A dynamic limit order market with fast and slow traders

Peter Hoffmann[1]

European Central Bank

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[1]The views expressed are those of the author and do not necessarily reflect those of the ECB.

Intro

- A very stylized model that helps to think about HFT
- Dynamic Limit Order Market
  - Traders choose endogenously between MO and LO
  - Private gains from trade
  - LOs face the risk of being picked off
- One additional ingredient: Speed
  - Traders are fast (FTs) or slow (STs)
  - Being fast helps to **avoid** adverse selection
- Efficiency, trading profits, order flow, social welfare

Key findings

- Introducing differences in speed affects the realization of gains from trade in two ways
- FTs face a lower risk of being "picked off"
  - FTs obtain higher profits from posting limit orders (outside option)
  - Reduced "order shading" leads to more trade
- STs face some traders with higher bargaining power
  - Affects trade-off between execution probability and profits conditional on execution
  - Trade decreases due to more cautious quotes
- Overall effect is positive unless there are few FTs and adverse selection is low

Key findings

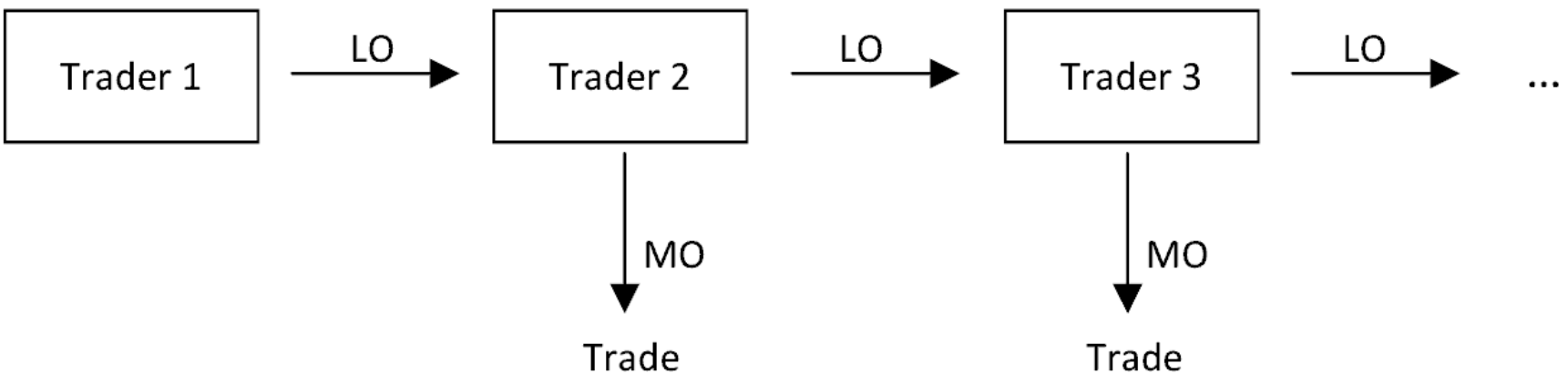
- FTs endogenously arise as "makers"
  - submit more LOs
  - are more likely to trade "passively" (and more so for large σ)
- The presence of FTs decreases STs' profits from LOs
- FTs execute MOs at better prices than STs
  - STs enjoy fewer profits from picking off stale quotes
  - STs are willing to accept worse quotes (lower outside option)
- In sum: STs are worse off
  - Social welfare loss with endogenous α as in Biais et al. (2012)
  - Different channel: FTs avoid adverse selection
  - Externality: loss in bargaining power
- Quick remarks on policy proposals

Setup - Foucault (1999)

- Dynamic limit order market
- Risk neutral agents arrive sequentially and choose between MO and LO
- Asset follows random walk
  $v_{t+1} = v_t + \varepsilon_{t+1}$, where $\varepsilon_{t+1} \in \{-\sigma, +\sigma\}$
- Private gains from trade: $y_t \in \{-L, +L\}$

Some intuition

- The limit order market can be seen as a sequential bargaining game over a surplus of $2L$
- Agents either accept outstanding offers (via MO) or make an offer (LO) to the next trader
- The bargaining power is determined endogenously by the expected profits obtained from posting market orders V^{LO} (outside option)
- Optimal quotes make agents indifferent between LO and MO

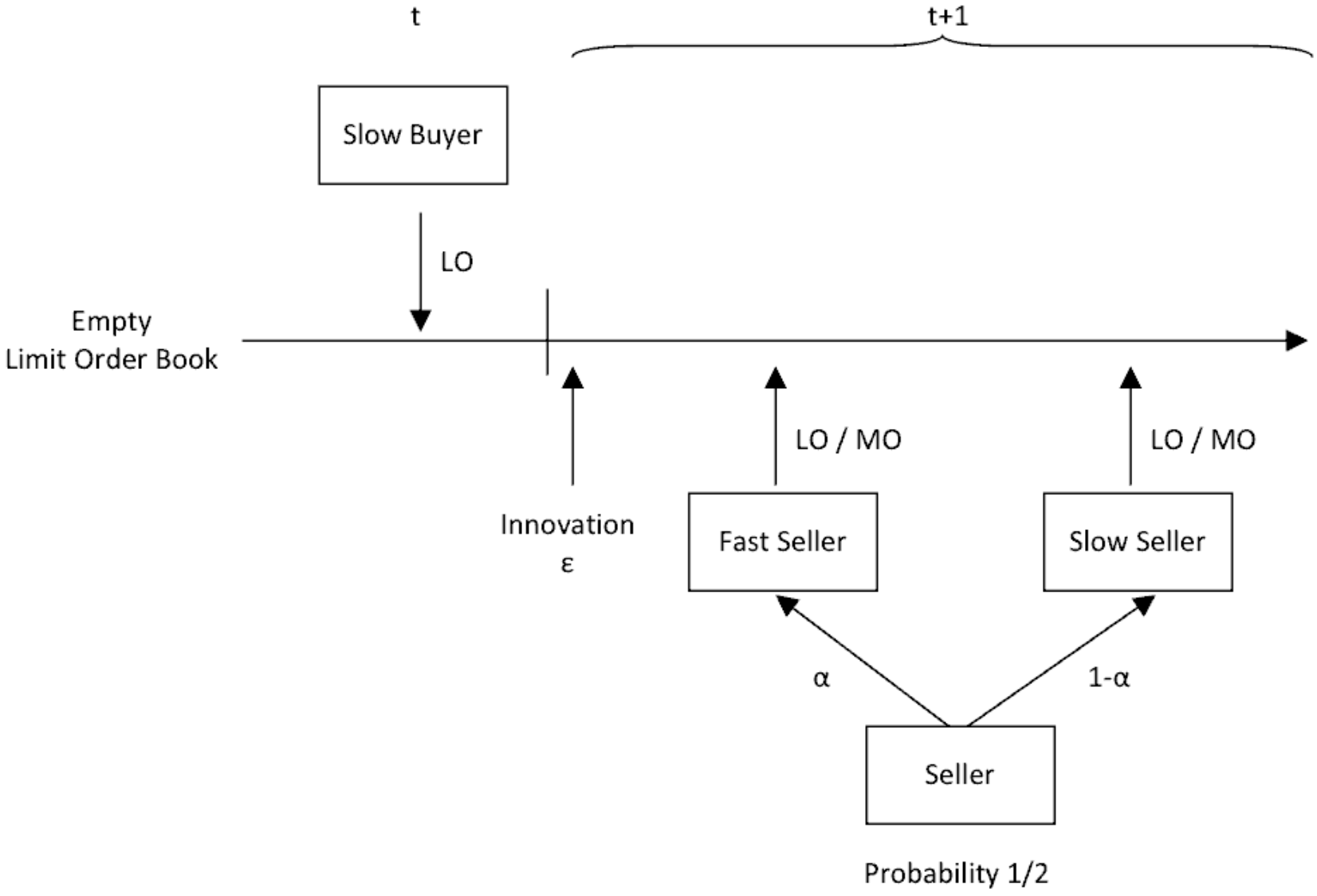


The role of adverse selection

- New information hits the market between trader arrivals
- Limit orders cannot revised once posted (imperfect monitoring)
- News render LOs stale (adverse selection)
- Two types of equilibria
  - high fill-rate ($\sigma < \bar{\sigma}$)
  - low fill-rate ($\sigma \geq \bar{\sigma}$)
- The latter equilibrium is inefficient because gains from trade are realized less frequently

Adding speed

- News lead to a "race" between traders
  - LO trader wants to revise outstanding order
  - MO trader wants to grab stale quote
- In the Foucault model, the MO trader always wins
- Now suppose that some agents are faster than others
- Let α denote the proportion of FTs
- Assumption: MO traders always win unless they are slower than LO traders
  - FTs can revise limit orders if the next agent is a ST
  - FTs cannot revise limit orders of the next agent in a FT
  - STs continue to be unable to revise orders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t+1
Slow Buyer
LO
Empty
Limit Order Book
Innovation
ε
LO / MO
LO / MO
Fast Seller
Slow Seller
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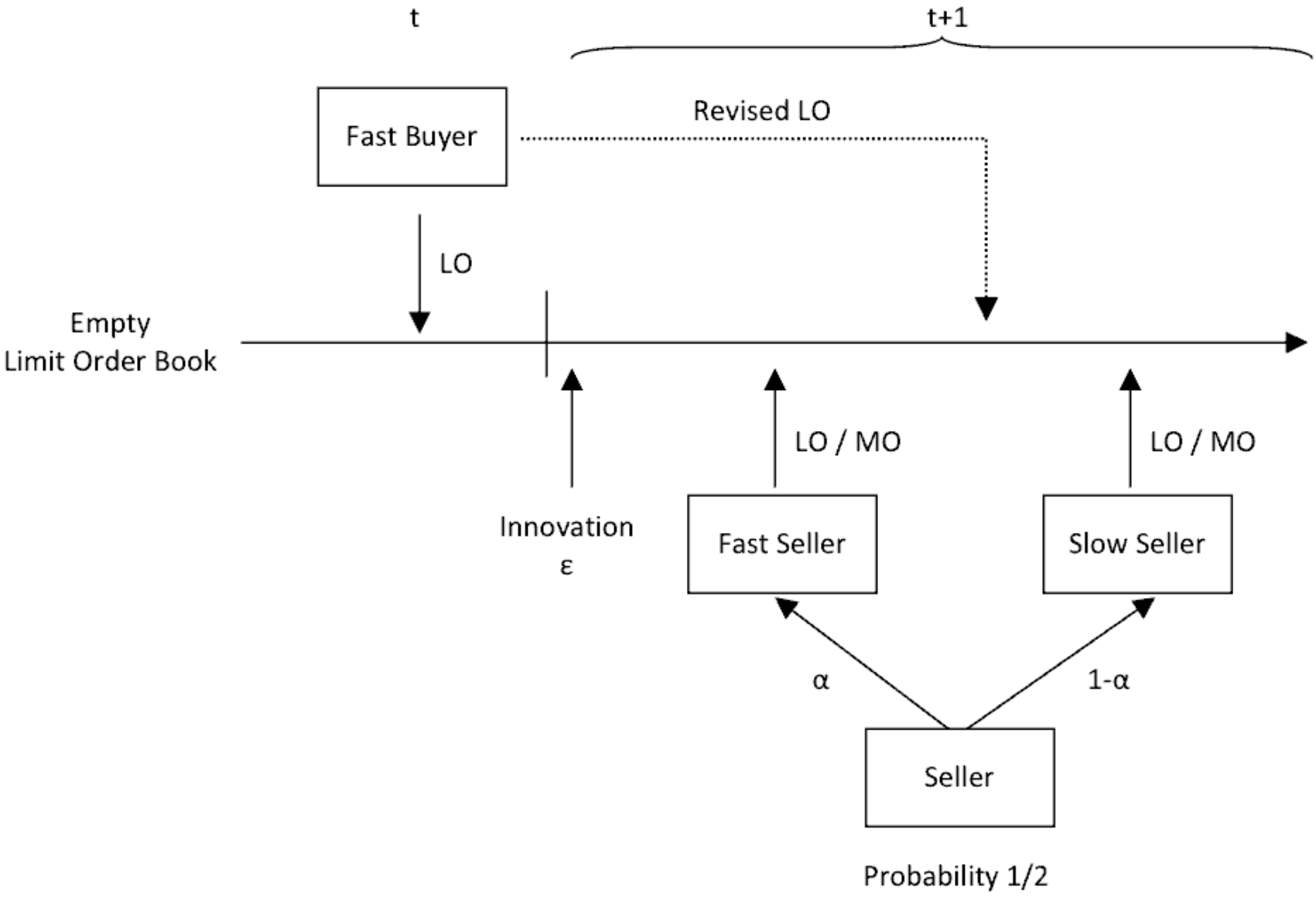
Seller
Probability 1/2


t
t+1
Fast Buyer
Revised LO
LO
Empty
Limit Order Book
Innovation
ε
LO / MO
LO / MO
Fast Seller
Slow Seller
α
1-α
Seller
Probability 1/2

Strategies

- Obviously, being fast is valuable: $V_{FT}^{LO*} > V_{ST}^{LO*}$
- Hence LO execution depends also on the next trader's type
- Relevant states at t+1 (provided a seller arrives)
  - $-\sigma/ST, -\sigma/FT, +\sigma/ST, +\sigma/FT$
- STs choose one quote $B_{t,ST}$
  - high or low fill-rate
  - specialized (only STs) or unspecialized (both STs and FTs)
- FTs choose initial and revised quotes $(B_{t,FT}, B_{t,FT}^{-\sigma}, B_{t,FT}^{+\sigma})$

Lemma

In equilibrium, FTs revised bid quotes are given by

$$B_{t,FT}^{-\sigma} = C_{ST}^{s*}(v_t - \sigma) \qquad B_{t,FT}^{+\sigma} = C_{ST}^{s*}(v_t + \sigma)$$

Equilibrium

Proposition

For fixed parameters (α, L, σ), there exists a unique Markov-perfect equilibrium in the limit order market. In equilibrium
a) STs employ a high fill-rate strategy for $\sigma < \sigma^*_{ST}(\alpha)$ and a low fill-rate strategy otherwise.
b) STs employ a specialized strategy for $\alpha < \alpha^*_S(\sigma)$ and an unspecialized strategy otherwise.
a) FTs employ a high fill-rate strategy for $\sigma < \sigma^*_{FT}(\alpha)$ and a low fill-rate strategy otherwise.

- Volatility σ induces order shading as in Foucault (1999)
- A low level of α leads to specialized strategies
  - Specialized quotes are less likely to execute but yield higher profits conditional on execution

Limit order profits

- STs can react in two possible ways to the presence of FTs
  - quote more aggressively to attract both FTs (unspecialized strategy)
  - only target STs (specialized strategy) and accept decreased execution probability
  - Either way, expected profits are lower than without FTs ($\alpha = 0$)

Corollary

$V_{FT}^{LO*} > V_0^{LO*} > V_{ST}^{LO*}$ for all $\alpha \in (0,1)$

Order flow

- On the equilibrium path, there are 4 possible events
  - i) ST-LO ii)ST-MO iii) FT-LO iv) FT-MO

- Stationary distribution: $\varphi^* = (\varphi_{ST}^{LO*}, \varphi_{ST}^{MO*}, \varphi_{FT}^{LO*}, \varphi_{FT}^{MO*})$
- Trading rate $TR^* = \varphi_{ST}^{MO*} + \varphi_{FT}^{MO*}$
- Limit-to-market order ratio: $LtM^* = \frac{\varphi_{ST}^{LO*} + \varphi_{FT}^{LO*}(2-\alpha)}{\varphi_{ST}^{MO*} + \varphi_{FT}^{MO*}}$
- Make-take ratio: $MT_k^* = \frac{\varphi_k^{LO*}\pi_{k,ST}^* + \varphi_k^{LO*}\pi_{k,FT}^*}{\varphi_{ST}^{LO*}\pi_{ST,k}^* + \varphi_{FT}^{LO*}\pi_{FT,k}^*}$

Trading rate

Corollary

The presence of FTs increases the trading volume except in a specialized high fill-rate equilibrium (i.e. if both σ and α are sufficiently low)

- Ability to revise limit orders mitigates the inefficiency rooted in the adverse selection problem (more trade)
- Higher outside option of FTs induces order shading by STs (less trade)
- Empirically, the "advent" of HFT is associated with more trading (no causality though)
  - Chordia et al (2011)
  - Jovanovic and Menkveld (2011)

LtM

Corollary

FTs are more likely than STs to submit limit orders ($LtM^*_{FT} > LtM^*_{ST}$) and their presence increases the overall message traffic ($LtM^* > LtM^*_0$).

- FTs mechanically submit more limit orders (revisions)
- Higher outside option lets FTs reject some quotes that STs find worth accepting
- Empirical evidence on AT/HFT message traffic
  - Hagströmer & Norden (2013), Malinova et al. (2012)
  - Hendershott et al. (2011)

Make-take ratio

Corollary

FTs are more likely than STs to trade via limit order, i.e. $MT^*_{FT} \geq 1 \geq MT^*_{FT}$. Moreover, MT^*_{FT} (MT^*_{ST}) is increasing (decreasing) in σ.

- FTs' ability to revise limit orders
  - Increases the chance of successful execution
  - Reduces the need for order shading
- Menkveld (2012), Hagströmer & Norden (2013), Malinova et al. (2012), Chaboud et al. (2013), Brogaard et al. (2012)
  - HFTs mostly trade passive, "natural" market makers
  - Passive HFTs faster than aggressive ones
  - Different if arbitrage opportunities can arise?

Market Orders

- Market order profits can be written as $V_k^{MO*} = L - E(\tau_k^*)$
- The transaction cost $E(\tau_k^*)$ reflects
  - bargaining power (outside option)
  - profits from "picking off" stale limit orders

Corollary

If $\sigma \in [8/15, \underline{\sigma})$ then $E(\tau_{ST}^*) > E(\tau^*) > E(\tau_0^*) > E(\tau_{FT}^*)$ for all $\alpha \in (0,1)$.

- FTs get better prices
  - Hendershott & Riordan (2012), Moallemi and Saglam (2011), etc.
- Speed discrepancies increase average trading costs
  - Not in line with most of the empirical literature
  - Difficult to disentangle speed from other benefits of automation

Welfare

- Now suppose that α is not exogenous but instead traders can become fast upon investing c (as in Biais et al. (2012))
- Trading profits are weighted averages of V_k^{LO*} and V_k^{MO*}

$$W_{ST}^* = \frac{\varphi_{ST}^{LO*}}{\varphi_{ST}^{MO*}+\varphi_{ST}^{MO*}} V_{ST}^{LO*} + \frac{\varphi_{ST}^{MO*}}{\varphi_{ST}^{MO*}+\varphi_{ST}^{MO*}} V_{ST}^{MO*}$$

$$W_{FT}^* = \frac{\varphi_{FT}^{LO*}}{\varphi_{FT}^{MO*}+\varphi_{FT}^{MO*}} V_{FT}^{LO*} + \frac{\varphi_{FT}^{MO*}}{\varphi_{FT}^{MO*}+\varphi_{FT}^{MO*}} V_{FT}^{MO*}$$

- Social Welfare is then given by

$$W^*(\alpha^*) = (1-\alpha^*)W_{ST}^*(\alpha^*) + \alpha^*(W_{FT}^*(\alpha^*) - c)$$

- The equilibrium level of investment satisfies $W_{ST}^*(\alpha^*) = W_{FT}^*(\alpha^*) - c$

Welfare

Corollary

Any positive equilibrium level of investment $\alpha^* > 0$ exceeds the socially optimal level α^+ and moreover yields a social welfare loss, i.e. $W(\alpha^*) < W(0)$.

- Although FTs may help increase trade, STs are always worse off
- Same conclusion as in Biais et al. (2012)
  - Different channel: speed helps to avoid adverse selection
  - Externality: STs loose bargaining power
- Note: Corner solution $\alpha^* = 1$ is always inefficient (same outcome as for $\alpha = 0$)
- This does NOT imply that $\alpha^+ = 0$!

Policy

- Ideally, one may want to implement α^+ (which can be positive)
- "Circulating" proposals
  - Minimum resting times
  - Limits on message traffic
- This would curb HFT, but also the associated benefits
  - In fact, quick order revisions are the reason for potential efficiency gains
- Rather directly tax HFT activity ?

Conclusions

- Introducing speed into a LOM with adverse selection has a number of effects
  - Speed partially eliminates "picking off" risks, but also makes STs more cautious
  - FTs emerge as makers, more likely to submit and trade via LO
  - FTs trade at more favourable prices than STs
  - STs face reduced profits due to lower bargaining power
  - Equilibrium investment is welfare reducing (externalities)
- Existing policy proposals probably sub-optimal

The 9th Annual Central Bank Workshop in Market Microstructure

- This year at the ECB in Frankfurt, 5-6 September (Th-Fr after EFA)
- Keynote: Darrell Duffie
- Policy Panel (Marco Pagano, Urich Bindseil)
- Key topics:
  - Fixed income markets (Money Markets, Bond Markets)
  - Long-run trends in MM, e.g. opacity, OTC vs. regulated markets, efficiency, automation
  - The impact of current regulatory initiatives on market structure, e.g. Transaction Taxes, Vickers/Volcker Rules, CCPs, LIBOR reform
- Submission deadline: April 30th (microstructure@ecb.europa.eu)