Secondary Issues and Party Politics
An Application to Environmental Policy[1]

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Abstract

The paper develops a political economy model to assess the interplay between party formation and an environmental policy dimension viewed as secondary to the redistributive dimension. We define being a secondary issue in terms of the intensity of preferences over this issue rather than in terms of the proportion of voters who care for the environment. Equilibrium policies are the outcome of an electoral competition game between endogenous parties.

We obtain the following results: i) The Pigouvian tax never emerges in an equilibrium; ii) The equilibrium environmental tax is larger when there is a minority of green voters; iii) Stable green parties exist only if there is a minority of green voters and income polarization is large enough relative to the saliency of the environmental issue. We also study the redistributive policies advocated by green parties.

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1 Introduction

The objective of this paper is to study electoral competition when the policy space is composed of a "frontline issue", redistribution, and of a "secondary issue", environmental policy. Frontline issues are those that are considered important enough that they drive the vote of a large fraction of the electorate. Prominent examples are the aggregate level of government spending or the degree of income redistribution. By opposition, secondary issues are not the main focus of a large fraction of the electorate. Such secondary issues include gun control, trade policy, foreign aid or environmental policy. Many authors then take the view that secondary issues are better studied in the context of special interest politics, and especially of lobbies. A recent paper by List and Sturm (2006) argues to the contrary that electoral incentives constitute an important determinant of policy choices for secondary issues as well. While List and Sturm (2006) focus on political accountability (how incentives for being reelected affect the incumbent's choices in both issues), our focus is on political compromise: how incentives for winning elections affect all politicians' willingness to compromise on both issues.

We choose environmental policy as the secondary dimension because we are especially interested in understanding the role that the formation of political parties plays on that policy domain. Our main motivation is the emergence in the last decades of "green parties" which are mainly focused on the environment. We wish to better understand how these parties survive in a political system where environmental issues are not frontline for a majority of voters, and what type of redistributive policy green parties advocate at equilibrium. More precisely, we wish to shed light on the following questions. Under which circumstances (if any) is the equilibrium environmental policy efficient? How is this policy affected by the proportion of voters who care about pollution? What are the necessary conditions to be satisfied for a green party to form at equilibrium? Can we have more than one green party at equilibrium? Who forms the constituency of a green party? Why is it that green parties are overwhelmingly associated with strong redistributive concerns?[1]

To the best of our knowledge, the political economy literature has not developed electoral competition models where the environment is secondary to another dimension. A large fraction of this literature assumes that the environmental policy is shaped by the action of lobbies and adopts mainly the menu auction approach first introduced by Bernheim and Whinston (1986) and popularized by Grossman and Helpman (2002). In this approach, elections are typically not explicitly modelled.[2]

[1] See Neumayer (2004) and the many references there in for empirical evidence.

[2] With the exception of Wilson and Damania (2005) who combine common agency and Downsian

Recent surveys of this literature include Heyes and Dijkstra (2001) and Oates and Portney (2003).

A second branch of the political economy literature, beginning with Congleton (1992) and sometimes referred to as "majority voting models", applies variants and extensions of the median voter model to diverse economic settings. For instance, Mc Ausland (2003) uses a majority voting model to analyze how inequality and openness to trade interact to determine voters' demand for environmental policy, and Jones and Manuelli (2001) and Kempf and Rossignol (2007) study voting over environmental policy in growth models. A small number of papers introduce however both the redistributive and the environmental dimensions, and use different political equilibrium concepts (sequential voting for Cremer et al. (2004), Party Unanimity Nash Equilibrium (PUNE) for Cremer et al. (2008)) but they do not model the environment as secondary.[3] As for List and Sturm (2006), we differ from them on two main accounts. First, they develop a political agency model with an incumbent, while we focus on electoral competition between parties. Second, they introduce term limits in order to generate - and test - empirical predictions, while our paper is exclusively theoretical.

We develop a two-dimensional model with endogenous parties based on Levy (2004). There is a continuum of voters who differ according to two traits: their income and their concern for the environment. Each trait can take two values, so that there are four groups of people. There are two goods in the economy, the numeraire and a polluting good. Public policy consists of two linear tax rates, one on income and one on the consumption of the polluting good. Tax proceeds are rebated lump sum to all citizens. Public policy is the result of electoral competition between political parties. This can be viewed as a two-stage process. In the first stage, representatives of the different groups form political parties. In the second stage, these parties simultaneously propose political platforms, composed of an income tax rate and an environmental tax rate, in order to win the elections. The party that gets a plurality of the votes wins the election and implements its proposed policy. The crucial assumption is that the set of policies that a party can commit to is endogenous. If a party is made of a single type of citizens, the only proposal it can commit to is their most-preferred policy. On the other hand, if a party is made of citizens of different types, the party can commit to any policy that belongs to the Pareto set of its founders. An equilibrium political state is a partition of citizens into parties and a vector of electoral platforms such that (i) no citizen has an incentive to split up the party he belongs to, or to merge it with another party and (ii) no party can make its members better-off by choosing another electoral platform.

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politics.

[3]Also related are the papers by Brett and Keen (2000) and Anesi (2006a), who study the earmarking of environmental taxes in different electoral competition models with two policy instruments.

We obtain the following results. The Pigouvian level of the environmental tax rate is never an equilibrium of this game. Surprisingly, the equilibrium environmental tax is larger when there is a minority of green voters than when they form a majority. As a result, we find that a green party, defined as a party that proposes green voters' most-preferred environmental policy, can only be part of an equilibrium political state (i.e., be stable) if there is a minority of green voters. This result suggests that the main reason for the emergence of green parties is not simply to be found in an increase in the number of voters holding green views. Rather, we obtain that, for a green party to be stable, it is necessary for income polarization to be large enough, compared to the saliency of the environmental issue, for the non-green citizens. Finally, we show that a larger income polarization increases the minimum income tax rate proposed by the green party.

Before proceeding, the connections between the present paper and our earlier work (Anesi and De Donder, 2009) are noteworthy. In the latter, we studied the role of party formation in a similar model where the second dimension is attitude towards racism instead of an environmental issue. We differ from that paper in two main respects. First, the focus of Anesi and De Donder (2009) was to understand why racist policies may emerge when a minority of people hold racist views. We then made strong assumptions on the distribution of types but none on the relative saliency of the two issues. By contrast, in the current paper we make no assumption on the distribution of types (green voters may or may not form a majority) but rather assume that the environmental dimension is secondary to the redistributive one. Second, we adopt a different collective choice model and, more specifically, a different stability concept for political parties. In contrast to Anesi and De Donder (2009) who allowed for deviations to smaller parties only and who considered purely policy-motivated politicians, we allow here for mergers between existing parties and we assume that politicians are both policy- and office-motivated.

The rest of the paper is organized as follows. Section 2 presents the economic environment and the political equilibrium concept. Section 3 explains in what sense environmental taxation represents a secondary issue. Sections 4 and 5 analyze, respectively, equilibrium taxes and the parties formed at equilibrium. Final remarks are made in Section 6.

2 The Model

2.1 The Economic Environment

There is a large citizenry with total mass equal to one, in an economy with two goods: the numeraire and a polluting good, which are both produced at constant marginal cost, normalized to unity.[4] Citizens are differentiated by their exogenous income, $\omega \in \{\omega_\ell, \omega_h\}$, with $\omega_\ell < \omega_h$, and their concern about pollution, $j \in \{g, n\}$: the "green voters" ($j = g$) care for the pollution associated with aggregate consumption of the polluting good, while the others ($j = n$) do not. Following Fredriksson (1997), we assume that the preferences of green voters over the two consumption goods are given by

$$c + V(x) - \alpha \bar{x}, \tag{1}$$

where c and x are individual consumptions of the numeraire and the polluting good, respectively, $\bar{x}$ is the aggregate consumption of the polluting good, and $\alpha \in (0, 1)$ is a parameter that measures the intensity of the green voters' concerns about pollution. This intensity is assumed to be the same for all green voters. The utility of a non-green voter is simply given by

$$c + V(x). \tag{2}$$

All individuals have the same taste for individual consumption of the polluting good, which is represented by the continuous function V with the following properties: $V(0) \geq 0$, $V' > 0$, $V'' < 0$, $\lim_{x\to 0} V'(x) = \infty$, and $\lim_{x\to\infty} V'(x) < 1$.

Let $\Theta \equiv \{\omega_\ell, \omega_h\} \times \{g, n\}$ be the type space, with generic element $\theta_i^j = (\omega_i, j)$. The fraction of the population that is of type θ_i^j is μ_i^j, where $\mu_i^j < 1/2$ for every $i = \ell, h$ and $j = g, n$. The proportion of voters with income level $\omega_i, i = \ell, h$ is denoted by $\mu_i = \mu_i^g + \mu_i^n$, whereas the proportion of green (respectively, non-green) voters is given by $\mu^g = \mu_\ell^g + \mu_h^g$ (respectively, $\mu^n = \mu_\ell^n + \mu_h^n$). Let $\bar{\omega} \equiv \mu_\ell \omega_\ell + \mu_h \omega_h$ be the aggregate income, and assume as usual that the median income is below the average ($\mu_\ell > 1/2$).

The policy that voters must choose is composed of a proportional income tax, $t \in [0, 1]$, and an environmental tax on the consumption of the polluting good, $e \in [0, 1]$. Tax revenues are used to finance a lump sum transfer to all citizens, which is then determined as a residual: $T = t\bar{\omega} + e\bar{x}$. Once a public policy (t, e) has been decided, citizens choose the consumption level that maximizes their direct utility ((1) for green and (2) for non-green citizens) subject to the individual budget constraint

$$c + (1 + e)x \leq (1 - t)\omega + T.$$

Solving the consumers' problem leads to the following characterization of the demand for the polluting good, $x(e)$:

$$V'(x) \equiv 1 + e.$$

Each individual's choice is too small to affect the average quantity of the public good, $\bar{x}$, so that with quasi-linear preferences they all end up consuming the same

[4] This assumption is made to simplify the exposition. All results carry through to the case of varying marginal costs.

amount of the good,[5] and $\bar{x}(e) = x(e)$. After appropriate rearrangements, the policy preferences of an individual of type (ω, j) can be represented by the following indirect utility function:

$$u(t, e, \omega, j) \equiv \begin{cases} \omega + t(\bar{\omega} - \omega) + V(x(e)) - (1+\alpha)x(e) & \text{if } j = g \\ \omega + t(\bar{\omega} - \omega) + V(x(e)) - x(e) & \text{if} j = n. \end{cases} \quad (3)$$

It is easy to obtain individual θ_i^j's most-preferred policy. Obviously, in the absence of incentive effects from income taxation, poor voters favor income confiscation $(t = 1)$, whereas rich voters prefer laissez-faire $(t = 0)$. As for the environmental policy, non green voters dislike any form of environmental taxation $(e = 0)$, while green voters's most-preferred tax rate is equal to the intensity of their dislike of pollution $(e = \alpha)$. Observe that, in our setting, the most-preferred environmental tax of an individual is independent of her income. This is due to the fact that all individuals consume the same quantity of the polluting good, so that environmental taxation is not redistributive.[6]

Since nobody would prefer to increase e above α, without loss of generality we restrict the policy space to be $P = [0, 1] \times [0, \alpha]$, with generic element (t, e). In this economy, the collective choice of a public policy (t, e) is made through electoral competition between endogenous political parties. We now turn to the description of the electoral competition side of the model.

2.2 Political Parties and Elections

We propose the following adaptation of Levy (2004). We present that approach in the context of our paper, but refer the reader to those papers for an in-depth discussion of the basic assumptions.

Each group of voters is represented by a single politician who is a perfect representative of her group, in that her policy preferences are given by (3). Politicians running alone are unable to commit to any proposal differing from their ideal policy. The key assumption of Levy (2004) is, however, that politicians can credibly commit to a larger set of policies by forming political parties (or *coalitions*, to use the language of game theory): the set of policies which a party can commit to is the Pareto set of its members. Formally, a *politician* is an element θ of Θ while a *party* is a non-empty subset S of Θ. A policy $(t, e) \in P$ is in the Pareto set of party S,

[5]We assume that even poor individuals have income (or unmodelled wealth) large enough to consume that amount.

[6]Modelling a situation where richer people consume more of the polluting good would be more in line with reality (see De Donder et al. (2007), section 4, for the case of energy consumption), but would complexify the analysis by adding a redistributive component to environmental taxation. Moreover, this added complexity would not generate additional insight about the political phenomena under study.

denoted by P_S, if there is no other policy (t', e') such that $u(t', e'; \theta) \geq u(t, e; \theta)$ for all $\theta \in S$ and $u(t', e'; \hat{\theta}) > u(t, e; \hat{\theta})$ for some $\hat{\theta} \in S$.

The political game we study has two steps. The first step is one of party formation, while the second step encompasses electoral competition, where all parties simultaneously choose a feasible policy and compete in a winner-takes-all election. We now describe how each step takes place, beginning with the electoral competition game.

Electoral Competition

A party structure is a partition of Θ into parties (i.e., we assume that all citizens belong to a political party). Let Π be the set of party structures. We assume that the result of the party formation stage is some arbitrary party structure $\pi \in \Pi$. Elections then proceed as follows. Every party $S \in \pi$ chooses an *electoral strategy* (or *platform*), namely a policy $(t_S, e_S) \in P_S \cup \{\varnothing\}$, where $\varnothing$ means that the party proposes no policy (we say that it does not run). In the case where no party runs for election, every politician receives a zero payoff. If at least one party runs, we assume that voters record their preferences sincerely over any list of candidate platforms, $\mathbf{p} \equiv ((t_S, e_S))_{S \in \pi}$, and that the election is by plurality rule with no abstention.[7] The election outcome is then a fair lottery between the policies in

$$W(\mathbf{p}) \equiv \left\{ (t_S, e_S) : S \in \arg\max_{S' \in \pi} V_{S'}(\mathbf{p}) \right\} ,$$

where $V_{S'}(\mathbf{p})$ denotes party S''s realized vote share. We assume that parties prefer not running to proposing a policy that will lose for sure.

Let $\psi_\theta(S)$ be the indicator function on 2^Θ taking on the value of 1 if $\theta \in S$ and 0 otherwise. Members of the winning party equally share an (arbitrarily small) non-policy benefit $\beta > 0$ (ego-rents, perks of office...). As a consequence, the expected utility of politician θ resulting from a profile of electoral strategies $\mathbf{p}$ is given by

$$U(\mathbf{p}, \theta) \equiv \frac{1}{|W(\mathbf{p})|} \sum_{(t_S, e_S) \in W(\mathbf{p})} \left[u(t_S, e_S, \theta) + \psi_\theta(S) \frac{\beta}{|S|} \right]$$

if there is at least one party $S \in \pi$ such that $(t_S, e_S) \neq \varnothing$, and $U(\mathbf{p}, \theta) = 0$ otherwise.

Given a party structure $\pi \in \Pi$, a vector of electoral strategies $\mathbf{p} = ((t_S, e_S))_{S \in \pi}$ is a $\pi-$*equilibrium* of the electoral-competition game if, for all $S \in \pi$, there is no $(t'_S, e'_S) \in P_S \cup \{\varnothing\}$, $(t'_S, e'_S) \neq (t_S, e_S)$, that satisfies

$$U((t'_S, e'_S), \mathbf{p}_{-S}; \theta) \geq U((t_S, e_S), \mathbf{p}_{-S}; \theta)$$

[7] Voters who are indifferent between several policies use a fair mixing device.

for all $\theta \in S$, with at least one strict inequality. Let $\delta(\pi)$ be the set of π-equilibrium policy outcomes.[8]

Stability of Party Structures

Up to this point, we have taken the party structure π as given. We now turn to the party formation stage and ask whether π is a stable party structure. First of all, note that there may exist multiple π-equilibria, and therefore multiple equilibrium outcomes ($\delta(\pi)$ may not be a singleton). Thus, π may satisfy stability conditions for one electoral outcome but not for others. As a consequence, we will not study the stability of π alone, but the stability of pairs $(\pi, \mathbf{p})$ where $\mathbf{p}$ is a π-equilibrium. We will refer to them as *political states*. Which of these should be considered as the set of equilibrium outcomes for the present model? The answer to this question depends on the stability requirements imposed on party structures. The stability condition we adopt in this paper is bi-core stability (Levy, 2004).[9]

Let π and π' be two party structures. π' is said to be *induced* from π if π' is formed by either breaking one party in π into two, or by merging two existing parties in π into one. Forming a new party made up of subsets of current parties is excluded on the basis that nobody would trust a politician who is willing to betray her current partners. Mergers involving more than two existing parties are also excluded but, in the context of the present paper, this restriction does not affect the results and is only maintained for expositional simplicity.

We say that a political state $(\pi, \mathbf{p})$ is *blocked* by another political state $(\pi', \mathbf{p}')$ if there exists $S \subseteq \Theta$ such that: (1) S can induce π' from π, and (2) for every $\theta \in S$, $U(\mathbf{p}', \theta) > U(\mathbf{p}, \theta)$. We are now ready to define equilibrium political states:

Definition 1 *Let $\pi^* \in \Pi$ be a party structure and $\mathbf{p}^*$ a profile of electoral strategies for π^*. The pair $(\pi^*, \mathbf{p}^*)$ is an equilibrium political state (EPS) if (i) $\mathbf{p}^*$ is a π^*-equilibrium, and (ii) there is no political state $(\pi, \mathbf{p})$ that blocks $(\pi^*, \mathbf{p}^*)$.*

Thus, an equilibrium situation is defined as one that meets two requirements: first, given the equilibrium party structure π^*, no party $S \in \pi^*$ can make all its

[8] Any profile of electoral strategies induces an electoral outcome which, due to the possibility of a tie, may be a lottery between several policies. As a consequence, $\delta(\pi)$ is a subset of the family of fair lotteries over P. Throughout the paper, we write $\langle x_1, \ldots, x_n \rangle$ for the random mixture between policies $x_1, \ldots, x_n$, but simply use x instead of $\langle x \rangle$.

[9] As its definition reveals, the bi-core stability concept does not allow for deviating coalitions to be farsighted – i.e., to consider whether a blocking state is itself stable or not. There are two justifications to this assumption. First, "farsighted" stability concepts – such as the equilibrium processes of coalition formation (Konishi and Ray, 2003, Anesi, 2006b), or the extended equilibrium binding agreements (Diamantoudi and Xue, 2007) – face serious tractability and existence problems in voting situations with infinite sets of states, as in our paper. Second, a large body of experimental literature rejects both individuals' and groups' farsightedness (the most recent contributions include Hey and Knoll, 2007, and Bone et al., 2009).

members better-off by deviating from its equilibrium announcement to a different platform in $P_S \cup \{\varnothing\}$; second, given the equilibrium platform profile $\mathbf{p}^*$, parties in π^* are stable in the sense that no coalition of politicians can make all its members strictly better-off by inducing another political state.

We are now in a position to apply this political equilibrium concept to our economic environment.

3 Environmental Policy as a Secondary Issue

Before we turn to the formal characterization of political equilibria, we first define what we mean by environmental policy being a "secondary issue" compared to redistribution. Unlike List and Strum (2006), our definition is not related to the number of people caring for the environment, but rather to the intensity of their preferences. To make this point more formally, some additional notation will prove handy. Since the indirect utility functions (1) and (2) are separable in t and e, we denote by Δ^j the difference in utility level, for an individual of type θ_i^j, $j = g, n$, between her most-preferred and her least-preferred environmental policy in the policy space P — i.e.

$$\begin{aligned}
\Delta^g &\equiv u(t,\alpha,\omega,g) - u(t,0,\omega,g) = V\left(x(\alpha)\right) - V\left(x(0)\right) - (1+\alpha)\left[x(\alpha) - x(0)\right], \\
\Delta^n &\equiv u(t,0,\omega,n) - u(t,\alpha,\omega,n) = V\left(x(0)\right) - V\left(x(\alpha)\right) - \left[x(0) - x(\alpha)\right].
\end{aligned}$$

Similarly, we denote by Δ_i the difference in utility level, for an individual of type θ_i^j, $i = \ell, h$, between her most-preferred and her least-preferred income taxation policy in the policy space P — i.e.

$$\begin{aligned}
\Delta_\ell &\equiv u(1,e,\omega_\ell,j) - u(0,e,\omega_\ell,j) = \bar{\omega} - \omega_\ell = \mu_h\left(\omega_h - \omega_\ell\right), \\
\Delta_h &\equiv u(0,e,\omega_h,j) - u(1,\alpha,\omega_h,j) = \omega_h - \bar{\omega} = \mu_\ell\left(\omega_h - \omega_\ell\right).
\end{aligned}$$

For future reference, note that Δ_ℓ (and similarly $\mu_\ell\Delta_\ell$) can be seen as a measure of income polarization, namely a measure of the saliency of the conflict between the rich and the poor. In the spirit of Esteban and Ray's (1994) original definition, polarization should indeed rise as inequality $(\omega_h - \omega_\ell)$ increases and the sizes of the two groups become closer to each other $(\mu_h \to 1/2)$.

We impose the following restriction on preferences: for all individuals θ_i^j, the difference in utility level from moving from the least-preferred to the most-preferred *taxation* policy is larger than the difference in utility from moving from the least-preferred to the most-preferred *environmental* policy. Formally, we impose the following assumption:[10]

[10] Observe that $\Delta_l < \Delta_h$, because, by assumption, $\mu_l > 1/2$.

A1 $\max\{\Delta^g, \Delta^n\} < \Delta_\ell$

Assumption A1 is the precise statement that environmental policy is a secondary issue compared to redistributive policy. This assumption imposes restrictions on preferences over extreme policy bundles. It guarantees that every citizen prefers a policy bundle comprising her ideal redistributive and worst environmental policies to a bundle involving her worst redistributive and ideal environmental policies. For instance, the non-green rich prefers no redistribution accompanied with a high pollution tax to the total confiscation of their income without pollution tax. By this assumption, we do not deny that there may exist people who would be ready to give up all their resources for higher pollution taxes, but we assume their mass is not electorally significant.[11]

4 Environmental Taxes

We start with the benchmark case where there is no party formation.[12]

Lemma 1 *Let* $\pi^0 \equiv \{\{\theta_h^g\}, \{\theta_\ell^g\}, \{\theta_\ell^n\}, \{\theta_h^n\}\}$. *Suppose A1 holds, and* $\mu^g \neq 1/2$.[13] *Then*

$$\delta\left(\pi^0\right) = \begin{cases} \{(1,\alpha)\} & \text{if } \mu^g > 1/2, \\ \{(1,0)\} & \text{if } \mu^g < 1/2. \end{cases}$$

In the absence of party formation, our model boils down to the standard citizen-candidate framework proposed by Osborne and Slivinski (1996),[14] where the only credible proposal by any citizen is her own most-preferred policy. In that case, the set of feasible policies is restricted to $\{(1,0),(1,\alpha),(0,0),(0,\alpha)\}$, and assumption A1 guarantees the existence of a transitive majority voting ordering over this set. Since poor outnumber rich citizens, any policy with income confiscation gets a majority compared to any policy with laissez-faire. If green voters outnumber non-green ($\mu^g > 1/2$), for any tax policy, a policy with $e = \alpha$ is favored by a majority to a policy without environmental tax ($e = 0$). The policy $(1, \alpha)$ is then a Condorcet

[11] List and Sturm (2006) report that the number of members in the three largest environmental organizations (Greenpeace, the Sierra Club and the National Wildlife Federation) between 1987 and 2000 varies from a minimum of 0.25 percent of the population in Mississippi to a maximum of just over 2 percent in Vermont.

[12] All proofs are relegated to the Appendix.

[13] We assume away the case where $\mu^g = 1/2$ first because this is a knife-edge situation, but, mainly because considering this case increases considerably the length of the proofs without adding any new insight. For the interested reader, with $\mu^g = 1/2$ we obtain that $\delta(\pi^0) = \{\langle(1,\alpha),(1,0)\rangle\}$ if $\mu_\ell^g = \mu_\ell^n > \mu_h$, and $\emptyset$ otherwise. The proof is available upon request.

[14] The model proposed by Besley and Coate (1997) differs in that it assumes that voters behave strategically.

winner among the four possible policies (i.e., it beats any other feasible option through pairwise majority comparisons). In the case where $\mu^g < 1/2$, the Condorcet winning policy is $(1, 0)$.

It is easy to see that the Condorcet winning policy is an equilibrium of the electoral competition game with partition π^0. More precisely, the candidate most preferring the Condorcet winner runs unopposed and obtains her most-preferred policy since, by definition, no other candidate can run with a different policy and defeat the Condorcet winner. The less easy part to prove in Lemma 1 is that there is no other equilibrium. To do that, we consider in turn cases where more than one candidate runs, (i.e., proposes his most-preferred policy), and we show that they can not constitute equilibria. We now briefly summarize how we proceed in the proof in order to give the reader a better feeling as to how the electoral competition stage gets solved in our model.

Since there is a strict transitive majority voting ordering over the four feasible policies, it is impossible for two candidates to run at equilibrium and to tie. Given our assumption that a losing candidate/party prefers no to run, we can rule out any situation with two candidates running. The same intuition carries through to the case where the four candidates run: given that poor outnumber rich voters, one rich candidate loses for sure if they run, and thus prefers not to run. This leaves only the possibility that three candidates run at equilibrium. Given that poor voters form a majority, it is impossible to have a three-way tie with two rich candidates running. With two poor and one rich candidates running, we show in the Appendix that one poor candidate has an incentive not to run to guarantee that the other poor candidate will win for sure. This is because, by assumption A1, a poor citizen prefers the policy favored by the poor citizen-candidate of the other environmental type to a random mixture between the three original policies. This proves that the only equilibrium under π^0 has one candidate running with the Condorcet winning policy. We then obtain the very intuitive result that, *in the absence of party formation, the pollution tax is larger when there is a majority of green voters in the electorate.*

We now turn to party formation. The main incentive to form a party is to enlarge the set of policies that may credibly be proposed to the voters. Figure 1 depicts the Pareto set of all potential parties. Intuitively, parties made exclusively of rich and poor green (resp., non-green) citizens may credibly propose any income tax rate ($t \in [0, 1]$) provided that it is coupled with the maximum (resp. minimum) preferred environmental tax rate $e = \alpha$ (resp., $e = 0$). Similarly, a party made exclusively of green and non-green poor (resp., rich) citizens may credibly propose any environmental tax rate ($e \in [0, \alpha]$) provided they also propose full confiscation, $t = 1$ (resp., laissez-faire, $t = 0$).

As for parties with two opposite types ($\{\theta^n_\ell, \theta^g_h\}$ and $\{\theta^g_\ell, \theta^n_h\}$), observe that the environmental tax rates associated to interior income tax rates differ according to which opposite types compose the party. This is due to the fact that, with a majority of poor voters ($\mu_\ell > 1/2$), rich citizens care more about income tax policy than poor citizens (in the sense that $\Delta_h > \Delta_\ell$, as noted in footnote 10). A rich non-green citizen will then compromise more on environmental policy (i.e., accepts (t, e) with $e > \alpha/2$ and $0 < t < 1$ when forming a party with the poor green citizen) than a poor non-green citizen, who will insist on a low value of e ($e < \alpha/2$ for $0 < t < 1$) when joining forces with rich green citizens. Formally, when $0 < t < 1$, we have that $(t, \alpha\mu_\ell) \in P_{\{\theta_\ell^g, \theta_h^n\}}$, while $(t, \alpha\mu_h) \in P_{\{\theta_\ell^n, \theta_h^g\}}$. Finally, the Pareto set of parties composed of three types can easily be obtained from the previous case; and the Pareto set of the four-type party is equal to the feasible set P.

We now proceed to a comparison between EPS when green voters are a majority and when they are not. But before we state those results, the following remark is in order. While EPS always exist in this model (see, for instance, the EPS described immediately after footnote 15), unicity is far from being guaranteed. Therefore, to make the equilibrium comparison meaningful, any statement about one or several equilibrium policies must be true for all the equilibria associated to the case under consideration. We start with the case where a majority of citizens are green.

Proposition 1 *Suppose A1 holds. If $\mu^g > 1/2$, then any environmental tax rate e^* that emerges in an EPS must satisfy $e^* \leq \alpha\mu_h$.*

The intuition for this result runs as follows. The poor green citizens are in a position of power since there are more poor than rich voters, and more green than non-green voters. As Lemma 1 shows, poor green citizens obtain their most favored policy when no party forms. This hinders the formation of any party containing poor green candidates. Take for instance a party composed of both green politicians. Poor green voters have a double incentive to disband such a party: they would not have to compromise on the income taxation issue and moreover they would not have to share the spoils of office (however small β is) with their partner.

On the other hand, poor green citizens are not powerful enough to win against all others because, by assumptions, $\mu_l^g < 1/2$. As a result, the citizen-candidate equilibrium depicted in Lemma 1 is not an EPS, since the rich green citizens have an incentive to form a party together with the poor non-green citizens in order to propose a compromise policy (a positive but not extreme income tax coupled with a low but positive environmental tax) that they both prefer to $(1, \alpha)$ and that obtains a majority of votes against $(1, \alpha)$.[15] Moreover, it can be shown that the party formed of θ_h^g and θ_l^n, proposing $(t, \alpha\mu_h)$ for some $0 < t < 1$, and running unopposed is an EPS. In a nutshell, the poor green voters are "too powerful" to form a stable party but "not powerful enough" to guarantee themselves against other parties. We then obtain that poor green voters can not obtain their most-preferred environmental

[15] This statement is made formally in Lemma 2, which is presented and proved in the Appendix.

policy ($e = \alpha$), and moreover that there is no EPS where the environmental tax is larger than $\alpha\mu_h$.

This reasoning does not carry through to the case where green citizens form a minority ($\mu^g < 1/2$). In that case, poor green voters have no incentive to break a party made of rich as well as poor green voters, but on the contrary have an incentive to join forces to increase environmental taxation. On the contrary, poor non-green voters become powerful enough that they are reluctant to form a party. This explains why the allocation described in the previous paragraph is not an EPS when $\mu^g < 1/2$, since the poor non-green voter would disband the party formed with the rich green candidate in order to win outright with her most-preferred policy (1,0). The next Proposition shows that all equilibrium political states exhibit a large environmental component ($e \geq \alpha\mu_\ell$) in that situation.

Proposition 2 *Suppose A1 holds. If $\mu^g < 1/2$, then any environmental tax rate $\bar{e}$ that emerges in an EPS must satisfy $\bar{e} \geq \alpha\mu_\ell$.*

Combining Propositions 1 and 2, we obtain the following surprising result: *Due to the party formation process, the environmental tax rate that emerges in a political equilibrium is larger when there is a minority of green voters.* This result illustrates very starkly that, given our modelling of party formation and electoral competition, an increase in the proportion of green voters need not result in more environmentally friendly policies. Another immediate consequence of Propositions 1 and 2 is that environmental quality is better when there is a minority of green voters.

Turning to the normative properties of EPS, observe first that, in our quasi-linear setting without income tax distortions, a utilitarian planner is indifferent between all values of the income tax rate. The optimal utilitarian environmental tax rate is given by its Pigouvian level, $e^* = \alpha\mu^g$. This Pigouvian level belongs to the Pareto sets of the grand four-member party, of several three-member parties, and also of two-member parties in the special case where $\mu^g \in \{\mu_h, \mu_\ell\}$.

We then obtain the following corollary to Propositions 1 and 2.

Corollary 1 *Every EPS is inefficient, in the sense that the Pigouvian tax rate $\alpha\mu^g$ is never implemented in equilibrium.*

The inefficiency of every EPS is driven by the link between the proportion of green voters and the equilibrium environmental tax rate. The presence of a majority of green citizens calls for a large Pigouvian tax ($e > \alpha/2$) but generates an equilibrium with a low tax rate ($e < \alpha/2$), and vice versa when green citizens form a minority.

To summarize, the main conclusion to draw from the discussion up to this point is the following: When party formation is taken into consideration, the explanation for the emergence of green policies is not to be found in an increase in the proportion of green voters. The next section will show that other factors, such as the saliency of the environmental issue and the income polarization may play an important role in explaining the emergence of green parties/policies.

5 Stable Green Parties

We now address the question of the existence of a green party, which is defined as a party offering the ideal environmental policy of green citizens. Formally, party $S \subseteq \Theta$ is a *stable green party* if there exists an EPS $(\pi, \mathbf{p})$ such that $S \in \pi$ and $e_S = \alpha$. We already know from the previous section that a stable green party exists only if there is a minority of green voters. The next proposition goes further.

Proposition 3 *A stable green party $S \subseteq \Theta$ exists only if $S = \{\theta_h^g, \theta_\ell^g\}$ and the following conditions hold: (a) $\mu^g < 1/2$, (b) $\mu_\ell \Delta_\ell \geq \Delta^n$, and (c) $\mu_\ell \Delta^g \geq \mu_h \Delta^n$. Furthermore, if (c) is replaced by (d) $\mu_h \Delta^g > \mu_\ell \Delta^n$, then $\{\theta_h^g, \theta_\ell^g\}$ is a stable green party.*

The only class of green party that may exist at equilibrium (i.e., be stable) is composed of the two types of green voters. This is a consequence of the political power of the poor non-green candidate, who belongs simultaneously to the majority of poor voters ($\mu_\ell > 1/2$) and to the majority of non-green voters ($\mu^g < 1/2$). On one hand, the poor non-green candidate does not wish to constitute a party with a green candidate, since he is powerful enough alone. On the other, he has enough electoral power to defeat a party composed of green and non-green citizens that would run against him. The only stable green party must then be made of the two green voters (who would not want to share power with and accommodate a third type) running against the two separate non-green candidates.

In words, Proposition 3 establishes three necessary conditions for the green party $\{\theta_h^g, \theta_\ell^g\}$ to be stable:

(a) The green citizens form a minority;

(b) Income polarization, measured by $\mu_\ell \Delta_\ell$, is large enough compared to the saliency of the environmental issue for the non-green citizens, measured by Δ^n;

(c) The saliency of the environmental policy for the green citizens is sufficiently large compared to the saliency of this issue for the non-green citizens.

The intuition for condition (a) is familiar from the previous subsection: Proposition 1 establishes that equilibrium environmental taxes cannot exceed $\alpha \mu_h$ when the green citizens form a majority. But, even when the green citizens form a minority, the green party has still to guard itself against two dangers, one external and one internal to the party.

The external danger is the majority coalition formed by the non-green voters. We show that such a threat can only be countered if condition (b) holds. Suppose, to the contrary, that income polarization is relatively low. This weakens the redistributive conflict between rich and poor non-green politicians, thereby causing the defeat of the green party: a non-green candidate can run and win the elections against the green party by getting the votes of all non-green voters (who outnumber green voters), or the non-green politicians can compromise on the redistributive issue and form a party that defeats the green party.

The internal danger faced by the green party consists in one of its two members being wooed away by the policy of a non-green candidate. More precisely, if the poor green and non-green voters were to both prefer the policy $(1, 0)$ to the compromise policy (t, α) proposed by the green party, then the policy $(1, 0)$ would be proposed by the poor non-green candidate who would win the elections for sure since poor voters form a majority. We show in the proof of Proposition 3 that this threat does not materialize if condition (c) is satisfied.

By making it easier for the green politicians to compromise on the income tax rate than for the non-green politicians, condition (c) has another effect on the stability of the green party. It also ensures that, whenever it faces a party made of the two non-green politicians, the green party can always find a policy that attracts some non-green voters and then defeat its opponent. Combined with (b), condition (c) therefore guarantees that non-green cannot coalesce in a party to defeat the green party.

In the first part of Proposition 3, (a), (b), and (c) establish only necessary conditions for the existence of a stable green party. However, the second part of the Proposition reveals that reinforcing (c) (replacing it by (d)) suffices to obtain existence.

Proposition 3 has interesting implications in terms of the electoral alliances between green and non-green voters and in term of the policies proposed by green parties.

First, as we explained above, Proposition 3 shows that a large enough income polarization is necessary for the emergence of a stable green party. Moreover, increasing income polarization dampens the external threat to the existence of the green party and also increases the minimum tax rate that the green party needs to propose in order to fend off the internal threat — i.e., to prevent the poor green citizen from siding with the poor non-green citizen rather than supporting (t, α). This is illustrated in Figure 2, where the set of policies (t, α) that are preferred by the poor green citizen to policy $(1,0)$ shrinks (i.e., $1-\Delta^g/\Delta_\ell$ increases) as income polarization – represented here by Δ_ℓ – increases relative to the saliency of the environmental issue for green voters. The role of income polarization is then summarized in the following

Remark 1 *A large enough income polarization, namely $\mu_\ell \Delta_\ell \geq \Delta^n$, is necessary to have a stable green party. Furthermore, when this condition holds, the minimum equilibrium income tax rate proposed by the green party increases and converges to one as income polarization becomes arbitrarily large.*

Note that this result is in line with the fact that, according to the empirical evidence, green parties are overwhelming associated with strong redistributive concerns (see Neumayer, 2004, and the many references therein).

A second implication of Proposition 3 is that a party involving a non-green politician never offers a green policy ($e=\alpha$) in equilibrium. Therefore, our model predicts that "Red-Green alliances" and (less common) "Blue-Green alliances" between green politicians and leftist/rightist non-green politicians typically fail to deliver the green type's most-preferred environmental policy.

A last implication is that a situation with two green parties is not stable. This is also in line with real world experience, where situations with several green parties coexisting (as in France in the 1990s) do not persist for long.

6 Conclusion

In this paper, we have built a model of electoral competition to assess the interplay between political party formation and environmental taxation viewed as secondary to the income taxation. We have defined being a secondary issue in terms of the intensity of preferences over this issue rather than in terms of numbers of voters who care for it. We have built on Levy (2004) for the political equilibrium concept, defined as the solution to a two stage game where politicians first form parties and where parties then compete by choosing a policy bundle in order to win the elections.

The first two propositions together establish that the equilibrium environmental tax is larger when the green voters represent a minority of the electorate than when they form a majority. The main driving force behind this result is that, when green voters form a majority, they are electorally too powerful to compromise and form a party with other citizens, but not powerful enough to prevent other types from merging into a party and defeating them. Observe that this result is very different from what we would obtain with, for instance, a median voter approach applied sequentially to the two dimensions. In that case, a majority of poor and green voters would simply translate into a confiscatory policy coupled with a large environmental tax rate. Contrasting these results shows the importance of taking into account the endogeneity of the political parties, both in terms of number of parties and of their constituency. It also shows very starkly that, at least within the confines of our model, the reason for the emergence of green parties and policies is not to be found in an increase in the proportion of voters who care for the environment. Rather, as Proposition 3 illustrates, the saliency of the environmental issue and income polarization play an important role in explaining why green parties and green policies emerge at equilibrium.

More precisely, our model suggests the existence of a positive relationship between income polarization and the existence of, and the degree of income redistribution proposed by, green parties. Although there exists a literature showing that higher income inequality is associated with worse environmental results (see, for instance, Boyce (1994), Heerink et al. (2001)), we are aware of no study linking income inequality and green parties' policies. We hope this paper contributes to drawing the attention of applied researchers to this issue.

The remaining results of the paper regarding the existence, the number and the policies of green parties, follow probably more closely our intuition. We obtain that there can only be one stable party, which is made of both types of green voters, who bargain over redistribution, but agree on environmental policy. This means that a situation with two green parties differing in redistributive policy, such as that experienced by France in the 1990s for instance, is not stable. We also obtain that green parties are associated with large redistribution, in the sense that there exists a lowerbound on the income tax rate proposed at equilibrium by any green party. This is in line with the numerous empirical evidence, surveyed by Neumayer (2004), which shows that green parties are located to the left on the redistributive dimension.

Appendix

Throughout this appendix, we will use the following notation:

$$\begin{aligned}
\pi^1 &\equiv \{\{\theta_h^g, \theta_\ell^g\}, \{\theta_\ell^n\}, \{\theta_h^n\}\} \quad , \quad \pi^2 \equiv \{\{\theta_h^g\}, \{\theta_\ell^g, \theta_\ell^n\}, \{\theta_h^n\}\} \\
\pi^3 &\equiv \{\{\theta_h^g\}, \{\theta_\ell^g, \theta_h^n\}, \{\theta_\ell^n\}\} \quad , \quad \pi^4 \equiv \{\{\theta_h^g, \theta_\ell^n\}, \{\theta_\ell^g\}, \{\theta_h^n\}\} \\
\pi^5 &\equiv \{\{\theta_h^g, \theta_h^n\}, \{\theta_\ell^g\}, \{\theta_\ell^n\}\} \quad , \quad \pi^6 \equiv \{\{\theta_h^g\}, \{\theta_\ell^g\}, \{\theta_\ell^n, \theta_h^n\}\} \\
\pi^7 &\equiv \{\{\theta_h^g, \theta_\ell^g\}, \{\theta_\ell^n, \theta_h^n\}\} \quad , \quad \pi^8 \equiv \{\{\theta_h^g, \theta_h^n\}, \{\theta_\ell^g, \theta_\ell^n\}\} \\
\pi^9 &\equiv \{\{\theta_h^g, \theta_\ell^n\}, \{\theta_\ell^g, \theta_h^n\}\} \quad , \quad \pi^{10} \equiv \{\{\theta_h^g, \theta_\ell^g, \theta_\ell^n\}, \{\theta_h^n\}\} \\
\pi^{11} &\equiv \{\{\theta_h^g, \theta_\ell^g, \theta_h^n\}, \{\theta_\ell^n\}\} \quad , \quad \pi^{12} \equiv \{\{\theta_h^g, \theta_\ell^n, \theta_h^n\}, \{\theta_\ell^g\}\} \\
\pi^{13} &\equiv \{\{\theta_h^g\}, \{\theta_\ell^g, \theta_\ell^n, \theta_h^n\}\} \quad , \quad \pi^{14} \equiv \{\{\theta_h^g, \theta_\ell^g, \theta_\ell^n, \theta_h^n\}\}
\end{aligned}$$

Proof of Lemma 1

Let $\succeq^m$ stand for the majority preference relation, and let $\succ^m$ and $\sim^m$ be its asymmetric and symmetric parts, respectively. Under Assumption A1, this relation is a transitive linear order over the set of politicians' ideal policies whenever $\mu^g \neq 1/2$:

If $\mu^g > 1/2$: $(1, \alpha) \succ^m (1, 0) \succ^m (0, \alpha) \succ^m (0, 0)$,

If $\mu^g < 1/2$: $(1, 0) \succ^m (1, \alpha) \succ^m (0, 0) \succ^m (0, \alpha)$.

With these useful observations in mind, we can now turn to the determination of π^0-equilibria.

• *One-candidate equilibria*

Consider first π^0-equilibria in which a single party runs. An immediate consequence of the above observations is that $(\varnothing, (1, \alpha), \varnothing, \varnothing)$ (resp., $(\varnothing, \varnothing, (1, 0), \varnothing)$) is the unique π^0-equilibrium in which a single party runs whenever $\mu^g > 1/2$ (resp. $\mu^g < 1/2$). As a consequence, $(1, \alpha) \in \delta(\pi^0)$ when $\mu^g > 1/2$, and $(1, 0) \in \delta(\pi^0)$ when $\mu^g < 1/2$.

• *Two-candidate equilibria*

Suppose that $\mu^g > 1/2$. As $\geq^m$ is a transitive linear order, there is no π^0-equilibrium in which two candidates run against each other. Indeed, one of them would lose for sure in such a situation and, by assumption, would choose not to run. The same argument applies when $\mu^g < 1/2$.

• *Three-candidate equilibria*

Note first that $\mu_\ell > 1/2$ rules out ties between θ_h^g, θ_ℓ^n, and θ_h^n, and between θ_h^g, θ_ℓ^g, and θ_h^n.

Suppose now that the three running parties are $\{\theta_h^g\}$, $\{\theta_\ell^g\}$, and $\{\theta_\ell^n\}$. Such a situation cannot be a π^0-equilibrium. Indeed, party $\{\theta_\ell^n\}$ could deviate to $\varnothing$, thereby enforcing policy $(1, \alpha)$ she strictly prefers to the fair lottery between the policies offered by the three candidates under Assumption A1. A similar argument shows that $\{\theta_\ell^g\}$, $\{\theta_\ell^n\}$, and $\{\theta_h^n\}$ running against each other cannot be a π^0-equilibrium.

• *Four-candidate equilibria*

Our assumption on the distribution of types, namely $\mu_\ell > 1/2$, rules out the case where four candidates tie when running.

In summary, the θ_ℓ^g-politician (resp. θ_ℓ^n-politician) running alone and offering her ideal policy $(1, \alpha)$ (resp. $(1, 0)$) is the unique π^0-equilibrium when $\mu^g > 1/2$ (resp. $\mu^g < 1/2$). This proves the lemma.

Proof of Proposition 1

We start with a series of useful lemmas.

Lemma 2 *Suppose A1 holds. There exists* $t_1 \in (0,1)$ *such that* $(t_1, \mu_h\alpha) \in \delta(\pi^4)$, *and*

$$\begin{aligned} u(t_1, \mu_h\alpha, \theta_h^g) &> u(1, \alpha, \theta_h^g), \\ u(t_1, \mu_h\alpha, \theta_\ell^n) &> u(1, \alpha, \theta_\ell^n). \end{aligned}$$

Proof: Note first that $(1, \alpha) \notin P_{\{\theta_h^g, \theta_\ell^n\}}$. From this (and the strict concavity of V), we can infer that there is $t_1 \in [0,1]$ such that $(t_1, \mu_h\alpha) \in P_{\{\theta_h^g, \theta_\ell^n\}}$ and

$$\begin{aligned} u(t_1, \mu_h\alpha, \theta_h^g) &> u(1, \alpha, \theta_h^g) \\ u(t_1, \mu_h\alpha, \theta_\ell^n) &> u(1, \alpha, \theta_\ell^n). \end{aligned}$$

Since $(t, \mu_h\alpha) \in P_{\{\theta_h^g, \theta_\ell^n\}}$, for any $t \in [0,1]$, $(t_1, \mu_h\alpha) \in P_{\{\theta_h^g, \theta_\ell^n\}}$. Consider now party structure π^4 and suppose $\{\theta_h^g, \theta_\ell^n\}$ runs alone and offers $(t_1, \mu_h\alpha)$. Since the θ_ℓ^g- and θ_ℓ^n-politicians strictly prefer $(t_1, \mu_h\alpha)$ to $(0,0)$ and $\mu_\ell > 1/2$, $\{\theta_h^n\}$ cannot profitably deviate by offering $(0,0)$. Similarly, $\{\theta_\ell^g\}$ cannot profitably deviate by offering $(1,\alpha)$, for politicians of type θ_h^g, θ_ℓ^n, and θ_h^n all strictly prefer $(t_1, \mu_h\alpha)$ to $(1,\alpha)$. This proves that $(t_1, \mu_h\alpha) \in \delta(\pi^4)$.

□

Lemma 3 *Suppose A1 holds. If* $\mu^g > 1/2$, *then* $(1,\alpha) \in \delta(\pi^5)$ *and there is no EPS involving* π^5.

Proof: Given that we assume that parties which are indifferent between running and not running do not run, the first part of the above statement means that there is a π^5-equilibrium which involves party $\{\theta_\ell^g\}$ running alone. Indeed, the Pareto sets of the other parties in π^5 do not contain $(1,\alpha)$.

To prove the lemma, note that for any $e \in [0, \alpha]$ the policy $(1,\alpha)$ defeats both $(0,e)$ and $(1,0)$ in a pairwise vote ($\mu^g > 1/2$). As a result, if $\{\theta_h^g, \theta_h^n\}$ [resp. $\{\theta_\ell^n\}$] runs alone, and then offers $(0,e)$ [resp. $(1,0)$], $\{\theta_\ell^g\}$ can profitably deviate by offering her ideal policy $(1,\alpha)$. Moreover, platform profiles of the form $((0,e),(1,\alpha),\varnothing)$ or $(\varnothing,(1,\alpha),(1,0))$ cannot be π^5-equilibria since $\{\theta_\ell^g\}$ wins for sure. For the same reason, $\{\theta_\ell^g\}$ running alone is a π^5-equilibrium as no other potential candidate can defeat it.

However, party $\{\theta_\ell^g\}$ running alone in π^5 cannot be an EPS. To see this note that $(1,\alpha)$ is defeated by $(t_1, \alpha\mu_h) \in P_{\{\theta_h^g, \theta_\ell^n, \theta_h^n\}}$ in pairwise vote (see Lemma 2).

Therefore, $\{\theta_h^g, \theta_h^n\}$ should coalesce with $\{\theta_\ell^n\}$ to induce π^{12}. Doing so, they could indeed implement $(t_1, \alpha\mu_h)$ which makes all of them strictly better-off and share the non-policy benefit β.

Consider now a profile of the form $((0,e), \varnothing, (1,0))$. An immediate implication of Assumption A1 is that voters of type θ_ℓ^g strictly prefer $(1,0)$ to $(0,e)$ for any $e \in [0,1]$. As $\mu_\ell > 1/2$, this implies that $\{\theta_\ell^n\}$ wins for sure. This is then not an equilibrium situation.

To complete the proof of Lemma 3, it then remains to show that the three parties in π^5 running at the same time is not an EPS. To see this, consider a platform profile $((0,e), (1,\alpha), (1,0))$ with $e \in [0,1]$. As $V(x(\alpha)) - x(\alpha) \leq V(x(e)) - x(e)$, we have

$$\begin{aligned}
& \frac{1}{3}\left[V(x(0)) - x(0) + V(x(\alpha)) - x(\alpha)\right] - \frac{2}{3}\left[V(x(e)) - x(e)\right] \\
\leq\ & \frac{1}{3}\left[V(x(0)) - x(0) - (V(x(e)) - x(e))\right] \\
\leq\ & \frac{1}{3}\Delta^n(\alpha) < \frac{1}{3}\mu_h(\omega_h - \omega_\ell) = \frac{1}{3}\left(\bar{\omega} - \omega_\ell\right) \qquad (4)
\end{aligned}$$

where the last inequality results from Assumption A1. Rearranging (4), we obtain

$$\begin{aligned}
& \frac{1}{3}\left[2\left(\bar{\omega} - \omega_\ell\right) + V(x(e)) - x(e) + V(x(0)) - x(0) + V(x(\alpha)) - x(\alpha)\right] \\
<\ & \bar{\omega} - \omega_\ell + V(x(e)) - x(e)
\end{aligned}$$

or, equivalently,

$$\frac{1}{3}u\left(0, e, \theta_\ell^n\right) + \frac{1}{3}u\left(1, \alpha, \theta_\ell^n\right) + \frac{1}{3}u\left(1, 0, \theta_\ell^n\right) < u\left(1, e, \theta_\ell^n\right).$$

This means that the θ_ℓ^n-politician strictly prefers the policy $(1,e) \in P_{\{\theta_\ell^g, \theta_\ell^n\}}$ to the fair lottery between $(0,e)$, $(1,\alpha)$, and $(1,0)$. Using a parallel argument we can deduce from $\Delta^g(\alpha) < \mu_h\left(\omega_h - \omega_\ell\right)$ that the same is true for politician θ_ℓ^g.

As a consequence parties $\{\theta_\ell^n\}$ and $\{\theta_\ell^g\}$ can profitably merge with each other to induce π^8. Indeed, $\mu_\ell > 1/2$ ensures that $\{\theta_\ell^n, \theta_\ell^g\}$ offering $(1,e)$ and winning with probability 1 is a π^8-equilibrium. This proves that there is no ESP involving π^5 and ends the proof of Lemma 3.

□

Lemma 4 *Suppose A1 holds. Then* $\delta\left(\pi^8\right) = P_{\{\theta_\ell^g, \theta_\ell^n\}}$.

Proof: By A1, all poor voters (and politicians) strictly prefer any policy in $P_{\{\theta_\ell^g, \theta_\ell^n\}}$ to any policy in $P_{\{\theta_h^g, \theta_h^n\}}$. As $\mu_\ell > 1/2$, this implies that any policy in

$P_{\{\theta_\ell^g,\theta_\ell^n\}}$ beats any policy in $P_{\{\theta_h^g,\theta_h^n\}}$ in a pairwise vote. Thus, a strategy profile is a π^8-equilibrium if, and only if, it is of the form $(\varnothing,(t,e))$ with $(t,e)\in P_{\{\theta_\ell^g,\theta_\ell^n\}}$. This establishes Lemma 4.

$\square$

We now return to the main proposition. The idea is to check that, for every $j=0,\ldots,14$, the following statement is true:

($\mathbf{P}_k$) Suppose $\mu^g>1/2$. If (t,e) is a policy that emerges with a positive probability in an EPS $\left(\pi^k,\mathbf{p}\right)$, then $e\leq\mu_h\alpha$.

($\mathbf{P}_k$) is evidently true for $k\in\{1,2,3\}$ since we know from Lemma 1 that politician θ_ℓ^g can always profitably induce π^0. Let us now turn to the other party structures.

• $k=0$

From Lemmas 1 and 2, we immediately see that $\{\theta_h^g\}$ and $\{\theta_\ell^n\}$ can profitably induce π^4 from π^0. This proves **($\mathbf{P}_0$)**.

• $k=4$

To show **($\mathbf{P}_4$)**, we have to check that $\{\theta_\ell^g\}$ can never win or tie for winning by offering $(1,\alpha)$, and that $\{\theta_h^g,\theta_\ell^n\}$ can never win or tie for winning by offering a policy of the form $(0,e)$, with $e\in(\mu_h\alpha,1]$. Note first that a tie between the three parties in π^4 is not consistent with our assumptions on the distribution of voters' types ($\mu_\ell>1/2$ and $\mu^g>1/2$). A three-candidate equilibrium is therefore impossible. Moreover, the platform profile $(\varnothing,(1,\alpha),(0,0))$ cannot be an equilibrium since $\{\theta_\ell^g\}$ wins for sure.

We know that $(t,\mu_h\alpha)\in P_{\{\theta_h^g,\theta_\ell^n\}}$ defeats $(1,\alpha)$ in a pairwise vote (recall the proof of Lemma 2). This guarantees that $\{\theta_\ell^g\}$ can never win with her ideal policy.

Let us now turn to party $\{\theta_h^g,\theta_\ell^n\}$. Under A1, the θ_ℓ^n- and θ_ℓ^g politicians strictly prefer $(1,\alpha)$ to any policy of the form $(0,e)$. This implies that $(1,\alpha)$ is preferred by a majority of voters to any policy $(0,e)$ with $e\in(\mu_h\alpha,1]$ ($\mu_\ell>1/2$), which in turn implies that $\{\theta_h^g,\theta_\ell^n\}$ can never win by offering such a policy.

Finally, $\{(1,e),(1,\alpha),\emptyset\}$ with $e\in(\mu_h\alpha,1]$ can not be an equilibrium, since a tie between $(1,e)$ and $(1,\alpha)$ would require that θ_h^g prefer the first to the latter, which is impossible.

• $k=5$

($\mathbf{P}_5$) is a direct consequence of Lemma 3.

• $k=6$

To show **($\mathbf{P}_6$)**, we have to check that neither $\{\theta_h^g\}$ nor $\{\theta_\ell^g\}$ can win with a positive probability in an EPS involving π^6. We distinguish between several cases:

(i) $\{\theta_\ell^g\}$ running alone and implementing $(1,\alpha)$ cannot be an equilibrium situation as policy $(t_1,\mu_h\alpha)$ (described in Lemma 2) makes politicians (and then voters) of types θ_ℓ^n, θ_h^n, and θ_h^g strictly better-off. Coalitions $\{\theta_\ell^n,\theta_h^n\}$ and $\{\theta_\ell^n\}$ can therefore profitably induce π^{12} to enforce that policy and grasp the non-policy benefit.

(ii) A strategy profile of the form $(\varnothing,(1,\alpha),(t,0))$ is also impossible in an EPS. Indeed, for this to be possible there should be a tie between the running candidates, namely $\{\theta_\ell^n,\theta_h^n\}$ and $\{\theta_\ell^g\}$. As $\mu_\ell > 1/2$, this would imply that the voters of type θ_ℓ^n prefer $(t,0)$ to $(1,\alpha)$, and then that $t>0$. It would also imply that the θ_h^g-voters would be indifferent between $(1,\alpha)$ and $(t,0)$. But these last statements are in contradiction with $(\varnothing,(1,\alpha),(t,0))$ being a π^6-equilibrium. Indeed, party $\{\theta_\ell^n,\theta_h^n\}$ could make all its members better-off by deviating to a platform $(t-\varepsilon,0)$, with $\varepsilon>0$ very small. Although the θ_ℓ^n politician would incur a small utility loss, she would be compensated by an increase in the non-policy benefit ($\beta/2$ instead of $\beta/4$) as, by continuity, the change in platform would attract θ_h^g-voters and ensure her party's victory.

(iii) As $(0,\alpha)$ is defeated by $(1,\alpha)$ in a pairwise vote, $\{\theta_h^g\}$ running alone or running against $\{\theta_\ell^g\}$ cannot be equilibrium situations.

(iv) Suppose now the strategy profile is $((0,\alpha),\varnothing,(t,0))$. For this profile to be a π^6-equilibrium, voters of type θ_ℓ^g must be indifferent between $(0,\alpha)$ and $(t,0)$. As $\beta>0$, however, there exists $\epsilon>0$ sufficiently small such that $\{\theta_\ell^n,\theta_h^n\}$ can profitably deviate by offering $(t+\epsilon,0)$. This would allow it to win and then to get β for sure, thus compensating its member of type θ_h^n for the small utility loss caused by the change in platform.

(v) Finally, the three parties in π^6 running at the same time cannot be an EPS. Indeed, coalition $\{\theta_\ell^g,\theta_\ell^n,\theta_h^n\}$ should deviate to π^{13}. To see this, define the policy (t_2,e_2) as follows:

$$t_2 \equiv \frac{1}{3}\left(1+t\right),$$

$$V\left(x\left(e_2\right)\right)-x(e_2) \equiv \frac{2}{3}\left[V\left(x\left(\alpha\right)\right)-x(\alpha)\right]+\frac{1}{3}\left[V\left(x\left(0\right)\right)-x(0)\right].$$

It is easy to see that (t_2,e_2) is a certainty equivalent of $\langle(0,\alpha),(1,\alpha),(t,0)\rangle$ for both non-green politicians. Now, define e_3 as follows

$$\begin{aligned} V\left(x\left(e_3\right)\right)-(1+\alpha)x(e_3) &\equiv \frac{2}{3}\left[V\left(x\left(\alpha\right)\right)-(1+\alpha)x(\alpha)\right] \\ &+ \frac{1}{3}\left[V\left(x\left(0\right)\right)-(1+\alpha)x(0)\right]. \end{aligned}$$

By definition, (t_2,e_3) is a certainty equivalent of $\langle(0,\alpha),(1,\alpha),(t,0)\rangle$ for the θ_ℓ^g-politician. Our curvature conditions further imply that $e_3<\alpha/3<\alpha\mu_\ell$ (the tie between the three candidates implies that $\mu_\ell^g=1/3$, and then $\mu_\ell>1/3$), and

$e_3 < e_2$. This implies that $(t_2, e_3) \in P_{\{\theta_\ell^g, \theta_\ell^n, \theta_h^n\}}$, and that both non-green politicians strictly prefer (t_2, e_3) to $\langle(0,\alpha),(1,\alpha),(t,0)\rangle$. For the θ_ℓ^n-politician to accept the deviation towards π^{13}, just pick $\epsilon > 0$ sufficiently small so that $(t_2, e_3 + \epsilon)$ belongs to $P_{\{\theta_\ell^g, \theta_\ell^n, \theta_h^n\}}$ and makes all members of $\{\theta_\ell^g, \theta_\ell^n, \theta_h^n\}$ strictly better-off (non-policy benefits remain unchanged for the green politician and increase for the non-green politicians).

• $k = 7$

First of all, note that there exists a sufficiently $\varepsilon > 0$ such that $u(1-\varepsilon, \alpha, \theta_\ell^n) > u(0, 0, \theta_\ell^n)$ and $u(1-\varepsilon, \alpha, \theta_\ell^g) > u(1, 0, \theta_\ell^g)$, thus implying that $(1-\varepsilon, \alpha) \in \delta(\pi^1)$. Indeed, our assumptions on the distribution of types ($\mu_\ell > 1/2$ and $\mu^g > 1/2$) guarantee that party $\{\theta_h^g, \theta_\ell^g\}$ cannot be defeated in π^1 when it offers $(1-\varepsilon, \alpha)$.

Consider now party structure π^7. Since $\mu^g > 1/2$, party $\{\theta_h^g, \theta_\ell^g\}$ must win for sure in a π^7 equilibrium. Suppose first that it implements a policy $(t, \alpha) \in P_{\{\theta_h^g, \theta_\ell^g\}}$ such that $t < 1$. Then, θ_ℓ^n can profitably induce π^1 and then $(1, \alpha)$, which is her ideal policy in $P_{\{\theta_h^g, \theta_\ell^g\}}$. Suppose now that $\{\theta_h^g, \theta_\ell^g\}$ implements $(1, \alpha)$. Then, θ_h^n can profitably induce π^1 and then $(1-\varepsilon, \alpha) \in \delta(\pi^1)$. As a consequence, there is no EPS involving π^7 and $(\mathbf{P}_7)$ evidently holds.

• $k = 8$

As $\mu_\ell > 1/2$, $\{\theta_\ell^g, \theta_\ell^n\}$ wins with a probability of 1 in π^8-equilibrium. But politician θ_ℓ^g can induce π^5, thereby enforcing her ideal policy and getting a benefit of β instead of $\beta/2$. Thus, there is no EPS involving π^8.

• $k = 9$

If condition $(\mathbf{P}_9)$ does not hold, then one of the following situations must arise.

(i) Suppose first that $\{\theta_\ell^g, \theta_h^n\}$ offers a policy $(0, e)$ with $e \in [\mu_h\alpha, \mu_\ell\alpha]$.

Then party $\{\theta_h^g, \theta_\ell^n\}$ can ensure its victory by offering $(0, e+\epsilon)$, with ϵ arbitrarily small. Both green politicians prefer this policy to $(0, e)$. Moreover, as ϵ is very small, the θ_ℓ^n-politician is compensated by an increase in her benefit of at least $\beta/4$:

$$u(0, e, \theta_\ell^n) - u(0, e+\epsilon, \theta_\ell^n) < \frac{\beta}{4}.$$

(ii) Suppose now that $\{\theta_\ell^g, \theta_h^n\}$ offers a policy (t, e) of the form $(t, \mu_\ell\alpha)$ with $t > 0$ or $(1, e)$ with $e > \mu_\ell\alpha$.

By the curvatures conditions imposed on V, $\{\theta_h^g, \theta_\ell^n\}$ has again a profitable deviation. To see this, take the indifference curves of politicians θ_h^g and θ_ℓ^n that pass through (t, e). These curves cross each other at another point, say (t', e'). It is easy to check that the unique intersection between the segment joining (t, e) to (t', e') and $P_{\{\theta_h^g, \theta_\ell^n\}}$ is a policy that enables $\{\theta_h^g, \theta_\ell^n\}$ to win for sure.

(iii) Finally, suppose $\{\theta_h^g, \theta_\ell^n\}$ offers a policy $(0, e)$ with $e > \alpha\mu_h$.

If $e \leq \alpha\mu_\ell$ then, by the same argument as in (i), $\{\theta_\ell^g, \theta_h^n\}$ has a profitable deviation. If $e > \alpha\mu_\ell$, then there exists a policy in $P_{\{\theta_\ell^g, \theta_h^n\}}$ which is strictly preferred to $(0, e)$ by the voters of type θ_ℓ^g, θ_h^n, and θ_ℓ^n. A deviation to this policy is therefore profitable to party $\{\theta_\ell^g, \theta_h^n\}$. As a result, $\{\theta_h^g, \theta_\ell^n\}$ cannot offer a pollution tax that exceeds $\alpha\mu_h$ in a π^9-equilibrium.

• $k = 10$

We first define the sets P_1, P_2, and P_3 as follows:

$$\begin{aligned}
P_1 &\equiv \left\{(t, e) \in P_{\{\theta_h^g, \theta_\ell^g, \theta_\ell^n\}} : u\left(t, e, \theta_\ell^n\right) \leq u\left(1, \alpha, \theta_\ell^n\right)\right\}, \\
P_2 &\equiv \left\{(t, e) \in P_{\{\theta_h^g, \theta_\ell^g, \theta_\ell^n\}} : u\left(t, e, \theta_h^g\right) \leq u\left(1, \alpha, \theta_h^g\right)\right\}, \\
P_3 &\equiv P_{\{\theta_h^g, \theta_\ell^g, \theta_\ell^n\}} \setminus \left(P_1 \cup P_2\right).
\end{aligned}$$

Under structure π^{10}, the three-member party must win for sure in an equilibrium, and then offer a policy in $P_{\{\theta_h^g, \theta_\ell^g, \theta_\ell^n\}} \equiv P_1 \cup P_2 \cup P_3$. We distinguish between three different cases.

(i) It offers a policy in P_1. Then $\{\theta_\ell^g, \theta_\ell^n\}$ can induce π^2, thus enforcing $(1, \alpha)$ and obtaining a benefit of $\beta/2$ instead of $\beta/3$.

(ii) It offers a policy in P_2. By the same argument as previously, $\{\theta_h^g, \theta_\ell^g\}$ can profitably induce π^1.

(iii) It offers a policy (t, e) in $P_3 \setminus P_{\{\theta_h^g, \theta_\ell^n\}}$. Substituting (t, e) to $(1, \alpha)$ in the proof of Lemma 2, we obtain that there exists a policy $(t', \mu_h\alpha)$ such that $(t', \mu_h\alpha) \in \delta\left(\pi^4\right)$, and

$$\begin{aligned}
u\left(t', \mu_h\alpha, \theta_h^g\right) &> u\left(t, e, \theta_h^g\right), \\
u\left(t', \mu_h\alpha, \theta_\ell^n\right) &> u\left(t, e, \theta_\ell^n\right).
\end{aligned}$$

This implies that $\{\theta_h^g, \theta_\ell^n\}$ can profitably induce π^4. Doing so, they indeed enforce a better policy and no longer share the non-policy benefit with θ_ℓ^g.

Suppose that $(t, e) \in P_3 \cap P_{\{\theta_h^g, \theta_\ell^n\}}$. This implies that (t, e) satisfies the conditions of Lemma 2, which in turn implies that $(t, e) \in \delta\left(\pi^4\right)$. Therefore, coalition $\{\theta_h^g, \theta_\ell^n\}$ can enforce the same policy without sharing the non-policy benefit with θ_ℓ^g.

This proves that there is no EPS involving party structure π^{10}.

• $k = 11$

From Lemma 3, we know that θ_ℓ^g's ideal policy $(1, \alpha) \in \delta\left(\pi^5\right)$. As $\beta > 0$, the θ_ℓ^g-politician has consequently a profitable deviation to π^5.

• $k = 12$

For $(\mathbf{P}_{12})$ to be true, it suffices to check that the big party in π^{12} never offers a policy $(0, e)$ with $e > \mu_h\alpha$, and that $\{\theta_\ell^g\}$ never wins in a π^{12}-equilibrium.

When $\{\theta_h^g, \theta_\ell^n, \theta_h^g\}$ offers a policy of the form $(0, e)$ with $e > \alpha\mu_h$, it is defeated with a probability of 1 by $\{\theta_\ell^g\}$ which offers $(1, \alpha)$. Indeed, under A1, voters of type θ_ℓ^n strictly prefer $(1, \alpha)$ to any policy $(0, e)$ with $e \in [0, 1]$, and $\mu_\ell > 1/2$. Therefore, if $\{\theta_h^g, \theta_\ell^n, \theta_h^g\}$ runs in a π^{12}-equilibrium, then it offers an environmental tax at most equal to $\alpha\mu_h$.

Let us now turn to party $\{\theta_\ell^g\}$. This party can only offer $(1, \alpha)$ which is defeated by $(t_1, \alpha\mu_h) \in P_{\{\theta_h^g, \theta_\ell^n, \theta_h^g\}}$ in pairwise vote (see Lemma 2). As a result, it can never win in a π^{12}-equilibrium.

• $k = 13$

Suppose that, contrary to **($\mathbf{P}_{13}$)**, a policy (t, e) with $e > \alpha\mu_h$ emerges in a π^{13}-equilibrium. This cannot be an EPS. To see this, suppose first that $t < 1$. Then (t, e) does not belong to the Pareto set of $\{\theta_\ell^g, \theta_\ell^n\}$. This implies that there exists a policy in $P_{\{\theta_\ell^g, \theta_\ell^n\}}$ that makes θ_ℓ^g and θ_ℓ^n strictly better-off and is a π^2-equilibrium policy. Indeed, it is easy to see that, under conditions A1 and $\mu_\ell > 1/2$, $(t, e) \in \delta(\pi^2)$ for every $(t, e) \in P_{\{\theta_\ell^g, \theta_\ell^n\}}$.

Suppose now that the policy (t, e) under consideration satisfies $t = 1$ and $e \geq \alpha\mu_\ell$. It is easy to see that any such a policy is a π^3-equilibrium policy (implemented by party $\{\theta_\ell^g, \theta_h^n\}$). Therefore, coalition $\{\theta_\ell^g, \theta_h^n\}$ can profitably induce π^3, thus enforcing the same policy (t, e) without sharing the non-policy benefit with θ_ℓ^n. If $e < \alpha\mu_\ell$, then $(t, e) \notin P_{\{\theta_\ell^g, \theta_h^n\}}$. There consequently exists $(t', e') \in P_{\{\theta_\ell^g, \theta_h^n\}}$ (with $e' = \alpha\mu_\ell$) that makes θ_ℓ^g and θ_h^n strictly better off. Substituting (t', e') to (t, e) in the previous reasoning proves **($\mathbf{P}_{13}$)**.

• $k = 14$

Suppose first that the grand party offers a policy (t, e) outside the Pareto set of $\{\theta_\ell^g, \theta_\ell^n\}$. This implies that there exists $(t'', e'') \in P_{\{\theta_\ell^g, \theta_\ell^n\}}$ such that $u\left(t'', e'', \theta_\ell^j\right) > u\left(t, e, \theta_\ell^j\right)$ for every $j \in \{g, n\}$. By Lemma 4, $\{\theta_\ell^g, \theta_\ell^n\}$ should then induce π^8 so as to enforce (t'', e'') and raise the benefit of its members.

To show that **($\mathbf{P}_{14}$)** is true, we must therefore show that the grand party implementing a policy $(1, e) \in P_{\{\theta_\ell^g, \theta_\ell^n\}}$, with $e > \alpha\mu_h$, is not an EPS. For every $e > \alpha\mu_h > 0$, there exists by continuity an $\varepsilon > 0$ such that $e - \varepsilon \geq 0$ and

$$u\left(1, e, \theta_\ell^g\right) - u\left(1, e - \varepsilon, \theta_\ell^g\right) < \frac{\beta}{2}.$$

Moreover, Lemma 4 establishes that $(1, e - \varepsilon) \in \delta(\pi^8)$. This proves that a deviation to π^8 is again profitable to coalition $\{\theta_\ell^g, \theta_\ell^n\}$, thus completing the proof of Proposition 1.

Proof of Proposition 2

Substituting θ_i^n to θ_i^g, $i \in \{\ell, h\}$, and μ^n to μ^g in the proof of Proposition 1, we can prove Proposition 2 in like manner.

Proof of Proposition 3

Our proof of Proposition 3 will proceed in three short steps. Given our previous findings, we already know that there cannot be a stable green party when there is a majority of green voters. We consequently assume throughout that $\mu^g < 1/2$.

Step 1: If $S \subseteq \Theta$ is a stable green party, then $S = \{\theta_h^g, \theta_\ell^g\}$.

We can directly infer from Lemma 1 that there is no stable green party in π^0, π^2, and π^4. In π^3, if $\{\theta_\ell^g, \theta_h^n\}$ offers $(1, \alpha)$ than $\{\theta_\ell^n\}$ can win for sure by offering its ideal policy: θ_h^n-voters strictly prefer $(1, 0)$ to $(1, \alpha)$ and $\mu^n > 1/2$.

Policy $(0, \alpha)$ is defeated by both $(1, 0)$ and $(1, \alpha)$ in pairwise vote. Therefore, there is no π^5-equilibrium in which $\{\theta_h^g, \theta_h^n\}$ runs alone, or against a single opponent, and offers $(0, \alpha)$. Under assumption A1, the θ_h^n-politician strictly prefers $(1, 0)$ to $\langle(0, \alpha), (1, \alpha), (1, 0)\rangle$. A parallel argument to that used to prove Lemma 3 would show that there is no three-candidate π^5-equilibrium.

The non-green party wins with a probability of 1 in any π^6- and π^7-equilibrium since $\mu^n > 1/2$.

The no EPS involving π^8. Indeed, θ_ℓ^n can profitably induce the π^5-equilibrium in which she implements *alone* her ideal policy. In π^9, party $\{\theta_h^g, \theta_\ell^n\}$ [resp. $\{\theta_\ell^g, \theta_h^n\}$] can never win by offering $(0, \alpha)$ [resp. $(1, \alpha)$], for there is a policy in the Pareto set of $\{\theta_\ell^g, \theta_h^n\}$ [resp. $\{\theta_h^g, \theta_\ell^n\}$] that allows the latter to win for sure.

Consider π^{10} and π^{13} now. Under these party structures, the three-member party must win for sure. Suppose it offers a policy of the form (t, α). As $\beta > 0$, inducing π^2 and enforcing $(1, \alpha)$ is strictly profitable to coalition $\{\theta_\ell^g, \theta_\ell^n\}$.

For a policy $(t, \alpha) \in P_{\{\theta_h^g, \theta_\ell^g, \theta_h^n\}}$ to be a π^{11}-equilibrium policy, both θ_ℓ^g and θ_h^n must prefer (t, α) to $(1, 0)$ (otherwise, $\{\theta_\ell^n\}$ could win by offering $(1, 0)$). But then, there exists a policy $(t', \alpha\mu_\ell)$ such that $\langle\varnothing, (t', \alpha\mu_\ell), \varnothing\rangle$ is a π^3-equilibrium, and both θ_ℓ^g and θ_h^n prefer $(t', \alpha\mu_\ell)$ to (t, α). Indeed, a brief inspection of the structure of preferences reveals that the analysis of EPS involving π^3 when $\mu^g < 1/2$ is symmetric to the analysis of EPS involving π^4 when $\mu^g > 1/2$. We can then deduce from Lemma 2 that such a policy exists. But this implies that coalition $\{\theta_\ell^g, \theta_h^n\}$ can profitably deviate by inducing π^3 and enforcing $(t', \alpha\mu_\ell)$.

In a π^{12}-equilibrium, the bigger party never offers $(0, \alpha)$. Since θ_ℓ^n voters strictly prefer $(1, \alpha)$ to $(0, \alpha)$, $\{\theta_\ell^g\}$ could indeed win for sure by offering $(1, \alpha)$.

Finally, the grand coalition is not a stable green party. Suppose the unique party in π^{14} offers a policy of the form (t, α). As $\beta > 0$, inducing π^8 and enforcing $(1, \alpha)$ is strictly profitable to coalition $\{\theta_\ell^g, \theta_\ell^n\}$.

Step 2: A stable green party exists only if $\mu_h \mu_\ell (\omega_h - \omega_\ell) \geq \Delta^n$ and condition (c) in the statement of Proposition 3 hold.

An immediate consequence of Step 1 is that the only party structure in which there can be a stable green party is π^1. A little reflection suggests that the analysis EPS involving π^1 when $\mu^g < 1/2$ is symmetric to the analysis of EPS involving π^6 when $\mu^g > 1/2$. Inspecting the case $k = 6$ in the proof of Proposition 1 thus reveals that, when $\mu^g < 1/2$, there is no EPS in which the green party runs against one or two rival candidates.

Our focus is therefore on EPS of the form $(\pi^1, \langle (t, \alpha), \varnothing, \varnothing \rangle)$ where $t \in [0, 1]$. $((t, \alpha), \varnothing, \varnothing)$ cannot be a π^1-equilibrium if one of the following conditions hold:

(i) θ_ℓ^g-voters strictly prefer $(1, 0)$ to (t, α) (party $\{\theta_\ell^n\}$ can offer $(1, 0)$ and win for sure since $\mu_\ell > 1/2$) or, equivalently, $t < 1 - \Delta^g / \Delta_\ell$;

(ii) θ_ℓ^n-voters strictly prefer $(0, 0)$ to (t, α) (party $\{\theta_h^n\}$ can offer $(0, 0)$ and win for sure since $\mu^n > 1/2$) or, equivalently, $t < \Delta^n / \Delta_\ell$;

(iii) θ_h^n-voters strictly prefer $(1, 0)$ to (t, α) (party $\{\theta_\ell^n\}$ can offer $(1, 0)$ and win for sure since $\mu^n > 1/2$) or, equivalently $t > 1 - \Delta^n / \Delta_h$.

For none of these three conditions to hold, t must then belong to the interval

$$T \equiv \left[\max \left\{ 1 - \frac{\Delta^g}{\Delta_\ell}, \frac{\Delta^n}{\Delta_\ell} \right\}, 1 - \frac{\Delta^n}{\Delta_h} \right].$$

Therefore, a necessary condition for $((t, \alpha), \varnothing, \varnothing)$ to be a π^1-equilibrium is that T is nonempty. But this is only the case if $\mu_\ell \Delta^g \geq \mu_h \Delta^n$ and $\mu_h \mu_\ell (\omega_h - \omega_\ell) \geq \Delta^n$.

Step 3: $\{\theta_h^g, \theta_\ell^g\}$ is a stable green party whenever $\mu_h \mu_\ell (\omega_h - \omega_\ell) > \Delta^n$ and $\mu_h \Delta^g > \mu_\ell \Delta^n$.

When $\mu_\ell \Delta^g > \mu_h \Delta^n$ and $\mu_h \mu_\ell (\omega_h - \omega_\ell) > \Delta^n$, there is a tax rate t that belongs to the interior of T. Then, it follows from the above argument that $((t, \alpha), \varnothing, \varnothing)$ is a π^1-equilibrium. What remains to be proved, therefore, is that $(\pi^1, ((t, \alpha), \varnothing, \varnothing))$ is an EPS.

Note first that coalition $\{\theta_h^g, \theta_\ell^g\}$ cannot be part of a deviating coalition: (t, α) belongs to the Pareto set of that coalition and forming a larger party with another politician would make their non-policy benefit decrease. Moreover, we know from Lemma 1 that neither θ_h^g nor θ_ℓ^g have an interest in inducing π^0.

Politicians θ_h^n and θ_ℓ^n inducing π^7 is then the only possible deviation. As $\mu^g < 1/2$, $\{\theta_\ell^n, \theta_h^n\}$ must run alone in a π^7-equilibrium. Suppose first that it offers a policy $(t', 0) \in P_{\{\theta_\ell^n, \theta_h^n\}}$ such that $u(t', 0, \theta_h^n) < u(0, \alpha, \theta_h^n)$. Then, tedious computations reveal that the set of policies $(t'', \alpha) \in P_{\{\theta_h^g, \theta_\ell^g\}}$ such that politicians of types θ_h^g, θ_ℓ^g, and θ_h^n strictly prefer (t'', α) to $(t', 0)$ is nonempty whenever $\mu_\ell \Delta^g > \mu_h \Delta^n$. This implies that the green party can profitably deviate by offering (t'', α), and then $(t', 0) \notin \delta(\pi^7)$.

A parallel argument shows that if $\{\theta_\ell^n, \theta_h^n\}$ offers a policy $(t', 0) \in P_{\{\theta_\ell^n, \theta_h^n\}}$ such that $u(t', 0, \theta_\ell^n) < u(1, \alpha, \theta_\ell^n)$, then the green party can also profitably deviate whenever $\mu_h \Delta^g > \mu_\ell \Delta^n$. As

$$\{t' \in [0,1] : u(t', 0, \theta_h^n) < u(0, \alpha, \theta_h^n)\} \cup \{t' \in [0,1] : u(t', 0, \theta_\ell^n) < u(1, \alpha, \theta_\ell^n)\} = [0,1]$$

whenever $\mu_h \mu_\ell (\omega_h - \omega_\ell) > \Delta^n$, this proves that there is no π^7-equilibrium, and then no possible deviation from π^1, when $\mu_h \mu_\ell (\omega_h - \omega_\ell) > \Delta^n$.

Combining Steps 1-3, we obtain the proposition.

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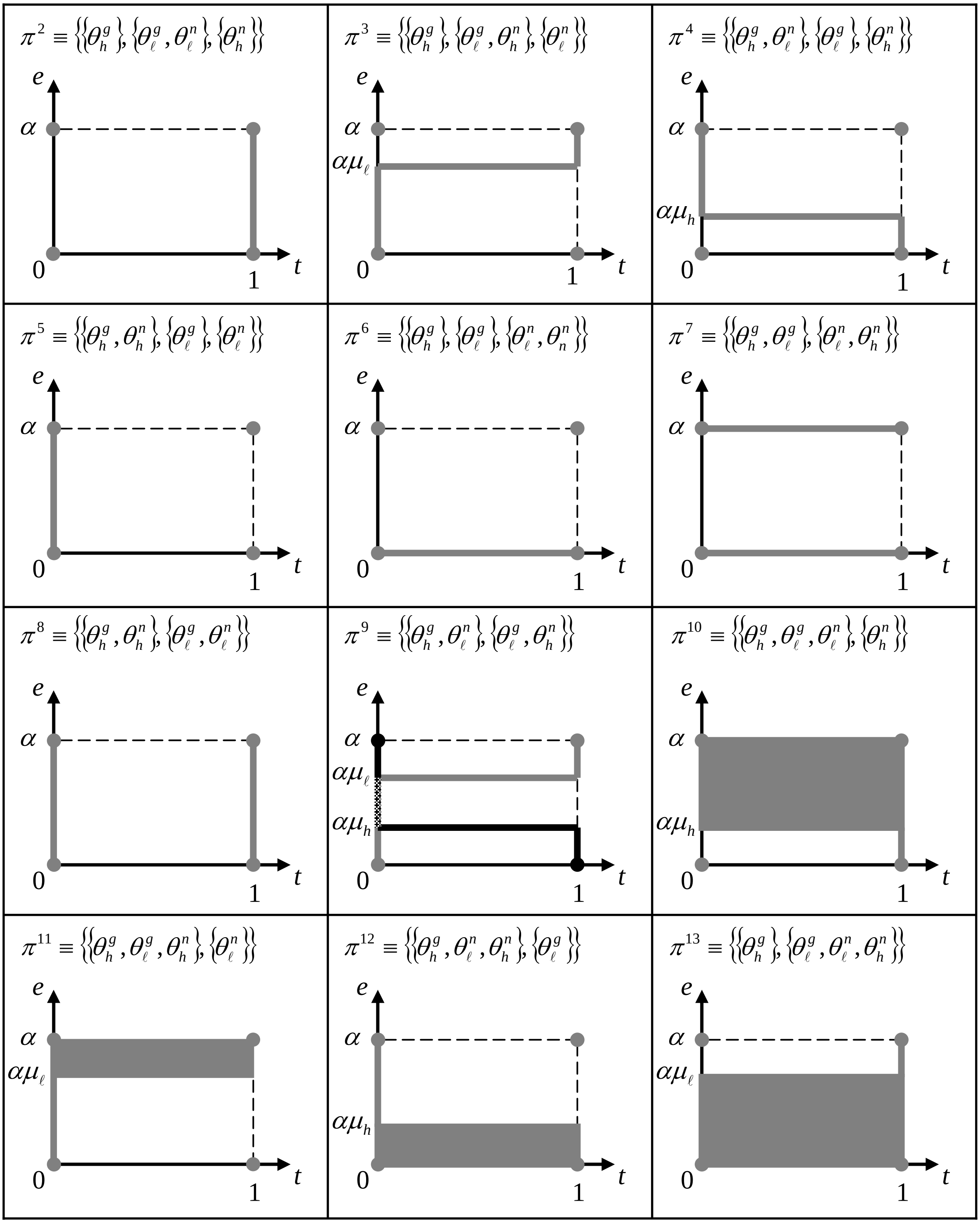
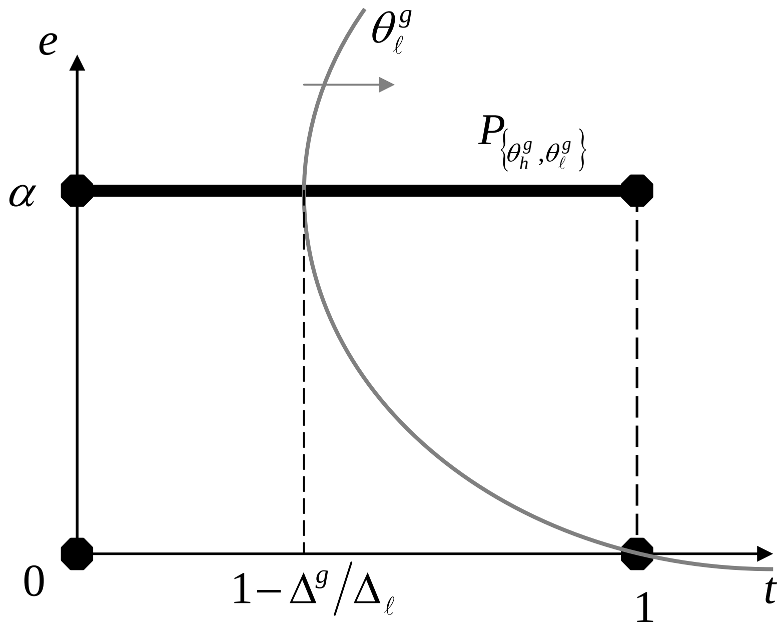

$\pi^2 \equiv \{\{\theta_h^g\},\{\theta_\ell^g,\theta_\ell^n\},\{\theta_h^n\}\}$
e α 0 1 t
$\pi^3 \equiv \{\{\theta_h^g\},\{\theta_\ell^g,\theta_h^n\},\{\theta_\ell^n\}\}$
e α $\alpha\mu_\ell$ 0 1 t
$\pi^4 \equiv \{\{\theta_h^g,\theta_\ell^n\},\{\theta_\ell^g\},\{\theta_h^n\}\}$
e α $\alpha\mu_h$ 0 1 t
$\pi^5 \equiv \{\{\theta_h^g,\theta_h^n\},\{\theta_\ell^g\},\{\theta_\ell^n\}\}$
e α 0 1 t
$\pi^6 \equiv \{\{\theta_h^g\},\{\theta_\ell^g\},\{\theta_\ell^n,\theta_n^n\}\}$
e α 0 1 t
$\pi^7 \equiv \{\{\theta_h^g,\theta_\ell^g\},\{\theta_\ell^n,\theta_h^n\}\}$
e α 0 1 t
$\pi^8 \equiv \{\{\theta_h^g,\theta_h^n\},\{\theta_\ell^g,\theta_\ell^n\}\}$
e α 0 1 t
$\pi^9 \equiv \{\{\theta_h^g,\theta_\ell^n\},\{\theta_\ell^g,\theta_h^n\}\}$
e α $\alpha\mu_\ell$ $\alpha\mu_h$ 0 1 t
$\pi^{10} \equiv \{\{\theta_h^g,\theta_\ell^g,\theta_\ell^n\},\{\theta_h^n\}\}$
e α $\alpha\mu_h$ 0 1 t
$\pi^{11} \equiv \{\{\theta_h^g,\theta_\ell^g,\theta_h^n\},\{\theta_\ell^n\}\}$
e α $\alpha\mu_\ell$ 0 1 t
$\pi^{12} \equiv \{\{\theta_h^g,\theta_\ell^n,\theta_h^n\},\{\theta_\ell^g\}\}$
e α $\alpha\mu_h$ 0 1 t
$\pi^{13} \equiv \{\{\theta_h^g\},\{\theta_\ell^g,\theta_\ell^n,\theta_h^n\}\}$
e α $\alpha\mu_\ell$ 0 1 t



<u>Figure 2:</u> Left-Wing Orientation of Stable Green Parties