Marcel Boiteux meets Fred Schweppe: nodal peak load pricing

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Abstract

Two different models are used in the academic literature on electric power markets: (*i*) peak-load pricing, which determines optimal dispatch, prices, and investment, hence long-term marginal costs, for a single market, i.e., ignoring congestion on the transmission grid, and (*ii*) nodal pricing, which examines interconnected markets, i.e., explicitly models congestion on the transmission grid, and *in its application* often considers only short term marginal costs.

In reality, power markets are interconnected *and* rely on long-term marginal costs for investments. This article therefore examines peak-load pricing for two interconnected markets. To the best of my knowledge, it is the first that does so. First, it shows how the optimal generation mix is modified when one includes congestion on the transmission grid. Second, it computes the marginal value of transmission capacity when long-term prices and investment in generation are taken into account.

Keywords: electric power markets, peak-load pricing, nodal prices

JEL Classification: L11, L94, D61

1 Introduction

Two separate strands of academic literature on electric power markets: (*i*) peak-load pricing, which determines optimal dispatch, prices, and investment, hence long-term marginal costs for a single market, i.e., ignoring congestion on the transmission grid (following Boiteux (1949)'s seminal analysis, see for example Borenstein and Holland (2005), Joskow and Tirole (2006), and Léautier (2012)); and (*ii*) nodal pricing, which examines interconnected markets, i.e., explicitly models congestion on the transmission grid, and *in its application* often considers only short term marginal costs (following Schweppe et al. (1986)'s seminal analysis, see for example Hogan (1992)), or Léautier (2001)).

In reality, power markets are interconnected *and* rely on long-term marginal costs for investments. This discrepancy between theory and reality leaves many questions open. First, how are the main results of peak load pricing models, in particular the optimal generation mix, modified when one includes congestion on the transmission grid?

Second, one particular issue of great practical interest is the value of transmission investments: most analyses simulate the short-term equilibrium of interconnected markets, and value transmission capacity using the expected difference in short-term marginal costs (e.g. Leuthold et al. (2012)). On the other hand, peak-load pricing indicates that, over the long-term, which is the relevant horizon to value long-lived investments such as transmission lines, prices should equal the long-run marginal costs, i.e., also cover capital costs. What is the value of transmission reinforcement when long-term prices are taken into account?

This article therefore examines peak-load pricing for two interconnected markets. To the best of my knowledge, it is the first that does so.

2 The model

Demand State of the world is $t \geq 0$, distributed according to cumulative distribution $F(.)$, and probability distribution $f(.) = F'(.)$. All customers are homogenous, individual demand is $D(p,t)$. Inverse demand is $P(q,t)$, that verifies:

$$P_q = \frac{\partial P}{\partial q} < 0 \text{ and } P_t = \frac{\partial P}{\partial t} > 0$$

The first condition simply requires inverse demand to be downward sloping, the second orders the states of the world without loss of generality.

Two markets, indexed by $n = 1, 2$. $p_n(t)$, $q_n^s(t)$, and $q_n^d(t)$ are respectively the price, production, and demand in market n in state t.

Total mass of customers normalized to 1, a fraction $\theta \in [0,1]$ of customers is located in market 1. Thus

$$q_1^d(t) = \theta D(p_1(t), t) \Leftrightarrow p_1(t) = P\left(\frac{q_1^d(t)}{\theta}, t\right),$$

and

$$q_2^d(t) = (1-\theta) D(p_2(t), t) \Leftrightarrow p_2(t) = P\left(\frac{q_2^d(t)}{1-\theta}, t\right).$$

The above structure implies that demands in both markets are perfectly correlated.

Supply One production technology (c_n, r_n) located in market n. Node 1 is the baseload market: $r_1 > r_2$ and $c_1 < c_2$. Investing and using both technologies is assumed to be economically efficient. Precise sufficient conditions are provided later.

Interconnection An interconnection links both markets. $\varphi(t)$ is the flow from market 1 to market 2 in state t. Φ is the transmission capacity on the interconnection, assumed identical for both directions. The transmission constraints are thus

$$|\varphi(t)| \leq \Phi.$$

$\eta(t)$ is the marginal value of transmission capacity in state t. Nodal pricing analysis (for example, Schweppe et al. (1986), Hogan (1992)) shows that

$$\eta(t) = |p_2(t) - p_1(t)|,$$

thus $\mathbb{E}[\eta(t)]$, the marginal value of transmission capacity, is

$$\mathbb{E}[\eta(t)] = \mathbb{E}[|p_2(t) - p_1(t)|].$$

Critical states of the world and marginal value functions

Definition 1 *1. For any (k,c), $\widehat{t}(k,c)$ is the first state of the world such that $P\left(k, \widehat{t}(k,c)\right) \geq c$, and*

$$\Psi(k,c) = \int_{\widehat{t}(k,c)}^{+\infty} (P(k,t) - c) f(t)\, dt.$$

2. For any (k, c_1, c_2),

$$\begin{aligned}\beta(k, c_1, c_2) &= \int_{\widehat{t}(k,c_1)}^{\widehat{t}(k,c_2)} (P(k,t) - c_1) f(t)\, dt + \int_{\widehat{t}(k,c_2)}^{+\infty} (c_2 - c_1) f(t)\, dt \\ &= \int_{\widehat{t}(k,c_1)}^{\widehat{t}(k,c_2)} \frac{\partial}{\partial t} P(k,t)(1 - F(t))\, dt.\end{aligned}$$

Ψ and β are continuously differentiable and decreasing in their first argument by inspection. Thus, for any (c, r), $\xi(c, r)$ defined by

$$\delta(\xi(c, r), c) = r$$

is unique (and assumed to exist for (c_1, r_1) and (c_2, r_2)).

3 The standard analyses

3.1 Peak load pricing

If the line is never congested, the standard peak-load pricing model applies. As long as the baseload generation is not at capacity, it is the only one producing. Baseload generation produces at capacity for states t such that

$$D(c_1, t) \geq k_1 \Longleftrightarrow P(k_1, t) \geq c_1 \Longleftrightarrow t \geq \widehat{t}(k_1, c_1).$$

Thus, for $t \leq \widehat{t}(k_1, c_1)$,

Generation	$q_1^s(t) = D(c_1, t)$	$q_2^s(t) = 0$
Demand	$q_1^d(t) = \theta D(c_1, t)$	$q_2^d(t) = (1-\theta) D(c_1, t)$
Flow	$\varphi(t) = (1-\theta) D(c_1, t)$	
Prices	$p_1(t) = c_1$	$p_2 = c_1$

.

For $t \geq \widehat{t}(k_1, c_1)$, and until peaking generation starts producing, price is set by the intersection of the (downward sloping) demand curve and the vertical supply curve, $p(t) = P(k_1, t)$. Peaking generation starts producing for states t such that

$$P\left(k_1,t\right) \geq c_2 \Longleftrightarrow t \geq \widehat{t}\left(k_1,c_2\right).$$

Thus, for $\widehat{t}\left(k_1,c_1\right) \leq t \leq \widehat{t}\left(k_1,c_2\right)$,

Generation	$q_1^s\left(t\right)=k_1$	$q_2^s\left(t\right)=0$
Demand	$q_1^d\left(t\right)=\theta k_1$	$q_2^d\left(t\right)=\left(1-\theta\right)k_1$
Flow	$\varphi\left(t\right)=\left(1-\theta\right)k_1$	
Prices	$p_1\left(t\right)=P\left(k_1,t\right)$	$p_2\left(t\right)=P\left(k_1,t\right)$

For $\widehat{t}\left(k_1,c_2\right)$, price is $p\left(t\right)=c_2$, until peaking generation reaches capacity. Thus, for $\widehat{t}\left(k_1,c_2\right) \leq t \leq \widehat{t}\left(k_1+k_2,c_2\right)$,

Generation	$q_1^s\left(t\right)=k_1$	$q_2^s\left(t\right)=\frac{D(c_2,t)}{2}-k_1$
Demand	$q_1^d\left(t\right)=\theta D\left(c_2,t\right)$	$q_2^d\left(t\right)=\left(1-\theta\right)D\left(c_2,t\right)$
Flow	$\varphi\left(t\right)=k_1\frac{-D(c_2,t)}{2}$	
Prices	$p_1\left(t\right)=c_2$	$p_2\left(t\right)=c_2$

.

Finally, when peaking generation is at capacity, price is set by the intersection of downward sloping demand and vertical supply curves. For $t \geq \widehat{t}\left(k_1+k_2,c_2\right)$,

Generation	$q_1^s\left(t\right)=k_1$	$q_2^s\left(t\right)=k_2$
Demand	$q_1^d\left(t\right)=\frac{k_1+k_2}{2}$	$q_2^d\left(t\right)=\frac{k_1+k_2}{2}$
Flow	$\varphi\left(t\right)=\frac{k_1-k_2}{2}$	
Prices	$p_1\left(t\right)=P\left(k_1+k_2,t\right)$	$p_2\left(t\right)=P\left(k_1+k_2,t\right)$

.

Optimal unconstrained total capacity $\left(k_1^U+k_2^U\right)$ is uniquely determined by:

$$\int_{\widehat{t}\left(k_1^U+k_2^U,c_2\right)}^{+\infty}\left(P\left(k_1^U+k_2^U,t\right)-c_2\right)f\left(t\right)dt=\Psi\left(k_1^U+k_2^U,c_2\right)=r_2$$

$\Leftrightarrow$

$$k_1^U + k_2^U = \xi\left(c_2, r_2\right) \tag{1}$$

while optimal unconstrained baseload capacity k_1^U is uniquely determined by:

$$\int_{\widehat{t}\left(k_1^U, c_1\right)}^{+\infty} \left(p\left(t\right) - c_1\right) f\left(t\right) dt = r_1$$

$\Leftrightarrow$

$$\int_{\widehat{t}\left(k_1^U, c_1\right)}^{\widehat{t}\left(k_1^U, c_2\right)} \left(P\left(k_1^U, t\right) - c_1\right) f\left(t\right) dt + \int_{\widehat{t}\left(k_1^U, c_2\right)}^{\widehat{t}\left(k_1^U + k_2^U, c_2\right)} \left(c_2 - c_1\right) f\left(t\right) dt + \int_{\widehat{t}\left(k_1^U + k_2^U, c_2\right)}^{+\infty} \left(P\left(k_1^U + k_2^U, t\right) - c_1\right) f\left(t\right) dt = r_1$$

$\Leftrightarrow$

$$\int_{\widehat{t}\left(k_1^U, c_1\right)}^{\widehat{t}\left(k_1^U, c_2\right)} \left(P\left(k_1^U, t\right) - c_1\right) f\left(t\right) dt + \int_{\widehat{t}\left(k_1^U, c_2\right)}^{+\infty} \left(c_2 - c_1\right) f\left(t\right) dt + \int_{\widehat{t}\left(k_1^U + k_2^U, c_2\right)}^{+\infty} \left(P\left(k_1^U + k_2^U, t\right) - c_2\right) f\left(t\right) dt = r_1$$

$\Leftrightarrow$

$$\beta\left(k_1^U, c_1, c_2\right) = r_1 - r_2. \tag{2}$$

Sofar, existence of k_1^U and k_2^U solutions of equations (1) and (2) has been assumed. Sufficient conditions for existence are:

Assumption 1 *1. If*

$$P\left(0,0\right) > c_2 \text{ and } \mathbb{E}\left[P\left(0,t\right)\right] > c_2 + r_2,$$

then $k_1^U + k_2^U = \xi\left(c_2, r_2\right) > 0$ *exists.*

2. If

$$c_2 + r_2 > c_1 + r_1 \text{ and } \beta\left(k_1^U + k_2^U, c_1, c_2\right) < r_1 - r_2,$$

then there exists $k_1^U \in \left(0, k_1^U + k_2^U\right)$ *solution of equation* (2).

If $P\left(0,0\right) > c_2$, the first unit produced is worth more than its short-run marginal cost in all states of the world, hence should be consumed. It implies that $\widehat{t}\left(0, c_2\right) = 0$. Second, since $\widehat{t}\left(0, c_2\right) = 0$,

$$\Psi\left(0, c_2\right) = \int_0^{+\infty} \left[P\left(0,t\right) - c_2\right] f\left(t\right) dt = \mathbb{E}\left[P\left(0,t\right)\right] - c_2,$$

Thus, $\mathbb{E}[P(0,t)] > c_2 + r_2$ implies that $\Psi(0,c_2) > r_2$. Since $\lim_{k \to +\infty} \Psi(k,c_2) = 0 < r_2$, and $\Psi(.,.)$ is continuous in its first argument, there exists $k^* > 0$ such that $\Psi(k^*,c_2) = r_2$.

Since

$$\beta(0,c_1,c_2) = \int_0^{+\infty} [c_2 - c_1] f(t)\,dt = c_2 - c_1,$$

$c_2 + r_2 > c_1 + r_1$ implies that $\beta(0,c_1,c_2) > r_1 - r_2$. Then, since $\beta(.,.)$ is continuous in its first argument, $\beta\left(k_1^U + k_2^U, c_1, c_2\right) < r_1 - r_2$ is sufficient to ensure existence of $k_1^U \in \left(0, k_1^U + k_2^U\right)$ solution of equation (2).

Assumption 2

$$\xi(c_2,r_2) > \xi(c_1,r_1).$$

(**This assumption may or may not be a consequence of the other assumptions. I have not yet found a proof one way or the other. It leads me to one family of solutions. If the reverse holds, we have another family of solutions, with similar results.)**

Figure 1 plots $\varphi(t)$ **(to be included)**. When only technology 1 is producing, $\varphi(t)$ increases until $(1-\theta)k_1^U$. Then technology 1 reaches capacity, which stabilizes the flows. Then, peaking technology 2 is turned on, and price is set at c_2: demand, hence $\varphi(t)$, decreases. When peaking technology 2 reaches capacity, demand, hence $\varphi(t)$, remains constant.

3.2 Nodal pricing

Until the line is congested, only the base load technology produces and serves both markets. The line becomes congested when

$$(1-\theta)D(c_1,t) = \Phi \Longleftrightarrow D(c_1,t) = \frac{\Phi}{1-\theta} \Longleftrightarrow P\left(\frac{\Phi}{1-\theta},t\right) = c_1 \Longleftrightarrow t = \widehat{t}\left(\frac{\Phi}{1-\theta},c_1\right).$$

Thus, for $t \leq \widehat{t}\left(\frac{\Phi}{1-\theta}, c_1\right)$

Generation	$q_1^s(t) = D(c_1, t)$	$q_2^s(t) = 0$
Demand	$q_1^d(t) = \theta D(c_1, t)$	$q_2^d(t) = (1-\theta) D(c_1, t)$
Flow	$\varphi(t) = (1-\theta) D(c_1, t)$	
Prices	$p_1(t) = c_1$	$p_2(t) = c_1$

.

When the line is congested, the flow is limited to Φ. As long as $p_2(t) < c_2$, technology 2 does not produce. $q_2^d = \Phi = (1-\theta) D(p_2(t), t)$, thus $p_2(t) = P\left(\frac{\Phi}{1-\theta}, t\right)$. For $\widehat{t}\left(\frac{\Phi}{1-\theta}, c_1\right) \leq t \leq \widehat{t}\left(\frac{\Phi}{1-\theta}, c_2\right)$,

Generation	$q_1^s(t) = \theta D(c_1, t) + \Phi$	$q_2^s(t) = 0$
Demand	$q_1^d(t) = \theta D(c_1, t)$	$q_2^d(t) = \Phi$
Flow	$\varphi(t) = \Phi$	
Prices	$p_1(t) = c_1$	$p_2(t) = P\left(\frac{\Phi}{1-\theta}, t\right)$

.

For $t \geq \widehat{t}\left(\frac{\Phi}{1-\theta}, c_2\right)$, peaking technology produces:

Generation	$q_1^s(t) = \theta D(c_1, t) + \Phi$	$q_2^s(t) = (1-\theta) D(c_2, t) - \Phi$
Demand	$q_1^d(t) = \theta D(c_1, t)$	$q_2^d(t) = (1-\theta) D(c_2, t)$
Flow	$\varphi(t) = \Phi$	
Prices	$p_1(t) = c_1$	$p_2(t) = c_2$

The expected value of the difference in prices is

$$\begin{aligned} \mathbb{E}[p_2(t) - p_1(t)] &= \int_{\widehat{t}\left(\frac{\Phi}{1-\theta}, c_1\right)}^{\widehat{t}\left(\frac{\Phi}{1-\theta}, c_2\right)} \left(P\left(\frac{\Phi}{1-\theta}, t\right) - c_1\right) f(t)\, dt + \int_{\widehat{t}\left(\frac{\Phi}{1-\theta}, c_2\right)}^{+\infty} (c_2 - c_1) f(t)\, dt \\ &= \beta\left(\frac{\Phi}{1-\theta}, c_1, c_2\right). \end{aligned}$$

Since $P\left(\frac{\Phi}{1-\theta}, t\right) < c_2$ for $t \in \left[\widehat{t}\left(\frac{\Phi}{1-\theta}, c_1\right), \widehat{t}\left(\frac{\Phi}{1-\theta}, c_2\right)\right]$,

$$\beta\left(\frac{\Phi}{1-\theta}, c_1, c_2\right) \leq \int_{\widehat{t}\left(\frac{\Phi}{1-\theta}, c_1\right)}^{+\infty} (c_2 - c_1) f(t)\, dt = (c_2 - c_1) \times \Pr(\textit{line constrained}),$$

which differs slightly from the commonly held view.

$\beta\left(\frac{\Phi}{1-\theta}, c_1, c_2\right)$ is in general not the marginal value of transmission capacity. The above analysis assumes that prices at each market are equal to the short-run marginal costs. This cannot be true in the long-run, as investment costs have to be recovered. Standard peak-load pricing suggests, that, if markets were not connected:

$$\mathbb{E}[p_n(t)] = c_n + r_n.$$

The question is then, what is the marginal value of transmission capacity, once prices are set to cover marginal costs?

One possible heuristic is to observe that generation at market 1 contributes to the reliability at market 2, hence to add r_2 the "missing money" at market 2, taken as the value of reliability. In that case, the marginal value of capacity would be

$$\gamma_1(\Phi) = \beta\left(\frac{\Phi}{1-\theta}, c_1, c_2\right) + r_2.$$

Another possible heuristic would be to add r_1 and r_2 to c_1 and c_2 on peak. The marginal value of capacity would be

$$\gamma_2(\Phi) = \beta\left(\frac{\Phi}{1-\theta}, c_1, c_2\right) + (r_2 - r_1)\Pr(\textit{peak})$$

These two heuristics produce different results: $\gamma_1(\Phi) > \beta\left(\frac{\Phi}{1-\theta}, c_1, c_2\right)$ while $\gamma_2(\Phi) < \beta\left(\frac{\Phi}{1-\theta}, c_1, c_2\right)$ since $r_1 > r_2$.

4 The complete analysis

Transmission capacity is fixed at Φ. We determine the optimal generation mix $(k_1, k_2)(\Phi)$. Economically, this recognizes that generation adjusts faster than transmission, which is empirically verified.

4.1 Optimal dispatch, pricing, and investment

The peak-load analysis presented above shows that maximum flow is $(1-\theta)\,k_1^U$. Thus, if $\Phi \geq (1-\theta)\,k_1^U$, the line is never congested. If $\Phi < (1-\theta)\,k_1^U$, two cases must be distinguished.

4.1.1 Light, one-way, congestion

Consider the case $\Phi < (1-\theta)\,k_1^U$. Then, $\Phi < (1-\theta)\,k_1$. The proof proceeds by contraction: if $\Phi > (1-\theta)\,k_1$, the line would never be congested, hence $k_1(\Phi) = k_1^U$, and $\Phi > (1-\theta)\,k_1^U$, which contradicts the hypothesis.

Supply, demand, flow and prices For low-demand states of the world, only baseload technology is producing and serving the entire market. Since $\Phi < (1-\theta)\,k_1$, the transmission line becomes congested *before* baseload generation produces at capacity:

$$\Phi < (1-\theta)\,k_1 \Leftrightarrow \frac{\Phi}{1-\theta} < k_1 \Leftrightarrow P\left(\frac{\Phi}{1-\theta}, t\right) > P(k_1, t) \Leftrightarrow \widehat{t}\left(\frac{\Phi}{1-\theta}, c_1\right) < \widehat{t}(k_1, c_1).$$

Thus, for $t \leq \widehat{t}\left(\frac{\Phi}{1-\theta}, c_1\right)$,

Generation	$q_1^s(t) = D(c_1, t)$	$q_2^s(t) = 0$
Demand	$q_1^d(t) = \theta D(c_1, t)$	$q_2^d(t) = (1-\theta)\,D(c_1, t)$
Flow	$\varphi(t) = (1-\theta)\,D(c_1, t)$	
Prices	$p_1(t) = c_1$	$p_2(t) = c_1$

.

For $t \geq \widehat{t}\left(\frac{\Phi}{1-\theta}, c_1\right)$, $p_1(t) = c_1$, while $p_2(t) = P\left(\frac{\Phi}{1-\theta}, t\right)$ as long as $p_2(t) \leq c_2$:

$$P\left(\frac{\Phi}{1-\theta}, t\right) \leq c_2 \Longleftrightarrow t \leq \widehat{t}\left(\frac{\Phi}{1-\theta}, c_2\right).$$

Thus, for $\widehat{t}\left(\frac{\Phi}{1-\theta}, c_1\right) \leq t \leq \widehat{t}\left(\frac{\Phi}{1-\theta}, c_2\right)$,

Generation	$q_1^s(t) = \theta D(c_1, t) + \Phi$	$q_2^s(t) = 0$
Demand	$q_1^d(t) = \theta D(c_1, t)$	$q_2^d(t) = \Phi$
Flow	$\varphi(t) = \Phi$	
Prices	$p_1(t) = c_1$	$p_2(t) = P\left(\frac{\Phi}{1-\theta}, t\right)$

.

For $t \geq \widehat{t}\left(\frac{\Phi}{1-\theta}, c_2\right)$, peaking technology produces. Then one technology reaches capacity. Either

$$\theta D(c_1, t) + \Phi = k_1 \Leftrightarrow D(c_1, t) = \frac{k_1 - \Phi}{\theta} \Leftrightarrow t = \widehat{t}\left(\frac{k_1 - \Phi}{\theta}, c_1\right)$$

or

$$D(c_2, t) - \Phi = k_2 \Leftrightarrow D(c_2, t) = \frac{k_2 + \Phi}{1 - \theta} \Leftrightarrow t = \widehat{t}\left(\frac{k_2 + \Phi}{1 - \theta}, c_2\right).$$

Lemma 1 *If $\xi(c_2, r_2) > \xi(c_1, r_1)$, then the baseload technology reaches capacity first.*

P roof. *The proof is presented in the Appendix. It proceeds by contradiction: if the peaking technology was constrained first, we would have $\xi(c_2, r_2) < \xi(c_1, r_1)$.* ■

Since baseload capacity reaches capacity first, for $\widehat{t}\left(\frac{\Phi}{1-\theta}, c_2\right) \leq t \leq \widehat{t}\left(\frac{k_1-\Phi}{\theta}, c_1\right)$,

Generation	$q_1^s(t) = \theta D(c_1, t) + \Phi$	$q_2^s(t) = (1-\theta) D(c_2, t) - \Phi$
Demand	$q_1^d(t) = \theta D(c_1, t)$	$q_2^d(t) = (1-\theta) D(c_2, t)$
Flow	$\varphi(t) = \Phi$	
Prices	$p_1(t) = c_1$	$p_2(t) = c_2$

.

When the baseload technology is constrained $p_1(t) = P\left(\frac{k_1-\Phi}{\theta}, t\right)$. $p_1(t)$ increases until it reaches c_2:

$$P\left(\frac{k_1 - \Phi}{\theta}, t\right) = c_2 \Leftrightarrow t = \widehat{t}\left(\frac{k_1 - \Phi}{\theta}, c_2\right).$$

Thus, for $\widehat{t}\left(\frac{k_1-\Phi}{\theta}, c_1\right) \leq t \leq \widehat{t}\left(\frac{k_1-\Phi}{\theta}, c_2\right)$

Generation	$q_1^s(t) = k_1$	$q_2^s(t) = (1-\theta) D(c_2, t) - \Phi$
Demand	$q_1^d(t) = k_1 - \Phi$	$q_2^d(t) = (1-\theta) D(c_2, t)$
Flow	$\varphi(t) = \Phi$	
Prices	$p_1(t) = P\left(\frac{k_1-\Phi}{\theta}, t\right)$	$p_2(t) = c_2$

Then, $p_1(t) = p_2(t) = c_2$ and the line is no longer constrained. $q_1^d(t) = \theta D(c_2, t)$, hence $\varphi(t) = k_1 - \theta D(c_2, t)$, and

$$q_2^s(t) = (1-\theta) D(c_2, t) - k_1 + \theta D(c_2, t) = D(c_2, t) - k_1.$$

This occurs until the peaking technology is constrained:

$$D(c_2, t) - k_1 = k_2 \Leftrightarrow t = \widehat{t}(k_1 + k_2, c_2).$$

Thus, for $\widehat{t}\left(\frac{k_1-\Phi}{\theta}, c_2\right) \leq t \leq \widehat{t}(k_1 + k_2, c_2)$,

Generation	$q_1^s(t) = k_1$	$q_2^s(t) = D(c_2, t) - k_1$
Demand	$q_1^d(t) = \theta D(c_2, t)$	$q_2^d(t) = (1-\theta) D(c_2, t)$
Flow	$\varphi(t) = k_1 - \theta D(c_2, t)$	
Prices	$p_1(t) = c_2$	$p_2(t) = c_2$

Finally, for $t \leq \widehat{t}(k_1 + k_2, c_2)$,

Generation	$q_1^s(t) = k_1$	$q_2^s(t) = k_2$
Demand	$q_1^d(t) = \theta(k_1 + k_2)$	$q_2^d(t) = (1-\theta)(k_1 + k_2)$
Flow	$\varphi(t) = (1-\theta) k_1 - \theta k_2$	
Prices	$p_1(t) = P(k_1 + k_2, t)$	$p_2(t) = P(k_1 + k_2, t)$

Total capacity $(k_1 + k_2)$ is determined by

$$\int_{\widehat{t}(k_1+k_2,c_2)}^{+\infty} \left(P\left(k_1 + k_2, t\right) - c_2\right) f\left(t\right) dt = \Psi\left(k_1 + k_2, c_2\right) = r_2,$$

hence

$$k_1 + k_2 = \xi\left(c_2, r_2\right) = k_1^U + k_2^U. \tag{3}$$

Baseload capacity $k_1\left(\Phi\right)$ is determined by:

$$\begin{aligned} r_1 &= \int_{\widehat{t}\left(\frac{k_1-\Phi}{\theta},c_1\right)}^{\widehat{t}\left(\frac{k_1-\Phi}{\theta},c_2\right)} \left(P\left(\frac{k_1-\Phi}{\theta},t\right) - c_1\right) f\left(t\right) dt + \int_{\widehat{t}\left(\frac{k_1-\Phi}{\theta},c_2\right)}^{\widehat{t}(k_1+k_2,c_2)} \left(c_2 - c_1\right) f\left(t\right) dt \\ &\quad + \int_{\widehat{t}(k_1+k_2,c_2)}^{+\infty} \left(P\left(k_1 + k_2, t\right) - c_1\right) f\left(t\right) dt \end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned} r_1 &= \int_{\widehat{t}\left(\frac{k_1-\Phi}{\theta},c_1\right)}^{\widehat{t}\left(\frac{k_1-\Phi}{\theta},c_2\right)} \left(P\left(\frac{k_1-\Phi}{\theta},t\right) - c_1\right) f\left(t\right) dt + \int_{\widehat{t}\left(\frac{k_1-\Phi}{\theta},c_2\right)}^{+\infty} \left(c_2 - c_1\right) f\left(t\right) dt + r_2 \\ &= \beta\left(\frac{k_1-\Phi}{\theta}, c_1, c_2\right) + r_2 \end{aligned}$$

$\Leftrightarrow$

$$\beta\left(\frac{k_1-\Phi}{\theta}, c_1, c_2\right) = r_1 - r_2 = \beta\left(k_1^U, c_1, c_2\right)$$

$\Leftrightarrow$

$$k_1 = \theta k_1^U + \Phi. \tag{4}$$

Total capacity is unchanged. An increase in transmission capacity leads to a one for one substitution of baseload generation capacity for peaking generation capacity. Equivalently, $k_1 < k_1^U$ since $\Phi < \left(1-\theta\right) k_1^U$: congestion on the transmission line reduces the optimal baseload capacity installed at market 1, and increases the optimal peaking capacity installed at market 2.

The transmission line is not congested for all $t \geq \widehat{t}\left(\frac{\Phi}{1-\theta}, c_1\right)$. For $t \geq \widehat{t}\left(\frac{k_1-\Phi}{\theta}, c_2\right)$, power prices in both markets are equal, hence power flow from market 1 to market 2 is reduced, thus the line is no longer congested. This suggests the marginal value of transmission is lower than previously assumed.

4.1.2 Severe, two-way congestion

The transmission constraint from market 2 to market 1 is

$$(1-\theta)\,k_1 - \theta k_2 \geq -\Phi$$

$\Leftrightarrow$

$$k_1 - \theta\,(k_1 + k_2) = \theta k_1^U + \Phi - \theta\left(k_1^U + k_2^U\right) = \Phi - \theta k_2^U \geq -\Phi$$

$\Leftrightarrow$

$$\Phi \geq \frac{\theta}{2} k_2^U.$$

This sets a lower bound on Φ for which the above solution is valid. Equivalently, if $\Phi < \frac{\theta}{2} k_2^U$, the transmission line becomes constrained in the reverse direction at some point, and the analysis must be amended.

Supply, demand, flows and prices Nothing changes for $t \leq \widehat{t}\left(\frac{k_1-\Phi}{\theta}, c_2\right)$, until both marginal costs are equal. Then, the reverse transmission constraint becomes binding

$$k_1 - \theta D\,(c_2, t) = -\Phi \Leftrightarrow D\,(c_2, t) = \frac{k_1 + \Phi}{\theta} \Leftrightarrow t = \widehat{t}\left(\frac{k_1 + \Phi}{\theta}, c_2\right)$$

before the peaking technology reaches capacity

$$(1-\theta)\,D\,(c_2, t) + \Phi = k_2 \Leftrightarrow D\,(c_2, t) = \frac{k_2 - \Phi}{1-\theta} \Leftrightarrow t = \widehat{t}\left(\frac{k_2 - \Phi}{1-\theta}, c_2\right).$$

Thus, for $\widehat{t}\left(\frac{k_1-\Phi}{\theta}, c_2\right) \leq t \leq \widehat{t}\left(\frac{k_1+\Phi}{\theta}, c_2\right)$,

Generation	$q_1^s\,(t) = k_1$	$q_2^s\,(t) = D\,(c_2, t) - k_1$
Demand	$q_1^d\,(t) = \theta D\,(c_2, t)$	$q_2^d\,(t) = (1-\theta)\,D\,(c_2, t)$
Flow	$\varphi\,(t) = k_1 - \theta D\,(c_2, t)$	
Prices	$p_1\,(t) = c_2$	$p_2\,(t) = c_2$

.

as previously, and for $\widehat{t}\left(\frac{k_1+\Phi}{\theta}, c_2\right) \leq t \leq \widehat{t}\left(\frac{k_2-\Phi}{1-\theta}, c_2\right)$,

Generation	$q_1^s(t) = k_1$	$q_2^s(t) = (1-\theta) D(c_2, t) + \Phi$
Demand	$q_1^d(t) = k_1 + \Phi$	$q_2^d(t) = (1-\theta) D(c_2, t)$
Flow	$\varphi(t) = -\Phi$	
Prices	$p_1(t) = P\left(\frac{k_1+\Phi}{\theta}, t\right)$	$p_2(t) = c_2$

.

Finally, for $t \geq \widehat{t}\left(\frac{k_2-\Phi}{1-\theta}, c_2\right)$,

Generation	$q_1^s(t) = k_1$	$q_2^s(t) = k_2$
Demand	$q_1^d(t) = k_1 + \Phi$	$q_2^d(t) = k_2 - \Phi$
Flow	$\varphi(t) = -\Phi$	
Prices	$p_1(t) = P\left(\frac{k_1+\Phi}{\theta}, t\right)$	$p_2(t) = P\left(\frac{k_2-\Phi}{1-\theta}, t\right)$

.

Optimal peaking capacity $k_2(\Phi)$ is such that

$$\Psi\left(\frac{k_2(\Phi)-\Phi}{1-\theta}, c_2\right) = r_2 = \Psi\left(\left(k_1^U + k_2^U\right), c_2\right)$$

$\Leftrightarrow$

$$k_2(\Phi) = \Phi + (1-\theta)\left(k_1^U + k_2^U\right). \tag{5}$$

$k_2(\Phi)$ is increasing one for one in Φ: as Φ increases, the reverse transmission constraint is partially relieved, and additional peaking capacity is added. $k_2(0) = (1-\theta)\left(k_1^U + k_2^U\right)$: when markets are isolated, technology 2 is the only technology available, hence is deployed following $\Psi(k_2, c_2) = r_2$. All demand is served using peaking technology, the only technology available.

Optimal baseload capacity $k_1(\Phi)$ is such that

$$\begin{aligned} r_1 \quad = \quad & \int_{\widehat{t}\left(\frac{k_1-\Phi}{\theta}, c_1\right)}^{\widehat{t}\left(\frac{k_1-\Phi}{\theta}, c_2\right)} \left(P\left(\frac{k_1-\Phi}{\theta}, t\right) - c_1\right) f(t)\, dt + \int_{\widehat{t}\left(\frac{k_1-\Phi}{\theta}, c_2\right)}^{\widehat{t}\left(\frac{k_1+\Phi}{\theta}, c_2\right)} (c_2 - c_1) f(t)\, dt \\ & + \int_{\widehat{t}\left(\frac{k_1+\Phi}{\theta}, c_2\right)}^{\widehat{t}\left(\frac{k_2-\Phi}{1-\theta}, c_2\right)} \left(P\left(\frac{k_1+\Phi}{\theta}, t\right) - c_1\right) f(t)\, dt + \int_{\widehat{t}\left(\frac{k_2-\Phi}{1-\theta}, c_2\right)}^{+\infty} \left(P\left(\frac{k_1+\Phi}{\theta}, t\right) - c_1\right) f(t)\, dt \end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned} r_1 &= \int_{\widehat{t}\left(\frac{k_1-\Phi}{\theta},c_1\right)}^{\widehat{t}\left(\frac{k_1-\Phi}{\theta},c_2\right)} \left(P\left(\frac{k_1-\Phi}{\theta},t\right)-c_1\right) f(t)\,dt + \int_{\widehat{t}\left(\frac{k_1-\Phi}{\theta},c_2\right)}^{+\infty} (c_2-c_1) f(t)\,dt \\ &+ \int_{\widehat{t}\left(\frac{k_1+\Phi}{\theta},c_2\right)}^{+\infty} \left(P\left(\frac{k_1+\Phi}{\theta},t\right)-c_2\right) f(t)\,dt \end{aligned}$$

$\Leftrightarrow$

$$\beta\left(\frac{k_1(\Phi)-\Phi}{\theta},c_1,c_2\right)+\Psi\left(\frac{k_1(\Phi)+\Phi}{\theta},c_2\right)=r_1. \tag{6}$$

The function

$$H(x,\Phi)=\beta\left(\frac{x-\Phi}{\theta},c_1,c_2\right)+\Psi\left(\frac{x+\Phi}{\theta},c_2\right)-r_1$$

is decreasing in x, since both β and Ψ are decreasing in their first argument. Equation (6) thus implicitly defines a unique $k_1(\Phi)$ for $\Phi<\frac{\theta}{2}k_2^U$.

While $k_1(\Phi)$ for $\Phi<\frac{\theta}{2}k_2^U$ cannot be explicitly characterized, a few properties can be derived, summarized in the following:

Lemma 2 *1. $k_1(.)$ is bounded above and below on $\left[0,\frac{\theta}{2}k_2^U\right]$:*

$$\theta k_1^U+\Phi\leq k_1(\Phi)\leq\theta\left(k_1^U+k_2^U\right)-\Phi.$$

2. $k_1(.)$ and $k_2(.)$ are continuous on $\left[0,\frac{\theta}{2}k_2^U\right]$, and

$$k_1(0)=\theta\xi(c_1,r_1)\,,\ \text{and}\ k_1\left(\frac{\theta}{2}k_2^U\right)=\theta\left(k_1^U+\frac{k_2^U}{2}\right).$$

3. Finally, $k_1(.)$ is continuously differentiable on $\left[0,\frac{\theta}{2}k_2^U\right]$, and

$$\left|\frac{dk_1}{d\Phi}\right|\leq 1.$$

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1.

$$H\left(\theta k_1^U+\Phi,\Phi\right)=\beta\left(k_1^U,c_1,c_2\right)+\Psi\left(k_1^U+\frac{2\Phi}{\theta},c_2\right)-r_1=\Psi\left(k_1^U+\frac{2\Phi}{\theta},c_2\right)-r_2.$$

$k_1^U + \frac{2\Phi}{\theta} \le k_1^U + k_2^U \Leftrightarrow \Psi\left(k_1^U + \frac{2\Phi}{\theta}, c_2\right) \ge \beta\left(k_1^U + k_2^U, c_2\right) = r_2$. *Thus,*

$$H\left(\theta k_1^U + \Phi, \Phi\right) \ge 0 \Leftrightarrow k_1(\Phi) \ge \theta k_1^U + \Phi.$$

Define $\bar{k} = k_1^U + k_2^U - k_2(\Phi) = \theta\left(k_1^U + k_2^U\right) - \Phi$.

$$H\left(\bar{k}, \Phi\right) = \beta\left(k_1^U + k_2^U - \frac{2\Phi}{\theta}, c_1, c_2\right) + \Psi\left(k_1^U + k_2^U, c_2\right) - r_1 = \beta\left(k_1^U + k_2^U - \frac{2\Phi}{\theta}, c_1, c_2\right) + r_2 - r_1.$$

Then, $k_1^U + k_2^U - \frac{2\Phi}{\theta} \ge k_1^U + k_2^U - k_2^U = k_1^U \Leftrightarrow \beta\left(k_1^U + k_2^U - \frac{2\Phi}{\theta}, c_1, c_2\right) \le \beta\left(k_1^U, c_1, c_2\right) = r_1 - r_2$. *Thus,*

$$H\left(\bar{k}, \Phi\right) \le 0 \Leftrightarrow k_1(\Phi) \le \bar{k} \Leftrightarrow k_1(\Phi) + k_2(\Phi) \le k_1^U + k_2^U.$$

2. $k_1(.)$ *is continuous on* $\left[0, \frac{\theta}{2} k_2^U\right)$ *by inspection. Since* $\theta k_1^U + \Phi \le k_1(\Phi) \le \theta\left(k_1^U + k_2^U\right) - \Phi$,

$$\theta\left(k_1^U + \frac{k_2^U}{2}\right) \le \lim_{\Phi \to \frac{\theta}{2} k_2^U, \Phi \le \frac{\theta}{2} k_2^U} k_1(\Phi) \le \theta\left(k_1^U + \frac{k_2^U}{2}\right)$$

hence $k_1\left(\frac{\theta}{2} k_2^U\right) = \theta\left(k_1^U + \frac{k_2^U}{2}\right)$. *For* $\Phi \ge \frac{\theta}{2} k_2^U$,

$$k_1\left(\frac{\theta}{2} k_2^U\right) = \theta k_1^U + \frac{\theta}{2} k_2^U = \theta\left(k_1^U + \frac{k_2^U}{2}\right),$$

thus $k_1(.)$ *is continuous at* $\Phi = \frac{\theta}{2} k_2^U$. *Similarly, for* $\Phi \ge \frac{\theta}{2} k_2^U$,

$$k_2\left(\frac{\theta}{2} k_2^U\right) = k_1^U + k_2^U - \theta k_1^U + \frac{k_2^U}{2} = (1-\theta)\left(k_1^U + k_2^U\right) + \frac{\theta}{2} k_2^U$$

while, for $\Phi < \frac{\theta}{2} k_2^U$

$$\lim_{\Phi \to \frac{\theta}{2} k_2^U} k_2(\Phi) = \frac{\theta}{2} k_2^U + (1-\theta)\left(k_1^U + k_2^U\right) = k_2\left(\frac{\theta}{2} k_2^U\right),$$

thus $k_2(.)$ *is continuous at* $\Phi = \frac{\theta}{2}k_2^U$.

$$\begin{aligned} H(x,0) &= \beta\left(\frac{x}{\theta},c_1,c_2\right)+\Psi\left(\frac{x}{\theta},c_2\right) \\ &= \int_{\widehat{t}\left(\frac{x}{\theta},c_1\right)}^{\widehat{t}\left(\frac{x}{\theta},c_2\right)}\left(P\left(\frac{x}{\theta},t\right)-c_1\right)f(t)\,dt+\int_{\widehat{t}\left(\frac{x}{\theta},c_2\right)}^{+\infty}(c_2-c_1)\,f(t)\,dt+\int_{\widehat{t}\left(\frac{x}{\theta},c_2\right)}^{+\infty}\left(P\left(\frac{x}{\theta},t\right)-c_2\right)f(t)\,dt \\ &= \int_{\widehat{t}\left(\frac{x}{\theta},c_1\right)}^{+\infty}\left(P\left(\frac{x}{\theta},t\right)-c_1\right)f(t)=\Psi\left(\frac{x}{\theta},c_1\right), \end{aligned}$$

thus, $k_1(0) = \theta\xi(c_1, r_1)$.

3. *Implicit differentiation of equation* (6) *with respect to* Φ *yields*

$$\frac{\partial\beta}{\partial k}\left(\frac{k_1'(\Phi)-1}{\theta}\right)+\frac{\partial\Psi}{\partial k}\left(\frac{k_1'(\Phi)+1}{\theta}\right)=0$$

$\Leftrightarrow$

$$k_1'(\Phi)=\frac{\frac{\partial\beta}{\partial k}-\frac{\partial\Psi}{\partial k}}{\frac{\partial\beta}{\partial k}+\frac{\partial\Psi}{\partial k}}.$$

$k_1'(\Phi)$ *is continuous on* $\left[0,\frac{\theta}{2}k_2^U\right]$ *since* $\frac{\partial\beta}{\partial k}$ *and* $\frac{\partial\Psi}{\partial k}$ *are continuous and* $\frac{\partial\beta}{\partial k}+\frac{\partial\Psi}{\partial k}<0$ *on* $\left[0,\frac{\theta}{2}k_2^U\right]$.

$$\frac{dk_1}{d\Phi}-1=-2\frac{\frac{\partial\Psi}{\partial k}}{\frac{\partial\beta}{\partial k}+\frac{\partial\Psi}{\partial k}}<0,$$

and

$$\frac{dk_1}{d\Phi}+1=2\frac{\frac{\partial\beta}{\partial k}}{\frac{\partial\beta}{\partial k}+\frac{\partial\Psi}{\partial k}}>0,$$

thus $\left|\frac{dk_1}{d\Phi}\right|\leq 1$.

■

Using the previous Lemma, we verify that the solution is internally consistent, i.e., that $(1-\theta)\,k_1(\Phi)-\theta k_2(\Phi)<-\Phi$ for all $\Phi<\frac{\theta}{2}k_2^U$:

$$(1-\theta)\,k_1(\Phi)-\theta k_2(\Phi)<(1-\theta)\left(\theta\left(k_1^U+k_2^U\right)-\Phi\right)-\theta\left(\Phi+(1-\theta)\left(k_1^U+k_2^U\right)\right)=-\Phi.$$

When both markets are isolated, technology 1 is used to serve demand. I do not believe that we can ascertain the sign of $\frac{dk_1}{d\Phi}$, since the line is congested in both directions. When the line is congested from market 1 to market 2, an increase in transmission capacity leads to an increase in the capacity installed in market 1. On the other hand, it also leads to an increase in capacity installed in market 2, when the line is congested from market 1 to market 2. Either effect may dominate. However, the net effect is lower than 1 in absolute value.

4.1.3 Summary

The previous analysis can be summarized in the following:

Proposition 1 *1. If $\Phi \geq (1-\theta) k_1^U$, the transmission line is never congested.*

2. If $\Phi \in \left[\frac{\theta}{2} k_2^U, (1-\theta) k_1^U\right)$, the transmission line is congested from market 1 to market 2. The total installed capacity is unchanged. As transmission capacity increases, baseload capacity is substituted one for one for peaking capacity.

3. If $\Phi \in \left[0, \frac{\theta}{2} k_2^U\right)$, the transmission line is congested in both directions. As transmission capacity increases, peak load capacity increases one for one. However, the impact on baseload capacity is undetermined.

4.2 Marginal value of transmission capacity

Proposition 2 *1. If $\Phi \geq (1-\theta) k_1^U$, $\mathbb{E}[\eta(t)] = 0$.*

2. If $\Phi \in \left[\frac{\theta}{2} k_2^U, (1-\theta) k_1^U\right)$, the marginal value of transmission capacity is equal to the expected difference in short term marginal costs, plus the difference in investment costs:

$$\mathbb{E}[\eta(t)] = \beta\left(\frac{\Phi}{1-\theta}, c_1, c_2\right) + r_2 - r_1.$$

3. If $\Phi \in \left[0, \frac{\theta}{2} k_2^U\right)$, the marginal value of transmission capacity is bounded above and below:

$$\beta\left(\frac{\Phi}{1-\theta}, c_1, c_2\right) + r_2 - r_1 \leq \mathbb{E}[\eta(t)] \leq \beta\left(\frac{\Phi}{1-\theta}, c_1, c_2\right) + r_2 - r_1 + 2\Delta$$

where

$$\Delta = \Psi(\xi(c_1, r_1), c_2) - \Psi(\xi(c_2, r_2), c_2) > 0.$$

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1. *If the line is unconstrained, the marginal value of capacity is equal to zero.*

2. *For* $\Phi \in \left[\frac{\theta}{2}k_2^U, (1-\theta)\, k_1^U\right)$, $\mathbb{E}\left[\eta\left(t\right)\right] = I$, *where*

$$\begin{aligned} I &= \int_{\widehat{t}\left(\frac{\Phi}{1-\theta},c_1\right)}^{\widehat{t}\left(\frac{\Phi}{1-\theta},c_2\right)} \left(P\left(\frac{\Phi}{1-\theta},t\right) - c_1\right) f\left(t\right) dt + \int_{\widehat{t}\left(\frac{\Phi}{1-\theta},c_2\right)}^{\widehat{t}\left(\frac{k_1-\Phi}{\theta},c_1\right)} \left(c_2 - c_1\right) f\left(t\right) dt \\ &\quad + \int_{\widehat{t}\left(\frac{k_1-\Phi}{\theta},c_1\right)}^{\widehat{t}\left(\frac{k_1-\Phi}{\theta},c_2\right)} \left(c_2 - P\left(\frac{k_1-\Phi}{\theta},t\right)\right) f\left(t\right) dt \\ &= \int_{\widehat{t}\left(\frac{\Phi}{1-\theta},c_1\right)}^{\widehat{t}\left(\frac{\Phi}{1-\theta},c_2\right)} \left(P\left(\frac{\Phi}{1-\theta},t\right) - c_1\right) f\left(t\right) dt + \int_{\widehat{t}\left(\frac{\Phi}{1-\theta},c_2\right)}^{+\infty} \left(c_2 - c_1\right) f\left(t\right) dt \\ &\quad + \int_{\widehat{t}\left(\frac{k_1-\Phi}{\theta},c_1\right)}^{\widehat{t}\left(\frac{k_1-\Phi}{\theta},c_2\right)} \left(c_1 - P\left(\frac{k_1-\Phi}{\theta},t\right)\right) f\left(t\right) dt + \int_{\widehat{t}\left(\frac{k_1-\Phi}{\theta},c_2\right)}^{+\infty} \left(c_1 - c_2\right) f\left(t\right) dt \\ &= \beta\left(\frac{\Phi}{1-\theta},c_1,c_2\right) - \beta\left(\frac{k_1-\Phi}{\theta},c_1,c_2\right) \end{aligned}$$

Thus,

$$\mathbb{E}\left[\eta\left(t\right)\right] = \beta\left(\frac{\Phi}{1-\theta},c_1,c_2\right) + r_2 - r_1$$

since $\beta\left(\frac{k_1-\Phi}{\theta},c_1,c_2\right) = r_1 - r_2$ *from equation* (4).

3. *For* $\Phi \in \left[0, \frac{\theta}{2}k_2^U\right)$,

$$\mathbb{E}\left[\eta\left(t\right)\right] = I + J$$

where

$$\begin{aligned} J &= \int_{\widehat{t}\left(\frac{k_1+\Phi}{\theta},c_2\right)}^{\widehat{t}\left(\frac{k_2-\Phi}{1-\theta},c_2\right)} \left(P\left(\frac{k_1+\Phi}{\theta},t\right) - c_2\right) f\left(t\right) dt + \int_{\widehat{t}\left(\frac{k_2-\Phi}{1-\theta},c_2\right)}^{+\infty} \left(P\left(\frac{k_1+\Phi}{\theta},t\right) - P\left(\frac{k_2-\Phi}{1-\theta},t\right)\right) f\left(t\right) dt \\ &= \int_{\widehat{t}\left(\frac{k_1+\Phi}{\theta},c_2\right)}^{+\infty} \left(P\left(\frac{k_1+\Phi}{\theta},t\right) - c_2\right) f\left(t\right) dt + \int_{\widehat{t}\left(\frac{k_2-\Phi}{1-\theta},c_2\right)}^{+\infty} \left(c_2 - P\left(\frac{k_2-\Phi}{1-\theta},t\right)\right) f\left(t\right) dt \\ &= \Psi\left(\frac{k_1+\Phi}{\theta},c_2\right) - r_2 \end{aligned}$$

since $\int_{\hat{t}\left(\frac{k_2-\Phi}{1-\theta},c_2\right)}^{+\infty}\left(P\left(\frac{k_2-\Phi}{1-\theta},t\right)-c_2\right)f(t)\,dt=r_2$ *from equation* (5)*. Thus,*

$$\begin{aligned}\mathbb{E}[\eta(t)] &= \beta\left(\frac{\Phi}{1-\theta},c_1,c_2\right)-\beta\left(\frac{k_1(\Phi)-\Phi}{\theta},c_1,c_2\right)+\Psi\left(\frac{k_1(\Phi)+\Phi}{\theta},c_2\right)-r_2\\ &= \beta\left(\frac{\Phi}{1-\theta},c_1,c_2\right)+2\Psi\left(\frac{k_1(\Phi)+\Phi}{\theta},c_2\right)-(r_1+r_2)\end{aligned}$$

by replacing $\beta\left(\frac{k_1-\Phi}{\theta},c_1,c_2\right)$ *from equation* (6)*.* $k_1(\Phi)\geq\theta k_1^U+\Phi\Leftrightarrow\frac{k_1-\Phi}{\theta}\geq k_1^U\Leftrightarrow-\beta\left(\frac{k_1-\Phi}{\theta},c_1,c_2\right)\geq r_2-r_1$*.* $k_1(\Phi)\leq\theta\left(k_1^U+k_2^U\right)-\Phi\Leftrightarrow\frac{k_1+\Phi}{\theta}\leq k_1^U+k_2^U\Leftrightarrow\Psi\left(\frac{k_1+\Phi}{\theta},c_2\right)\geq r_2$*. Thus,*

$$\mathbb{E}[\eta(t)]\geq\beta\left(\frac{\Phi}{1-\theta},c_1,c_2\right)+r_2-r_1.$$

Since $(k_1(\Phi)+\Phi)$ *is increasing,* $\Psi\left(\frac{k_1(\Phi)+\Phi}{\theta},c_2\right)\leq\Psi\left(\frac{k_1(0)}{\theta},c_2\right)=\Psi(\xi(c_1,r_1),c_2)=r_2+\Delta$*, where*

$$\Delta=\Psi(\xi(c_1,r_1),c_2)-\Psi(\xi(c_2,r_2),c_2)>0.$$

Thus,

$$\mathbb{E}[\eta(t)]\leq\beta\left(\frac{\Phi}{1-\theta},c_1,c_2\right)+r_2-r_1+2\Delta.$$

■

None of the heuristics presented in Section 2 is actually correct.

A practical implication: for the slightly congested line, the marginal value of transmission capacity can be much lower than the expected difference in short-term marginal costs. Consider the following example: technology 1 is nuclear, and technology 2 is Combined Cycle Gas Turbine. The International Energy Agency (*IEA* (2010)) provides the following estimates for the costs:

	1	2
c_n	11	49
r_n	34	8

The marginal cost difference is 38 €/MWh. If the line is congested 50% of the time, this corresponds $\beta(\Phi)=19$ € *per MW per hour* on average, or, multiplying by 8,760 hours per year, 166,440

€ *per MW per year*, or € 1.7 million *per MW* discounted in perpetuity at 10%. This value exceeds most estimates of the cost of transmission (need references here), hence expansion should be undertaken.

However, when the marginal value is properly computed, it becomes

$$\mathbb{E}\left[\eta\left(t\right)\right] = -7 \text{ € } \textit{per MW per hour},$$

which suggests no expansion should be undertaken.

The marginal value of capacity on a slightly congested line is the expected difference in short-term marginal costs plus the difference in investment costs. It is thus lower than the expected difference in short-term marginal costs.

5 Conclusion