retailers through negotiated listing fees. The ban turns wholesale prices into floor prices, thus increasing resale price and lessening consumers welfare. However, if producers do not collude, this mechanism does not hold. These results are robust even if the listing fees are two-part tariff.

Jel Codes: L13, L41, L42.
Keywords: Vertical Restraints, Loss Leaders, Retailing Sector, Collusion.

1 Introduction

In most European countries or in North America, retailer’s pricing strategies are submitted to general competition laws just like producers. Below cost pricing strategies may be banned but only if it can be proved that a firm used it for predatory purposes. Thus, it is not rare to observe loss leaders in most of these countries, as for example in Wal-Mart Supermarkets1. Yet some few countries like France, Ireland, Belgium or

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Portugal, also as some States of America, have adopted in the mid-nineties a specific regulation for retailer’s pricing strategies: it is forbidden to resale at a loss. Such a rule aims at protecting both small shops and producers. Small shops would, thanks to this rule, be protected against the fierce competition of bigger retailers and producers would not see anymore their goods offered at too low prices, thus hurting their brand reputation.

However, loss-leader pricing strategies may have several different motivations. A large literature in industrial economics has been devoted to analyse below cost pricing strategies. Three main explanations have been pointed out. First, a below cost pricing strategy may be used for predatory purposes. The underlining reasoning may be described in a two periods context. A firm may choose to set its price below its cost (thus realizing losses) in a first period to eliminate its rival and then benefit from the monopoly profit in a second period. Of course particular assumptions are necessary for such a strategy to be an equilibrium (see for example Milgrom and Roberts, 1982, or Telser, 1966). Loss leaders may also be used by a retailer to attract consumers imperfectly informed about prices of items sold in their shop. This idea is formalized by Gerstner and Hess (1987) or Lal and Matutes (1994). Here, loss leaders are used through advertisements to extract a greater part of consumers surplus. Following the same basic idea, Walsh and Whelan (1999) prove that when retailers are differentiated by their location around a circle and when consumers have information about prices of some of the products but not all, retailers sell at a loss some products whose prices are known by consumers to attract them, and then set their monopoly prices for some other goods. In such a case, to resell goods at a loss is a way to compensate consumers for their imperfect information. The latter explanation does not rely on competition. A retailer sells several complements and substitutes goods, and thus, may find profitable to sell some goods at below cost prices to increase the demand for complements goods sold with positive margins (see Ramsey, (1927), Bliss (1988) and Chambolle, (2004)). Hence, forbidding resale at a loss as a predatory pricing strategy may appear as a too strong regulation.

In France the ban of resale at a loss has been introduced by the Galland law in 1996. It is clarified that a retailer can’t set the price of a good below the unit price invoiced by the supplier of this good, this unit price being defined as the one appearing on the bill, and excluding all the anticipated rebates and reductions that are not included in the invoice. Since then, for instance, the retailers cannot deduce from this price the anticipated amount of listing fees the producers usually pay at the
end of the year, and which are in proportion to the size and quality of shelf space dedicated to their products in the stores. Other reductions and rebates negotiated or paid at the end of the year are also excluded from the reference unit price. Yet these rebates forced by the buyer after the original contract had been signed and agreed may make up a large part of the final price, as shown by table 1: this table splits up the total margin of the retailer (i.e. the difference between the unit resale price and the unit price really paid by the retailer to the producer) into "observable margins", i.e. the difference between the resale price and the wholesale price invoiced and appearing on the bill, and "hidden margins", i.e. the amount of the rebates and fees paid by the producer to the retailer. These margins are given on average, by industry. Negative figures (for cosmetics and drinks, in 1995) mean that the products were sold at a loss.

<table>
<thead>
<tr>
<th>Industry</th>
<th>1995</th>
<th>1997</th>
<th>1999</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grocery</td>
<td>(26; 74)</td>
<td>(19; 81)</td>
<td>(12; 88)</td>
</tr>
<tr>
<td>Fresh and dairy products, frozen items</td>
<td>(50; 50)</td>
<td>(46; 58)</td>
<td>(34; 66)</td>
</tr>
<tr>
<td>Cosmetics, cleaning products</td>
<td>(−6; 106)</td>
<td>(18; 82)</td>
<td>(14; 86)</td>
</tr>
<tr>
<td>Drinks</td>
<td>(−1; 101)</td>
<td>(18; 82)</td>
<td>(11; 89)</td>
</tr>
<tr>
<td>Others (non food products)</td>
<td>(61; 39)</td>
<td>(58; 42)</td>
<td>(56; 44)</td>
</tr>
</tbody>
</table>

Source: Ilec, France.

According to professionnals and consumers, the Galland law had inflationary effects in France. A first statistical measure of this effect, led by Nielsen’s consumer panel for the retailer Leclerc\(^2\), launched the debate: it showed an average increase of 4.14 % of the prices of 1500 items (mostly national brands) in two months, after the application of the law. This price increase was inequally supported by the different items: the highest increase was for the liquids\(^3\) (+5.27 %), grocery products (+4.64 %), and cosmetics products (+4.81 %) whereas it was less important for fresh and dairy products (+1.54 %) or other non-food products (+1.15%). Furthermore, these price increases were mostly on well known brands. On the contrary, some private labels prices decreased. A counter-test led by the DGCCRF (Competition Division of the Ministry of Economics) concluded that the increase was far less than what Nielsen showed (+0.5 % on average for the same period), but this study did not consider the same type of products: the panel of products included, for each item, one national

\(^2\)Linéaires n°1529, 1997, march, 6\(^{th}\).

\(^3\)This figure is to be considered with care as the taxes on alcohol increased over the same period.
brand, one private label, and one discount product. However, these two studies seem to indicate that the law had, on average, an inflationary effect.

This inflationary effect is twofold. Of course, the first direct effect of the law is to increase the resale price of the goods that were sold at a loss. This limited effect should be easily measurable, and can be compensated by decreases in the prices of other goods, as it is simply due to a new constraint in a multiproduct optimal pricing problem. But the ban could also have other indirect effects, relying on the ability of producers to use the ban as a mean to fix minimum resale prices, or “floor prices”. The ban of resale at a loss might then turn into a facilitating practice, and have indirect pro-collusive effects on producers’ competition, like other vertical restraints (see for instance Rey and Stiglitz, 1988 and 1995, and Rey and Vergé, 2003). This paper explores the supports of such a critics and thus brings an original analysis of below cost pricing retailers’ strategies in considering both interbrand and intrabrand competition within a vertical chain. The framework is developed in section 2. Section 3 develops the equilibrium analysis and exhibits the indirect effect of the ban, which is inflationary when the products are complements. Section 4 proposes an extension where producers collude in a cartel: in that framework, the ban can be inflationary even when the goods are substitutes. Section 5 introduces two-part tariffs listing fees. Section 6 concludes.

2 The model

We consider the market for two horizontally differentiated goods produced by two producers, A and B. The producers cannot sell directly to the consumers and have to sell the goods through a downstream independent retail industry, where two retailers 1 and 2 are competing. We denote $q_{Ki}$ the quantity and $p_{Ki}$ the price of good $K$ ($K \in \{A, B\}, K \neq L$) sold by the retailer $i$ ($i \in \{1, 2\}, i \neq j$) on the market. We assume that the representative consumer has a quadratic utility function given by:

$$U(q) = \sum_{K,i} q_{Ki} - \frac{1}{2} \sum_{K,i} q_{Ki}^2 - a \sum_i q_{Ai} q_{Bi} - b \sum_K q_{K1} q_{K2} - c \sum_K q_{K1} q_{L2}$$

(1)

Normalizing the revenue, we thus derive the inverse demand for good $K$ sold by
the retailer $i$ as follows:

$$p_{Ki} = 1 - q_{Ki} - aq_{Li} - bq_{Kj} - cq_{Lj} \text{ with } i \neq j \text{ and } K \neq L$$

(2)

Parameter $a$ measures the degree of substitutability of the products (interbrand competition): products are substitutes if $a > 0$, and complements if $a < 0$. We assume that $|a| < 1$. Parameter $b$ represents the intrabrand competition when the two retailers offer the same product $K$ and we assume that $b \in [0, 1]$. Finally, we assume that the competition between different products sold through different retailers is represented by a parameter $c$ combining both inter-brand and intra-brand competition. For simplicity reasons, we assume that $c = a.b$. However, our results would qualitatively not change without this assumption.

We assume that retailers compete in prices. Without loss of generality we normalise producing and retailing costs to zero. To represent vertical contracting between producers and retailers, we use a two-stages model focusing on the real timing of vertical negotiations. As in most countries, the commercial law requires general terms of sale to be published and non-discriminatory, we assume that (unit) wholesale prices are set independently by the producers and published before any negotiation with the retailers. Wholesale price $w_K (K \in \{A, B\})$ is then the same for both retailers 1 and 2. Once the wholesale prices are published, each retailer secretly bargains with each supplier over the listing fees transferred from the producer to the retailer. These fees are assumed to be proportional to the quantities exchanged, and paid after some delay, for instance at the end of the year as it is usual: this implies that they cannot be integrated to the threshold of resale at a loss. The reference price constituting the threshold of resale at a loss for good $K$ is thus the wholesale unit price $w_K$. We assume that the fees are secretly and bilaterally negotiated: this assumption seems consistent with the reality of vertical negotiations (Allain and Chambolle, 2003). Following Horn and Wolinsky (1988), we assume that the firms have “passive beliefs”: if producer $K$ and retailer $i$ do not come to an agreement, the disagreement point corresponds to a situation where all the other pairs operate at the anticipated equilibrium level. This assumption is common in literature on secret multilateral negotiations\(^4\). In the last stage wholesale prices and listing fees are common knowledge, and firms interact on the product market. The timing of the game is as follows:

Stage 1: Producers simultaneously set their wholesale prices $w_A$ and $w_B$. 

\(^4\)For a detailed presentation of different sets of beliefs and among others the passive beliefs, see McAfee and Schwartz (1994).
Stage 2: Listing fees \( f_{Ai} \) and \( f_{Bi}, \ i \in \{1, 2\} \) are secretly and bilaterally negotiated through a Nash bargaining. Both producers have the same exogenous bargaining power denoted \( \alpha \ (\alpha \in [0, 1]) \).

Stage 3: Retailers compete in prices.

This game is now solved in the following section.

3 Equilibrium analysis: the basic mechanisms

In this section, we compare the equilibria in the game with regulatory constraint (no resale at a loss) to those in the benchmark case (without regulatory constraint).

3.1 Equilibrium in the game with no restriction

We solve the game for its symmetric subgame-perfect Nash equilibria. The last stage of the game determines the optimal retail prices as a function of wholesale prices \( w_A \) and \( w_B \), and of the four values of the listing fees \( f_{Ki}, \ K \in \{A, B\}, \ i \in \{1, 2\} \). The resolution of the second-stage Nash program gives the optimal values of the listing fees (see appendix A1). Interestingly, the anticipated profits of the producers in the first stage do not depend on the wholesale prices: there is a continuum of solution pairs \((w^*_K, f^*_Ki)\) for \( K \in \{A, B\} \) and \( i \in \{1, 2\} \) with \( w^*_A = w^*_B \) satisfying the Nash conditions. All the solutions lead to the same transfer from retailer \( i \) to producer \( K \)

\[
w^*_K - f^*_Ki\]

There is no commitment value of the first stage of the game, as the outcome of the game is completely determined in the second stage by the negotiation of the listing fees. Downstream price at \( i \)'s store is then the same for both goods \( A \) and \( B \), it is positive and smaller than 1:

\[
p^*_Ki = 1 + \frac{a(1-a)-1}{(2-b)(1-a+a)}.
\]

Depending on the value of \( f^*_Ki \), \( p^*_Ki \) may be higher or lower than the wholesale price \( w^*_K \): this equilibrium may be with or without resale at a loss. Moreover, whatever the wholesale prices, the outcomes for the two producers, on the one hand, and for the two retailers on the other hand are the same.

The equilibrium unit price a retailer \( i \) has to pay to a producer \( K \), \( w^*_K - f^*_Ki \), is a strictly increasing function in \( \alpha \). The higher \( w^*_K - f^*_Ki \) is, the higher the final price \( p^*_Ki \) is, according to the double-marginalization effect. Thus, the final price is also a strictly increasing function of the producer’s power \( \alpha \).
3.2 Forbidding resale at a loss

Let us now consider the case where resale at a loss is forbidden. The pricing strategies of the retailers are then constrained: they have to set retail prices higher than the wholesale prices. We characterise situations where it is profitable for the firms to use this constraint strategically.

**Lemma 1** The only constrained equilibrium candidate is such that the producers behave like vertically integrated firms: they fix the wholesale prices that maximise the vertical structures’ profits, and the retailers set zero margins.

**Proof:** see appendix A2. ■

Prices are thus \( w_A = w_B = p = \frac{1-a}{2} \), with \( p \) the retail price of all the goods. This partial result is quite intuitive, as if the constraint is binding, then retailers set retail prices equal to the wholesale prices and get profit only through the listing fees. The producers behave then as vertically integrated firms and internalise intrabrand competition: the price depends on \( a \) but no more on \( b \). We denote this strategy as "floor pricing" strategy. This strategy maximises the profit of each vertically integrated structure, but does not achieve the optimal profit of the whole industry as producers still behave competitively. Complete collusion prices chosen by an integrated firm owning both products as well as both outlets would be \( p_{K_1} = \frac{1}{2} \).

**Lemma 2** The constrained equilibrium candidate exists only if producers have few bargaining power.

**Proof:** see appendix A3. ■

For small values of \( \alpha \), the optimal constrained wholesale price is higher than the optimal non constrained retail price, so that the constraint is really binding. This lemma is quite intuitive since on the one hand the optimal constrained wholesale price is independent on \( \alpha \), while on the other hand the optimal final price in the unconstrained case is an increasing function of the producers’ bargaining power. The optimal constrained wholesale price is also the price that maximises each vertical structure’s profit: as such, it does not depend from the sharing of this profit among the upstream firm and its retailers. On the contrary, in the unconstrained case, the optimal resale price is an increasing function of the producers’ bargaining power: the unit net margin of producer \( K \), \( \bar{w} - \bar{f} \), increases with \( \alpha \). Yet this increase is partially passed on to the consumers by the retailers who set higher resale prices, increasing
their margins to the detriment of the total profit of the industry. This is a classical double-marginalization effect.

To know whether this candidate is indeed an equilibrium, it has to be profitable for the producers to choose the associate values of the wholesale prices in the first stage of the game. We study the profitability of the strategy in appendix.

**Lemma 3** The floor pricing strategy is profitable for the producers only if their bargaining power is high.

**Proof**: see appendix A4. ■

This result comes from the fact that, for small values of $\alpha$, even if the floor pricing strategy is profitable for the whole industry, the retailers gain more from this strategy than the producers. This comes from the combination of two effects.

The first one translates the asymmetry in the degree of competition upstream and downstream: in this model, for $a = b$, competition is indeed fiercer downstream than upstream. If producers have no bargaining power ($\alpha = 0$), the negotiation in the first stage leads to a situation where the producers set zero-margins and get no profit, whereas the retailers get all the profit. Everything happens as if retailers were vertically integrated and get the input at zero cost. On the contrary, if $\alpha$ gets close to 1, the producers do not manage to absorb the whole profit, as after the negotiation, retailers can set a strictly positive margin, and thus get a positive profit. The imperfection of downstream competition reduces the competition between producers in the first stage: the manufacturers perceive a less elastic demand than when they directly compete, or when the retailers are perfectly competitive. Everything happens as if competition were stronger among retailers than among producers, and thus, when producers have all the bargaining power, they manage to reach a higher level of total profit than when the retailers have all bargaining power. This effect leads the sum of upstream and downstream unconstrained profits to increase with $\alpha$.

The second effect is that the double marginalization inefficiency increases with $\alpha$: for $\alpha = 0$, there is no double marginalization as producers set zero margins; but for $\alpha = 1$, both upstream and downstream firms set positive margins, and the double margin inefficiency reduces the total profit.

For small values of $\alpha$, the first effect dominates the second, and this is the reason why the sum of upstream and downstream unconstrained profits increases with $\alpha$, at least for small values of $\alpha$. In that framework, producers’ profit is more sensitive to $\alpha$ than retailers’ profit: the share of total profit captured by the producers varies from
0 for \( \alpha = 0 \) to \( \frac{2}{3} \) for \( \alpha = 1 \). The negotiation thus leads the producers to get a share of the total profit that is higher than \( \alpha \), for small values of \( \alpha \). On the contrary, in the constrained case, producers get a share of total profit that is exactly \( \alpha \). Consequently, producers’ profit is higher in the unconstrained case than in the constrained case for small values of \( \alpha \), and gets smaller for high values of \( \alpha \) : the floor pricing strategy is profitable for the producers only for high values of \( \alpha \).

To know whether this strategy can appear in equilibrium, we now have to find in which cases the floor pricing strategy can exist and is profitable for the producers.

**Proposition 4** The “floor pricing” strategy never appears in equilibrium.

**Proof**: see appendix A5.

Notice that the floor pricing strategy would increase the total profit of vertical structures, but the gains would be completely shifted onto retailers and producers would get smaller profits than with standard pricing. In our framework, the ban of resale at a loss thus cannot be used as a mean to increase the total profits of the industry to the detriment of the consumers.

## 4 Collusion among producers

Let us now turn to the case where, rather than competing, producers collude to fix their wholesale prices in the first stage of the game. We assume that producers and retailers still bargain bilaterally in the second stage. We focus at collusion among producers, in a framework where downstream firms cannot collude and still fix the resale prices competitively. This situation can arise either if upstream collusion is explicit (i.e. within an explicit upstream cartel), or even in a tacit collusion framework, in an infinitely repeated game where producers have an infinite horizon whereas retailers live only a finite time and are periodically replaced (see appendix 8 for a discussion of the sustainability of collusion in that dynamic setting).

As we mentioned in the previous section, when resale at a loss is not forbidden, the equilibria do not depend on the wholesale prices chosen by the producers in the first stage. Thus whether the producers compete or collude to set their wholesale prices in the first stage of the game, equilibria are unchanged. The inability of producers to collude when they bargain with retailers just hinders them from achieving the collusion outcomes.
Lemma 5  When producers and retailers secretly and bilaterally bargain over hidden margins, producers are unable to collude over wholesale prices.

This result comes from the timing of the game. In fact, interbrands competition is realized through the bargaining with retailers over listing fees rather than through the wholesale price fixing at the first stage of the game. The wholesale price fixing by producers is neutral since equilibria of the unconstrained game are only determined by ex post bargaining over listing fees. This is an interesting property of negotiated hidden margins, since in a more classical framework with take-it-or-leave-it linear wholesale pricing, collusion among producers would be possible and would lead to an inefficient resale price $p \in \left[\frac{1}{2}, \frac{3}{4}\right]$ depending on the strength of downstream competition.

On the contrary, if resale at a loss is forbidden, the wholesale prices chosen by producers may determine equilibria if the constraint is binding. Hence, if producers collude rather than compete, this may modify equilibria outcomes. When producers collude, the prices maximizing the sum of their profits are $\tilde{w}_K^C = p^C = \frac{1}{2}$, where the upper script $C$ denotes the collusion outcomes. We naturally have $\tilde{w}_K^C > \tilde{w}_K$, since when they collude, each producer internalizes the marginal effect of the level of his wholesale price on the other producer’s profit. Thus, in the collusion case, the vertical structure’s profit is higher than in the competition case. When the constraint is binding, retailers set the final prices at the wholesale prices level ($\tilde{p}^C = \tilde{w}_K^C$), and thus, because resale at a loss is forbidden, producers are able to maintain this collusion price at the downstream level.

Lemma 6  When producers collude in the first stage of the game, the constrained equilibrium candidate exists if retailers’ bargaining power is less than a threshold $\alpha^C_e(a, b)$.

Proof: see appendix A6.

The value of the threshold is increasing in $a$ and $b$. When the producers have relatively few bargaining power, i.e. $\alpha \leq \alpha^e(a, b)$ the optimal constrained wholesale price is higher than the optimal non constrained retail price, so that the constraint is really binding. This constraint is more binding when intra- or interbrands competition is more intense: this comes from the fact that collusion is more profitable in these cases. To know whether this candidate is indeed an equilibrium, it has to be profitable for the producers to choose the associate values of the wholesale prices in the first stage of the game. We show that this strategy is now profitable for the producers when the goods are close substitutes, and thus the floor pricing strategy appears in equilibrium for high values of $a$. 

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Proposition 7  When producers collude in the first stage of the game, collusion with “floor pricing” strategy emerges in equilibrium if and only if producers’ bargaining power and interbrands competition are intermediate: $1 - b \leq a \leq 1 - b/2$ and $\alpha^p_C \leq \alpha \leq \alpha^v_C(a, b)$.

Proof : see appendix A6. ■

Notice however that when the goods are close substitutes, the “floor pricing” strategy increases the total profit of vertical structures, but for the producers to find this strategy profitable, their negotiation power has to be high enough (see appendix A6). For sufficient competition among products ($a$ high enough), there exists some intermediary values of the parameter $\alpha$ such that the floor pricing strategy exists and is profitable for the producers: floor pricing appears in equilibrium. The discussion of the sustainability of collusion in an infinitely repeated game with a discount factor $\delta$ is discussed in appendix A8.

5 Two-part listing fees

In this section, the listing fees are assumed to be two-part tariffs: the marginal component is denoted $f'^*_Ki$ and the fixed fee $F'^*_Ki$. Just as with linear tariffs, there is no commitment value for the wholesale prices in the first stage of the game since the outcome is completely determined in the second stage by the bargaining over the listing fees. Here, the producers’ negotiation power $\alpha$ has no influence on the level of $f'^*_Ki$. Thus, whatever the producers’ bargaining power towards retailers, the final price and thus the sum of all profits remain the same. The level of $\alpha$ just determines the part of the vertical structures’ profit the producers capture. Since with the fixed part $F'^*_Ki$ the producers capture a part of the vertical structure profit, the level of $f'^*_Ki$ simply maximizes the vertically integrated structure’s profit. On equilibrium, $f'^*_Ki$ would be zero if products were perfect substitutes and positive as long as $a \in [0, 1]$. Moreover, when retailers buy the goods at a strictly positive unit cost, the final prices they set are higher than if this buying unit price were zero. Indeed, a positive $f'^*_Ki$ reduces the downstream retailers competition and thus increases the vertical structure’s profit. When resale at a loss is forbidden, the equilibrium is unchanged whether the listing fees are linear or two-parts tariffs and whether the producers compete or collude. $^5$A similar result is obtained in Shafer (1991).
Proposition 8 When producers compete, equilibrium with “floor pricing” never exists. When producers collude, equilibrium with “floor pricing” appears if products are substitutes and intra-brand and inter-brands competition parameters $a$ and $b$ are high enough.

Proof: see appendix A7.

The result obtained when firms bargain over linear listing fees are thus extended to two-part listing fees. Here again, forbidding resale at a loss may be used by producers to practice collusion at their upstream level but also, through a price floor mechanism that hinders competition between retailers, at the downstream level. This strategy allows to eliminate the double-marginalization effect since the competition between retailers disappears and to raise the vertical structures profits.

6 Conclusion

This paper shows that in a vertical structure with imperfect competition at upstream and downstream levels, colluding producers supplying retailers with substitute goods may find profitable to choose a price high enough to become a floor price for retailers constrained by the loss leading banning law. This mechanism allows colluding producers to raise final prices up to the price that would be chosen by an industry where both producers and retailers collude over the final prices. This strategy allows producers both to eliminate double-marginalization effects and to improve vertical structures profits and in particular the part of industry profit the producers may capture. The resale at a loss banning is thus used as a ”resale price maintenance” (or at least a floor price) vertical restraint since producers can force retailers to set their final prices to the wholesale price they choose. The latter thus prefer renouncing to their observable margin for higher hidden margins. In that case, the loss leader banning induces an increase of final prices, hurting consumers’ surplus. Our analysis thus allows to sustain the idea that the resale at a loss banning may have inflationary effects. However, such an indirect effect of the ban does not appear when producers compete, whether the products are substitutes or complements. These results are robust for two-part listing fees.
References


A Appendix

A.1 Equilibria of the game without constraint

We solve the game by backward induction. We look for symmetric equilibria only. Consider the subgame where listing fees $f_{Ki}$, $K \in \{A, B\}$, $i \in \{1, 2\}$ and both wholesale prices $w_A$ and $w_B$ are fixed. Each retailer $i$ anticipates downstream demands $q_{Ki}(p_{Ki}, p_{Li}, p_{Kj}, p_{Lj})$, and maximises her (concave) profit:

$$\max_{p_{Ai}, p_{Bi}} \Pi_i = (p_{Ai} - w_A + f_{Ai})q_{Ai} + (p_{Bi} - w_B + f_{Bi})q_{Bi}.$$ 

The sufficient first order conditions determine the optimal prices $p_{Ki}$ ($\{K, L\} = \{A, B\}, \{i, j\} = \{1, 2\}$) chosen by the retailers, as functions of $(w_K - f_{Ki})$. The second stage of the game is the Nash-bargaining over the listing fees. The Nash program of the negotiation between producer $K$ and retailer $i$ is as follows:

$$\max_{f_{Ki}} \Pi_K - \Pi_i^{sq} \alpha (\Pi_i - \Pi_i^{sq})^{1-\alpha}$$

where $\alpha$ is the exogenous Nash bargaining power of the producer and $(1 - \alpha)$ the exogenous Nash bargaining power of the retailer, $\Pi_K$ (resp. $\Pi_i$) is the profit of producer $K$ (resp. retailer $i$) and $\Pi_K^{sq}$ (resp. $\Pi_i^{sq}$) is the *statu quo* profit earned by producer $K$ (resp. retailer $i$) if the negotiation breaks, *i.e.* if producer $K$ only deals with retailer $j$ (resp. retailer $i$ only deals with producer $L$). Following Horn and Wolinsky (1988), we assume that the firms have ”passive beliefs”: if producer $K$ and retailer $i$ do not come to an agreement, the disagreement point corresponds to a situation where all
the other pairs operate at the anticipated equilibrium level. The statu quo profits thus are:

\[
\Pi^\text{sq}_K = (w_K - f^*_K)q^*_K(w_K, w_L, f^*_K, f^*_L, f^*_L)
\]

\[
\Pi^\text{sq}_i = (p_L - w_L + f^*_i)q^*_i(w_K, w_L, f^*_K, f^*_L, f^*_L)
\]

The simplified bilateral Nash program in the unconstrained case is written:

\[
\alpha \frac{d\Pi_K}{df^*_K}[\Pi_i - \Pi^\text{sq}_i] + (1 - \alpha) \frac{d\Pi_i}{df^*_K}[\Pi_K - \Pi^\text{sq}_K] = 0
\]  

(3)

The resolution of the four Nash programmes gives a unique solution: given the value of the wholesale prices, the optimum listing fees are:

\[
f^*_K = w^*_K - \frac{a(1 - \alpha)}{1 - \alpha + \alpha}
\]  

(4)

These values fully determine the producers’ profits in the first stage. These profits do not depend from the wholesale prices, thus there is a continuum of pairs \((w_K, f^*_K)\) leading to the equilibrium: any value of \(w_A\) and \(w_B\) can be chosen, given that the value of the listing fee determined by the bargaining in the second stage will compensate this value. In particular, the listing fee can be negative if the producer chooses a low value of the wholesale price. Downstream price is then the same for both goods \(A\) and \(B\) at both retailers’ stores, and is denoted \(p^*\):

\[
p^* = 1 + \frac{a(1 - \alpha) - 1}{(2 - b)(1 - a + \alpha)}
\]

Profits realized by the producers and the retailers are, for \(K \in \{A,B\}\) and \(i \in \{1,2\}:\)

\[
\Pi^*_K = \frac{2a(1 - \alpha)(1 - a(1 - \alpha))}{(1 + \alpha)(1 + b)(2 - b)(1 - a + \alpha)^2}
\]

\[
\Pi^*_i = \frac{2(1 - b)(1 - a(1 - \alpha))}{(1 + \alpha)(1 + b)(2 - b)(1 - a + \alpha)^2}
\]

A.2 Constrained equilibria: proof of lemma 1

We solve the game under the constraint that retailers cannot resale any product at a loss. This constraint does not change anything for the equilibria of the subgames with \(w_K < p^*_K\), but all equilibria with \(w_K > p^*_K\) are destroyed by this constraint, and new equilibria may appear.
The candidates for these new "constrained" equilibria saturate the constraint. At the second stage of the constrained game, given the values of $w_A$ and $w_B$, the firms anticipate that the final prices will be $\tilde{p}_A = w_A$ and $\tilde{p}_B = w_B$. The simplified bilateral Nash program of the negotiation between producer $K$ and retailer $i$ is written:

$$\alpha \frac{d\tilde{\Pi}_i}{df_{KI}} [\tilde{\Pi}_i - \tilde{\Pi}_{i}^{eq}] + (1 - \alpha) \frac{d\tilde{\Pi}_K}{df_{KI}} [\tilde{\Pi}_K - \tilde{\Pi}_K^{eq}] = 0$$  \hspace{1cm} (5)$$

The resolution of the Nash program gives the following optimal listing fees:

$$\tilde{f}_{A1} = \tilde{f}_{A2} = (1 - \alpha) w_A$$
$$\tilde{f}_{B1} = \tilde{f}_{B2} = (1 - \alpha) w_B$$

In the first stage, anticipating the constraint, producer $K$ maximises his profit by fixing the wholesale prices that maximises the profit of the vertical structure $(K,1,2)$, given $w_L$. In equilibrium, the producers thus set the following wholesale prices:

$$\tilde{w} = \tilde{w}_A = \tilde{w}_B = 1 - \frac{1}{2 - a}$$  \hspace{1cm} (6)$$

We now have to verify that it is then optimal for the retailers to set $\tilde{p} = \tilde{w} = 1 - \frac{1}{2 - a}$. They will set zero margins only if they are on the decreasing side of their profit function. The constraint has to be actually binding for this candidate to be an equilibrium.

### A.3 Constrained equilibria: proof of lemma 2

We now have to verify that this candidate is indeed an equilibrium. First, we need to have $\tilde{w} > p^*$ (else the retailers would benefit from setting positive margins). We study the difference $\text{exist} = \tilde{w} - p^*$:

$$\text{exist} \geq 0$$
$$\Leftrightarrow \alpha \leq \frac{(1 - a)(b - a)}{2 - b - a(2 - a)}$$

If $b \leq a$, this condition is never fulfilled and $\tilde{w}$ is never higher than $p^*$, so that there is no possible constrained equilibrium. If $b \geq a$, we denote $\alpha_{\text{exist}} = \frac{(1 - a)(b - a)}{2 - b - a(2 - a)}$ and we have $\text{exist} \geq 0$ for all $\alpha \in [0, \alpha_{\text{exist}}]$. This is natural since $\tilde{p} = \tilde{w}$ is independent of $\alpha$, whereas
$p^*$ is a strictly increasing function of $\alpha$. This means that, in the unconstrained case, the higher the producers’ exogenous market power, the higher the net input prices $w_K - f_{Ki}$ and the higher the final price $f_i$ fixed by the retailers on the market.

The constrained equilibrium profits are:

\[
\tilde{\Pi}_K = \frac{2\alpha(1-a)}{(2-a)^2(1+a)(1+b)}
\]
\[
\tilde{\Pi}_i = \frac{(1-a)}{\alpha}\tilde{\Pi}_K
\]

A.4 Proof of lemma 3

We now compare producers’ profits in the constrained and unconstrained case, to determine which strategy they choose in the first stage.

We study $\tilde{\Pi}_K - \Pi_K^*$: the difference is positive for $a$ higher than a threshold $\alpha_P(a, b) \in [0, 1]$. The floor pricing strategy is thus profitable for producers only if their bargaining power is higher than $\alpha_P(a, b)$.

\[
\alpha_P(a, b) = \frac{-4 + 2b + 8a - 2ab - 4a^2 + a^3 + (2 - a)\sqrt{8 - 4b - 8a(2 - b) + 4a^2(3 - b) - 4a^3 + a^4}}{2(2 - b)}
\]

A.5 Proof of proposition 4

Yet we have shown in lemma 2 that $\alpha$ has to be smaller than $\alpha_e(a, b)$ for the constraint to be binding. As $\alpha_e(a, b) \leq \alpha_P(a, b)$ for all possible values of the parameters $(a, b)$, these conditions are incompatible: when the floor pricing strategy exists, it is never profitable for the producer: $\Pi_K^* > \tilde{\Pi}_K$.

A.6 Proof of proposition 7

Using the same method than in appendix A3, we show that the constrained strategy exists only if $\alpha \leq \alpha_C(a, b) = \frac{b(1-a)}{2 - 2a - b}$ (with $\alpha_C(a, b) > \alpha_e(a, b)$). Notice that for $a \geq 1 - \frac{b}{2}$, this threshold is negative so that the constrained strategy never exists.

The constrained strategy is profitable for colluding producers if:
\[ \alpha \geq \alpha^C_P = \frac{-4 + 8a - 4a^2 + 2b - 2ab + 4(1 - a)\sqrt{2 - 2a + a^2 - b + ab}}{2(2 - b)} \]

The constrained equilibrium exists if and only if \( \alpha^C_P \leq \alpha \leq \alpha^P_e(a, b) \), which is possible only if \( \alpha^C_P \leq \alpha^P_e(a, b) \) : this happens if and only if \( 1 - \frac{b}{2} \geq a \geq 1 - b \).

### A.7 Proof of proposition 8

The optimal prices \( p_{Ki} (\{K, L\} = \{A, B\}, \{i, j\} = \{1, 2\}) \) chosen by the retailers, as functions of \( (w_K - f_{Ki}) \) are the same as in the previous section since the fix fees do not change the first order conditions. However, in the second stage the Nash bargaining is influenced by the fix fee \( F_{Ki} \). In fact, in the first order conditions, the difference between equilibrium profit and status quo of both the producer and the retailer involved in the bargaining. The equilibrium two part listing fees are:

\[ F'_{Ki} = \frac{1 - a - \alpha (2 - a - b)}{(1 + a)(1 + b)(3 - a - b)^2} \]  
\[ f'_{Ki} = w'_{Ki} - \frac{(1 - a)}{(3 - a - b)} \]  

The equilibrium marginal component \( f_{Ki} \) does not depend on producers’ negotiation power \( \alpha \). On the contrary, the equilibrium fix fee \( F_{Ki} \) decreases in \( \alpha \).

Since final prices only depend on \( f_{Ki} \), their level will no more be influenced through \( \alpha \).

\[ p'_{Ki} = 1 - \frac{1}{(3 - a - b)} \]  

And producers’ and retailers’ profits are:

\[ \Pi^*_A = \Pi^*_B = \frac{2\alpha (2 - a - b)}{(1 + a)(1 + b)(3 - a - b)^2} \]  
\[ \Pi^*_i = \Pi^*_2 = \frac{(1 - \alpha)}{\alpha} \Pi^*_A \]

Let us now turn to the constrained case.

Just like in the previous section, equilibria with \( p'_{Ki} > w_K \) are destroyed by this constraint, and new equilibria may appear.
Candidates for constrained equilibria verify:

\[
\begin{cases}
  p_A \leq w_A \\
  p_B \leq w_B
\end{cases}
\]  

(11)

The Nash bargaining is changed by the two-part tariff fee assumption. The optimum listing fees are such that:

\[
\begin{align*}
\tilde{f}_{A1} &= \tilde{f}_{A2} = (1 - \alpha) w_A - \frac{F_{Ai}}{q_{Ai}} \\
\tilde{f}_{B1} &= \tilde{f}_{B2} = (1 - \alpha) w_B - \frac{F_{Bi}}{q_{Bi}}
\end{align*}
\]

Thus in the second stage, there are an infinite number of two-part tariff equilibria. Replacing the optimum listing fees in the producers’ profit functions, we find that the optimal producers’ wholesale prices and profits are the same as those emerging without the fix fee. If, instead of competing, the producers collude, the wholesale prices and profits also remain unchanged with those emerging without the fix fee.

Proof of proposition 7:

A constrained equilibrium exists if and only if \( \tilde{w}_K - p_{Ki}^* > 0 \). Using (9) and (6), we prove that a constrained equilibrium exists if \( \frac{(b-1)}{(2-a)(3-a-b)} > 0 \). Thus whatever \( a \in ]-1,1[ \) and \( b \in [0,1] \), a constrained equilibrium never exists.

In case of collusion, a constrained equilibrium exists if and only if \( \tilde{w}_K^C - p_{Ki}^C > 0 \). We prove that a constrained equilibrium exists if \( a+b-1 > 0 \). Moreover, this strategy is always profitable for producers since \( \Pi_{K} - \Pi_{K}^* = \frac{\alpha(1-a-b)^2}{(1+a)(1+b)(3-a-b)^2} > 0 \).

A.8 Collusion sustainability

We consider the infinitely repeated game of described in section 4, and assume that all the firms have the same discount factor \( \delta \). In that setting, the sustainability of collusion classically depends on the value of \( \delta \).

CASE 1:

We first turn to the case where the constrained equilibrium exists when both producers collude and when one producer deviates. Let \( \alpha^d_e \) be the limit value of \( \alpha \) such that the deviating price \( w_{Ki}^d = \frac{2-a}{4} \) is equal to the final price \( p^* \) in the competition game. Thus \( \alpha^d_e = \frac{(1-a)(a(2-b)-2b)}{(a(2+b)-2(2-b))} \).
Then, when \( \alpha_C \leq \alpha \leq \min[\alpha_C, \alpha_e] \), the constrained equilibrium exists when both producers collude and when one producer deviates. This zone exists if and only if the following conditions on subsituability parameters hold:

\[
\begin{align*}
\alpha_C \leq \alpha_C &\quad \Rightarrow \quad 1 - a \leq b \\
0 \leq \alpha_C &\quad \Rightarrow \quad b \leq 2 - 2a \\
\alpha_C \leq \alpha_C &\quad \Rightarrow \quad \frac{2(2+2a-a^2)}{(2+a)^2} \leq b
\end{align*}
\]

In this interval, the tacit collusion game may be represented by the following matrix:

\[
\begin{array}{ccc|c|c}
B & A & NC & C & C \\
NC & \left(\frac{2a(1-a)(1-a(1-a))}{(1+a)(1+b)(2-b)(1-a+a)^2}, \frac{2a(1-a)(1-a(1-a))}{(1+a)(1+b)(2-b)(1-a+a)^2}\right) & \left(\frac{\alpha(2-a(2+a))}{(1-a)^2(1+b)}, \frac{\alpha(2-a)^2}{8(1-a^2)(1+b)}\right) \\
C & \left(\frac{\alpha(2-a)^2}{8(1-a^2)(1+b)}, \frac{\alpha(2-a(2+a))}{(1+a)(1+b)(2-b)(1-a+a)^2}\right) & \left(\frac{\alpha}{2(1+a+b+ab)}, \frac{\alpha}{2(1+a+b+ab)}\right)
\end{array}
\]

In an infinite game, with a discount factor \( \delta \), we show that collusion is sustainable if:

\[
\delta \geq \hat{\delta}_1(\alpha, a, b)
\]

Notice that \( \hat{\delta}_1(\alpha, a, b) \) is increasing in \( b \) and in \( a \): collusion becomes more difficult when products and shops become closer substitutes, as the degree of competition increases and the deviation profit become higher. Furthermore, \( \hat{\delta}_1(\alpha, a, b) \) is decreasing in \( \alpha \): producers’ bargaining power facilitates collusion. To illustrate case 1, we show that for \( a = 0.5, b = 0.9 \) and \( \alpha = 0.8 \), collusion is sustainable for \( \delta \geq 0.798 \).

CASE 2:

We now turn to the case where the constrained equilibrium exists when producers collude but not when one producer deviates. This happens if and only if \( \max[\alpha_C, \alpha_e] \leq \alpha \leq \alpha_C \).

This zone exists if and only if the following conditions on subsituability parameters hold:

\[
\begin{align*}
\alpha_C \leq \alpha_C &\quad \Rightarrow \quad 1 - a \leq b \\
0 \leq \alpha_C &\quad \Rightarrow \quad b \leq 2 - 2a \\
\alpha_C \leq \alpha_C &\quad \Rightarrow \quad \frac{2(2-a)}{2+a} \leq b \text{ and } a \geq 2/3
\end{align*}
\]

Yet \( \frac{2(2-a)}{2+a} \geq 2 - 2a \), so that these conditions are not compatible. This zone never exists.

CASE 3: If \( \alpha \in [\alpha_C, 1] \)

In this interval, the collusion equilibrium does not exist. The tacit collusion is not sustainable.