Storing wind for a rainy day:

What kind of electricity does Denmark export?

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Abstract

On windy days, Denmark tends to export electricity to its neighbours, and to import power on calm days. Storing electricity in this way thus allows the country to deal with the intermittency of wind generation. We show that this kind of behaviour is theoretically optimal when a region with wind and thermal generation can trade with one based on hydro power. However, annual trends in Denmark's trade follow its output of thermal generation, Nordic production of hydro power, and the amount of water available to Scandinavian generators, not wind generation. We estimate the cost of volatility in Denmark's wind output to equal between 4% and 9% of its market value.

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1. Introduction

The last decade has seen a remarkable increase in the number of wind installations throughout the world, as part of a coordinated effort to shift towards a higher share of renewable generation. In Europe, Denmark is amongst the leading countries in wind generation, in terms of installed capacity relative to population and demand (Cossent et al, 2009; Gøransson and Johnsson, 2008; Eriksen et al., 2005). It is therefore not surprising that the country has often been used as a case study to investigate the consequences and challenges of high rates of wind generation, from both a technical and economic perspective.

The Danish electricity system is divided between two separate zones, East and West, which are synchronized to the NORDEL (Nordic) and UCTE (Western European) grid systems, respectively. Although East Denmark is the more densely populated zone of the two, it is West Denmark that exhibits the highest rates of wind generation (21% as opposed to 8% for East Denmark, Østergaard 2008). The two zones are independent from each other, in the sense that they cannot communicate for the purposes of power balancing or wind integration. In 2003, towards the middle of our period, Western Denmark had approximately 2400 MW wind power installed as opposed to Eastern Denmark with a more modest 1100MW, which are expected to rise to 3860 MW and 1850 MW, respectively, by the year 2020 (Lund and Münster, 2003).

The management of the highly intermittent wind-generated output in a country with practically no domestic capacity for hydro storage, like Denmark, is of great interest to the UK and other regions that are committed to increase their use of renewable generation significantly, and likely to obtain much of the increase from wind power. Sharman (2005) argues that the Danish experience is unique and very hard to replicate, as it relies heavily on the availability of "privileged" interconnections of West Denmark's to Norway and Sweden, both of which equipped with hydro-generation capacities, and therefore able to adjust their output very rapidly to changing flows of wind output. Moreover, a high correlation coefficient between wind output and net power outflows may suggest that a significant part of the wind-generated output is exported. Bach (2009) calculates that 77% of the power produced in East Denmark in 2006 was exported, while CEPOS (2009) gives an eight-year average for West Denmark of 57%. For CEPOS, the Danish consumers who are subsidising wind power are not seeing the benefit of the low-carbon electricity it produces. An alternative view, presented by Mignard et al (2007), is that this claim is "unwarranted", because there is a stronger correlation between the production of electricity from (so-called) local Combined Heat and Power plants and exports from West Denmark than there is between wind production and exports.

Our aim is to examine Denmark's electricity production and trade, on different timescales, in the context of an economic model that shows how wind generation should be co-optimised with hydro and thermal plants. In our setting, the wind and thermal plants are in one country (i.e. Denmark), and the hydro stations in another (i.e. Norway). We will show in the model that the average level of thermal generation and of exports from our wind-thermal country is inversely correlated with the amount of hydro generation available, and that this result is reflected in monthly data between 1997 and 2007. We will also show that short-term fluctuations in the level of wind generation are highly correlated with exports from the wind-thermal country, and that this is also reflected in hourly data from January 2000 to May 2007.

The rest of the paper is organised as follows: Section two summarises the related literature. Section three develops a theoretical model to explain the trade flows between a wind- and hydro generating region. The model extends Førsund's (2007) generalised bathtub framework, making easier to account for the impact of varying wind output on the trade flows between the two regions of interest. Section four tests the theoretical implications of the model for the volumes of production and trade, using data for Denmark and its neighbours. The fifth section assesses the cost of fluctuations in wind output, in terms of differences between average prices and those actually received for wind output and Danish exports. Finally, section six concludes.

2. Previous Work

Most of the studies on the integration of wind-generated output with other forms of generation appear in the engineering literature and focus on the technical challenges entailed in the integration and storage of large amounts of highly variable and intermittent wind output. In the economics literature, the majority of the studies on wind generation analyse the possible impact of renewables on prices for a particular country or region. For Denmark, Østergaard (2008) models the impact of the geographic dispersion of wind generation on the variability of wind output and, consequently, the need for operational reserve capacity. His findings suggest an explicit negative correlation between the two, implying that local fluctuations in wind output can be partly offset by spreading wind turbines over a sufficiently large area[1].

The benefits of interconnection in the integration of wind output are also discussed in Oswald et al (2008) for the UK, who, however, find that UK wind speeds can be highly and positively correlated with these in neighbouring countries during times of low output. As a result, wind output can be low during periods of high demand, implying again the need for sufficient available reserve capacity.

Rosen et al (2007) present a model analysis of the short- and long-term effects of high levels of wind generation on output composition for Germany. Their results suggest that wind output acts mainly as a substitute for intermediate- and base-load plants (such as coal, lignite and nuclear plants), whereas gas-fired plants are used as stand-by capacities to balance fluctuations in wind output. Similar findings are reached in Weigt and Hirschhausen (2009), who use hourly wind generation data for Germany over the period 2006 to 2008, to assess the potential of wind energy as a substitute of installed conventional generation capacities. They also discuss the impact of wind generation on market prices, finding it to lead to significant reductions, especially during peak hours.

3. The Model

The model we use is an extended version of the multi-period "bathtub" framework presented in Førsund (2007) and Førsund et al (2008), adjusted to allow for the

[1] However, the argument cannot be readily generalised to reduce the maximum reserve capacity needed in the area, as Østergaard acknowledges the presence of times with zero wind production in all interconnected areas.

inclusion of thermal, wind and hydro generation over several time periods, indexed by $t = 1\ldots T$. In its simplest form, the original model provides a stylised way to analyse the effect of competing hydro and thermal generators on prices and demand patterns for each type of plant[2]. The quantities of generating output for each type of plant (and, the resulting price of electricity) can be obtained as solutions to a constrained optimisation problem that maximises a social welfare function subject to a series of capacity constraints.

We assume that generation takes place within two regions, A and B. The two regions are connected to each other with an interconnector which, for simplicity, we assume has unlimited capacity (so that trade will equalise their prices). Region B is equipped with hydro plants of total fixed capacity $\bar{g}^h$. Region A produces thermal and wind-generated output, with total fixed capacities of $\bar{g}^{Th}$ and $\bar{g}^w$, respectively. The thermal generators have a collective variable cost function given by $\phi(.)$. Hydro and wind generators have no variable costs. While the wind capacity is fixed, its ability to generate electricity depends on the strength of the wind, represented by a stochastic variable $\omega_t \in [0,1]$. All generators behave competitively.

There is demand in each region, $D_t^A\left(p_t^A\right)$ and $D_t^B\left(p_t^B\right)$, which varies between periods and is sensitive to the wholesale price in that region in that period. We can invert the demand functions to obtain the marginal value of power, given by $p_t^A\left(D_t^A\right)$ and $p_t^B\left(D_t^B\right)$. Free trade between the two regions will eliminate any arbitrage

[2] In a more recent article, Førsund et al (2008) extend the original model to include wind and hydro generation (but not thermal).

opportunities and therefore in equilibrium there will be only one price p_t^* across the two regions.

We assume that the thermal and hydro producers act as price takers and choose how to allocate their output between the periods in such a way as to maximize their total profits subject to their capacity constraints. From a social planner's point of view, therefore, the optimisation problem at hand takes the form:

$$\max_{g_t^{Th}, g_t^h, g_t^w, D_t^A, D_t^B} \sum_{t=1}^{T} \int_0^{D_t^A} p_t^A(q)\,dq + \int_0^{D_t^B} p_t^B(q)\,dq - \phi\left(g_t^{Th}\right) \qquad (1)$$

subject to

$$g_t = g_t^h + g_t^{Th} + g_t^w \geq D_t^A + D_t^B \qquad (1.1)$$

$$g_t^h \leq \overline{g}^h \qquad (1.2)$$

$$g_t^{Th} \leq \overline{g}^{Th} \qquad (1.3)$$

$$g_t^w \leq \omega_t \overline{g}^w \qquad (1.4)$$

$$\sum_{t=1}^{T} g_t^h \leq W \qquad (1.5)$$

The first two integrals in the objective function represent the value of electricity consumption in the two regions, and the variable cost of thermal output is subtracted from this.

The energy balance constraint (1.1) is written as an inequality, but in equilibrium, as in Førsund (2007), generators will not spill output. Constraints (1.2)-(1.4) reflect binding capacity constraints faced by each producer – the per-period output from hydro and thermal stations must be less than their capacity, while the output from the wind generator must be less than its capacity multiplied by the strength of the wind in that period, ω_t. The last equation (1.5) is the water balance constraint facing the hydro producer. It must allocate a fixed amount of water, *W*, across the periods. We assume that the amount of water available is known at the start of the optimisation, and that there are no other reservoir constraints.[3] If reservoir and transmission constraints do not bind, then storing water can eliminate arbitrage opportunities and impose a common price across regions and time periods.

The objective Lagrange function for maximisation takes then the form:

$$\begin{aligned}\max_{g_t^{Th},g_t^h,g_t^w,D_t^A,D_t^B} L = \sum_{t=1}^{T}\int_0^{D_t^A} p_t^A(q)\,dq + \int_0^{D_t^B} p_t^B(q)\,dq - \phi\left(g_t^{Th}\right) \qquad (2)\\ -\lambda_t\left(D_t^A + D_t^B - g_t^h - g_t^{Th} - g_t^w\right) - \theta_t^{Th}\left(\bar{g}^{Th} - g_t^{Th}\right) + \theta_t^h\left(\bar{g}^h - g_t^h\right) + \theta_t^w\left(\omega_t\bar{g}^w - g_t^w\right)\\ + \mu\left(W - \sum_{t=1}^{T} g_t^h\right)\end{aligned}$$

The top line is the welfare function from equation (1), the second line gives the per-period constraints (energy balance, and generation from each type of plant below its effective capacity), and the third line gives the inter-temporal constraint on hydro generation. The first order conditions include:

$$\frac{\partial L}{\partial D_t^A} = p_t^A(D_t^A) - \lambda_t \le 0 \qquad (2.1)$$

$$\frac{\partial L}{\partial D_t^B} = p_t^B(D_t^B) - \lambda_t \le 0 \qquad (2.2)$$

$$\frac{\partial L}{\partial g^{Th}} = \lambda_t - \phi'\left(g_t^{Th}\right) - \theta_t^{Th} \le 0 \qquad (2.3)$$

$$\frac{\partial L}{\partial g_t^h} = \lambda_t - \theta_t^h - \mu \le 0 \qquad (2.4)$$

[3] For simplicity we assume that reservoir constraints are not binding. A more generalised discussion of the impact of reservoir constraints on the optimization set up can be found in Førsund (2007).

$$\frac{\partial L}{\partial \mu} = W - \sum_{t=1}^{T} g_t^w \geq 0 \quad (2.5)$$

$$\frac{\partial L}{\partial \theta_t^w} = \omega_t \overline{g}^w - g_t^w \leq 0 \quad (2.6)$$

Equations 2.1 and 2.2 ensure that, in the absence of transmission constraints, the prices in the two regions will be equal within each period. The prices must equal system lambda, the marginal cost of providing power. The next two equations show how that marginal cost relates to the opportunity costs of different kinds of capacity. Equation 2.3 requires system lambda to equal the opportunity cost of thermal plant in that period, given by the derivative of its variable cost plus the shadow value of the capacity constraint on thermal plant in that period – if the latter is binding. Equation 2.4 comes from the hydro plant, which has an opportunity cost given by the shadow value of its capacity constraint in that period and the shadow value of water – which does not vary across periods. In the absence of binding capacity constraints, this means that system lambda will be the same in each period, equal to the shadow value of water, and that this will apply in both regions.[4]

Equation 2.5 reproduces the water balance constraint, which will hold with equality. Similarly, equation 2.6 will hold with equality, ensuring that the potential wind output is all used in each period, unless it exceeded the level of demand at a price of zero!

[4] It would be straightforward to add a constraint, and shadow value, for the capacity of the interconnector between the regions. The two regional prices would diverge whenever the flow needed to equalise them exceeded the interconnector capacity. We chose to keep the model as simple as possible, while still covering the key issues relevant to this paper.

The shadow value of water is the most important variable in determining the level of output. In the absence of relevant binding constraints, this value will equal system lambda, and hence the marginal cost of thermal generation and the marginal value of demand, in each period. If more water was available, its shadow value would fall, and the lower system lambda would show both that demand would increase and that the marginal cost of thermal output would decrease, because less was required, crowded out by the hydro generation.

$$\sum_{t=1}^{T}\left(D_t^A(\mu)+D_t^B(\mu)-\omega_t\overline{g}_t^w-g_t^{Th}(\mu)\right)-W=0 \tag{3}$$

Equation (3) adds up all the energy balance constraints and substitutes for wind and hydro generation, assuming no other constraints are binding. If so, we can differentiate it totally to obtain:

$$\frac{\partial\mu}{\partial W}=\frac{-1}{\sum_{t=1}^{T}\left(\frac{1}{\phi''\left(g_t^{Th}\right)}-\frac{\partial D_t^A}{\partial\mu}-\frac{\partial D_t^B}{\partial\mu}\right)} \tag{4}$$

where the first term in the denominator is the inverse of the slope of the marginal cost function for thermal generation, thus giving the increase in output obtained for an incremental rise in price. A change in wind output (in any period) will also affect the shadow value of water, with an almost identical formula:

$$\frac{\partial\mu}{\partial\omega_t\overline{g}^w}=\frac{-1}{\sum_{t=1}^{T}\left(\frac{1}{\phi''\left(g_t^{Th}\right)}-\frac{\partial D_t^A}{\partial\mu}-\frac{\partial D_t^B}{\partial\mu}\right)} \tag{5}$$

We can calculate the net exports from our thermal-wind country (country A), adding up the relevant items:

$$X_t^A = \omega_t \overline{g}_t^w + g_t^{Th}(\mu) - D_t^A(\mu) \tag{6}$$

If the amount of water available in the hydro system (country B) changes, this will lead (in the absence of other binding constraints) to an equal change in country A's net exports in each period:

$$\frac{\partial X_t^A}{\partial W} = \frac{-\left(\dfrac{1}{\phi''\left(g_t^{Th}\right)} - \dfrac{\partial D_t^A}{\partial \mu}\right)}{\sum_{t=1}^{T}\left(\dfrac{1}{\phi''\left(g_t^{Th}\right)} - \dfrac{\partial D_t^A}{\partial \mu} - \dfrac{\partial D_t^B}{\partial \mu}\right)} \tag{7}$$

In the simplified case in which country B's demand for power is completely insensitive to price, a one unit increase in the amount of water available in country B will reduce country A's exports by $1/T$ units in each period. The most efficient way of using the water is to distribute it evenly over the periods, and if none of it is to be used inside country B (demand is not dependent on the price), then all can be used to allow a higher demand or a lower level of thermal generation in the other country. If the amount of wind available in country A changes in period *t*, then this will affect its net exports differently in that period and in any other period, denoted *s*:

$$\frac{\partial X_t^A}{\partial \omega_t \overline{g}^w} = 1 - \frac{\dfrac{1}{\phi''\left(g_t^{Th}\right)} - \dfrac{\partial D_t^A}{\partial \mu}}{\sum_{t=1}^{T}\left(\dfrac{1}{\phi''\left(g_t^{Th}\right)} - \dfrac{\partial D_t^A}{\partial \mu} - \dfrac{\partial D_t^B}{\partial \mu}\right)} \tag{8}$$

$$\frac{\partial X_s^A}{\partial \omega_t \overline{g}^w} = \frac{-\left(\frac{1}{\phi''\left(g_t^{Th}\right)} - \frac{\partial D_t^A}{\partial \mu}\right)}{\sum_{t=1}^{T}\left(\frac{1}{\phi''\left(g_t^{Th}\right)} - \frac{\partial D_t^A}{\partial \mu} - \frac{\partial D_t^B}{\partial \mu}\right)} \tag{9}$$

On the same simplifying assumption as before (demand in country B is completely price-insensitive), a one unit increase in the amount of wind available in country A will lead to an increase in its exports of $1 - 1/T$ in that period, and a reduction of $1/T$ in every other period. In other words, most of the additional wind output is exported, substituting for hydro production in country B, but this then increases the amount of water available for use in other periods, reducing its shadow value and allowing hydro generation to substitute for thermal generation in country A. With a fixed amount of water and a fixed demand for power in country B, all of the exported power would eventually be re-imported. We illustrate some of these results graphically in the appendix.

Note that if the amount of wind generation capacity increases, as opposed to the amount of output within any one period, this will lead to an expected reduction in the output from the thermal plants in every period, and hence a reduction in the shadow price of water.

It can therefore be seen that, in such a setting, "exporting" (in the sense of "storing") wind output to a reservoir to "import it back" when demand is peaking, becomes a defendable and, in fact, a socially optimal strategy. The next section looks into the trade patterns between Denmark and Norway and investigates to what extent the flows of output between the two countries could be justified in the context of the model that has been outlined in this section.

4. Patterns of production and trade

In this section we assess the theoretical implications of the previously discussed bathtub model, using data to analyse the trade flows of thermal-, hydro- and wind-generated output between Denmark and its neighbours over the period 2000 to 2007. Our dataset compiles information from Energinet.dk (www.energinet.dk), for hourly observations on the production, and consumption for East and West Denmark, as well as information on the trade flows of each of the two zones with their trading partners (Norway, Sweden and Germany); and from Nordel, for monthly data on Nordic reservoir levels and output. The information on Danish production distinguishes between three types of output, reflecting the class of the generating plant: wind, local CHP plants (which tended to follow their heat demand rather than participating actively in the electricity market) and primary plants, also usually CHP, but larger, and active participants in the wholesale market. Hourly Net Exports are, therefore, readily attainable as the difference between total output and consumption.

Figure 1 goes about here

We start with the big picture, trends in production and exports on an annual scale. Our hypothesis is that thermal production will tend to be inversely correlated with the amount of water available to hydro generators. Figure 1 gives twelve-month moving totals for thermal and wind production in Denmark (left hand scale) and hydro inflows and production in Norway, Sweden and Finland (right hand scale), derived from Nordel's annual statistical yearbooks.[5] At the top of the figure, the hydro inflows are clearly volatile, even after taking 12-month totals. The output levels are smoother, but also closely follow the inflows, with a bit of a lag (the industry cannot take advantage of a wet year until after it has occurred).

In the middle of the diagram, Denmark's production of thermal electricity follows the inverse pattern to Nordic hydro generation – when the latter is high, Denmark generates relatively less power. Note that the scales of the two series are different, and an equal distance on the page implies two and a half times as much hydro generation as thermal: the change in Denmark's generation is only part of the response to hydro conditions further north.

At the bottom of the diagram, the 12-month average of Denmark's net exports clearly follows the same pattern as the country's thermal generation. When thermal generation is high, so are exports. There is no apparent correlation between the level of exports and of wind generation, however. Wind output has been gradually increasing throughout the period. Exports and thermal output have cycled around a trend which appears essentially flat since 2000.

These visual impressions are confirmed by analysis of the correlations between the variables, shown in table 1. The top panel gives correlation coefficients between annual values for the whole sample (1996 to 2007), while the bottom panel drops the first four years, in which wind generation was very low (the industry was in its infancy) but exports were (for this period) unusually high.

[5] The Nordel yearbooks give the reservoir levels in each country at regular intervals through the year, together with monthly hydro production, from which it is straightforward to derive the inflows.

Table 1: Correlation coefficients between Danish power and Nordic hydro variables

full period	Water inflows	Hydro output	Danish exports	Danish thermal	Danish wind
Water inflows	1	0.829	-0.826	-0.780	0.318
Hydro output		1	-0.874	-0.792	0.250
Danish exports			1	0.977	-0.529
Danish thermal				1	-0.692
Danish wind					1
2000-2007	Water inflows	Hydro output	Danish exports	Danish thermal	Danish wind
Water inflows	1	0.783	-0.781	-0.800	0.089
Hydro output		1	-0.888	-0.807	-0.259
Danish exports			1	0.973	0.113
Danish thermal				1	-0.101
Danish wind					1

Source: authors' calculations from Nordel data, using annual totals

It is clear that water inflows and hydro generation are highly correlated, and that there is a strong negative correlation between these variables (especially the latter) and Denmark's exports. There is a very high correlation between Denmark's thermal generation and its exports, and basically no correlation between wind generation and any of the other variables. In the latter part of the sample, a year with above average wind output will see slightly higher exports and slightly lower thermal generation, but the correlation is very weak.

Moving to an intermediate scale, figure 2 shows the average pattern of production and trade on a monthly scale, relative to a centred 12-month moving average. Denmark clearly produces more electricity from both wind and thermal stations in the winter months, when demand is also higher. The country also exports more power in the winter, on average. This would appear to go against one of the predictions of our model, which is that the thermal country should export in the off-peak periods. The model assumes that the marginal cost curve is constant, however, which is not the case in Denmark. Specifically, the high demand for heat in the winter months effectively shifts the thermal marginal cost curve down and to the right. The need to produce heat means that Denmark also produces power, even though relatively more of this is exported than in the summer months with lower demand.

Figure 2 to go about here

Moving to the most detailed scale, figure 3 shows the average hourly pattern of Denmark's output and consumption over the day – demand is clearly higher in the daytime than overnight. The output from wind plants is about 20% higher during the day (8 am to 8 pm) than the night (10 pm to 6 am), as is the output from the large primary stations. The stations with the greatest variation in production are the smaller local stations, which tend to follow their heat loads rather than market signals – their daytime output was 60% above their night-time production. Total production exceeds demand, on average, in most hours of the day, as Denmark was a net exporter during this period. The difference is greater when demand is low, again consistent with our model – the thermal producer is most likely to export off-peak.

Figure 3 to go about here

The analysis so far has been in terms of averages, but the most important factor about wind generation is its volatility. We therefore end this section by calculating the deviations from the normal level of generation (and other variables) by time of day, as measured by a set of hour-specific 28-day centred moving averages.[6] When the correlations are performed on this basis, a different picture emerges. Table 2 shows that deviations from the normal level of net exports are strongly and positively correlated with deviations in wind output from its mean value. This in turn suggests that Danish exports are on average higher when there is "excess wind". Although the correlation of net exports with thermal output is positive, it is significantly weaker. Finally, deviations in Danish exports are found to be uncorrelated with deviations in consumption.

Table 2: Correlation coefficients between deviations from normal levels

	Net Exports	Primary Stations	Local CHP	Wind	Consumption
Net Exports	1	0.300	0.248	0.650	0.049
Primary Stations		1	0.490	-0.311	0.622
Local CHP			1	0.022	0.692
Wind				1	0.053
Consumption					1

The deviations in output of primary stations are negatively correlated with those of the wind stations, implying that when high levels of wind production depress market prices, the primary stations respond to this signal, unlike the local stations, which (until recently) did not receive it. There is a strong correlation between deviations in consumption and in the production from thermal stations, reflecting the way that the weather can drive both electricity and heat demand, and that this affects the stations'

[6] That is, we take the hour 1 wind output for 14 January, and subtract the average of the hour 1 wind outputs for 1 January to 28 January, and so on.

production choices. Figures 4 to 6 present these deviations in graphical form, showing clearly that there is a stronger relationship between the deviation in wind output and net exports than for the two types of thermal power.

Figures 4 to 6 about here.

5. The cost of storing wind abroad

The theoretical model and data analysis presented in the earlier sections make clear that there is (and should be) a negative correlation between the level of thermal output in Denmark and the amount of water available in Norwegian reservoirs. In the short term, there is also a clear relationship between the amount of wind generation and the net exports of electricity from Denmark. Effectively, Denmark is storing some of its wind in its neighbours' power systems. Does it have to pay for the privilege? When the wind is high, power prices will be relatively low (for the time of day and time of year), whereas they will tend to be relatively high when there is little wind. Twomey and Neuhoff (2009) discuss some of the implications of this within a single electricity market, in that wind generators will tend to receive less than the time-weighted average price of power, adjusted for their seasonal output patterns.[7] Denmark could suffer from a similar effect with the wind power that it exports to its neighbours. In this section we use data on the value, volume and prices of power flows between the two Danish markets (East and West) and its neighbouring trade partners (Norway, Sweden and Germany) to recover estimates of this cost of "storing wind".

[7] Green and Vasilakos (2009) simulate this effect for the case of Great Britain in 2020, when wind generation might make up around a quarter of generation, finding that onshore generators might receive 8% less than the time-weighted average price.

First, we report the "market value" of wind power in Denmark,[8] based on Nord Pool prices weighted by the actual amounts of generation, compared to the same prices, weighted by the average amount of generation in each hour in each month. For this series, we calculate the average wind output for the 31 hours just after midnight in January 2001 and weight each of the 31 individual prices by this amount when calculating the weighted average for the period. We then do the same for the 31 hours starting at 1 a.m., and so on. We call this the "smoothed" series. The weighting thus captures variations in the average pattern of wind output over the year and over the day, but excludes day-to-day variation around this pattern. The results are shown in table 3.

Table 3 about here

We report results for West and East Denmark separately, since the former has a higher proportion of wind generation. At first, when wind output is relatively lower, the time-weighted average price is slightly below the smoothed average price for wind generation. This is not surprising, since wind output is higher in the winter months, when we would expect also to see higher prices. In the later years, the average price weighted by the smoothed output series is below the time-weighted average price. In all years, the price weighted by the actual pattern of output is below the smoothed series. The difference captures the effect of the co-variance between the level of wind output and the wholesale price – when wind generation is above average (for the time of day and year) the price is likely to be below average, as noted by Twomey and Neuhoff (2009). The effect is much greater for West Denmark than for East

[8] Note that most Danish wind generators receive a feed-in tariff with pre-set rates, rather than the wholesale market prices we present here.

Denmark, where there is proportionally less wind generation. The cost of intermittency in West Denmark varies between 5% and 15% of the time-weighted average price, while in East Denmark it varies between 1.7% and 7%. Over the eight year period, the average cost in West Denmark is 9% of the actual price, and that in East Denmark is 4.4%. These costs are noticeable but not unbearable.

We take advantage of this difference between the two halves of the country to estimate the cost of storing wind power in Denmark's neighbours. We cannot simply compare the prices received for exporting and importing power, not least because Denmark normally receives a higher price for (winter) exports than it pays for (summer) imports. However, we can make a comparison based on the actual revenues received for exports (and sums paid for imports), relative to the sum that might have been received, had the terms of trade not been worsened by the correlation between wind output and market prices. Assume, for instance, that the unweighted average price at 11 a.m. for the hours in which East Denmark exports one month is DKK200/MWh. Since we would expect that exports would be higher in hours when the neighbouring countries had above-average prices, we might find that the export-weighted average price for this hour in this month would be DKK 210/MWh, that is 5% higher. Assume that the absolute level of prices in West Denmark is slightly different, in which case it might have an unweighted average price at 11 a.m. of DKK 220/MWh. If the region is able to trade on similar terms to East Denmark, then the actual export-weighted price would also be 5% higher than the unweighted average price, or some DKK 231/MWh. If, in fact, the actual average price of exports is only DKK 227/MWh, then we take this to mean that a negative correlation between wind generation and market prices has depressed the actual revenues received by West

Denmark by some DKK 4/MWh. If the country exported 1500 MWh during these hours, then the implied cost of storage would be DKK 6,000.

In its general form, therefore, we define the cost of storage as the loss of revenues caused by deviations of the hourly average price from its smoothed (monthly averaged) value for imports and exports: $C_{TOTAL} = C_I + C_X$, where $C_I = Y_I \times (p^I{}_h - p^I{}_m)$ and $Y_I, p^I{}_h, p^I{}_m$ are used to denote net flows, hourly and monthly price averages for imports, respectively (and similarly for exports). The estimates of import (export) prices are derived as the ratio of the actual import (export) values and trade flows, obtained using actual hourly Elspot prices and data on physical exchanges, both available from our dataset. We separate the international flows into net exports from Denmark and transits between the three neighbouring countries, using Bialek's (1996) tracing algorithm. This splits outflows from a node (here, a region of Denmark) in proportion to the inflows. If West Denmark is importing 300 MW from Germany, for example, and exporting 800 MW to Norway and 400 MW to Sweden, we assume that there is a transit of 200 MW from Germany to Norway, and a transit of 100 MW from Germany to Sweden. Denmark's net exports would consist of 600 MW to Norway and 300 MW to Sweden. We calculate the cost of storage based on these net exports (and net imports) alone.

Table 4 about here

Table 4 presents our estimates of the cost of storage via trade. The first columns, [2]-[4] present information on total wind output and net trade flows for the two Danish regions, while column [5] gives the average price received by wind generators in each year (dividing total revenue by total wind output). Column [6] gives the actual prices received for exports from East Denmark, valued at the average of the price in East Denmark and in the receiving region. Where the prices differ, the congestion surplus is divided between the two transmission system operators, and we effectively take a "Denmark plc" approach in combining this income with the price that the generator would receive. Column [7] gives the average price received for exports, but weighted by the smoothed volume series which excludes the impact of day-to-day variations. Note that the actual price received for exports is generally greater than the smoothed price – this implies that exports follow a need for power in the importing country, reflected in its price, rather than a surplus of power in the exporter. Columns [8] and [9] repeat these figures for West Denmark, with a similar pattern. Columns [10] to [13] present the average prices for imports, weighted by actual and smoothed flows. In this case, the actual prices are sometimes below the smoothed prices, implying that Denmark is taking advantage of cheap power available abroad, but in other years, the actual prices are greater, implying that Denmark needs to import when the market is tight.

Columns [14] and [15] give the actual values for exports and imports in millions of DKK, while column [16] gives our estimate of the cost of storage via trade. To reiterate, this is based on comparing the actual and smoothed prices for East Denmark and then calculating the revenues that West Denmark would have obtained, had it been able to trade on the same terms, relative to its smoothed price, as East Denmark did. This assumes that East Denmark's revenues are not reduced by its own needs to store power, and is thus an under-estimate of the true cost of doing so. Our annual estimates of the cost of storage can be aggregated to provide a total estimate of DKK

524 million for the period 2001-2008. This can be compared to the wind output in West Denmark over the same period of 36.9 TWh. The implicit cost of storage, due to the greater volatility of West Denmark's wind output, is thus just over DKK 14/MWh, or 5% of the average price of traded power.[9]

6. Conclusions

There have been suggestions that much of Denmark's output of wind power is exported, with the implication that Danish consumers therefore do not benefit from the low-carbon electricity that it produces. In this paper, we present a theoretical model, adapted from the work of Førsund (2007), in which we show how production by thermal and hydro generators should be optimised to take advantage of wind power. A thermal producer should generate more at off-peak times, when the level of water available to the hydro producers is low, and when wind output is low. The last is a transient effect, given the variability of the wind.

Our analysis of the Danish experience shows that this model works on all scales. First, we have shown that Danish exports of power respond to the hydrological situation in Norway, Sweden and Finland, and that exports are greatest when hydro generation is relatively low, because inflows of water have been low. This year-to-

[9] This ignores the cost due to the volatility of East Denmark's wind output and net exports. We calculated the Neuhoff-Twomey effect for East Denmark to equal 4.4% of the average revenues, compared to 9% in West Denmark. The difference between the two regions is 4.1% of the nationwide average revenues. This is comparable to our estimate of the differential cost of storage through trade. While the actual cost varies from year to year, on average, this implies that it might be appropriate to add around 4% of the average price to give an overall estimate of the cost of storing wind via trade.

year pattern of exports is driven by the amount of power that Denmark's fossil-fuelled thermal power stations produce.

At a daily level, Denmark's net exports of power are higher at off-peak times, showing that the country takes advantage of its neighbours' hydro storage to effectively flatten its load-curve. It exports more overnight, and less during the day-time peak, thereby reducing the fluctuations in its own power output. There is a tendency for wind generation to be higher during the day, when Denmark's net exports are relatively lower.

However, the critics of the Danish experience are correct to say that Denmark relies heavily on its neighbours to absorb short term fluctuations in its wind output. We have shown that the correlation between short-term deviations (from the norm for that time of day and year) in wind power and in net exports are much higher than for thermal generation. When it is windy in Denmark, the country's exports rise. What the critics seem not to appreciate, however, is that much of this power is stored in the countries further north, which reduce their own generation to absorb it, and then bought back at times of relatively low wind output. In other words, Denmark tends to store its wind in the form of water.

This would not be possible without the cooperation of its neighbours, and, indeed, the presence of neighbours with large systems and (to the north) a lot of storage hydro. Furthermore, the Danes do have to pay for this storage, in that the actual prices at which Western Denmark trades are relatively worse, compared to those under average conditions, than for Eastern Denmark, which is less affected by variations in wind output. We estimate the cost of this effect at just over DKK 14/MWh of wind output over the period between 2001 and 2008. This is just under 5% of the average market price received by the wind output. Denmark is therefore able to deal with the problem of intermittency in a reasonably cost-effective way.

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Appendix: A graphical presentation

We can illustrate some of these results in the two-period case, shown in figures A1-A3, adapting Førsund's (2007) presentation.

Figure A1 goes about here

Figure A1 shows what might happen in country A if there was no trade (and no wind), but Country B is included to help comparison with the later figures. Demand and output in the Peak period are measured from the left-hand origin – the thermal marginal cost curve (MC) rises with the quantity produced, whereas the demand is invariant to price, for simplicity, and equal to D_P^A. In the absence of trade or storage, thermal production (g_P^{Th}) has to equal demand, and the price in this period would be given by the intersection of demand and marginal cost, at p^P. For the off-peak period, demand and output are measured from the right-hand origin. The thermal marginal cost curve is identical (albeit a mirror image of the curve for the peak period), whereas demand (also price-invariant) is lower, at D_{OP}^A. Again, thermal production (g_{OP}^{Th}) equals demand, and the intersection between this demand and the marginal cost curve is therefore at the lower level of p^{OP}.

The two demands from country A are separated by a distance equal to the total demand (over the two periods) in country B. Once again, the (price-insensitive) demand in the peak period is measured from left to right, while the off-peak demand is measured from right to left. The vertical line near the centre of the diagram divides it between the total demand in the peak period $\left(D_P^A + D_P^B\right)$ and the total demand in the off-peak period $\left(D_{OP}^{A}+D_{OP}^{B}\right)$; since the peak demand exceeds the off-peak demand, the line is nearer to the right hand side of the figure than the left. The final feature of the diagram is the thick two-headed arrow that shows the total amount of water available in country B across the two periods. We have chosen to make this exactly equal to Country B's consumption, which implies that neither country will be a net exporter, summed over the two periods. This is somewhat artificial (though inevitable in a truly autarkic situation), but means that we can start from a position in which each country exactly meets its own demand in each period from its own production.

In figure A1, we ignore the price of power inside country B – in fact, with a completely price-insensitive demand and a pure hydro system, the price would be undetermined in autarky. We remedy this in figure A2, in which we introduce trade between the two countries. Country A exports power in the off-peak period, and imports the same amount in the peak period. The amount of trade is chosen to equalise the marginal cost of thermal production across the two periods, and so the price in each period, p^*, is identical. This is also the case for the level of thermal output in country A – the peak output, g_{P}^{Th}, is identical to the off-peak output, g_{OP}^{Th}. Compared to figure A1, the double-headed arrow showing the amount of water available has shifted to the left – country B uses more of its water in the peak period, and less in the off-peak. In the peak period, hydro exports from country B can replace relatively expensive thermal output from high on country A's marginal cost curve. In the off-peak period, the water is replaced by relatively cheaper thermal output from low on country A's marginal cost curve.

Figure A2 about here

Figure A3 shows what happens if there is some wind generation in country A. In this case, we assume that the wind blows only in the off-peak period. This gives a further boost to country A's exports in that period, and its off-peak imports. With less thermal generation required overall, the price drops from p^* to p^{**}: the old price is shown as a dotted line above the dashed line of the new price. The total generation from country A in the off-peak period is higher than in figure A2, but it should be clear that the thermal generation is lower, just as it is in the peak period. It should also be straightforward to picture the case in which the wind generation occurs in the peak period. Country A would import less in that period, there would be more water available for the off-peak period in country B, and so country A would export less then.

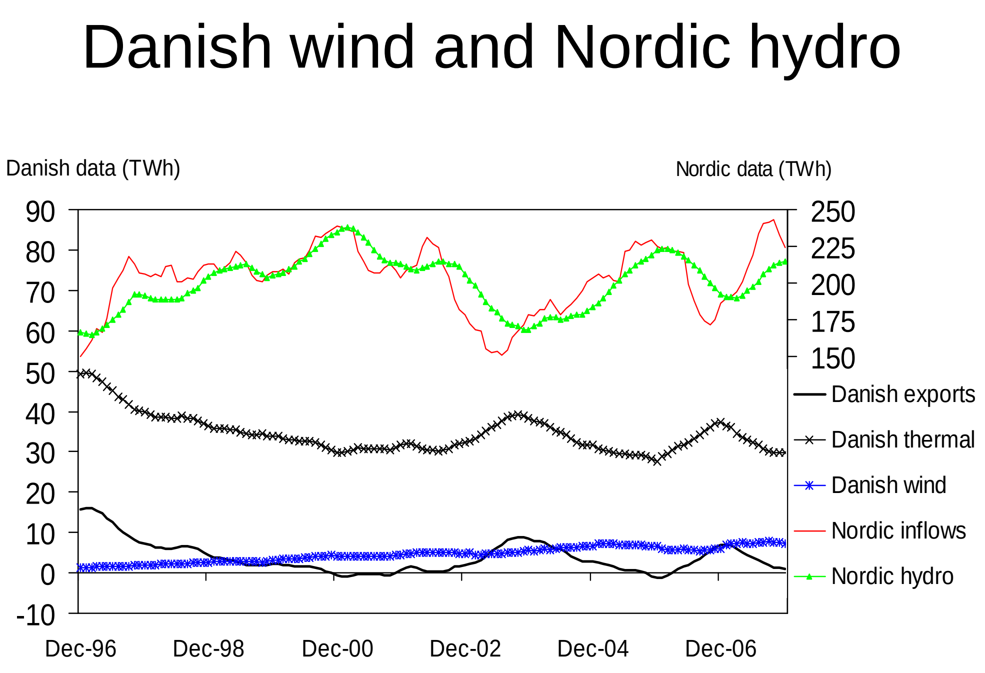

Figure 1

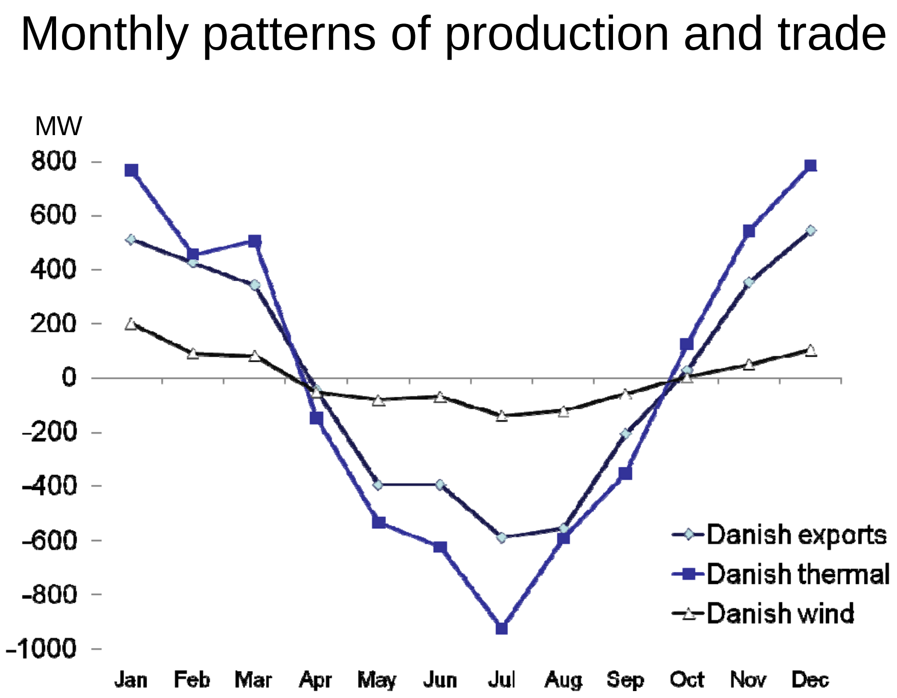

Figure 2

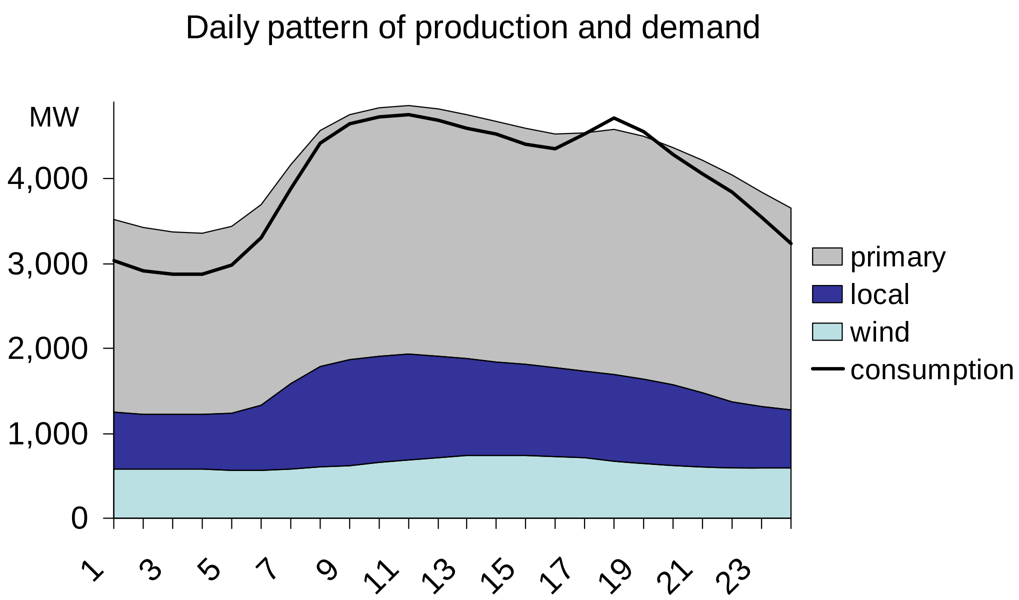

Figure 3

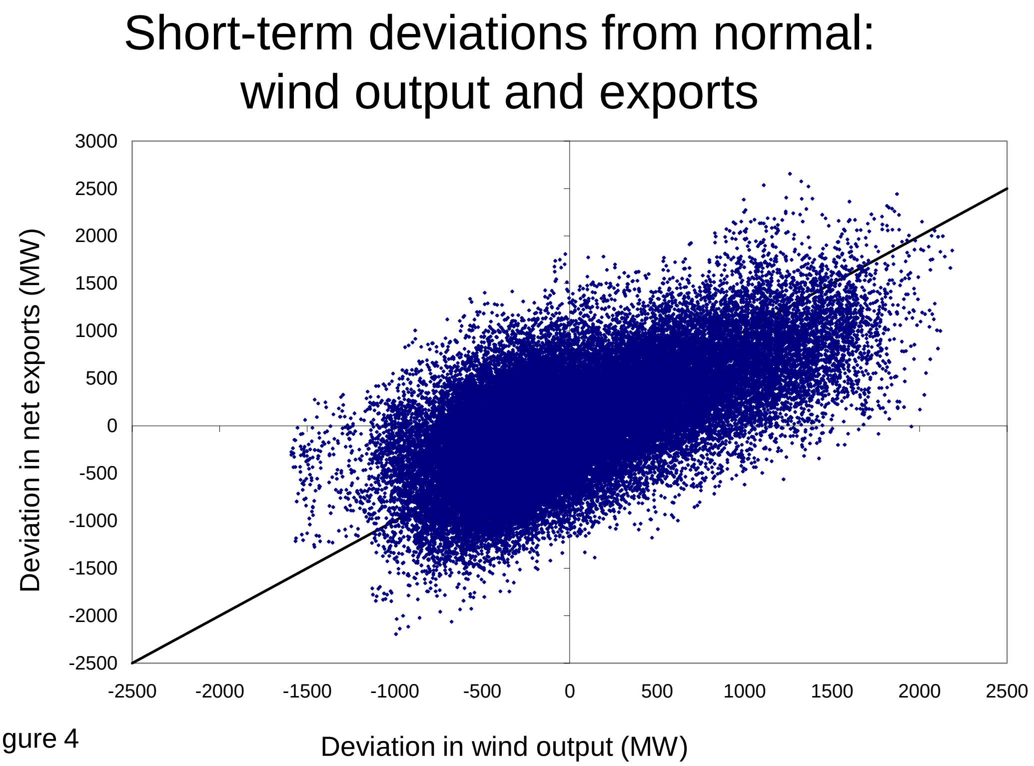

Figure 4

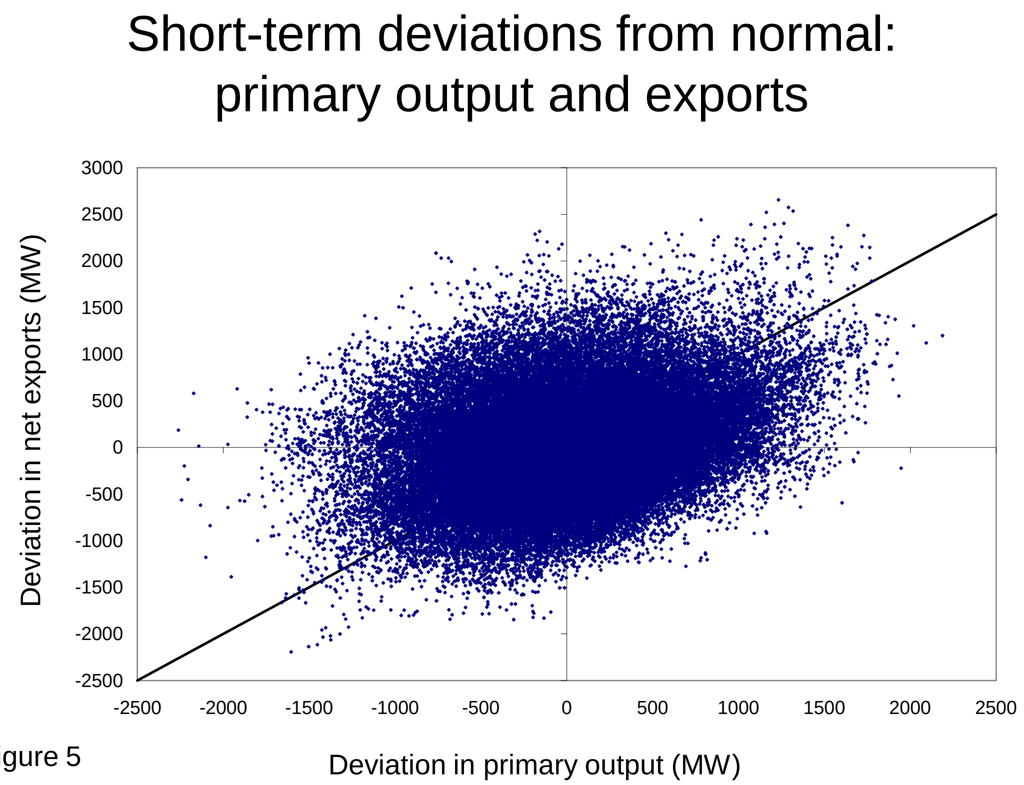

Figure 5

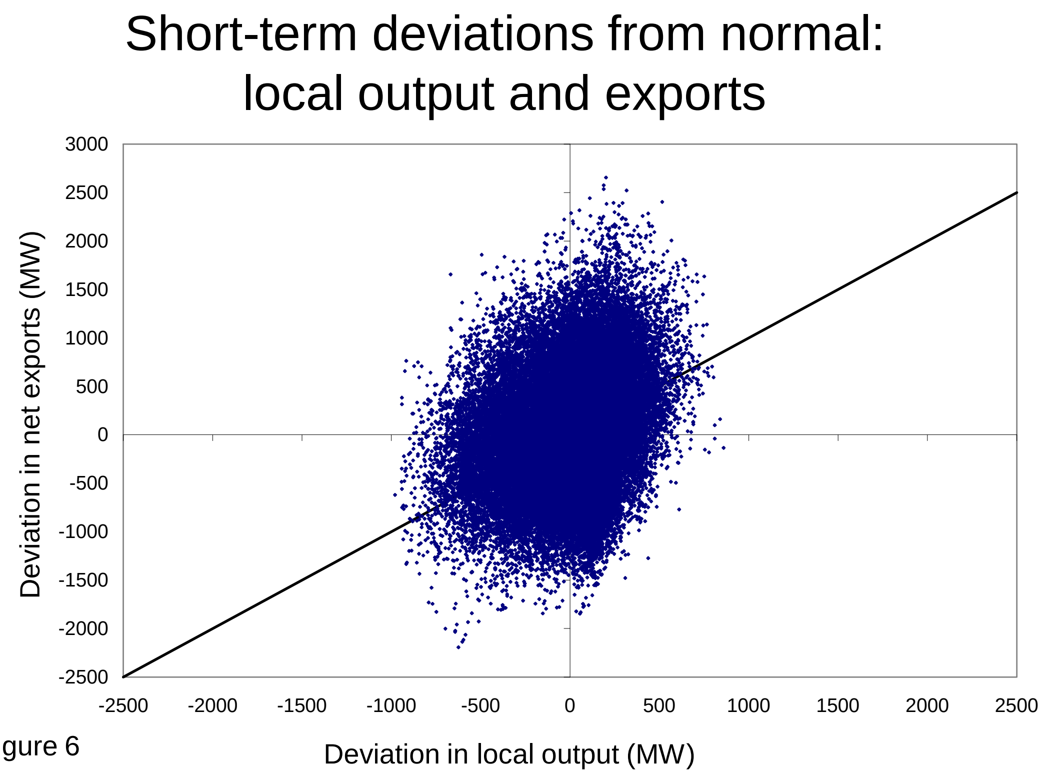

Figure 6

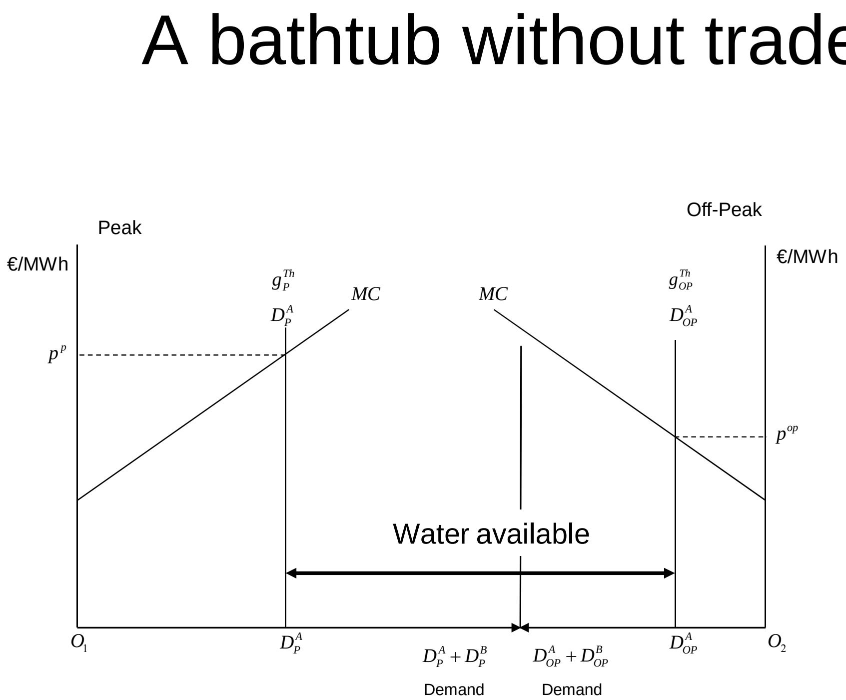

Figure A1

A bathtub with trade

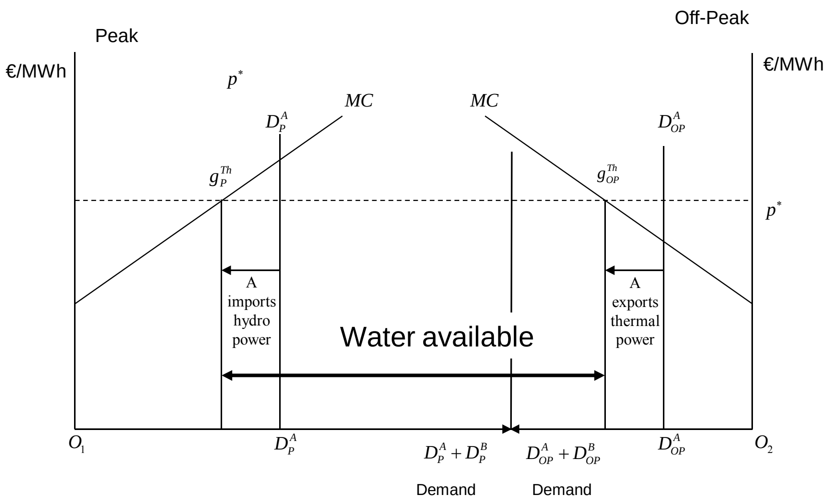

Figure A2

A bathtub with trade and wind

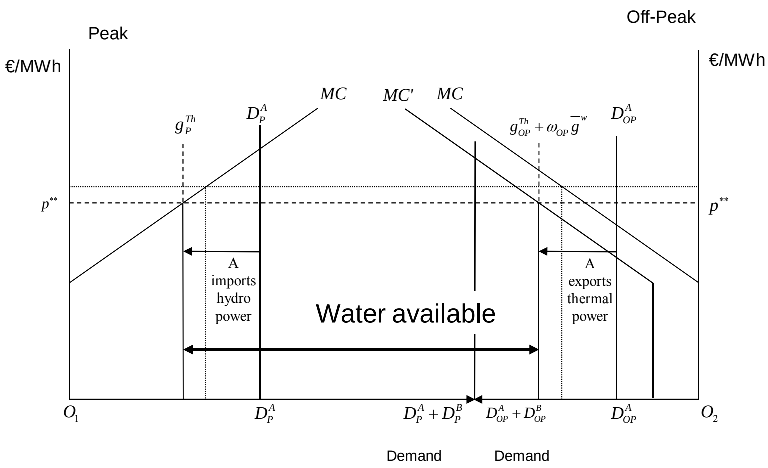

Figure A3

Table 3: Prices for wind generation in Denmark, 2001-2008, DKK/MWh

	West Demark				East Denmark			
year	Time-weighted	Wind: smoothed	Wind: actual	penalty	Time-weighted	Wind: smoothed	Wind: actual	penalty
2001	176.87	178.55	169.34	5.2%	175.44	177.00	173.91	1.7%
2002	189.22	191.49	171.48	10.5%	212.36	214.22	207.35	3.2%
2003	250.27	252.25	214.07	15.1%	273.45	275.41	261.70	5.0%
2004	214.28	213.52	201.86	5.5%	210.94	210.84	206.92	1.9%
2005	277.44	266.89	245.04	8.2%	251.89	250.12	235.85	5.7%
2006	329.54	320.48	302.43	5.6%	361.97	349.44	334.59	4.2%
2007	241.37	238.74	213.49	10.6%	245.94	243.88	226.95	6.9%
2008	420.70	409.31	381.62	6.8%	422.28	411.55	396.02	3.8%

Table 4: Estimated cost of storage via trade for Denmark, 2001-2008

Year	Wind output TWh	Exports TWh	Imports TWh	Wind price DKK	Export Prices - East		Export Prices - West		Import Prices - East		Import Prices – West		Export value mDKK	Import value mDKK	Cost of storage mDKK
					Actual DKK	Smoothed DKK	Actual DKK	Smoothed DKK	Actual DKK	Smoothed DKK	Actual DKK	Smoothed DKK			
[1]	**[2]**	**[3]**	**[4]**	**[5]**	**[6]**	**[7]**	**[8]**	**[9]**	**[10]**	**[11]**	**[12]**	**[13]**	**[14]**	**[15]**	**[16]**
2001	4.32	4.20	3.63	170.31	179.88	166.14	190.49	190.15	167.34	170.33	158.09	154.96	791	595	62
2002	4.88	5.30	3.23	179.20	357.74	359.85	222.28	231.18	152.78	155.65	135.57	133.49	1,380	479	34
2003	5.56	9.14	0.60	224.33	309.28	303.20	272.76	277.63	249.67	232.62	152.67	140.88	2,594	148	67
2004	6.58	5.51	2.65	203.17	204.98	200.21	218.26	216.83	217.45	217.17	217.01	215.20	1,188	576	18
2005	6.61	2.66	4.03	242.82	272.67	247.19	253.56	267.76	229.72	240.97	257.86	258.35	683	971	105
2006	6.10	7.56	0.62	310.28	363.87	359.59	380.11	372.82	370.04	372.68	277.42	259.36	2,830	211	-8
2007	7.17	4.32	3.37	216.51	284.83	259.01	279.92	298.56	206.02	212.00	178.13	172.09	1,209	663	166
2008	6.98	3.26	4.71	385.30	422.90	397.23	423.89	424.74	399.59	412.14	385.75	395.27	1,382	1,860	80
Totals	48.20	41.95	22.84										12,057	5,503	524

Source: Energinet data and authors' calculations