Optimal Coverage of Large Risks: Theoretical Analysis and Application to Oil Spills [1]

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Abstract

In this paper, we consider a firm within an industry that has to cover a large-scale risk induced by its activity. It bears also a risk of bad reputation and has to decide the amount of money to be invested in prevention. The optimal coverage contract combines a deductible, coinsurance from a standard insurance company and some financial investment. When insurance is available, optimal prevention increases if firms have the opportunity to send signals about their risk-reducing activities to potential investors. We apply our results to the current international regime that regulates maritime oil transport.

Key-Words: capital markets, catastrophe, insurance, prevention, oil spill.

JEL Classification: D80, G22, Q50.

1 Introduction

There are different reasons that explain why firms the activity of which present large-scale risks should use hedging mechanisms. Some of them organize some risk spreading within the industry they belong to by creating a compensation fund. Nevertheless, the aggregate risk is still variable because of the possibly huge consequences of an incident and because of the limited number of contributing members in the Fund. Consequently, the mutuality principle is no longer sufficient to spread all the risk on the oil firms.

Doherty (2000) provides several arguments that insurance is profitable for firms, and stresses the fact that insurance mechanisms have to be completed by some investment on capital markets when dealing with large risks. Froot (2001) also provides different reasons why markets are more efficient than insurers in global risk reductions. One important point is that securitization may reduce transaction costs such as administrative fees or costs related to agency issues.

In harmony with Doherty and Dionne (1993), Schlesinger (1999), Doherty and Schlesinger (2002) and Mahul (2002), we show that insurance combined with a financial hedging performs better than standard insurance only. However, our economic context is different from these studies. In our framework, each individual firm bears a percentage of the aggregate risk of the industry (organised as a pool) and an individual risk of bad reputation that is positively correlated to the aggregate risk and non insurable. To date, the litterature has focused essentially on risks that can be split into idiosyncratic risk, specific to the individual and easily insurable, and a systematic risk, independent from the idiosyncratic one.

Losses induced by reputation constitute an important variable in our model; reputation and its impact on firms' value has become a major concern for firms involved in environmentally risky activities as shown by Lanoie et al. (1998). Hence taking into account losses induced by (bad) reputation constitutes a progress compared to the previous literature.

In the second part of the paper, we focus on the compensation system implemented when an oil spill is registered in the territorial sea of any member of the 1992 Civil Liability Convention. This Convention regulated the maritime transport of oil in most countries of the world[1], except mainly for the United States, which has its own Convention[2]. Since oil spills can create severe damages to the environment but also to human activities near the coast, they may result in very large claims, which cannot be covered without a compensation system adapted to such catastrophic losses. Furthermore, the economic litterature on catastrophic risks does not provide formal analyses about the impact of hedging on the prevention of large risks. We examine both compensation and prevention in this paper. We also take into account the (bad) reputational effect that oil firms have to bear each time an oil spill is announced.

The 1992 International Oil Pollution Compensation Fund (1992 IOPC Fund) participates in the compensation of victims of an oil spill if the payment already granted by the insurer of the owner of the tanker is not sufficient. The contributions of oil firms to the Fund are proportional to the quantity of oil received in a year and they are due each time an oil spill has occurred in the territorial waters of a member, whatever the flag of the tanker and whatever the citizenship of the oil firm. Hence the IOPC Fund enables the compensation of victims even if the owner of the tanker is not a citizen of a member state and empirical evidence shows that the IOPC Fund seems to be rather efficient in minimizing the time between the oil spill event and the effective compensation of victims. However, funds are levied at random dates and expenses are not smoothed through time. In particular, the 1992 IOPC Fund, as it stands, does not rely on the risk transfer principle. By defining contributions on the basis of the aggregate risk of the pool, only the mutuality principle (Borch, 1962; Wilson, 1968) is applied. Hence we explain why, within the current international regime, oil firms would benefit from the capital markets and from utilizing appropriate financial instruments. Financial mechanisms can improve and complement hedging that could be provided by insurance policies. Precisely, small and medium spills could be managed by classical insurance, while large oil spills should

[1] 81 states ratified the 1992 Civil Liability Convention.

[2] The 1990 Oil Pollution Act. See Ketkar (1995) and Kim (2002) for details on this legislation.

be covered by adequate investment on the financial markets.

The paper is organized as follows. In the second section, we present the basis model and introduce standard insurance mechanisms in order to define the optimal insurance contract that a firm can buy from an insurer. It entails a deductible with coinsurance for all losses higher than the deductible. In the third section, we show that financial hedging may be a good way to cover the residual risk still retained by firms after insurance. When incorporating this point in the insurance contract, the risk premium asked by the insurer decreases and more (standard) insurance becomes available for small and medium incidents, while capital markets are useful for hedging large damages. Another important point is that financial markets may provide incentives to invest in prevention by allowing firms to give positive signals to potential investors. The fourth section presents an application of our theoretical results to the maritime transport of oil. First, we describe the current regime of the IOPC Fund. Then we discuss the characteristics of the financial assets that would fit with the joint hedging strategy obtained in our model. Section five concludes. Proofs are given in Appendix.

2 Optimal coverage of large-scale risks

We analyze the introduction of standard insurance, first, and of insurance and financial hedging, second, in the large-scale risks management.

2.1 (Catastrophe) Risk mutualization and the basis model

Consider n firms from a given industry. We denote $\widetilde{x}_i$ the risk of loss borne by Society and due to Firm i's activity. This random loss $\widetilde{x}_i$ takes the strictly positive value x_i with probability p_i and equals zero with probability $(1 - p_i)$. Probability p_i of incident is affected by the level of prevention e_i decided by the firm: $p_i = p(e_i)$ with $p'(e_i) < 0$. The cost of prevention is defined as $c(e_i) = e_i$. The aggregate risk of the industry is

$\widetilde{X} = \sum_{i=1}^{n} \widetilde{x}_i$ with values in $[0, L]$ [3] and with distribution function $F(X/e)$, where e is the vector of all individual investments in prevention: $e \equiv (e_1, ...e_n)$. An increase in the level of individual prevention of, at least, one firm improves the distribution in the sense of the first order stochastic dominance, but at a decreasing rate: $F_{e_i} > 0$, $F_{e_i e_i} \leq 0, \forall X \in]0, L[$ and $F_{e_i}(0/e) = F_{e_i}(L/e) = 0$.

Let us consider now that all firms are involved in a compensation fund at which mutualisation of losses is applied. Each time an accident is registered, the Fund calls for contributions by each firm. The percentage of contribution of Firm i, denoted α_i, is applied to the level of the agregate loss X of the pool, up to a maximum value $\widehat{X}$, which is assumed to be less than the amount of losses registered if all firms would have an accident in the same period[4]: $\widehat{X} < L$. In other words, firms benefit from a kind of limited liability.

In addition to the risk $\alpha_i \widetilde{X}$, Firm i bears a second risk related to (bad) reputation. Each time an incident occurs, the whole industry is affected by the harsh public opinion. Nevertheless, the effect of bad reputation is stronger for the firm the activity of which is directly linked to the accident because of the bad advertising which is made around its brand. Finally, each firm bears a bad reputational effect composed of an individual effect, which is zero if the firm is not implied in the accident, and a general effect which is positive for any incident. Formally, the random variable describing the total reputational effect is denoted $-g(\widetilde{X}, \widetilde{x}_i)$ with $0 < g_X < g_{x_i}$ and $g_{XX} < 0$. The preferences of the firm are represented by a Von Neumann Morgenstern utility function $u(.)$ and the firm owns an initial non random wealth w_i.

[3] Here, L is simply equal to the sum of the strictly positive individual values: $L = \sum_{i=1}^{n} x_i$. We assume that n is sufficiently large so as to consider $\widetilde{X}$ as a continuous variable.

[4] This reflects the fact that compensation funds are almost always upperbounded.

If no insurance is available, the firm can only choose the level of prevention e_i:

$$\max_{e_i} R = \int_0^{\widehat{X}} u(w_i - \alpha_i X - g(X, \widetilde{x_i}))f(X/e)dX + \int_{\widehat{X}}^{L} u(w_i - \alpha_i \widehat{X} - g(X, \widetilde{x_i}))f(X/e)dX - e_i, \tag{1}$$

where $g(X, \widetilde{x_i})$ is the expected value of the reputational effect evaluated with respect to $\widetilde{x_i}$:

$$g(X, \widetilde{x_i}) = p(e_i)g(X, x_i) + (1 - p(e_i))g(X, 0), \qquad \forall X \in [0, L] \tag{2}$$

In the course of the text, we adopt the following notations: $w_f = w_i - \alpha_i X - g(X, \widetilde{x_i})$, $\widehat{w_f} = w_i - \alpha_i \widehat{X} - g(X, \widetilde{x_i})$, $g_{e_i} = g_{e_i}(X, \widetilde{x_i})$ and $g_X = g_X(X, \widetilde{x_i})$. For given prevention levels of the other oil firms, the optimal level of prevention e_i^* of Firm i satisfies the following first order condition:

$$- \int_0^{\widehat{X}} g_{e_i}.u'(w_f)f(X/e)dX - \int_{\widehat{X}}^{L} g_{e_i}.u'(\widehat{w_f})f(X/e)dX + \int_0^{\widehat{X}} (\alpha_i + g_X).u'(w_f)F_{e_i}(X/e)dX + \int_{\widehat{X}}^{L} g_X.u'(\widehat{w_f})F_{e_i}(X/e)dX = 1 \tag{3}$$

It is obtained thanks to a differentiation of (1) with respect to e_i and thanks to integrations by part of the terms in $f_{e_i}(.)$. The right term of Equality (3) is the expected marginal cost of prevention. From our assumptions, this amount is certain and equal to one. The left-hand-side term is the expected marginal benefit of prevention. First, increasing prevention reduces the risk of bad reputation (first and second term) because the probability for Firm i to be directly involved in an accident (probability p_i) decreases as e_i increases. Second, prevention has also a positive impact on the aggregate risk of the Fund since it improves its distribution. Firm i will benefit from an aditionnal reduction of bad reputation due, this time, to the reduction of the aggregate risk of the pool (third and fourth term). Lastly, the presence of α_i in the third member of the left-hand-side term represents the direct benefit of prevention: increasing prevention reduces the risk $\alpha_i\widetilde{X}$ borne by Firm i.

This first order condition is useful to discuss the impact of a variation of the upper bound $\widehat{X}$ of the pool on the willingness of firms to invest in prevention.

Proposition 1 *An increase of the upper bound $\widehat{X}$ of the funds available for clean-up and compensation through the pool induces an increase in the level of prevention choosen by Firm i, other things being equal.*

Having to pay more for large accidents is similar for the firm to bearing more risk. Thus the marginal benefit of prevention increases, while the monetary marginal cost of prevention remains unchanged.

Hence increasing the maximum level of contribution by firms to the Fund may be a good way to increase both the available funds in case of an incident and ex ante prevention. However, this fragilizes the mutuality principle since more aggregate loss is borne by each individual firm. Furthermore, small firms may have some difficulties to fulfill their commitments if their contributions become too high.

We propose now to introduce standard insurance as a way to increase contributions to the Fund without deteriorating the financial condition of the firms.

2.2 Optimal standard insurance contract

The idea is that firms may be able to contribute more to the Fund if their random contributions were insured.

Assume that the firm can transfer a part or the whole of its risk $\alpha_i\widetilde{X}$ to an insurer. The compensation function is denoted $C(\alpha_i X)$ and is defined over $[0, \alpha_i L]$. The Von Neumann Morgenstern utility function of the insurer is denoted $v(.)$ with $v'(.) > 0$ and $v''(.) \leq 0$ and W is his initial wealth. Let us denote λ the marginal administrative cost of insurance: each time, the insurer is paying an indemnity $C(\alpha_i X)$, this costs him $(1+\lambda)C(\alpha_i X)$. Nevertheless, since we also consider risk aversion, the insurance premium denoted Q may be higher than this value: We have $Q = (1+\delta)E\left[C(\alpha_i X)\right]$

where δ represents the administrative costs of the insurer plus the risk premium per unit of transferred risk and E the expectation operator over X. Thus we have $\delta > \lambda$ for a risk-averse insurer.

The maximization program of the oil firm subject to the participation constraint of the insurer becomes[5]

$$\begin{aligned}\max_{C(.)} R^C &= \int_0^L u(w_i - \alpha_i(X.1_{\{X\leq\hat{X}\}} + \widehat{X}.1_{\{X>\hat{X}\}}) \\ &\quad + C(\alpha_i X) - Q - g(X, \widetilde{x_i}))f(X/e)dX - e_i \\ &\quad \text{subject to} \int_0^L v(W + Q - (1+\lambda)C(\alpha_i X))f(X/e)dX \geq v(W)\end{aligned} \tag{4}$$

with $Q = (1+\delta)E\left[C(\alpha_i X)\right]$. We use optimal control to solve this maximization program. The random variable X plays the role of time, $C(.)$ is the control variable while the state variable is $z(X) = \int_0^X v(W + Q - (1+\lambda)C(\alpha_i t))f(t/e)dt$. Its evolution is described by the system:

$$\begin{cases} \dot{z}(X) = v(W + Q - (1+\lambda)C(\alpha_i X))f(X/e) \\ z(0) = 0 \\ z(L) = v(W) \end{cases}$$

The Hamiltonien of Program (4) is

$$H(X) = \left(u(w_f^C(X, \widehat{X})) - e_i + \mu(X)v(W + Q - (1+\lambda)C(\alpha_i X))\right).f(X/e), \tag{5}$$

with $w_f^C(X, \widehat{X}) = w_i - \alpha_i(X.1_{\{X\leq\hat{X}\}} + \widehat{X}.1_{\{X>\hat{X}\}}) + C(\alpha_i X) - Q - g(X, \widetilde{x_i})$ and μ the Lagrange function. The contract C^* that maximizes H is presented in Proposition 2 hereafter.

[5] Function $\mathbf{1}_{\{.\}}$ is the indicator function, taking value one if the condition into brackets is satisfied, zero otherwise.

Proposition 2

(i) The optimal insurance contract displays a positive deductible when administrative costs are linear in the level of indemnities. Marginal compensations for damages beyond the level of deductible but lower than $\widehat{X}$ *are given by*

$$C^{*\prime}(\alpha_i X) = \frac{\left(1 + \frac{g_X}{\alpha_i}\right) R_u\left(w_f^C\right)}{R_u\left(w_f^C\right) + (1+\lambda) R_v(W_f^C)}, \tag{6}$$

with $R_u\left(w_f^C\right)$ *and* $R_v(W_f^C)$ *the absolute risk aversion ratios of, respectively, the insured and the insurer. For damages higher than* $\widehat{X}$*, marginal indemnities are given by*

$$\widehat{C}^{*\prime}(\alpha_i X) = \frac{\frac{g_X}{\alpha_i}.R_u\left(w_f^C\right)}{R_u\left(w_f^C\right) + (1+\lambda) R_v\left(W_f^C\right)}. \tag{7}$$

(ii) The optimal contract presents a disappearing deductible for losses lower than $\widehat{X}$ *if the insurer is risk-neutral and an upper limit for losses beyond a level* $\overline{X}$*, with* $\widehat{X} < \overline{X}$*.*

This result holds whatever the sign of $u'''(.)$*.*

(iii) If the insurer is risk averse, the coverage may display a coinsurance rate smaller than one for damages beyond the deductible and an upper limit of coverage.

Equation (6) is close to the one that Raviv (1979) obtained in a model with one insurable risk and to that obtained by Gollier (1996) with background risk. Nevertheless in our model, the risk of bad reputation is uninsurable and it depends positively on the insurable risk (we have $g_X > 0$). Thus we should expect that the insured firm accepts to pay for a higher coverage of the first risk in order to protect itself against its background risk if it is prudent in the sense of Kimball (1990). We obtain a similar result, but prudence is not necessary. In our model the second risk, g, is completely defined by the first one, X, so that for a given x_i, both variables have the same distribution. Formally, the fact that the insured firm asks for more insurance than in a case without reputational effect is illustrated by the presence of g_X, positive, at the numerator of $C^{*\prime}(.)$. It is as if the insured firm would bear an individual "aggregate" risk, $\alpha_i X + g(X, \widetilde{x}_i)$, which cannot be completely insured. Besides, the presence of the uninsurable reputational risk explains why indemnities can increase with X even if the insurable loss borne by the firm (its contribution to the Fund) is fixed and equal to $\alpha_i \widehat{X}$. This result is also due to the positive correlation between g and $\alpha_i X$. Indeed for $g_X = 0$, we would have $\widehat{C}^{*\prime}(.) = 0$ for any X larger than $\widehat{X}$ and $\widehat{X} = \overline{X}$.

What is different from the literature on background risk is that we are dealing with catastrophe risks. Hence an insurer whose portfolio contains the aggregate risk of the Fund bears an additional risk of insolvency following a catastrophe that he has accepted to cover. Besides, empirical facts show that reinsurance groups that accept to cover pollution damages ask for high insurance premia, which entails high risk premia. It is often argued that the management of large risks entails additional transaction costs, due to risks of insolvency or to the complexity of audits and of claims settlements. This may justify the significant increase in the price of classical insurance. In such an economic environment, it is unreasonable to assume that the insurer behaves as a risk-neutral agent. He is more likely to be risk averse and the loading factor of the insurance premium related to the management of catastrophe risks may be sufficiently high to argue that, in most cases, the optimal insurance contract displays coinsurance between the insurer and the insured firm beyond a deductible level. We obtain this result with equation (6): R_v must be sufficiently high to counterbalance the effect of g_X/α_i at the numerator. In other terms, a disappearing deductible, which induces that indemnities increase more rapidly than the loss, is seldom the best contract. This result is rather intuitive since such a contract would compell the insurer to pay really high indemnities in the case of a catastrophic event. Figure 1 displays the coinsurance contract.

Figure 1 about here

It is still interesting to notice that a contract with coinsurance beyond a deductible may also be the best risk sharing when the insurer bears convex administrative costs, as shown by Raviv (1979). If the convexity assumption is not the most plausible one when dealing with classical risks such as car- or house-insurance risks, it is much more closer to reality when one focuses on large risks. Consequently, convex costs may also explain the optimality of coinsurance in the management of large risks[6]. Nevertheless, in our setting convexity would not be a sufficient condition because of the reputanional effect. Still there, the insurer should be sufficiently risk-averse.

Another important result of this section deals with the optimal level of prevention when insurance is available. Formally, after a differentiation of (4) with respect to e_i, integrations by part applied to the terms in $f_{e_i}(.)$ yield the following first order condition for the optimal level of prevention, denoted now e_i^C:

$$\begin{aligned} 1 &= -\int_0^L (g_{e_i} + Q_{e_i}).u'(w_f^C(X,\widehat{X}))f(X/e^C)dX \\ &\quad +\alpha_i \int_0^{\widehat{X}} (1 + \frac{g_X}{\alpha_i} - C'(\alpha X)).u'(w_f^C)F_{e_i}(X/e^C)dX \\ &\quad +\alpha_i \int_{\widehat{X}}^L (\frac{g_X}{\alpha_i} - \widehat{C}'(\alpha_i X)).u'(\widehat{w_f^C})F_{e_i}(X/e^C)dX \end{aligned} \tag{8}$$

Proposition 3 *When the insurer can obtain information on the risk-reducing activities of the firm, the optimal level of prevention decided by the firm increases compared to a situation where no insurance is available.*

This last result is not surprising. The insurer can obtain information about the level of prevention decided by the firm. Consequently, the insurer is able to define a premium which depends on the level of prevention chosen by the firm[7]. If insurance is available, an increase in the level of prevention decreases the level of the premium.

[6] This assumption is not retained here. With a cost function more general than the one we are using, the parameter λ would be replaced by the first derivative of the cost function and the second derivative would appear at the denominator of Equation (6).

[7] This characteristic implies that the insurance contract is self-enforcable: Once the contract is underwritten, the insured firm has no incentive to choose a level of care lower than the one considered by the insurer when defining the price of insurance. This is consistent with the large-scale risk insurance sector. Indeed only a few companies are specialized in such a coverage so that the sector is relatively concentrated. In such a situation if a firm cheats, the insurer will break the contract and the firm will

In our model, an increase in prevention has also an effect on the marginal indemnities through its impact on the non insurable risk. Indeed, the marginal level of the bad reputation risk g appears in $C^{*\prime}(\alpha_i X)$. Hence as in standard models with complete information on prevention, the firm improves the prevention when it has access to insurance.

Finally, when only standard insurance is available, insurers may ask for high risk premia for accepting to manage a catastrophe risk and the optimal contract displays some coinsurance: As the damage increases, firms are less well covered at the margin and they have to bear more and more residual risk. Finally our aim, which was to use insurance coverage in order to provide more available funds in the case of a huge incident (which means that the cap $\widehat{X}$ of the pool could have been increased) can be difficult to achieve. As a limit case, if standard insurance is too costly and non compulsory, oil firms may prefer not to be insured at all.

3 Providing a better hedging strategy through capital markets

In this section, the issue is to find complementary mechanisms that are able to diversify risks over a wider range of individuals and to transfer risk to agents such as financial investors. In this way, it will be possible to reduce the residual risk borne by the firm after (standard) insurance and to increase available funds for victims in case of an accident. In a first paragraph, we provide our results related to combined hedging strategies. In the second subsection, we discuss the financial implications.

have many difficulties to find another insurer who accepts to offer it an insurance contract at a same price.

3.1 The optimal joint strategy

A more sophisticate hedging strategy would consist in combining several coverage instruments. Doherty and Dionne (1993) and Mahul (2002) provide such an approach by dividing the risk into two components: an idiosyncratic risk, which can be related to the specific activities of a given firm, and a systematic risk, related to the risk of the industry as a whole. While the individual risk can be insured by a standard insurance policy, the systematic risk is managed through a participating contract.

Our framework is different from the ones of Doherty and Dionne (1993) and Mahul (2002) because 1) The firm does not bear an insurable idiosyncratic risk since the effect of bad reputation, which plays this role, is non insurable, 2) The individual risk of the firm is correlated to the risk of the Fund, while in the quoted analyses both are independent, and 3) Prevention is absent from the models of Doherty and Dionne and of Mahul while it plays an important role in our work.

Now, assume that the Fund, representing all contributing firms, has to pay for all oil spills, whatever their size. This means that no upper limit of compensation exists ($\widehat{X}$ does no longer hold) and, as a direct consequence, that oil firms are no longer protected by limited liability. Nevertheless, the firm can still transfer part of its risk to an insurer, and it can also invest on financial markets in order to cover losses (contributions to the Fund) in excess of the insurance coverage[8].

Our objective is to limit the implication of the insurer in the coverage of large risks in order to mitigate his insolvency risk. Recall that in the previous section we have shown that sufficient risk aversion of the insurer leads him to offer a contract with an upper limit of insurance when dealing with catastrophe risks. We take this result as given

[8] Another way to deal with such a configuration would be to let firms be protected by limited liability, that means that individual contributions are still limited to $\alpha_i \widehat{X}$. Nevertheless the Fund would have to pay for all damages so that it would have strong incentives to go on financial markets in order to find the additionnal needed funds. Such a setting is consistent when dealing with natural hazards (see, for instance, the earthquakes hedging system in California). However it would have no impact on the firms in terms of prevention when risks can be controlled by them. Thus we do not consider it here.

here: We denote $I(.)$ the indemnity schedule and $\overline{X}$ the level of damage such that any contribution higher than $\overline{X}$ induces the same indemnity cap. Formally, when an accident occurs and after having contributed to the Fund, the firm obtains an indemnity $I(\alpha_i X)$ if its contribution is less than $\alpha_i \overline{X}$ and the fixed amount $\overline{I} = I(\alpha_i \overline{X})$ for any larger contribution.

Still assume that the firm can sell to an external investor a part β of its residual risk minus the deductible[9], (which is always borne by the firm in order to avoid moral hazard problems[10]), for any damage higher than $\overline{X}$: $\alpha_i X - \overline{I} - D^\beta$.[11] The price of this risk transfer is denoted π. In this model, financial markets can obtain some information about environmental policies adopted by the firms[12]. We have $\pi = \pi(\beta, e_i)$ with $\pi_\beta > 0$ and $\pi_{e_i} < 0$. Lastly, the insurer's unit loading factor of insurance δ depends now on β: if the insured commits to cover the worst states of nature on financial markets, the insurer takes into account this information when evaluating the insurance premium. The consequences of a catastrophe are now split between the insurer and financial markets. As a direct consequence, the costs of risk management for the insurer are lower than in the previous case because of a decrease in the risk premium. Formally, we have $\delta(0) > \delta(\beta) \geq \lambda$. Such a behavior implies that firms communicate with the insurer on their financial strategy. From an empirical point of view, this is rather usual when

[9] By buying and selling puts and calls of appropriate underlying securities. See the discussion in the section 4 of the paper.

[10] Indeed this allows us to avoid some discontinuity of the indemnity schedule at point D^β (see Figure 2).

[11] Thanks to the forthcoming results on the design of the optimal indemnity function for losses less than the upper bound $\overline{X}$, we are to show that the scalar $\alpha_i X - \overline{I} - D^\beta$ is always positive. Indeed, D^β is the deductible imbedded in the indemnity function so that $I(\alpha_i X) = h(\alpha_i X) - D^\beta$. Furthermore we will have that $h(\alpha_i X) > D^\beta$ for any loss $\alpha_i X$ in $\left]D^\beta, \alpha_i L\right]$ and that $0 \leq h'(\alpha_i X) < 1$ at optimum. Hence we have that $\overline{I} < \alpha_i X - D^\beta$.

[12] See Blacconiere and Patten (1994), Cormier and Magnan (1997) and Lanoie et al. (1998) for details about how those informations are released on financial markets and their impact. See also Freedman and Stagliano (1991) who show that firms with a high level of disclosure about their risk-reducing activities suffer from a smaller decrease in their stock price after an incident.

looking at the pollution insurance market. Insurers ask for more and more informations about the risk-reducing activities of the firms and firms collaborate most of the time in order to obtain adequate coverage.

The maximization program of the oil firm becomes

$$\begin{aligned}\max_{(I,\beta)} R^{\beta} &= \int_{0}^{\overline{X}} u(w_i - \alpha_i X - Q^{\beta} + I(\alpha_i X) - g(X, \widetilde{x_i}) - \pi(\beta, e_i)) f(X/e) dX \qquad (9) \\ &+ \int_{\overline{X}}^{L} u(w_i - \alpha_i X - Q^{\beta} + \overline{I} + \beta \left[\alpha_i X - D^{\beta} - \overline{I}\right] \\ &- g(X, \widetilde{x_i}) - \pi(\beta, e_i)) f(X/e) dX - e_i \\ &\text{subject to} \\ &\int_{0}^{\overline{X}} v(W + Q^{\beta} - (1+\lambda) I(\alpha_i X)) f(X/e) dX \\ &+ v(W + Q^{\beta} - (1+\lambda).\overline{I})(1 - F(\overline{X}/e)) \geq v(W),\end{aligned}$$

with $Q^{\beta} = (1 + \delta(\beta))E\left[I(\alpha_i X)\right]$ the insurance premium. The firm has to choose the combined hedging strategy $(I(.), \beta)$ that maximizes its expected net utility subject to the participation of the insurer.

Proposition 4

(i) The optimal indemnity function displays a positive deductible D^{β}. Marginal indemnities for losses between the deductible level and the bound $\alpha_i \overline{X}$ are given by:

$$I^{*\prime}(\alpha_i X) = \frac{\left(1 + \frac{g_X}{\alpha_i}\right) R_u\left(w_f^{\beta}\right)}{R_u\left(w_f^{\beta}\right) + (1+\lambda).R_v\left(W_f^{\beta}\right)}, \qquad (10)$$

with $R_u\left(w_f^{\beta}\right)$ and $R_v\left(W_f^{\beta}\right)$ the absolute risk aversion ratios of, respectively, the insured and the insurer, $w_f^{\beta} = w_i - \alpha_i X - Q^{\beta} + I^*(\alpha_i X) - g(X, \widetilde{x_i}) - \pi(\beta, e_i)$ and $W_f^{\beta} = W + Q^{\beta} - (1+\lambda) I^*(\alpha_i X)$.

(ii) If positive hedging is provided by financial markets ($\beta > 0$), the optimal marginal insurance coverage is higher than the one obtained when only standard insurance is available: $I^{*\prime}(\alpha_i X) > C^{*\prime}(\alpha_i X)$ for any loss $\alpha_i X$ partially covered and less than $\alpha_i \overline{X}$.

Besides, we have $D^\beta < D$ if and only if:

$$Q^\beta_\delta.\delta'(\beta).\left(u''(w^\beta_f) + \gamma(1+\lambda).v''\left(W^\beta_f\right)\right) < -\pi_\beta.u''(w^\beta_f) \tag{11}$$

Condition (11) is an inequality between the marginal benefit of reporting one unit of risk from the standard insurance market (letf-hand-side term) to the financial markets (right-hand-side term). Thus increasing the indemnity of small losses through standard insurance is valuable if the insurer is sufficiently sensitive to the fact that the insured is looking for alternative coverage.

Actually, Point (ii) suggests that firms should use the wide diversification capability of financial markets to manage the potential large consequences driven by catastrophe risks and they should buy standard insurance for small and medium losses.

Figure 2 displays an example of an optimal combined contract.

Figure 2 about here

For convenience we use in what follows the notation $g(X, \widetilde{x_i}) \equiv g$ and $\pi(\beta, e_i) \equiv \pi$.

Proposition 5 *Partial financial hedging is optimal if and only if*

$$\begin{aligned}
\pi_\beta.\int_0^L u'(w^\beta_f)f(X/e)dX &= \int_0^{\overline{X}} I^*_\beta(\alpha_i X).u'(w^1_f)f(X/e)dX \\
&+ \int_{\overline{X}}^L \left[(\alpha_i X - D^\beta - \overline{I}) - \beta.D^\beta_\beta\right].u'(w^2_f)f(X/e)dX \\
&- \int_0^L Q^\beta_\delta.\delta'(\beta).u'(w^\beta_f)f(X/e)dX,
\end{aligned} \tag{12}$$

$$with \begin{cases} w^1_f = w_i - \alpha_i X - Q^\beta + I(\alpha_i X) - g - \pi \\ w^2_f = w_i - \alpha_i X - Q^\beta + \overline{I} + \beta\left[\alpha_i X - D^\beta - \overline{I}\right] - g - \pi \\ w^\beta_f = w_i - \alpha_i X - Q^\beta + I(\alpha_i X).\mathbf{1}_{\{X \leq \overline{X}\}} + \left[\overline{I} + \beta\left(\alpha_i X - D^\beta - \overline{I}\right)\right].\mathbf{1}_{\{X > \overline{X}\}} - g - \pi \end{cases}$$

Function $\mathbf{1}_{\{.\}}$ is the indicator function, which takes value one if the condition into brackets is satisfied, zero otherwise.

Equation (12) is obtained by differentiating (9) with respect to β. Each term on both sides are positive[13]. (Partial) external financing is optimal if the expected marginal cost of an increase in β (left-hand-side-term) equals the expected marginal benefit, obtained thanks to an increase in the coverage of the small and medium losses (first member in the right-hand-side-term), to the direct increase of the coverage of large losses (second member) and to the decrease of the price of standard insurance (third member).

4 Application to oil spill risks

Let us now apply the results of the theoretical analysis to a specific case study: the hedging of oil spill risks. This case is of particular interest notably because its functionning is already based on risk mutualization. Nevertheless, upper limits of compensation are often reached for large oil spills and no additional transfer principle is considered.

4.1 The context

Since 24 May 2002, International maritime transport (except for the United States) is exclusively regulated by the 1992 Civil Liability Convention (CLC in the course) and by the 1992 International Oil Pollution Compensation Fund (IOPC Fund) Convention[14].

The Fund is financed by contributions of the oil industry of member states receiving more than 150,000 tons of oil per year after sea transport. The contribution of each

[13]In particular, by writing $I(\alpha_i X)$ as being equal to $h(\alpha_i X) - D^\beta$ for indemnified contributions, we have that:

$$I^*_\beta = h_\beta(\alpha_i X) - D^\beta_\beta$$

Because $D^\beta > D$, $I(0) = C(0) = h(0) = 0$ and $\partial I'/\partial\beta > 0$, we have that $h_\beta(\alpha_i X) > 0$, so that $I^*_\beta > 0$.

[14]Actually, the first Civil Liability Convention dates from 1969 and the Fund was created in 1971. Both were amended in 1992. For details, see the companion paper of Schmitt and Spaeter (2007).

company is proportional to the annual tonnage received by sea and is directly payable to the Fund. It corresponds to α_i in our setting. Contributions, decided each year by the Assembly of the Fund, cover administrative costs and estimated compensation payments for passed pollutions. Hence no provision is made ex ante and each oil firm pays an ex post indemnity equal to the part α_i of the losses induced by all oil spills registered until this date: The sum of these oil spills can be considered as the aggregate loss of the IOPC Fund, which is shared among its members (X in our model). In this spirit, there is a kind of loss mutualization.

It is important to notice that, under the 1992 Civil Liability Convention, only the owner of the tanker is held financially liable for the catastrophe. The convention obliges him to buy pollution insurance, provided by P&I Clubs which are non-profit making mutual insurance associations. These mutual groups offer insurance depending on the size of the boat and not directly on the damages that may be induced by a wreck. And the Fund complements this coverage if it is less than total damages up to a given cap (that could be $\widehat{X}$ in our setting).

This International compensation regime has improved the protection of sea environment against oil pollution by inducing a decrease of the number of large oil spills in the last two decades[15]. Also, it facilitates claims settlement for victims of pollution and it has increased the compensation available for them. Although claims for damage to the ecosystem are not admissible, compensation is granted to a wide range of costs (clean-up operations, property damages, economic losses, ...).

Nevertheless this regime also shows its limits regarding the total compensation available for victims[16] and the incentives to enhance environmental prevention through the

[15]The number of large oil spills (spilling more than 700 tons) was 7.3 per year on average during the 1990s compared to 24.2 during the 1970s. (source : ITOPF Handbook 2003-2004) However, the level of losses eligible for compensation has increased dramatically in some huge incidents. This can be explained by the higher than average increase of population in coastal areas and the development of tourism.

[16]Only partial compensation was available to victims after the wrecks of Nakhodka (1997), Erika (1999) and Prestige (2002). In the case of Erika, the percentage of compensation was the highest

chartering of safe boats. Indeed, while the shipowner is solely held liable through the Civil Liability Convention, the whole oil industry participates in compensations through the IOPC Fund Convention: no direct compensation between the owner of the oil leaked from the boat and victims can be established. Moreover, the contribution to the Fund is upper bounded, and this kind of limited liability may induce oil firms to charter boats with medium, or even low, levels of quality.

Besides, the shipowner is also protected by limited liability, which benefits mostly low market value firms as shown by Dionne and Spaeter (2003) and by Schmitt and Spaeter (2005, forthcoming). Consequently, risk-reducing activities may still be worsened.

As a last important point, the 1992 Fund was recently complemented by a Supplementary Fund, which entered into force the 3rd March 2005. Its aim is to significantly increase the levels of compensation if compensation available through the Civil Liability Convention and the IOPC Fund should prove insufficient. Only states that receive more than 1 million tons of oil are able to ratify the new convention so that the number of members of the Supplementary Fund will be significantly lower than the number of the 1992 Fund. Small countries are excluded from this new additional compensation mechanism even if a large oil spill is registered in their territorial waters. This new Supplementary Fund also casts doubt on the fact that contributions to the Funds are relatively fair since fewer firms will contribute significantly more.

4.2 Optimal hedging of oil spill risks

In order to improve the victims' compensation, the cap of the IOPC Fund has been increased just after the Erika incident, once again during the year 2003, and was still increased at the end of 2004 through the implementation of the Supplementary Fund[17].

one among these three catastrophes: About 80% of the total losses estimated by the experts of the IOPC Fund. Nevertheless, insurance from P&I Clubs amounted only to 7% of the total available funds. Concerning the Prestige incident, the Executive Committee of the IOPC Fund decided in May 2003 to limit compensation to 15% of the loss actually suffered by the respective claimants.

[17]Under the Supplementary Fund Convention, the total amount of compensation available for pollution damage in the States that have ratified the Protocol is 750 million SDR (US$ 1 100 million),

Following our theoretical framework, an increase of $\widehat{X}$ yields an increase in the level of prevention by the firms, other things being equal. Nevertheless, the impact of one individual firm on the distribution of the agregate risk may be negligible if other firms do not change their behavior. In practice, the increase of the IOPC Fund cap did not seem to have had a significant effect on the shartering policies in the oil industry. Therefore, it is interesting to discuss the possibility to provide insurance to firms in order to eliminate this cap $\widehat{X}$. This would induce a kind of unlimited liability.

In our setting, prevention is observable: This may look as a bold hypothesis knowing that maritime oil transport is largely subcontracted to shipowners. However, charterers get a precise information on the safeness of a boat through the classification society that has checked it. When a given boat is chartered, its capacity and its safeness become common knowledge because maritime authorities diffuse the results of the control. More generally, it is fair to assume that insurers have some information about the risk-reducing activities of the firms they insure. Indeed, policies are often conditioned on the adoption of adequate mitigation measures by firms that are candidates to pollution insurance.

As a direct consequence, if insurance were available for the coverage of the agregate risk of the IOPC Fund, the insurance premium would depend on the safeness of the chartered boats in our approach. This is still consistent with the maritime oil insurance sector. Indeed only a few companies are specialized in such a coverage so that the sector is relatively concentrated. In such a situation if a firm cheats, the insurer will break the contract and the firm will have many difficulties to find another insurer who accepts to offer it an insurance contract with a fair price. Besides, cheating will have a negative impact on the reputation of the oil firm.

Another important point that should be discussed deals with the question of who should buy insurance. In our setting, we assumed that firms purchase insurance for their own random contribution to the fund. An alternative would be to give this job to the

including the 203 million SDR (US$ 300 million) available under the 1992 Conventions (Jacobsson, 2004).

Fund. This could significantly decrease the transaction costs. Nevertheless, the staff who is in charge of the management of the IOPC Fund in London has no decision power about the strategies of oil transport or of boat chartering that the oil firms may adopt, so that it is reasonable to assume that oil firms take insurance decisions, rather than the IOPC Fund. Besides, firms are more able to disclose information to the financial markets about their individual risk-reducing activities. This is an important point that enhances, among others, the interest firms should put in financial hedging as a way to complement classical insurance. Indeed, investors are sensitive to the fact that firms insure their risks (Doherty (2000)) and they provide more easier access to financing. Foulon et al. (1999) propose some empirical results that confirm the importance firms and investors give to information disclosure in the paper industry. Blacconiere and Patten (1994) also obtain interesting conclusions by studying empirically the impact of the Bhopal incident in India (1984) on the stock value of chemical firms in general and on Union Carbide India Limited (UCIL), which was responsible of the pesticides leak, in particular.

Finally, the oil industry present characteristics that fit well with our setting. Thus, by applying our results to this sector we argue that it would benefit from a re-organisation of its compensation system in the spirit of a joint hedging mechanism. To go further in this direction, it is important to notice that what one commonly calls the 1992 IOPC Fund is composed of two distinct funds actually. The first one, the general Fund, is dedicated to the payment of the current administrative costs and to the compensation of small oil spills (less than 4 millions SDRs with one Special Drawing Right = US$ 1.52145 on 16 May 2007), while the main claims Fund is dedicated to large oil spills. Finally the general Fund should negociate some coverage conditions offered by standard insurers, while the main claims Fund should rather be managed through interventions on capital markets.

4.3 Some financial insights

So far, we have discussed the conditions under which financial hedging may supplement insurance mechanisms to enhance the management of oil pollution risk. Two main topics remain to be discussed. First of all, we have to explain why these conditions are likely to be met empirically, i.e. why financial instruments may be attractive to both investors and oil companies. Then, we have to discuss the design of the financial instruments adapted to oil pollution risks and its consequences on the level of environment prevention.

One reason that is often evoked to explain the limited ability of reinsurance companies to handle catastrophe risk is the insufficient available funds of the sector compared to the size of capital markets (see for instance Froot (2001)). This problem of credit risk translates into the inability of the reinsurer to fulfil its obligation to oil companies if a catastrophe should occur. Although losses incurred under the current international maritime regime are far lower than hurricane or earthquake losses, the implementation of the Supplementary Fund[18] will make oil companies even more sensitive to oil pollution risk. According to some executives of the Marsh Company, a world leader of business risk management and insurance broking, even the biggest oil companies are now aware of the needs to hedge this kind of risk. Indeed, the Supplementary Fund introduces a third tier that sets the total amount of compensation payable for any incident to a combined total of 750 million Special Drawing Rights (just over US$1,000 million) including the amount of compensation paid under the existing CLC/Fund Convention. This is more than three times as large as the current limit. Furthermore, this third tier will be taken over by a few companies since the supplementary fund is likely to be ratified only by European countries and Japan. This in turn means that the mutuality principle is weakened and splitting the total risk by issuing adequate financial instruments is likely to become less costly than insurance coverage.

[18] As already mentionned earlier in the paper, recall that only countries receiving more than 1,000,000 tons a year can join the Supplementary Fund convention, while its cap is significantly higher than the one of the 1992 IOPC Fund.

On the supply side, investors are likely to be attracted by instruments of which distributions of payoffs have a weak (nil) correlation with other financial assets. This offers a unique opportunity for diversification. Actually, this argument applies to all kinds of catastrophe risk. The only difference is the anthropic feature of oil pollution risk: It depends on weather conditions, on the location of the wreck but also on human activity through the safeness of the chartered boats or an act of sabotage. But one can hardly imagine that this could challenge the opportunity to invest in such assets. The amount X of compensation available is assessed by independent experts and eligibility for compensation could be easily rejected if a specific incident should be proven to be deliberate. Opportunities to influence financial quotations seem to be very unlikely.

The development of catastrophe assets is also explained by their ability to reduce transaction costs, especially they mitigate the moral hazard problem at the expense of increasing the basis risk. For instance, the CBOT contracts are defined on various industry indice losses, so that an individual firm (an insurance company) has no (or a weak) incentive to declare excess losses because it will only marginally benefit from this behavior. This argument of agency cost mitigation is less appealing in the context of oil pollution risks since the IOPC Fund already applies the mutuality principle. The total amount of compensation X is assessed from all incidents that occurred within a year and its estimation is not contested by oil companies. In this perspective, there is no clear advantage of financial instruments compared to reinsurance. Note however that financial instruments suitable with the coverage of oil pollution risk do not increase basis risk.

Figure 2 depicts the optimal hedging strategy when both insurance policies and financial instruments are available. It also shows the shape of the payoffs of the financial instruments that will fit with oil pollution risks. The coverage provided by financial instruments ($\beta(\alpha_i X - D^{\beta} - \overline{I})$) corresponds to the design of a call option. The underlying asset would correspond to X (or more precisely $\alpha_i X$), that is the total compensation paid by the oil industry during a year within the international regime. The strike price could be set to $\widehat{X}$. One main difference would be the slope of the payoff (β) which is lower than 1, the slope of usual option contracts. This simply means that instead of getting the difference between the index value and the striking price, the buyer of the option would get only a percentage β from it.

To enhance the attractiveness and the liquidity of such contracts, one can imagine to build stop loss contracts. In the option context, this would correspond to a bundle of call options. For instance, one can create a bull spread by buying a call option on the index X with a certain strike price and selling a call option on the same index with a higher strike price. This is a good example of the advantages of securitization which allows to decompose and repackage risk (see Doherty and Schlesinger (2002) for a general presentation). Indeed, the call option on X can be decomposed in a set of adequate bull spreads, as illustrated on Figure 3.

Figure 3 about here

This decomposition has at least two advantages. On the one hand, the investor would limit his risk exposure. On the other hand, it facilitates standardization. Indeed, among the characteristics of the call option that we described just above, most of them were specific to each company. The percentage of contribution to the IOPC fund α_i depends on the level of activity of Firm i whereas $\overline{X}$ results from the maximization program of Firm i. The decomposition of total risk enables each firm to limit its basis risk by selling the desired risk exposure.

Earlier on, we have supposed that the cost π of the financial instruments depends on the level of prevention e_i of Firm i. Unless tailored-made financial contracts are proposed to a firm, a successful market of oil pollution hedging instruments requires standardization. The latter is reckoned as a key advantage of financial markets since it enables to reduce transaction costs and increase liquidity. This implies that the cost π will depend on the prevention adopted by the whole sector (e) and not only on e_i.

5 Conclusion

The new firms' management of risks tries to encompass all types of risks. Firms have to cope with numerous sources of uncertainties, linked to the production process, to unanticipated market evolutions, non expected internal organization issues and also with uncertainties related to the existence of large risks. Large risks are often catastrophe risks. These are characterized by low frequency but may induce very large economic consequences, irreversible ecological damages and sometimes loss of human lifes.

In this paper, we have shown that transferring part of the aggregate risk of an industry, namely the part related to catastrophic losses, to investors that have access to capital markets makes standard insurance of small and medium damages less costly. The joint strategy, which consists in using the properties of standard insurance for risks that are reasonably insurable and the wide capability of financial markets to diversify risk across many people in the world for catastrophic losses, seems to be a good compromise. Moreover if firms can send to markets signals on their environmental policies, financing hedging creates additionnal incentives to invest in risk-reducing activities.

Compared to other existing researchs (Doherty and Dionne (1993), Mahul (2002), Doherty and Schlesinger (2002)), the originality of our theoretical analysis deals with the introduction of prevention and of a bad reputation risk, which cannot be insured but which influences the coverage strategy of the insured for the insurable one and which is positively correlated to the insurable risk.

In the second part of the paper, we apply our findings to the maritime transport of oil. To manage oil spills, the 1992 IOPC Fund calls for ex post contributions by each oil firm belonging to a member state of the Fund. However, no insurance mechanism is designed and only the mutuality principle is applied: The individual contribution corresponds to a percentage of the aggregate risk of the Fund. Because of the limited number of members and also of the huge financial consequences induced by some oil spills, the aggregate risk cannot be fully spread across the oil firms. Hence it is useful to think about other diversification and/or coverage instruments that would help to smooth the payments of firms through time and also to increase the funds available for compensation.

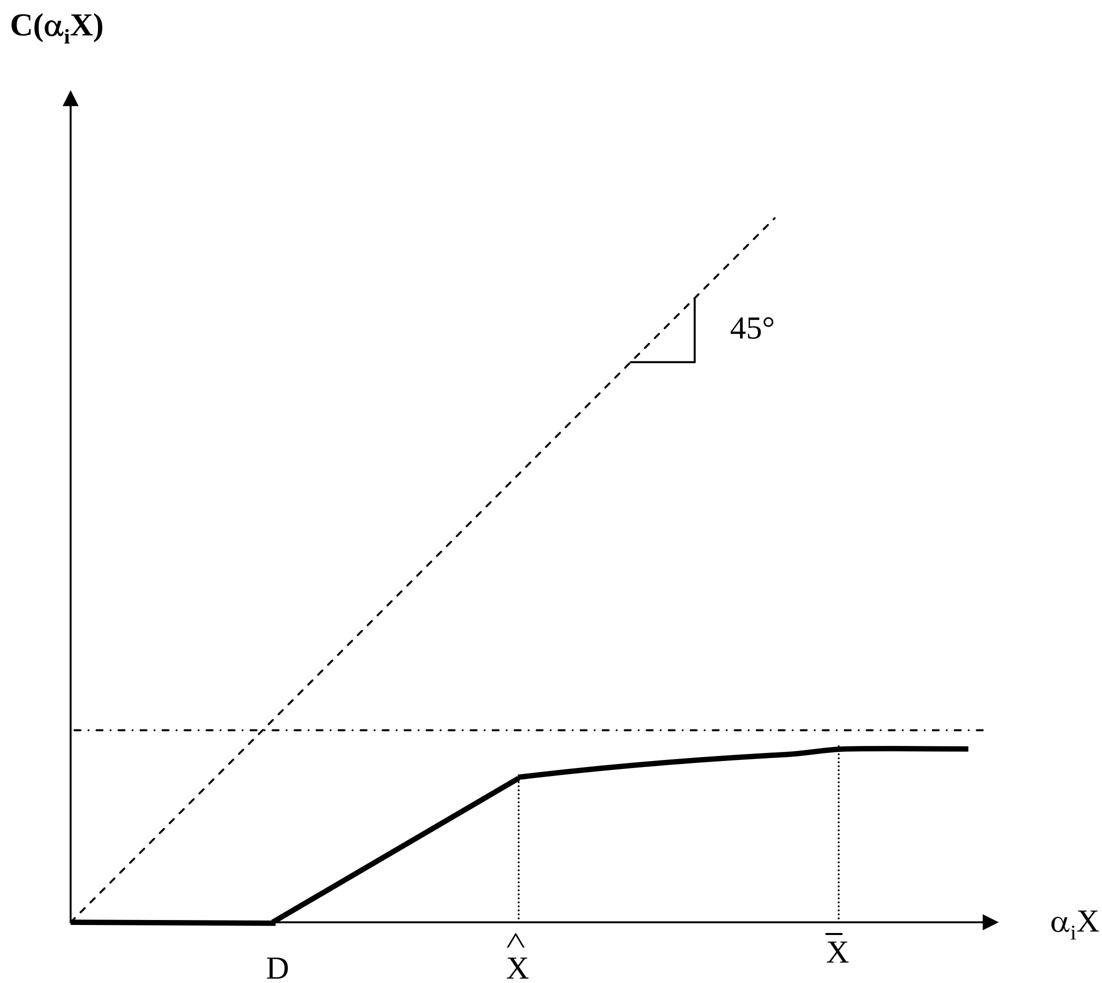

Figure 1. Optimal Compensation Function when only standard insurance is available and the insurer is « sufficiently » risk averse

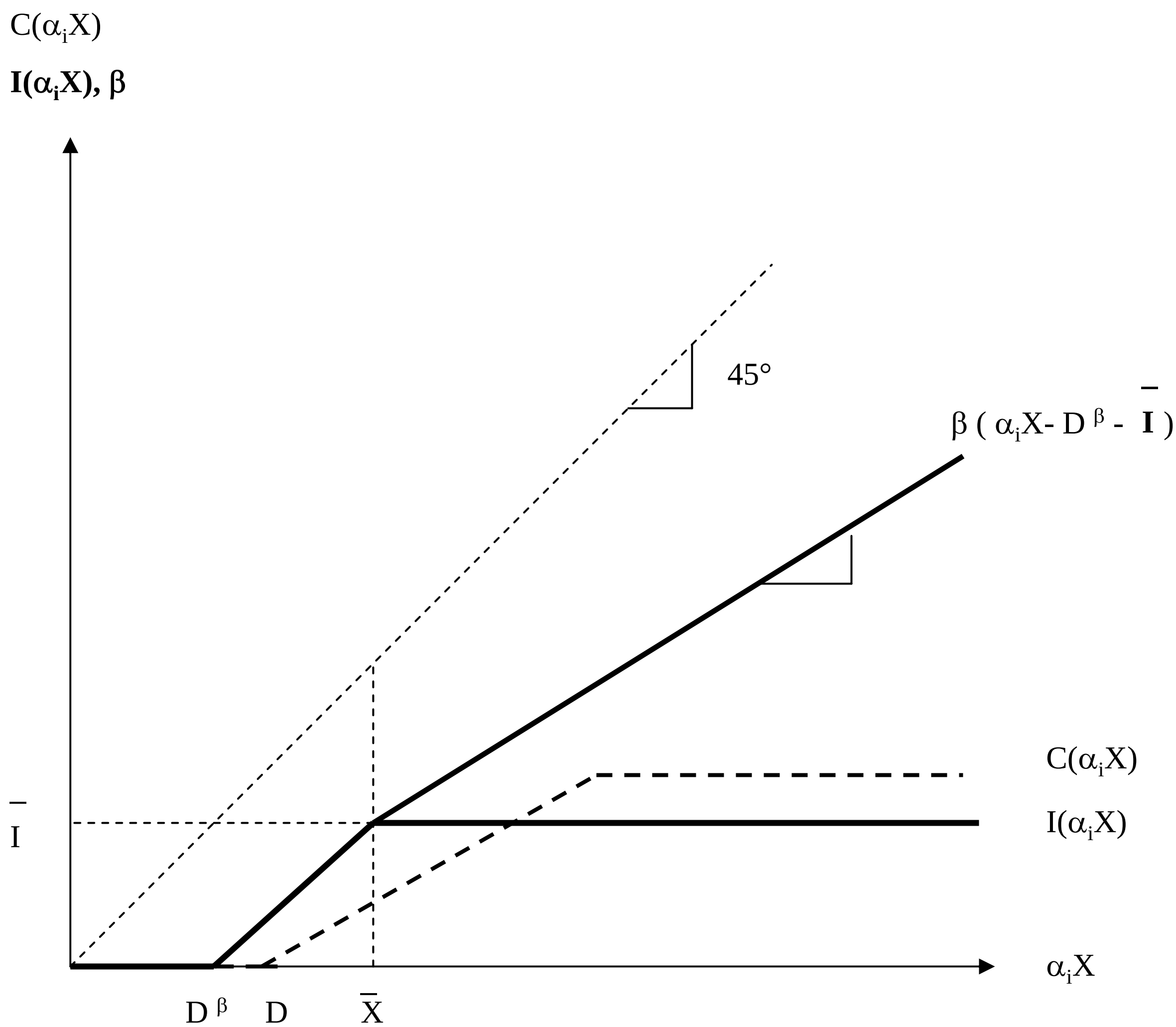

Figure 2. An example of optimal hedging strastegy

(At fixed premium)

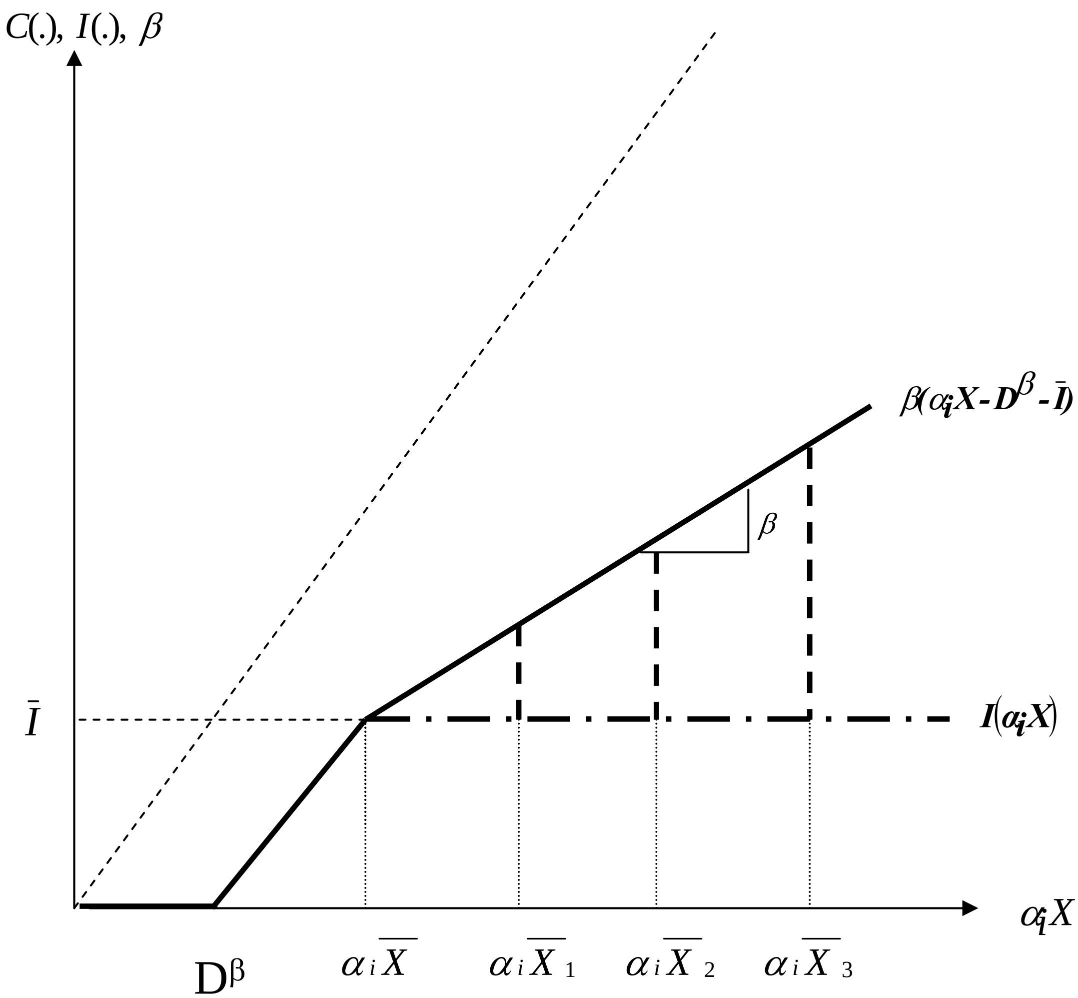

Figure 3. An example of stop loss contracts with strike prices X, X_1, X_2 and X_3

APPENDIX

Proof of Proposition 1

Recall that $w_f = w_i - \alpha_i X - g(X, \widetilde{x_i})$, $\widehat{w_f} = w_i - \alpha_i \widehat{X} - g(X, \widetilde{x_i})$, $g_{e_i} = g_{e_i}(X, \widetilde{x_i})$, $g_X = g_X(X, \widetilde{x_i})$. The effect on prevention of a variation in $\widehat{X}$ is obtained thanks to a total differentiation of R_{e_i} given by (3) with respect to (w.r.t) e_i and to $\widehat{X}$:

$$\frac{de_i}{d\widehat{X}} = \frac{\alpha_i}{|R_{e_ie_i}|}\left[\int_{\widehat{X}}^{L} g_{e_i}.u''(\widehat{w_f})f(X/e)dX + u'(w_i - \alpha_i\widehat{X} - g(\widehat{X}, \widetilde{x_i})).F_{e_i}(\widehat{X}/e) - \int_{\widehat{X}}^{L} g_X.u''(\widehat{w_f})F_{e_i}(X/e)dX\right] \quad (13)$$

With $R_{e_ie_i}$ the derivative of R_{e_i} with respect to e_i.

From (2) we have that $g_{e_i} < 0$. By definition we also have that $g_X > 0$: An increase in the aggregate loss X of the Fund deteriorates the reputation of all firms. Finally, with F_{e_i} positive the numerator of (13) is strictly positive for a risk-averse, or risk-neutral, oil firm. The denominator is obtained thanks to a differentiation of (3) w.r.t. e_i. With $g = g(X, \widetilde{x_i})$ and[19] $w_f(X, \widehat{X}) = w_i - g(X, \widetilde{x_i}) - \alpha_i X.\mathbf{1}_{\{X \le \widehat{X}\}} - \alpha_i \widehat{X}.\mathbf{1}_{\{X > \widehat{X}\}}$ we have:

$$\begin{aligned} R_{e_ie_i} = & -\int_0^L g_{e_ie_i}.u'(w_f(X,\widehat{X}))f(X/e)dX + \int_0^L g_{e_i}^2.u''(w_f(X,\widehat{X}))f(X/e)dX \\ & + \int_0^L g_{Xe_i}.u'(w_f(X,\widehat{X}))F_{e_i}(X/e)dX - \int_0^L g_{e_i}.u'(w_f(X,\widehat{X}))f_{e_i}(X/e)dX \\ & - \int_0^{\widehat{X}} (\alpha_i + g_X).g_{e_i}.u''(w_f)F_{e_i}(X/e)dX - \int_{\widehat{X}}^L g_X.g_{e_i}.u''(\widehat{w_f})F_{e_i}(X/e)dX \\ & + \int_0^{\widehat{X}} (\alpha_i + g_X).u'(w_f)F_{e_ie_i}(X/e)dX + \int_{\widehat{X}}^L g_X.u'(\widehat{w_f})F_{e_ie_i}(X/e)dX \end{aligned}$$

[19] Function $\mathbf{1}_{\{.\}}$ is the indicator function, taking value one if the condition into brackets is satisfied, zero otherwise.

From the definition (2) of $g(X, \widetilde{x_i})$, we have that $g_{e_iX} = g_{Xe_i} = 0$. Finally, an integration by part of the second term in the second line leads to:

$$\begin{aligned} R_{e_ie_i} &= -\int_0^L g_{e_ie_i}.u'(w_f(X,\widehat{X}))f(X/e)dX + \int_0^L g_{e_i}^2.u''(w_f(X,\widehat{X}))f(X/e)dX \\ &\quad -2\int_0^{\widehat{X}}(\alpha_i + g_X).g_{e_i}.u''(w_f)F_{e_i}(X/e)dX - 2\int_{\widehat{X}}^L g_X.g_{e_i}.u''(\widehat{w_f})F_{e_i}(X/e)dX \\ &\quad + \int_0^{\widehat{X}}(\alpha_i + g_X).u'(w_f)F_{e_ie_i}(X/e)dX + \int_{\widehat{X}}^L g_X.u'(\widehat{w_f})F_{e_ie_i}(X/e)dX \end{aligned}$$

By assumption, we have $F_{e_ie_i} \leq 0$, $g_X > 0$ and $u'' < 0$. Besides, $g_{e_ie_i}$ is equal to $p_{e_ie_i}.(g(X, x_i) - g(X, 0))$ (see Equation (2)). Function $g(X, .)$ is increasing in x_i and $p_{e_ie_i}$ is positive or equal to zero, so that $g_{e_ie_i}$ is positive. Finally $R_{e_ie_i}$ is negative (the second order conditions are satisfied) and $de_i/d\widehat{X}$ given by (13) is positive. Proposition 1 is demonstrated.

Proof of Proposition 2

The optimality conditions related to optimal control that must be satisfied are

$$\begin{cases} (i)\ H_z = -\mu'(X) \\ (ii)\ H_\mu = \dot{z}(X) \\ (iii)\ z(0) = 0 \\ (iv)\ z(L) = v(W) \end{cases}$$

and $H_C = 0, \forall X$ such that $0 < C(\alpha_i X) < \alpha_i X$. From (5) we have $H_z = 0$ so that μ is constant. Conditions (ii), (iii) and (iv) are also satisfied. Because $f(X/e)$ is always positive by definition, it is possible to work with the simplified Hamiltonien $H^* = H/f(X/e)$. We have for any X such that $0 < C(\alpha_i X) < \alpha_i X$:

$$H_C^* = 0$$

$$\Longleftrightarrow u'(w_f^C(X, \widehat{X})) - \mu(1+\lambda)v'(W_f^C) = 0 \tag{14}$$

with $w_f^C(X,\widehat{X}) = w_i - \alpha_i \left(X.\mathbf{1}_{\{X \leq \widehat{X}\}} + \widehat{X}.\mathbf{1}_{\{X > \widehat{X}\}}\right) + C(\alpha_i X) - Q - g(X, \widetilde{x}_i)$ and $W_f^C = W + Q - (1+\lambda)C(\alpha_i X)$.

First, we have to show that the optimal contract displays a positive deductible. Let us define as $J(X)$ the function given by (14) and evaluated at $C(\alpha_i X) = 0$ and $K(X)$ the same function but evaluated at $C(\alpha_i X) = \alpha_i X$. By differentiating them with respect to (w.r.t.) X it is easy to show that $J(X)$ is increasing in X and $K(X)$ is decreasing. Moreover, both functions are equal at point $X = 0$. Denote them m at this point: $m = u'(w_i - Q) - \mu(1+\lambda)v'(W+Q)$. Two cases must be considered : either m is negative or m is positive (the trivial case for which $m = 0$ is not analyzed).

♦ $m > 0$

Since $J(.)$ is increasing in X, m is the smallest value it can take. Thus J is always positive and $C(\alpha_i X) = 0$ is never optimal[20]. Besides, $K(.)$ is decreasing in X. Then there exists a positive level of damage $\underline{X}$ such that K is positive on $[0, \underline{X}]$ and $C(\alpha_i X) = \alpha_i X$ is optimal on this interval. For damages higher than $\underline{X}$, K becomes negative: from this point, coverage must be constant and an upper limit of insurance is optimal.

♦ $m < 0$

In this case, $K(X)$ is always negative and full coverage is never optimal. Besides, there exists a level of damage D such that $J(X)$ is negative on $[0, D]$, so that a positive deductible is optimal and it presents partial coverage for any damage higher than D. A positive deductible D is optimal.

Following Raviv (1979), we can show that, at fixed insurance premium, a contract with full insurance of small losses and an upper limit for larger damages is always stochastically dominated by pure coinsurance when insurance is costly (namely when $\lambda > 0$). The intuition is that the risk averse insured prefers a transfer of indemnities of small damages to higher ones when insurance is costly. In the same spirit, a deductible contract dominates a pure coinsurance contract in the sense of the second order stochastic dominance (Gollier and Schlesinger (1996)). Hence, the optimal contract displays a strictly

[20] We have $H^*_{CC} = u''(w_f^C) + \mu(1+\lambda)^2 v''(W_f^C) < 0$. The second order conditions are satisfied and the result holds.

positive deductible as long as the marginal cost of insurance λ is positive.

Now, we have to define the optimal marginal indemnities beyond the deductible level. This is done first on $\left]D, \widehat{X}\right[$ and second on $\left]\widehat{X}, L\right[$. By differentiating Equality (14) w.r.t. X on $\left]D, \widehat{X}\right[$ and using it to define μ we must have, for any loss partially covered on $\left]D, \widehat{X}\right[$:

$$(-\alpha_i + \alpha_i.C^{*\prime}(\alpha_i X) - g_X).u''(w_f^C) + (1+\lambda)^2.\alpha_i.C^{*\prime}(\alpha_i X).\mu.v''(W_f^C) = 0$$

$$\Leftrightarrow C^{*\prime}(\alpha_i X) = \frac{(1+\frac{g_X}{\alpha_i}).u''(w_f^C)}{u''(w_f) + (1+\lambda).\frac{v''(W_f^C).u'(w_f^C)}{v'(W_f^C)}}$$

$$\Leftrightarrow C^{*\prime}(\alpha_i X) = \frac{(1+\frac{g_X}{\alpha_i}).R_u}{R_u + (1+\lambda).R_v}$$

With $R_u = -u''(w_f^C)/u'(w_f^C)$ and $R_v = -v''(W_f^C)/v'(W_f^C)$. Equation (6) of Point i) is demonstrated. Equation (7) in Point i) is obtained thanks to an identical reasoning, but with X in $\left]\widehat{X}, L\right[$ and $\widehat{w_f^C} = w_i - \alpha_i\widehat{X} + C(\alpha_i X) - Q - g(X, \widetilde{x}_i)$.

If the insurer is risk neutral we have R_v equal to zero and $C^{*\prime}(\alpha_i X) = 1 + \frac{g_X}{\alpha_i}$ for losses less than $\widehat{X}$ and $C^{*\prime}(\alpha_i X) = \frac{g_X}{\alpha_i}$ for losses higher than $\widehat{X}$. Since all terms are positive, we have $C^{*\prime}(\alpha_i X) > 1$ for any X in $\left]0, \widehat{X}\right[$ such that $0 < C^*(\alpha_i X) < \alpha_i X$: The deductible disappears progressively. For losses in $\left]\widehat{X}, L\right[$ $C^{*\prime}$ is still positive but it decreases as X increases because g_{XX} is negative. Consequently, from a level of damage $\overline{X}$ larger than $\widehat{X}$, marginal indemnities are close to zero and the compensation function displays a kind of upper limit. This is Point (ii).

Now, if the insurer is risk averse and asks for a large risk premium , which means that δ is large, the value of $C^{*\prime}(\alpha_i X)$ may be less than one so that coinsurance for any partially indemnified loss on $\left]D, \widehat{X}\right[$ is optimal. Indeed, from the Arrow-Pratt theorem, we know that the higher the risk premium the higher the coefficient of absolute risk aversion R_v. Other things being equal, an increase in R_v increases the denominator of Point $C^{*\prime}(\alpha_i X)$, and the marginal indemnity can be less than one for a sufficiently high ratio R_v. Point (iii) of Proposition 2 is demonstrated.

Proof of Proposition 3

A differentiation of (8) w.r.t. e_i and C yields:

$$\begin{aligned}\frac{de_i}{dC} &= \frac{1}{-R^C_{e_ie_i}}.\left[-\int_0^L (g_{e_i}+Q_{e_i}).(1-Q_C).u''(w^C_f(X,\widehat{X}))f(X/e^C)dX\right. \\ &\quad -\int_0^L Q_{e_iC}.u'(w^C_f(X,\widehat{X}))f(X/e^C)dX \\ &\quad +\alpha_i\int_0^{\widehat{X}}(1+\frac{g_X}{\alpha_i}-C'(\alpha_iX))(1-Q_C).u''(w^C_f)F_{e_i}(X/e^C)dX \\ &\quad \left.+\alpha_i\int_{\widehat{X}}^L(\frac{g_X}{\alpha_i}-\widehat{C}'(\alpha_iX))(1-Q_C).u''(\widehat{w^C_f})F_{e_i}(X/e^C)dX\right] \qquad (15)\end{aligned}$$

Marginal compensations $C'(\alpha_iX)$ are always lower than or equal to $1+\frac{g_X}{\alpha_i}$ at optimum (see Equation (6)), while $\widehat{C}'(\alpha_iX)$ is always lower than or equal to $\frac{g_X}{\alpha_i}$ (see Equation (7)). The premium Q is equal to $\int_0^L(1+\delta)C(X,\widehat{X})f(X/e^C)dX$, with $C(X,\widehat{X})\equiv C(.)$ on $\left[0,\widehat{X}\right]$ and $C(X,\widehat{X})\equiv\widehat{C}(.)$ on $\left]\widehat{X},L\right]$. Consequently, $Q_{e_i}=\int_0^L(1+\delta)C(X,\widehat{X})f_{e_i}(X/e^C)dX = -\alpha_i\int_0^L(1+\delta)C_X(X,\widehat{X})F_{e_i}(X/e^C)dX$, which is negative. We also have that $Q_C=(1+\delta)$ and Q_{e_iC} equals zero. Equation (15) becomes:

$$\begin{aligned}\frac{de_i}{dC} &= \frac{\delta}{-R^C_{e_ie_i}}.\left[\int_0^L (g_{e_i}+Q_{e_i}).u''(w^C_f(X,\widehat{X}))f(X/e^C)dX\right. \\ &\quad -\alpha_i\int_0^{\widehat{X}}(1+\frac{g_X}{\alpha_i}-C'(\alpha_iX)).u''(w^C_f)F_{e_i}(X/e^C)dX \\ &\quad \left.-\alpha_i\int_{\widehat{X}}^L(\frac{g_X}{\alpha_i}-\widehat{C}'(\alpha_iX)).u''(\widehat{w^C_f})F_{e_i}(X/e^C)dX\right]\end{aligned}$$

The second order conditions of this problem are satisfied (the computation is similar to the one presented in the proof of Proposition 1), so that $R^C_{e_ie_i}$ is negative. Finally, $\frac{de_i}{dC}$ is positive and Proposition 3 is demonstrated.

Proof of Proposition 4

The control variable is $I(\alpha_i X)$ and the state variable is $z(X) = \int\limits_0^X v(W + Q^\beta - (1 + \lambda)I(\alpha_i t))f(t/e)dt$.The simplified Hamiltonian of Program (9) is

$$H^{\beta*} = u(w_f^1).\mathbf{1}_{\{X \leq \overline{X}\}} + u(w_f^2).\mathbf{1}_{\{X > \overline{X}\}} - e_i + \gamma(X)v(W_f^\beta), \tag{16}$$

with $\gamma(X)$ the Lagrange function, $w_f^1 = w_i - \alpha_i X - Q^\beta + I(\alpha_i X) - g(X, \widetilde{x}_i) - \pi(\beta, e_i)$, $w_f^2 = w_i - \alpha_i X - Q^\beta + \overline{I} + \beta\left[\alpha_i X - D^\beta - \overline{I}\right] - g(X, \widetilde{x}_i) - \pi(\beta, e_i)$ and $W_f^\beta = W + Q^\beta - (1 + \lambda)I(\alpha_i X)$. The level D^β is the optimal deductible in this model for a given β. Still here, the Lagrange function γ is a constant ($H_z^{\beta*} = 0$). We have for any X in $\left]0, \overline{X}\right[$ such that $0 < I(\alpha_i X) < X$:

$$H_I^{\beta*} = 0$$

$$\Leftrightarrow u'(w_f^1) - \gamma(1 + \lambda)v'(W_f^\beta) = 0 \tag{17}$$

Thanks to a proof similar to that proposed for Proposition 2, we can first show that the optimal level D^β is positive as long as insurance is costly ($\lambda > 0$).

Second, by differentiating Equality (17) w.r.t. X and using it to define γ we must have, for any X in $\left]D^\beta, \overline{X}\right[$ such that $0 < I(\alpha_i X) < X$,

$$(-\alpha_i + \alpha_i.I^{*\prime}(\alpha_i X) - g_X).u''(w_f^1) + (1 + \lambda)^2.\alpha_i.I^{*\prime}(\alpha_i X).\gamma.v''(W_f^\beta) = 0,$$

$$\Leftrightarrow I^{*\prime}(\alpha_i X) = \frac{(1 + \frac{g_X}{\alpha_i}).u''(w_f^1)}{u''(w_f^1) + (1 + \lambda).\frac{v''(W_f^\beta).u'(w_f^1)}{v'(W_f^\beta)}}$$

$$\Leftrightarrow I^{*\prime}(\alpha_i X) = \frac{(1 + \frac{g_X}{\alpha_i}).R_u(w_f^1)}{R_u(w_f^1) + (1 + \lambda).R_v(W_f^\beta)} \tag{18}$$

With $R_u = -u''(w_f^1)/u'(w_f^1)$ and $R_v = -v''(W_f^\beta)/v'(W_f^\beta)$. Point i) is demonstrated. For point ii), we know that δ is decreasing in β and that $\delta(0) = \lambda$. The marginal indemnities $I^{*\prime}$ and $C^{*\prime}$ (given by (6)) differ from the term $\delta(\beta)$ that appears in the insurance premium and which reflects adminsitrative costs λ plus a risk premium. By definition, we have that $\delta(0) > \delta(\beta)$. From the Arrow-Pratt theorem, we deduce that the ratio of absolute risk aversion decreases as β increases, other thinks being equal. Hence $I^{*\prime}$ is always higher than $C^{*\prime}$ when β is positive.

Concerning the deductible level, a total differentiation of (17) evaluated at $X = D^{\beta}$ leads to:

$$\frac{dD^{\beta}}{d\beta} = \frac{Q_{\delta}^{\beta}.\delta'(\beta).\left(u''(w_f^{\beta}) + \gamma(1+\lambda).v''\left(W_f^{\beta}\right)\right) + \pi_{\beta}.u''(w_f^{\beta})}{-H_{ID^{\beta}}^{\beta *}}$$

with $H_{ID^{\beta}}^{\beta *}$ the derivative of (17) evaluated at $X = D^{\beta}$ with respect to D^{β}. Since D^{β} is the optimal level of deductible and because the second order conditions are satisfied, we have that $H_{I}^{\beta *}$ is positive for any icnrease in the level of deductible and is negative for any decrease in the deductible. Thus we have that $H_{ID^{\beta}}^{\beta *} > 0$ and $\frac{dD^{\beta}}{d\beta}$ is negative if and only if

$$Q_{\delta}^{\beta}.\delta'(\beta).\left(u''(w_f^{\beta}) + \gamma(1+\lambda).v''\left(W_f^{\beta}\right)\right) < -\pi_{\beta}.u''(w_f^{\beta}).$$

Proposition 4 is demonstrated.

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