A dynamic limit order market with fast and slow traders

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HFT Conference Paris, 18-19 April 2013

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1 The views expressed are those of the author and do not necessarily reflect those of the ECB.
A very stylized model that helps to think about HFT

Dynamic Limit Order Market
- Traders choose endogenously between MO and LO
- Private gains from trade
- LOs face the risk of being picked off

One additional ingredient: Speed
- Traders are fast (FTs) or slow (STs)
- Being fast helps to avoid adverse selection

Efficiency, trading profits, order flow, social welfare
Key findings

- Introducing differences in speed affects the realization of gains from trade in two ways
- FTs face a lower risk of being “picked off”
  - FTs obtain higher profits from posting limit orders (outside option)
  - Reduced “order shading” leads to more trade
- STs face some traders with higher bargaining power
  - Affects trade-off between execution probability and profits conditional on execution
  - Trade decreases due to more cautious quotes
- Overall effect is positive unless there are few FTs and adverse selection is low
Key findings

- FTs endogenously arise as “makers”
  - submit more LOs
  - are more likely to trade “passively” (and more so for large $\sigma$)
- The presence of FTs decreases STs’ profits from LOs
- FTs execute MOs at better prices than STs
  - STs enjoy fewer profits from picking off stale quotes
  - STs are willing to accept worse quotes (lower outside option)
- In sum: STs are worse off
  - Social welfare loss with endogenous $\alpha$ as in Biais et al. (2012)
  - Different channel: FTs avoid adverse selection
  - Externality: loss in bargaining power
- Quick remarks on policy proposals
Dynamic limit order market
Risk neutral agents arrive sequentially and choose between MO and LO
Asset follows random walk
\[ \nu_{t+1} = \nu_t + \varepsilon_{t+1}, \text{ where } \varepsilon_{t+1} \in \{-\sigma, +\sigma\} \]
Private gains from trade: \( y_t \in \{-L, +L\} \)
Some intuition

- The limit order market can be seen as a sequential bargaining game over a surplus of $2L$
- Agents either accept outstanding offers (via MO) or make an offer (LO) to the next trader
- The bargaining power is determined endogenously by the expected profits obtained from posting market orders $V^{LO}$ (outside option)
- Optimal quotes make agents indifferent between LO and MO
The role of adverse selection

- New information hits the market between trader arrivals
- Limit orders cannot be revised once posted (imperfect monitoring)
- News renders LOs stale (adverse selection)
- Two types of equilibria
  - High fill-rate ($\sigma < \bar{\sigma}$)
  - Low fill-rate ($\sigma \geq \bar{\sigma}$)
- The latter equilibrium is inefficient because gains from trade are realized less frequently
Adding speed

- News lead to a “race” between traders
  - LO trader wants to revise outstanding order
  - MO trader wants to grab stale quote
- In the Foucault model, the MO trader always wins
- Now suppose that some agents are faster than others
- Let $\alpha$ denote the proportion of FTs
- Assumption: MO traders always win unless they are slower than LO traders
  - FTs can revise limit orders if the next agent is a ST
  - FTs cannot revise limit orders of the next agent in a FT
  - STs continue to be unable to revise orders
Empty Limit Order Book

\[ t \]

**Fast Buyer**

\[ \text{LO} \]

Innovation \( \varepsilon \)

**Fast Seller**

\[ \alpha \]

**Slow Seller**

\[ 1 - \alpha \]

Probability 1/2

\[ t+1 \]

Revised LO

\[ \text{LO} / \text{MO} \]
Strategies

- Obviously, being fast is valuable: \( V_{FT}^{LO*} > V_{ST}^{LO*} \)
- Hence LO execution depends also on the next trader’s type
- Relevant states at \( t+1 \) (provided a seller arrives)
  - \(-\sigma/ST, -\sigma/FT, +\sigma/ST, +\sigma/FT\)
- STs choose one quote \( B_{t,ST} \)
  - high or low fill-rate
  - specialized (only STs) or unspecialized (both STs and FTs)
- FTs choose initial and revised quotes \((B_{t,FT}, B_{t,FT}^{-\sigma}, B_{t,FT}^{+\sigma})\)

Lemma

In equilibrium, FTs revised bid quotes are given by
\[
B_{t,FT}^{-\sigma} = C_{ST}^{s^*}(v_t - \sigma) \quad B_{t,FT}^{+\sigma} = C_{ST}^{s^*}(v_t + \sigma)
\]
Equilibrium

Proposition

For fixed parameters \((\alpha, L, \sigma)\), there exists a unique Markov-perfect equilibrium in the limit order market. In equilibrium:

a) STs employ a high fill-rate strategy for \(\sigma < \sigma_{ST}^{*}(\alpha)\) and a low fill-rate strategy otherwise.

b) STs employ a specialized strategy for \(\alpha < \alpha_{S}^{*}(\sigma)\) and an unspecialized strategy otherwise.

a) FTs employ a high fill-rate strategy for \(\sigma < \sigma_{FT}^{*}(\alpha)\) and a low fill-rate strategy otherwise.

- Volatility \(\sigma\) induces order shading as in Foucault (1999)
- A low level of \(\alpha\) leads to specialized strategies
  - Specialized quotes are less likely to execute but yield higher profits conditional on execution
Limit order profits

- STs can react in two possible ways to the presence of FTs
  - quote more aggressively to attract both FTs (unspecialized strategy)
  - only target STs (specialized strategy) and accept decreased execution probability
  - Either way, expected profits are lower than without FTs ($\alpha = 0$)

**Corollary**

$$V^{LO*}_{FT} > V^{LO*}_0 > V^{LO*}_{ST} \text{ for all } \alpha \in (0,1)$$
On the equilibrium path, there are 4 possible events

- i) ST-LO
- ii) ST-MO
- iii) FT-LO
- iv) FT-MO

Stationary distribution: \( \varphi^* = (\varphi_{ST}^{LO*}, \varphi_{ST}^{MO*}, \varphi_{FT}^{LO*}, \varphi_{FT}^{MO*}) \)

Trading rate \( TR^* = \varphi_{ST}^{MO*} + \varphi_{FT}^{MO*} \)

Limit-to-market order ratio: \( LtM^* = \frac{\varphi_{ST}^{LO*} + \varphi_{FT}^{LO*}(2-\alpha)}{\varphi_{ST}^{MO*} + \varphi_{FT}^{MO*}} \)

Make-take ratio: \( MT^*_k = \frac{\varphi_k^{LO*}\pi_{k,ST}^{*} + \varphi_k^{LO*}\pi_{k,FT}^{*}}{\varphi_{ST}^{LO*}\pi_{ST,k}^{*} + \varphi_{FT}^{LO*}\pi_{FT,k}^{*}} \)
Trading rate

**Corollary**

The presence of FTs increases the trading volume except in a specialized high fill-rate equilibrium (i.e. if both $\sigma$ and $\alpha$ are sufficiently low)

- Ability to revise limit orders mitigates the inefficiency rooted in the adverse selection problem (more trade)
- Higher outside option of FTs induces order shading by STs (less trade)
- Empirically, the “advent” of HFT is associated with more trading (no causality though)
  - Chordia et al (2011)
  - Jovanovic and Menkveld (2011)
FTs are more likely than STs to submit limit orders ($LtM_{FT}^* > LtM_{ST}^*$) and their presence increases the overall message traffic ($LtM^* > LtM_0^*$).

- FTs mechanically submit more limit orders (revisions)
- Higher outside option lets FTs reject some quotes that STs find worth accepting
- Empirical evidence on AT/HFT message traffic
  - Hagströmer & Norden (2013), Malinova et al. (2012)
  - Hendershott et al. (2011)
Corollary

FTs are more likely than STs to trade via limit order, i.e. $MT_{FT}^* \geq 1 \geq MT_{FT}^*$. Moreover, $MT_{FT}^*$ ($MT_{ST}^*$) is increasing (decreasing) in $\sigma$.

- FTs’ ability to revise limit orders
  - Increases the chance of successful execution
  - Reduces the need for order shading

  - HFTs mostly trade passive, “natural” market makers
  - Passive HFTs faster than aggressive ones
  - Different if arbitrage opportunities can arise?
Market Orders

- Market order profits can be written as $V_{k}^{MO^*} = L - E(\tau^*_k)
- The transaction cost $E(\tau^*_k)$ reflects
  - bargaining power (outside option)
  - profits from “picking off” stale limit orders

**Corollary**

If $\sigma \in [\frac{8}{15}, \sigma]$ then $E(\tau^*_{ST}) > E(\tau^*) > E(\tau^*_0) > E(\tau^*_{FT})$ for all $\alpha \in (0, 1)$.

- FTs get better prices
  - Hendershott & Riordan (2012), Moallemi and Saglam (2011), etc.
- Speed discrepancies increase average trading costs
  - Not in line with most of the empirical literature
  - Difficult to disentangle speed from other benefits of automation
Welfare

- Now suppose that $\alpha$ is not exogenous but instead traders can become fast upon investing $c$ (as in Biais et al. (2012))
- Trading profits are weighted averages of $V_{k}^{LO*}$ and $V_{k}^{MO*}$
  \[
  W_{ST}^{*} = \frac{\phi_{ST}^{LO*}}{\phi_{ST}^{MO*} + \phi_{ST}^{MO*}} V_{ST}^{LO*} + \frac{\phi_{ST}^{MO*}}{\phi_{ST}^{MO*} + \phi_{ST}^{MO*}} V_{ST}^{MO*}
  \]
  \[
  W_{FT}^{*} = \frac{\phi_{FT}^{LO*}}{\phi_{FT}^{MO*} + \phi_{FT}^{MO*}} V_{FT}^{LO*} + \frac{\phi_{FT}^{MO*}}{\phi_{FT}^{MO*} + \phi_{FT}^{MO*}} V_{FT}^{MO*}
  \]
- Social Welfare is then given by
  \[
  W^{*}(\alpha^{*}) = (1 - \alpha^{*}) W_{ST}^{*}(\alpha^{*}) + \alpha^{*}(W_{FT}^{*}(\alpha^{*}) - c)
  \]
- The equilibrium level of investment satisfies $W_{ST}^{*}(\alpha^{*}) = W_{FT}^{*}(\alpha^{*}) - c$
Welfare

**Corollary**

Any positive equilibrium level of investment $\alpha^* > 0$ exceeds the socially optimal level $\alpha^+$ and moreover yields a social welfare loss, i.e. $W(\alpha^*) < W(0)$.

- Although FTs may help increase trade, STs are always worse off
- Same conclusion as in Biais et al. (2012)
  - Different channel: speed helps to avoid adverse selection
  - Externality: STs lose bargaining power
- Note: Corner solution $\alpha^* = 1$ is always inefficient (same outcome as for $\alpha = 0$)
- This does NOT imply that $\alpha^+ = 0$!
Ideally, one may want to implement $\alpha^+$ (which can be positive)

“Circulating” proposals

- Minimum resting times
- Limits on message traffic

This would curb HFT, but also the associated benefits

- In fact, quick order revisions are the reason for potential efficiency gains

Rather directly tax HFT activity?
Conclusions

- Introducing speed into a LOM with adverse selection has a number of effects
  - Speed partially eliminates “picking off” risks, but also makes STs more cautious
  - FTs emerge as makers, more likely to submit and trade via LO
  - FTs trade at more favourable prices than STs
  - STs face reduced profits due to lower bargaining power
  - Equilibrium investment is welfare reducing (externalities)

- Existing policy proposals probably sub-optimal
The 9th Annual Central Bank Workshop in Market Microstructure

- This year at the ECB in Frankfurt, 5-6 September (Th-Fr after EFA)
- Keynote: Darrell Duffie
- Policy Panel (Marco Pagano, Urich Bindseil)
- Key topics:
  - Fixed income markets (Money Markets, Bond Markets)
  - Long-run trends in MM, e.g. opacity, OTC vs. regulated markets, efficiency, automation
  - The impact of current regulatory initiatives on market structure, e.g. Transaction Taxes, Vickers/Volcker Rules, CCPs, LIBOR reform
- Submission deadline: April 30th (microstructure@ecb.europa.eu)