Cournot versus Supply Functions: What does the data tell us?[1]

Bert WILLEMS

Tilburg University, TILEC & CentER
K.U.Leuven, Energy Institute & CES
b.r.r.willems@uvt.nl

Ina RUMIANTSEVA

Hannes WEIGT

Dresden University of Technology, Faculty of Business and Economics
Chair of Energy Economics and Public Sector Management
ina.rumiantseva@mailbox.tu-dresden.de
hannes.weigt@tu-dresden.de

Abstract

This paper compares two popular models of oligopolistic electricity markets, Cournot and the Supply Function Equilibrium (SFE), and then tests which model best describes the observed market data. Using identical demand and supply specifications, both models are calibrated to the German electricity market by varying the contract cover of firms. Our results show that each model explains an identical fraction of the observed price variations. We therefore suggest using Cournot models for short-term analysis, since these models can accommodate additional market details, such as network constraints, and the SFE model for long-term analysis (e.g., the study of a merger) since it is less sensitive to the calibration parameters selected.

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1 Introduction

Within the last two decades, as electricity markets throughout the world have undergone significant structural changes because of liberalization, a demand for sophisticated market analyses has emerged. Typically, such analyses are especially complex because the decentralized structure of the new markets increases the interactions between market players, and factors such as regulatory guidelines, market design (and redesign) and the particular nature of electricity itself must be considered. In addition, desire on the part of regulators and politicians to determine market outcomes, interest in simulating expected market situations and modeling the behavior of market players have fostered the development of novel approaches.

Since classical economic tools, such as the Herfindahl-Hirschman Index (HHI), are largely unsuitable for electricity markets, specific tools and models must be designed (Borenstein, et al. 1999).[2] Ventosa et al. (2005) identify three basic trends: optimization models, equilibrium models and simulation models. This paper will focus on two equilibrium models for oligopolisic wholesale electricity markets: the Cournot model and the Supply Function Equilibrium model (SFE). While standard Cournot models are simple to calculate, the results often do not represent reasonable market outcomes. For realistic values of demand elasticities, prices are too high and output too low. In Cournot models for electricity markets, it is, therefore, usually assumed that a fixed percentage of sales are covered by forward contracts.[3]

Supply Function Equilibrium models (e.g. Klemperer and Meyer, 1989) on the other hand are deemed to represent electricity markets more realistically because they assume that generators, instead of one single quantity, compete by bidding complete supply functions in an oligopolistic market with demand uncertainty. The SFE approach has been used in several applications to analyze electricity markets since its first application by Green and Newbery (1992) for England and Wales. The major drawbacks of SFE models are that they are difficult to calculate, often have multiple equilibria, often give unstable solutions and require strong simplifications with respect to market and cost structures.

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[2] See Twomey et al. (2005) for an overview of the basic characteristics and economic analysis tools suitable for electricity markets.

[3] Alternatively, one could assume that generators are vertically integrated with the retail sector for a portion of their activities.

Given the strengths and weaknesses of both approaches, we set out to discover whether the two models offer similar predictions and ranges of feasible outcomes. We ask whether the added complexity of SFE models relative to Cournot models can be compensated for by more robust, realistic predictions. In order to investigate this issue, we calibrate the two models with an identical dataset from the German electricity market, and then compare the results with the observed market clearing prices.

The remainder of our paper is structured as follows: In the next section, an overview of the theory of SFE and Cournot modeling is given. Section 3 describes the implementation of the models and the underlying assumptions. Section 4 presents a dataset for the German market and the model calibration methods. Section 5 presents and discusses the simulation results. We find that Cournot approaches allow for more complex market models while SFE models are less sensitive to adjustments. Section 6 closes with a summary and our conclusions about the suitability of each approach for short- and long-term economic analysis and policy-making.

2 Literature Review

2.1 Theoretical Background

Two major trends have emerged to analyze oligopolistic electricity market outcomes, in particular, wholesale markets.[4] Both approaches assume that companies are profit- maximizing, but differ in the assumption regarding the free choice variables and the behavior of the remaining market participants. We now discuss these differences.

In both models the profit function of the firms is equal to the revenue minus generation costs:

$$p\,q_i - c_i(q_i) \tag{1}$$

with q_i the output of firm i, p the price of electricity, and $c_i(.)$ the production cost of the firm i. For the paper we assume that the inverse demand function depends on total production $\sum_j q_j$, the time period t and a random error component ξ. Hence we have the following.

$$p = p(\sum_j q_j, t, \xi) \tag{2}$$

[4] Another classic approach is the Bertrand model. As this is seldom used in electricity markets, given the difficulty of presenting capacity constraints, we do not consider it in detail.

Generally speaking, a firm i can submit a bid function to the power exchange that represents its willingness to supply a specific quantity q at a given price p, a given time period t and -- in case the firms observes the demand shock before bidding -- the error term ξ.

$$q_i = B_i(p,t,\xi) \tag{3}$$

The market clearing condition determines the production quantities and prices in each time period t and for each demand shock ξ. That is, they are determined jointly by equations (2) and (3).

Cournot and SFE models make different assumptions regarding the strategy space and the information set of the bidding firms. In a Cournot equilibrium *the demand realization* (t,ξ) is known by the firm before bidding. The firms therefore bid the following:

$$q_i = B_i^C(t,\xi) \tag{4}$$

It follows from equation (4) that the firm might bid a high quantity during peak hours and a low quantity during off-peak hours. For a given time period t and demand shock ξ each firm maximizes profits by setting production quantities and sales q_i knowing that the market price is a result of its own output and the output of its competitors q_{-i}. The Cournot approach yields a direct outcome in terms of price and quantities for a given demand realization. Cournot models generally provide a unique Nash equilibrium.

In the *SFE models* (Klemperer and Meyer, 1989), firms cannot condition their bids on the demand realization (t,ξ). It is assumed that firms do not know the size of the demand shock ξ, and are not allowed to submit a different bid for different time periods t. The bidding function depends solely on the price:

$$q = B_i^{SFE}(p) \tag{5}$$

Each firm maximizes (expected) profits by bidding a supply curve $B_i^{SFE}(p)$, assuming that the supply function of its competitors $B_{-i}^{SFE}(p)$ remains fixed. Hence, the quantity that the competitors produce depends on the market price and, thus, indirectly on the output decision of the firm itself. In order to solve the SFE-model, it is assumed that the stochastic demand shock and the time period shift the demand function horizontally:.

$$p = p(\sum_j q_j - \xi - f(t)) \tag{6}$$

Under these assumptions the (stochastic) optimization problem that each firm must solve can be rewritten as a differential equation. Typically, a range of feasible equilibrium supply functions is found when solving a set of differential equations, one for each firm. It can be shown that every

SFE supply function lies between the Cournot and the Bertrand solution for any realized demand shock. Delgado and Moreno (2004) show, however, that only the least-competitive equilibrium is coalition proof when the number of firms is sufficiently large. SFE models require simplified assumptions of the markets' supply structure to obtain feasible solutions. This is a serious drawback of SFE models.

Whether a more realistic representation of the bidding process is to neglect the price dependency as in the Cournot model, or to neglect the market state dependency as in the SFE model is unclear a priori.

2.2 Use of Cournot and SFE models in electricity markets

2.2.1 Cournot models

Although Cournot models are ubiquitous, they often overestimate observed market prices and underestimate market quantities. As the model outcome is based only on quantity competition, the results are highly sensitive to assumptions about demand elasticity: in equilibrium, firm i sets its output such that its markup is proportional to its market share s_i and inversely proportional to the demand elasticity ε of the total market.

$$\frac{P - c_i'}{P} = -\frac{s_i}{\varepsilon} \tag{7}$$

Given that most electricity markets have few oligopolistic firms (s_i is large) and low, short-term demand elasticities, markups are accordingly very high.

Following Allaz and Villa (1993), we allow for forward contracts that can be used to predict more realistic outcomes. Firms can both sell energy in a spot market and sell a certain amount of their supply in the forward market. In a two-stage game, the oligopolists first determine the quantity to be sold in the forward market before entering the spot market and playing a Cournot game. Thus, forward sales reduce the oligopolists' available quantity in the spot market, resulting in a more competitive market. When using a single-stage game, the impact of forward contracts can be simulated by accounting for the contract volume F_i in the profit function:

$$p(\sum_j q_j)(q_i - F_i) - c(q_i) \tag{8}$$

This results in a reduced markup on marginal costs since the contracting factor $f_i = F_i / q_i$ must be considered:

$$\frac{P - C_i'}{P} = \frac{s_i}{\varepsilon}\left(1 - f_i\right) \tag{9}$$

By varying the contracting factor, a "bundle" of feasible Cournot solutions that resemble SFE outcomes can be generated.

The role of forward contracts, when comparing Cournot results with real market outcomes, is clearly demonstrated by Bushnell et al. (2005). They look at California, PJM and New England markets and compare price data of the power exchanges with competitive model outcomes, a standard Cournot model and a Cournot model using contract cover as an approximation for vertical arrangements. They conclude that neglecting the contract cover yields results that vastly exceed observed market prices. Ellersdorfer (2005) analyses the competitiveness of the German electricity market using a multi-regional two-stage Cournot model. He shows the extent to which cross-border network extensions and increased forward capacities enhance competition and decrease market power.

Cournot approaches are often preferred when technical characteristics such as network constraints (voltage stability, loop flows) or generation characteristics (start-up costs, ramping constraints, unit commitment) need to be considered. The impact of congestion on market prices and market power has been analyzed in several studies. Smeers and Wei (1997) use variational inequalities to describe the Cournot model. Willems (2002) discusses the assumptions necessary to include transmission constraints in Cournot models. Neuhoff et al. (2005) summarize different characteristics of Cournot network models. They show that although all models predict the same outcomes, in cases of competitive markets they vary with respect to assumptions about market design and expectations of generators.

A more general overview of Cournot models used to analyze market power issues appears in Bushnell et al. (1999). Other reviews of electricity market models with respect to network issues are given in Day et al. (2002) and Ventosa et al. (2005).

2.2.2 SFE models

In general, SFE models are frequently used in determining market power issues. Bolle (1992) makes a theoretical application to electricity markets by analyzing the possibility of tacit collusion when bidding in supply functions. He concludes that if firms coordinate on bidding the highest feasible supply function, a decrease in market concentration does not necessarily result in convergence of aggregated profits to zero. Green and Newbery (1992) present an empirical analysis for England and Wales using symmetric players. They compare the duopoly of National Power and PowerGen with a hypothetical five firm oligopoly, concluding that the latter results in a range of supply functions closer to marginal costs.

Baldick and Hogan (2002) set up another model of the England and Wales markets which incorporates price caps, capacity constraints, and varies the time horizon by which supply functions must remain fixed. They show that the latter characteristic has a large impact on market competitiveness. Evans and Green (2005) simulate the same market for the period of April 1997 to March 2004, assuming linear marginal cost functions to determine linear supply functions of asymmetric firms. They conclude that the change to a decentralized market has no impact on short-term prices but that the reduction in concentration does. Sioshansi and Oren (2007) study the Texas balancing market, using supply function equilibria with capacity constraints. They find that the larger firms more or less behave according to the SFE for incremental bids. They argue that SFE models with capacity constraints are an interesting tool to study balancing markets since demand is very inelastic, and hence the supply elasticity of competitors is a major component in determining the elasticity of the residual demand of the firms. These SFE models with capacity constraints are also useful because they reflect the actual bidding behavior of the firms, *i.e.* "hockey stick bidding": firms bid (too) low for low levels of supply and have a steep supply function for larger levels of supply.

Several theoretical contributions have extended Klemperer and Meyer (1989) by incorporating typical market characteristics. Holmberg (2005) considers the problem of asymmetric companies by simplifying the supply structure to constant marginal costs. He shows that under this setup there is a unique SFE that is piece-wise symmetric. Anderson and Hu (2005) propose a numerical approach using piece-wise linear supply functions and a discretization of the demand distribution. They show that the approach also has good convergence behavior in models with capacity constraints. Holmberg (2006) studies capacity constraints on generation units and shows that a unique, symmetric SFE exists with symmetric producers, inelastic demand, price cap, and capacity constraints. Green (1996), Rudkevich (2005) and Baldick, et al (2004) develop the theory of linear supply functions. These linear supply functions are easier to solve, can also be used in asymmetric games, and generally give stable and unique equilibria. However, they do not take capacity constraints into account. Boisseleau et al. (2004) use a piece-wise linear supply function and describe a solution algorithm which obtains an equilibrium even when there are capacity constraints.

2.3 Comparing the models

Only a few authors have compared the equilibria in both models. Related to our paper is recent work by Vives (2007), who studies the properties of two auction mechanisms: one where firms bid supply functions and one where they bid à la Cournot. Firms are assumed to have private information about their costs. Solving for a Bayesian equilibrium in linear supply functions, Vives (2007) shows that supply functions aggregate the dispersed information of the players, while the Cournot model does not. Hence, Cournot games might be socially less efficient, since the dispersed information is not used efficiently. Our paper differs since we are less interested in understanding the properties of auction mechanisms. In our model, firms have perfect information about their costs, firms sell forward contracts, and supply functions are not restricted to linear bid functions.

Hu et al. (2004) use a bilevel game to model markets for delivery of electrical power on looped transmission networks with the focus on the function of an ISO. Within this analysis they compare supply function and Cournot equilibria and show that in case of transmission congestion, SFE need not be bounded from above by Cournot equilibria as is the case for unconstrained networks. They conclude that in the presence of congestion, Cournot games may be more efficient than supply function bidding.

Ciarreta and Gutiérrez-Hita (2006) undertake a theoretical analysis of collusion in repeated oligopoly games, using a supergame model that is designed both for supply function and quantity competition. They show that depending on the number of rivals and the slope of the market demand, collusion is easier to sustain under supply function rather than under quantity competition. An experimental approach by Brandts, Pezanis-Christou and Schram (2003) includes the impact of forward contracts on electricity market outcomes with differences due to Cournot and supply function competition. They show that the theoretical outcomes of SFE models (mainly their feasibility range between Cournot and marginal costs) can be reproduced by experimental economics. Furthermore, they find that introducing a forward market significantly lowers prices for both types of competition.

3 Model formulation

The Cournot and SFE supply models are found by simultaneously solving a set of equations describing the market equilibrium for different demand realizations *k*.. In order to compare the results, we use identical assumptions with respect to the demand and supply structure of the market: the market demand is assumed to be linear whereas the marginal cost curve is a cubic function based on the actual power plant costs (see Section 4).

The *demand equation* describes for demand realization k, how the demand D_k^o that strategic players (oligopolies) face depends on a demand shock Δ_k and the price p_k:

$$D_k^O = \alpha - \gamma \cdot p_k - \Delta_k \tag{10}$$

with $k = 1,...,K$ the index of the aggregate demand shock, both due to the stochastic demand component and the deterministic time component: $\Delta_k = \xi + f(t)$ with $\Delta_k < \Delta_{k+1}$, $\Delta_1 = 0$ and $\Delta_K = \alpha$.

The *energy balance equation* describes that for all demand realizations k, demand should equal total supply:

$$D_k^O = S_k^O \tag{11}$$

where $S_k^O = \sum_{i=1}^{n} q_{ik}$

The *marginal cost equation* relates output of firm *i* with the marginal cost of that production plant given by:

$$c_{k,i} = \lambda_{0,i} + \lambda_{1,i} q_{k,i} + \lambda_{2,i} q_{k,i}{}^2 + \lambda_{3,i} q_{k,i}{}^3 \tag{12}$$

The *continuity equation* (13) imposes continuity of the supply function. It describes the relation between the slope of the supply function β, the production levels, and the price level of the firms. Continuity implies that the arc-slope of the supply function can be written as the weighted sum of the slopes at the two endpoints of the interval. That is, the following holds:

$$q_{i,k+1} - q_{i,k} = (p_{k+1} - p_k)\left[(1 - \xi_{ik})\beta_{ik} + \xi_{ik}\beta_{ik+1}\right] \tag{13}$$

with $0 < \xi_{ik} < 1$. This formulation specifies a piecewise linear supply function.[5]

The *pricing equation* describes the first order conditions of each player *i* for each demand shock *k*. It requires that a player's marginal revenue and marginal cost are equal:

$$(q_{ik} - F_{ik})\frac{dp_{ik}^R}{dq_{ik}} = p_k - c_{ik} \tag{14}$$

F_{ik} is the amount of contracts signed in equilibrium by firm i in realization k, and $\frac{dp_{ik}^R}{dq_{ik}}$ is the slope of its inverse residual demand function. We allow the firms to sign fixed-capacity contracts specified as a quantity (in MW) which is independent of the demand shock k: $F_{ik} = f_i$. A

[5] This formulation is based on Anderson and Hu (2005). It rewrites their equations (13) and (19) by eliminating p_{ik}.

typical fixed contract is a baseload contract (the firm commits to sell forward a fixed amount of energy).[6]

The pricing equation differs for the two models. In the Cournot equilibrium, each player assumes the production of the other players as given, and therefore the slope of the residual inverse demand function depends only on the slope of the demand function ($1/\gamma$).

Firm i signs f_i fixed contracts and the pricing equation becomes:

$$q_{ik} - f_i = (p_k - c_{ik})\gamma \qquad (15)$$

By contracts, for the SFE model the slope of the residual demand function depends on the slope of the demand function and the slope of the supply functions of the competitors. With the assumed fixed contract cover f_i the pricing equation becomes:

$$q_{ik} - f_i = (p_k - c_{ik})(\sum_{j \neq i} \beta_{jk} + \gamma) \qquad (16)$$

A Cournot equilibrium is a solution of equations (10), (11), (12) and (15). A SFE is a solution of equations (10), (11), (12), (13) and (16). The equilibrium is described as the price and demand level for realization *k,* D_k, p_k and for each firm *i* a description of production, marginal costs and a slope of the demand function $\beta_{ik}, q_{ik}, c_{ik}$. The solution of these equations is not straightforward, given the non-linearities in equations (16) and (13).

The model is solved using the COIN-IPOPT solver in GAMS (Wächter and Biegler, 2006).[7]

4 Data

4.1 An oligopolistic market: Germany

For this paper, we chose Germany because its oligopolistic market structure lends itself to valid representations for both Cournot and SFE models. The German market consist of two large firms (E.ON and RWE) owning about 50% of generation capacity, two smaller firms (Vattenfall and

[6] An earlier version of this paper also considered load-following contract. For these contracts, certain fraction of the sales is contracted forward for each realization of the demand shock: $F_{ik} = \phi_i q_{ik}$. Such load-following contracts are typically signed between a generator and a retailer. In this case, the generator does not sign a contract for a fixed quantity, but promises to fulfill all demand by the retailer.

[7] The CONOPT solver which was used by Anderson and Hu (2005) did not always converge.

EnBW) each with about 15% of the market, and a competitive “fringe” that acts as a price-taker.[8] Germany is a winter peaking system with a large share of nuclear and coal units and a significant share of wind production in the North and East. The transmission grid is well connected with the rest of Europe, and the country is a net exporter of electrical energy.

We study the winter period since strategic behavior is more likely to occur when capacity is scarce. As the models assume that the underlying cost structures remain constant for the duration of the bidding period we restrict our analysis to January and February of 2006. Most of Germany’s electricity is traded bilaterally, but voluntary power exchanges selling standardized products are gaining favor. The main price index for Germany is the day ahead price at the European Energy Exchange (EEX). Figure 1 shows the price distribution during the observation period. Within the sample period, we consider four ‘time periods’: January peak and off-peak hours, and February peak and off-peak hours.[9] For each time period the observed demand, wind production and net import amounts, as well as prices are used to estimate the corresponding demand function for Germany (See section 4.3).

[8] Due to the characteristics of electricity markets small companies can have market power potentials (Sioshansi and Oren, 2007). Neglecting the strategic behavior of the fringe may lead to price underestimations.

[9] All hours in our sample which are between 8am and 8pm are classified as peak hours; the others are classified as off-peak.

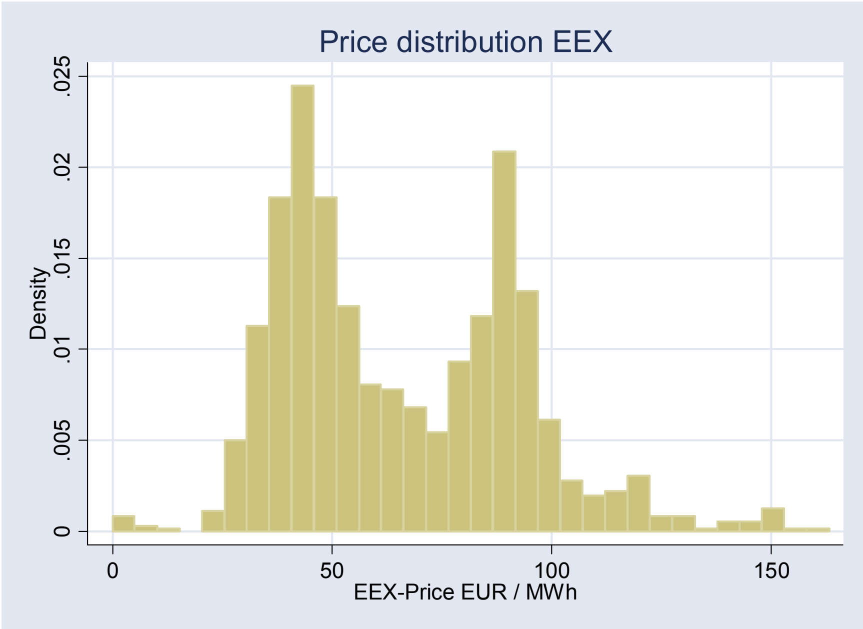

Figure 1 Price distribution on the EEX.

4.2 Approximation of the cost functions of the generators

The marginal costs of generator *i* is described by a function $c_i(q)$, which takes into account the generation portfolio of each player. Generation capacities and ownership are obtained from public sources, mainly VGE (2006). More than 300 power plants are considered summing to 100 GW of fossil and hydro generation. Wind, biomass and solar capacities are not considered within the firm's generation portfolios. Plant capacities are decreased by seasonal availability factors following Hoster (1996). Using a type-specific algorithm based on Schröter (2004) with construction year as proxy, we calculate a plant-specific efficiency to derive marginal costs. Fuel prices are taken from Bafa (2006) and resemble average monthly cross border prices for gas, oil and coal. We include the price of CO_2-emission allowances in the cost estimate based on fuel type and plant efficiency. Allowance prices are taken from the EEX. The marginal cost functions are estimated for January and February respectively.

In our model, the marginal cost functions of the generators are simplified to a cubic function[10]

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[10] We decided not to incorporate production capacity constraints in the model. To our feeling the theoretical literature on supply function equilibria with capacity constraints is not yet sufficiently developed, especially for situations with multiple asymmetric firms. For instance Holmberg (2008) shows

$$\tilde{c}_i(q) = \lambda_{i0} + \lambda_{i1}q + \lambda_{i2}q^2 + \lambda_{i3}q^3 \tag{17}$$

where the parameters of the function are found by minimizing the weighted squared difference of the parameterized function and the true cost function, subject to the condition that marginal cost should be upward sloping. The following optimization problem is solved

$$\begin{aligned} &\min_{\lambda_0,..,\lambda_3} \int \left(\tilde{c}_i(q) - c_i(q)\right)^2 dF(c(q)) \\ &\text{s.t.} \qquad \tilde{c}_i\,'(q) \geq 0 \end{aligned} \tag{18}$$

with $F(\cdot)$ the cumulative density function of the prices in the EEX. Figure 2 shows the approximate marginal cost function for Germany's four largest players averaged for both months.

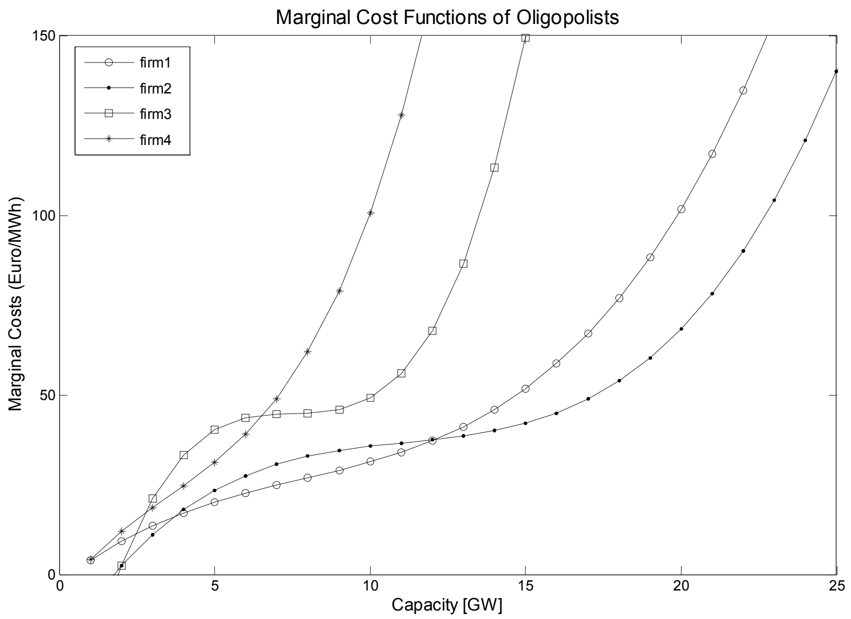

Figure 2: Competititve supply functions

that with symmetric players, inelastic demand, a price cap and (binding) capacity constraints there exists a unique, symmetric equilibrium. Delgado (2006) shows that there might be multiple equilibria in oligopoly models with capacity constraints and that it is no longer evident that the firms will co-ordinate on the Cournot equilibruim.

4.3 The demand function faced by the oligopolists

The final demand D for energy is served by wind production Q_W, by imports Q_I, by production of the four German oligopolists Q_i and by the fringe generator Q_F.

$$D = Q_W + Q_F + Q_I + \sum_{i=1}^{4} Q_i \tag{19}$$

We assume that the final demand for electricity D does not depend on the price, but varies through time *t* and has a random component: $D = \alpha_t^D + \varepsilon_t^D$. Wind production might also vary through time and have a random component: $Q_W = \alpha_t^W + \varepsilon_t^W$.

The production level of the fringe generators depends on the price they can obtain for their output. The supply of the fringe is determined by the inverse of its marginal cost function.

$$Q_F(p) = MC^{-1}(p) \tag{20}$$

We approximate the marginal cost function of the fringe linearly, and rewrite the supply function as

$$Q_F(p) = \alpha_F + \gamma_F p \tag{21}$$

We use two approaches to determine the parameters α_F and γ_F. In the first, we use a weighted least squares regression (the "weights" are derived from the price density function of the EEX prices). This approach typically underestimates the marginal cost for large prices as no capacity constraint is considered. In the second approach, we assume that the fringe is always producing at full capacity ($\gamma_F = 0$) which underestimates marginal costs for low prices. The fringe marginal cost function is derived using information on monthly fuel prices and generation capacities. We obtain a distinct estimate for January and February.

German imports are determined by the difference of the price in Germany and the neighboring regions. If the price in Germany is high relative to the price in neighboring regions, imports increase. We estimate imports by regressing the following equation:

$$Q_{It} = \gamma_I p_t - \sum_j \gamma_j p_{jt} + \sum_z \beta_z \delta_{zt} + \varepsilon_t^I \tag{22}$$

where p_t is the price in Germany, p_{jt} is the price in border country j, δ_{zt} is a vector of time dummies (day of week) for all hours t in the relevant period.[11] We use a two-stage least squares estimator to address the endogeneity of the German price p with respect to imports. As instruments, we use the total demand level in Germany D_t and German wind production Q_t^W. As explained in Bushnell et al. (2005), the demand level is a valid instrument for import levels since demand is inelastic in the short-term, and does not depend on the price level in Germany. The import elasticities are estimated for each of the four periods we consider. Note that we do not explicitly model cross-border capacity constraints. The results of the regression can be found in the appendix.

Combining equations (19) to (22), we can rewrite the residual demand function for the oligopolists D^O:

$$D_t^O(p) = \alpha_t^O - (\gamma_F + \gamma_I) p_t + \varepsilon_t^O \tag{23}$$

Our four German oligopolists, therefore, face an elastic demand function due to import elasticity and the supply of the fringe generators ($\gamma_F + \gamma_I$). An increase in the German electricity prices will increase the import levels and the production by the fringe generators. In the model, the demand shock α_t and the random component ε_t for each time period t are transferred into a constant demand intercept α^O and a positive shock Δ_k for a specified set of demand realizations k (see equation (10)). We choose the intercept of the demand level such that when the shock is zero, 98% of the observations in the German market are below the demand function (2% of the outliers are not taken into account).

Within the sample period, the demand elasticity varies according to the considered period. The estimation results show that demand is more elastic during off-peak hours and in January, and less so in February and during peak hours. The higher demand elasticity probably reflects the fact that cross-border transmission lines are less congested in off-peak hours, and that imports create more competitive pressure within Germany.

[11] Hourly price data was used from the Netherlands, France, Austria, Poland, Sweden, East Denmark and West Denmark.

5 Results of the models

Combining the price data of the German power exchange, the demand, the import and the wind data, Figure 3 shows the aggregate supply function of German thermal production during January and February 2006. The aim of this paper is to test whether a Cournot model or a SFE approach is capable to explain this observed aggregated supply function taking into account the cost of the firms and strategic behavior of the four largest generation firms.

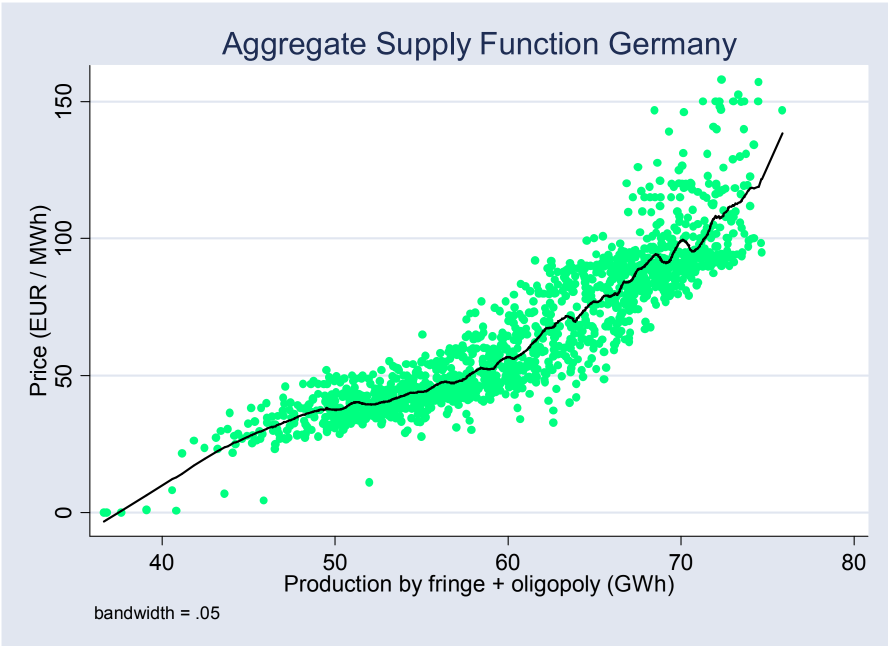

Figure 3: Aggregate supply function of the oligopolists and the fringe generator

The Cournot model produces a *single equilibrium* given the demand realization and the contract cover. The SFE model gives, for a given contract cover, a *bundle of equilibria.* Which equilibrium firms choose, depends on how the firms coordinate. We assume that firms co-ordinate on the least competitive SFE equilibrium, *i.e.* the equilibrium where prices are highest.[12] The outcome of both models depends on the amount of contracts that firms have signed. We use the contract coverage as a calibration parameter. The contract coverage f_i is specified relative to

[12] This approach is also taken by most other studies who use the SFE-model for policy studies. (*e.g.* Green and Newbery (1992). See also Delgado and Moreno, 2004).

the total installed (and available) capacity for each firm ($q_i^{\max}$) – and not relative to the demand level:

$$f_i = \phi_i \, q_i^{\max} \tag{24}$$

with ϕ_i the contract coverage of firm *i* in percent.

There is no reliable data on the long-term commitments and contracting positions of German electricity firms. Only about 20% of electrical power is traded on the EEX power exchange spot market. The uncontracted position of firms might, however, be significantly different from this number as (i) an unknown amount of energy is traded in short-term OTC markets, (ii) the net position of each firm might be significantly smaller than the gross level of trade in the market, (iii) only contracts which specify a fixed price reduce market power, whereas contracts indexed on the spot price have no effect. We therefore conclude that the contract cover that calibrates the model should not be interpreted as the actual amount of forward contracts that firms have signed. Furthermore, the model itself neglects important market aspects like start-up times, capacities, and network constraints. The calibration parameter might pick up some of these effects as well.

To test numerically which model predicts the market outcomes more realistically, we use the observed price-demand results at the German sport market EEX as a benchmark. To compare the models' prices with the prices observed in the market, we conduct a non-linear least squares regression:

$$P_t = \tilde{P}_t(\phi) + \varepsilon \tag{25}$$

with P_t the average observed price on the market, $\tilde{P}_t$ the prediction of the model, and ε as error-term. If the error-term is normally distributed this estimator of the contract cover will converge to the true value of the regression. For the regression we minimize the (squared) error between the prices we observe and the prices predicted by the model:

$$\min_{\phi}(P_t^{Obs} - \tilde{P}_t(\phi))^2 \tag{26}$$

The model predictions $\tilde{P}_t$ are calculated for the same demand shock Δ_k as observed in the market defined by equation (10).[13] By measuring goodness-of-fit in this way, we measure only the errors in the supply side of the model conditional on the demand functions being correctly defined.

The first two lines of Table 1 show the results of the regression analyses. The optimal contract coverage for the Cournot model is one where firms contract 50% of their installed capacity. This is about twice as much as for SFE. At the optimal contract cover, the standard deviation of the error term in the regression is about 9.41 EUR/ MWh for the Cournot model and 9.31 EUR / MWH for the SFE model. Hence both models perform equally well in predicting the market outcome. An alternative measure of the goodness-of-fit is the R-squared term. It is a (relative) measure of how much of the variation in observed prices is explained by the model:

$$R^2 = 1 - \frac{V^{\text{model}}}{V^{\text{data}}} \tag{27}$$

with V^{model} the variance of the error term of the regression and V^{data} the variance of the observed market prices. Assuming that the error is normally distributed we can calculate a confidence interval for contract covers, expressed as a variance of the estimate. The variance of contract cover for the SFE model is higher than the variance of the Cournot model. This reflects the fact that the SFE-model is less sensitive to changes in the contract cover, while the results of the Cournot model depend more heavily on the contract cover.

Figure 4 shows the Cournot solution and the bundle of SFE solutions for the contract positions found in the regression. That is Cournot firms sign contract for 49.8% of their installed generation capacity, while SFE firms contract 27.4%. We only consider the SFE solution with the highest price in our analysis thus neglecting the remainder of the bundle. In the mid-load range both models produce the same outcome. However, during peak load the SFE solution is above the Cournot one and in off-peak situations it is vice versa. In general the Cournot solution follows a more linear trend whereas the SFE solution is slightly backward bending.

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[13] This means that for the observed price and quantity P_t^{obs}, $Q_t^{O,obs}$, the price and quantity predictions $\tilde{P}_t$, $\tilde{D}_t^O$ are such that $D_t^{O,obs} - \tilde{D}_t^O = \gamma(p_t^{obs} - \tilde{p}_t)$.

	Contract Cover	Var	Correction term τ	Var	Std Dev Error	R^2	Number of Observations
Cournot	49.8%	0.234	-	-	9.41	0.85	1361
SFE	27.4%	0.664	-	-	9.31	0.85	
Cournot	50.4%	0.311	0.064	0.001	9.39	0.85	
SFE	25.5%	0.876	-0.106	0.001	9.26	0.85	

Table 1: Results of the regression analysis

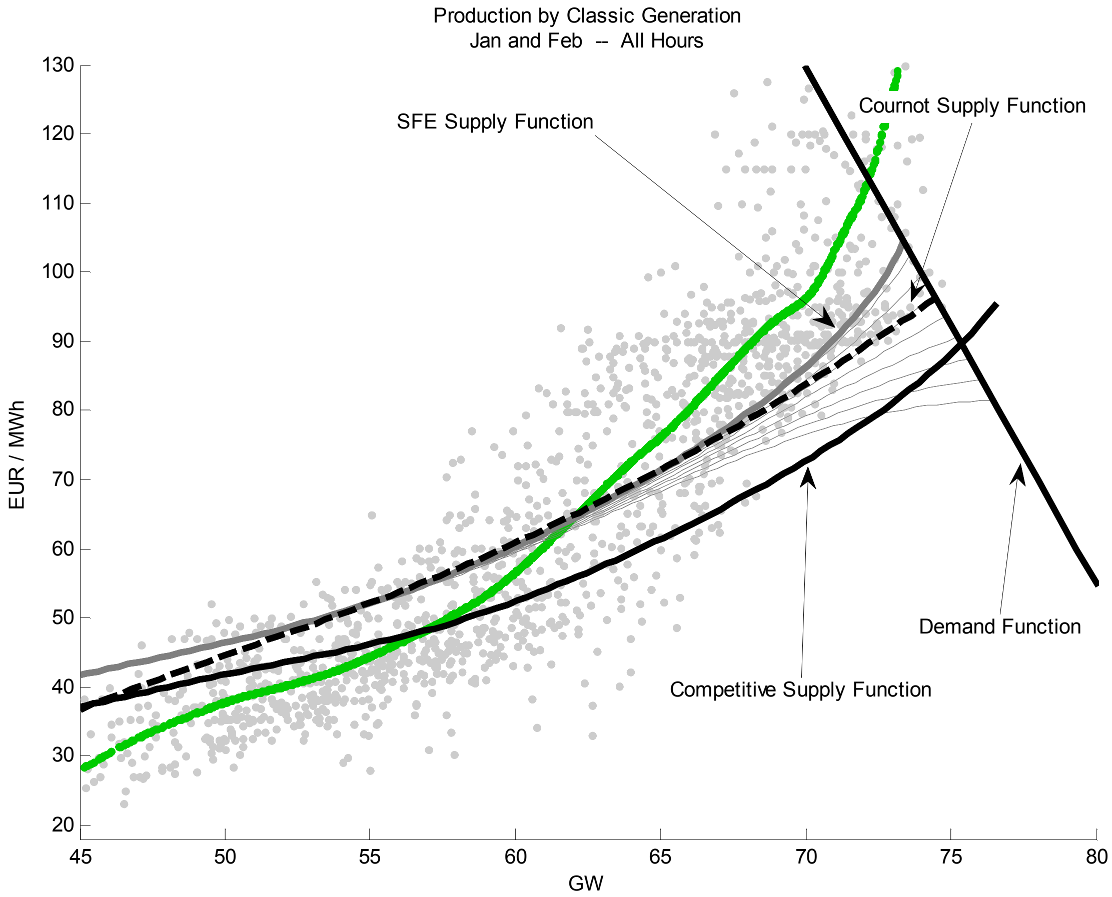

Figure 4 Unique Cournot outcome and bundle of SFE outcomes at the calibrated contract cover

As both models neglect start-up and capacity restrictions we adjust the regression analysis by introducing a constant term τ depending on the difference between marginal fuel costs and marginal costs including start-up and capacity restrictions:

$$P_t = \tilde{P}_t(\phi) + \tau\,(MC_t^{\text{mod}} - MC_t^{startup}) + \varepsilon \tag{28}$$

with MC^{mod} the marginal costs as used in the Cournot and SFE model and $MC^{startup}$ the corresponding marginal costs of a model approach including start-up costs and restrictions.[14]

If the start-up adjusted marginal costs reflect the real costs better than the marginal costs used in the model, then we should observe a negative τ. As the start-up adjusted costs are on average larger than the modeled marginal costs, the price cost mark-up of the firms is slightly smaller and we, therefore, expect that the model predicts that a lower amount of fixed contracts is signed.

The last two lines of Table 1 show the results of the regression including the term reflecting start-up costs. For the SFE-model, the impact of the marginal cost has the expected sign, but is (in absolute terms) relatively small.[15] Correcting for the start-up costs, the contract cover has decreased. For the Cournot model, the parameter has the wrong sign, but remains very small in absolute terms. For both models the optimal contract coverage only varies slightly with an increase in the according variance and the R-squared value remains constant. Thus the fit of the models cannot be increased.

In order to test which of the approaches performs better under varying assumptions we conduct a series of robustness tests. The aim of these robustness checks is to test whether the relative performance of the Cournot and the SFE model depend on the particular assumptions we make. We will give some hints about which specifications perform better, but this is not the aim of the robustness tests.

- We will allow for different contract coverage during peak and off-peak periods, vary marginal generation costs on monthly basis, and estimate different import elasticities for peak and off-peak periods in each month respectively.
- For the behavior of the fringe, we consider the two cases as explained in section 4.3: one where the fringe has an elastic supply function and one where the fringe always produces at full capacity. The elastic supply function is on average the best representation of the supply of the fringe generator, but neglects capacity constraints. The inelastic supply function might be a better representation of the supply by the fringe during peak periods when capacity constraints play a larger role.
- We introduce load-following contracts in the Cournot simulations allowing for an extra flexibility in the contracting options of the firms.

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[14] The hourly values for $MC^{startup}$ are obtained from Hischhausen and Weigt (2007).

[15] In the SFE-model, firms are unable to make bids conditional on the time period, and are therefore not able to reflect the start-up costs in their bids.

We first assume an elastic fringe (Table 3). We observe that the contract coverage is higher during off-peak periods for both the Cournot and SFE approach. Likewise the variance during off-peak is higher – particularly for the SFE model – indicating that the results are less sensitive to the chosen coverage. The Cournot model still yields a higher optimal contract cover than the SFE model. Surprisingly, the R-squared values decrease compared to the average base case, which does not allow for contract levels that are differentiated according to the time of day. The reason for this is that the R-squared is a relative measure of the quality of fit based on the underlying sample data, and not an absolute measure. Another reason that makes it hard to compare the R-squared measures is that we drop 2% of the outliers in the peak and 2% of the off-peak hours, which is different than dropping 2% over the total sample.

Furthermore, we observe that the off-peak outcome of the oligopolistic models is only slightly better than the competitive solution which has an R-squared of 0.71. However, during peak hours, the competitive equilibrium is a bad predictor of market results. (R=0.05) indicating that during peak hours, it is more important to consider strategic company behavior. The impact of start-up and capacity restrictions has the expected sign in most cases, although the coefficient is relatively small.

	Contract Cover	Var	Correction term τ	Var	Std Dev Error	R	Number of Observations
Cournot Peak	46.4	0.458	-0.033	0.001	11.26	0.77	727
Cournot Off-Peak	56.8	0.851	0.056	0.003	5.53	0.84	631
SFE Peak	23.0	0.739	-0.120	0.001	10.18	0.81	727
SFE Off-Peak	41.9	4.130	-0.180	0.003	6.03	0.81	631

Table 2: Results of the regression analysis for different periods, elastic fringe

When we fix the fringe output to its maximum generation capacity, the resulting residual demand function for the oligopolists will be steeper in both peak and off-peak. However, during off-peak the residual demand level is lower as a larger fraction of the total demand will be satisfied by the fringe (see Table 4). We observe that during peak times the optimal contract cover increases for both approaches and the variance goes down. Particularly, for the SFE model the decreased demand elasticity results in a twice as high contracting. During off-peak the optimal contract level decreases.

The impact of start-up and capacity restrictions reverses if we fix the fringe output. However, the impact remains below 20% of the marginal cost difference (MC^{mod} - $MC^{startup}$). Hence we conclude that the prices that we observe are only marginally driven by start-up costs (as simulated in our model).

Comparing both fringe assumptions, the optimal model is probably one where the inelastic fringe is used during peak hours – the production capacity is binding – and the elastic fringe during off-peak hours – the production capacity is not binding. In that case we have similar amount of contracts being signed during peak and off-peak periods. This implies that during off-peak hours, a larger fraction of total demand is covered by contracts. The Cournot model has a coverage 10 to 15 percent higher than the SFE approach; and corrections for start-up and capacity constraints have the expected sign in three out of four cases.

	Contract Cover	Var	Correction term τ	Var	Std Dev Error	R	Number of Observations
Cournot Peak	53.7	0.071	-0.025	0.001	10.48	0.80	726
Cournot Off-Peak	47.7	0.227	0.234	0.001	5.10	0.86	629
SFE Peak	42.2	0.141	0.048	0.001	12.13	0.73	726
SFE Off-Peak	32.8	1.111	0.103	0.001	5.23	0.85	629

Table 3: Results of the regression analysis for different periods, inelastic fringe

As a last robustness check, we analyse the impact of load following contracts for the Cournot approach.[16] The regressions show that the optimal amount of load following contracts is equal to 0%. This means that the model can be calibrated well during both peak and off-peak by relying only on fixed contracts.

6 Conclusion

This paper compares the classical Cournot model with the SFE approach to test whether the higher complexity of SFE results in a better representation of strategic market outcomes. Both models are tested using the dataset of Germany's electricity market and the same assumptions regarding demand and generation. The modeling results are then compared to observed market outcomes. We calibrate the model by changing the amount of fixed capacity contracts that firms sign and the behavior of the fringe.

[16] Load following contracts can be understood as a representation of vertically integrated generators that aim to satisfy the demand of their customers.

The results indicate that the Cournot approach can be calibrated well to the observed market outcomes by assuming a relatively high level of fixed-capacity contracts. For the SFE model, the best fit is found when firms sign few contracts. Using the R-squared coefficient of a non-linear least squares regression as a measure, the calibrated SFE and Cournot models perform equally well, *i.e.*, they explain the same percentage of the price variation in the market. We conclude therefore that the SFE model does not significantly outperform the Cournot model as a tool to study the German electricity market. However, the SFE models rely less on calibration parameters than the Cournot model, and appear, therefore, to give more robust, “realistic” predictions.

To solve the models numerically, especially the SFE model, we made several simplifying assumptions with respect to the generation and demand data. These assumptions may bias the quantitative and qualitative results of our models. For example, the linearization of import and fringe behavior can lead to a general overestimation of demand elasticity especially for high-demand periods, resulting in wholesale prices that are too low. The neglect of start-up and ramping issues leads to an overestimation of costs during off-peak periods. The general assumption of continuous supply function may lead to an underestimation of generation costs close to peak capacity. We conduct a robustness check, testing for different fringe behavior, analyzing the impact of start-up costs and splitting the sample in different time periods. However, the results do not differ significantly for both models.

We do not know whether our results extend to other electricity markets, but we conjecture that the difference between the two models becomes less pronounced as markets become less concentrated and more dependent on imports. We observe that the SFE may give better results for markets that are less import-dependent and more concentrated than Germany and that therefore have a lower demand elasticity.

Given the limited flexibility of SFE approaches to incorporate technical characteristics (unit commitment, start-up costs and network issues), we suggest that Cournot models should be the preferred option when electricity market characteristics must be modelled in technical detail. Thus, Cournot models are aptly suited for the study of market rules or congestion allocation mechanisms. However, when long-term aspects play a role, for instance in a merger study, SFE models might become more relevant because they are less sensitive with respect to calibration parameters. Furthermore, for long-term simulations, one cannot assume that contract positions are exogenous, increasing the complexity of Cournot models.

We foresee that both models (Cournot and SFE) will continue to be used for practical purposes, with each model being further tailored for its specific range of applications. We hope that this paper will assist policy-makers, regulators, and industry actors to understand the advantages and disadvantages of the different models.

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Appendix

	January Peak		February Peak		January Off-Peak		February Off-Peak		January & February	
	Coef.	t	Coef.	t	Coef.	t	Coef.	t	Coef.	t
EEX (Germany)	**0.09**	**3.34**	**0.06**	**5.23**	**0.40**	**2.52**	**0.14**	**4.03**	**0.13**	**5.81**
day_1	1.49	1.77	0.50	1.19	0.62	1.18	0.26	0.81	1.00	3.13
day_2	1.22	1.51	0.08	0.21	0.26	0.41	0.04	0.13	0.77	2.47
day_3	0.72	0.91	-0.03	-0.08	0.77	1.37	0.37	1.19	0.73	2.34
day_4	0.01	0.01	0.40	1.05	0.73	1.09	-0.13	-0.37	0.37	1.02
day_5	-0.36	-0.44	0.67	1.78	1.06	1.95	0.52	1.57	0.55	1.70
day_6	0.64	0.97	0.18	0.65	0.08	0.15	-0.15	-0.46	0.34	1.16
Netherlands	0.00	-0.25	-0.02	-5.71	-0.14	-2.65	-0.05	-2.57	-0.02	-3.78
France	-0.06	-3.26	-0.02	-5.15	-0.06	-3.14	-0.05	-4.88	-0.04	-6.95
Austria	-0.02	-0.9	0.00	-0.22	-0.14	-1.72	-0.03	-1.31	-0.06	-3.29
Poland	0.00	-0.43	-0.01	-0.89	0.01	0.77	0.03	2.24	-0.01	-1.28
DKeast	-0.02	-1.92	-0.01	-2.89	0.03	1.39	-0.02	-1.48	-0.01	-2.81
DKwest	-0.05	-1.75	0.02	2.12	-0.10	-2.00	0.01	0.23	-0.01	-1.12
Sweden	0.05	2.72	-0.01	-0.94	-0.01	-0.19	-0.03	-0.79	0.02	1.77
_cons	-3.28	-6.57	-3.25	-13.97	-2.89	-6.31	-3.00	-11.38	-3.12	-13.97

Table 4: 2-SLS regression with demand and wind input as instruments (see equation 22)

	January	February	January & February
Firm 1			
λ_0	-2.369	-2.916	-2.614
λ_1	7.308	6.881	7.075
λ_2	-0.720	-0.577	-0.643
λ_3	0.031	0.024	0.028
Firm 2			
λ_0	-20.674	-20.697	-20.411
λ_1	13.842	13.576	13.613
λ_2	-1.200	-1.108	-1.143
λ_3	0.037	0.033	0.034
Firm 3			
λ_0	-58.565	-51.792	-60.969
λ_1	40.472	37.287	41.759
λ_2	-5.431	-4.873	-5.529
λ_3	0.244	0.218	0.245
Firm 4			
λ_0	-5.726	-5.975	-6.035
λ_1	11.549	10.852	11.482
λ_2	-1.675	-1.267	-1.534
λ_3	0.158	0.125	0.145

Table 5: Coefficients of the cubic marginal cost functions (see equation 17)

	January	February	January & February
Elastic Fringe			
α_F	8.439	4.856	7.610
γ_F	0.077	0.140	0.094
Inelastic Fringe			
α_F	17.930	17.930	17.930
γ_F	0.000	0.000	0.000

Table 6: Values for fringe supply function (see equation 21)