Long-run Cost Functions for Electricity Transmission

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Conference on "The Economics of Energy Markets," IDEI Toulouse, January 28 and 29, 2010

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Introduction

Electricity transmission differs from other transportation and network systems (pipelines, railroads, the road system, telecommunications). The physical laws of electricity make transmission complex and unusual.

No characterization of cost function for transmission grids. Two reasons:

First is that there is no full agreement on an obvious output.

Second is that the effects of Kirchhoff's laws lead to bewildering irregularities in the relationship between outputs and capacities.

Transmission cost functions: strange properties that make them interesting for an audience outside electricity. Where else can you expect to have negative marginal costs?

Knowledge of transmission cost functions help to plan cost-minimizing transmission systems over a wide range of potential outputs.

It could also help to assess investments in renewable energies, and in the implementation of incentive mechanisms for transmission investment.

We study long-run electricity transmission cost functions based upon a definition of transmission output in terms of point-to-point transactions

Build on Hogan, Rosellón and Vogelsang (HRV) (2010) model (which combines the merchant and regulatory approaches): FTR-based cost functions exhibit very normal economic properties in a variety of circumstances if topology is given, and when capacity of lines can be changed.

One purpose of our study is to establish that the problem of non well-behaved, non-continuous transmission cost functions is related to demand changes that lead to a change in network topology (as suggested by HRV, 2010).

We test the behavior of FTR-based cost functions for distinct network topologies. Two basic cases:

First adjust line capacities, but nodes, lines, impedances and thus PTDFs do not change.

Second case allows for changes in line impedances (and PTDFs) correlated to the changes of line capacities.

Characterization of Electricity Transmission Outputs

The definition of the output for electricity transmission is difficult since the physical flow through a meshed transmission network is complex and highly interdependent among transactions (Bushnell and Stoft,1997, and Hogan, 2002a, 2002b).

Under a network with loop flows, outputs could be defined as bilateral trades between pairs of nodes that aggregate to net injections at all nodes. This idea derives from the FTR literature.

An FTR (obligation) q_{ij} represents the right to inject electricity in the amount of q at node i and to take delivery of the same amount at node j. The FTR does not specify the path taken between i and j. It is a flow concept.

Whereas in directed networks like natural gas or oil an additional unit of output can normally be associated with a well-defined cost parameter or function, additional output in electricity networks depends on the grid conditions.

Model, Topologies and Data

The network topology (nodes and lines) is described by the network incidence matrix (Léautier, 2000).

For a given network topology we assume that the line capacity is variable so that it can be changed between 0 and ∞, but at a cost.

Transmission cost function *c(.)*: least costs combination of line capacities *k* necessary to satisfy Q_{ij} (the matrix consisting of a specific set of FTR combinations q_{ij}):

$$c(Q_{ij}) = \min_{k_i} \sum_{i,j} f_{ij}(k_{ij})$$

Minimization is subject to technical restrictions representing the network's power-flow characteristics:

$$\left| pf_{ij} \right| \leq k_{ij} \quad \forall ij$$

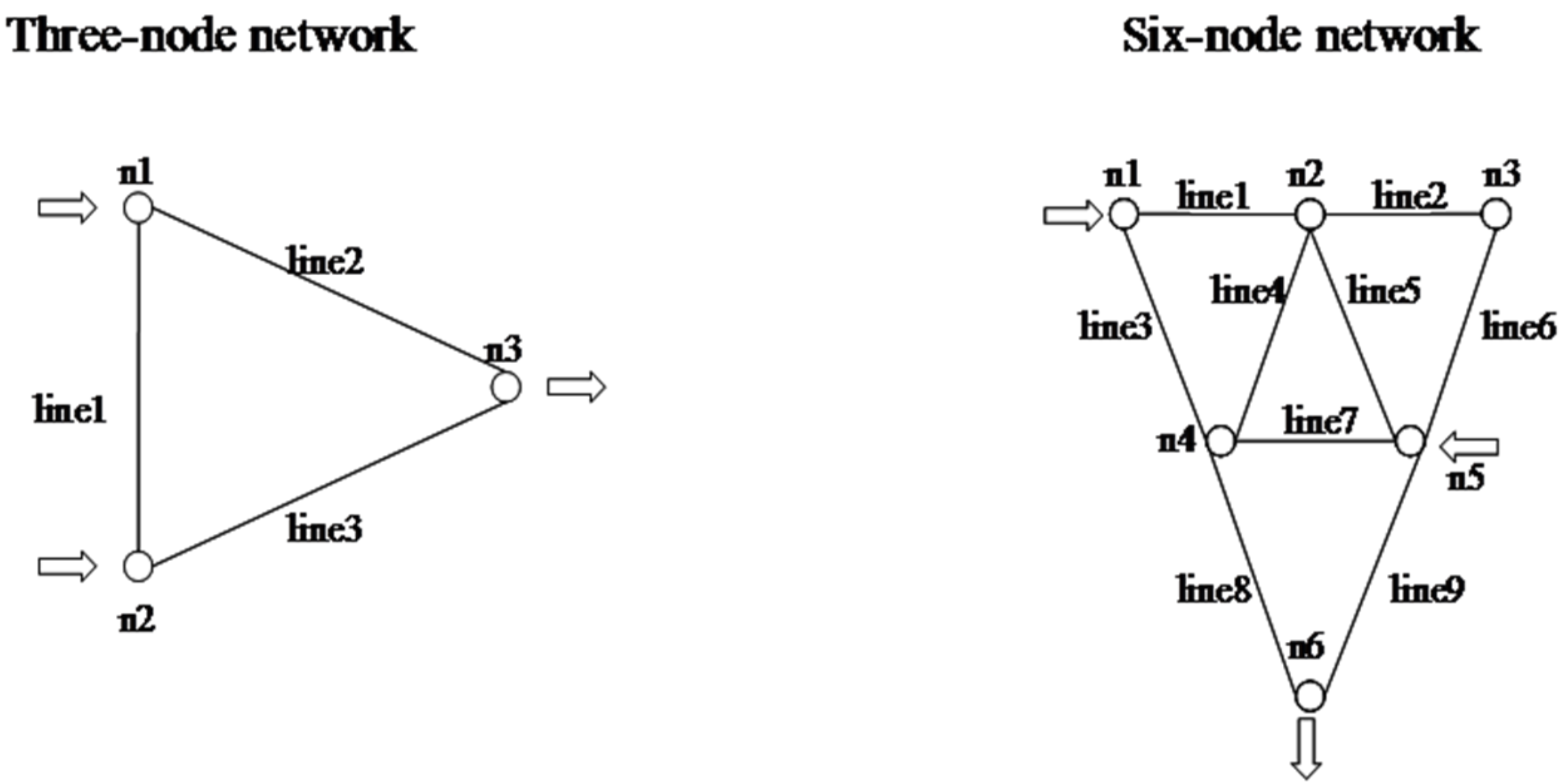
line capacity constraint

$$\sum_j q_{ij} - \sum_j q_{ji} = \sum_j pf_{ij} \quad \forall i$$

energy balance constraint

We incorporate model in General Algebraic Modeling System (GAMS) as a non-linear minimization tool.

Figure 1: Two network topologies



Four forms of line extension costs functions $f_{ij}(k_{ij})$: constant marginal cost, decreasing marginal cost (economies of scale), increasing marginal costs (diseconomies of scale), and lumpy behavior.

Linear function (constant marginal cost): $f_{ij} = b_{ij}\, k_{ij}$

Logarithmic function (economies of scale): $f_{ij} = \ln\left(a_{ij} + b_{ij}\, k_{ij}\right)$

Quadratic function (diseconomies of scale): $f_{ij} = b_{ij}\, k_{ij}^2$

Lumpy function: $f_{ij} = b_{ij}\, k_{ij} \qquad k_{ij} \in Z^+$

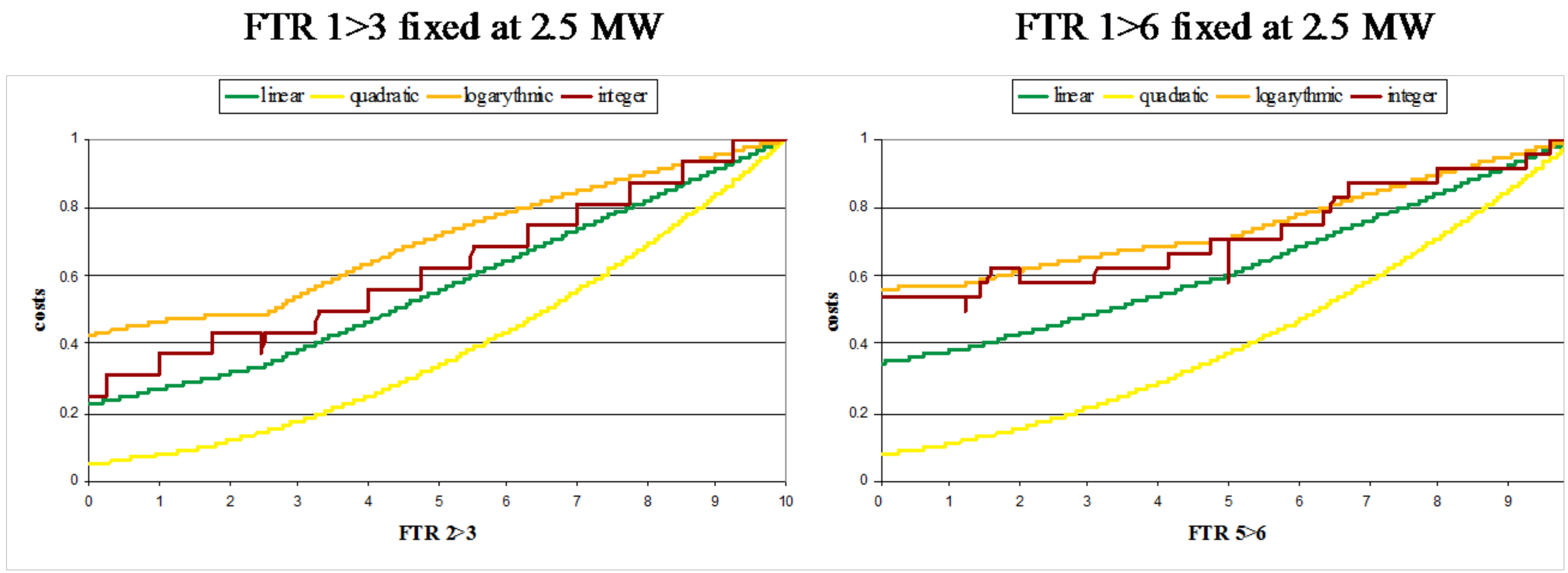
Table 1: Scenario overview for cost function calculation

	Fixed line reactances	Variable line reactances
Starting line reactances	1	
Line extension functional parameters	$a_{ij} = b_{ij} = 1$	$a_{ij} = b_{ij} = 1$
Starting capacity values [MW]	$k_{ij} = 0$	$k_{ij} = 2$
Three node network		
FTR range [MW]	FTR 1 to 3: 1 to 5 FTR 2 to 3: 1 to 10	
Six node network		
FTR range [MW]	FTR 1 to 6: 1 to 5 FTR 5 to 6: 1 to 10	

Scenarios and Results

Fixed line reactances: extending one FTR

Figure 1: Cost function fixing one FTR, three-node and six-node network, fixed reactances

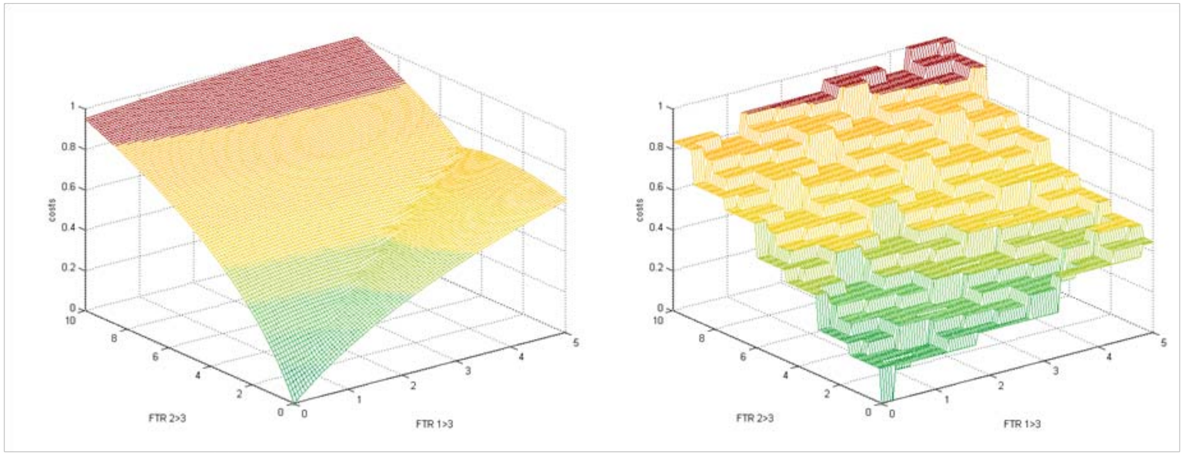


Source: Own calculation

Fixed line reactances: global cost function

Figure 1: Global cost function, three-node network, fixed reactances

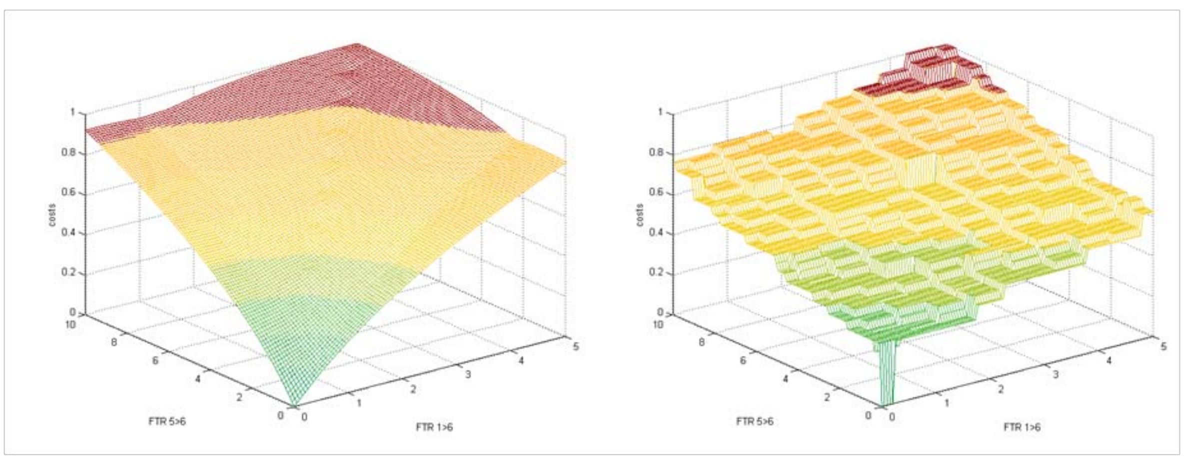
Logarithmic extension costs | Lumpy extension costs



Source: Own calculation

Figure 2: Global cost function, six-node network, fixed reactances

Logarithmic extension costs | Lumpy extension costs



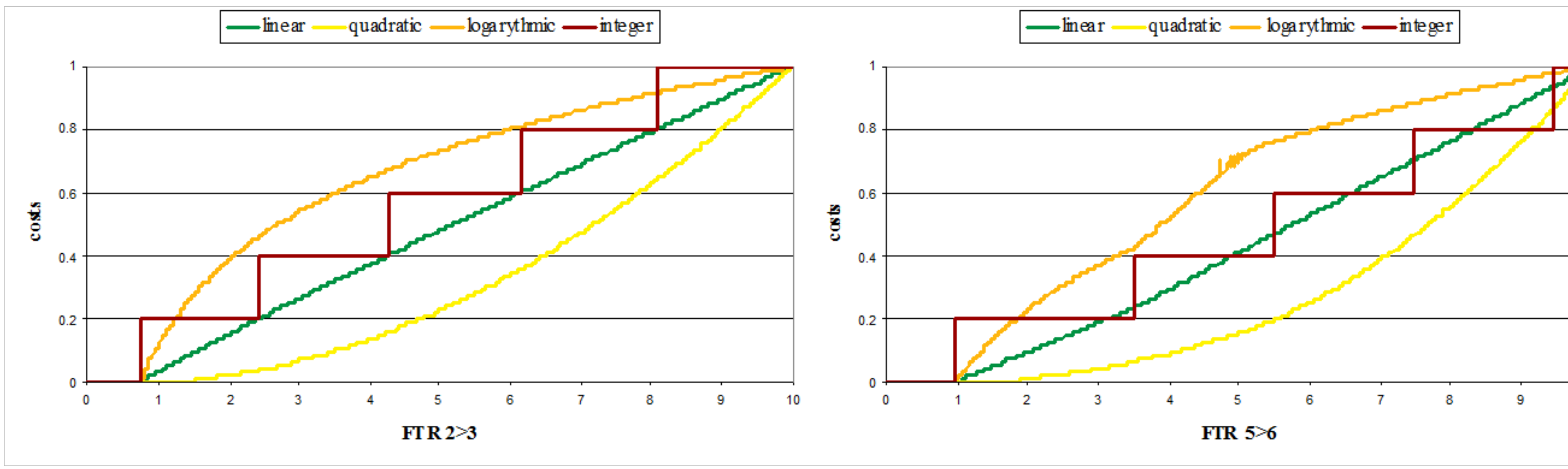
Source: Own calculation

Variable line reactances: extending one FTR

Figure 1: Cost function fixing one FTR, three-node and six-node network, variable reactances

FTR 1>3 fixed at 2.5 MW

FTR 1>6 fixed at 2.5 MW

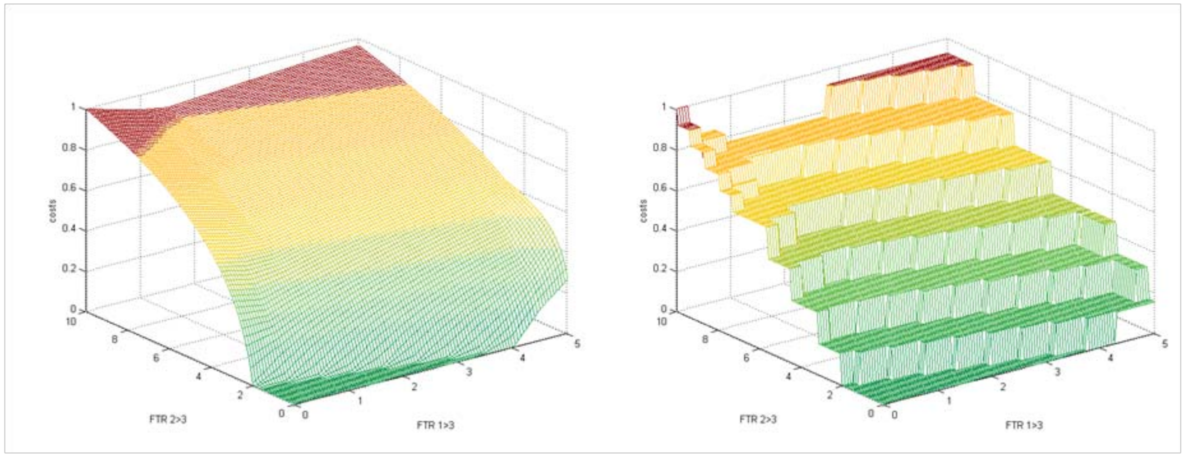


Source: Own calculation

Variable line reactances: global cost function

Figure 1: Global cost function, three-node network, variable reactances

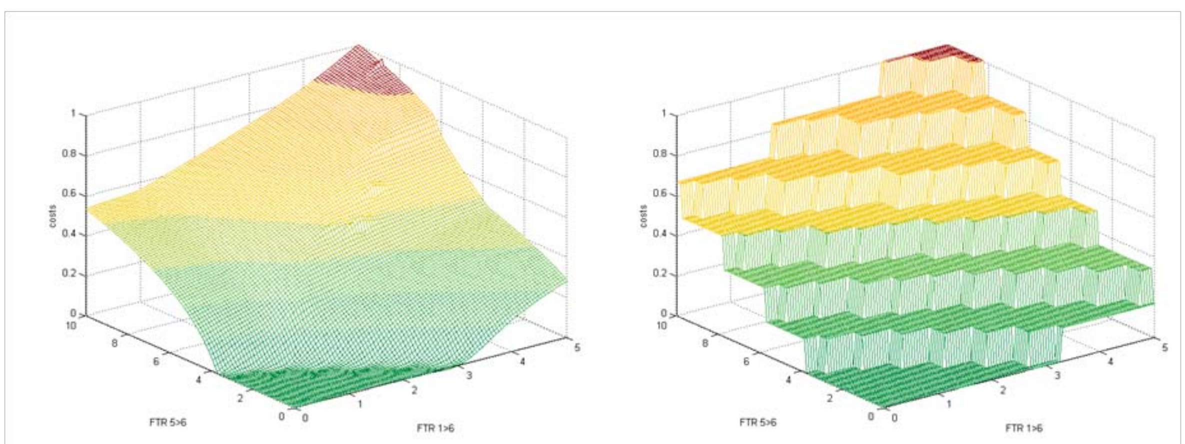
Logarithmic extension costs **Lumpy extension costs**



Source: Own calculation

Figure 2: Global cost function, six-node network, variable reactances

Logarithmic extension costs **Lumpy extension costs**



Source: Own calculation

Table 1: Overview of results

	Fixed Reactance		Variable Reactance	
	1 FTR Fixed	**2 FTRs**	**1 FTR Fixed**	**2 FTRs**
Three nodes	• Resulting capacity cost for increasing an FTR value will show a "kink" at the level of the fixed FTR. • In the lumpy extension case, the kink is represented by a jump in the step function. • In the quadratic case the slope of the line extension close to the origin is almost horizontal, thus the loop-flow kink does not occur.	• The kink of lines moves gradually with increasing FTRs. • The lumpy investment case is a combination of cost reductions based on counter-flows and capacity steps depending on net injections. • The quadratic case shows no signs of kinks.	• No clear correlation between counter-flows and kinks as the canceling-out point of counter-flows changes with the alteration of network characteristics. • The cost function will resemble the extension cost of increasing capacity on the most utilized line causing a continuous cost function.	• The resulting cost function can show a decreasing part at a specific FTR range attributed to the missing of a sufficient counter-flow.
Six nodes	• Lines 2, 5, and 6 will cancel out their flows at the same FTR level due to the network symmetry, and thus three counter-flow kinks are obtained. • The lumpy-cost curve shows the impact of several interacting loop flows: the cost function first increases and then decreases again.	• Resulting global cost function shows three kinks. • The lumpy investment case is highly fragmented due to the interaction of counter-flows and capacity steps.	• By extending the most utilized lines the power-flow share on those lines also increases, and the need to extend other lines is reduced. • Kinks can occur when the extension includes further lines or when it switches the extended line(s). • In the logarithmic case more capacity is added on a line than is actually utilized (due to decreasing marginal extension costs), altering the power-flow pattern so that less capacity is needed on other lines.	• The dominant effect of a single line is canceled out by the increased number of loop-flowed lines in the system. • Within the observation range the cost functions show a continuous behavior with increasing global costs for increased FTR values.

Conclusions regarding the relationship of kinks, negative slopes and loop-flows for the non-lumpy cost functions.

First, smoothness is gained with variable reactances in the three-node case and in the global cost-function case:

> Whereas in the fixed-reactance scenario it was clear that for specific FTR combinations the power-flow on one line will fall to zero and cause kinks in both the 1-FTR and global-cost function cases, this is not valid in the variable-reactance scenario because the canceling-out points can change with the alteration of network characteristics.

Additionally, for the global-cost function case the counter-flow structure may imply decreasing ranges of the cost function.

Lumpy extension case: gains from considering variable reactances will imply an increasing stepwise function.

Results then indicate that taking into account the full characteristics of electricity networks provides a cost framework that is closer to the well-behaved continuous functionality.

Comparative analysis suggests piece-wise continuity of cost functions. This property crucial is for the application of price-cap incentive mechanisms to real-world expansion projects.

Redefinition via a FTR approach only takes into account the overall extension costs, and thus avoids line specific discontinuities.

More realistic scenario when cost minimization occurs over the optimal design of the network (location and number of links and nodes denoted by transfer-admittance matrix *H*)

H becomes a variable, and a more complicated cost minimization problem results. The problem also leads to new goods (FTRs) for new nodes, and new goods change the costs for all of the old goods.

The method would calculate the cost function for the changed network, and then compare it to the cost function of the original network. If the new cost function lies everywhere below (above) the original cost function, the new topology dominates (is dominated by) the original one.

Application of HRV (2010)

Rosellón, J. and H. Weigt (2010), "A Dynamic Incentive Mechanism for Transmission Expansion in Electricity Networks – Theory, Modeling and Application," (winner of the Reimut Jochimsen Prize)

Simplified model of the BENELUX:

- Implementation of the HRV model to meshed electricity networks.

-Covering 15 nodes and 28 lines.

- Including 8 plant types (nuclear, lignite, coal, CCGT, gas/oil, hydro, pump) with fixed marginal costs.

-Initial congestion between Belgium and France, and Germany and the Netherlands.

- Neglecting wind capacities.

- 20 periods, 8% interest rate.

- Only network upgrades possible at linear extension costs of 100 € per km per MW capacity.

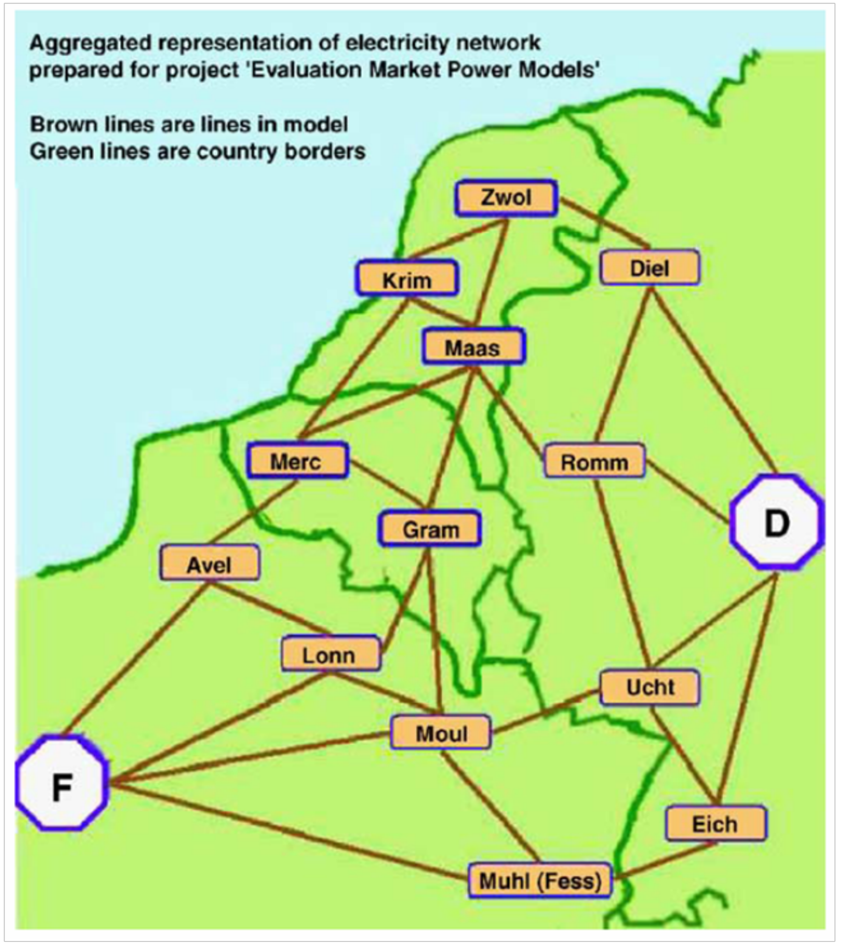
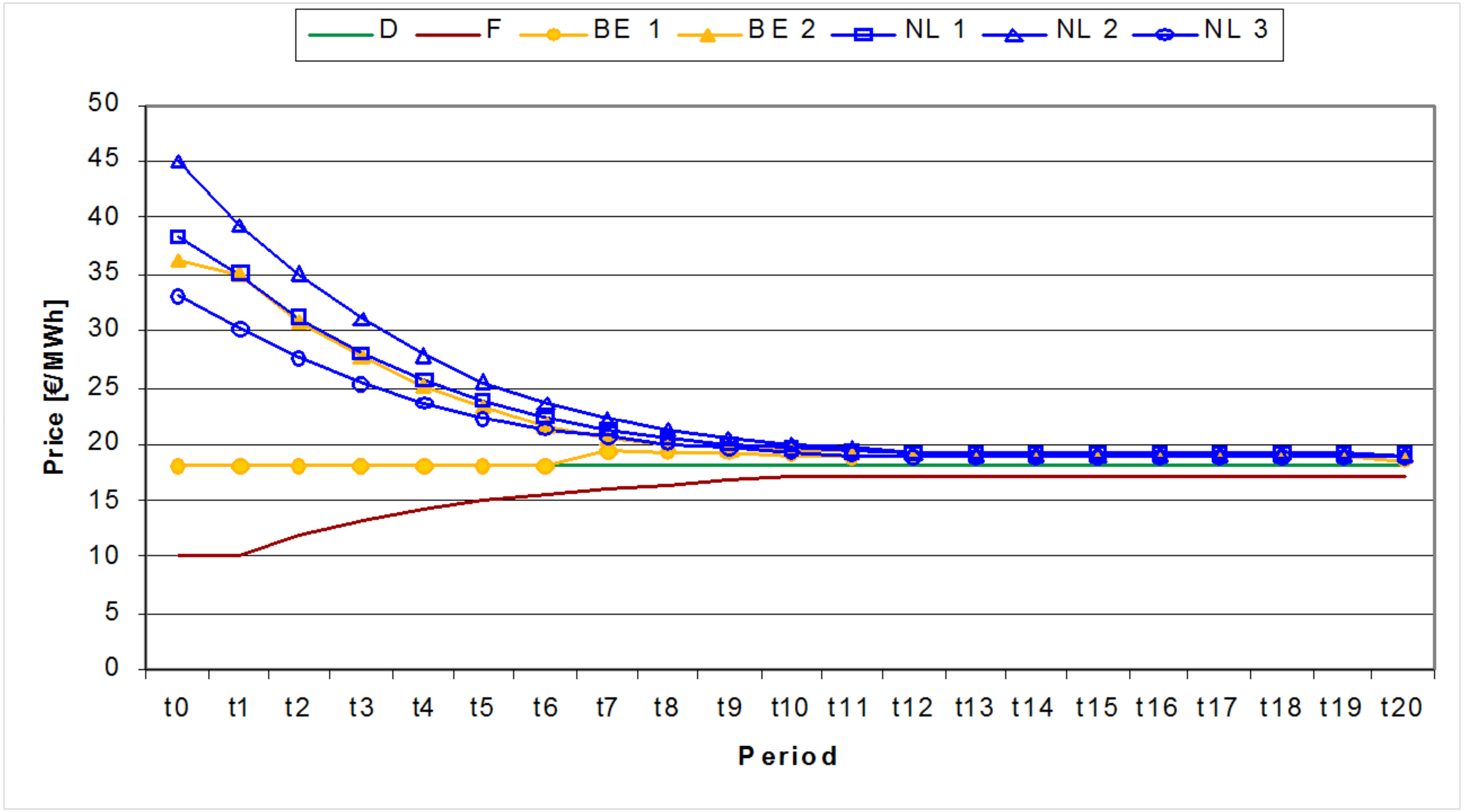


Figure 1: Price development in the European model



Source: based on Rosellon and Weigt (2007). "D" stands for Germany, "F" for France, "BE" for Belgium, and "NL" for the Netherlands.

Total transmission capacity is significantly increased, the Transco´s profits are augmented, and there is convergence towards the welfare optimum.

Table 1: Comparison of regulatory approach with welfare maximization (values refer to the last period)

	No grid extension	Regulatory Approach	Welfare Maximization
Consumer rent [Mio€/h]	10.37	10.31	10.30
Producer rent [Mio€/h]	0.65	0.99	1.02
Congestion rent [T€/h]	107.8	20.20	7.13
Total welfare [Mio€/h]	11.13	11.32	11.33
Total extension sum [Mio€]	-	285.27	305.26
Total grid capacity [GW]	33.4	60.9	62.64
Average price [€/MWh]	28,4	18,5	18,1

Conclusions

Our simulations suggest that FTR-based cost functions remain piecewise continuous – as well as piecewise differentiable – over the entire FTR range.

Regions with negative marginal costs could cause concern. They seem to shrink as one moves to more realistic (multi-node) networks and include changes in reactances.

The introduction of a link between capacity and reactance appears to reduce the impact of loop-flows in terms of significant kinks. Smoothness of non-lumpy cost functions is gained with variable reactances.

For modeling purposes, the logarithmic and lumpy behaviors produce high degrees of nonlinearities with non-smoothness, and require further calculations and solver capabilities, the quadratic functions show a generally continuous behavior, and the linear extension functions fall somewhere between.

Most suitable for modeling, therefore, is using the linear functions with the piecewise, linear nature of the resulting global costs function, which let us derive global optima.

The ultimate research challenge is to identify the changes in network topology: number of configurations increases significantly with the number of nodes.

We could further combine our analytical approach with actual (engineering) data to estimate actual cost functions in ways pioneered for telecommunications networks (Gasmi et al., 2002).

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