Relaxing competition through speculation: Committing to a negative supply slope*

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Abstract

We analyze how firms can take strategic speculative positions in derivatives markets to soften competition in the spot market. In our game, firms first choose a portfolio of call options with a spectrum of strike prices; then they compete with supply functions under demand uncertainty. In equilibrium firms sell call options with low strike prices and buy call options with high strike prices to commit to downward sloping supply functions in the spot market. Strategic speculation is anti-competitive and increases the volatility of commodity prices.

Keywords: supply function equilibrium, option contracts, strategic commitment, speculation

JEL codes: C73, D43, D44, G13, L13, L94

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1 Introduction

The trade in commodity derivatives is widespread and trading volumes often surpass that of the underlying commodities. Ideally derivatives markets can improve market efficiency as they allow firms to manage risk and as they aggregate distributed information across market agents. However, the effect of derivatives speculation on the liquidity and variability of spot markets is a point of debate.[1] Another issue is whether it is beneficial for competition that strategic producers use financial contracts as a commitment device (Allaz and Vila, 1993; Mahenc and Salanié, 2004). We contribute to this discussion by showing that the type of derivatives that producers use matters, especially if producers are not restricted to compete with Cournot or Bertrand strategies in the spot market.

We show that in equilibrium, producers will use financial derivatives to commit to a downward sloping supply function, i.e. to produce more when prices are low and less when prices are high. The reason for doing so is that by committing to a downward sloping supply function a firm reduces the elasticity of its competitors' residual demand curves (Figure 1). This induces its competitors to increase their mark-ups in the spot market and to reduce total output. Such a soft response by competitors is profitable for the firm. In equilibrium all firms use a similar strategy. A downward sloping aggregate supply will increase the volatility of the spot price and can result in prices higher than the monopoly price; but anti-competitive behaviour is mitigated by increased demand uncertainty.

In the paper, firms use call option contracts as a commitment device. A call option gives the buyer the right, but not the obligation, to procure one unit of the good in the spot market from the seller at a predetermined price, the option's strike price. Producers can commit to a supply function with a negative slope by selling call options at low strike prices and buying call options at high strike prices. Selling call options at a low strike price makes it costly for the firm to withhold capacity during low demand periods, while buying call options with a high strike price, gives the producers an incentive to withhold capacity in high demand periods, so as to receive additional income from the option contracts. This trading strategy is called a *bear call spread* when traders use it to speculate on a lower commodity price.

We generalize Allaz and Vila's (1993) analysis of strategic contracting of risk-neutral producers by considering a general strategy space both at the contracting stage and in the spot market. In the first stage, we allow producers to choose a portfolio of call option contracts with a spectrum of strike prices, which is disclosed. In the second stage, firms compete with supply functions in the spot market under demand uncertainty as in Klemperer and Meyer (1989) and Green and Newbery (1992). Thus we extend the model by Chao and Wilson (2005), who consider the influence of exogenous option contracts on supply function competition in the spot market. As is common in the rest of the literature on strategic

[1] Cox and Ross (1976), Turnovsky (1983), and Korniotis (2009) argue that financial markets stabilize commodity prices and that commodity prices are not drive by speculation but reflect underlying economic factors. Figlewski (1981), Hart and Kreps (1986), and Stein (1987) argue that this is not the case.

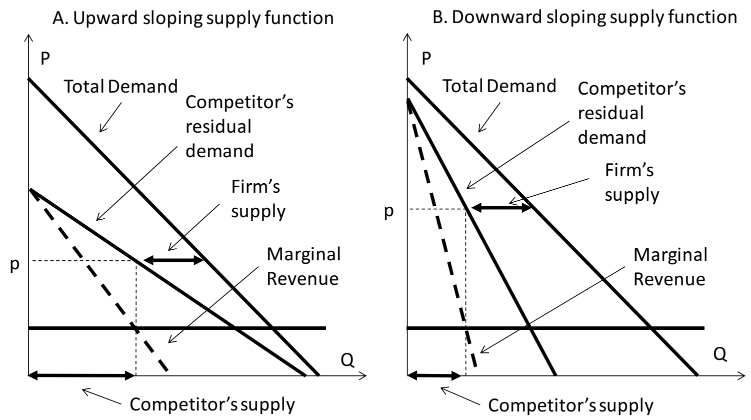

Figure 1: The effect of bidding upward and downward sloping supply functions on a competitor's residual demand function and the equilibrium prices.

contracting, this paper focuses on short-run effects and disregard the relation between contracts, entry and investment decisions.[2] We also assume that contract positions are observable.[3]

The main results of our paper have some parallels in the literature on delegation games. Singh and Vives (1984) and Cheng (1985) analyze a game where firms in the first-stage delegate decisions to a manager in order to commit to either a Bertrand or Cournot strategy, and then compete with this strategy in the second stage. In this game, firms unilaterally prefer to play Cournot when demand is certain. By committing to a Cournot strategy, the residual demand function of competitors becomes less elastic which makes its competitors softer. They will set a higher mark-up, which is beneficial for the firm. Reisinger and Ressner (2009) show however that Bertrand is preferable for sufficiently high demand uncertainties. Thus as in our model, firms' preferred slope of their supply increases when the demand uncertainty increases.

The structure of the paper is as follows. Section 2 summarizes the previous literature. The model of strategic option contracting is introduced in Section 3 and we analyze it in Section 4. Section 5 concludes the paper.

[2] Newbery (1998) shows that producers may use contract sales to keep output high and spot prices low to deter entry. Murphy and Smeers (2010) show that the impact of forward contracts on competition is ambiguous once one endogenizes investment decisions. Argenton and Willems (2010), show how firms use standard forward contracts to exclude potential more efficient entrants, and Petropoulos et al. (2010) show that financial contracts might also lead to overinvestments by incumbent firms, reducing the overall efficiency of the market.

[3] For risk-neutral firms, strategic contracting only materializes when contract positions are observable (Hughes and Kao, 1997). Financial trading is anonymous in most markets, and a firm's contract positions are normally not revealed to competitors. Still competitors can get a rough estimate of changes in the firm's forward position by analyzing changes in the turnover in the forward market and the forward price (Ferreira, 2006). Ferreira's theoretical argument is also relevant in practice. As an example, even if individual contract positions are not disclosed in the Dutch gas market, an empirical study by van Eijkel and Moraga-González's (2010) find that firms are able to infer competitors' positions and that contracts in the Dutch gas market are used for strategic reasons rather than for hedging reasons.

2 Literature review

Most models of strategic commitment in oligopolistic models can be analyzed in the framework developed by Fudenberg and Tirole (1984) and Bulow et al., (1985). A firm will commit to a certain strategy if it softens the response of its competitors. Firms are willing to commit to aggressive (tough) spot market bidding if it results in a soft response from competitors, i.e. strategies are substitutes. On the other hand, firms are willing to commit to passive (soft) spot market bidding if this results in a soft response from competitors.

The seminal paper on strategic forward contracting by Allaz and Vila (1993) is an example of a game where strategies are substitutes. They analyze a two-stage Cournot model of a homogeneous duopoly product market. The set-up introduces a prisoners' dilemma for producers: Each strategic producer will use forward contracts as a commitment device to commit the producer to a large output[4]. This gives it a Stackelberg first-mover advantage, so that competitors reduce their output. But when they all increase their forward sales, all producers end up worse off. As a result, competition is tougher on the spot market and welfare is improved as compared to a situation without forward trading. Thus endogenous contracting is pro-competitive when firms compete with quantities. The Allaz and Vila effect has also been confirmed in experiments by Brandts et al. (2008). However, introducing a forward market worsens competition when strategies are complements, i.e. when an aggressive commitment results in an aggressive spot market response from competitors. Mahenc and Salanié (2004) analyze a market with differentiated goods and price competition, and show that a commitment to low mark-ups, due to forward sales, is met with a tough response, that is competitors also lower their mark-ups. To avoid the tough response, firms buy in the forward market (negative contracting) in order to soften competition in the spot market. This increases mark-ups in the spot-market. Thus, forward trading reduces social welfare when strategies are complements.

In a more generalized form of spot market competition, producers compete with supply functions under demand uncertainty, as in the supply function equilibrium (SFE) model (Klemperer and Meyer, 1989).The setting of the SFE model is obviously well-suited for markets where producers sell their output in a uniform-price auction, as in wholesale electricity markets (Anderson and Philpott, 2002b; Baldick and Hogan, 2002; Green and Newbery, 1992; Holmberg and Newbery, 2010). Vives (2009) notes that competition in supply functions has also been used to model bidding for government procurement contracts, management consulting, airline pricing reservation systems, and provides a reduced form in strategic agency and trade policy models. In addition, Klemperer and Meyer (1989) argue that although most markets are not explicitly cleared by uniform-price auctions, firms typically face a uniform market price and they need predetermined decision rules for its lower-level managers on how to deal with changing market conditions.

[4]The fact that selling forward contracts makes a strategic producer more interested in output and less in mark-ups in the spot market, i.e. tough, has been established both empirically (Wolak, 2000; Bushnell et al., 2008) and theoretically (von der Fehr and Harbord, 1992; Newbery, 1998; Green, 1999).

Thus firms actually commit to supply functions also in the general case.

Bertrand and Cournot competition can be seen as two extreme forms of supply function competition. Thus, it is not surprising that the competitive effects of strategic forward contracting are ambiguous in supply function markets. Newbery (1998) considers cases where the demand variation is bounded, so that the market has multiple equilibria. He shows that the outcome depends on how firms coordinate their strategies in the spot market. Green (1999) shows that forward contracting strategies are neutral (neither pro- nor anti-competitive) in markets with linear marginal costs and linear demand if producers coordinate to linear supply function equilibria. Holmberg (2011) considers a sufficiently wide support of the demand shock density, so that the spot market has a unique equilibrium. He shows that contracting strategies in markets with supply function competition are substitutes (pro-competitive) when marginal costs are convex and residual demand is concave and that the reverse is true when marginal costs are concave and residual demand is convex. Herrera-Dappe (2008) calculates asymmetric contracting equilibria and in his setting forward trading will decrease welfare.[5]

The difference in our setting compared to previous studies of strategic contracting, is that option contracting may have an impact on whether actions are strategic complements or substitutes. Willems (2005) generalizes Allaz and Vila's (1993) result by considering firms that sell a bundle of option contracts. As in Allaz and Vila (1993) a firm can commit to be tough by selling more contracts. However strategies are stronger strategic substitutes if option contracts are used, the incentive to sell contracts is therefore increased, and the equilibrium is more competitive. By being tough a firm lowers the spot price, which reduces the number of option contracts that are exercised by competitors.[6] This softens the competitors' actions more than with forward contracts. In addition, by selling physical call options a firm can commit to a positively sloped supply function instead of playing Cournot. As this makes the competitors' residual demand more elastic, it toughens their response. Actions become weaker strategic substitutes or even complements. Hence, firms will commit less or nothing at all. Therefore the market becomes less competitive with physical options compared to financial options. In our current paper we show that in a supply function model, the physical and the financial option contracts are equivalent, as both contracts affect the slope of the offer functions in the second stage. We also show that firms prefer contracting curves with a negative slope, so that the slope of competitors' residual demand becomes less elastic.

Willems et al. (2009) give our theoretical result some empirical support. They test what type of forward contracting is congruent with the observed data for the German electricity market. They compare standard forward contracts and load

[5] Anderson and Xu (2005), Anderson and Hu (2008), Aromi (2007), Chao and Wilson (2005) and Niu et al. (2005) have also analyzed how exogenously given forward or option contracts influence supply function competition. But they do not analyze to what extent contracting is strategically driven.

[6] A similar mechanism occurs for forward contracts, if there is imperfect arbitrage between the forward and the spot market. In that case the firms' contract sales influence the forward price and therefore also its competitors' contract sales as in Green (1999) and Holmberg (2011).

following contracts. With the load following contracts, the firm has sold a set of option contracts such that for each price level in the spot market, the same fraction of output is hedged. The German data indicates that the pure forward contract (the option contract with zero strike price) fits the observed market outcomes better than the load following contracts which imply a number of call options with positive strike price being sold. This is in line with the predictions of our paper. Firms do not have an incentive to *sell* option contracts with high strike prices, but to buy those contracts.

3 Model

Producers' trading strategies in the electricity market are modeled as a subgame perfect Nash equilibrium (SPNE) of a two-stage game. In the first stage, risk-neutral producers commit by strategically choosing a portfolio of call option contracts with a spectrum of strike prices. In the second stage firms compete in the spot market. Firms' contracting decisions are made simultaneously. Similar to Allaz and Vila (1993) producers announce their contracting decisions; and risk-neutral, non-strategic consumers ensure that the price of each option contract rules out any arbitrage opportunities.[7] We model the second stage spot market as a uniform-price auction in which sellers simultaneously submit supply functions. After these offers have been submitted, an additive demand shock is realized. The distribution of the shock is common knowledge.

A physical call option gives the buyer the right, but not the obligation, to procure one unit of the good in the spot market from the seller at a predetermined strike price r.[8] In stage 1, firm $i \in \{1, 2, ..., N\}$ decides how many physical option contracts to sell at each strike price. The decision is represented by the differentiable option distribution function $X_i(r) : [0, \infty) \to \mathbb{R}$, the amount of option contracts sold with a strike price equal to or below r. Firm i can decide to go short ($X_i(r) > 0$) or long ($X_i(r) < 0$). We define the option density function by $x_i(r) = X_i'(r)$. We also introduce $X_{-i}(r) = \sum_{j \neq i} X_j(r)$. Let $\sigma(r)$ be the price of an option with strike price r in the contracting market. Hence, a producer's revenue from sales of call options in the contracting market is given by:

$$\int_0^{\overline{p}} \sigma(r) \cdot dX_i(r) + \sigma(0) \cdot X_i(0),$$

where we have assumed that 0 and $\overline{p}$ are the lowest and highest prices in the market, respectively. $X_i(0)$ is the amount of sold forward contracts and $\sigma(0)$ is the forward price.

Each producer's supply decision in stage 2, the spot market, is represented by a monotonic and differentiable net-supply function (output net of physical option contracts) denoted by $S_i(p)$. Let $S_{-i}(p) = \sum_{j \neq i} S_j(p)$ be the aggregate net-supply

[7] This assumption is used in most studies of strategic contracting, e.g. Allaz and Vila (1993) and Newbery (1998).

[8] As in Chao and Wilson (2005), option contracts are physical in our model specification, but we also show that the results under financial contracts are identical.

function of all firms, except firm i. The total output function of firm i after stage 2 is equal to $Q_i(p) := S_i(p) + X_i(p)$. We let $Q_{-i}(p) := \sum_{j \neq i} Q_j(p)$.

As in Klemperer and Meyer (1989), the electricity demand $D(p, \varepsilon)$ is realized after offers to the spot market have been submitted. The demand function is differentiable and depends on the spot price p and is subject to an exogenous additive shock, ε. Hence,

$$D(p, \varepsilon) = D(p) + \varepsilon. \tag{1}$$

Conditional on the symmetric information available to consumers and producers in stage 1, the shock density and the probability distribution are denoted by $f(\varepsilon)$ and $F(\varepsilon)$, respectively. The shock density has support on $[0, \overline{\varepsilon}]$ and on this interval we assume that $f(\varepsilon) > 0$. As long as the net-supply of each firm is optimal ex-post (after the shock has been realized), the probability distribution of the shock has itself no influence on optimal bidding in the spot market. However, the distribution function will have an impact on the contract prices, and contracting positions that the firms take.

Due to uncertainties in the demand curve, firms face an unknown residual demand curve in the spot market. The additive demand shock will shift the residual demand curve of firm i horizontally, so residual demand curves do not cross each other. As in Anderson and Philpott (2002a), we use the market distribution function $\psi_i(p, s)$ to characterize the residual demand curves.[9] For given offers of the competitors this function returns the probability that an offer (p, s) from firm i is rejected. In our setting this implies that

$$\psi_i(p, s) = F(\underbrace{Q_{-i}(p) + X_i(p) + s - D(p)}_{\varepsilon}). \tag{2}$$

Note that ε and ψ_i are constant along each realized residual demand curve.

The market clearing price in the spot market is for any given demand shock, ε, defined implicitly by the market clearing condition: aggregate supply should be equal to total demand. The price function $P(\varepsilon)$ maps the demand shock ε, to the market equilibrium price p.

$$P(\varepsilon) : \varepsilon \mapsto p : \sum_{i=1}^{N} Q_i(p) = D(p) + \varepsilon.$$

To guarantee existence of an equilibrium price, we assume as in Klemperer and Meyer (1989) that all agent's profits will be zero if the market does not clear.

A producer's revenue from selling in the spot market is given by:

$$P(\varepsilon) \cdot S_i(P(\varepsilon)).$$

All call options that are in the money will be exercised (i.e. those with a strike price r below the spot market price $P(\varepsilon)$). Thus for a shock outcome ε, producer i receives an additional revenue flow due to the exercise of options

[9] Anderson and Philpott's method is more general (it is not necessarily restricted to additive demand shocks) than the approach used by Klemperer and Meyer (1989). In our case Anderson and Philpott's method simplifies the proof of ex-post optimality of the net-supply curves.

$$\int_0^{P(\varepsilon)} r \cdot dX_i(r).$$

Firm $i \in \{1, 2, ..., N\}$ has a cost function, $C_i(\cdot)$, which is common knowledge, increasing, convex and twice continuously differentiable. There are no capacity constraints. Thus the total production cost for firm i when shock ε is realized is given by[10]

$$C_i(Q_i(P(\varepsilon))).$$

In stage 1, firm i's expected profit from trading in the contract and the spot markets is equal to the sum of contract revenue, expected spot market revenue, and expected revenue from exercised option contracts, minus the expected production cost:

$$\begin{aligned} E_\varepsilon [\pi_i(\varepsilon)] &= \int_0^{\overline{p}} \sigma(r) \cdot dX_i(r) + \sigma(0) X_i(0) + E_\varepsilon [P(\varepsilon) \cdot S_i(P(\varepsilon))] \\ &\quad + E_\varepsilon \left[\int_0^{p(\varepsilon)} r \cdot dX_i(r) \right] - E_\varepsilon [C_i(Q_i(P(\varepsilon)))]. \end{aligned} \tag{3}$$

Risk-neutral, non-strategic consumers trade in the contract market and ensure that the following non-arbitrage condition is satisfied for each strike price r.

$$\forall r: \quad \sigma(r) = E_\varepsilon [\min(P(\varepsilon) - r, 0)]. \tag{4}$$

Hence, the price of the option σ should equal the expected savings for consumers, which are able to obtain the good at the strike price r instead of paying the spot price $P(\varepsilon)$.

4 Analysis

The subgame perfect Nash equilibrium of the game with strategic contracting is solved by backward induction. Sequentially rational spot market bids in stage 2 are analyzed in Section 4.1. In Section 4.2, we rely on non-arbitrage conditions to derive the expected profit in stage 1 given the contracting position of firms. We can then derive conditions for optimal contracting in stage 1 in Section 4.3.

4.1 The spot market

In the last stage of the game, producers observe/infer competitors' portfolio of option contracts and then submit net-supply function offers to the uniform-price auction of the spot market. In the subgame equilibrium of the spot market, each firm i chooses its net-supply function $S_i(p)$ to maximize the firm's expected profit

[10]Note that we solve the first stage of the game for general cost functions, but restrict ourselves to zero costs once we solve the first stage equilibrium.

given the competitors' spot market bids $S_{-i}(p)$. We rewrite the pay-off in (3) to emphasize that the revenue from sold contracts is now sunk.[11]

$$\pi_i(p,s) = \underbrace{\int_0^{\overline{p}} \sigma(r)\,dX_i(r) + \sigma(0)\,X_i(0)}_{\text{Sunk in second stage}} + \int_0^p r dX_i(r) \qquad (5)$$
$$+ \underbrace{p\,s}_{\text{Spot Revenue}} - \underbrace{C_i\left[X_i(p)+s\right]}_{\text{Production Cost}}.$$

A firm chooses its bidfunction $S_i(p)$ to maximize its expected profit which can be written as the following line integral (Anderson and Philpott, 2002a):

$$\max_{S_i(p)} \int_{S_i(p)} \pi_i(p,s) d\psi_i(p,s). \qquad (6)$$

Proposition 1 [12] *If the first order derivatives of π_i and the second order derivatives of ψ_i exist, and $S_i(p)$ is differentiable solution of the program in Equation 6, then for all points on the function $S_i(p)$, the residual demand function and the iso-profit functions are tangent, i.e. $\forall(s,p)$ such that $s = S_i(p)$,*

$$\frac{\partial \pi_i}{\partial s} \cdot \frac{\partial \psi_i}{\partial p} - \frac{\partial \pi_i}{\partial p} \cdot \frac{\partial \psi_i}{\partial s} = 0. \qquad (7)$$

Proof. We can rewrite equation 6 as

$$\max_{S(p)} \int_0^{\overline{p}} \underbrace{\pi_i(p,S(p)) \cdot \left[\frac{\partial \psi_i}{\partial s}(p,S(p)) \cdot S'(p) + \frac{\partial \psi_i}{\partial p}(p,S(p))\right]}_{L(p,S(p),S'(p))} dp.$$

Next we use the Euler equation $\frac{\partial L}{\partial S} - \frac{d}{dp}(\frac{\partial L}{\partial S'}) = 0$, and

$$\frac{\partial L}{\partial S} = S'\left[\frac{\partial \pi_i}{\partial s}\frac{\partial \psi_i}{\partial s} + \pi_i \frac{\partial^2 \psi_i}{\partial s^2}\right] + \left[\frac{\partial \pi_i}{\partial s}\frac{\partial \psi_i}{\partial p} + \pi_i \frac{\partial^2 \psi_i}{\partial s \partial p}\right].$$
$$\frac{d}{dp}\left(\frac{\partial L}{\partial S'}\right) = S'\left[\frac{\partial \pi_i}{\partial s}\frac{\partial \psi_i}{\partial s} + \pi_i \frac{\partial^2 \psi_i}{\partial s^2}\right] + \left[\frac{\partial \pi_i}{\partial p}\frac{\partial \psi_i}{\partial s} + \pi_i \frac{\partial^2 \psi_i}{\partial s \partial p}\right]$$

[11] Note that the pay-off is identical for financial contracts. With such contracts all physical sales take place in the spot market, but on the other hand the producer has to pay the insurance $(p-r)X_i(p)$ to the counter-parties of the contracts that are in the money. Thus

$$\pi_i(p,s) = \underbrace{\int_0^{\overline{p}} \sigma(r)\,dX_i(r) + \sigma(0)\,X_i(0)}_{\text{Sunk in second stage}} - \underbrace{\int_0^{\overline{p}} \max\{p-r,0\} dX_i(r) - pX_i(0)}_{\text{Insurance Payment}}$$
$$+ \underbrace{p\,(X_i(p)+s)}_{\text{Spot Revenue}} - \underbrace{C_i\left[X_i(p)+s\right]}_{\text{Production Cost}},$$

which gives an identical pay-off.

[12] This is the same first-order condition as derived for uncontracted firms by Anderson and Philpott (2002a)

to derive the necessary first order condition. This is possible as the first order derivatives of π_i and the second order derivatives of ψ_i exist. Simplifying the Euler equation gives immediately equation 7. ■

In our setting, the residual demand of firm i, $D(p) + \varepsilon - Q_{-i}(p) - X_i(p)$, is a function of the price and the demand shock is additive. Thus the possible realizations of the residual demand curve never cross each-other. Hence, $\psi_i(p, s)$ is constant along any realized residual demand curve, and each contour of $\psi_i(p, s)$ corresponds to a realization of the residual demand curve. Thus the net-supply curve $S_i(p)$ is ex-post optimal and it crosses each residual demand curve at a point where the latter is tangent to the firm's iso-profit line as in Fig. 2.

From (2) and (5) we have

$$\frac{\partial \psi_i}{\partial s} = f(Q_{-i}(p) + X_i(p) + s - D(p)) \tag{8}$$

$$\frac{\partial \psi_i}{\partial p} = \left[Q'_{-i}(p) + X'_i(p) - D'(p)\right] \cdot f(Q_{-i}(p) + X_i(p) + s - D(p)) \tag{9}$$

$$\frac{\partial \pi_i}{\partial s} = p - C'_i(X_i(p) + s) \tag{10}$$

$$\frac{\partial \pi_i}{\partial p} = s + X'_i(p) \cdot [p - C'_i(X_i(p) + s)] \tag{11}$$

We can use equations (8-11) and (7) to derive a generalized version of the first-order condition in Klemperer and Meyer (1989) that considers contracts:

$$\forall i: \quad S_i(p) + [p - C'_i(Q_i(p))] \left[D'(p) - Q'_{-i}(p)\right] = 0. \tag{12}$$

The equation in (12) can be rewritten as follows

$$\frac{p - C'_i(Q_i)}{p} = \frac{-S_i/p}{\frac{\partial D}{\partial p} - \frac{\partial Q_{-i}}{\partial p}} = -\frac{1}{\varepsilon_{\text{Residual Demand}}}.$$

An intuitive interpretation of the Klemperer-Meyer condition is that each producer acts as a monopolist with respect to its residual demand curve (net of option contracts) for each shock outcome and the optimal price of a producer is given by the inverse elasticity rule (Tirole, 1988) for each shock outcome.

The following definition provides the notation for solutions to the system of ordinary differential equations (ODE).

Definition 2 *The tuple* $\left\{\check{S}_i(p)\right\}_{i=1}^{N} = (\check{S}_1(p), ..., \check{S}_i(p), ..., \check{S}_N(p))$ *is an ODE solution of the second stage game, if each bid function* $\check{S}_i(p)$ *is a continuously differentiable function on the price interval* $[0, \overline{p})$, *and the bid functions solve jointly the system of the ordinary differential equations (12).*

By definition an ODE solution satisfies the necessary first-order condition of a local profit maximum. To ensure that the solution is also a global profit maximum

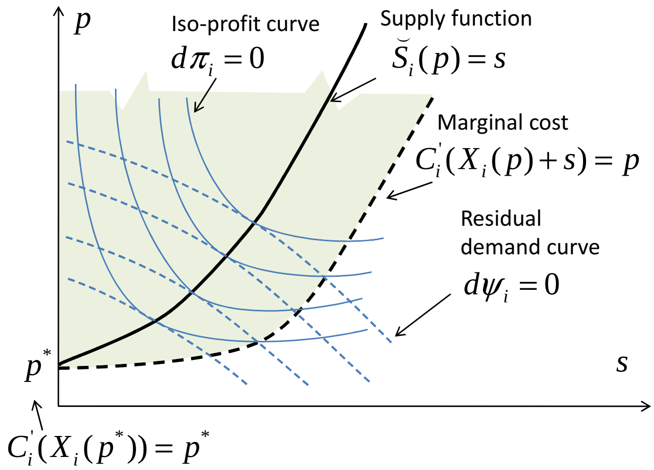

Figure 2: Iso-profit lines are tangent to residual demand curves at points along ex-post optimal net-supply curves.

we need to verify a global second-order condition. The proposition below states that the ODE solution is a subgame equilibrium if the spot supply functions $\check{S}_i(p)$ are monotonic and in the shaded area in Figure 2.[13]

Proposition 3 *The ODE solution $\left\{\check{S}_i(p)\right\}_{i=1}^{N}$ constitutes an SFE in the second-stage, if for each firm $i \in \{1, \ldots, N\}$ the spot market sales are non-negative $\check{S}_i(p) \geq 0$, strictly monotonic $\check{S}_i'(p) > 0$, and the spot price is not below the marginal cost of production $p \geq C_i'\left(X_i(p) + \check{S}_i(p)\right)$.*

Proof. Consider an arbitrary firm i. It takes its own contract position as given and assumes that its competitors bid $\check{S}_{-i}(p)$ as net-supply. We prove that bidding $\check{S}_i(p)$ is profit maximizing for firm i.

It is never profitable to sell in the spot market at prices below marginal cost. Hence spot prices need to be above marginal costs, and from (2) it follows that $\frac{\partial \psi_i}{\partial s} > 0$. Hence $\check{S}_i(p)$ is the best response to $\check{S}_{-i}(p)$ when each iso-profit curve π^* is (weakly) to the right of the residual demand curve ψ^* to which it is tangent. This can be seen in Figure 3. We prove this by looking at the horizontal distance between the iso-profit and the residual demand function.

At each point $\left(p^*, \check{S}_i(p^*)\right)$ there is an iso-profit line and a residual demand function which are tangent. The property $\frac{\partial \pi_i}{\partial s} > 0$ implies that the iso-profit line passing through this point can be written as a function of the price, $s_i^{\pi}\left(p; \pi_i^*\right)$. Similarly, the residual demand curves can be written as a function of price $s_i^{\psi}\left(p; \psi^*\right)$. Define $\gamma(p)$ as the difference between those curves $\gamma\left(p\right) := s_i^{\pi}\left(p, \pi_i^*\right) - s_i^{\psi}\left(p, \psi^*\right)$. See Figure 3. We now prove $\gamma(p) \geq 0$ for all $p \neq p^*$, which ensures that the iso-profit curve is (weakly) to the right of the residual demand curve. We do this be showing that $\gamma\left(p\right)$ is pseudo-convex, i.e. $\gamma'\left(p\right) \leq 0$ if $p \leq p^*$ and $\gamma'\left(p\right) \geq 0$ if $p \geq p^*$. We will use the fact that $\check{S}_{-i}(p^*)$ is an ODE-solution to prove this. From

[13]The proof generalizes Holmberg et al. (2008) for spot markets without contracting.

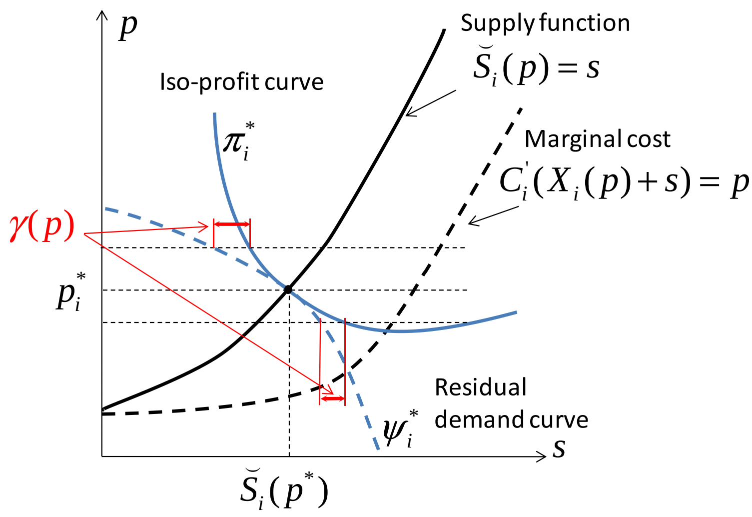

Figure 3: Second-stage profit optimization.

(8), (10), and (12) we have that

$$\begin{aligned}\gamma'(p) &= -\frac{s_i^\pi + X_i'(p)\cdot\left[p - C_i'(X_i(p)+s_i^\pi)\right]}{p - C_i'(X_i(p)+s_i^\pi)} + \check{Q}_{-i}'(p) + X_i'(p) - D'(p) \\ &= \frac{\check{S}_i(p)}{p - C_i'\left(X_i(p)+\check{S}_i(p)\right)} - \frac{s_i^\pi\left(p,\pi_i^*\right)}{p - C_i'(X_i(p)+s_i^\pi\left(p,\pi_i^*\right))}.\end{aligned}$$

Thus $\gamma(p)$ has the desired property when $\check{S}_i(p) \geq s_i^\pi(p,\pi_i^*)$ for $p \geq p^*$ and $\check{S}_i(p) \leq s_i^\pi(p,\pi_i^*)$ for $p \leq p^*$. In Figure 3 this implies that the iso-profit line should be left of $\check{S}_i(p)$ for prices above p^*, and to the right for prices below. The curves have this property near p^* as $\check{S}_i'(p) > 0$ and as $s_i^\pi(p,\pi_i^*)$ is perpendicular to $\check{S}_i(p)$ at p^*. Finally, we realize that $s_i^\pi(p,\pi_i^*)$ cannot cross $\check{S}_i(p)$ at prices other than p^*, because $s_i^\pi(p,\pi_i^*)$ would need to be perpendicular to the strictly monotonic curve $\check{S}_i(p)$ at such crossings as well, but $s_i^\pi(p,\pi_i^*)$ cannot have this shape as it is a function of the price. Thus we have proven that $\check{S}_i(p^*)$ is the best response for the shock outcome ε^*. We can use the same argument for all shocks and all firms and we can conclude that the ODE solution is a Nash equilibrium. ■

The following proposition shows if firms are identical and hold identical contract positions then, under certain conditions, bidding in the second stage will be symmetric. We will use this proposition later to uniquely define market outcomes.

Proposition 4 *Consider a market where N producers have symmetric costs $C(q)$ and hold symmetric differentiable contract positions $X(p)$. If there exists a price p^* such that $p^* = C'(X(p^*))$ and $\underline{\varepsilon} + D(p^*) \leq NX(p^*) < \overline{\varepsilon} + D(p^*)$, then there is no asymmetric ODE solution that could constitute an SFE.*

Proof. Note that (12) can be written as:

$$S_i(p) + \left[p - c'(S_i(p))\right]\left[d'(p) - S_{-i}'(p)\right] = 0. \tag{13}$$

if we introduce the variables $d'(p) := D'(p) - (N-1)X'(p)$ and $c'(S_i(p)) := C'(X(p) + S_i(p))$.

We now can use a similar argument as in Proposition 3 by Klemperer and Meyer (1989), but have to generalize their claim to an N firms setting. To achieve this we rewrite the differential equation above on the standard form:[14]

$$S_i'(p) = \frac{d'(p)}{N-1} - \frac{S_i(p)}{p - c'(S_i(p))} + \frac{1}{N-1} \sum_{k=1}^{N} \frac{S_k(p)}{p - c'(S_k(p))}. \tag{14}$$

Step 1 First we analyze the case where no firm produces at price p^*, hence output is symmetric and zero at this price level $S_i(p^*) = 0$ for $i = 1, \ldots, N$. As in Klemperer and Meyer (1989), we prove that solutions that are symmetric at p^* are also symmetric for larger price levels. Make the contradictory assumption that solutions become asymmetric. Without loss of generality we assume that firm 1 has the highest net-supply and firm 2 has the lowest net-supply at some price $\tilde{p} > p^*$, i.e. $S_2(\tilde{p}) \leq S_i(\tilde{p}) \leq S_1(\tilde{p})$ and $S_2(\tilde{p}) < S_1(\tilde{p})$. Subtracting KM equations in (14) for differentiable net-supply offers from firm 1 and firm 2 yields:

$$S_1'(p) - S_2'(p) = \frac{S_2(p)}{p - c'(S_2(p))} - \frac{S_1(p)}{p - c'(S_1(p))}.$$

But this would imply that $S_1'(p) < S_2'(p)$ whenever $S_1(p) > S_2(p)$, which is inconsistent with $S_1(p^*) = S_2(p^*) = 0$ and the asymmetry supposition that $S_2(\tilde{p}) < S_1(\tilde{p})$ for some $\tilde{p} > p^*$.

Step 2 Thus the only remaining asymmetric alternative is when $S_i(p^*) \neq 0$ for some firm(s). We now show that this is impossible. In case $S_i(p^*) > 0$ and $c'(S_i(p^*)) > p^*$, this would imply that firm i is sometimes selling below marginal cost, so there is a profitable deviation for firm i. Similarly, we can rule out cases for which $S_i(p^*) < 0$ and $c'(S_i(p^*)) < p^*$. If $S_i(p^*) \neq 0$, and $c'(S_i(p^*)) = p^*$,[15] then this would according to (14) imply that at least some of the supply functions are not differentiable at p^*. Thus they are not ODE solutions according to Definition 1, and we can rule out that asymmetric ODE solutions can constitute SFE. ■

4.2 First Period Profit function under perfect arbitrage

The no-arbitrage condition (4) is an identity, which is true for any contracting choices of the producers. By reversing the order of integration and using the

[14] Baldick and Hogan (2002) show how the Klemperer and Meyer equations for uncontracted firms can be written in the standard form for N firms.

[15] This can occur if $c'(Q)$ is constant for low levels of production.

arbitrage condition, we can rewrite the contracting revenue of firm i:

$$\begin{aligned}
& \int_0^{\overline{p}} \sigma\left(r\right) \cdot dX_i\left(r\right) + \sigma(0) \cdot X_i(0) & (15) \\
= \; & \int_0^{\overline{p}} E_\varepsilon\left[\min\left(P\left(\varepsilon\right) - r, 0\right)\right] \cdot dX_i\left(r\right) + E_\varepsilon\left[P(\varepsilon)\right] \cdot X_i\left(0\right) \\
= \; & E_\varepsilon\left[\int_0^{P(\varepsilon)} \left(P\left(\varepsilon\right) - r\right) \cdot dX_i\left(r\right) + P(\varepsilon) \cdot X_i\left(0\right)\right] \\
= \; & E_\varepsilon\left[P\left(\varepsilon\right) \cdot X_i(P(\varepsilon))\right] - E_\varepsilon\left[\int_0^{P(\varepsilon)} r \cdot dX_i\left(r\right)\right].
\end{aligned}$$

This expression shows that under perfect arbitrage the contracting revenue is equal to the expected revenue that the firm forgoes by not selling its output directly into the spot market, minus the expected revenue that the firm receives from the exercised options. Substituting the expected contract revenue (15) in the pay-off (3) and simplifying we obtain:

$$E_\varepsilon\left[\pi_i\left(\varepsilon\right)\right] = E_\varepsilon\left[P(\varepsilon) \cdot Q_i(P(\varepsilon)) - C_i(Q_i(P(\varepsilon)))\right], \tag{16}$$

where we use the fact that $Q_i(p) = S_i(p) + X_i(p)$. Thus firm i's pay-off does not depend on the contract position directly, but the firm will decide its contract level in stage 1 for strategic reasons. By selling more or less contracts (X_i), it can strategically change the price it might be able to charge in the spot market (P) or affect its output (Q_i).

4.3 Strategic contracting

The next step is to determine the option contracts that firm 1 will sell in stage 1 to maximize its profit. It will do so taking into account the option sales of its competitors and how contracts influence the equilibrium that will be played in the sub-game. Its stage 1 profit is given by (16) $pQ_1(p) - C_1\left(Q_1(p)\right)$. The spot price is determined by the subgame equilibrium of the second stage satisfying (12). Moreover, clearing of the spot market requires that spot demand must equal spot supply. For simplicity we assume further that costs are set to zero $C_i(Q_i) \equiv 0$. Thus firm 1's optimal contracting level is determined by the following optimization problem.

$$\max \int_0^{\overline{p}} p \cdot Q_1(p) \cdot f\left(\varepsilon\left(p\right)\right) \cdot \underbrace{u(p)\; dp}_{d\varepsilon}. \tag{17}$$

$$s.t. \begin{cases} \forall i \in \{1, \ldots, N\}: & \sum_{j \neq i} Q_j'(p) = D'(p) + \frac{Q_i(p) - X_i(p)}{p} \\ & \varepsilon' = u(p) \\ & D\left(p\right) + \varepsilon(p) = \sum_i \; Q_i\left(p\right) \end{cases}$$

Note that in the constraints of the optimal control problem we have replaced net-supply $S_i\left(p\right)$ in the Klemperer and Meyer equation by $Q_i(p) - X_i(p)$. In equilibrium, firm 1 takes other firms' contracting decisions $\{X_i(p) : i \neq 1\}$ as given. In the optimal control problem, $X_1(p)$ only appears in the first constraint if $i = 1$. This implies that this constraint is never going to be binding, as firm 1 can freely choose $X_1(p)$ to satisfy this equation without influencing other constraints or the objective function. In the objective function and in the second and third constraints, competitors' total output matters, but not how it is divided between these firms. For cases $i \neq 1$, we can therefore sum up the remaining $(N-1)$ equations of the first constraint into one constraint. Using that $F(\overline{\varepsilon}) = 1$ and integration by parts we can now rewrite the dynamic optimization problem as follows:

$$\max \int_0^{\overline{p}} (pQ_1)' \left[1 - F(\varepsilon)\right] dp \tag{18}$$

subject to

$$\begin{aligned} (N-1)pQ_1' + (N-2)pQ_{-1}' &= (N-1)pD' + Q_{-1} - X_{-1} && (19) \\ Q_{-1} + Q_1 &= D + \varepsilon, && (20) \end{aligned}$$

where, as before, the subscript $_{-i}$ refers to the sum of a variable over all firms, excluding firm i. Thus firm 1's expected profit is given by the integral of its marginal profit $\frac{\partial}{\partial p}\left(p\, Q_1(p)\right)$ at price p, weighted by $1 - F(\varepsilon(p))$, the probability that the realized price is larger than p, and this also makes sense intuitively.

The Proposition below states an equation from which firm's optimal contracting can be calculated. Note that $\frac{d}{d\varepsilon}\left(\frac{1-F(\varepsilon)}{f(\varepsilon)}\right) \leq \frac{1}{N-1}$ is a fairly non-restrictive condition, as it is satisfied for all probability distributions with a decreasing inverse hazard rate, which is equivalent to an increasing hazard rate. This includes most probability distributions that one encounters in practice, such as the normal distribution, and the uniform-distribution.

Proposition 5 *Differentiable solutions to the dynamic optimization problem defined by (18), (19) and (20) are symmetric and satisfy the equations*

$$\begin{aligned} \frac{1 - F(NQ - D)}{f(NQ - D)} &= \frac{1}{N-1}\left[(N-1)^2 (pQ)' - (N-2)\left(NQ + pD'\right)\right] && (21) \\ X &= -p\,(N-1)\,Q' + pD' + Q. \end{aligned}$$

Provided that competitors' contract $X_i(p) = X\,(p)$ for $i \neq 1$, firm 1 globally maximizes its expected profit by also choosing $X_1(p) = X\,(p)$ if and only if $\frac{d}{d\varepsilon}\left(\frac{1-F(\varepsilon)}{f(\varepsilon)}\right) \leq \frac{1}{N-1}$ for $\forall \varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$.

Proof. <u>**Step 1**</u> We first simplify the dynamic optimization problem by rewriting the constraints and then substitute them into the objective function. By adding the constraint (19) and $N-1$ times constraint (20) we get the following:

$$(N-1)(pQ_1)' = (N-1)\,(pD)' - X_{-1} + (N-1)\varepsilon - (N-2)(pQ_{-1})'. \tag{22}$$

We use the identity in equation (20) to write $(pQ_{-1})'$ as a function of $(pQ_1)'$.

$$(pQ_{-1})' = (pD)' + (p\varepsilon)' - (pQ_1)',$$

which we can substitute into (22), to give an expression for the marginal profit $(pQ_1)'$

$$(pQ_1)' = (pD)' - X_{-1} + \varepsilon - (N-2) \cdot p \cdot \varepsilon'.$$

Substituting this marginal profit into the objective function in (18) gives the following optimization problem:

$$\max \int_0^{\overline{p}} \{\underbrace{(pD)' - X_{-1}}_{h_1(p)} + \varepsilon - (N-2) \cdot p \cdot \varepsilon'\} \left[1 - F(\varepsilon)\right] dp. \tag{23}$$

Step 2 We now derive the first order conditions of the optimal solution by applying integration by parts to the last term of the integrand. First we rewrite (23) as the sum of two integrals:

$$\max \int_0^{\overline{p}} \{h_1(p) + \varepsilon\} \left[1 - F(\varepsilon)\right] dp \ - (N-2) \int_0^{\overline{p}} p \cdot (G(\varepsilon) - G(\overline{\varepsilon}))' dp,$$

where $G(\varepsilon) = \int_0^{\varepsilon} (1 - F(t)) dt$. Note that $(G(\overline{\varepsilon}))'$ is zero. The second term can be rewritten using integration by parts:

$$\max \int_0^{\overline{p}} \underbrace{\{[h_1(p) + \varepsilon] \left[1 - F(\varepsilon)\right] + (N-2) \left(G(\varepsilon) - G(\overline{\varepsilon})\right)\}}_{\theta(p,\varepsilon)} dp.$$

This function only depends on $\varepsilon(p)$, and we can maximize the integral by maximizing $\theta(p, \varepsilon)$ for each p.

$$\begin{aligned} \frac{\partial \theta(p, \varepsilon)}{\partial \varepsilon} &= (N-1)\left(1 - F(\varepsilon(p))\right) - \left(h_1(p) + \varepsilon(p)\right) f(\varepsilon(p)) \qquad (24) \\ &= f(\varepsilon(p)) \cdot \left[\frac{(N-1)\left(1 - F(\varepsilon(p))\right)}{f(\varepsilon(p))} - \left(h_1(p) + \varepsilon(p)\right)\right]. \end{aligned}$$

Thus the first order condition of this optimization problem is:

$$\frac{\left[1 - F(\varepsilon)\right](N-1)}{f(\varepsilon)} - \left[h_1(p) + \varepsilon\right] = 0. \tag{25}$$

Step 3 We want to know under what circumstances solutions to this condition globally maximizes profits. Let $\widetilde{\varepsilon}(p)$ be a solution to this equation for a given contracting choice of the competitors, $X_{-1}(p)$. We see from (24) that $\frac{\partial \theta(p,\varepsilon)}{\partial \varepsilon}$ has the same sign as $\frac{(N-1)(1-F(\varepsilon(p)))}{f(\varepsilon(p))} - (h_1(p) + \varepsilon(p))$. Thus provided that $\frac{d}{d\varepsilon}\left(\frac{1-F(\varepsilon)}{f(\varepsilon)}\right) \leq \frac{1}{N-1}$ we realize that for all price levels p:

$$\frac{\left(1 - F(\varepsilon)\right)(N-1)}{f(\varepsilon)} - \left(h_1(p) + \varepsilon\right) \begin{array}{l} \leq 0 \text{ if } \varepsilon > \widetilde{\varepsilon}(p) \\ = 0 \text{ if } \varepsilon = \widetilde{\varepsilon}(p) \\ \geq 0 \text{ if } \varepsilon < \widetilde{\varepsilon}(p). \end{array}$$

Accordingly, $\widetilde{\varepsilon}(p)$ globally maximizes θ at each price if $\frac{d}{d\varepsilon}\left(\frac{1-F(\varepsilon)}{f(\varepsilon)}\right) \leq \frac{1}{N-1}$. But if this condition on the inverse hazard rate is violated at some $\varepsilon^* \in [\underline{\varepsilon}, \overline{\varepsilon}]$ then $\widetilde{\varepsilon}(\cdot)$ will be at a profit minimum at the price level where $\widetilde{\varepsilon}(p) = \varepsilon^*$.

Step 4. The next step is to verify that solutions must be symmetric to our problem. In an equilibrium where all firms choose contracting optimally, equations corresponding to (25) must be simultaneously satisfied for all firms. This is only possible if $\forall i,\ h(p) = h_i(p)$. Thus it follows from the definition of $h_i(p)$ in (23) that contracting must be symmetric in equilibrium, i.e. $X(p) = X_i(p)$.

Step 5. Finally we rewrite the first order condition in (25), which is a function of $\varepsilon(p)$, to derive a relation that can be used to determine the optimal output that firms commit to in equilibrium. Substituting the definition of $h_1(p)$ in (23) into (25) we find:

$$\frac{[1 - F(\varepsilon)]\,(N-1)}{f(\varepsilon)} - [(pD)' - X_{-1} + \varepsilon] = 0. \tag{26}$$

As firms costs and their contracting positions are symmetric, it follows from Proposition 4 that differentiable net-supply offers to the spot market are symmetric as well. In a symmetric equilibrium it follows from the constraints in (22) and (20) that:

$$\begin{aligned} Q &= Q_i = \{D + \varepsilon\}/N \\ X &= X_i = (p \cdot D)' + \varepsilon - (N-1)(pQ)'. \end{aligned}$$

Equation (26) can now be written as a function of Q, using the last two expressions and eliminating ε.

$$\frac{1 - F(NQ - D)}{f(NQ - D)} = \frac{1}{N-1}\left[(N-1)^2(pQ)' - (N-2)\,(NQ + pD')\right].$$

■

Before presenting analytical solutions for contracting in equilibrium, we will have a brief look at the residual demand function that firm 1 faces to get some intuition. We get the residual demand by summing the constraints (19) and (20):

$$Q_1 = \underbrace{D + \varepsilon}_{I} - \underbrace{X_{-1}}_{II} - \underbrace{(N-1)\,p\,|D'|}_{III} - \underbrace{(N-1)\,Q_1'p}_{IV} - \underbrace{(N-2)pQ_{-1}'}_{V} \tag{27}$$

The first term is the demand function for different realizations of ε. The other terms correspond to the output of the competitors. II is the production that other firms have sold in the contract market. It corresponds to the Stackelberg effect. By being first movers, competitors can reduce the residual demand of firm 1. Competitors' net-sales in the spot market are proportional to the slope of their residual demand, which explains terms III and IV. If firm 1's bid function is flatter (Q_1' is large), the competitors will act more aggressively (and the residual demand that firm 1 faces decreases). Term V is an interaction effect between competitors of firm 1. If one competitor sets a flatter supply function, then other competitors will be more competitive as well, and the residual demand that firm 1 faces decreases.

It follows from (27) that it is going to be 'costly', either in terms of a reduced quantity or a reduced price, to set a positive slope $Q_1' > 0$, because it makes competitors' residual demand curves more elastic, which increases their output.

Thus we would intuitively expect that firm 1 would find it optimal to keep this slope relatively small or even negative. To achieve this and still sell a significant amount, it will be optimal to offer a relatively large quantity at $p = 0$ and then to keep output fairly inelastic or even backward bending in the whole price range. This is confirmed by the result below, which we derive for demand shocks that are Pareto distributed of the second-order (Holmberg, 2009), so that $\frac{1-F(\varepsilon)}{f(\varepsilon)} = \alpha\varepsilon + \beta$. This is a family of probability distributions with a wide range of properties. For example, we note that $\alpha = 0$ gives the exponential distribution and $\alpha = -1$ corresponds to uniformly distributed demand.

Proposition 6 *If the demand function $D(p) = -D_1 p$ is linear with $D_1 > 0$ and demand shocks are Pareto distributed of the second-order, so that $\frac{1-F(\varepsilon)}{f(\varepsilon)} = \alpha\varepsilon + \beta$ and $f(\varepsilon) = \beta^{1/\alpha}(\alpha\varepsilon + \beta)^{-1/\alpha - 1}$for $\varepsilon > 0$, where $\beta > 0$ and $\alpha \in \left(-\infty, \frac{1}{(N-1)N}\right)$, then*

$$Q(p) = \frac{\beta(N-1)}{1-\alpha(N-1)N} + \frac{\alpha(N-1)-(N-2)}{1-\alpha(N-1)N+(N-1)^2} D_1 p$$
$$X(p) = \frac{\beta(N-1)}{1-\alpha(N-1)N} + \frac{2\alpha(N-1)-2N+2}{1-\alpha(N-1)N+(N-1)^2} D_1 p$$

is a unique solution to (21), and it is a SPNE.

Proof. When demand shocks are Pareto distributed of the second-order, we can simplify equation (21) as follows:[16]

$$(N-1)[\alpha(NQ + D_1 p) + \beta] = Q + (N-1)^2 pQ' + (N-2)pD_1$$

This equation can be rewritten in the form

$$aQ + pQ' = g(p) \tag{28}$$

with

$$a = \frac{1-\alpha(N-1)N}{(N-1)^2}$$

and

$$g(p) = \frac{\beta}{N-1} + \frac{\alpha(N-1)-(N-2)}{(N-1)^2} pD_1.$$

We multiply both sides of (28) with the integrating factor p^{a-1} and then integrate both sides. As long as $a > 0$ or equivalently $\alpha < \frac{1}{(N-1)N}$, we have that $p^a Q(p)$ is zero at $p = 0$, so

$$\begin{aligned} Q(p) &= p^{-a}\int_0^p g(t)t^{a-1}dt \\ &= \frac{\beta}{a(N-1)} + \frac{\alpha(N-1)-(N-2)}{(a+1)(N-1)^2} D_1 p. \\ &= \frac{\beta(N-1)}{1-\alpha(N-1)N} + \frac{\alpha(N-1)-(N-2)}{1-\alpha(N-1)N+(N-1)^2} D_1 p. \end{aligned} \tag{29}$$

[16] Note that we use the fact that $(N-1)^2(pQ)' = (N-1)^2 pQ' + Q + (N-2)NQ$

We have from Proposition 5 that the contract level of the firm is given by the following expression:

$$\begin{aligned}
X(p) &= Q(p) - ((N-1)\,Q'(p) - D'(p)) \cdot p \\
&= \frac{\beta\,(N-1)}{1-\alpha\,(N-1)\,N} \\
&+ \frac{(2-N)\,\alpha\,(N-1) + (N-2)^2 - 1 + \alpha\,(N-1)\,N - (N-1)^2}{1-\alpha\,(N-1)\,N + (N-1)^2} D_1 p \\
&= \frac{\beta\,(N-1)}{1-\alpha\,(N-1)\,N} + \frac{2\alpha\,(N-1) - 2N + 2}{1-\alpha\,(N-1)\,N + (N-1)^2} D_1 p.
\end{aligned}$$

Now we want to check whether this solution is also a SPNE. It follows from Proposition 5 that this solution (globally) maximizes each firm's contracting decisions as, $\frac{d}{d\varepsilon}\left(\frac{1-F(\varepsilon)}{f(\varepsilon)}\right) = \alpha < \frac{1}{N-1}$. Thus according to Proposition 3 it also constitutes a Nash equilibrium as mark-ups are positive and

$$\begin{aligned}
S(p) = Q\,(p) - X\,(p) &= \frac{\alpha\,(N-1) - (N-2) - 2\alpha\,(N-1) + 2N - 2}{1-\alpha\,(N-1)\,N + (N-1)^2} D_1 p \\
&= \frac{-\alpha\,(N-1) + N}{1-\alpha\,(N-1)\,N + (N-1)^2} D_1 p
\end{aligned}$$

is positive and has a positive slope, because $D_1 > 0$ and $\alpha\,(N-1)\,N < 1$. Accordingly $Q(p)$ is sub-game perfect. ∎

Hence, when demand is linear, the contracting and output functions are also linear for a Pareto distribution of the second order. The net-supply is up-ward sloping, but the contracting function is backward bending; producers sell forward contracts and buy call options for strike prices above zero. The output function is also backward bending for $N > 2$ or when $\alpha < 0$. Hence firms produce less, although the demand shock increases. As a result prices increase steeply as demand shocks increases. Even in the alternative case where total output is forward bending (duopoly $N = 2$ and $\alpha \geq 0$) , the total output curve is still very steep. That is, the slope of the total output as a function of price is less than $|D'|$. This is lower than in the monopoly case without contracting, where the optimal output equals $p\,|D'|$. [17]

It follows from Proposition 6 that $Q'\,(p)$ becomes less negative when α increases, which corresponds to a higher demand uncertainty.[18] This uncertainty mitigates the anti-competitive consequences of contracts, but in some cases $(N > 2)$ total supply is backward bending also for the highest demand uncertainty that we consider $\alpha = \frac{1}{(N-1)N}$. Demand becomes certain in the other limit when $\alpha \to -\infty$.

[17] Moreover, $\alpha \geq 0$ implies that the support of the probability density is infinite (Holmberg, 2009). Hence, there exist high prices for which, total output in an oligopoly market with contracting is lower than the monopoly output without contracting.

[18] For $\alpha \leq 0$, the demand shock range is $\left[0, \frac{\beta}{|\alpha|}\right]$, so a less negative α increases the range of demand shocks. For $\alpha \geq 0$, a larger α increases the thickness of the tail of the demand density (Holmberg, 2009).

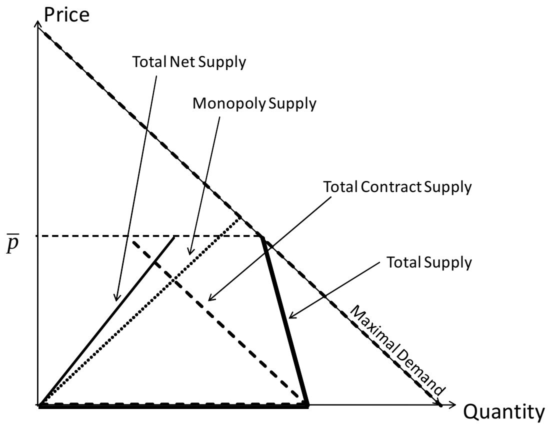

Figure 4: Total contracting and output for the SPNE of a duopoly market with uniformly distributed demand.

In this limit we have $Q(p) \to \frac{-|D'|p}{N}$, which is less than the monopoly output. Thus for sufficiently small α, social welfare is lower than in a monopoly market without contracts.

It is also straightforward to verify that total contracting in the market at price $p = 0$, increases with the number of firms. This ensures that the market becomes more competitive for low shock outcomes. However, the total output function will bend backwards more, as the number of firms increase. $NQ'(p)$ decreases with more firms in the market. Thus for $\alpha \geq 0$ (when the support of the shock density is infinite) the market will be less competitive for the highest shock outcomes if there are more firms in the market. Hence, our finding that firms have incentives to use option contracts to commit to fairly inelastic or even a backward bending output, becomes more apparent in markets with more firms. We can explain this with the interaction effect between competitors in (27). If one competitor sets a flatter supply function, then other competitors will be more competitive as well. Thus the payoff of having an inelastic or even backward bending output is greater in markets with more competitors.

5 Conclusion

Option contracts are very useful to hedge the risk of agents. However, in an oligopoly market they will also be used strategically by producers. Solving for a subgame perfect Nash equilibrium of a two-stage game with contracting and then spot market competition, we show that the strategic interaction implies that risk-neutral producers sell call option contracts at low strike prices and buy them at high strike prices. This trading strategy is called a bear call spread. Traders use it when speculating on a lower commodity price, but it also commits strategic firms to a fairly inelastic or even backward bending supply function in the spot market. This makes competitors' residual demand less elastic, so their mark-ups are high and the strategic firm can increase its profit at competitors' expense.

We show that the anti-competitive effects of strategic option contracting are most apparent when the number of firms is large and demand uncertainty is small. When demand uncertainty is small, strategic contracting results in spot prices higher than the monopoly price, which hurt both producers and consumers. When demand uncertainty becomes larger, it is optimal for firms to offer supply functions that have a less negative slope, as this allows them to benefit more from high demand realizations. Thus to avoid the anti-competitive effect of speculation, this suggests that option contracts should not be traded near delivery when firms have a good estimate of demand in the spot market. When more firms are active in the market, competition is improved for low demand realizations, but speculative strategic behavior for high demand realizations offsets this effect. As the number of firms increases a strategic interaction effect between competitors kicks in: If a negative supply slope induces a competitor to choose a less competitive supply function with a less positive slope, then this response will induce other competitors to further increase their mark-ups. Thus the payoff of having an inelastic or even backward bending output is greater in markets with more competitors. Unlike spot markets with Cournot competition (Willems, 2005), our results do not depend on whether contracts are financial or physical.

Oren (2005) recommends that electricity markets should use call options with high strike prices in other to steady the revenue flow of peak power plants in electricity markets. These plants have a high marginal cost and are used infrequently, so such a recommendation makes much sense from a hedging perspective. But our results indicate that there are also drawbacks with introducing option contracts with high strike prices, because especially large producers with market power have incentives to misuse them, i.e. to go short at high strike prices. This is something that market monitors should scrutinize.

In our model producers are risk neutral and arbitrage in the financial market is perfect. Therefore, commitment is costless. As this is not the case in practice, our results should be seen as a limiting case. With risk aversion, firms are expected to reduce tail risk and to hold contracting portfolios that are closer to their actual output, and therefore to offer supply functions that are less steep. Also transaction costs in the financial market are likely to reduce the profitability of speculative positions.

In practice firms will also use other commitment tactics than financial contracts, for instance by delegating decisions to managers, merging with downstream firms, and irreversible investments. We believe that the main intuition of our paper, that firms would like to commit to downward sloping supply functions, remains valid in those settings. In this sense our result has parallels in Zöttl (2010), who models the strategic (irreversible) investments of firms, where firms compete in quantities in a spot market with random demand. He shows that firms will over-invest in technology with low marginal costs (base-load), but choose total investment capacities that are too low from a welfare viewpoint.

6 References

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