Intergenerational Equitiy in Long-run Decision Problems Reconsidered*

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Abstract: Although the individuals' lifetime is finite, intergenerational equity is discussed in infinitely lived agent models. We argue to explicitly consider the generations' life-cycles for three reasons: First, from infinitely lived agent specifications, underlying assumptions about individual and social time preference rates cannot be unambiguously deduced. Second, within infinitely lived agent models the distribution among generations living at the same time cannot be captured. Third, a utilitarian social planner's solution may not be implementable in overlapping generations market economies. Re-examining the recent debate on climate change, we conclude that Stern's and Nordhaus' infinitely lived agent models are not suitable to discuss issues of intergenerational equity, as they cannot adequately address these three aspects.

Keywords: Climate Change, infinitely lived agents, intergenerational equity, overlapping generations, time preference

JEL-Classification: D63, H23, Q54

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1 Introduction

How much should a government invest in public infrastructure? How much in basic research? Or recently heavily debated: How much CO_2 should be mitigated? All these decision problems exhibit two characteristics: A classical public good problem and an intergenerational equity problem. Economists would agree that the public good problem should be solved by internalizing the externalities, for example, via a Pigouvian tax. However, as in long-run decision problems stocks, such as stocks of pollutants or knowledge, are affected which, in turn, will influence the well-being of later generations, it is not clear how high the respective tax should be. This, of course, depends on the planner's objective and, in particular, on the weight attached to the utility of future generations. Most of the times the planner is supposed to maximize the utility of an infinitely lived agent which is interpreted as the sum of the utilities of the present and all future generations, but there is an ongoing dispute about the right social rate of time preference. The discussion finds its extreme positions in the normative approaches of early authors, for example, Fisher (1930), Pigou (1920), Ramsey (1928), respectively in the recent climate change debate of Cline (1992) and Stern (2007), who argue for low social time preference rates on ethical grounds, and in the positive approach by, for example, Manne et al. (1995) and Nordhaus (2007), who hold that social preferences are reflected by market outcomes.

This paper argues that the discussion should explicitly consider the life cycles of the different generations for three reasons: First, from the infinitely lived agent specification, the underlying assumptions about the time preference rates of individual households and the social planner cannot be unambiguously deduced. Thus, it may not be clear whether a supposed positive approach is positive or just an arbitrary normative choice. Second, the desideratum of a social planner to treat all generations equally possesses two different aspects: treating all generations alive equally at a each point in time and treating generations living today equally to those to be born in the future. In particular the first aspect cannot be captured in an infinitely lived agent framework. Third, the optimal solution of a social planner maximizing a utilitarian welfare function in an overlapping generations economy may not be implementable in a market economy via a tax/subsidy regime. Infinitely lived agent models neglect this fact.

In order to substantiate these claims, we set up a 'selfish' overlapping generations economy in continuous time. To keep the focus on intergenerational equity we abstract from any externalities.[1] We show analytically under what conditions an overlapping generations economy is observationally equivalent to a classical Ramsey-Cass-Koopmans economy. Without an operative bequest motive of the individual households, which is suggested by a number of empirical studies (e.g., Hurd 1987, Hurd 1989, Laitner and Ohlsson 2001), no conclusions on intergenerational equity considerations are possible from observing solely the market interest rate.

We also show that the infinitely lived agent model can be interpreted as an 'unconstrained social planner' that possesses full power to allocate the resources in order to maximize a utilitarian social welfare function. That is, the infinitely lived agent's rate of time preference reflects the unconstrained social planner's weights for the different generations' utilities. However, in an overlapping generations model without bequests, this solution would require major redistribution from old to young, at least in the case where the social time preference is weakly lower than that of the individual households. In this case, there is a trade off between equality among the generations living at the same point in time and equality between generations of today and in the future. The closer is the time preference of the social planner to that of the individuals, the more equally treated are the generations alive at a given time at the expense of a more unequal treatment of generations living at different times. Both aspects are only reconciled when individuals and the social planner exhibit a time preference rate of zero.

The unconstrained social planner's optimum may be difficult to implement due to the substantial redistribution requirements. Therefore, we define the problem of a 'constrained social planner' that is not able to discriminate individual households by age but can only influence prices. One could think of a democratically legitimized government. Our analysis shows that the time preference rate of a Ramsey consumer in the observationally equivalent infinitely lived agent economy differs from the weight a constrained social planner attaches to the different generations' well-being. This questions the interpretation of the infinitely lived agent's objective function as a social welfare function consisting of the weighted sum of subsequent generations' utility.

Applying our results to the recent debate on climate change mitigation, we conclude that intergenerational equity should rather be discussed within an overlapping generations framework, as the infinitely lived agent model fails to cover important aspects.

There are several papers that examine the relation between infinitely lived agent models

[1] This seems justified as the infinitely lived agent models in the climate change discussion calibrate preference parameters without explicitly considering the externalities the model is applied to.

and overlapping generations models.

Aiyagari (1985) proved that under certain assumptions the overlapping generations model with two-period lived individuals is observationally equivalent to an infinitely lived agent model in discrete time. The equivalence between a Ramsey-Cass-Koopmans economy and a model with finitely lived consumers in continuous time was also established by Calvo and Obstfeld (1988). However, their main concern was with time inconsistencies in fiscal policy arguing that these may arise if a government's fiscal tools are too limited to allow it to decentralize the command optimum. Our paper differs from Aiyagari (1985) in that it derives the equivalence between the Ramsey-Cass-Koopmans economy and the overlapping generations model in continuous time and provides the explicit mapping between the two frameworks with respect to the different rates of time preference. In contrast to Calvo and Obstfeld (1988) our focus is on aspects of intergenerational equity rather than time inconsistency of policies. In fact, all policies considered in this paper are time consistent. Two further differences are worth mentioning. First, we literally model finite life spans of individuals rather than using the specification where each individual possesses a certain probability of death at each point in time. The important difference in our context is that the strictly finitely lived selfish individuals would never make an investment that exceeds their lifetime, whereas the consumer of the "death-probability"-type would make investments with arbitrarily long gestation periods if the rate of return is sufficiently high. Second, switching to a discrete time two-period overlapping generations model, Calvo and Obstfeld (1988) argue that even a constrained planner that is not able to discriminate transfers by age can implement the command optimum given the time horizon is infinite rather than finite. We also consider a constrained social planner. However, our analysis shows that in a continuous time setting with infinite planning horizon the first best is not implementable implying that Calvo and Obstfeld (1988)'s result is strongly connected to their discrete-time-Diamond (1965)-setup.

Also in environmental economics applications, such as Howarth (1998), Howarth (2000), Gerlagh and Keyzer (2001), Gerlagh and van der Zwaan (2000), and Stephan and Müller-Fürstenberger (1997), it has been observed that infinitely lived agent models can be calibrated to yield similar outcomes as overlapping generations models. These papers use numerical simulations of intergrated assessment models, whereas we derive the relation analytically in a continuous time setup, however, without explicitly considering environmental externalities.

Formally, our overlapping generations model is most closely related to d'Albis (2007) who examines the influences of demographic structure on capital accumulation and growth.

In addition to his paper, we allow for exogenous technological change and have a clear focus on intergenerational equity. More remotely, our paper relates to the literature on overlapping generations and debt neutrality such as Barro (1974), Blanchard (1985) and Weil (1989).

The paper is structured as follows. In section 2, we develop the overlapping generations model in continuous time. We derive conditions for observational equivalence of the decentralized overlapping generations economy and an infinitely lived agent model in section 3. In section 4, we examine the relation between the latter and two social planner solutions, unconstrained and constrained. In Section 5 we apply our results to the recent debate on climate change mitigation. We conclude in section 6.

2 An Overlapping Generations Growth Model in Continuous Time

In this section, we present an overlapping generations growth model in continuous time. The key elements are that we assume each generation to live a finite time span and only to care for own lifetime consumption. We analyze the long-run individual and aggregate dynamics of a decentralized economy, assuming market equilibria on all markets at all times.

2.1 Households

We assume a continuum of households, each living the finite time span T. All households exhibit the same intertemporal preferences irrespective of their time of birth $s \in (-\infty, \infty)$. In fact, households maximize their welfare U, which is the discounted lifetime utility derived from consumption

$$U(s) = \int_{t-T}^{t} \frac{c(t,s)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \exp\left[-\rho^H (t-s)\right] \, dt \; , \tag{1}$$

where $c(t, s)$ is the consumption at time t of households born at time s, σ is the constant intertemporal elasticity of substitution, and ρ^H is the constant rate of time preference of the households. At any time alive, each household is endowed with one unit of labor, which is supplied inelastically to the labor market at wage $w(t)$ over the whole life span

T. Households can accumulate assets $b(t,s)$, which earn interest $r(t)$. The households' budget constraint is given by:[2]

$$\dot{b}(t,s) = r(t)b(t,s) + w(t) - c(t,s) \ , \qquad t \in [s, s+T] \ . \tag{2}$$

Households are born without assets and are not allowed to be indebted at time of death. We assume that households are not altruistic and, therefore, do not bequeath assets to succeeding generations. Thus, the following boundary conditions apply for all generations s:

$$b(s,s) = 0 \ , \qquad b(s+T,s) \geq 0 \ . \tag{3}$$

As intertemporal welfare U of a household born at time s can always be increased by consumption at time $s+T$, the latter inequality will hold with equality in the household optimum.

Maximizing equation (1) subject to conditions (2) and (3) yields the well known Euler equation

$$\dot{c}(t,s) = \sigma \left[r(t) - \rho^H \right] c(t,s) \ , \qquad t \in [s, s+T] \ . \tag{4}$$

The behavior of a household born at time s is characterized by the system of differential equations (2) and (4) and the boundary conditions for the asset stock (3).

At any time $t \in (-\infty, \infty)$ the size of the population $N(t)$ increases at the constant rate ν. Without loss of generality, we normalize the population at time $t = 0$ to one:

$$N(t) = \exp[\nu t] \ . \tag{5}$$

Due to the finite life span T of each household, equation (5) implies the following birth rate γ:[3]

$$\gamma = \frac{\nu \exp[\nu T]}{\exp[\nu T] - 1} \ . \tag{6}$$

[2] Throughout the paper, partial derivatives are denoted by subscripts (e.g., $F_k(k,l) = \partial F(k,l)/\partial k$), derivatives with respect to time t are denoted by dots and derivatives of functions depending on one variable only are denoted by primes.

[3] The equation is derived by solving $\int_{t-T}^{t} \gamma \exp[\nu s] \, ds = N(t)$, which states that at time t all households alive sum up to the population size $N(t)$.

2.2 Firms

We assume a continuum of identical competitive firms $i \in [0,1]$. All firms produce a homogeneous consumption good under conditions of perfect competition from capital $k(t,i)$ and 'effective labor' $A(t)l(t,i)$, where $A(t)$ characterizes the technological level of the economy and grows exogenously at a constant rate ξ. Without loss of generality, we normalized technological progress at $t = 0$ to one:

$$A(t) = \exp[\xi t] \ . \tag{7}$$

All firms have access to the same production technology $F(k(t,i), A(t)l(t,i))$, which exhibits constant returns to scale and positive but decreasing marginal productivity with respect to both inputs capital and effective labor. Furthermore, F satisfies the Inada conditions.

Constant returns to scale production and symmetry of the firms allow us to work with a representative firm, whose decision variables are interpreted as aggregate variables. Introducing aggregate capital in terms of effective labor

$$k(t) = \frac{\int_0^1 k(t,i)\, di}{A(t) \int_0^1 l(t,i)\, di} \ , \tag{8}$$

yields the intensive form production function $f\big(k(t)\big) = F\big(k(t), 1\big)$. Profit maximization of the representative firm yields for the wage $w(t)$ and the interest rate $r(t)$:

$$w(t) = A(t) \left[f\big(k(t)\big) - f'\big(k(t)\big)k(t) \right] \ , \tag{9a}$$

$$r(t) = f'\big(k(t)\big) \ . \tag{9b}$$

2.3 Aggregate Economy and Market Equilibrium

Investigating the aggregate economy, we first introduce aggregate household variables per effective labor, which are derived by integrating over all living individuals and dividing by the the technological level and the labor force of the economy according to the scheme

$$x(t) = \frac{\int_{t-T}^{t} x(t,s) \gamma \exp[\nu s]\, ds}{A(t) \int_0^1 l(t,i)\, di} \ , \tag{10}$$

where $x(t)$ and $x(t,s)$ denote aggregate per effective labor respectively individual household variables.

The economy consists of three markets: the labor market, the capital market and the consumption good market. We assume the economy to be in market equilibrium at all times t. In particular, this implies that labor demand equals the population size, i.e., $\int_0^1 l(t,i)\,di = N(t)$, and capital in terms of effective labor equals aggregate assets in terms of effective labor, i.e., $k(t) = b(t)$.

Then, the dynamics of the aggregate economy is characterized by[4]

$$\dot{c}(t) = \left\{\sigma\left[f'\big(k(t)\big) - \rho^H\right] - (\nu+\xi)\right\} c(t) + \frac{\gamma}{\exp[\xi t]}\left[c(t,t) - \frac{c(t,t-T)}{\exp[\nu T]}\right] \ , \tag{11a}$$

$$\dot{k}(t) = f\big(k(t)\big) - (\nu+\xi)k(t) - c(t) \ . \tag{11b}$$

Introducing the definition $\Delta c(t) = \frac{\gamma}{\exp[\xi t]}\left[c(t,t) - \frac{c(t,t-T)}{\exp[\nu T]}\right] - \nu c(t)$, we can rewrite equation (11a) as

$$\frac{\dot{c}(t)}{c(t)} = \sigma\left[f'\big(k(t)\big) - \rho^H\right] - \xi + \frac{\Delta c(t)}{c(t)} \ . \tag{11c}$$

We will see later that the term $\frac{\Delta c(t)}{c(t)}$ gives the difference between the decentralized overlapping generations economy and the observationally equivalent Ramsey type economy.

2.4 Steady State

Our analysis will concentrate on the long-run steady state growth path of the economy, in which both consumption per effective labor and capital per effective labor are constant over time, i.e., $c(t) = c^\star$, $k(t) = k^\star$.[5] Then, from equations (9) follows that also the interest rate $r(t) = r^\star = f'(k^\star)$ is constant in the steady state, and the wage $w(t)$ grows exponentially at the rate of technological progress. Thus, we can introduce the wage rate in terms of the technological level, $\tilde{w}$, which is constant in the steady state

$$w^\star = \tilde{w}(t) = \exp[-\xi t]w(t) = \left[f(k^\star) - f'(k^\star)k^\star\right] \ . \tag{12}$$

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[4] Observe that $\dot{x}(t) = -(\nu+\xi)x(t) + \exp[-(\nu+\xi)t]\int_{t-T}^{t} \dot{x}(t,s)g\exp[\nu s]\,ds + g\left[x(t,t) - \frac{x(t,t-T)}{\exp[(\nu+\xi)T]}\right]$.

[5] d'Albis and Augeraud-Véron (2007) show in a similar overlapping generations model with an AK production specification that the economy converges oscillatory to a unique steady state.

Introducing the constants

$$\phi = \frac{1-\exp[-(r^\star-\xi)T]}{1-\exp[-(r^\star-\sigma(r^\star-\rho))T]} \ , \quad \psi = w^\star\phi\frac{r^\star-\sigma(r^\star-\rho)}{r^\star-\xi} > 0 \ , \tag{13}$$

we can express the consumption path and wealth of an individual household in terms of the technological level $A(t)$ to yield:

$$\tilde{c}(t-s) = c^\star(t,s)\exp[-\xi t] = \psi\exp\left[\big(\sigma(r^\star-\rho)-\xi\big)(t-s)\right] \ , \tag{14a}$$

$$\begin{aligned}\tilde{b}(t-s) = b^\star(t,s)\exp[-\xi t] &= \frac{w^\star}{r^\star-\xi}\big\{\phi\exp\left[\big(\sigma(r^\star-\rho)-\xi\big)(t-s)\right] \\ &\quad +(1-\phi)\exp[(r^\star-\xi)(t-s)]-1\big\} \ .\end{aligned} \tag{14b}$$

Thus, in the steady states, the individual household variables in terms of the technological level depend only on the age, $t-s$, of a household but not on calender time t. Applying the aggregation rule (10), we derive for the aggregate values per effective labor:

$$c^\star = g\psi\frac{\exp[(\sigma(r^\star-\rho)-\nu-\xi)T]-1}{\sigma(r^\star-\rho)-\nu-\xi} \ , \tag{15a}$$

$$\begin{aligned}k^\star &= \frac{w^\star}{r^\star-\xi}\left\{g\phi\frac{\exp[(\sigma(r^\star-\rho)-\nu-\xi))T]-1}{\sigma(r^\star-\rho)-\nu-\xi)}\right. \\ &\quad \left. + g(1-\phi)\frac{\exp[(r^\star-\nu-\xi)T]-1}{r^\star-\nu-\xi}-1\right\} \ .\end{aligned} \tag{15b}$$

As $r^\star = f'(k^\star)$ and $w^\star = [f(k^\star)-f'(k^\star)k^\star]$, equation (15b) is an implicit equation for the capital stock per effective unit of labor in the steady state $k^\star$. The following proposition elaborates on the solutions of this implicit equation.

Proposition 1 (Existence and uniqueness of the steady state)
There exists a $k^\star > 0$ that solves equation (15b) if

$$\lim_{k\to 0}[-kf''(k)] > \frac{1}{\sigma T} \ . \tag{16a}$$

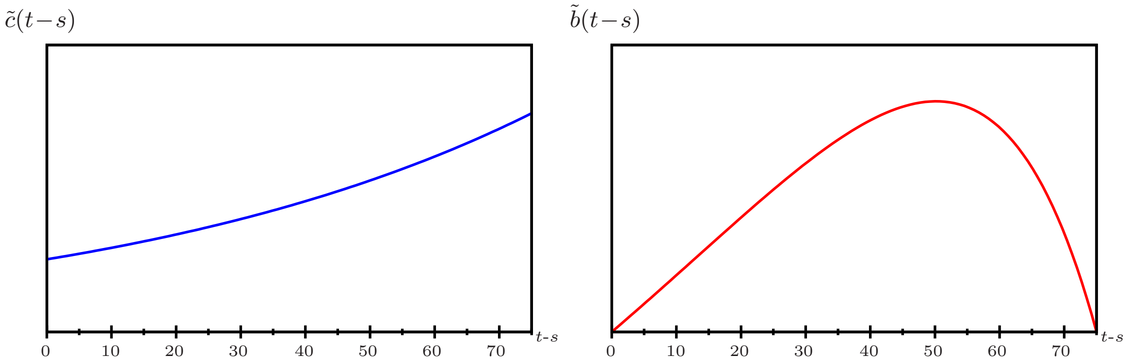

Figure 1: Steady state paths of consumption (left) and asset (right) for individual households in terms of the technological level of the economy.

$k^\star$ is unique if

$$\begin{aligned} s(k) &\leq \zeta(k) \quad \textit{and} \quad \frac{d}{dk}\left(\frac{s(k)}{\zeta(k)}\right) \geq 0 \text{ , \textit{where}} \qquad (16b) \\ s(k) &= \frac{kf'(k)}{f(k)} \quad \textit{(share of capital in output)} \\ \zeta(k) &= \frac{-f'(k)[1-s(k)]}{kf''(k)} \quad \textit{(elasticity of substitution betw. capital and labor).} \end{aligned}$$

The proof is given in the appendix.

For the remainder of this paper we assume conditions (16b) to hold. Although we cannot solve the implicit equation (15b) analytically and, therefore, cannot calculate the steady state interest rate $r^\star$, we can give a lower bound as the following proposition states.

Proposition 2 (Lower bound of steady state interest rate)
If there exists a $k^\star > 0$, then

$$f'(k^\star) = r^\star > \rho + \frac{\xi}{\sigma} \qquad (17)$$

The proof can be found in the appendix.

To illustrate the dynamics of the decentralized overlapping generations economy in the steady state, Figure 1 shows steady state paths for individual consumption and assets in terms of the technological level of the economy.[6] The individual consumption path grows exponentially over the lifetime of each generation. Individual household assets show an inverted U shape, i.e., households are born with no assets, accumulate assets in their youth and consume their wealth towards their death.

3 Decentralized Overlapping Generations Versus Infinitely Lived Agent Economy

Now, we investigate if and under what conditions the long-run steady state of the decentralized overlapping generations economy introduced in the previous section is *observationally equivalent* to the long-run steady state of an economy, in which population growth and the production side of the economy remain unchanged, but an infinitely lived agent chooses consumption and asset accumulation such as to maximize the intertemporal welfare of a Ramsey dynasty. Before we proceed, we define the term 'observational equivalence'.

Definition 1 (Observational Equivalence)
Two economies are observationally equivalent in the long-run steady state if the aggregate paths per effective labor of both, consumption $c(t)$ and assets $b(t)$, converge to the same long-run steady state.

We assume that the infinitely lived agent exhibits the same intertemporal elasticity of substitution σ as the households in the overlapping generations economy, but does not need to exhibit the same rate of time preference. More formally, we seek the rate of time preference $\bar{\rho}$ of the infinitely lived agent such that the aggregate paths per effective labor of the decentralized overlapping generations economy, $c(t)$ and $b(t)$, as given by the system of differential equations (11), are identical to the optimal paths of consumption and assets per effective labor of the infinitely lived agent in the long-run steady state

Defining $\bar{c}(t) = \exp[\xi t]c(t)$ as the aggregate consumption per capita, the welfare $\bar{U}$ of the Ramsey dynasty is given by:

$$\bar{U} = \int_0^\infty \frac{\bar{c}(t)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \exp[\nu t] \exp[-\bar{\rho} t] \, dt \ . \tag{18}$$

[6] The calculations use the following model specifications: $f(k) = k^\alpha$, $\alpha = 0.3$, $\rho = 0.03$, $\sigma = 1$, $\xi = 0.015$, $\nu = 0$, $T = 75$.

Again, we assume that all markets clear, and thus the budget constraint equals

$$\dot{k}(t) = f(k(t)) - (\nu + \xi)k(t) - c(t) \ , \tag{19}$$

which is identical to (11b).

Maximizing (18) subject to the budget constraint (19) and the transversality condition

$$\lim_{t\to\infty} \left\{ k(t)\exp[(\nu+\xi)t]\exp\left[-\int_0^t r(t')dt'\right]\right\} = 0 \ , \tag{20}$$

yields the standard Euler equation for aggregated consumption per effective labor:

$$\frac{\dot{c}(t)}{c(t)} = \sigma\big[r(t) - \bar{\rho}\big] - \xi \ . \tag{21}$$

As technology and population growth is identical in both economies, the equation of motion for the aggregated capital per effective labor is identical in the infinitely lived agent economy (19) and the decentralized overlapping generations economy (11b). Thus, the two economies are observationally equivalent in the long-run steady state if the two Euler equations (21) and (11c) coincide in the steady state. This implies the following relationship between the time preference rates ρ of the individual household in the decentralized overlapping generations economy and $\bar{\rho}$ of the infinitely lived agent:

$$\bar{\rho} = \rho - \frac{1}{\sigma}\frac{\Delta c(t)}{c(t)} \ . \tag{22}$$

Whether the time preference rate $\bar{\rho}$ of the infinitely lived agent exceeds that of an individual household of the decentralized overlapping generations economy ρ depends on whether $\frac{\Delta c(t)}{c(t)}$ is positive or negative. Although we cannot determine the sign of $\frac{\Delta c(t)}{c(t)}$ in general, we can explicitly calculate it in the steady state. The following proposition gives the result.

Proposition 3 (Equivalence between decentralized OLG and ILA ecnonomy) *The steady state of a decentralized overlapping generations economy is observationally equivalent to the steady state of an infinitely lived agent economy iff*

$$\bar{\rho} = r^{\star}(\rho) - \frac{\xi}{\sigma} > \rho \ . \tag{23}$$

Proof: From equations (14a) and (15a) we derive in the steady state $\frac{\Delta c(t)}{c(t)} = \xi - \sigma(r^{\star} - \rho)$. Inserting into equation (22) yields the equality of (23). Note that $r^{\star}$ in the steady state depends implicitly on the time preference rate ρ of the individual households, as can be seen from equation (15b). The inequality follows directly from Proposition 2. $\square$

Proposition 3 states that, for the two economies to be observationally equivalent in the long-run steady state, the rate of time preference $\bar{\rho}$ of the infinitely lived agent has to be higher than that of the individual households of the decentralized overlapping generations economy, ρ. The interpretation is straightforward. Given the same time preference rate and real interest rate, the net present value of one unit of investment in assets is lower for the household in the overlapping generations economy than for the infinitely lived agent, due to the finite lifetime of the former. Thus, for both to invest the same amount and, therefore, for the interest rate $r^\star$ to coincide, the rate of time preference ρ of the individual households in the overlapping generations economy has to be lower than the time preference rate of the infinitely lived agent $\bar{\rho}$.

As we cannot solve for $r^\star$ analytically, we cannot directly determine the spread $\Delta\rho = \bar{\rho} - \rho$ of the time preference rates in the steady state. However, we can analyze how the spread reacts to a change in the exogenously given parameters. The following proposition gives the results.

Proposition 4 (Comparative statics of the spread of $\boldsymbol{\Delta\rho}$)
In the observationally equivalent steady state, the following conditions for the spread $\Delta\rho = \bar{\rho} - \rho$ of the time preference rates hold:

$$\frac{\partial \Delta\rho}{\partial \xi} < 0 \quad \textit{if} \quad \xi < \frac{\sigma(r^\star - \rho) + r^\star - \nu}{2} \,, \tag{24a}$$

$$\textit{or} \quad \xi > \frac{\sigma(r^\star - \rho) + r^\star - \nu}{2} \quad \wedge \quad \sigma(r^\star - \rho) - \xi - \nu > 0 \,,$$

$$\frac{\partial \Delta\rho}{\partial \nu} > 0 \,, \tag{24b}$$

$$\frac{\partial \Delta\rho}{\partial \sigma} < 0 \quad \textit{if} \quad \xi = 0 \,, \tag{24c}$$

$$\frac{\partial \Delta\rho}{\partial T} < 0 \,, \tag{24d}$$

$$\lim_{T\to\infty} \bar{\rho} = \rho + \frac{\nu}{\sigma} - \frac{1}{\sigma(\xi + \nu - \sigma(r^\star - \rho))} \,. \tag{24e}$$

The proof is given in the appendix.

To give some intuition, recall equation (23). For a given time preference rate of the individual household, ρ, the spread $\Delta\rho$ increases in the interest rate corrected by the ratio of the rate of technological progress and the intertemporal elasticity of substitution. In this way, it is very plausible that a marginal increase of the rate of technological progress has an ambiguous effect on $\Delta\rho$. On the one hand, the ratio $\frac{\xi}{\sigma}$ would increase, leading to a lower $\bar{\rho}$ of the infinitely lived agent in the corresponding Ramsey-Cass-Koopmans economy. On the other hand, the reaction of the interest rate is ambiguous. Clearly, an increase in ξ would imply a higher marginal productivity of capital, however, the adjustment of the individuals' saving rates depends on their preferences, that is, whether the income or substitution effect prevails. If the substitution effect is strong enough the interest rate would fall as a consequence of a higher rate of technological progress and the spread $\Delta\rho$ would decline. Proposition 4 states parameter constellations that are sufficient for this case to occur. With a sufficiently strong income effect, the interest rate would increase in the rate of technological progress. In this situation no clear statements are possible with respect to the behavior of $\Delta\rho$ without specifying the production function any further.

Proposition 4 also states that $\Delta\rho$ unambiguously increases in the rate of population growth. The decisive factor is that the young generations which possess little savings receive a stronger weight relative to the older generations with high amounts of capital. The amount of capital per effective labor declines and, as a consequence, the interest rate and $\Delta\rho$ increases.

It is also very intuitive that the interest rate decreases with an increase in the elasticity of intertemporal substitution because with a higher σ, the individual household is more willing to accept deviations from a uniform pattern of consumption over time. Hence, the agents would save more leading to a lower interest rate. In this way, the spread $\Delta\rho$ would unambiguously decline if there was no technological progress. If ξ is positive, the term $\frac{\xi}{\sigma}$, which is decreasing in σ, allows no clear statement with respect to the behavior of $\Delta\rho$.

Finally, we can say that the finitely lived individuals would increase their savings if they lived a marginal unit of time longer. Staying with the previously given intuition, the net present value of investment in assets increases for the overlapping generations household if its lifetime increases. However, although $\Delta\rho$ declines in T, it will not converge to zero in the limit $T \to \infty$. The reason is that newly born generations in the overlapping generations economy are constrained in their saving behavior by their income, whereas in the Ramsey dynasty all the assets are shared equally among the agents alive.[7]

[7] With $T = \infty$ our overlapping generations economy corresponds to the one presented by Weil (1989).

4 Utilitarian Overlapping Generations Versus Infinitely Lived Agent Economy

In this section, we investigate the conditions under which an overlapping generations economy, governed by a social planner maximizing a social welfare function, is observationally equivalent to an economy in which an infinitely lived agent decides about investment and consumption.

We assume a utilitarian social welfare function in which the social planner trades off the weighted lifetime utility of different generations. The weight consists of two components. First, the lifetime utility of the generation born at time s is multiplied by its cohort size. That is, the social planner considers the total lifetime utility of each generation. Second, the social planner exhibits a social rate of time preference $\rho^S \geq 0$, i.e., the total lifetime utility of the generation born at time s is discounted at a constant rate.[8]

We assume a social planner maximizes a utilitarian welfare function, as described above, from $t = 0$ onward. Thus, the social welfare function consists of two parts. First, the weighted intergal of the *remaining* lifetime utility of all generations which are already alive at time $t = 0$ and the weighted integral of all future generations

$$\begin{aligned} W = & \int_{-T}^{0} \left\{ \int_{0}^{s+T} \frac{c(t,s)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \exp\left[-\rho^H (t-s)\right] \, dt \right\} \gamma \exp[\nu s] \exp\left[-\rho^S s\right] \, ds \\ & + \int_{0}^{\infty} \left\{ \int_{s}^{s+T} \frac{c(t,s)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \exp\left[-\rho^H (t-s)\right] \, dt \right\} \gamma \exp[\nu s] \exp\left[-\rho^S s\right] \, ds \ . \end{aligned} \tag{25}$$

The term in brackets is the (remaining) lifetime utility $U(s)$ of a household born at time s (cf. equation 1), the functional form of which is a given primitive for the social planner. The term $\gamma \exp[\nu s]$ denotes the cohort size of the generation born at time s and ρ^s is the social planner's rate of time preference. Changing the order of integration equation (25) can be written as

$$W = \int_{0}^{\infty} \left\{ \int_{t-T}^{t} \frac{c(t,s)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \gamma \exp\left[\left(\rho^S - \rho^H - \nu\right)(t-s)\right] \, ds \right\} \exp\left[\left(\nu - \rho^S\right) t\right] \, dt$$

[8] We are well aware that this social welfare function is an arbitrary *normative* choice, as many other social welfare functions are conceivable. Nevertheless, it represents the de facto standard in the economic literature (see e.g. Burton (1993), Calvo and Obstfeld (1988)), and is also the interpretation applied to the infinitely lived agent models in the climate change debate.

$$= \int_0^\infty \left\{ \int_0^T \frac{c(t,t-a)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \gamma \exp\left[\left(\rho^S - \rho^H - \nu\right) a\right] \, da \right\} \exp\left[\left(\nu - \rho^S\right) t\right] \, dt \ , \quad (26)$$

where $t-s$ is substituted by age a in the second line.

In the following, we consider two different scenarios. In the *unconstrained* utilitarian OLG economy, a social planner maximizing the social welfare function (26) decides about investment and households' consumption. Thus, the social planner is in command of a centralized economy. In contrast, in the *constrained* utilitarian OLG economy the social planner relies on a market economy, in which the households own the firms and the capital stock, and decide about investment and consumption such as to maximize their individual lifetime utility (1). The social planner can only influence prices by a tax/subsidy regime to maximize social welfare (25).

4.1 Unconstrained Utilitarian Overlapping Generations Economy

In general, the outcome of the decentralized overlapping generations economy studied in section 2 will not maximize the social welfare function W. Thus, we start by determining the unconstrained social planner's optimal allocation by solving

$$\max_{c(t,s)} \int_0^\infty \left\{ \int_0^T \frac{c(t,t-a)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \gamma \exp\left[\left(\rho^S - \rho^H - \nu\right) a\right] \, da \right\} \exp\left[\left(\nu - \rho^S\right) t\right] \, dt \ , \quad (27)$$

subject to the budget constraint (19) and the transversality condition (20).

Following the approach of Calvo and Obstfeld (1988), the optimization problem (27) can be interpreted as two nested optimization problems. Defining

$$V\left(\bar{c}(t)\right) = \max_{\{c(t,t-a)\}_{a=0}^T} \int_0^T \frac{c(t,t-a)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \gamma \exp\left[\left(\rho^S - \rho^H - \nu\right) a\right] \, da \ , \quad (28)$$

subject to

$$\bar{c}(t) \geq \exp[-\nu t] \int_{t-T}^t c(t,s) \gamma \exp[\nu s] \, ds = \int_0^T c(t,t-a) \gamma \exp[-\nu a] \, da \ , \quad (29)$$

the first maximization problem is to distribute aggregate consumption per capita at time t optimally among all generations alive at time t. Then, we obtain for the second maximization problem

$$\max_{\bar{c}(t)} \int_0^\infty V\left(\bar{c}(t)\right) \exp[\nu t] \exp\left[-\rho^S t\right] \, dt \ , \tag{30}$$

subject to the budget constraint (19) and the transversality condition (20).

From the first maximization problem we learn how the social planner optimally distributes consumption between all generation alive at the *same* time t. The following proposition states the result.

Proposition 5 (Optimal consumption distribution at one point in time)
The optimal solution of the maximization problem (28) subject to condition (29) reads:

$$c(t, t-a) = \frac{\sigma\left(\rho^S - \rho^H\right) - \nu}{\gamma\left(\exp\left[\left(\sigma\left(\rho^S - \rho^H\right) - \nu\right) T\right] - 1\right)} \bar{c}(t) \exp\left[\sigma\left(\rho^S - \rho^H\right) a\right] \ . \tag{31}$$

As a consequence, all households receive the same amount of consumption at time t irrespective of age for $\rho^H = \rho^S$, and receive less consumption the older (younger) they are at a given time t for $\rho^H > \rho^S$ ($\rho^H < \rho^S$).

The proof is given in the appendix.

Proposition 5 says that the difference between the households' rate of time preference, ρ^H, and the social rate of time preference, ρ^S, determines the distribution of consumption between households of different age at a given time t. In particular, if $\rho^H > \rho^S$,[9] at any instant of time the consumption profile with respect to the individuals' age is the opposite of that of the decentralized solution, as can be seen from the Euler equation (4). That is, in the social planner's solution households receive less consumption the older they are, whereas they would consume more the older they are in the decentralized overlapping generations economy.

In addition, Proposition 5 poses an "equality-trade-off" to the social planner if households exhibit a positive rate of time preference. Then, a social rate of time preference of zero would weigh the lifetime utilities of today's and future generations equally, but also implies that at each point in time the young enjoy higher consumption than the old. In contrast, a social rate of time preference equal to that of the individuals yields an equal

[9] We do not take up a stance on the relationship between the individual and the social rate of time preference, but merely hint at the resulting consequences. This is in line with Burton (1993), who argues that "...they represent profoundly different concepts" (p. 121/122) and, thus, may differ. However, if they are supposed to differ, than it is usually assumed that $\rho^H > \rho^S$ (see also Heinzel and Winkler 2007: Sec. 2).

distribution of consumption among the generations alive at each point in time at the expense of an unequal treatment of lifetime utilities of different generations. The trade-off vanishes only if the individuals' rate of time preference is zero. Of course, this trade-off can only be captured in an overlapping generations model which explicitly considers the life cycles of different generations.

We now turn to the second maximization problem (30) subject to the budget constraint (19) and the transversality condition (20). Observe that it is formally equivalent to a Ramsey consumer who exhibits the instantaneous utility function $V\big(\bar{c}(t)\big)$ and the time preference rate ρ^S.[10] We obtain $V\big(\bar{c}(t)\big)$ by inserting the optimal consumption profile (31) into equation (28)

$$V\big(\bar{c}(t)\big) = \left[\frac{\gamma\left(\exp\left[\left(\sigma\left(\rho^S - \rho^H\right) - \nu\right)T\right] - 1\right)}{\sigma\left(\rho^S - \rho^H\right) - \nu}\right]^{\frac{1}{\sigma}} \frac{\bar{c}(t)^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} \ . \tag{32}$$

As the social planner's maximization problem (30) is invariant against affine transformations of the objective function, it is identical to the maximization problem of the Ramsey consumer who maximizes (18) subject to the budget constraint (19) and the transversality condition (20) if the social planner's time preference rate ρ^S is equal to the time preference rate of the Ramsey consumer ρ^R.

Proposition 6 (Unconstrained utilitarian OLG and ILA ecnonomy)
An overlapping generations economy, in which a social planner solves maximization problem (27) subject to the budget constraint (19) and the transversality condition (20) is observationally equivalent to an infinitely lived agent economy, in which a Ramsey consumer maximizes (18) subject to (19) and (20) iff

$$\rho^S = \rho^R \ . \tag{33}$$

Proof: Equation (33) follows directly from inserting equation (32) into maximization problem (30). □

Proposition 6 says that, in aggregate terms, the social planner's problem (27) is equivalent to an infinitely lived agent maximizing (18) if the social planner and the Ramsey consumer exhibit the same rate of (social) time preference. Note that this holds in general and not only for the long-run steady state. One might argue that this equivalence supports the interpretation of the utility function (18) of a Ramsey consumer as a social

[10] This was already shown by Calvo and Obstfeld (1988).

welfare function (25). There is, however, a crucial difference. While the outcome of the infinitely lived agent economy can easily be decentralized, the decentralization of the unconstrained social planner's problem as a market outcome needs a transfer scheme that not only depends on time, but also discriminates with respect to age for each point in time. We consider such a transfer scheme hardly implementable in democratic societies. In fact, discrimination by race, gender or age is mostly considered as inequitable. As a consequence, in the following section we examine the situation where the social planner cannot discriminate transfers by age but only influence prices via taxes and subsidies. As we shall show, the social optimum cannot be achieved in this case.

4.2 Constrained Utilitarian Overlapping Generations Economy

In constrast to redistribution that explicitly depends on age, it is plausible that a government can impose taxes/subsidies on capital and labor income. Hence, we extend the market system by a tax/subsidy regime, but do not consider fully dictatorial solutions, in which the social planner can directly dictate the consumption path of individual generations.

We use $\tau_r(t)$ and $\tau_w(t)$ to denote the tax/subsidy on returns on savings and on labor income, respectively.[11] The individual households of the overlapping generations economy base their optimal consumption and saving decisions on the effective interest rate $r^{\mathrm{e}}(t)$ and the effective wage $w^{\mathrm{e}}(t)$:

$$\begin{aligned} r^{\mathrm{e}}(t) &= r(t) - \tau_r(t) \ , & \text{(34a)} \\ w^{\mathrm{e}}(t) &= w(t)\big[1 - \tau_w(t)\big] \ . & \text{(34b)} \end{aligned}$$

The individual budget constraint now reads

$$\dot{b}(t,s) = r^{\mathrm{e}}(t)b(t,s) + w^{\mathrm{e}}(t) - c(t,s) \ . \tag{34c}$$

Given this budget constraint, individual households choose consumption paths $c^{\mathrm{e}}(t,s)$ which maximize lifetime utility (1). Thus, the optimized consumption path $c^{\mathrm{e}}(t,s)$ is a function of the taxes/subsidies $\tau_r(t)$ and $\tau_w(t)$:

$$c^{\mathrm{e}}(t,s) = c^{\mathrm{e}}\left(t, \tau_r(t), \tau_w(t); s\right) \ . \tag{35}$$

[11] Following the standard convention, $\tau_i(t)$ is positive if it is a tax and negative if it is a subsidy.

Note that for given subsidy schemes $\{\tau_r(t), \tau_w(t)\}_{t=0}^{\infty}$ the individual household's optimal paths of consumption and assets can be characterized as in the decentralized OLG economy by (2) and (4) when using $r^{\mathrm{e}}(t)$ and $w^{\mathrm{e}}(t)$ instead of $r(t)$ and $w(t)$, respectively. Applying the aggregation rule (10) yields aggregate consumption per effective labor $c^{\mathrm{e}}(t) = c^{\mathrm{e}}(t, \tau_r(t), \tau_w(t))$.

To analyze observational equivalence of the aggregate long-run steady state dynamics, which is implied by maximizing social welfare (25) on the one hand and achieved by an infinitely lived agent maximizing the welfare of a Ramsey dynasty on the other hand, we have to restrict redistribution to mechanisms which do not alter the aggregate budget constraint (11b) of the economy. This implies that the redistribution scheme has to yield a balanced government budget at all times. A balanced budget at all times t, which implies that the aggregate budget constraint (11b) of the economy remains unchanged, requires

$$\tau_w(t)w(t) = -\tau_r(t)\bar{b}(t) \ . \tag{36}$$

Assuming a balanced aggregate budget at all times allows us to skip $\tau_w(t)$ and only use $\tau_r(t)$ as an argument in functions that depend on the tax/subsidy scheme. Then, we can write the social planner's problem as follows:

$$\max_{\tau_r(t)} \int_0^{\infty} \left\{ \int_{t-T}^{t} \frac{c^{\mathrm{e}}(t, \tau_r(t); s)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \gamma \exp[(\nu + \rho^H - \rho^S)s] \, ds \right\} \exp[-\rho^H t] \, dt \ , \tag{37}$$

subject to the budget constraint (19). Introducing V, the aggregate instantaneous utility of all generations alive at time t

$$V(t, \tau_r(t)) = \int_{t-T}^{t} \frac{c^{\mathrm{e}}(t, \tau_r(t); s)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \gamma \exp[(\nu + \rho^H - \rho^S)s] \, ds \ , \tag{38}$$

and denoting the costate variable for the capital stock as λ, the present value Hamiltonian is given by

$$\mathcal{H} = V(t, \tau_r(t)) \exp[-\rho t] + \lambda(t) \left[f(k(t)) - (\nu + \xi)k(t) - c^{\mathrm{e}}(t, \tau_r(t)) \right] \ . \tag{39}$$

The first order conditions for an optimal solution yield:

$$\lambda(t) = \exp[-\rho t]\frac{V_{\tau_r}(t,\tau_r(t))}{c^{\mathrm{e}}_{\tau_r}(t,\tau_r(t))} \ , \tag{40a}$$

$$\dot\lambda(t) = \lambda(t)\left[c^{\mathrm{e}}_k(t,\tau_r(t))+\nu+\xi-f'(k(t))\right] + V_k(t,\tau(t))\exp[-\rho t] \ . \tag{40b}$$

In general, this system of integro-differential equations yields no closed form solutions. However, for the long-run steady state, we can establish conditions such that the outcome of the optimization problem (37) is observationally equivalent to an infinitely lived agent who maximizes the welfare of a Ramsey dynasty (18). The following proposition gives the result.

Proposition 7 (Constrained utilitarian OLG and ILA ecnonomy)
The long-run steady state of an overlapping generations economy, in which a social planner solves maximization problem (37) subject to the budget constraint (19) is observationally equivalent to the long-run steady state of an infinitely lived agent economy, in which a Ramsey consumer maximizes (18) subject to (19) iff

$$\rho^S = \frac{\hat V_k(\tau_r^\star)}{\hat V_{\tau_r}(\tau_r^\star, k^\star)} c^{\mathrm{e}}_{\tau_r}(\tau_r^\star) - c^{\mathrm{e}}_k(\tau_r^\star) + \rho^R \ , \tag{41}$$

with

$$c^e(\tau_r^\star) = \gamma\hat\psi(\tau_r^\star)\frac{\exp[(\sigma(r^\star-\tau_r^\star-\rho)-\nu-\xi)T]-1}{\sigma(r^\star-\tau_r^\star-\rho)-\nu-\xi} \ , \tag{42a}$$

$$\hat V(\tau_r^\star) = \gamma\frac{\hat\psi(\tau_r^\star)^{\frac{\sigma-1}{\sigma}}}{\frac{\sigma-1}{\sigma}}\frac{\exp\left[\left((\sigma-1)(r^\star-\tau_r^\star-\rho)-\frac{\sigma-1}{\sigma}\xi-\nu-\rho+\hat\rho\right)T\right]-1}{(\sigma-1)(r^\star-\tau_r^\star-\rho)-\frac{\sigma-1}{\sigma}\xi-\nu-\rho+\hat\rho} \ , \tag{42b}$$

$$\hat\psi(\tau_r^\star) = (w^\star+\tau_r^\star k^\star)\frac{[r^\star-\tau_r^\star-\sigma(r^\star-\tau_r^\star-\rho)]\{1-\exp[-(r^\star-\tau_r^\star-\xi)T]\}}{\{1-\exp[-(r^\star-\tau_r^\star-\sigma(r^\star-\tau_r^\star-\rho))T]\}(r^\star-\tau_r^\star-\xi)} \ . \tag{42c}$$

The proof is given in the appendix. As $\hat V(\tau_r^\star)$ and $c^{\mathrm{e}}(\tau_r^\star)$ depend on ρ^H via the lifetime utility maximization of the individual households, equation (41) defines a relation between the individual time preference rate ρ, the time preference rate of the social planner ρ^S and the time preference rate ρ^R which an infinitely lived agent has to exhibit for the two economies to be observationally equivalent in the steady state. In particular Proposition 7 states that, in general, the time preference rate of the infinitely lived agent cannot be interpreted as the social time preference rate of the constrained planner in an observationally equivalent overlapping generations economy. Or put differently, a constrained social planner whose social time preference rate is identical to the time preference rate of the infinitely lived agent prefers an aggregate consumption and capital accumulation path that is different from the one in the infinitely lived agent economy. As it seems realistic that a planner such as a democratically legitimized government cannot arbitrarily redistribute between the generations alive at each point in time, the result in Proposition 7 questions the validity of the infinitely lived agent framework as a tool to derive policy advice.

Another point is interesting. As indicated earlier, the individual households' savings and consumption paths correspond to the ones in the decentralized economy for $w(t) = w^e(t)$ and $r(t) = r^e(t)$. As the optimal tax in steady state $\tau_r^\star$ is constant, the individual households' life cycles are qualitatively unchanged by the tax/subsidy scheme. In particular, this implies that for $\rho^H > \rho^S$ the constrained social planner is not able to attain the first best allocation of the unconstrained social planner which would require a reversal of the decentralized consumption profile of the individual households.

This result stands in contrast to Calvo and Obstfeld (1988) who show that even a planner that cannot discriminate (lump sum) taxes between different generations is able to reach the first best. The reason for the difference is that Calvo and Obstfeld (1988) use an overlapping generations model of the Diamond (1965)-type with only two generations alive at each point in time. As only one generation, the young, make the saving decision one instrument is sufficient to effectively target transfers to one of the generations. In our model there exists a continuum of generations at each point in time which makes it impossible to effectively direct transfers to different generations when only market prices can be influenced. However, the tax/subsidy scheme available to the constrained social planner would also suffice to reach the first best in a Diamond (1965)-type overlapping generations model.

5 Stern Versus Nordhaus on Intergenerational Equity and the Choice of the Social Rate of Time Preference

In the following, we apply the results derived in sections 3 and 4 to critically review recent approaches to the evaluation of climate change mitigation scenarios. We will focus on Stern (2007) and its critique by Nordhaus (2007), which both use in principle a Ramsey-Cass-Koopmans growth model in which an infinitely lived agent maximizes the welfare of a Ramsey dynasty. If one interprets the utility of the Ramsey consumer as a social welfare function, intergenerational equity concerns are closely related to the choice of the time preference rate $\bar{\rho}$ of the Ramsey consumer, as it determines the weight given to the utility of later "generations". In fact, there exists a long-lasting debate about the "right" choice of the time preference rate $\bar{\rho}$.[12]

5.1 Stern's and Nordhaus' Arguments in a Nutshell

The publication of Stern (2007) triggered a lot of interest among fellow economists (e.g., Beckerman and Hepburn 2007, Dasgupta 2007, Mendelsohn 2006, Tol and Gary 2006), as Stern, although using fundamentally the same methodological approach and the same data, projected considerably higher damages from global warming and, therefore, advocates much stronger and more immediate policy actions than comparable studies.[13] Despite some minor details, the difference in results mainly stems from using an exceptionally low time preference rate of $\bar{\rho} = 0.1\%$ per year in combination with an intertemporal elasticity of substitution of $\sigma = 1$, which leads to a steady state real interest rate of $r^{\star} = 1.4\%$ per year (in contrast, Nordhaus 2007 assumes parameter values which yield a real interest rate $r^{\star} = 5.5\%$ in the DICE-2007 model). As the real interest rate can be interpreted as the opportunity costs of investments into climate change mitigation, Stern's real interest rate justifies substantially higher emission abatement levels than Nordhaus' interest rate, which can be achieved by setting substantially higher tax rates on carbon emissions. Stern justifies his approach similarly to Ramsey (1928) that "discounting is ethically indefensible", but chooses a positive $\bar{\rho}$ which corresponds to a hazard rate that earth or at least humankind might cease to exist.

In his critique of the Stern review, Nordhaus (2007) takes the perspective that in an integrated assessment model the baseline scenario, the scenario in which no (additional) environmental policy is enacted, should compute empirically observable values, in particular for the real interest rate. He, therefore, advocates to choose σ and $\bar{\rho}$ such as to yield real interest rates in the "sensible" range of 4–7% per annum. Nevertheless, he clearly states that the objective function is to be interpreted as a utilitarian social welfare function, which considers the utility derived from consumption of different generations.

[12] Excellent overviews of the issue of time preference, intergenerational equity and discounting include Lind (1982), Portney and Weyant (1999), Frederick et al. (2002) and Groom et al. (2005).

[13] For example, in the baseline scenario without controlling climate change, Stern (2007) estimates the social cost per ton of carbon in 2015 at $350, while Nordhaus (2007) derives $35 per ton of carbon (both in 2005 prices).

5.2 Discussion of Nordhaus' Approach

As a matter of fact, the developed countries and also most countries in transition exhibit some kind of market economy in which generations of different age interact. This interaction yields some (aggregate and risk-free) long-run market interest rate $r^{\star}$. Observation alone, however, does not give any hint how this rate emerged (see Aiyagari 1985). Nordhaus seems to assume that the setting of an infinitely lived Ramsey consumer is an appropriate description for the baseline scenario, in which no (additional) environmental policy is enacted. The validity of this assumption hinges very much on how altruistic the individuals are. Only if bequests are operative, i.e. the Ricardian equivalence holds, the interest rate conveys the opportunity cost of investments in the very long run, which exceeds the lifetime of individual households. If the bequest motive is not operative – as suggested by a number of empirical studies (e.g., Hurd 1987, Hurd 1989, Laitner and Ohlsson 2001) – our decentralized overlapping generations economy, calibrated to yield a long-run interest rate of $r^{\star} = 5.5\%$, may be a more accurate description of the economy and the underlying behavioral assumptions.

As is clear from our analysis, a market interest rate $r^{\star} = 5.5\%$ can be achieved in a decentralized overlapping generations economy by an infinite set of combinations of individual preference parameters σ and ρ. For each individual set of preference parameters one achieves a different observationally equivalent infinitely lived agent specification, which differ in σ and $\bar{\rho}$ (in the appendix we calculate various combinations of ρ and σ which result in $r^{\star} = 5.5\%$ for different life spans T of the individual households). Moreover, from the observed market interest rate alone, we cannot infer the preferences of individual households for intergenerational equity. For example, an observed market interest rate $r^{\star} = 5.5\%$ is consistent with our assumption of an overlapping generations economy with purely selfish individuals. The central point, however, is that in a selfish OLG economy individuals would never pay voluntarily for investments if their benefits accrue beyond their own lifetime, whereas a Ramsey consumer can make investments with arbitrarily long gestation periods. Thus, calibrating the infinitely lived agent model to reproduce the observed market interest rate in the case that the Ricardian equivalence does not hold is an arbitrary normative choice.

Moreover, given the social welfare function (25) we consider in this paper, each different set of individual parameters ρ and σ, which yields a long-run interest rate $r^{\star} = 5.5\%$ in the decentralized overlapping generations economy, would yield a different rate of time preference $\hat{\rho}$ of the social planner, even if we assume an unconstrained social planner who can freely redistribute across generations, and thus $\hat{\rho} = \bar{\rho}$. For example, for $\sigma = 0.79$ and $\rho = 1\%$ we achieve $\hat{\rho} = \bar{\rho} = 2.97\%$, whereas for $\sigma = 1.42$ and $\rho = 3\%$ we achieve $\hat{\rho} = \bar{\rho} = 4.09\%$.

Thus, if one advocates a purely descriptive approach, it is not enough to observe the long-run interest rate $r^\star$. However, in most economies we do observe subsidies on different kinds of long-run investments such as education, retirement savings and research & development. As a consequence, it might be possible, at least in principle, not only to observe the market interest rate, but also subsidies on long-term investments. If these subsidies are being enacted by a constrained social planner, i.e. a government, which has been elected democratically by the individual households, we can interpret the observed subsidies as the revealed preference of the households on intergenerational equity. If, in addition, one knows the individual preference parameters σ and ρ, one can infer a social rate of time preference $\hat{\rho}$ by virtue of Proposition 7. Under the assumption that the time preference rate ρ of the individual households ranges from $\rho = 1\%$ to $\rho = 3\%$ per annum (in the appendix we calculate values for ρ ranging from 0–5% in 0.5 steps), Nordhaus' infinitely lived agent specification is in the steady state observationally equivalent to an overlapping generations economy in which the contstrained social planner exhibits a time preference rate $\hat{\rho}$ in the range of 2.15–2.86%, and subsidizes investment between 1.14–2.24%. This leads to an effective interest rate r^{e} between 6.64% and 7.74%.

5.3 Discussion of Stern's Approach

As outlined before, there is some doubt that observed market data alone is suitable to infer preferences on intergenerational equity. If one takes this stance, one has to make some (arbitrary) choice for the time preference rate $\hat{\rho}$ of the social planner. In general, such a normative choice of the social welfare function implies redistribution between different generations, and thus does not reproduce observed market data such as the interest rate $r^\star$. In fact, we interpret Stern such that he claims the social rate of time preference should equal $\hat{\rho} = 0.1\%$. Moreover, we interpret Stern's choice of a very low time preference rate in the spirit of Ramsey (1928) to treat all generations alike. In an overlapping generations model, however, treating all generations equally can have different meanings. First, it may imply that all generations alive at the *same time* have to be treated equally. Second, it may imply that generations living *today* have to be treated equally to generations which are just to be born in the *future*. In particular, the first meaning cannot be captured in an infinitely lived agent framework.

Stern's choice of a very low time preference rate $\bar{\rho}$ addresses the second meaning of intergenerational equity. As seen in Proposition 6, the time preference rate $\bar{\rho}$ of the Ramsey consumer is identical to the weight factor $\hat{\rho}$ an unconstrained social planner imposes on the lifetime utility of different generations in an observationally equivalent overlapping generations economy. But as seen in Proposition 5, the social planners optimum may infer redistribution among the generations alive. If the individual time preference rate ρ exceeds the time preference rate of the social planner $\hat{\rho}$ (what we consider to be likely in the case of $\hat{\rho} = 0.1\%$), the social planner would distribute consumption such that, at a given time t, consumption is higher the younger is an individual. Such a redistribution may not only be impossible to implement but also result in treating generations highly unequal, as redistribution discriminates by age.

Addressing intergenerational equity in the first sense, however, does not imply a social planner's rate of time preference of zero, but to choose the time preference rate $\hat{\rho}$ equal to the individual rate of time preference ρ. As seen from Proposition 5, only for $\hat{\rho} = \rho$ the unconstrained social planner distributes consumption equally among all generations alive. Thus, if one considers a sensible range for the individual time preference rate ρ between 1–3% and sets $\hat{\rho} = \rho$, the time preference rate $\bar{\rho}$ of the observationally equivalent infinitely lived agent economy ranges from 1–3% in case of an unconstrained social planner, and between 0.75–2.25% if one assumes a constrained social planner (see appendix). Both ranges are a long way off Stern's $\bar{\rho} = 0.1\%$.

The two different meanings of intergenerational equity are only compatible with each other if the individual rate of time preference is close to zero. The time preference of individual households, however, is not subject to normative choice but a given primitive, which has to observed. Empirical evidence does not support that individuals' time preferences are close to zero. Another possible avenue to reconcile equity considerations among generations living at the same time and between generations living at different times is the concept of a constrained social planner, who redistributes consumption between generations by a tax/subsidy scheme but treats all generations alive equally, as the taxes and subsidies do not discriminate by age. This, however, implies that the time preference rate of the social planner $\hat{\rho}$ is, in general, not equivalent to the time preference rate $\bar{\rho}$ of the observationally equivalent Ramsey consumer. As shown in the appendix, $\bar{\rho} = 0.1\%$ implies a $\hat{\rho}$ in the range of 0.12–0.25%, or the other way round, for $\hat{\rho} = 0.1\%$ an observationally equivalent Ramsey consumer has to exhibit $\bar{\rho}$ ranging from 0.06–0.08%, depending on the rate of time preference ρ of the individual households.

5.4 Stern Versus Nordhaus: A Final Remark

In our opinion, both Nordhaus' and Stern's approach neglect important aspects of intergenerational equity. While Nordhaus tries to deduce preferences on intergenerational equity by reproducing market interest rates in a infinitely lived agent model, Stern clearly advocates a normative approach but misses the clash between equity among generations alive and between generations living at different times. Summing up, we argue that assumptions about individual preference parameters ρ and σ and the time preference rate $\hat{\rho}$ of a social planner should be discussed in the more encompassing framework of an overlapping generations economy. On this basis, assumptions about intergenerational equity, which are only implicitly captured in an infinitely lived agent setup, become obvious and are open for debate.

6 Conclusions

Although the lifetime of individuals is finite, questions of intergenerational equity are most often discussed within infinitely lived agent frameworks, which are interpreted as a utilitarian social welfare function. In this paper, we analyzed to what extend this interpretation is justified. In particular, we examined under which conditions the infinitely lived agent economy is observationally equivalent to (i) a decentralized overlapping generations economy and (ii) an overlapping generations economy in which a social planner maximizes a utilitarian welfare function. Further, we applied our results to the recent dispute between Stern (2007) and Nordhaus (2007) in the discussion on the mitigation of climate change. We concluded that Stern's and Nordhaus' infinitely lived agent models are not suitable to discuss issues of intergenerational equity, as they cannot adequately address three important aspects: First, from infinitely lived agent specifications, underlying assumptions about individual and social time preference rates cannot be unambiguously deduced. Second, within infinitely lived agent models the distribution among generations living at the same time cannot be captured. Third, a utilitarian social planner's solution may not be implementable in overlapping generations market economies.

Our analysis employs two central assumptions. First, we assume individual households to be selfish. Although several empirical studies support this view, extending the model to include different degrees of altruism is an interesting venue for future research. Second, we assume a specific utilitarian social welfare function. Although commonplace in the literature, other functional specifications are conceivable. However, these extentions would not change our final conclusion, which is line with Solow (1986) and Howarth (1998), that infinitely lived agent models are inappropriate for the analysis of intergenerational equity in long-run decision problems which should rather be examined in overlapping generations frameworks.

A Appendix

A.1 Proof of Proposition 1

As in steady state all aggregate per-effective-labor variables are constant and an individual's per-efficiency variables only depend on that person's age but not on the time-periods in which she lives. I.e. per-efficiency consumption is the same for persons at time t and $t + t'$ as long as they are of the same age. This allows us to reformulate the variables in functions of age rather than functions of time and use the rationale of d'Albis (2007) to proof the existence and uniqueness of the steady state. Denote a person's age at time t, $(t - s)$, by ε. We can then write per-efficiency-unit consumption as

$$\frac{c(t,s)}{A(t)} = \tilde{c}(\varepsilon) = \frac{R(-\varepsilon)^{-\sigma}\exp[-(\xi+\rho\sigma)\varepsilon]}{\int_0^T R(-z)^{1-\sigma}\exp[-\rho z]^{\sigma}dz} w^{\star}\int_0^T \exp[(\xi - r^{\star})z]dz. \tag{A.1}$$

Similarly to d'Albis (2007), we define

$$H((1-\sigma)r^{\star}) \quad := \quad \int_0^T R(-z)^{1-\sigma}\exp[-\rho z]^{\sigma}dz, \tag{A.2}$$

$$G(r^{\star} - \xi) \quad := \quad w^{\star}\int_0^T \exp[(\xi - r^{\star})z]dz. \tag{A.3}$$

In this way, we can rewrite (A.1) as

$$\tilde{c}(\varepsilon) = \frac{G(r^{\star}-\xi)}{H((1-\sigma)r^{\star})}\exp[(\sigma r^{\star} - \xi)\varepsilon]\exp[-\rho\varepsilon]^{\sigma}. \tag{A.4}$$

With respect to assets, we have

$$\begin{aligned}
\frac{b(t,s)}{A(t)} = \tilde{b}(\varepsilon) \quad &= \quad \exp[-\xi\varepsilon]\int_{\varepsilon}^T \exp[\xi z](\tilde{c}(z) - w^{\star})\exp[-r^{\star}(z-\varepsilon)]dz \\
&= \quad \int_{\varepsilon}^T (\tilde{c}(z) - w^{\star})\exp[(\xi - r^{\star})(z-\varepsilon)]dz.
\end{aligned}$$

Consequently, total assets per effective labor is

$$\begin{aligned}
b = \frac{B(t)}{L(t)A(t)} \quad &= \quad \int_0^T g\exp[\nu(t-\varepsilon)]\tilde{b}(\varepsilon)d\varepsilon\exp[-\nu t] \\
&= \quad \int_0^T g\exp[(r^{\star} - \nu - \xi)\varepsilon]\int_{\varepsilon}^T (\tilde{c}(z) - w^{\star})\exp[(\xi - r^{\star})z]dz\, d\varepsilon.
\end{aligned}$$

Changing the order of integration. we have

$$
\begin{aligned}
b &= \int_0^T g(\tilde{c}(\varepsilon) - w^\star) \exp[(\xi - r^\star)\varepsilon] \int_0^\varepsilon \exp[(r^\star - \nu - \xi)z] dz \, d\varepsilon \\
&= \frac{g}{r^\star - \xi - \nu} \int_0^T (\tilde{c}(\varepsilon) - w^\star) \exp[(\xi - r^\star)\varepsilon](\exp[(r^\star - \nu - \xi)\varepsilon] - 1) \, d\varepsilon \\
&= \frac{g}{r^\star - \xi - \nu} \left(\int_0^T (\tilde{c}(\varepsilon) - w^\star) \exp[-\nu\varepsilon] \, d\varepsilon - \underbrace{\int_0^T (\tilde{c}(\varepsilon) - w^\star) \exp[(\xi - r^\star)\varepsilon] \, d\varepsilon}_{=0, \text{ as } b(T)=0} \right).
\end{aligned}
$$

Inserting $\tilde{c}(\varepsilon)$ yields

$$
b = \frac{g}{r^\star - \xi - \nu} \left(\frac{G(r^\star - \xi)}{H((1-\sigma)r^\star)} \underbrace{\int_0^T \exp[(\sigma r^\star - \xi - \nu)\varepsilon] \exp[-\rho\varepsilon]^\sigma d\varepsilon}_{H(\xi+\nu-\sigma r^\star)} - \underbrace{\int_0^T w^\star \exp[-\nu\varepsilon] \, d\varepsilon}_{G(\nu)} \right) \tag{A.5}
$$

We can write the wage per effective labor as

$$
w = g \int_0^T w^\star \exp[-\nu\varepsilon] \, d\varepsilon = gG(\nu) \tag{A.6}
$$

Note that, of course, $w = w^\star$. Solving (A.6) for g and inserting into (A.5), we obtain

$$
b = \frac{w}{r^\star - \xi - \nu} \left(\frac{G(r^\star - \xi)}{H((1-\sigma)r^\star)} \frac{H(\xi + \nu - \sigma r^\star)}{G(\nu)} - 1 \right) \tag{A.7}
$$

In steady state we must have

$$
k^\star = b = \phi(k^\star) = \frac{f(k^\star) - f'(k^\star)k^\star}{f'(k^\star) - \xi - \nu} \left(\underbrace{\frac{G(f'(k^\star) - \xi)}{H((1-\sigma)f'(k^\star))} \frac{H(\xi + \nu - \sigma f'(k^\star))}{G(\nu)}}_{J(f'(k^\star))} - 1 \right), \tag{A.8}
$$

which corresponds to equation (15b).

We will now establish that

Lemma 1
Function $J : \mathbb{R} \to \mathbb{R}_{++}$ given by

$$
J(r^\star) = \frac{G(r^\star - \xi)}{G(\nu)} \frac{H(\xi + \nu - \sigma r^\star)}{H((1-\sigma)r^\star))} \tag{A.9}
$$

is (i) strictly convex, (ii) $\lim_{r^\star \to +\infty} J(r^\star) = \lim_{r^\star \to -\infty} J(r^\star) = +\infty$.

For later reference it is useful to show the first derivative of function J:

$$J'(r^\star) = [\sigma h(\xi + \nu - \sigma r^\star) + (1-\sigma)h((1-\sigma)r^\star) - g(r^\star - \xi)]J(r^\star) \tag{A.10}$$

where

$$g(r^\star - \xi) = \frac{-G'(r^\star - \xi)}{G(r^\star - \xi)} = \frac{\int_0^T \varepsilon \exp[(\xi - r^\star)\varepsilon]d\varepsilon}{\int_0^T \exp[(\xi - r^\star)\varepsilon]d\varepsilon} > 0 \tag{A.11}$$

$$h(x) = \frac{-H'(x)}{H(x)} = \frac{\int_0^T \varepsilon \exp[-\rho\varepsilon]^\sigma \exp[-x\varepsilon]d\varepsilon}{\int_0^T \exp[-\rho\varepsilon]^\sigma \exp[-x\varepsilon]d\varepsilon} > 0 \tag{A.12}$$

The second derivative of J can be written as

$$J''(r^\star) = [-\sigma^2 h'(\xi + \nu - \sigma r^\star) + (1-\sigma)^2 h'((1-\sigma)r^\star) - g'(r^\star - \xi)]J(r^\star) \tag{A.13}$$

$$+[\sigma h(\xi + \nu - \sigma r^\star) + (1-\sigma)h((1-\sigma)r^\star) - g(r^\star - \xi)]^2 J(r^\star) \tag{A.14}$$

We can now apply the same reasoning as d'Albis (2007) to verify (i). Further we have $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} h(x) = 0$ and $\lim_{x\to-\infty} g(x) = \lim_{x\to-\infty} h(x) = T$. Hence, from (A.10) we see that

$$\lim_{x\to\infty} \frac{J'(r^\star)}{J(r^\star)} = -\lim_{x\to-\infty} \frac{J'(r^\star)}{J(r^\star)} = \sigma T \tag{A.15}$$

and (ii) follows. □

According to above's considerations, we have

$$\phi(k^\star) = \frac{f(k^\star) - f'(k^\star)k^\star}{f'(k^\star) - \xi - \nu}(J(f'(k^\star)) - 1), \tag{A.16}$$

which corresponds to equation (63) of d'Albis (2007). As $J(r^\star)$ exhibits the same properties as the $J(x)$ of d'Albis (2007), this paper's steady state proof directly applies when considering that in our model the golden rule capital stock satisfies $f'(k^{gr}) = \xi + \nu$. □

A.2 Proof of Proposition 2

The proof shows that $\sigma(r^\star - \rho^H) - \xi > 0$ is a necessary condition for aggregate assets to be strictly positive in steady state. Consequently, if $k^\star > 0$ the real interest rate in steady state must be larger than $\rho^H + \frac{\xi}{\sigma}$.

Consider the individual household's wealth profile over her lifetime as given by equation (14b):

$$\tilde{b}(a) = \frac{w^\star}{r^\star - \xi}\left\{\phi \exp\left[\left(\sigma(r^\star - \rho^H) - \xi\right)a\right] + (1-\phi)\exp[(r^\star - \xi)a] - 1\right\}, \tag{A.17}$$

where a denotes the household's age. Due to the transversality condition we have $r^\star - \xi > 0$. From (13) we obtain

$$\phi \begin{cases} < 1, & \text{if } \sigma(r^\star - \rho^H) - \xi < 0, \\ = 1, & \text{if } \sigma(r^\star - \rho^H) - \xi = 0, \\ > 1, & \text{if } \sigma(r^\star - \rho^H) - \xi > 0. \end{cases}$$

This allows us to directly infer from (A.17) that $\sigma(r^\star - \rho^H) - \xi = 0$ implies $b(a) = 0$ for all $a \in [0, T]$. As in this situation no individual household possesses a strictly positive wealth level, the economy's capital stock does not exceed zero. To show that $\sigma(r^\star - \rho^H) - \xi < 0$ precludes strictly positive capital stocks, we use the second derivative of $b(a)$.

$$\begin{aligned} \frac{d^2\,\tilde{b}(a)}{d\,a^2} \quad = \quad & \frac{w^\star}{r^\star - \xi}\left\{\phi\left(\sigma(r^\star - \rho^H) - \xi\right)^2 \exp\left[\left(\sigma(r^\star - \rho^H) - \xi\right)a\right]\right. \\ & \left. +(1-\phi)(r^\star - \xi)^2 \exp[(r^\star - \xi)a]\right\}. \end{aligned} \tag{A.18}$$

Since $\sigma(r^\star - \rho^H) - \xi < 0$ involves $\phi < 1$, it further implies that $\frac{d^2\tilde{b}(a)}{d\,a^2} > 0$. Hence the household's wealth profile is strictly convex. Given the boundary conditions $\tilde{b}(0) = 0 = \tilde{b}(T)$ this implies that over her entire lifetime each household exhibits non-positive wealth. This, however, contradicts strictly positive aggregate savings. Hence we can conclude that $\sigma(r^\star - \rho^H) - \xi \leq 0$ precludes $k^\star > 0$.

It is further obvious from (A.17) and (A.18) that $\sigma(r^\star - \rho^H) - \xi > 0$ does not contradict strictly positive wealth of the individual households and, consequently, is a necessary condition for $k^\star > 0$. □

A.3 Proof of Propositon 4

Comparative statics of the spread $\bar{\rho} - \rho$ means examining $-\frac{\Delta\bar{c}(t)}{\bar{c}(t)}$. Equivalently we could use (23) and study the changes of $\bar{\rho}$ with respect to the parameters of interest. Let x be such a parameter. We would then have:

$$\frac{\partial\bar{\rho}}{\partial x} = \frac{dr^\star}{dk^\star}\frac{dk^\star}{dx} - \frac{\partial\left(\frac{\xi}{\sigma}\right)}{\partial x}. \tag{A.19}$$

The variation in the interest rate in x will be determined via the implicit function theorem from equation (15b), respectively from (A.8). More precisely, with the notation of appendix A.1 we define:

$$F(k^\star, x) := k^\star - \phi(k^\star) = k^\star - \underbrace{\frac{f(k^\star) - f'(k^\star)k^\star}{f'(k^\star) - \xi - \nu}}_{M(k^\star,(x))} \Bigg(\underbrace{\frac{G(f'(k^\star) - \xi)}{H((1-\sigma)f'(k^\star))} \frac{H(\xi + \nu - \sigma f'(k^\star))}{G(\nu)}}_{J(f'(k^\star),x)} - 1 \Bigg) = 0, \tag{A.20}$$

and receive

$$\frac{dk^\star}{dx} = -\frac{\frac{\partial F(k^\star,x)}{\partial x}}{\frac{\partial F(k^\star,x)}{\partial k^\star}}. \tag{A.21}$$

As we concentrate on model specifications that yield unique steady states, i.e. that satisfy conditions (16b), we have

$$\frac{\partial F(k^\star, x)}{\partial k^\star} = 1 - \phi'(k^\star) > 0. \tag{A.22}$$

Recall that the sufficient conditions (16b) guarantee $\phi'(k^\star) < 1$. Due to the strict concavity of the production function, $\frac{dr^\star}{dk^\star} < 0$, and consequently, the sign of $\frac{dr^\star}{dx}$ is that of $\frac{\partial F(k^\star,x)}{\partial x}$.

A.3.1 Variation of $\bar{\rho}$ in ξ

Using (A.19) with $x = \xi$ we receive

$$\frac{\partial \bar{\rho}}{\partial \xi} = \frac{dr^\star}{dk^\star} \frac{dk^\star}{d\xi} - \frac{1}{\sigma}. \tag{A.23}$$

Hence if $\frac{\partial F(k^\star,\xi)}{\partial \xi} < 0$, $\bar{\rho}$ is declining in ξ. Otherwise the sign is ambiguous and cannot be determined without a further specification of the production function.

$$\frac{\partial F(k^\star, \xi)}{\partial \xi} = -\frac{\partial M(k^\star, \xi)}{\partial \xi}(J(f'(k^\star), \xi) - 1) - M(k^\star, \xi)\frac{\partial J(f'(k^\star), \xi)}{\partial \xi} \tag{A.24}$$

As

$$\frac{\partial M(k^\star, \xi)}{\partial \xi} = w^\star (r^\star - \xi - \nu)^{-2} > 0,$$

$\frac{\partial F(k^\star,\xi)}{\partial \xi} < 0$ iff

$$\frac{\partial J(f'(k^\star),\xi)}{\partial \xi} = J(f'(k^\star),\xi)\left(\frac{\frac{\partial G(r^\star-\xi)}{\partial \xi}}{G(r^\star-\xi)} + \frac{\frac{\partial H(\xi+\nu-\sigma r^\star)}{\partial \xi}}{H(\xi+\nu-\sigma r^\star)}\right) > 0. \tag{A.25}$$

This inequality is given whenever

$$\frac{\frac{\partial G(r^\star-\xi)}{\partial \xi}}{G(r^\star-\xi)} + \frac{\frac{\partial H(\xi+\nu-\sigma r^\star)}{\partial \xi}}{H(\xi+\nu-\sigma r^\star)} = \frac{\int_0^T \varepsilon \exp[(\xi-r^\star)\varepsilon]d\varepsilon}{\int_0^T \exp[(\xi-r^\star)\varepsilon]d\varepsilon} - \frac{\int_0^T \varepsilon \exp[(\sigma(r^\star-\rho)-\xi-\nu)\varepsilon]d\varepsilon}{\int_0^T \exp[(\sigma(r^\star-\rho)-\xi-\nu)\varepsilon]d\varepsilon} > 0$$

or equivalently

$$\frac{\exp[(\xi-r^\star)T]}{\exp[(\xi-r^\star)T]-1} - \frac{1}{\xi-r^\star} - \frac{\exp[(\sigma(r^\star-\rho)-\xi-\nu)T]}{\exp[(\sigma(r^\star-\rho)-\xi-\nu)T]-1} + \frac{1}{\sigma(r^\star-\rho)-\xi-\nu} > 0. \tag{A.26}$$

As in general,

$$\begin{aligned} \frac{d\left(\frac{\exp[XT]}{\exp[XT]-1} - \frac{1}{X}\right)}{dX} &> 0\ \text{, if } X<0\ \text{, and} \\ \frac{d\left(\frac{\exp[XT]}{\exp[XT]-1}\right)}{dX} &< 0, \end{aligned}$$

(A.26) holds iff

$$\xi - r^\star < \sigma(r^\star-\rho) - \xi - \nu \ \wedge\ \sigma(r^\star-\rho)-\xi-\nu > 0\,\text{, or} \tag{A.27}$$

$$\xi - r^\star > \sigma(r^\star-\rho) - \xi - \nu \quad \Leftrightarrow \quad \xi < \frac{\sigma(r^\star-\rho)+r^\star-\nu}{2} \tag{A.28}$$

A.3.2 Variation of $\bar{\rho}$ in ν

Concerning the rate of population growth, we obtain

$$\frac{\partial \bar{\rho}}{\partial \nu} = \frac{dr^\star}{dk^\star}\frac{dk^\star}{d\nu}. \tag{A.29}$$

To determine the behavior of $k^\star$ with respect to the population growth rate, it is convenient to write

$$\phi(k^\star,\nu) = b = \int_0^T g\exp[-\nu\varepsilon]\tilde{b}(\varepsilon)d\varepsilon,$$

where $\tilde{b}(\varepsilon) = \frac{b(\varepsilon)}{A(t)} = \frac{b(t,s)}{A(t)}$. As $\tilde{b}(\varepsilon)$ does not depend on ν, it follows that

$$\frac{\partial \phi(k^\star, \nu)}{\partial \nu} = -\int_0^T g\varepsilon \exp[-\nu\varepsilon]\tilde{b}(\varepsilon) d\varepsilon < 0.$$

Consequently,

$$\frac{\partial \bar{\rho}}{\partial \nu} > 0.$$

A.3.3 Variation of $\bar{\rho}$ in σ

With respect to the intertemporal elasticity of substitution, $\bar{\rho}$ changes according to

$$\frac{\partial \bar{\rho}}{\partial \sigma} = \frac{dr^\star}{dk^\star}\frac{dk^\star}{d\sigma} + \frac{\xi}{\sigma^2}. \tag{A.30}$$

As the last summand is positive, the sign of $\frac{\partial \bar{\rho}}{\partial \sigma}$ depends on the reaction of the OLG's savings with respect to a change in σ. The partial derivative can be written as

$$\frac{\partial \phi(k^\star, \sigma)}{\partial \sigma} = \int_0^T g \exp[-\nu\varepsilon] \frac{\partial \tilde{b}(\varepsilon)}{\partial \sigma} d\varepsilon,$$

where

$$\tilde{b}(\varepsilon) = \int_0^\varepsilon \left(w^\star - \frac{G(r^\star - \xi)}{H((1-\sigma)r)} \exp[(\sigma(r^\star - \rho) - \xi)z] \right) \exp[(r^\star - \xi)(\varepsilon - z)] dz,$$

and hence,

$$\frac{\partial \tilde{b}(\varepsilon)}{\partial \sigma} = -\exp[(r^\star - \xi)\varepsilon] \frac{G(r^\star - \xi)}{H((1-\sigma)r)} H_\varepsilon((1-\sigma)r^\star) \left(\frac{\frac{\partial H_\varepsilon((1-\sigma)r^\star)}{\partial \sigma}}{H_\varepsilon((1-\sigma)r^\star)} - \frac{\frac{\partial H((1-\sigma)r^\star)}{\partial \sigma}}{H((1-\sigma)r^\star)} \right)$$

where

$$H_\varepsilon((1-\sigma)r^\star) := \int_0^\varepsilon \exp[(\sigma(r^\star - \rho) - r^\star)z] dz.$$

The derivative is positive if

$$\frac{\frac{\partial H_\varepsilon((1-\sigma)r^\star)}{\partial \sigma}}{H_\varepsilon((1-\sigma)r^\star)} - \frac{\frac{\partial H((1-\sigma)r^\star)}{\partial \sigma}}{H((1-\sigma)r^\star)} < 0, \tag{A.31}$$

or

$$\frac{\varepsilon \exp[(\sigma(r^\star - \rho) - r^\star)\varepsilon]}{\exp[(\sigma(r^\star - \rho) - r^\star)\varepsilon] - 1} - \frac{T \exp[(\sigma(r^\star - \rho) - r^\star)T]}{\exp[(\sigma(r^\star - \rho) - r^\star)T] - 1} < 0. \tag{A.32}$$

As

$$\frac{d\left(\frac{T\exp[XT]}{\exp[XT]-1}\right)}{dT}\begin{cases} >0\ ,\ XT\neq 0,\\ =0\ ,\ \text{else,}\end{cases}$$

and $\varepsilon < T$, inequality (A.32) holds for all ages ε and consequently the partial derivative of total savings in the OLG-economy must also be positive. According to (A.21), we receive

$$\frac{dr^\star}{dk^\star}\frac{dk^\star}{d\sigma} < 0.$$

This implies that for positive rates of technological progress, the sign of $\frac{\partial\bar{\rho}}{\partial\sigma}$ cannot be determined without further specification of the production function. However, if $\xi = 0$,

$$\frac{\partial\bar{\rho}}{\partial\sigma} < 0.$$

A.3.4 Variation of $\bar{\rho}$ in T

From (A.19) we obtain

$$\frac{\partial\bar{\rho}}{\partial T} = \frac{dr^\star}{dk^\star}\frac{dk^\star}{dT}, \tag{A.33}$$

i.e. the variation of $\bar{\rho}$ with respect to the individual lifetimes depends only on the changes in the interest rate. According to the considerations at the beginning of this section of the appendix, the signum of $\frac{dr^\star}{dk^\star}\frac{dk^\star}{dT}$ is the same as that of $\frac{\partial F(k^\star,T)}{\partial T}$.

$$\frac{\partial F(k^\star,T)}{\partial T} = -\frac{\partial b}{\partial T} = \int_0^T g\exp[-\nu\varepsilon]\frac{\partial\tilde{b}(\varepsilon)}{\partial T}d\varepsilon + g\exp[-\nu T]\underbrace{\tilde{b}(T)}_{=0}.$$

If he lived a marginal unit of time longer, an individual at age ε would change his saving behavior according to

$$\frac{\partial\tilde{b}(\varepsilon)}{\partial T} = -\exp[r^\star\varepsilon]\frac{G(r^\star-\xi)}{H((1-\sigma)r^\star)}\left(\frac{\frac{\partial G(r^\star-\xi)}{\partial T}}{G(r^\star-\xi)} - \frac{\frac{\partial H((1-\sigma)r^\star)}{\partial T}}{H((1-\sigma)r^\star)}\right)\int_0^\varepsilon \exp[(\sigma(r^\star-\rho)-r^\star)z]dz.$$

This reveals that, dependent on the sign of

$$\frac{\frac{\partial G(r^\star-\xi)}{\partial T}}{G(r^\star-\xi)} - \frac{\frac{\partial H((1-\sigma)r^\star)}{\partial T}}{H((1-\sigma)r^\star)},$$

all agents would either increase or decrease their savings, however, in different intensities dependent on their age. Consequently, $\frac{\partial \tilde{b}(\varepsilon)}{\partial T} > 0$ iff

$$\frac{(\xi - r^\star)\exp[(\xi - r^\star)T]}{\int_0^T \exp[(\xi - r^\star)z]dz} - \frac{(\sigma(r^\star-\rho)-r^\star)\exp[(\sigma(r^\star-\rho)-r^\star)T]}{\int_0^T \exp[(\sigma(r^\star-\rho)-r^\star)z]dz} < 0$$

As

$$\frac{d\left(\frac{X\exp[XT]}{\exp[XT]-1}\right)}{dX}\begin{cases} > 0\ ,\ XT \neq 0, \\ = 0\ ,\ \text{else}, \end{cases}$$

we can state that $\frac{\partial\bar{\rho}}{\partial T} < 0$ iff

$$\xi - r^\star < \sigma(r^\star-\rho) - r^\star \quad \Leftrightarrow \quad \xi < \sigma(r^\star-\rho).$$

This must always be the case as shown in the proof of proposition 3.

A.3.5 Infinite lifetimes of individuals, $\lim_{T\to\infty}\ \bar{\rho}$

From the individual's Euler equation, the new transversality condition

$$\lim_{t\to\infty} b(t,s)\exp[-\int_s^t r(t')dt'] = 0,$$

and with the assumption $\sigma(r(t)-\rho) - r(t) < 0\ ,\ \forall t$, we can derive the consumption path of an individual born at time s as

$$c(t,s) = \frac{\sigma(r(s)-\rho)-r(s)}{\xi - r(s)} w(s)\exp[\int_s^t \sigma(r(t')-\rho)dt' + \xi s].$$

In steady state, aggregate consumption per capita with infinite lifetimes writes

$$\bar{c}(t) = g\exp[(\sigma(r^\star-\rho)-\nu)t]\frac{\sigma(r^\star-\rho)-r^\star}{\xi - r^\star} w^\star \int_{-\infty}^t \exp[(-\sigma(r^\star-\rho)+\xi+\nu)s]ds \quad \text{(A.34)}$$

Note that when individuals possess infinite lifetimes such a summation over different generations is only valid if the growth rate of the population is positive. Since nobody dies, zero population growth means that no new generations will be born and consequently the OLG would be equivalent to the Ramsey-Cass-Koopmans economy. In order to determine the growth rate of aggregate consumption per capita, we calculate the time derivative of (A.34):

$$\dot{\bar{c}}(t) = (\sigma(r^\star - \rho) - \nu)\bar{c}(t) + gc(t,t). \tag{A.35}$$

Together, equations (A.34), (A.35), and the assumption $\sigma(r^\star - \rho) - \xi - \nu < 0$ yield

$$\frac{\dot{\bar{c}}(t)}{\bar{c}(t)} = \sigma(r^\star - \rho) - \nu + \frac{1}{\xi + \nu - \sigma(r^\star - \rho)}.$$

A comparison with the Euler equation of the Ramsey-Consumer reveals that

$$\bar{\rho} = \rho + \frac{\nu}{\sigma} - \frac{1}{\sigma(\xi + \nu - \sigma(r^\star - \rho))}.$$

A.4 Proof of Proposition 5

The optimization problem (28) subject to condition (29) is an isoperimetric problem (Chiang 1992: 280–282), which can be solved by introducing the stock variable

$$y(a) = \int_0^a c(t, t - a')\gamma \exp[-\nu a']\, da' \ . \tag{A.36}$$

As $y_n(a) = c(t, t-a)\gamma \exp[-\nu a]$, the present value Hamiltonian reads

$$\mathcal{H} = \frac{c(t, t-a)^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}\gamma \exp\left[\left(\rho^S - \rho^H - \nu\right) a\right] + \lambda(a) c(t, t-a)\gamma \exp[-\nu a] \ , \tag{A.37}$$

where $\lambda(a)$ denotes the co-state variable of the stock y. The first order conditions yield

$$\lambda(a) = c(t, t-a)^{-\frac{1}{\sigma}} \exp\left[\left(\rho^S - \rho^H\right) a\right] \ , \tag{A.38a}$$

$$\dot{\lambda}(a) = 0 \ , \tag{A.38b}$$

which imply that

$$c(t, t-a) = X(t) \exp\left[\sigma\left(\rho^S - \rho^H\right) a\right] \ , \tag{A.39}$$

with some function $X(t)$ that does not depend on age a. Inserting equation (A.39) into condition (29) and solving for $X(t)$, we obtain

$$X(t) = \frac{\sigma\left(\rho^S - \rho^H\right) - \nu}{\gamma\left(\exp\left[\left(\sigma\left(\rho^S - \rho^H\right) - \nu\right)\tau\right] - 1\right)}\bar{c}(t) \tag{A.40}$$

Inserting $X(t)$ back into equation (A.39) yields equation (31). $\square$

A.5 Proof of Proposition 7

The steady state of an OLG economy, in which a constraint social planner solves maximization problem (37), is observationally equivalent to the steady state of an economy, in which an infinitely lived agent maximizes the welfare of a Ramsey dynasty (18), if they exhibit the same stock of capital $k^\star$. To see this, recall that $k^\star$ determines the interest rate $r^\star$ and the wage $w^\star$ via the production function f. This also implies that the aggregate consumption per effective labor c is the same in both economies, as the budget constraint is identical for the constraint planner and the Ramsey consumer.

We first solve for the Euler equation for the constraint social planner. In the steady state both effective capital k and the investment subsidy τ_r are constant over time:

$$k(t) = k^\star \qquad \tau_r(t) = \tau_r^\star \ . \tag{A.41}$$

As a consequence, in the steady state V and c^{e} depend at most directly on t but not indirectly via τ_r and k. Moreover, we can directly compute the equations for $V(t, \tau_r^\star)$ and $\hat{c}(t, \tau_r^\star)$. In the constrained utilitarian OLG economy, individual steady state consumption and aggregate per capita steady state consumption given by equations (14a) and (15a) when we replace $r^\star$ an $w^\star$ by $r^{\mathrm{e}} = r^\star - \tau_r^\star$ and $w^{\mathrm{e}} = w^\star + \tau_r^\star k^\star$. Introducing $\hat{\psi}(\tau_r^\star, k^\star)$, which is ψ of equation (13) with $r^\star$ an $w^\star$ replaced by r^{e} and w^{e}, and which is given by equation (42c), we derive equation (42a) for $c^{\mathrm{e}}(\tau_r^\star)$, which does not depend on t. For $V(t, \tau_r^\star)$ we derive:

$$\begin{aligned}
V(t,\tau_r^\star) \;&= \\
&= \gamma \frac{\hat{\psi}^{\frac{\sigma-1}{\sigma}}}{\frac{\sigma-1}{\sigma}} \int_{t-T}^{t} \exp\left[\left(\frac{\sigma-1}{\sigma}\xi+\nu+\rho-\hat{\rho}\right) s + (\sigma-1)(r^{\mathrm{e}}-\rho)(t-s)\right] ds \\
&= \gamma \frac{\hat{\psi}^{\frac{\sigma-1}{\sigma}}}{\frac{\sigma-1}{\sigma}} \frac{\exp\left[\left((\sigma-1)(r^\star-\tau_r^\star-\rho)-\frac{\sigma-1}{\sigma}\xi-\nu-\rho+\hat{\rho}\right)T\right]-1}{(\sigma-1)(r^\star-\tau_r^\star-\rho)-\frac{\sigma-1}{\sigma}\xi-\nu-\rho+\hat{\rho}} \exp\left[\left(\frac{\sigma-1}{\sigma}\xi+\nu+\rho-\hat{\rho}\right) t\right] \\
&= \hat{V}(\tau_r^\star) \exp\left[\left(\frac{\sigma-1}{\sigma}\xi+\nu+\rho-\hat{\rho}\right) t\right]
\end{aligned} \tag{A.42}$$

where $\hat{V}(\tau_r^\star, k^\star)$ is given by equation (42b).

Inserting $\hat{V}$ and c^{e} into equation (40a) and differentiating with respect to time yields:

$$\dot{\lambda}(t) = \left(\frac{\sigma-1}{\sigma}\xi + \nu - \hat{\rho}\right)\lambda(t) \ , \tag{A.43}$$

Inserting equations (A.43) and (40a) into equation (40b) yields the Euler equation

$$\hat{\rho} = \frac{\hat{V}_k(\tau_r^\star)}{\hat{V}_{\tau_r}(\tau_r^\star)} c^{\mathrm{e}}_{\tau_r}(\tau_r^\star) - c^{\mathrm{e}}_k(\tau_r^\star) + f'(k^\star) - \frac{\xi}{\sigma} \ . \tag{A.44}$$

From the Euler equation of the infinitely lived agent (21) we know that:

$$\bar{\rho} = r^\star - \frac{\xi}{\sigma} \ . \tag{A.45}$$

Taking into account that $f'(k^\star) = r^\star$ yields equation (41). □

A.6 Numerical calculations

In addition to the parameters given in the tables, we have chosen a Cobb-Douglas production function $f(k) = k^\alpha$ with $\alpha = 0.3$.

A.6.1 Nordhaus

ILA → util. OLG

We calculate what the ILA specifications $\xi = 0.02$, $\sigma = 0.5$, $r^\star = 0.055$ imply for an observationally equivalent utilitarian OLG economy for different individual time preference rates ρ for given lifetime $T = 75$.

ρ	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
$\tau_r^\star$	-0.66%	-0.89%	-1.14%	-1.40%	-1.67%	-1.95%	-2.24%	-2.54%	-2.86%	-3.17%	-3.5%
$\hat{\rho}$	1.86%	2.00%	2.15%	2.31%	2.48%	2.66%	2.86%	3.06%	3.27%	3.49%	3.70%

dec. OLG → ILA

For $\xi = 0.02$, we calculate what combinations of individual parameters lead to $r^\star = 0.055$ in a decentralized OLG economy and what $\bar{\rho}$ exhibits the observationally equivalent ILA economy for different lifetimes T of the individual households.

T=50:

ρ	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
σ	0.98%	1.08%	1.20%	1.35%	1.54%	1.80%	2.15%	2.69%	3.59%	5.39%	10.77%
$\bar{\rho}$	3.46%	3.64%	3.83%	4.02%	4.20%	4.39%	4.57%	4.76%	4.94%	5.13%	5.31%

T=75:

ρ	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
σ	0.65%	0.71%	0.79%	0.89%	1.02%	1.19%	1.42%	1.78%	2.37%	3.56%	7.11%
$\bar{\rho}$	2.41%	2.69%	2.97%	3.25%	3.53%	3.81%	4.09%	4.37%	4.66%	4.94%	5.22%

T=100:

ρ	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
σ	0.53%	0.59%	0.65%	0.73%	0.84%	0.98%	1.17%	1.47%	1.95%	2.93%	5.86%
$\bar{\rho}$	1.75%	2.09%	2.43%	2.77%	3.11%	3.45%	3.79%	4.14%	4.48%	4.82%	5.16%

const. OLG $\rightarrow$ ILA

For $\xi = 0.02$, $T = 75$ and given combination of σ and ρ, which yield $r^\star = 0.055$ in the decentralized OLG, we calculate $r^\star$ and $\tau_r^\star$ in a constraint social planner OLG economy for different $\hat{\rho}$, and what $\bar{\rho}$ exhibits the observationally equivalent ILA economy.

$\rho = 0.01$, $\sigma = 0.79$, $T = 75$:

$\hat{\rho}$	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
$r^\star$	2.39%	2.68%	3.10%	3.59%	4.16%	4.81%	5.55%	6.36%	7.24%	8.18%	9.17%
$\tau_r^\star$	-0.52%	-0.73%	-0.85%	-0.84%	-0.68%	-0.39%	0.03%	0.57%	1.22%	1.96%	2.78%
$\bar{\rho}$	-0.14%	0.15%	0.57%	1.06%	1.63%	2.28%	3.02%	3.83%	4.71%	5.65%	6.64%

$\rho = 0.015$, $\sigma = 0.89$, $T = 75$:

$\hat{\rho}$	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
$r^\star$	2.19%	2.44%	2.81%	3.25%	3.78%	4.40%	5.11%	5.91%	6.78%	7.72%	8.71%
$\tau_r^\star$	-0.33%	-0.62%	-0.85%	-0.93%	-0.85%	-0.61%	-0.24%	0.27%	0.90%	1.63%	2.45%
$\bar{\rho}$	-0.06%	0.19%	0.56%	1.00%	1.53%	2.15%	2.86%	3.66%	4.53%	5.47%	6.46%

$\rho = 0.02$, $\sigma = 1.02$, $T = 75$:

$\hat{\rho}$	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
$r^\star$	2.03%	2.23%	2.54%	2.93%	3.41%	3.99%	4.67%	5.45%	6.31%	7.25%	8.25%
$\tau_r^\star$	-0.08%	-0.45%	-0.79%	-0.97%	-0.98%	-0.82%	-0.50%	-0.03%	0.57%	1.30%	2.12%
$\bar{\rho}$	0.06%	0.26%	0.57%	0.96%	1.44%	2.03%	2.70%	3.50%	4.34%	5.28%	6.28%

$\rho = 0.025$, $\sigma = 1.19$, $T = 75$:

$\hat{\rho}$	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
$r^\star$	1.93%	2.07%	2.30%	2.62%	3.05%	3.58%	4.23%	4.98%	5.83%	6.77%	7.78%
$\tau_r^\star$	0.26%	-0.21%	-0.67%	-0.97%	-1.09%	-1.02%	-0.77%	-0.35%	0.24%	0.95%	1.78%
$\bar{\rho}$	0.24%	0.38%	0.61%	0.93%	1.36%	1.89%	2.54%	3.29%	4.14%	5.08%	6.10%

$\rho = 0.03$, $\sigma = 1.42$, $T = 75$:

$\hat{\rho}$	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
$r^\star$	1.91%	1.98%	2.12%	2.35%	2.69%	3.16%	3.76%	4.49%	5.33%	6.28%	7.30%
$\tau_r^\star$	0.61%	0.11%	-0.45%	-0.90%	-1.16%	-1.21%	-1.04%	-0.67%	-0.12%	0.59%	1.42%
$\bar{\rho}$	0.24%	0.38%	0.61%	0.93%	1.36%	1.89%	2.54%	3.29%	4.14%	5.08%	6.10%

A.6.2 Stern

ILA $\rightarrow$ util. OLG

We calculate what the ILA specifications $\xi = 0.013$, $\sigma = 1$, $r^\star = 0.014$ imply for an observationally equivalent utilitarian OLG economy for different individual time preference rates ρ for given lifetime $T = 75$.

ρ	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
$\tau_r^\star$	-0.09%	-0.11%	-0.13%	-0.15%	-0.17%	-0.20%	-0.23%	-0.27%	-0.31%	-0.36%	-0.42%
$\hat{\rho}$	0.12%	0.13%	0.13%	0.14%	0.15%	0.16%	0.17%	0.19%	0.20%	0.22%	0.25%

dec. OLG → ILA

For $\xi = 0.013$ and $\sigma = 1$, we calculate the resulting $r^\star$ in a decentralized OLG economy for different time preference rates ρ and lifetimes T of the individual households and what $\bar{\rho}$ exhibits the observationally equivalent ILA economy.

T=50:

ρ	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
$r^\star$	4.98%	5.28%	5.89%	5.91%	6.26%	6.62%	6.99%	7.38%	7.77%	8.18%	8.96%
$\bar{\rho}$	3.68%	3.98%	4.29%	4.61%	4.96%	5.32%	5.69%	6.08%	6.47%	6.88%	7.30%

T=75:

ρ	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
$r^\star$	3.60%	3.91%	4.25%	4.61%	4.99%	5.39%	5.80%	6.23%	6.67%	7.12%	7.58%
$\bar{\rho}$	2.30%	2.61%	2.95%	3.31%	3.69%	4.09%	4.50%	4.93%	5.37%	5.82%	6.28%

T=100:

ρ	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
$r^\star$	2.92%	3.25%	3.61%	4.00%	4.41%	4.84%	5.28%	5.74%	6.20%	6.67%	7.15%
$\bar{\rho}$	1.62%	1.95%	2.31%	2.70%	3.11%	3.54%	3.98%	4.44%	4.90%	5.37%	5.85%

util. OLG → ILA

For $\xi = 0.013$, $\sigma = 1$, $T = 75$ and given combination of ρ and $r^\star$ in the decentralized OLG, we calculate $r^\star$ and $\tau_r^\star$ in a constraint social planner OLG economy for different $\hat{\rho}$, and what $\bar{\rho}$ exhibits the observationally equivalent ILA economy.

$\rho = 0.01$, $T = 75$, $r^\star = 0.0425$:

$\hat{\rho}$	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
$r^\star$	1.38%	1.66%	2.05%	2.50%	3.02%	3.63%	4.32%	5.09%	5.92%	6.81%	7.74%
$\tau_r^\star$	-0.10%	-0.40%	-0.62%	-0.68%	-0.59%	-0.34%	0.04%	0.56%	1.18%	1.89%	2.68%
$\bar{\rho}$	0.08%	0.36%	0.75%	1.20%	1.72%	2.33%	3.02%	3.79%	4.62%	5.51%	6.44%

$\rho = 0.015$, $T = 75$, $r^\star = 0.0461$:

$\hat{\rho}$	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
$r^\star$	1.37%	1.64%	2.00%	2.41%	2.91%	3.49%	4.15%	4.90%	5.73%	6.62%	7.57%
$\tau_r^\star$	-0.11%	-0.45%	-0.72%	-0.84%	-0.81%	-0.62%	-0.28%	0.19%	0.79%	1.49%	2.27%
$\bar{\rho}$	0.07%	0.34%	0.70%	1.11%	1.61%	2.19%	2.85%	3.60%	4.43%	5.32%	6.23%

$\rho = 0.02$, $T = 75$, $r^\star = 0.0499\%$:

$\hat{\rho}$	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
$r^\star$	1.37%	1.62%	1.94%	2.33%	2.78%	3.33%	3.96%	4.69%	5.50%	6.39%	7.34%
$\tau_r^\star$	-0.12%	-0.50%	-0.83%	-1.01%	-1.03%	-0.90%	-0.62%	-0.20%	0.36%	1.03%	1.80%
$\bar{\rho}$	0.07%	0.29%	0.59%	0.94%	1.36%	1.86%	2.46%	3.15%	3.94%	4.81%	5.75%

$\rho = 0.025$, $T = 75$, $r^\star = 5.39$:

$\hat{\rho}$	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
$r^\star$	1.36%	1.59%	1.89%	2.24%	2.66%	3.16%	3.76%	4.45%	5.24%	6.11%	7.05%
$\tau_r^\star$	-0.13%	-0.55%	-0.93%	-1.17%	-1.26%	-1.19%	-0.98%	-0.61%	-0.11%	0.53%	1.27%
$\bar{\rho}$	0.06%	0.29%	0.59%	0.94%	1.36%	1.86%	2.46%	3.15%	3.94%	4.81%	5.75%

$\rho = 0.03$, $\sigma = 1.42$, $T = 75$:

$\hat{\rho}$	0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%	4.5%	5%
$r^\star$	1.36%	1.57%	1.83%	2.15%	2.53%	3.00%	3.55%	4.20%	4.95%	5.80%	6.72%
$\tau_r^\star$	-0.14%	-0.60%	-1.03%	-1.33%	-1.48%	-1.48%	-1.34%	-1.04%	-0.59%	-0.01%	0.70%
$\bar{\rho}$	0.06%	0.27%	0.53%	0.85%	1.23%	1.70%	2.25%	2.90%	3.65%	4.50%	5.42%

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